Pion structure as a problem in Minkowski space

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Perceiving the Emergence of Hadron Mass through AMBER@CERN
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References: W. de Paula et al, PRD 103 (2021) 014002, E. Ydrefors et al (under revision in PLB)

Introduction

- The pion plays a crucial role within QCD, because its Goldstone boson nature is associated with the dynamical generation of the mass of hadrons and nuclei.
- From the theory side, the current challenge is to extract from Euclidean calculations (e.g. LQCD) Minkowskian physical quantities, e.g. PDFs.
- Recently, the momentum distributions have been a target of intense investigation and will be as well at the planned EIC.
- In this work the pion was studied by solving the Bethe-Salpeter equation directly in Minkowski space.
- Within this approach it is straightforward to compute physical quantities defined on the LF.

Theoretical framework

- The BS amplitude, describing the 0^- quark-antiquark bound state, obeys the equation [1]

$$\Phi(k,p) = S(k+p/2)\int \frac{d^4k'}{(2\pi)^4} S^{\mu\nu}(q)\Gamma_\mu(q)\phi(k',p)\widehat{\Gamma}_\nu(q)S(k-p/2);$$
$$\widehat{\Gamma}_\nu(q) = C\Gamma_\nu(q)C^{-1}, \tag{1}$$

where we currently use bare progagators for the quarks and gluons, i.e.

$$S(k) = i\frac{\not{k}+m}{k^2-m^2+i\epsilon}, \quad S^{\mu\nu}(q) = -i\frac{g^{\mu\nu}}{q^2-\mu^2+i\epsilon}, \tag{2}$$

and the quark-gluon vertex is described by

$$\Gamma^\mu(q) = ig\frac{\mu^2-\Lambda^2}{q^2-\Lambda^2+i\epsilon}\gamma^\mu. \tag{3}$$

characterized by the scale parameter Λ.

[1] W. de Paula et al, Eur. Phys. C (2017) 77

- The BS amplitude is decomposed as

$$\Phi(k,p) = \sum_{i=1}^{4} S_i(k,p)\phi_i(k,p) \tag{4}$$

where S_i is a Dirac structure.

- Each scalar function ϕ_i written in terms of NIR:

$$\phi_i(k,p) = \int_{-1}^{1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{[k^2 + z'p\cdot k - \gamma' - \kappa^2 + i\epsilon]^3} \tag{5}$$

- After LF projection following system of coupled integral equations is obtained:

$$\int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{[\gamma + \gamma' + m^2z^2 + (1-z^2)\kappa^2]^2} = iMg^2 \sum_j \int_0^\infty d\gamma' \int_{-1}^{1} dz' \mathcal{L}_{ij}(\gamma,z;\gamma'z')g_j(\gamma,z'), \tag{6}$$

which are solved for the coupling constant g^2 and the Nakanishi weight functions g_i.

- Once the g_i's are known physical observables can be calculated as integrals over these functions.

Method for solving the integral equations

- Basis expansion:

$$g_i(\gamma, z) = \sum_{k=0}^{N_z} \sum_{n=0}^{N_\gamma} A_{kn}^i G_{2k+r_i}^i(z) \mathcal{L}_n(\gamma), \tag{7}$$

where

$$\begin{aligned} G_n^\lambda(z) &= (1-z^2)^{(2\lambda-1)/4} \Gamma(\lambda) \sqrt{\frac{n!(n+\lambda)}{2^{1-2\lambda}\pi\Gamma(n+2\lambda)}} C_n^\lambda(z) \\ \mathcal{L}_n(\gamma) &= \sqrt{a} L_n(a\gamma) e^{-a\gamma/2}. \end{aligned} \tag{8}$$

with $C_n^\lambda(z)$ Gegenbauer polynomial and $L_n(a\gamma)$ Laguerre polynomial.

- The system of integral equations when transformed to a generalized eigenvalue problem.

