

Quarks, gluons and emergence of hadron mass

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Outline

Preliminaries

Quark propagator

Quark-gluon vertex

The story of quarks and gluons

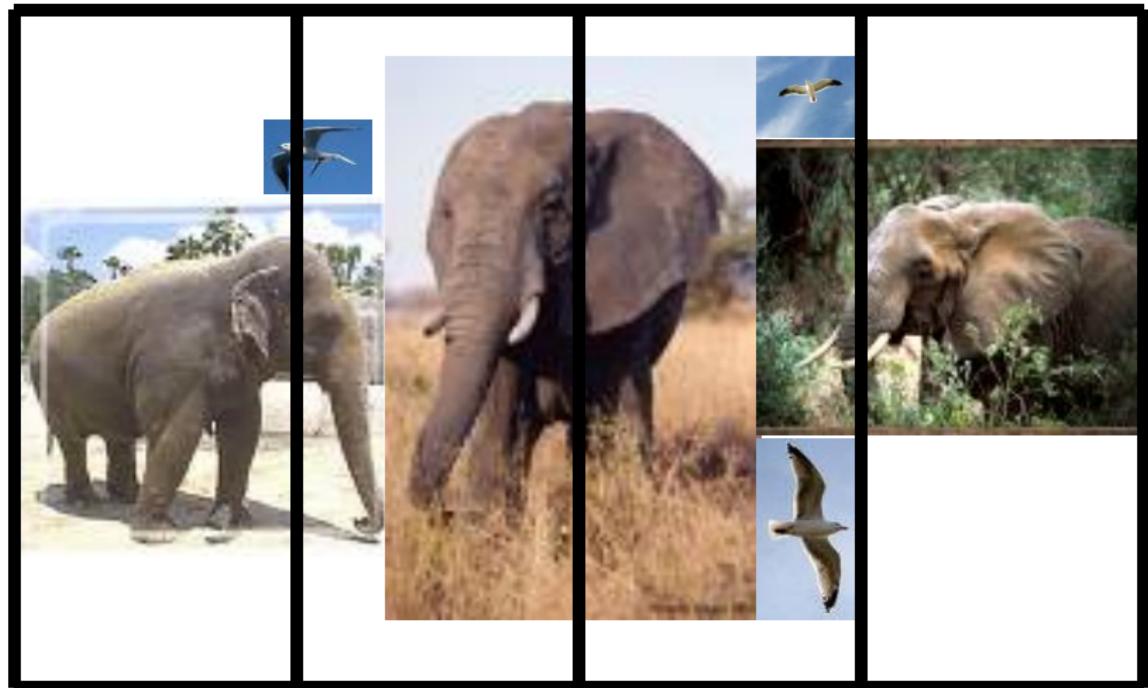
Quark Quark Quark!



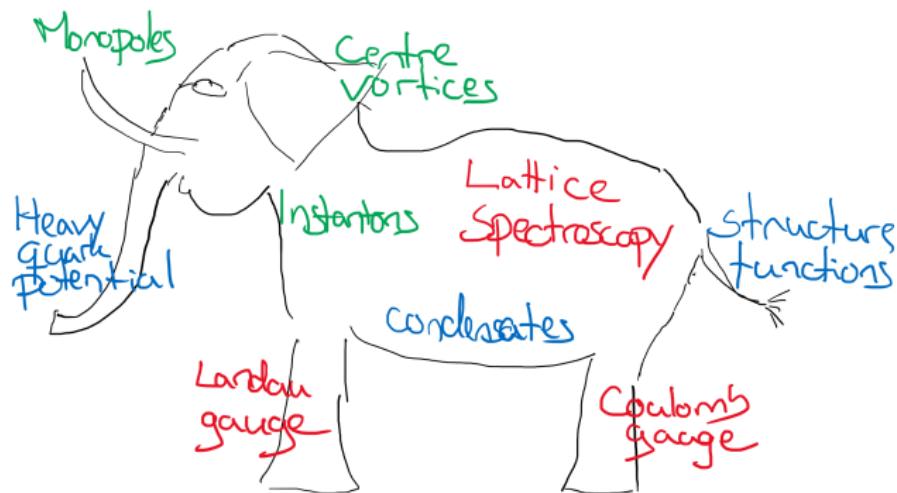
The story of quarks and gluons



The story of quarks and gluons



Understanding confinement, $D\chi SB$, EHM, ...



