# Emergent Hadronic Mass via meson GPDs:

## towards its experimental assessment

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# Introduction

# Introduction: EHM and the energy-momentum tensor

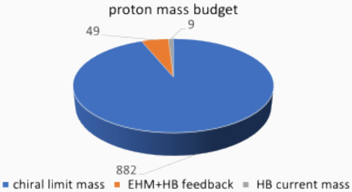
**Question:** *How does hadronic mass emerge from field theory?*

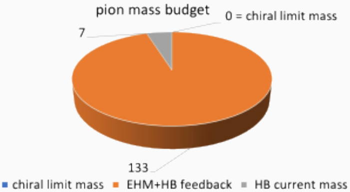
$$\mathcal{L}_{\rm QCD} \xrightarrow{\mathbb{R}^{1,3}\rtimes SO(1,3)} \text{EMT: } T^{\mu\nu} \;,\; g_{\mu\nu}\left\langle h(p)\right| T^{\mu\nu}\left|h(p)\right\rangle = -m_h^2$$

Energy momentum tensor trace

[See *e.g.*: C.D.Roberts:2102.01765]

$$g_{\mu\nu}T^{\mu\nu} = \frac{1}{4}\beta\left(\alpha_\mu\right)G_a^{\mu\nu}G^a_{\mu\nu} + \left[1+\gamma\left(\alpha_\mu\right)\right]\sum_i m_{\mu,i}\bar{\psi}_i\psi_i$$




**Pions** provide us with the clearest window onto EHM.

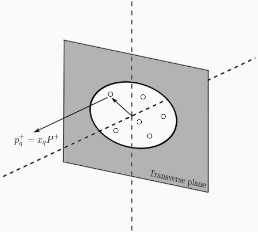
[C.D.Roberts:2102.01765, A.C.Aguilar-EPJA:10(55)2019]

**Question:** *How can we gain insights into $T^{\mu\nu}$?*

## Generalised parton distributions (GPDs)

**Probabilistic interpretation:**
probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction "$x$" of the hadron's light-cone momentum.

[M.Burkardt-PRD:071503(62)2020]




**EMT:** $\langle \pi(p') | T^{\mu\nu} | \pi(p) \rangle = 2P^\mu P^\nu \theta_2(t) + \frac{1}{2}\left(t g^{\mu\nu} - t^\mu t^\nu\right)\theta_1(t)$

**GPD MM:** $\int_0^1 dx x H^q(x,\xi,t) = A_{2,0}^q(t) + 4\xi^2 A_{2,2}^q(t)$

[X.Ji-PRL:610(78)1997]

**Further properties:**

1. PDFs as forward limit.
2. Electromagnetic and gravitational FFs as Mellin moments.

# Introduction

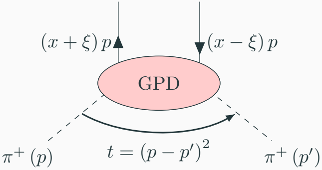
1. Studying EHM is intimately related with:
   - QCD energy momentum tensor
   - Pions: DCSB Nambu Goldston modes

2. Generalised parton distributions:
   - EMT gravitational form factors.
   - Unique tool for studying hadron structure.

*Can we build "theoretically-complete" pion GPD models?*

# GPD modelling

# GPD modelling: definition and properties



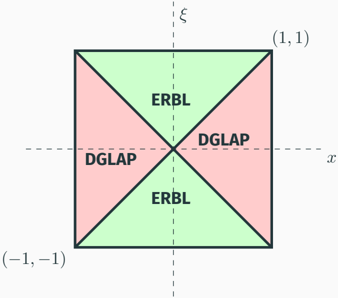

$x$: Momentum fraction of $p$.

$\xi$: Fraction of momentum longitudinally tranfered.

$t$: Momentum transfer.

**Kinematics:**
[M.Diehl-Phys.Rept:41(388)2003]

- **DGLAP** ($|x| > |\xi|$): Emits/takes a quark ($x > 0$) or antiquark ($x < 0$).
- **ERBL:** ($|x| < |\xi|$): Emits pair quark-antiquark.