Valence LF wave function

- The spin components of the valence LF wave function of the pion read

$$\psi_{\uparrow\downarrow}(\gamma,z) = \psi_2(\gamma,z) + \frac{z}{2}\psi_3(\gamma,z) + \frac{i}{M^3}\int_0^\infty d\gamma' \frac{\partial g_3(\gamma',z)/\partial z}{\gamma+\gamma'+z^2m^2+(1-z^2)\kappa^2},$$
$$\psi_{\uparrow\uparrow}(\gamma,z) = \frac{\sqrt{\gamma}}{M}\psi_4(\gamma,z), \tag{9}$$

  with the LF amplitudes given by

$$\psi_i(\gamma,z) = -\frac{i}{M}\int_0^\infty d\gamma' \frac{g_i(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2}. \tag{10}$$

- The probability density is then given by

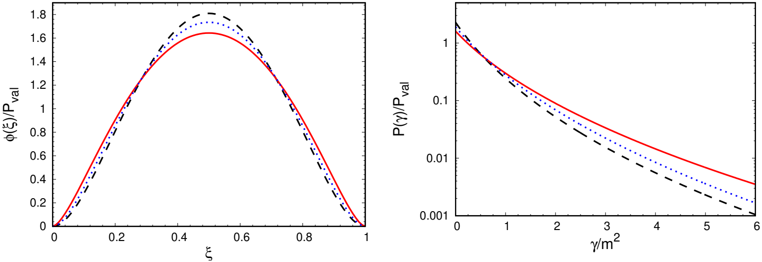
$$\mathcal{P}_{val}(\gamma,z) = \frac{N_c}{16\pi^2}\left[|\psi_{\uparrow\downarrow}(\gamma,z)|^2 + |\psi_{\uparrow\uparrow}(\gamma,z)|^2\right] \tag{11}$$

Static properties

Set	m (MeV)	B/m	μ/m	Λ/m	P_{val}	$P_{\uparrow\downarrow}$	$P_{\uparrow\uparrow}$	f_π (MeV)
I	187	1.25	0.15	2	0.64	0.55	0.09	77
II	255	1.45	1.5	1	0.65	0.55	0.10	112
III	255	1.45	2	1	0.66	0.56	0.11	117
IV	215	1.35	2	1	0.67	0.57	0.11	98
V	187	1.25	2	1	0.67	0.56	0.11	84
VI	255	1.45	2.5	1	0.68	0.56	0.11	122
VII	255	1.45	2.5	1.1	0.69	0.56	0.12	127
VIII	255	1.45	2.5	1.2	0.70	0.57	0.13	130
IX	255	1.45	1	2	0.70	0.57	0.14	134
X	215	1.35	1	2	0.71	0.57	0.14	112
XI	187	1.25	1	2	0.71	0.58	0.14	96

- The set VIII gives an f_π in good agreement with the experimental value.
- The valence probability is 64-71%, i.e. rather large contributions beyond the valence component.
- $P_{\uparrow\uparrow}/P_{val} \sim 19\%$, so relativistic effects in the pion quite important.

Valence LF-momentum distributions



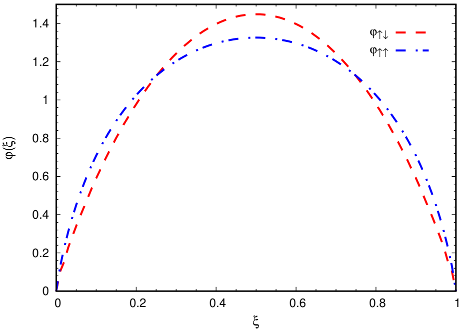
- Result in red reproduce experimental f_π and two other cases shown for comparison.
- Here

$$\phi(\xi) = \int_0^\infty d\gamma \mathcal{P}(\gamma, z), \quad P(\gamma) = \int_{-1}^1 dz \mathcal{P}(\gamma, z) \tag{12}$$

where $P(\gamma, z)$ is valence probability distribution.

