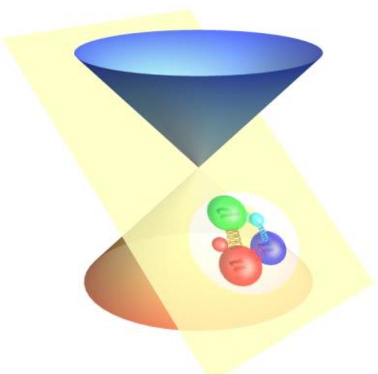
Light Meson Structure from Basis Light-front Quantization

Jiangshan Lan*, Hengfei Zhao*, Kaiyu Fu*,
Chandan Mondal*, Xingbo Zhao*, James P. Vary†



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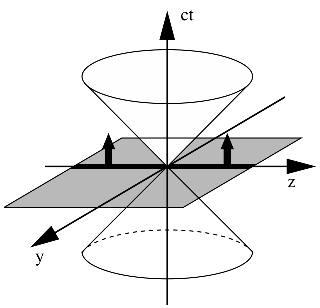
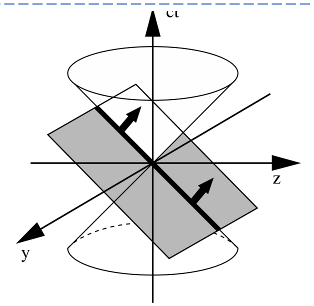
Perceiving the Emergence of Hadron Mass through AMBER@CERN 4/28/2021

Outline

- Basis Light-front Quantization approach
- Application to π and K
  - Leading Fock sector (based on NJL interacton)
  - With one dynamical gluon
- Summary and Future Plan

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\psi(t)\rangle = H\lvert\psi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\psi(x^+)\rangle = \frac{1}{2}P^-\lvert\psi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Advantages:

- Frame-independent wave functions
- Direct access to parton distributions
- Simple vacuum structure
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\boldsymbol{\pi}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}gg\rangle + d|q\bar{q}q\bar{q}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i(2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

Light-front QCD Hamiltonian

$$
\begin{aligned}
P^-_{LFQCD} = & \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A^i_a (\mathrm{i}\partial^\perp)^2 A_{ia} \\
& - \frac{1}{2} g^2 \int d^3x \,\mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
& + \frac{1}{2} g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2} \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
& - g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
& + g^2 \int d^3x \,\mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right] \frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
& + \frac{1}{2} g^2 \int d^3x \overline{\widetilde{\psi}} \widetilde{\not{A}} \frac{\gamma^+}{\mathrm{i}\partial^+} \widetilde{\not{A}} \widetilde{\psi} \\
& + g \int d^3x \overline{\widetilde{\psi}} \widetilde{\not{A}} \widetilde{\psi} \\
& + 2g \int d^3x \,\mathbf{Tr}\left(\mathrm{i}\partial^\mu \widetilde{A}^\nu \left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

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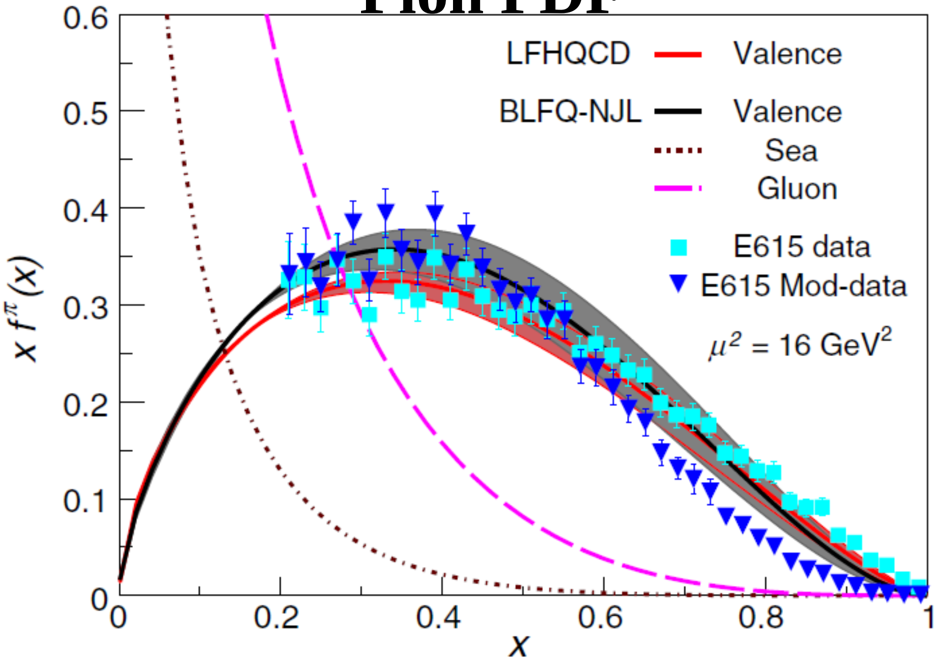
PDF from BLFQ and QCD Evolution for Light Mesons

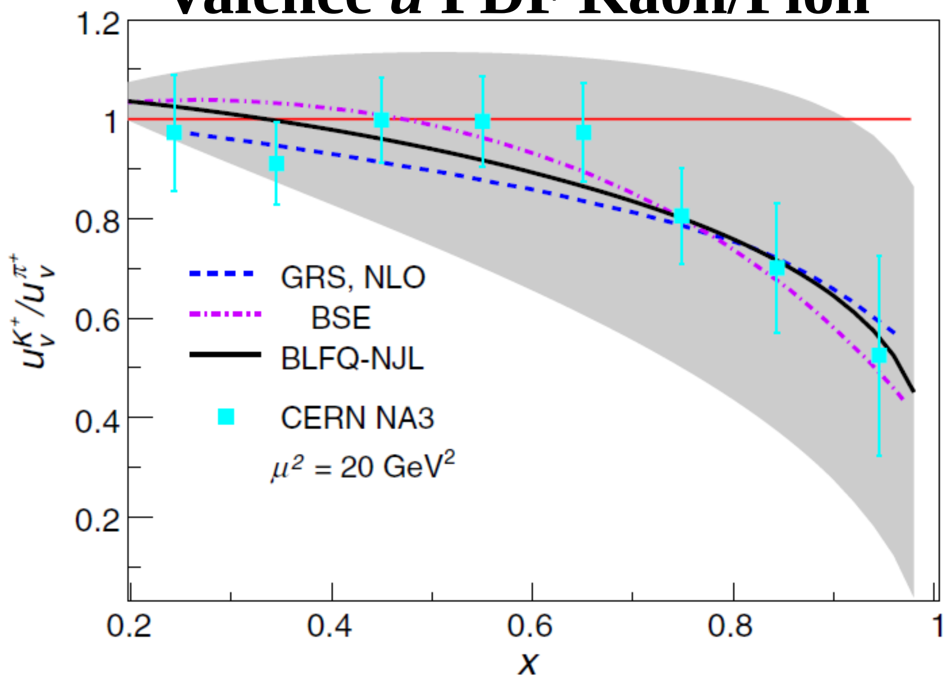
$$P_{\text{eff}}^{-} = \frac{\overrightarrow{k_{\perp}^{2}} + m_q^2}{x} + \frac{\overrightarrow{k_{\perp}^{2}} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big) + H_{\text{eff}}^{\text{NJL}}$$

PDF for the valence quark result from the light-front wave functions obtain by diagonalizing the effective Hamiltonian.

Pion PDF



Valence *u* PDF Kaon/Pion

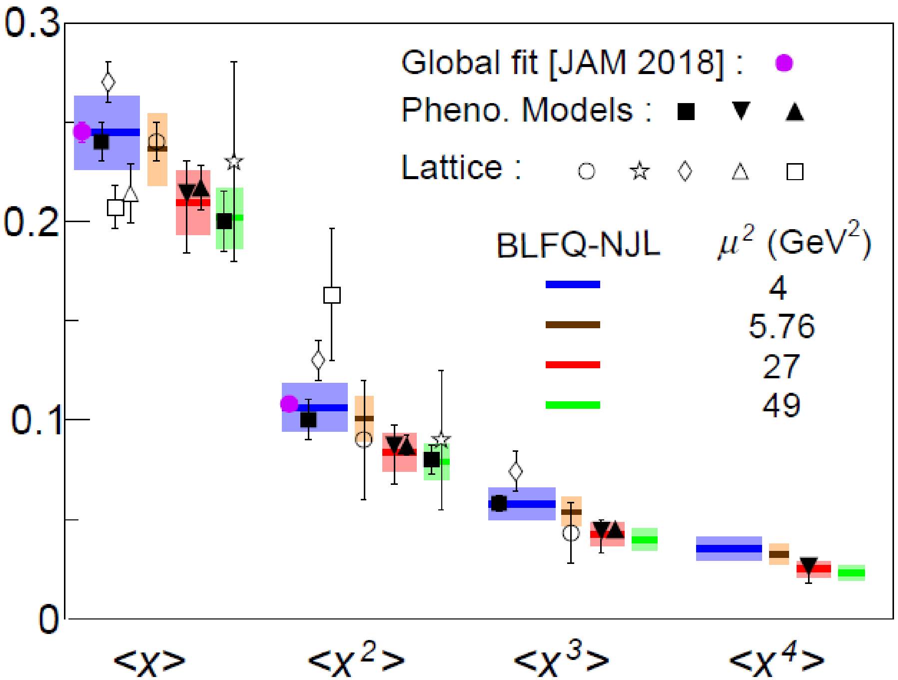


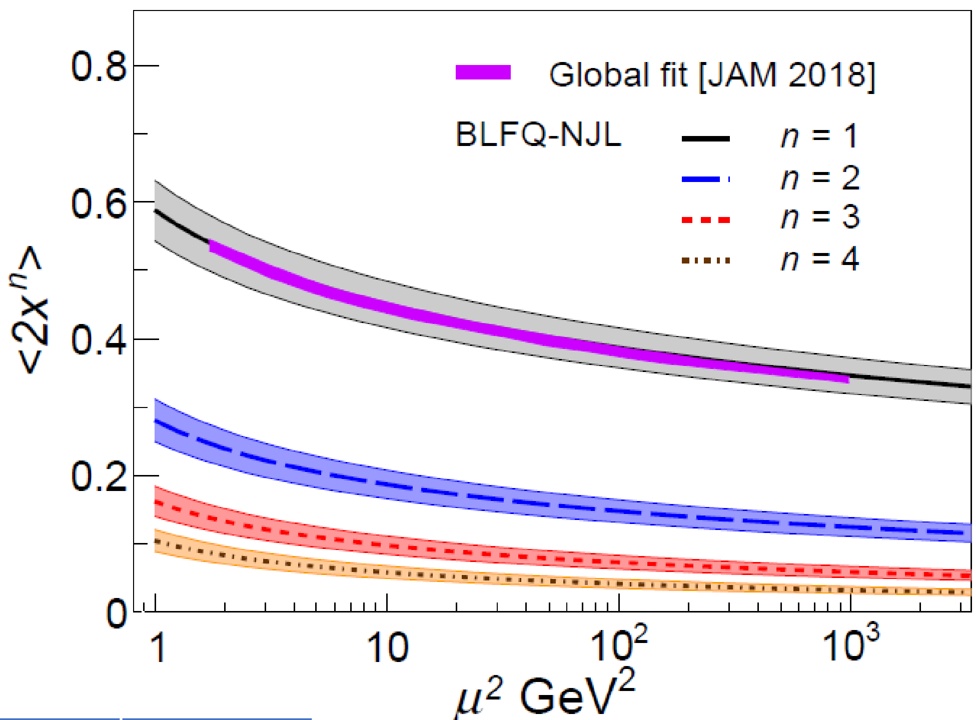
[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

Agree with experimental results

The moments of pion valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x, \mu^2),\ \ n = 1, 2, 3, 4.$$





⟨x⟩ @ 4 GeV²	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019’]	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