Emergent hadron mass in Landau gauge

Constituent quarks and gluons

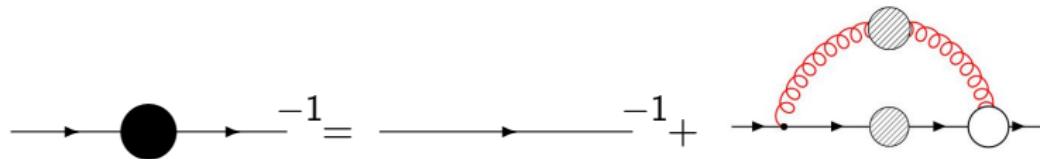
- ▶ Constituent quarks \sim description of hadron spectrum
- ▶ Constituent gluons \sim glueballs, oddballs, hybrids?
- ▶ Momentum-dependent “masses” needed
- ▶ Masses \leftrightarrow singularities (gauge independent)

Emergent hadron mass in Landau gauge

Constituent quarks and gluons

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Dyson–Schwinger (gap) equation



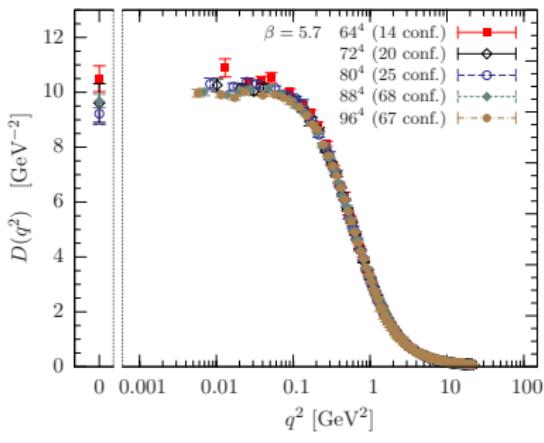
Lattice systematics

- ▶ Volume $V \rightarrow \infty$
- ▶ Lattice spacing $a \rightarrow 0$
 \implies distortions of momentum variables
- ▶ Fermion discretisation: Wilson, staggered, TmWilson, overlap, domain wall, ...
- ▶ Flavour effects: $N_f = 0, 2, 2 + 1, 2 + 1 + 1$
 - ▶ Pure YM: first order phase transition at $T_c \sim 270$ MeV
 - ▶ $N_f = 2 + 1(+1)$: analytic crossover at $T_{pc} \sim 150$ MeV
- ▶ Gauge group [SU(2) vs SU(3)]
 - ▶ Similar at $T = \mu = 0$
 - ▶ Different symmetries at $\mu \neq 0$

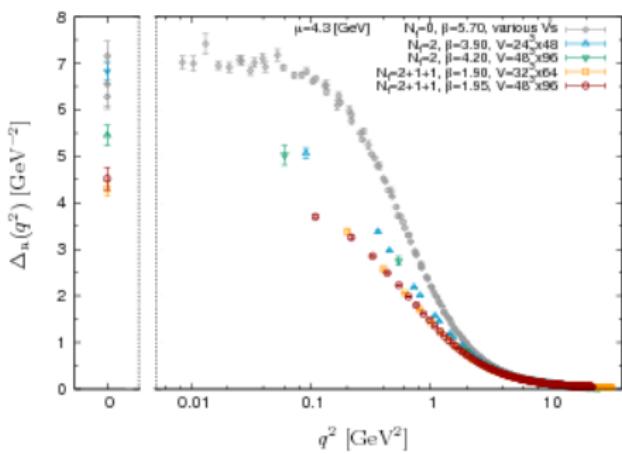
Gluon propagators — history

- ▶ IR slavery vs IR suppressed?
- ▶ Massive vs scaling solution?