- $\phi(\xi)$ is pdf at initial scale. Evolved PDFs are in progress.

Pion distribution amplitude

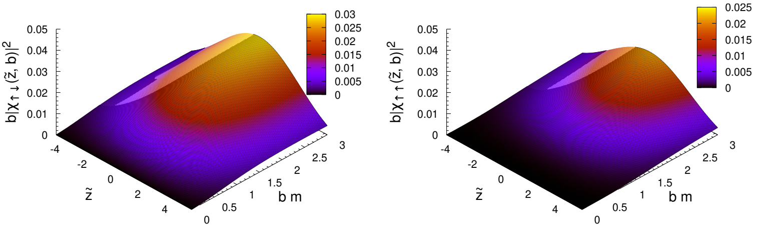


- Spin components of the DA, defined by

$$\phi_{\uparrow\downarrow(\uparrow\uparrow)}(\xi) = \frac{\int_0^\infty d\gamma \psi_{\uparrow\downarrow(\uparrow\uparrow)}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \psi_{\uparrow\downarrow(\uparrow\uparrow)}(\gamma, z)} \tag{13}$$

- Aligned component (in blue) more wide than the anti-aligned one (in red).

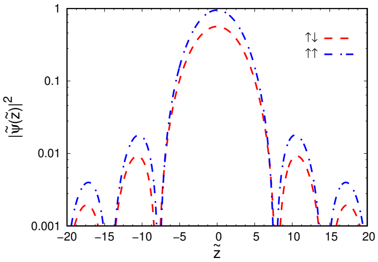
- The space-time structure of the pion, can be studied in terms of the Ioffe-time ($\tilde{z} = x^- p^+/2$) and the impact parameter $\mathbf{b} = \mathbf{x}_\perp$.
- It is done by performing the Fourier transform of the valence wave function.

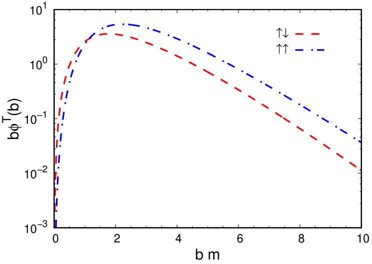


Here an exponential factor has been factored out, i.e.

$$\tilde{\psi}_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, \mathbf{b}) = e^{-b\kappa - \frac{i}{2}\tilde{z}} \chi_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, b) \tag{14}$$

Integrated amplitudes





- $\tilde{z}$ dependence:

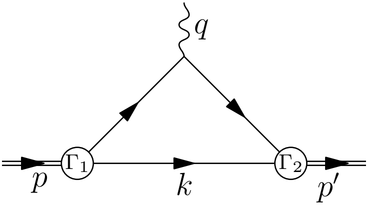
$$\Psi_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}) = \frac{\int_0^\infty dbb\tilde{\psi}_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, b)}{\int_0^\infty dbb \int_{-\infty}^\infty d\tilde{z}\tilde{\psi}_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, b)} \tag{15}$$

- Transverse dependence:

$$\phi^T_{\uparrow\downarrow(\uparrow\uparrow)}(b) = \frac{\int_{-\infty}^\infty d\tilde{z}\tilde{\psi}_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, b)}{\int_0^\infty dbb \int_{-\infty}^\infty d\tilde{z}\tilde{\psi}_{\uparrow\downarrow(\uparrow\uparrow)}(\tilde{z}, b)}; \quad \sim e^{-\kappa b} \text{ at large } b \tag{16}$$

- Transverse wave function should be accesible by Euclidean calculations, since it only depends on transverse coordinates.

Covariant electromagnetic form factors



- In impulse approximation, with bare photon vertex $i\gamma^\mu$,

$$(p+p')^\mu F(Q^2) = -i\frac{N_c}{4M^2+Q^2}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[(-\not{k}-m)\bar{\Phi}_2(k_2;p')(\not{p}+\not{p}')\Phi_1(k_1;p)], \tag{17}$$

  where $Q^2 = -(p-p')^2$.