# GPD modelling: definition and properties

- **Support:**
  [M.Diehl et al.-PLB:359(428)1998]

$$(x,\xi) \in [-1,1] \otimes [-1,1]$$

- **Polynomiality:** Order-$m$ Mellin moments are degree-$(m+1)$ polynomials in $\xi$.
  [X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^{1} dx x^m H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\,\xi^k$$

**Lorentz invariance**

- **Positivity:**
  [P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x,\xi,t=0)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)} \quad , \qquad |x| \geq \xi$$

**Positivity of Hilbert space norm**

- **Low energy soft-pion theorem**
  [M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

**PCAC/Axial-Vector WTI**

# GPD modelling: general strategy

**Goal:** Build pion GPD models fulfilling all these constraints.

**Problem:** Different modelling strategies and different problems

1. **Overlap representation** [M.Diehl et al.-NPB:33(569)2001]
   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$
   } Polynomiality ? Positivity ✓

2. **Double Distribution representation** [D.Müller et al.-Fort.Phys:2(42)1994, JLAB-THY-00-33]
   Relying on Radon transform, $\mathcal{R}$
   } Polynomiality ✓ Positivity ?

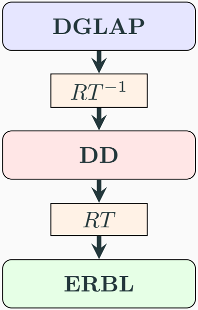
**Solution:**

**Covariant extension:** given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied.[N.Chouika et al.-EPJC:906(77)2017]

# GPD modelling: covariant extension

**Covariant extension:** given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

$$H\left(x,\xi,t\right)=\mathcal{R}\left[h\left(\beta,\alpha,t\right)\right]+\frac{1}{|\xi|}D^{+}\left(\frac{x}{\xi},t\right)+sgn\left(\xi\right)D^{-}\left(\frac{x}{\xi},t\right)$$




1. Build positive DGLAP GPD ⇒ **How?**
2. Covariant extension: ERBL GPD
3. Soft pion theorem: fix $D^{\pm}\left(\alpha,0\right)$

| **GPD properties** | | | |
|---|---|---|---|
| Support [Diehl-PLB(1998)] | ✓ | Positivity [Poby.-PRD(2002), Pire-EPJC(1999)] | ✓ |
| Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)] | ✓ | Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)] | ✓ |

# GPD modelling: positivity saturated models

**Question:** *How can we build a positive DGLAP GPD?*

**PDF**
Non-perturbative QCD

↓

**DGLAP GPD**
Positivity

↓

**ERBL GPD**
Polynomiality
[Chouika-EPJC(2018)]

↓

**Fix D-terms**
$t$-dependence
[Kumano-PRD(2018)]

**1.** Overlap representation [M.Diehl-NPB:33(569)2001]

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \int \frac{d^2k_\perp}{16\pi^3}\Psi^{q*}\left(x_-, k^2_{\perp,-}\right)\Psi^q\left(x_+, k^2_{\perp,+}\right)$$

**2.** Assume factorisation of the LFWF
[J.-L.Zhang et al.-PLB:136158(815)2021] (See Khépani's talk)

$$\Psi^q\left(x, k^2_\perp\right) \propto \varphi(x)\,\phi\left(k^2_\perp\right)$$

↓ (Overalp rep.)

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}\,\Phi(x,\xi,t)$$

↓ ($t=0$)

$$H^q(x,\xi,0)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}$$

**Positivity saturated**

# Pion GPDs

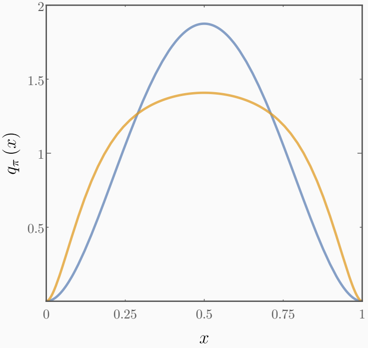
**1.** Chiral symmetry yields factorized LFWF:
[J.-L.Zhang et al.-PLB:136158(815)2021] (See Khépani's talk)

$$\Psi_\pi^q\left(x,k_\perp^2\right)\propto\sqrt{q_\pi\left(x\right)}\frac{M^2}{\left(k_\perp^2+M^2\right)^2}$$

**2.** Pion GPD saturating positivity

$$H_\pi^q\left(x,\xi,t\right)\big|_{\mathrm{DGLAP}}=\frac{\sqrt{q_\pi\left(x_-\right)q_\pi\left(x_+\right)}}{\left(1+z^2\right)^2}\left[3+\frac{1-2z}{1+z}\frac{\operatorname{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}}\right]$$