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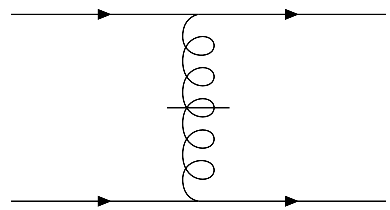
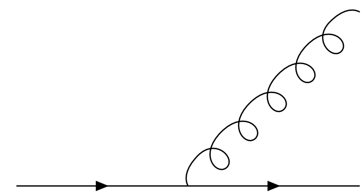
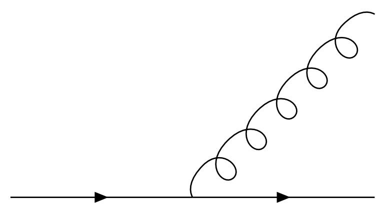
$$|\pi\rangle = |q\bar{q}\rangle + \cdots$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

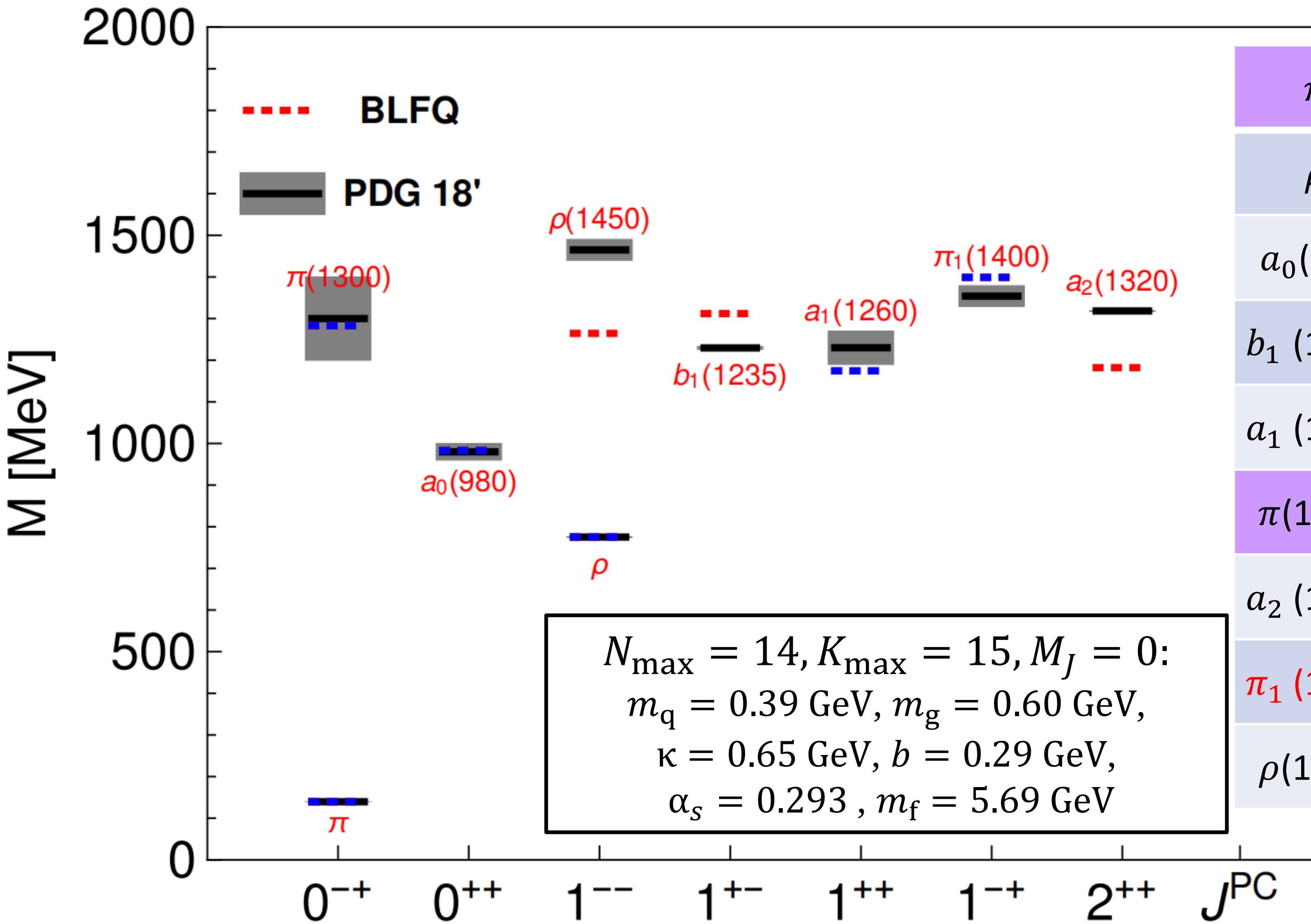
Interaction Part of Hamiltonian

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$\boldsymbol{H}_{\mathbf{int}}$	$\vert q\bar{q}\rangle$	$\vert q\bar{q}g\rangle$
$\langle q\bar{q}\vert$		
$\langle q\bar{q}g\vert$		0

$$P^- = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2}$$
$$- \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + \boldsymbol{H}_{\mathbf{int}}$$

Mass spectrum



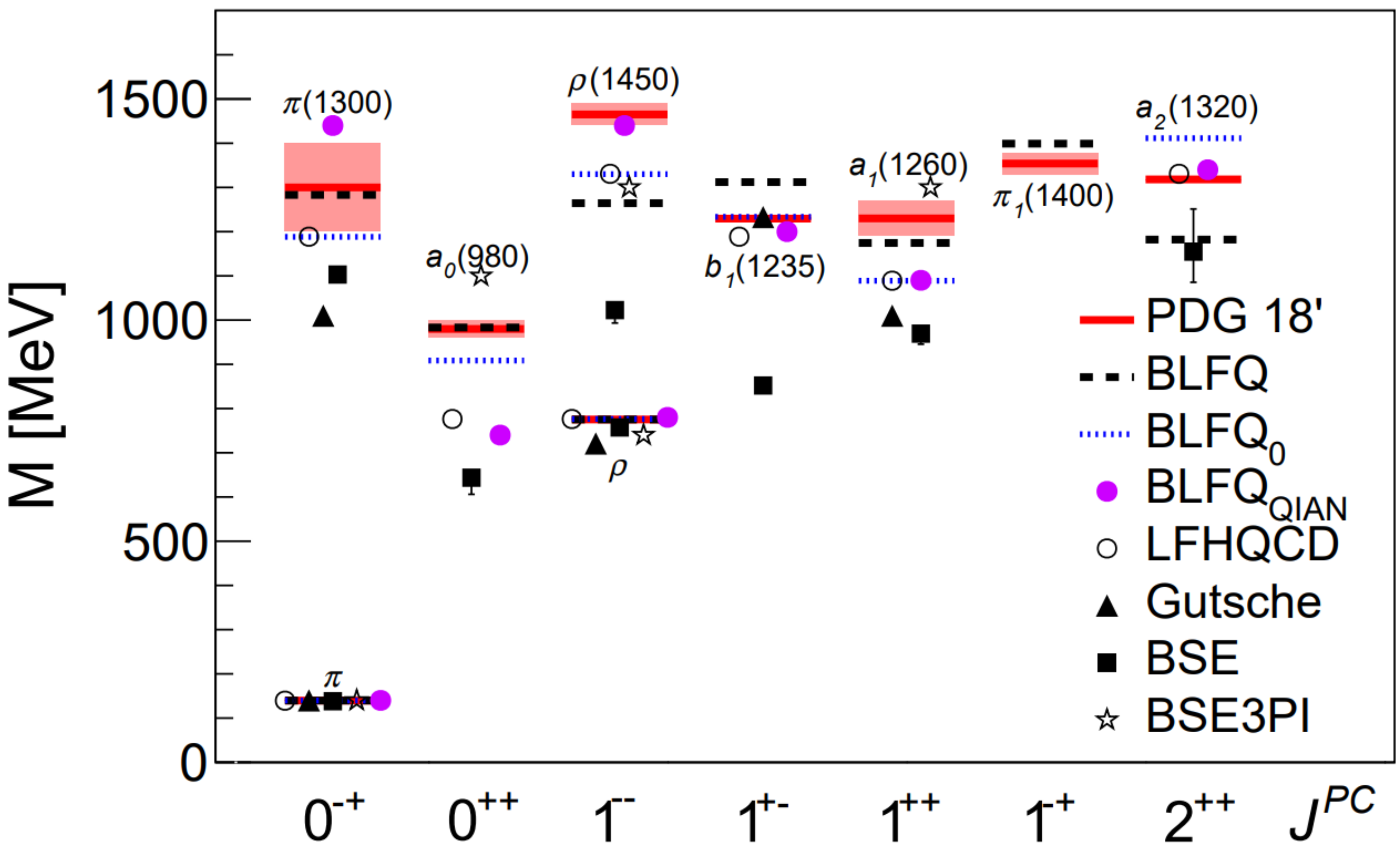
	norm $q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.306	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

Parameters fixed by fitting six blue states

Preliminary

[Lan, et al, in preparation]

Mass spectrum



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- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, et al, in preparation]

Preliminary

Pion mass, *DC*, *Radii*

$$\langle r_c^2 \rangle = -6 \frac{\partial}{\partial Q^2} F(Q^2)|_{Q^2 \to 0}$$

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

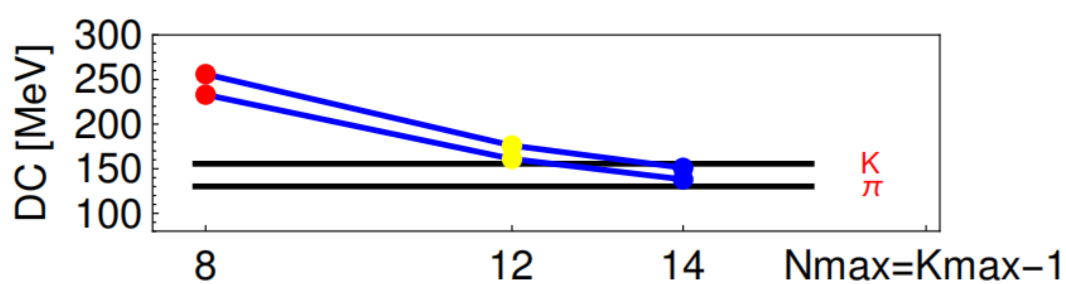
$$\langle 0|\bar{\psi}(0)\gamma^+\gamma_5\psi(0)|P(p)\rangle = \mathrm{i}p^+ f_P,$$

$$\langle 0|\bar{\psi}(0)\gamma^+\psi(0)|V(p,\lambda)\rangle = e_\lambda^+ M_V f_V.$$

	m_{π^+} [MeV]	m_{ρ^+} [MeV]	f_{π^+} [MeV]	f_{ρ^+} [MeV]	$\sqrt{\langle r_c^2 \rangle}\|_{\pi^+}$ [fm]	norm $q\bar{q}$
BLFQ	**139.57**	**775.26**	**138.2**	**129.0**	**0.516~1.456**	**0.492**
PDG [Tanabashi, et al, PRD(2018)]	***139.57***	***775.26±0.25***	***130.2±1.7***	***221±2***	***0.672±0.008***	
BLFQ-NJL [Jia, Vary, PRC(2018)]	*139.57*	*775.23±0.04*	*202.10/$\sqrt{2}$*	*100.12/$\sqrt{2}$*	*0.68±0.05*	