[Bogolobsky, Ilgenfritz, Müller-Preussker, Sternbeck (2009)]

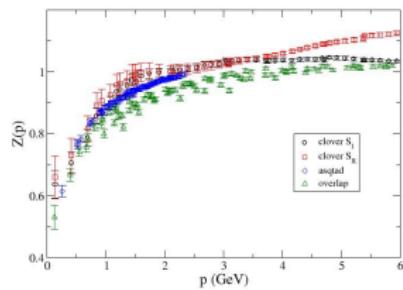
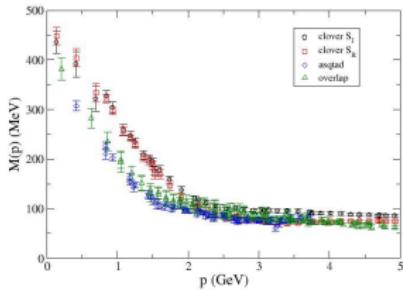


[Ayala et al, 1208.0795]



Quark propagator

$$S(p) = \frac{Z(p^2)}{ip + M(p^2)} = \frac{1}{ipA(p^2) + B(p^2)}$$

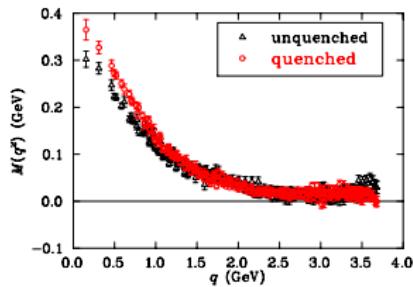
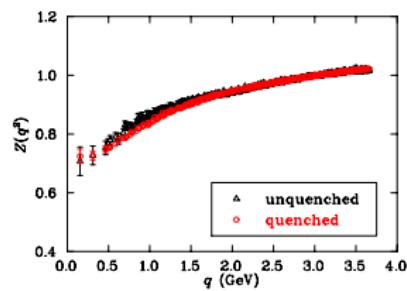


Quenched, 3 different discretisations

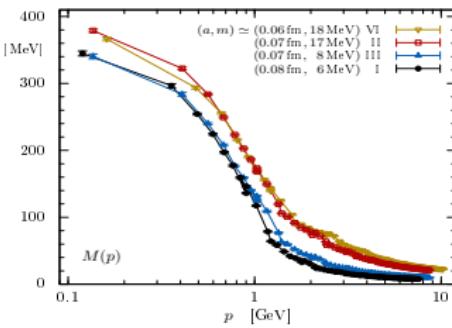
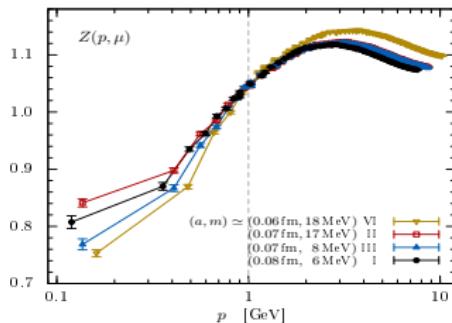
- ▶ Fair agreement between discretisations despite crude correction for lattice artefacts
- ▶ $M_q \sim 300\text{MeV}$ in infrared
- ▶ Infrared suppression of $Z(q)$ a significant factor

Quark propagator for $N_f = 2$

Staggered [Bowman et al 2005]



Wilson–Clover [Oliveira, Silva, JIS, Sternbeck 2019]



Quarks and gluons are not enough!

- ▶ Inserting lattice gluon propagator in gap equation with a bare vertex gives insufficient χ_{SB}
- ▶ Abelian (Ball–Chiu) vertex also gives insufficient enhancement
- ▶ Vertex related to ghost self-energy via Slavnov–Taylor identity
- ▶ Nontrivial tensor structure may be crucial
- ▶ Effective charge most naturally defined from quark–gluon vertex?