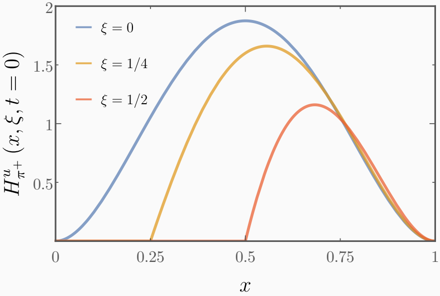
$$z=-t\left(1-x\right)^2/4M^2\left(1-\xi^2\right)$$



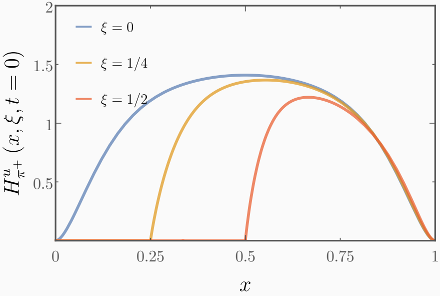

**Two models:**

- Algebraic model
  $q_\pi\left(x\right)=30x^2\left(1-x\right)^2$
- Realistic model (DSE)
  [M.Ding et al.-PRD:054014(101)2020]
  $q_\pi\left(x\right)=\mathcal{N}_q x^2\left(1-x\right)^2$
  $\times\left[1+\gamma x\left(1-x\right)+\rho\sqrt{x\left(1-x\right)}\right]$

# Pion GPDs

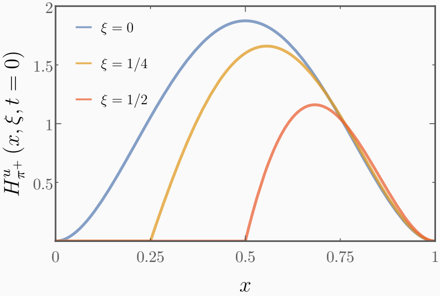


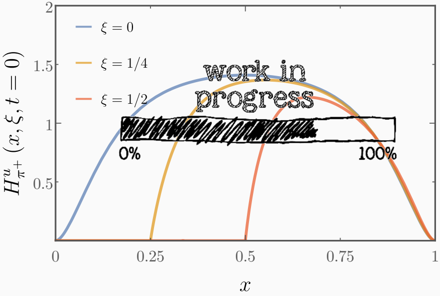
$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
0
0.5
1
1.5
2
0.25
0.75
$x$

$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
0
0.5
1
1.5
2
0.25
0.75
$x$

# Pion GPDs


$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
$x$
0
0.25
0.5
0.75
1
0.5
1
1.5
2


$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
work in progress
0%
100%
$H^u_{\pi^+}(x, \xi, t = 0)$
$x$
0
0.25
0.5
0.75
1
0.5
1
1.5
2

# Pion GPDs: covariant extension

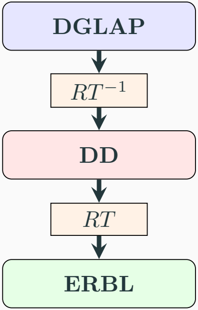
**Covariant extension:**

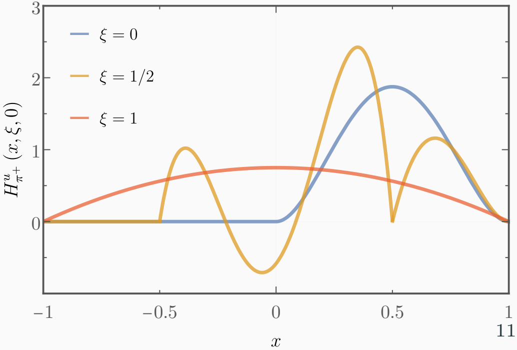
$$H^q(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|}D^+\left(\frac{x}{\xi},t\right) + sgn(\xi)\,D^-\left(\frac{x}{\xi},t\right)$$

Fix D-terms with soft pion theorem:

[M.V.Polyakov-NPB:231(555)1999, C.Mezrag at al.-PLB:190(741)2015]

$$H^{I=0}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) - H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = 0$$
$$H^{I=1}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) + H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = \varphi\left(\frac{1+x}{2}\right)$$