BLFQ

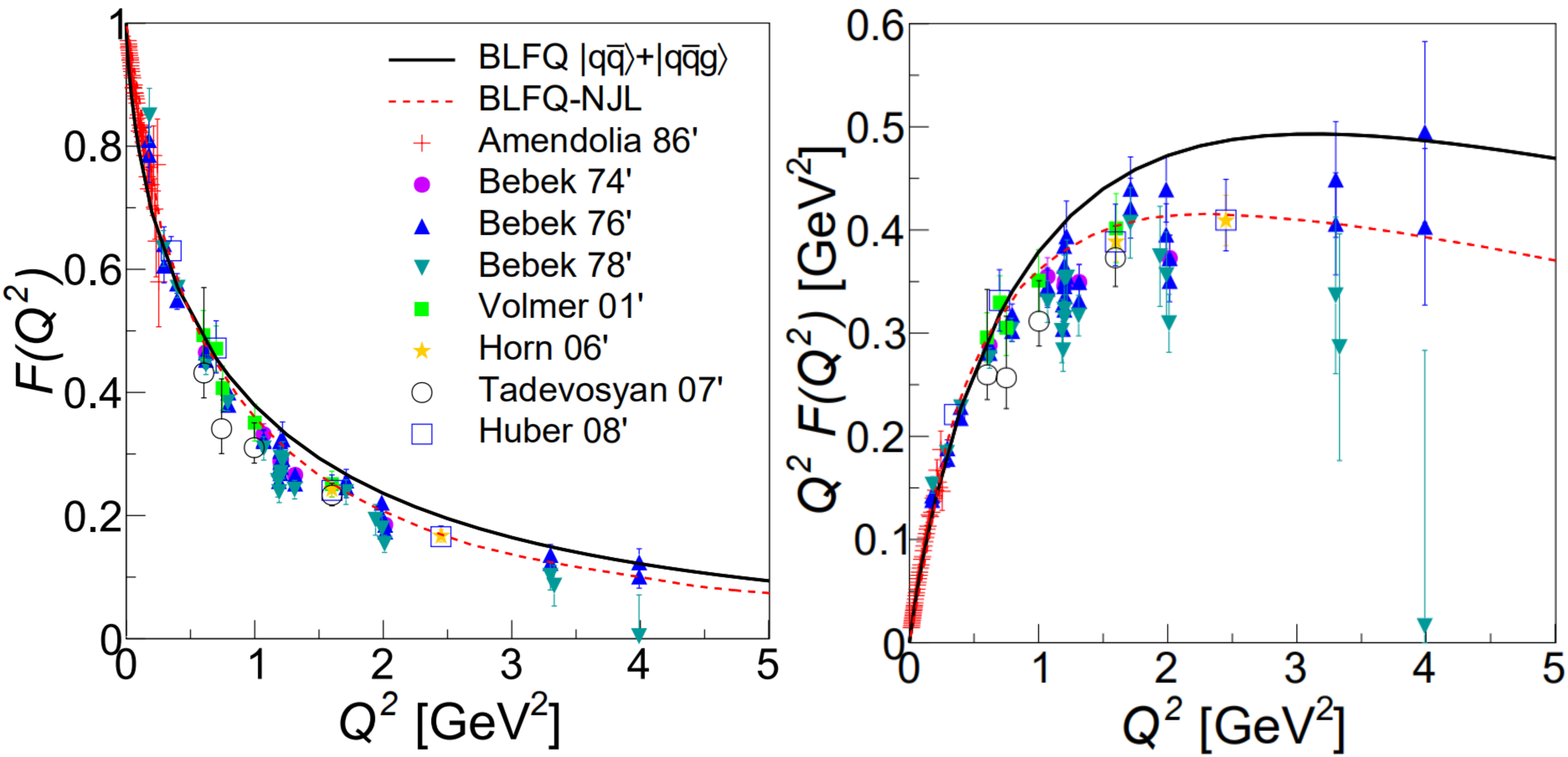
$N_{\max} = 14, K_{\max} = 15, M_J = 0$
$m_q = 0.39$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha_s = 0.293$, $m_f = 5.69$ GeV



Preliminary

[Lan, et al, in preparation]

Pion Electromagnetic Form Factor

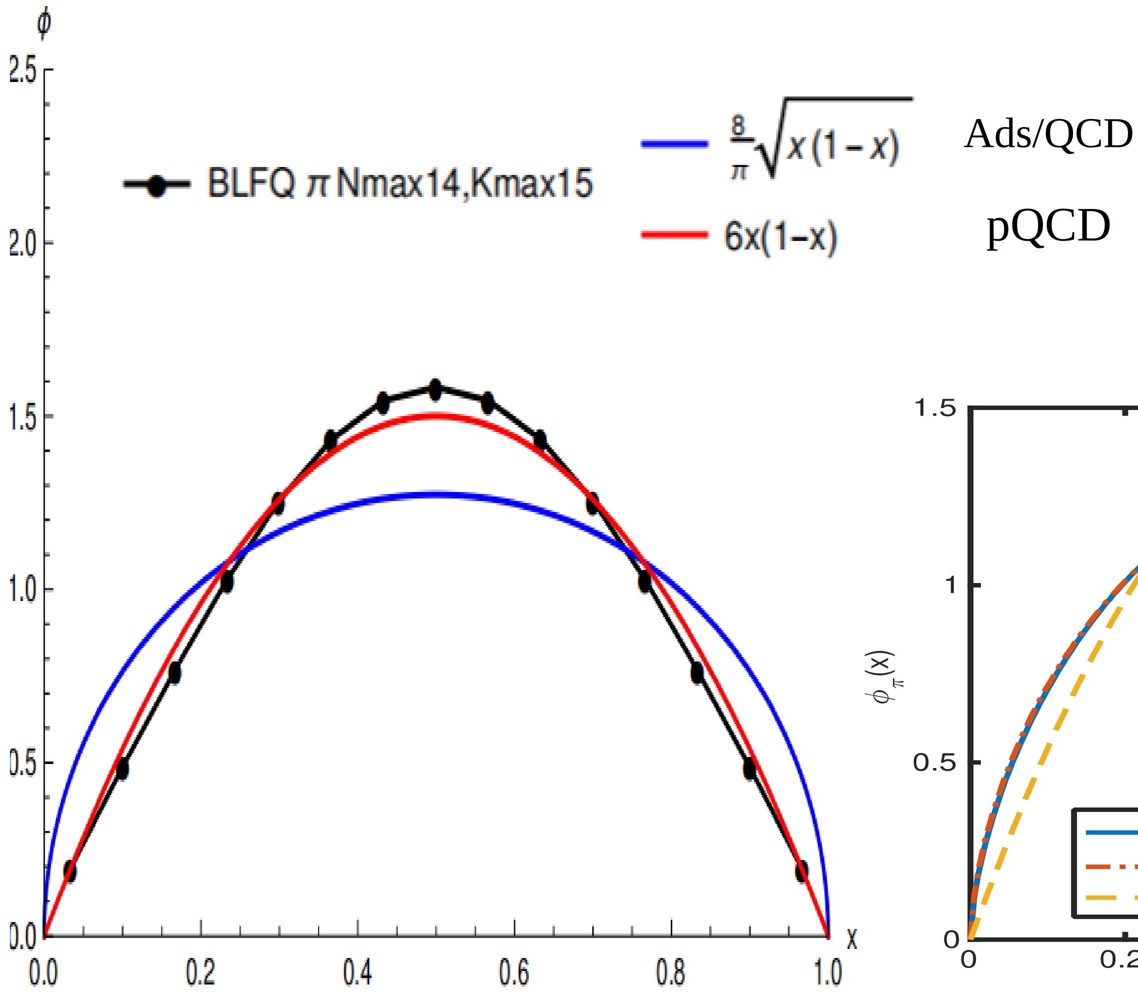


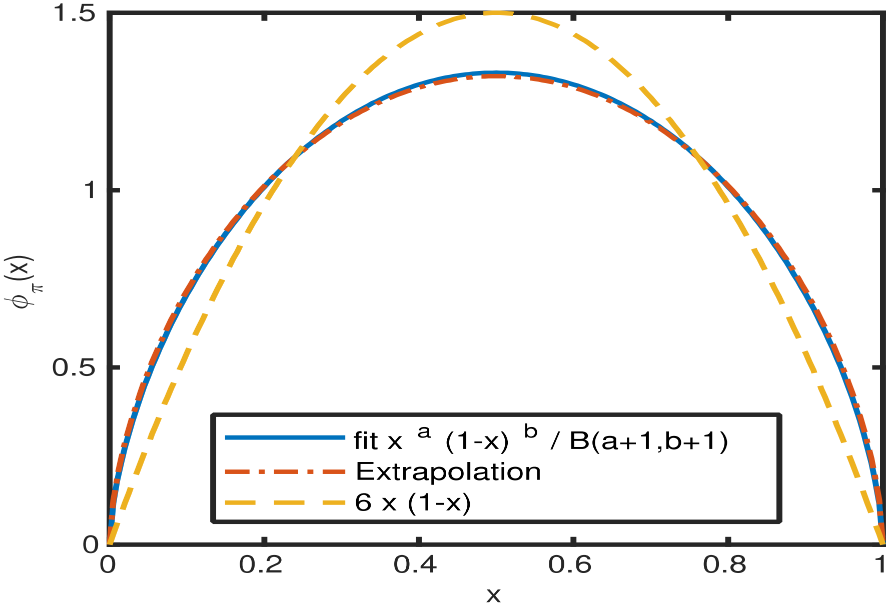
- No parameter fitting for form factor
- $F(Q^2) \propto 1/Q^2$ for large Q^2, consistent with pQCD

Preliminary

[Lan, et al, in preparation]

Pion PDA

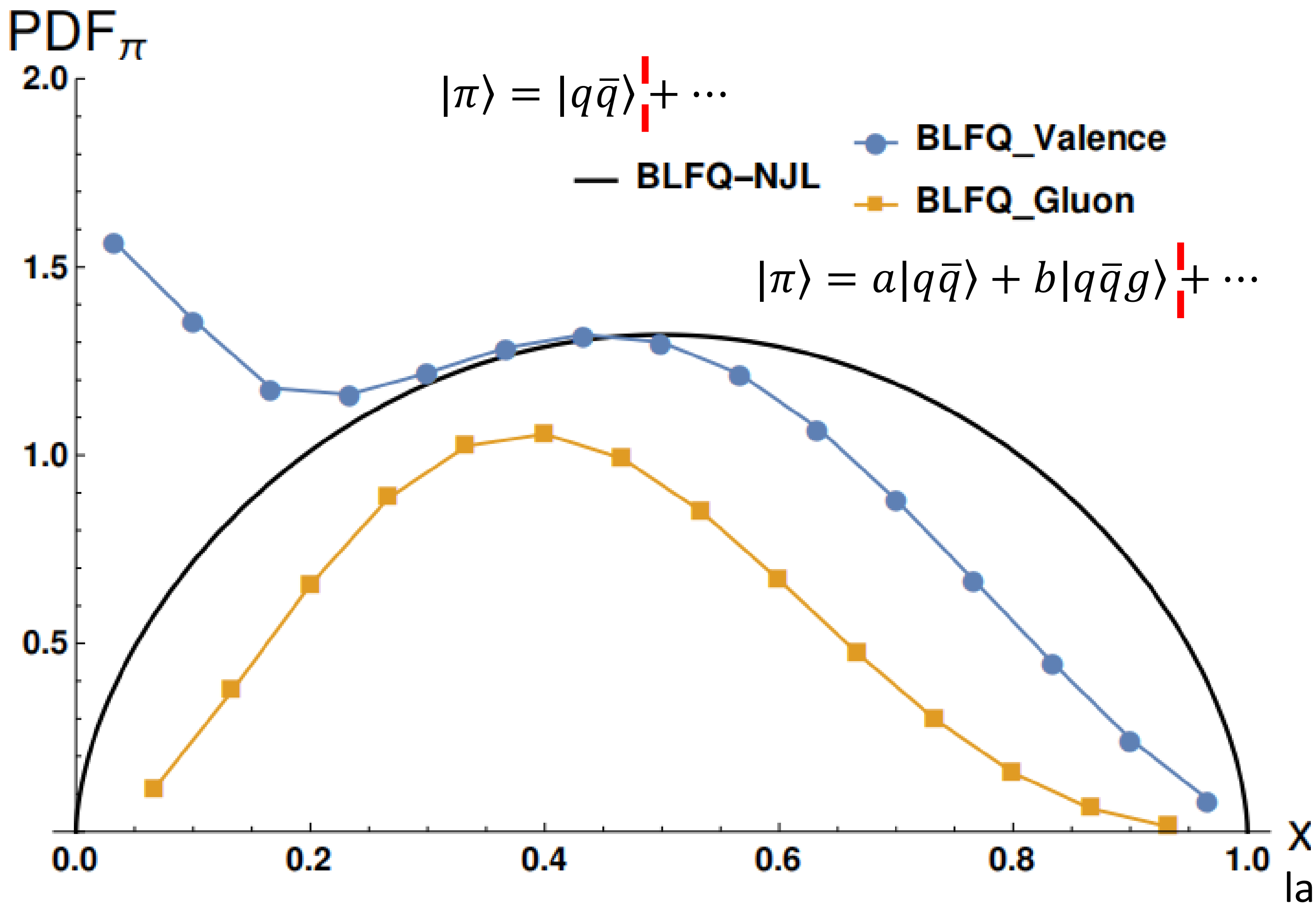




[Jia and Vary, PRC 99, 035206 (2019)]