- After using the NIR and computing the trace, etc one obtains

$$F(Q^2) = \frac{N_c}{32\pi^2}\sum_{ij}\int_0^\infty d\gamma\int_{-1}^1 dz g_j(\gamma,z)\int_0^\infty d\gamma'\int_{-1}^1 dz' g_i(\gamma',z')\int_0^1 dy y^2(1-y)^2\frac{c_{ij}}{M_{cov}^8} \tag{18}$$

Valence electromagnetic form factor

- The valence electromagnetic FF, obtained from the matrix element of γ^+, can be written as

$$F_{val}(Q^2) = \frac{N_c}{16\pi^3}\int d^2k_\perp \int_{-1}^{1} dz \Big[\psi^*_{\uparrow\downarrow}(\gamma', z)\psi_{\uparrow\downarrow}(\gamma, z) + \frac{\vec{k}_\perp \cdot \vec{k}'_\perp}{\gamma\gamma'}\psi^*_{\uparrow\uparrow}(\gamma', z)\psi_{\uparrow\uparrow}(\gamma, z)\Big];$$
$$F_{val}(0) = p_{val}, \tag{19}$$

where $\vec{k}'_\perp = \vec{k}_\perp + \frac{1}{2}(1+z)\vec{q}_\perp$ and e.g. $\gamma = |k_\perp|^2$.

- Total FF is $F(Q^2) = F_{val}(Q^2) + F_{nval}(Q^2)$.
- Asymptotically,

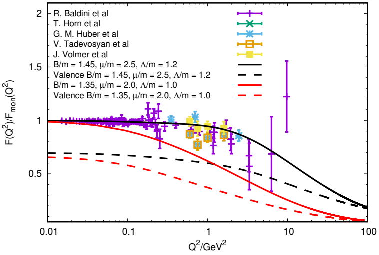
$$F_{val} \sim \frac{N_c}{16\pi^2}\int_{-1}^{1} dz \psi_{\uparrow\downarrow}\left(\frac{(1+z)^2}{4}Q^2, z\right)\int_0^\infty d\gamma \psi_{\uparrow\downarrow}(\gamma, z); \quad Q^2 \to \infty, \tag{20}$$

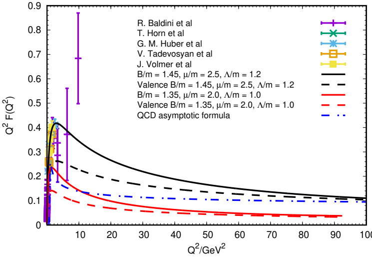
obtained by neglecting the aligned $(\uparrow\uparrow)$ contribution.

Results for the pion radius

Set	m	B/m	μ/m	Λ/m	P_{val}	f_π	r_π (fm)	r_{val} (fm)	r_{nval} (fm)
I	255	1.45	2.5	1.2	0.70	130	0.663	0.710	0.538
II	215	1.35	2	1	0.67	98	0.835	0.895	0.703

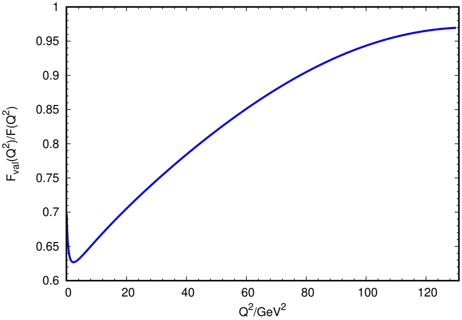
- $r_\pi^2 = -6dF(Q^2)/dQ^2|_{Q^2=0}$,
  $r_{val}^2 = -6/P_{val}dF_{val}(Q^2)/dQ^2|_{Q^2=0}$,
  $r_{nval}^2 = -6/P_{nval}dF_{val}(Q^2)/dQ^2|_{Q^2=0}$ with $F_{nval}(Q^2) = F(Q^2) - F_{val}(Q^2)$
- The set I gives $f_\pi = 130$ MeV and $r_\pi = 0.633$ fm in very good agreement with experimental data of $f_\pi^{PDG} = 130.50(1)(3)(13)$ MeV and $r_\pi^{PDG} = 0.659 \pm 0.004$ fm.
- For the radii $r_{val} > r_{nval}$, i.e. the higher Fock components generate a more compact charge distribution.