Tensor structure

12 independent form factors:

$$\Lambda_\mu(p, q, k) = \sum_{i=1}^4 \lambda_i(p^2, k^2, q^2) L_{i,\mu} + \sum_{i=5}^{12} \tau_i(p^2, k^2, q^2) T_{i,\mu}$$

$$L_{1,\mu} = \gamma_\mu$$

$$L_{2,\mu} = -P P_\mu$$

$$L_{3,\mu} = -i P_\mu$$

$$L_{4,\mu} = -i \sigma_{\mu\nu} P_\nu$$

$$T_{1,\mu} = -i \ell_\mu$$

$$T_{2,\mu} = -P \ell_\mu$$

$$T_{3,\mu} = \not{q} q_\mu - q^2 \gamma_\mu$$

$$T_{4,\mu} = -i [q^2 \sigma_{\mu\nu} P_\nu + 2 q_\mu \sigma_{\nu\lambda} p_\nu k_\lambda]$$

$$T_{5,\mu} = -i \sigma_{\mu\nu} q_\nu$$

$$T_{6,\mu} = (qP) \gamma_\mu - \not{q} P_\mu$$

$$T_{7,\mu} = -\frac{i}{2} (qP) \sigma_{\mu\nu} P_\nu - i P_\mu \sigma_{\nu\lambda} p_\nu k_\lambda$$

$$T_{8,\mu} = -\gamma_\mu \sigma_{\nu\lambda} p_\nu k_\lambda - p_\mu k_\lambda + k_\mu p_\lambda$$

with $P_\mu \equiv p_\mu + k_\mu$, $\ell_\mu \equiv (pq)k_\mu - (kq)p_\mu$

q = gluon momentum; p, k = quark momenta

Transverse projection and abelian vertex

Transverse projection: $\Gamma_\mu^P(p, q) \equiv P_{\mu\nu}(q)\Gamma_\nu(p, q)$

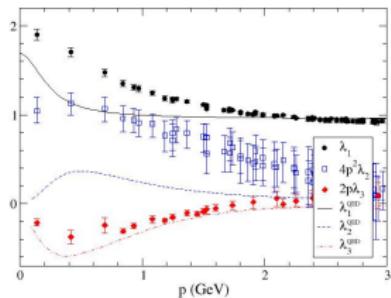
$$\begin{aligned}\lambda'_1 &= \lambda_1 - q^2\tau_3 & ; & \quad \lambda'_2 = \lambda_2 - \frac{q^2}{2}\tau_2; \\ \lambda'_3 &= \lambda_3 - \frac{q^2}{2}\tau_1 & ; & \quad \lambda'_4 = \lambda_4 + q^2\tau_4.\end{aligned}\tag{1}$$

Abelian vertex

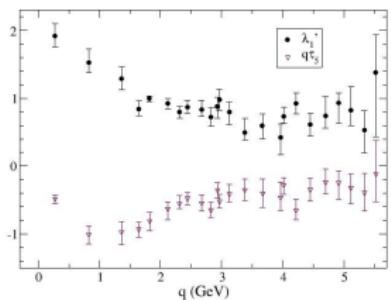
$$\begin{aligned}\lambda_1 &= \frac{1}{2}\left(A(p^2) + A(k^2)\right); & \lambda_2 &= \frac{A(p^2) - A(k^2)}{2(p^2 - k^2)}; \\ \lambda_3 &= \frac{B(p^2) - B(k^2)}{p^2 - k^2} & ; & \lambda_4 = 0.\end{aligned}\tag{2}$$

The quark–gluon vertex (quenched)