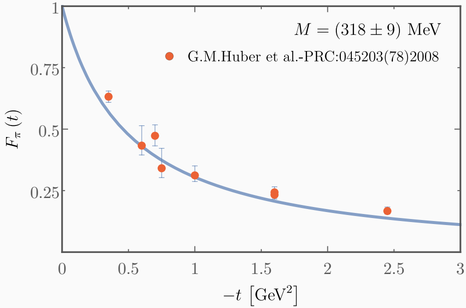


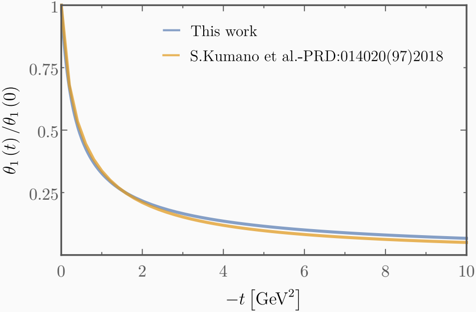



# Pion GPDs: Mellin moments and form factors

D-term t-dependence:

$$d^{\pm}(t) = 1/\left(1 - t/4M^2\right)$$




Great agreement between experimental data for $F_\pi(t)$ and $\theta_1(t)$ and model prediction with one single free parameter.

Comparison with available data for $\theta_2(t)$ is not as good as for $\theta_1(t)$. This is currently being investigated.

# Phenomenology of pion GPDs

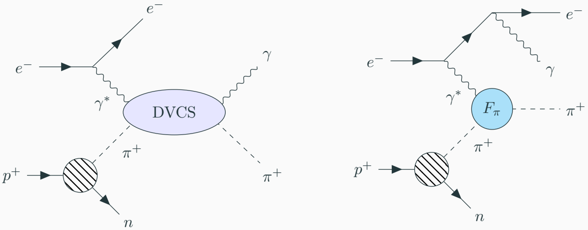
# Phenomenology of pion GPDs: Sullivan process

We have stablished a way of bulding pion GPD models **fulfilling all of the QCD theoretical constraints**, so...

*What about phenomenology?*

DVCS amplitudes: parametrized by hadron GPDs.
[X.Ji-PRD:7114(55)1997]

**Sullivan process** [J.D.Sullivan-PRD:1732(5)1972]




The Sullivan process has already been employed for extracting $\pi$-EFFs.[G.M.Huber at al.-PRC:045203(78)2008]

Can we probe pion GPDs?
[D.Amrath at al.-EPJC:179(58)2008]

# Phenomenology of pion GPDs: Sullivan process

In fact... this has been advocated in the recent EIC-Yellow report
[EICYR:phys.ins-det/2103.05419]