- Endpoint behavior agrees with pQCD

Pion initial PDF



$\mu^2_{0\mathrm{BLFQ-NJL}} = 0.240\ \mathrm{GeV}^2$ $\quad \langle x\rangle_{\mathbf{gluon}} = \mathbf{0}$; $\quad \langle x\rangle_{\mathbf{valence}\,u} = \mathbf{0.5}$ $\quad (1-x)^{0.596}$

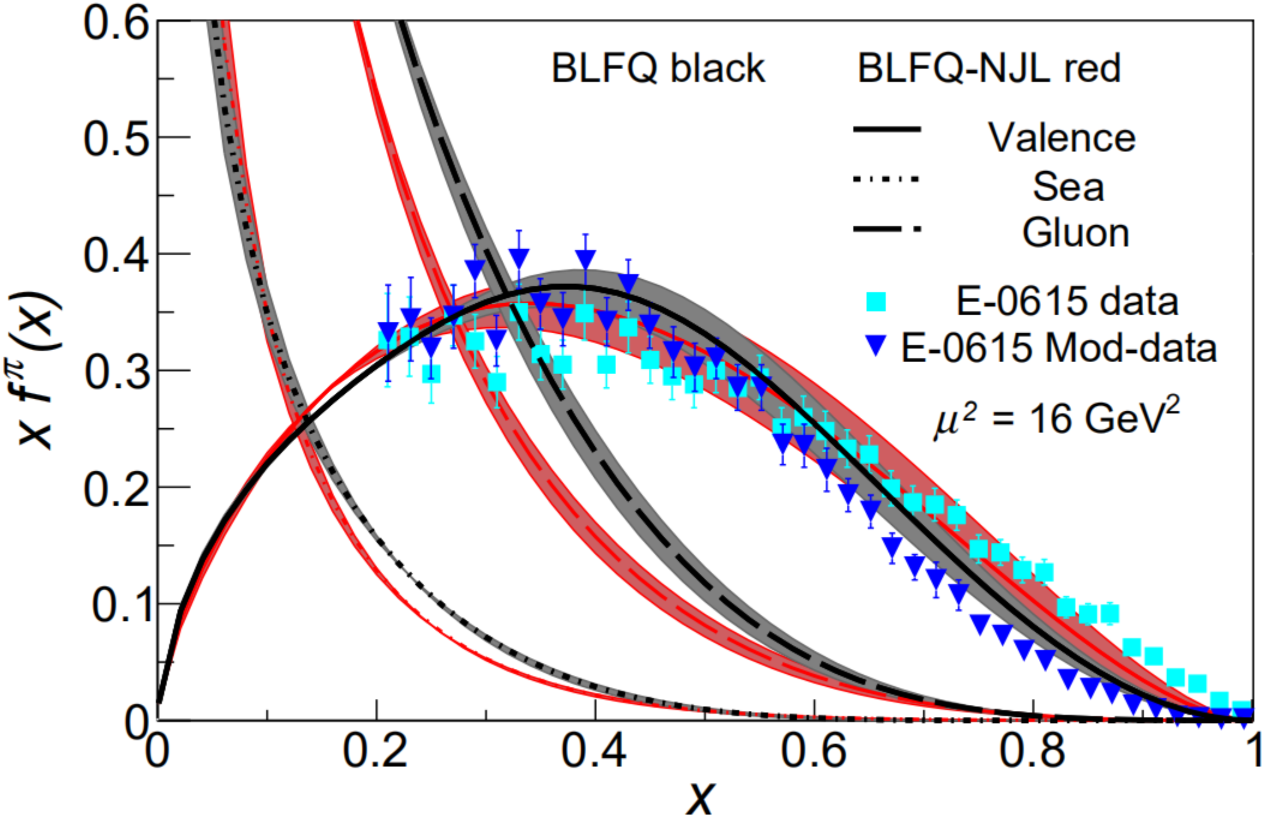
$\mu^2_{0\mathrm{BLFQ}} = 0.34\ \mathrm{GeV}^2$ $\quad \langle x\rangle_{\mathbf{gluon}} = \mathbf{0.216}$; $\quad \langle x\rangle_{\mathbf{valence}\,u} = \mathbf{0.392}$ $\quad (1-x)^{1.4}$

[Lan, et al, in preparation]

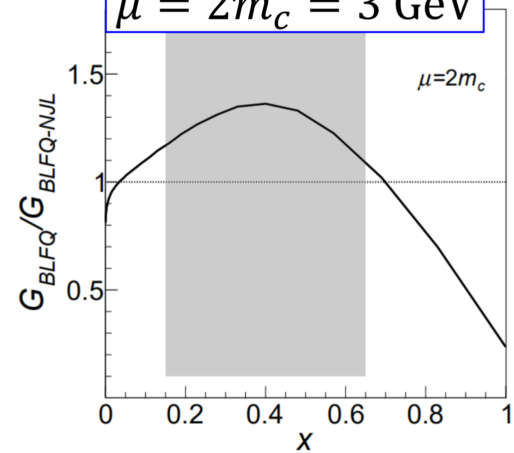
Preliminary

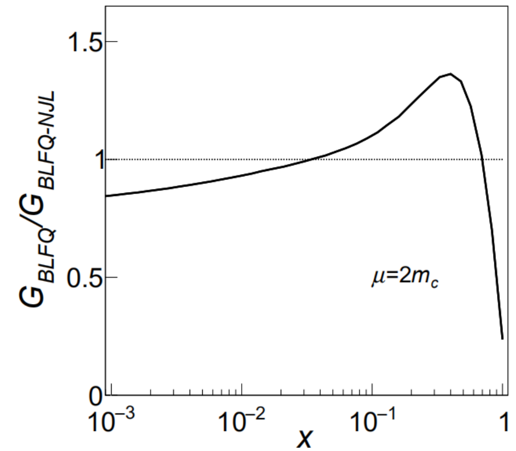
Pion PDF

$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$



$\mu = 2m_c = 3$ GeV





$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ	**0.483**	**0.421**	**0.096**
BLFQ-NJL	0.489	0.398	0.113
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

- Large-x behavior $(1-x)^{1.77}$ closer to pQCD

[Lan, et al, in preparation]

Preliminary

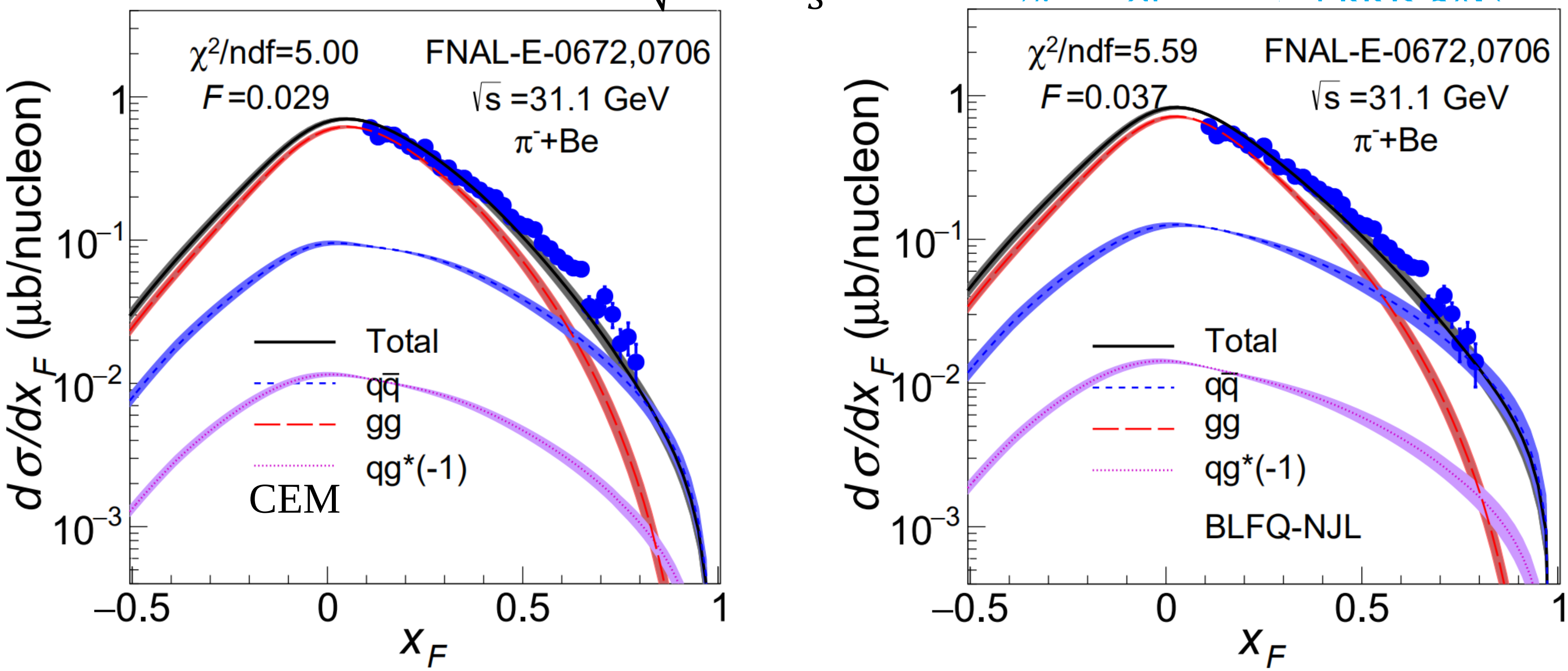
J/ψ production cross section

[Wen-Chen Chang, et al, PRD 102 (2020) 054024];
[P. Nason, et al, NPB 303 (1988) 607];
[M. L. Mangano, et al, NPB 405 (1993) 507]

$$\pi^{\pm} N \to J/\psi \ X$$

[nCTEQ 2015]

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{s\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{s}}} \hat{\sigma}_{ij}(s, M_{c\bar{c}}^2, \mu_R^2) f_i^{\pi^{\pm}}(x_1, \mu_F) f_j^N(x_2, \mu_F)$$



Agree with experimental data (**FNAL E672, E706**).