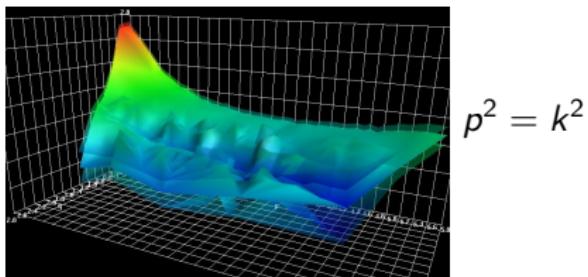
$$q^2=0$$



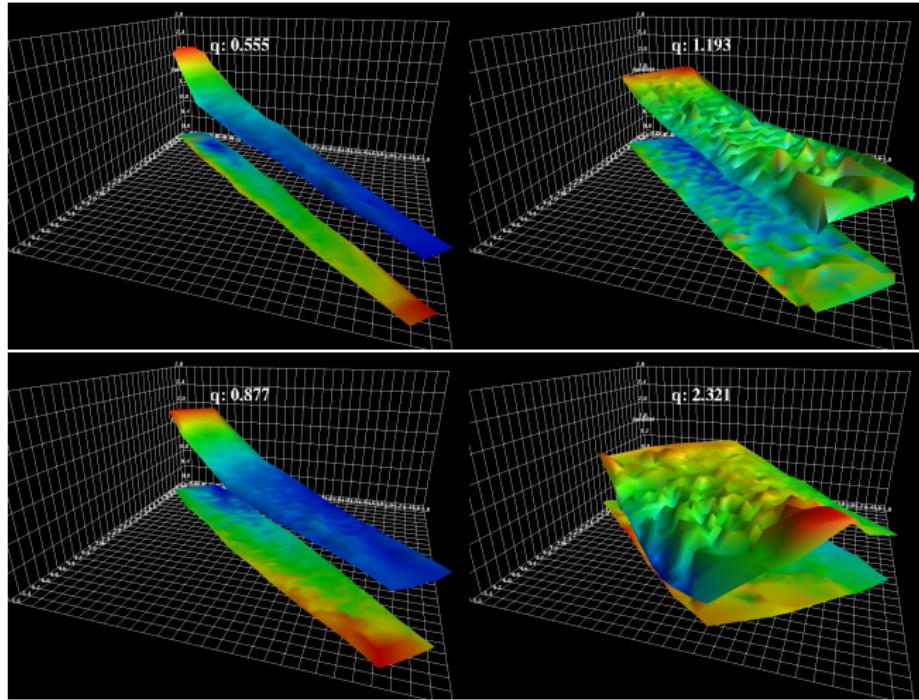
$$q^2=4p^2=4k^2$$



- ▶ Enhancement of vector part over and above abelian
- ▶ No enhancement of scalar part?
- ▶ Substantial contribution from “chromomagnetic” term