| Science Question | Key Measurement | Key Requirements |
|---|---|---|
| What are the quark and gluon energy contributions to the pion mass? | Pion structure function data over a range of $x$ and $Q^2$. | • Need to uniquely determine $e+p \to e' + X + n$ (low $-t$)<br>• CM energy range ∼10-100 GeV<br>• Charged and neutral currents desirable |
| Is the pion full or empty of gluons as viewed at large $Q^2$? | Pion structure function data at large $Q^2$. | • CM energy ∼100 GeV<br>• Inclusive and open-charm detection |
| What are the quark and gluon energy contributions to the kaon mass? | Kaon structure function data over a range of $x$ and $Q^2$. | • Need to uniquely determine $e+p \to e' + X + \Lambda/\Sigma^0$ (low $-t$)<br>• CM energy range ∼10-100 GeV |
| Are there more or less gluons in kaons than in pions as viewed at large $Q^2$? | Kaon structure function data at large $Q^2$. | • CM energy ∼100 GeV<br>• Inclusive and open-charm detection |
| Can we get quantitative guidance on the emergent pion mass mechanism? | Pion form factor data for $Q^2$ = 10-40 $(\mathrm{GeV}/c)^2$. | • Need to uniquely determine exclusive process $e+p \to e' + \pi^+ + n$ (low $-t$)<br>• $e+p$ and $e+D$ at similar energies<br>• CM energy ∼10-75 GeV |
| What is the size and range of interference between emergent-mass and the Higgs-mass mechanism? | Kaon form factor data for $Q^2$ = 10-20 $(\mathrm{GeV}/c)^2$. | • Need to uniquely determine exclusive process $e+p \to e' + K + \Lambda$ (low $-t$)<br>• L/T separation at CM energy ∼10-20 GeV<br>• $\Lambda/\Sigma^0$ ratios at CM energy ∼10-50 GeV |
| What is the difference between the impacts of emergent- and Higgs-mass mechanisms on light-quark behavior? | Behavior of (valence) up quarks in pion and kaon at large $x$. | • CM energy ∼20 GeV (lowest CM energy to access large-x region)<br>• Higher CM energy for range in $Q^2$ desirable |
| What is the relationship between dynamically chiral symmetry breaking and confinement? | Transverse-momentum dependent Fragmentation Functions of quarks into pions and kaons. | • Collider kinematics desirable (as compared to fixed-target kinematics)<br>• CM energy range ∼20-140 GeV |
| **More speculative observables** | | |
| What is the trace anomaly contribution to the pion mass? | Elastic $J/\Psi$ production at low $W$ off the pion. | • Need to uniquely determine exclusive process $e+p \to e' + J/\Psi + \pi^+ + n$ (low $-t$)<br>• High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$)<br>• CM energy ∼70 GeV |
| Can we obtain tomographic snapshots of the pion in the transverse plane? What is the pressure distribution in a pion? | Measurement of DVCS off pion target as defined with Sullivan process. | • Need to uniquely determine exclusive process $e+p \to e' + \gamma + \pi^+ + n$ (low $-t$)<br>• High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$)<br>• CM energy ∼10-100 GeV |
| Are transverse momentum distributions universal in pions and protons? | Hadron multiplicities in SIDIS off a pion target as defined with Sullivan process. | • Need to uniquely determine SIDIS off pion $e+p \to e' + h + X + n$ (low $-t$)<br>• High luminosity ($10^{34}\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$)<br>• $e+p$ and $e+D$ at similar energies desirable<br>• CM energy ∼10-100 GeV |

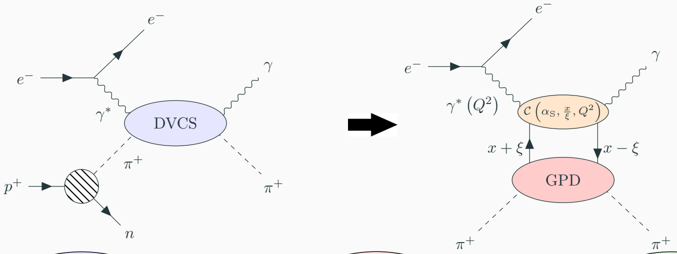
Let us see if that would be feasible in a future electron-ion collider.

# Phenomenology of pion GPDs: Sullivan process

One pion exchange approximation: [D.Amrath at al.-EPJC:179(58)2008]

- $-t < 0,6\,\mathrm{GeV}^2$
- $\sigma_L >> \sigma_\perp$

} Met at EIC [EICYR:phys.ins-det/2103.05419]




**Phenomenology**

1. $\sigma$-DVCS [D.Amrath et al.-EPJC(2008)]
2. Asymmetry
3. …

=

**QCD Evolution**

APFEL++

[V.Bertone et al.CPComm(2014), V.Bertone et al.:hep-ph/1708.00911]