[Lan, et al, in preparation] Preliminary

J/ψ production cross section

[Wen-Chen Chang, et al, PRD 102 (2020) 054024];
[P. Nason, et al, NPB 303 (1988) 607];
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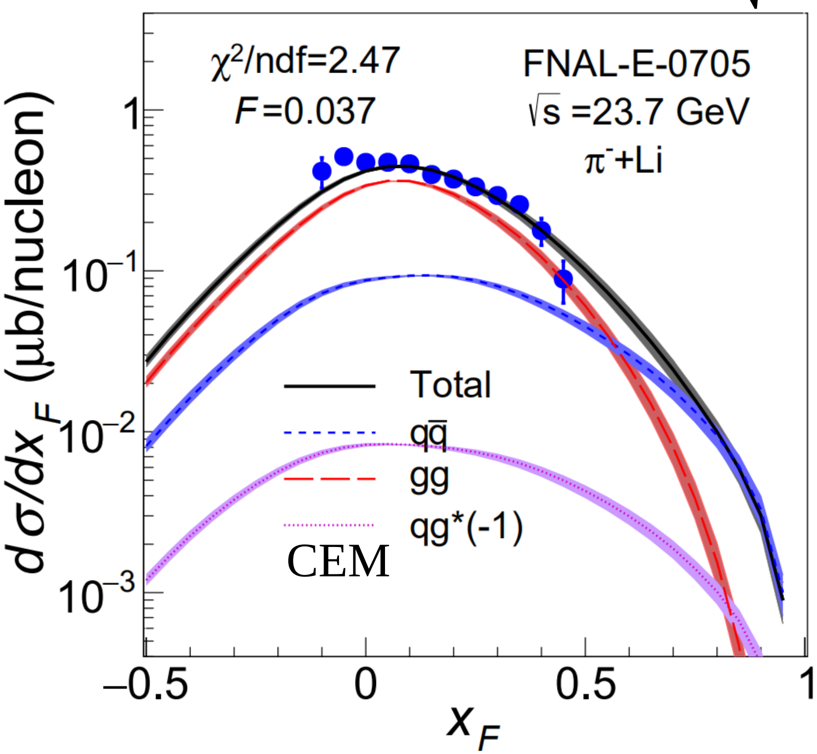

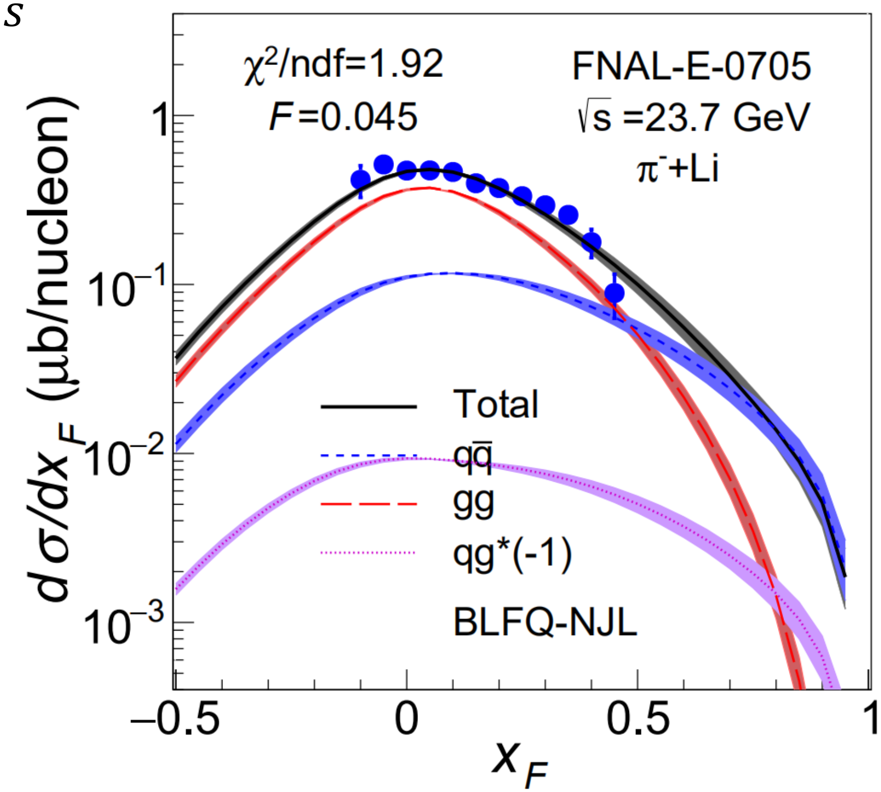

Agree with experimental data (**FNAL E705**).

7%Li6+93%Li7

[Lan, et al, in preparation]

Preliminary

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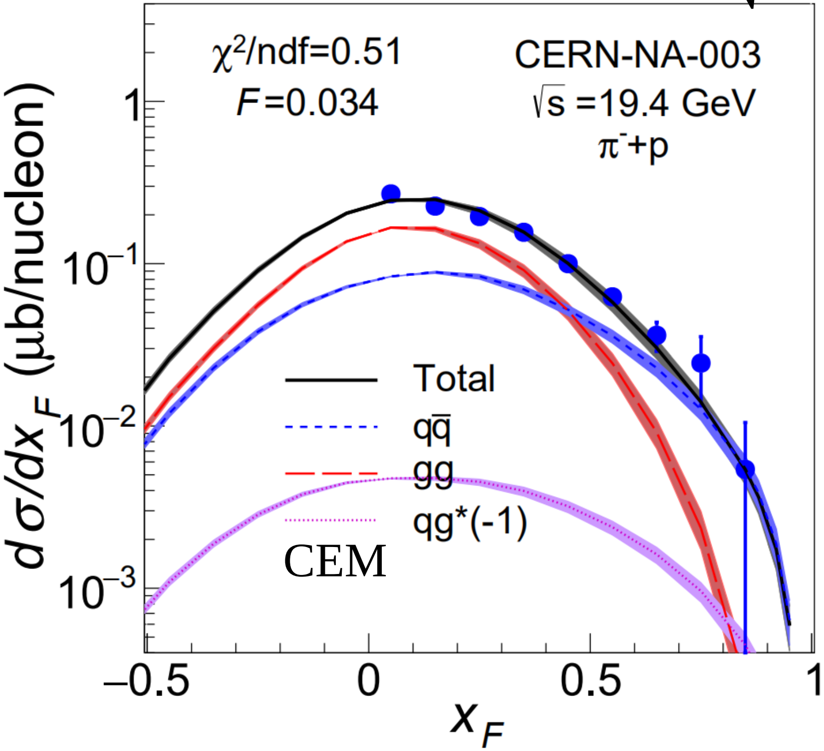

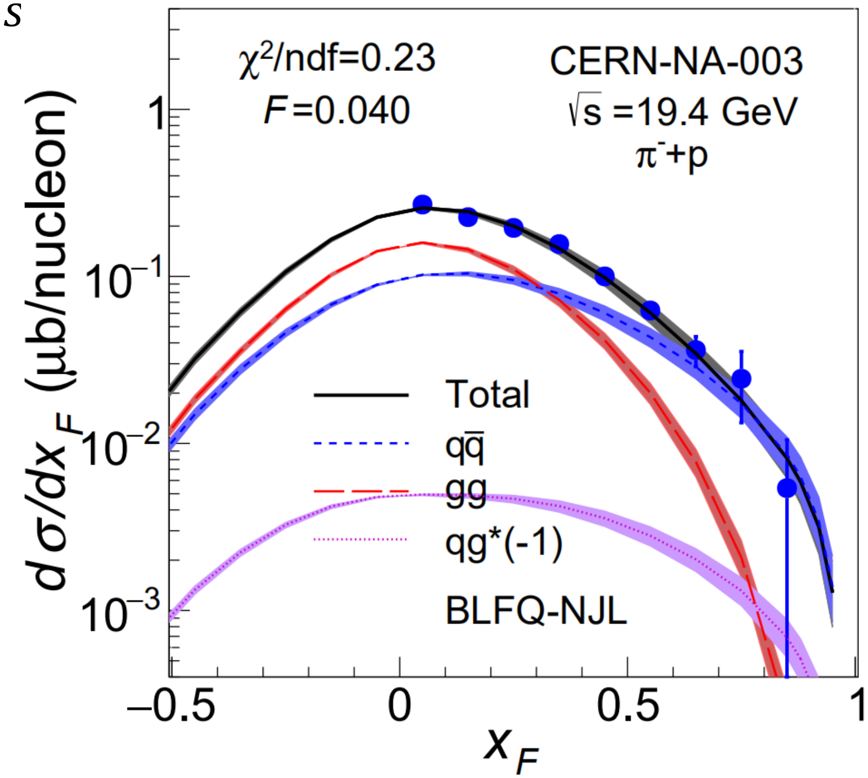

Agree with experimental data (**CERN NA3**).

[Lan, et al, in preparation] Preliminary

J/ψ production cross section

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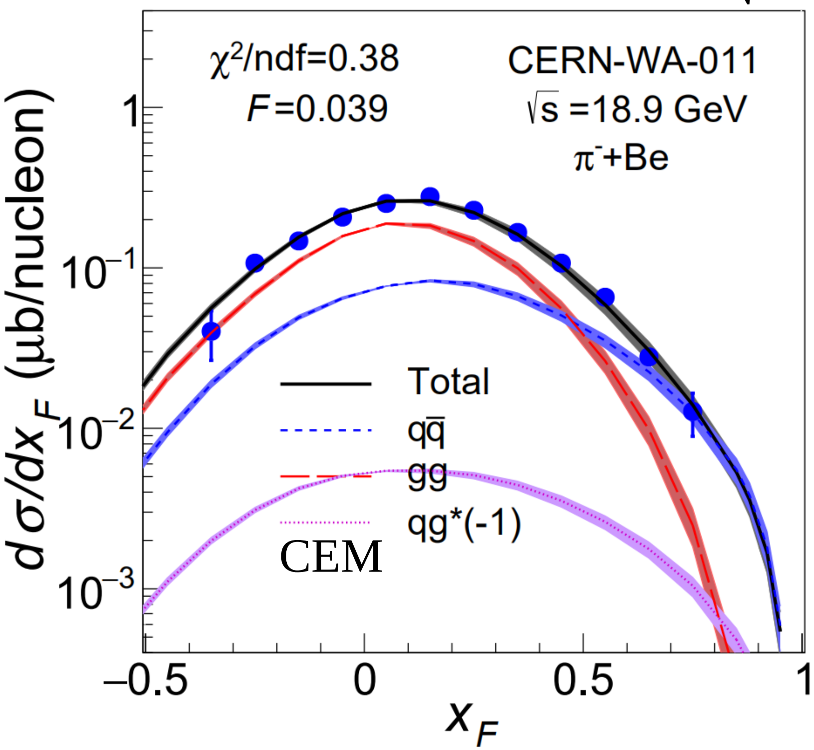

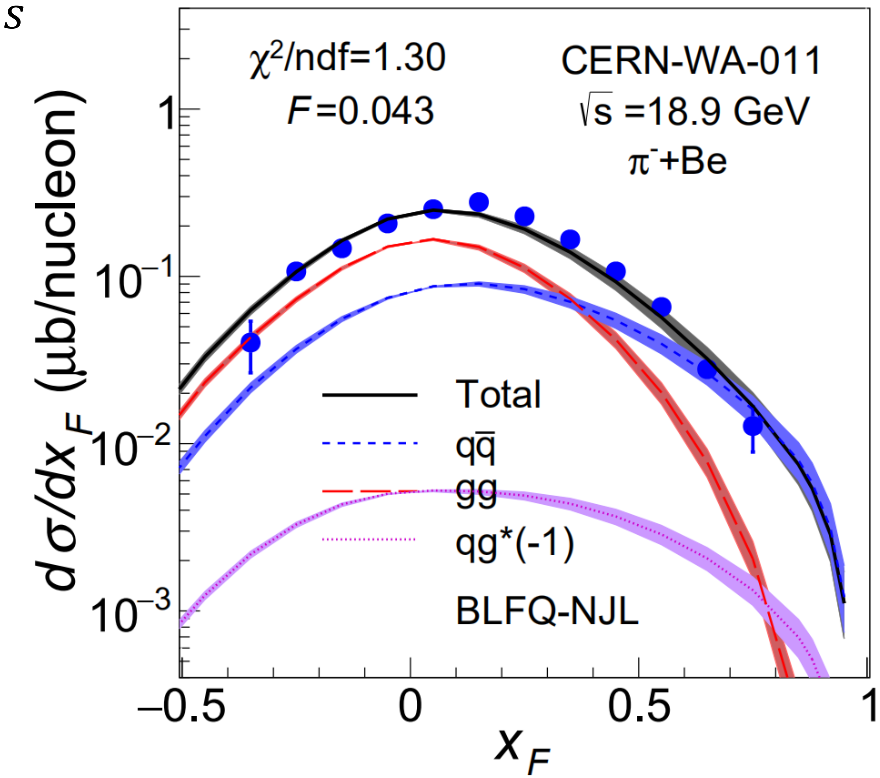

Agree with experimental data (**CERN W11**).