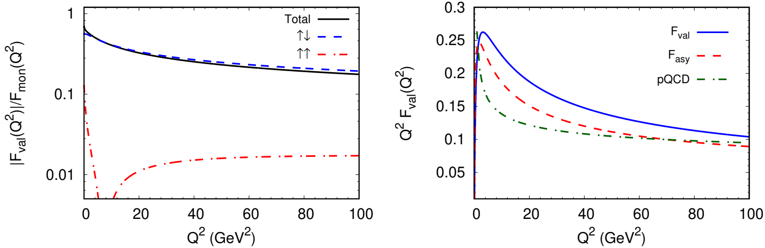


- Despite the simplicity of our model we have a good agreement with experimental data for all Q^2.
- As seen in right figure, our results agree quite well with the one of pQCD for large Q^2.

Valence vs covariant FF

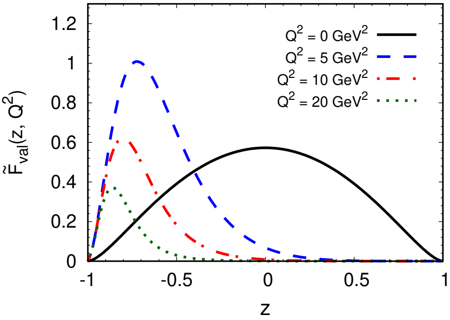


- Beyond-valence contributions important especially for small or moderate Q^2. At $Q^2 = 100\,\mathrm{GeV}^2$ the valence part exhausts 95% of the FF.



- At $Q^2 = 0$, spin-aligned contribution about 20% and decreasing with increasing Q^2. Almost neglible at large momentum transfers.
- Zero in spin-aligned FF due to relativistic spin-orbit coupling leading to the term $\vec{k}_\perp \cdot \vec{k}'_\perp$.
- Difference between exact formula and approximate formula decreases with increasing Q^2, as expected.

Sliced valence FF



- Sliced valence FF defined through

$$F_{val}(Q^2) = \int_{-1}^{1} dz \tilde{F}_{val}(z, Q^2) \tag{21}$$

- Sliced FF symmetric for $Q^2 = 0$.
- Cumulates close to $z = -1$ for increasing Q^2, i.e. $q\bar{q}$ pair is collinear to keep the pion in the final state.

Conclusion

- The pion has been studied by solving the fermion-antifermion BS equation directly in Minkowski space, through the use of the NIR.
- The spin contributions to several physical quantities defined on the LF plane has been analyzed.
- Furthermore, the image of the pion in the space consisting of the Ioffe-time and the transverse variables has been constructed.
- The beyond-valence contributions turn out to be important, i.e. the valence probability is of the order of 70%.
- The covariant FF (including all Fock components) and also its valence contribution has been computed. A very good agreement with experimental data was found.
- However, for the moment the adopted model has several limitations:
  - Quark and gluon propagators are the constituent ones (i.e. no momentum-dependent self-energies)
  - Relatively simple quark-gluon vertex
  - No confinement

Outlook

- Calculation of transverse momentum distribution including all Fock-components included in the model.
- Implementation of dressing functions for quarks and gluons.
- Implementation of a more realistic quark-gluon vertex.
- Calculations for the case of total angular momentum $J = 1$, so the ρ-meson can be studied.