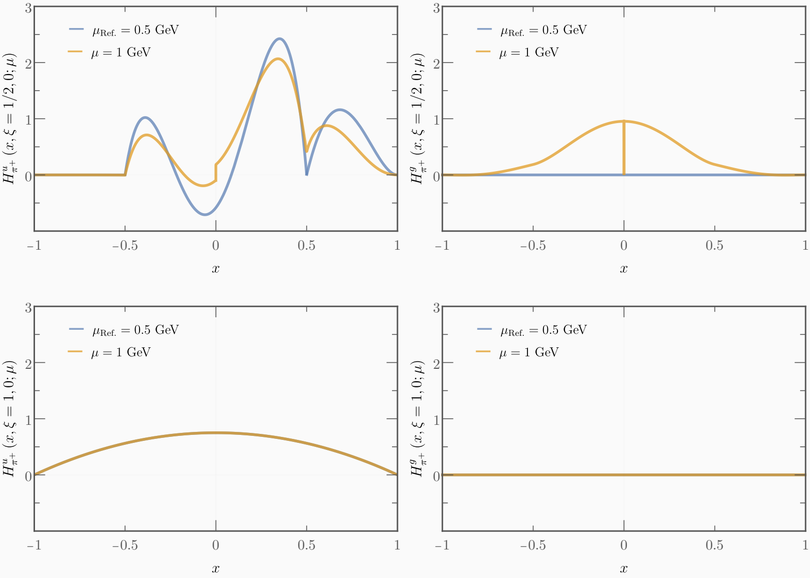
+

**LO/NLO CFFs**

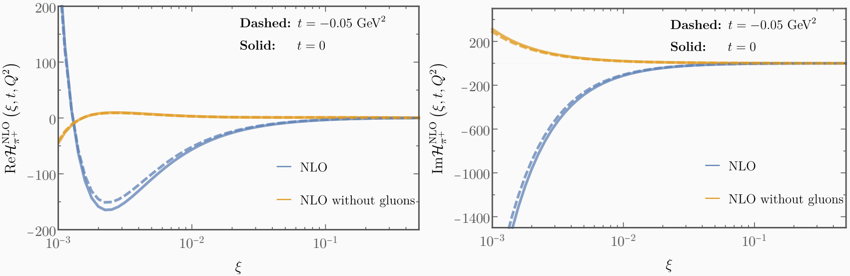
PARTONS

[B.Berthou et al.-EPJC(2018)]

# Phenomenology of pion GPDs: QCD evolution



# Phenomenology of pion GPDs: Compton Form Factors

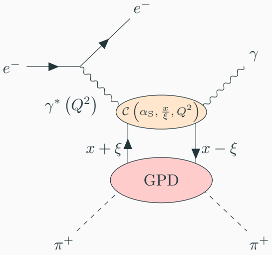


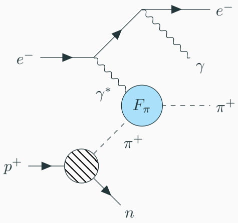

Gluon dominance makes essential at least NLO accuracy in any phenomenlogical analysis of DVCS at an EIC.

*Can we measure DVCS?*

One pion exchange approximation: [D.Amrath at al.-EPJC:179(58)2008]

$-t < 0,6\,\mathrm{GeV}^2$ $\sigma_L >> \sigma_\perp$

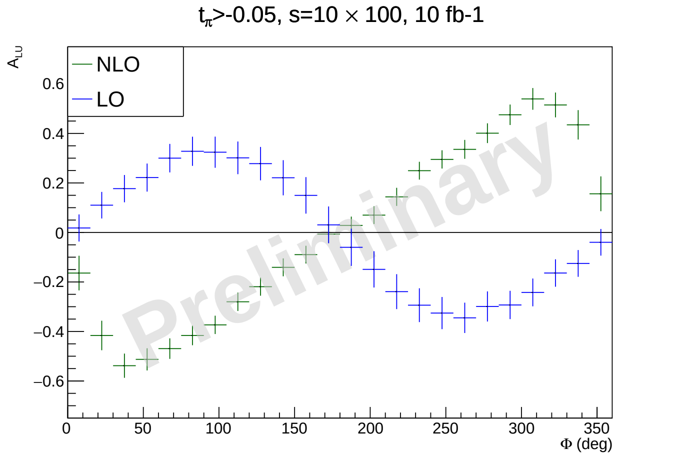



Changing lepton polarisation one can (formally) access interference between DVCS and BH amplitudes.

*Is it experimentally feasible?*

# Phenomenology of pion GPDs: Asymmetry (EIC)




Non-zero asymmetry: optimism about measuring DVCS on pions at future EIC.

# Summary and perspectives

# Summary and perspectives

### Summary

1. GPDs can provide insight into EHM.
2. Pion GPD models fulfilling every theoretical constraint
   - Polynomiality: Covariant extension.
   - Positivity
   - PCAC/AV-WTI: Soft pion theorem.
   - Agreement with experimental data for EFFs and GFFs.
3. DVCS on virtual pions influenced by gluon content
   - Higher order analysis needed for phenomenology.
4. Pion structure to be tested at future electron-ion colliders
   - Insights into EHM could be gained experimentally.

### Perspectives

1. Exploit realistic pion PDF (*currently at work*)
2. Extension of the computating chain
   - Higher order analysis
   - Baryons

# Thank you!