[Lan, et al, in preparation] Preliminary

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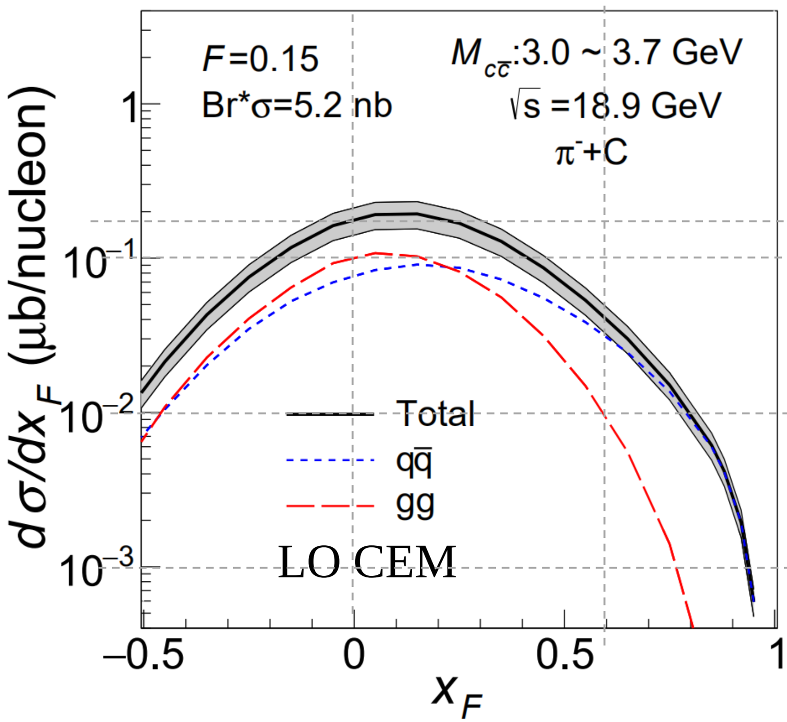
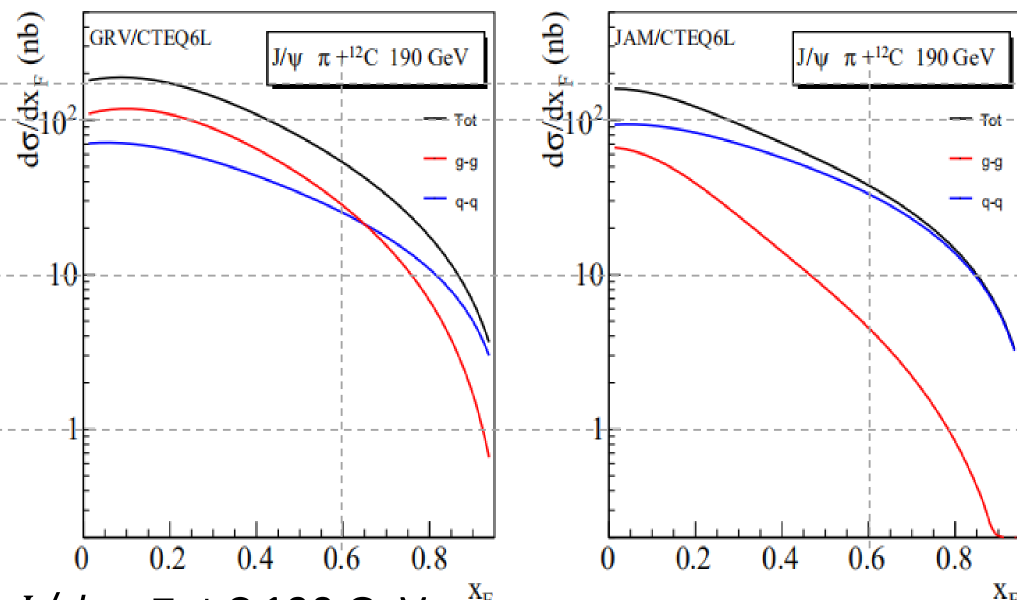


(from COMPASS++/AMBER Proposal Phase-I)



- F: set $Br * \sigma = 5.2$ nb from J/ψ, π^- +C 190 GeV
- Band: F with ±20%

[Lan, et al, in preparation]

J/ψ production cross section

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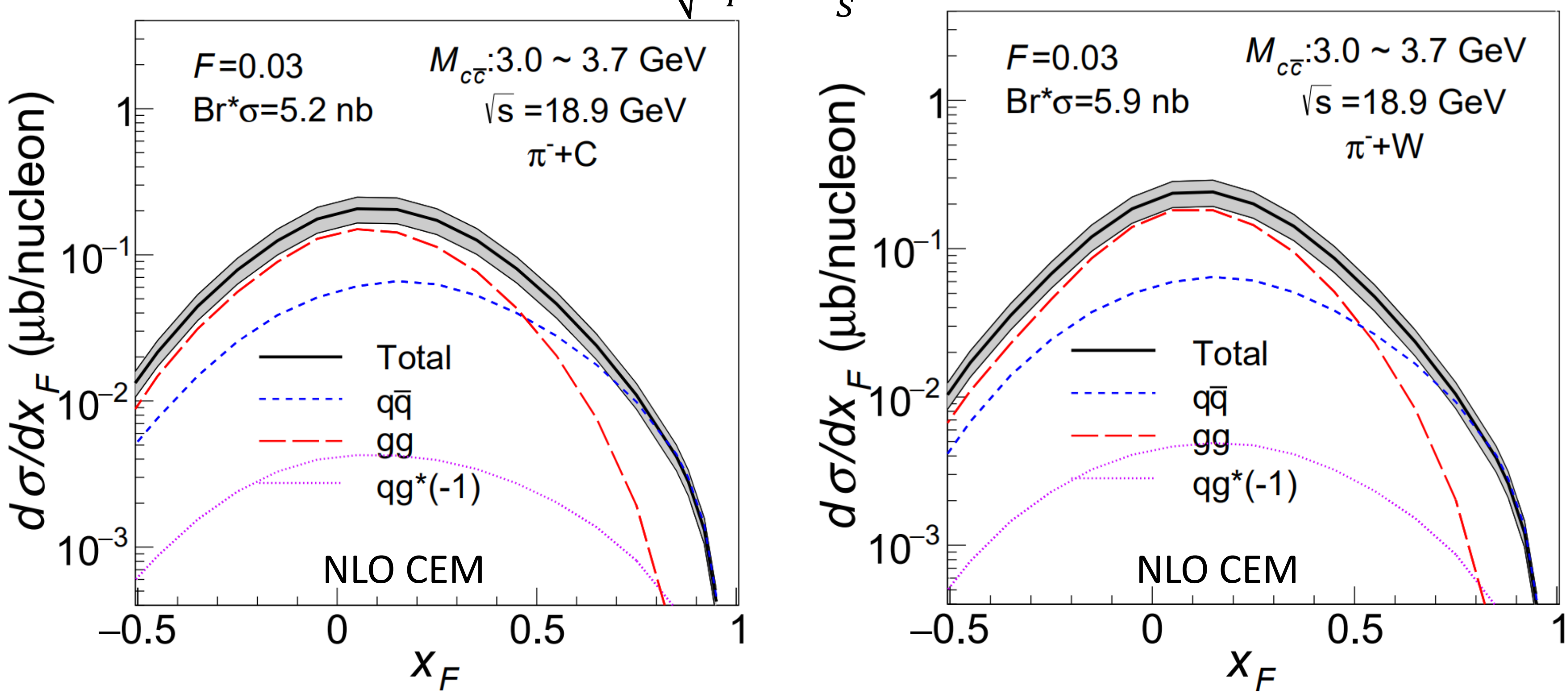
[nCTEQ 2015]

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{s\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{s}}} \hat{\sigma}_{ij}(s, M_{c\bar{c}}^2, \mu_R^2) f_i^{\pi^{\pm}}(x_1, \mu_F) f_j^N(x_2, \mu_F)$$



- F: the same of F from J/ψ, π^- +C 190 GeV
- Band: F with ±20%

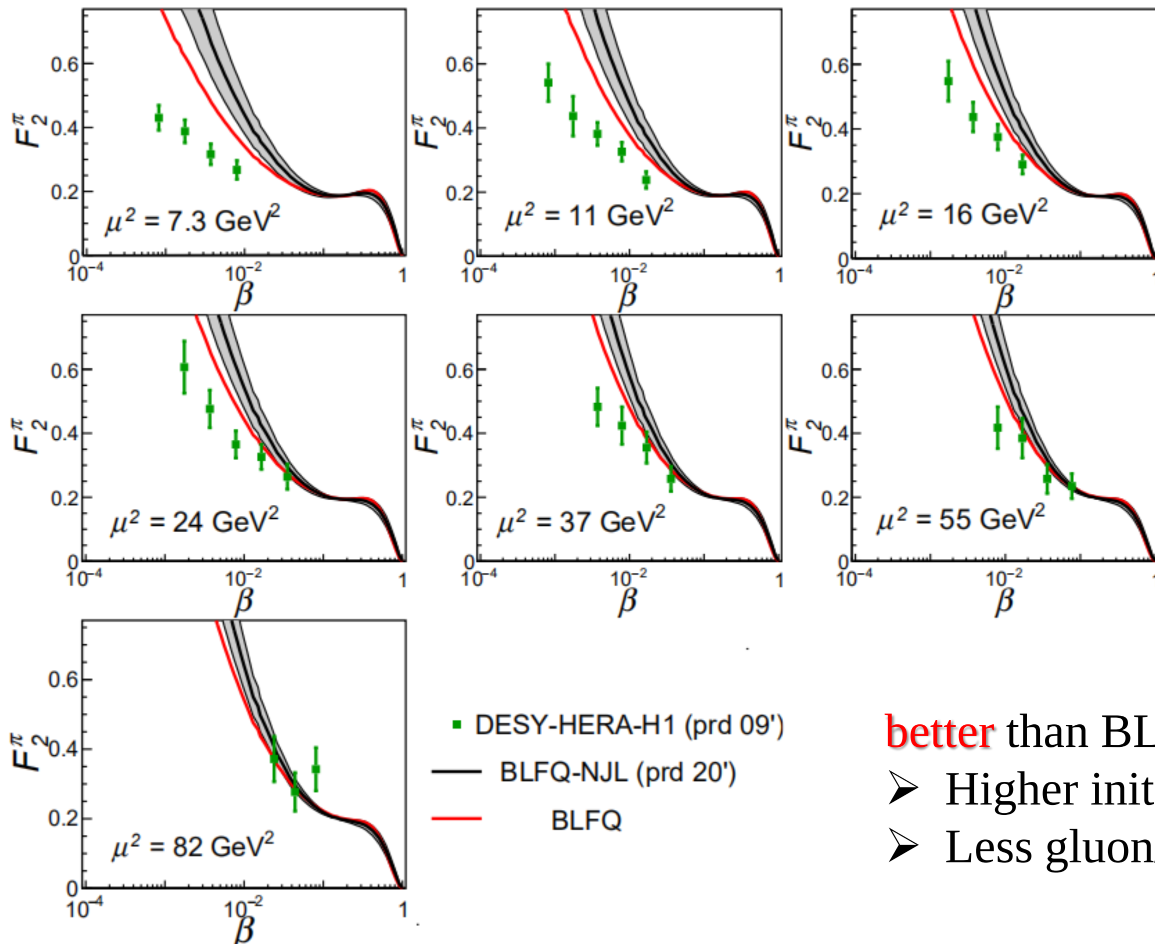
[Lan, et al, in preparation]

Pion Structure function

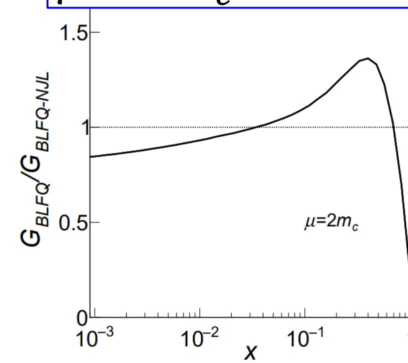
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$F_2^{\pi}(\beta,\mu^2) = \sum_{q,g} e_q^2 \beta\{f_q^{\pi}(\beta,\mu^2) + f_{\bar{q}}^{\pi}(\beta,\mu^2) + \frac{\alpha_s(\mu^2)}{2\pi}[C_{q,2}\otimes(f_q^{\pi} + f_{\bar{q}}^{\pi}) + 2C_{g,2}\otimes f_g^{\pi}]\}$$