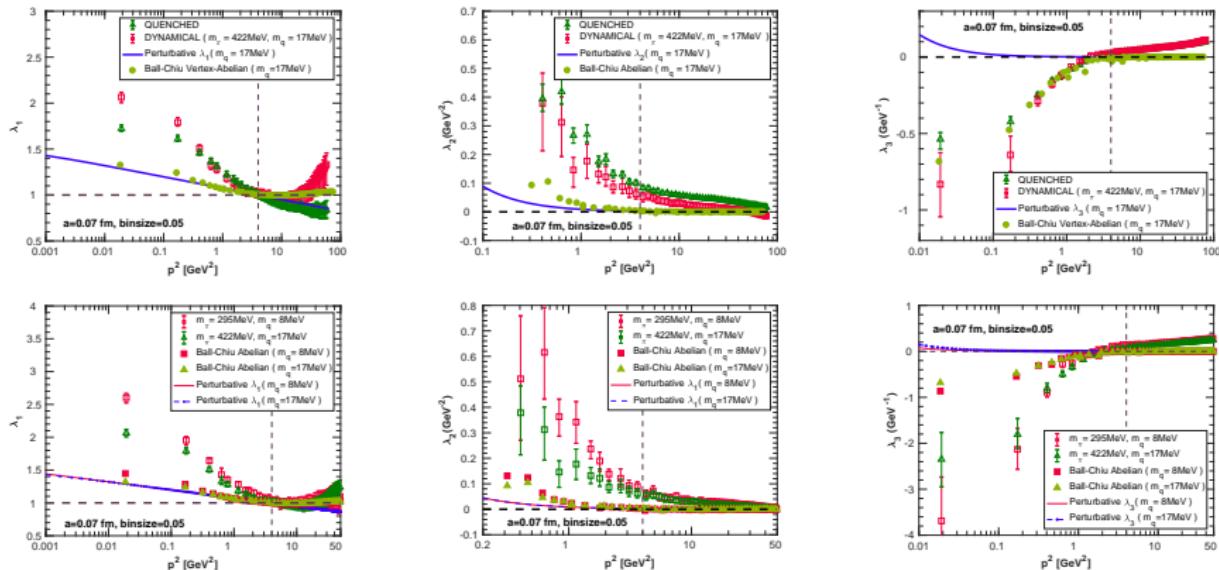


Vertex in general kinematics



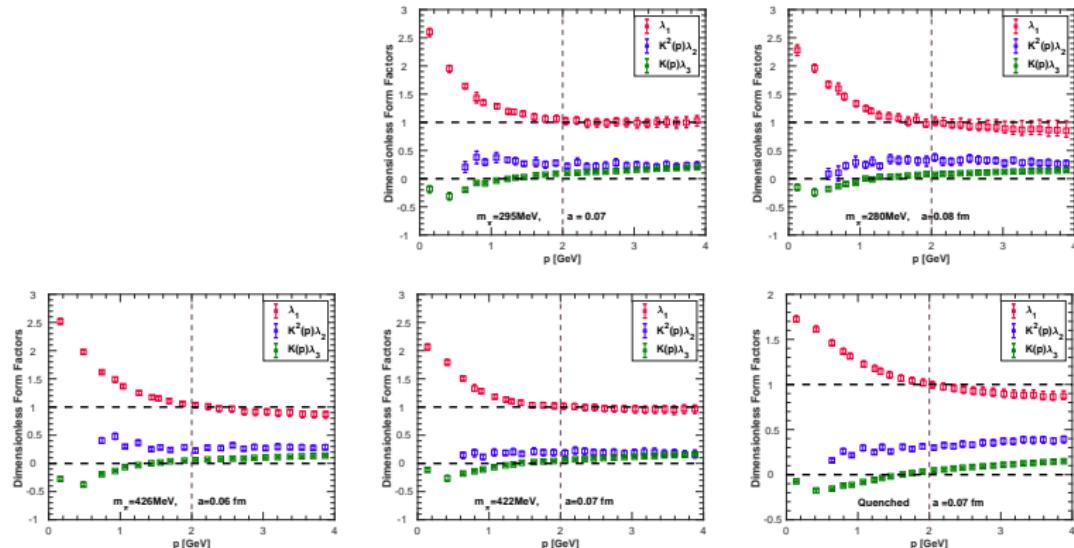
Quark-gluon vertex with $N_f = 2$ ($q^2 = 0$)

[Kızılersü, Oliveira, Silva, JIS, Sternbeck, 2102.02945]



- ▶ Moderate quark mass and (un)quenching effects
- ▶ Other kinematics to be done

Quark-gluon vertex with $N_f = 2$



- ▶ Greater infrared enhancement with lighter quarks
- ▶ Moderate lattice spacing effects
- ▶ Significant contribution from χ SB form factor λ_3

Outlook

Quark propagator

- ▶ Infrared (constituent) mass a **robust prediction**
- ▶ Still need to control all uncertainties

Quark-gluon vertex

- ▶ Enhancement in infrared — driver of chiral symmetry breaking
- ▶ Stronger enhancement
 - ▶ with dynamical quarks
 - ▶ for smaller quark masses
 - ▶ towards continuum limit
- ▶ Significant contribution from χ SB form factor λ_3
- ▶ Other kinematics, form factors **in progress**