$\mu = 2m_c = 3$ GeV



better than BLFQ-NJL results

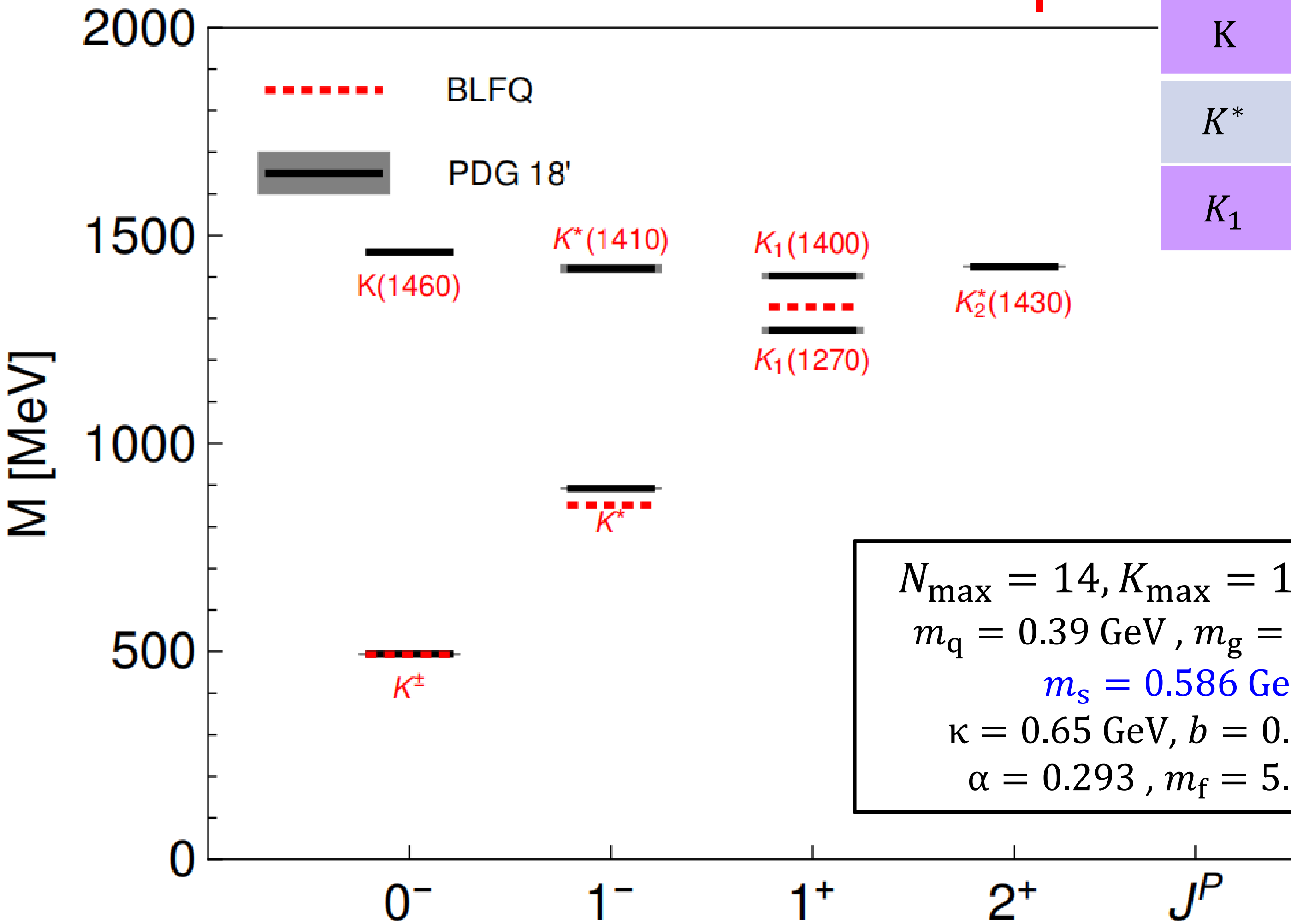
- Higher initial scale
- Less gluon/sea quark at small x

in future $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \cdots$

Kaon Spectrum

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$

	Norm1	DC[MeV]
K	0.588	151
K^*	0.602	142
K_1	0.419	13

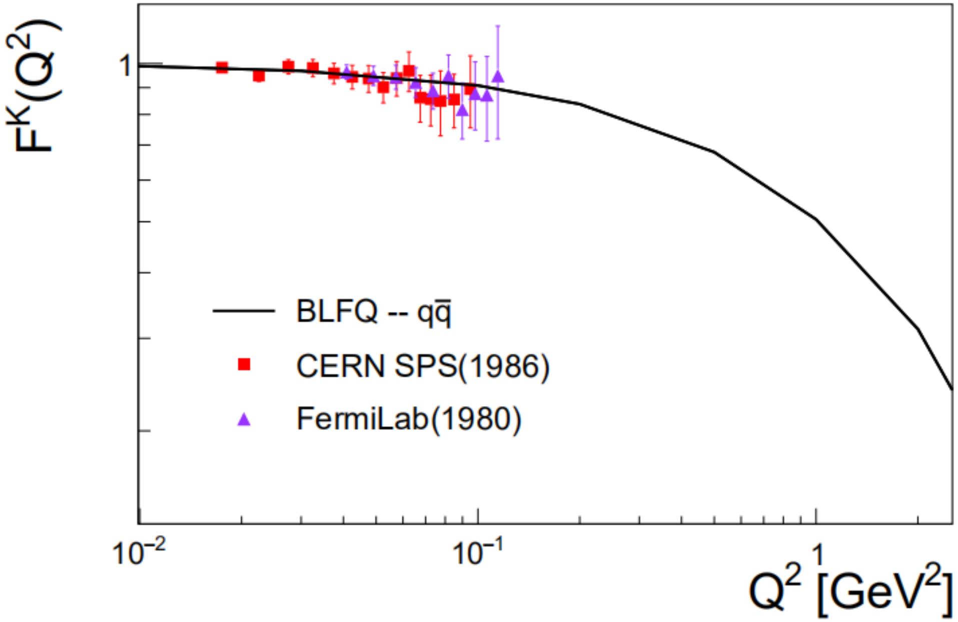


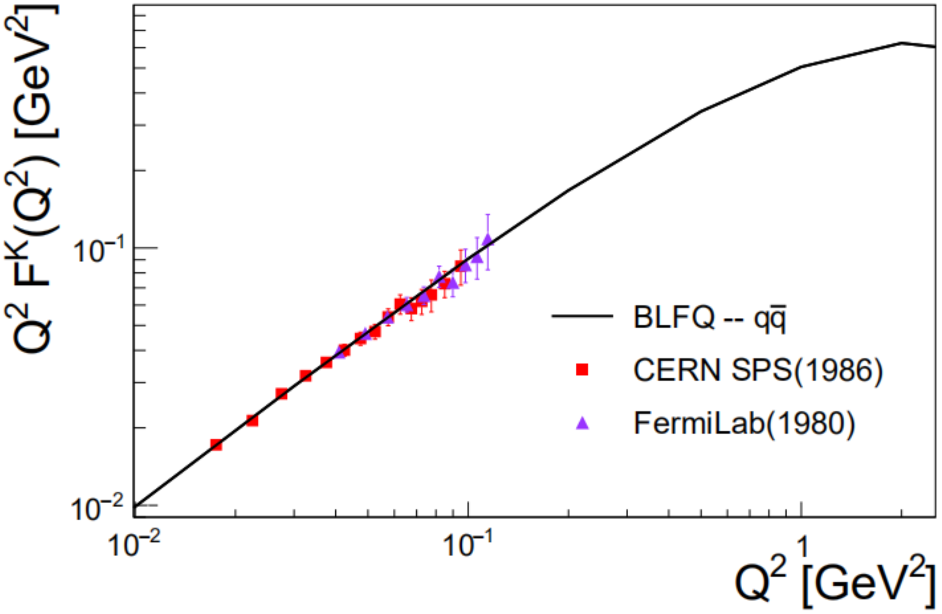
$N_{\max} = 14, K_{\max} = 15, M_J = 0$
$m_q = 0.39$ GeV , $m_g = 0.60$ GeV,
$m_s = 0.586$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

Preliminary

Kaon Form Factor

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

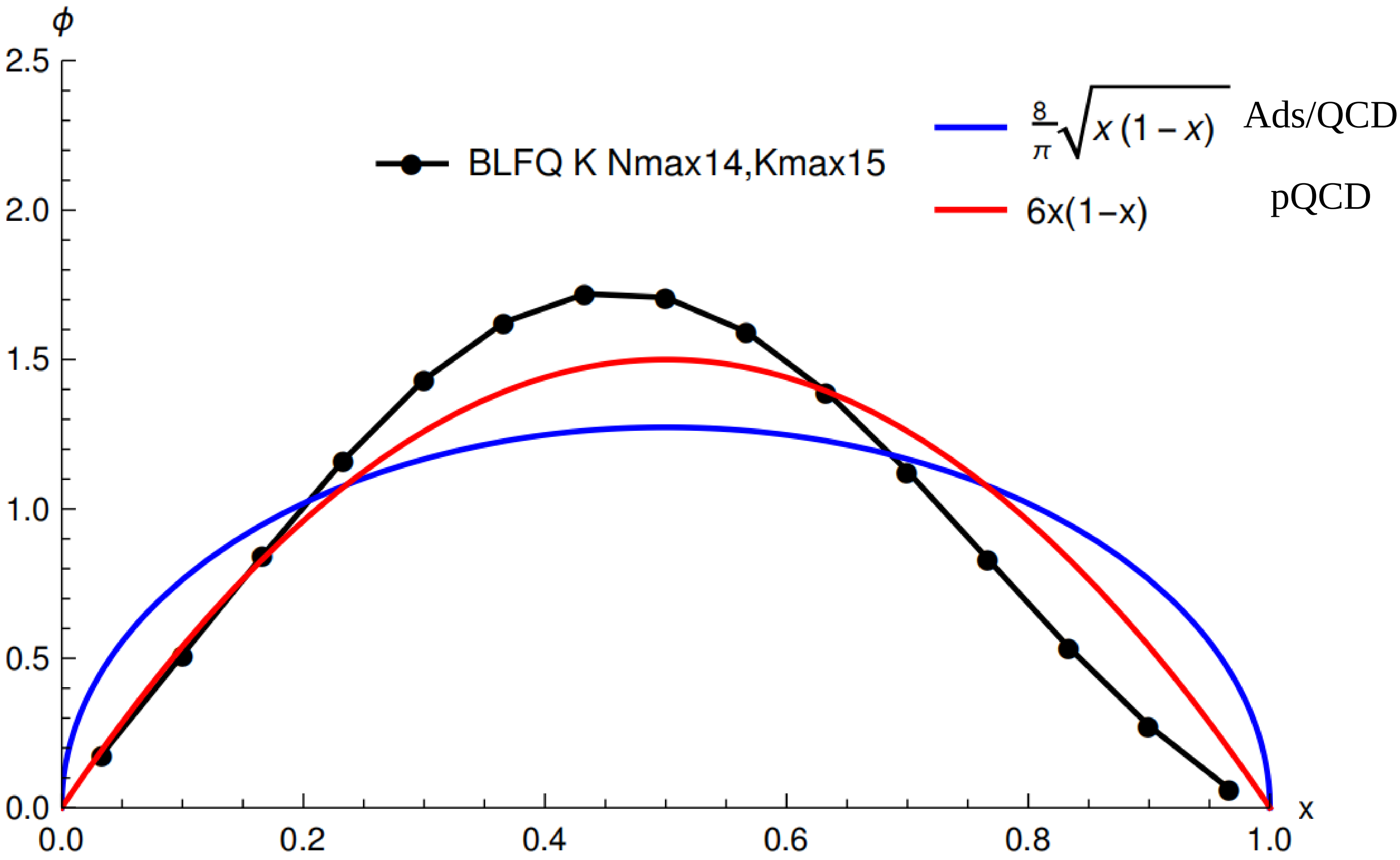




Preliminary: based on leading Fock Sector WF

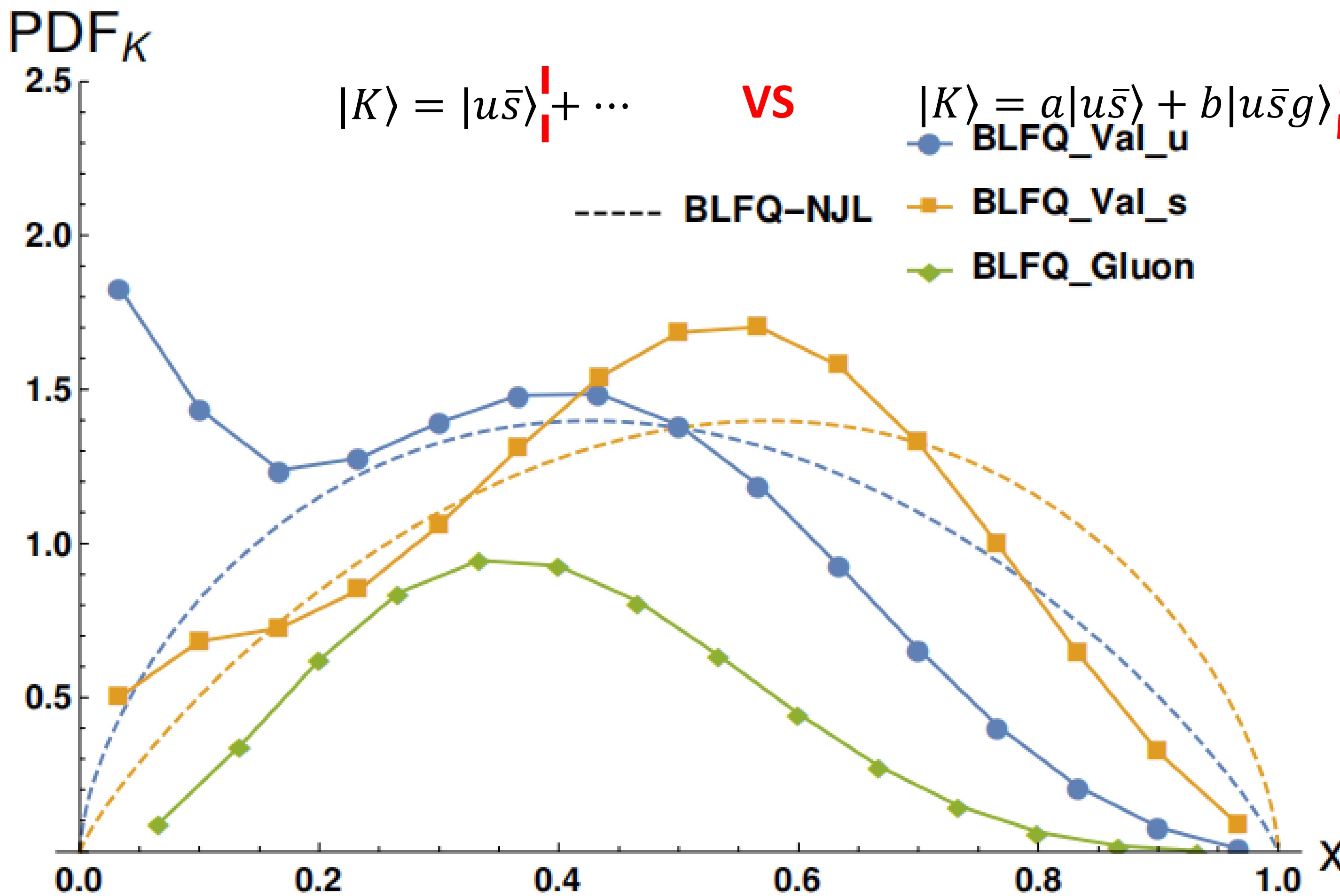
[Lan, et al, in progress]

Kaon PDA



Preliminary

Kaon initial PDF



u large x

$\mu^2_{0\text{BLFQ-NJL}} = 0.247\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = \mathbf{0}$; $\langle x\rangle_{\text{valence } u} = \mathbf{0.468}$; $\langle x\rangle_{\text{valence } s} = \mathbf{0.532}$ $(\mathbf{1}-x)^{\mathbf{0.8546}}$

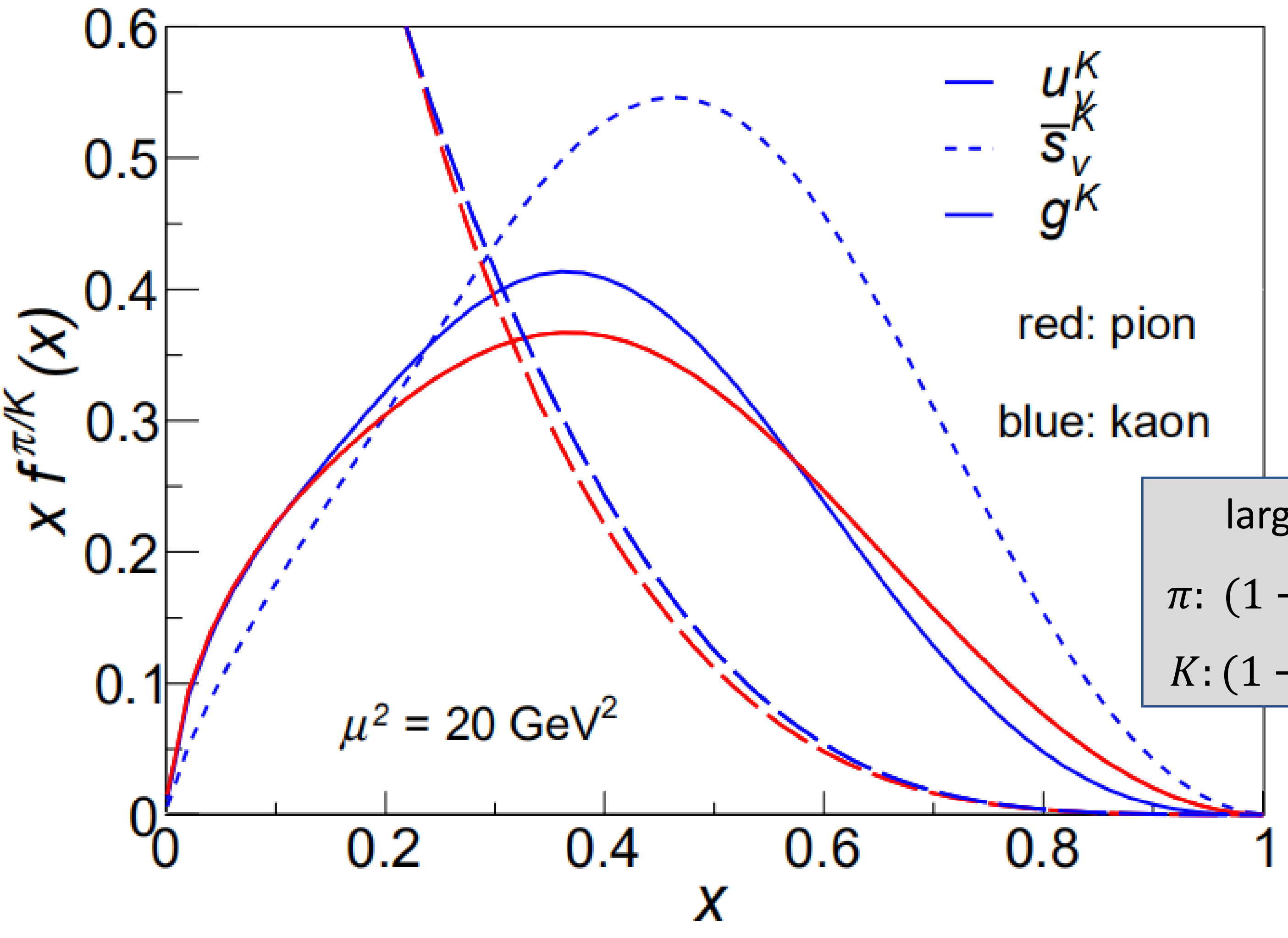
$\mu^2_{0\text{BLFQ}} = 0.47\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = \mathbf{0.162}$; $\langle x\rangle_{\text{valence } u} = \mathbf{0.353}$; $\langle x\rangle_{\text{valence } s} = \mathbf{0.485}$ $(\mathbf{1}-x)^{\mathbf{1.92}}$

[Lan, et al, in progress]

Preliminary

Kaon PDF

$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



large x

π: $(1-x)^{1.80}$

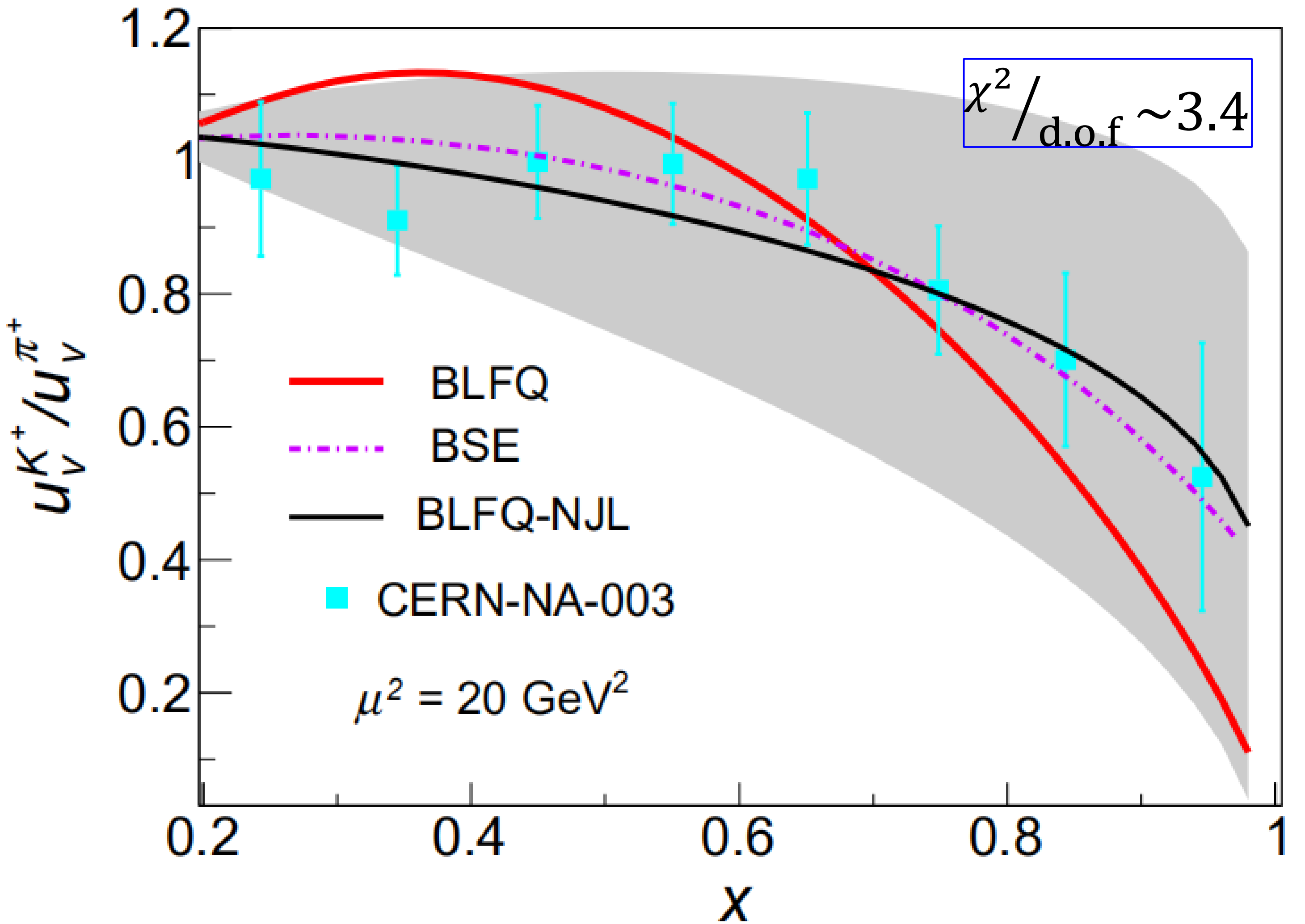
K: $(1-x)^{2.46}$

Preliminary

[Lan, et al, in progress]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **vs** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$

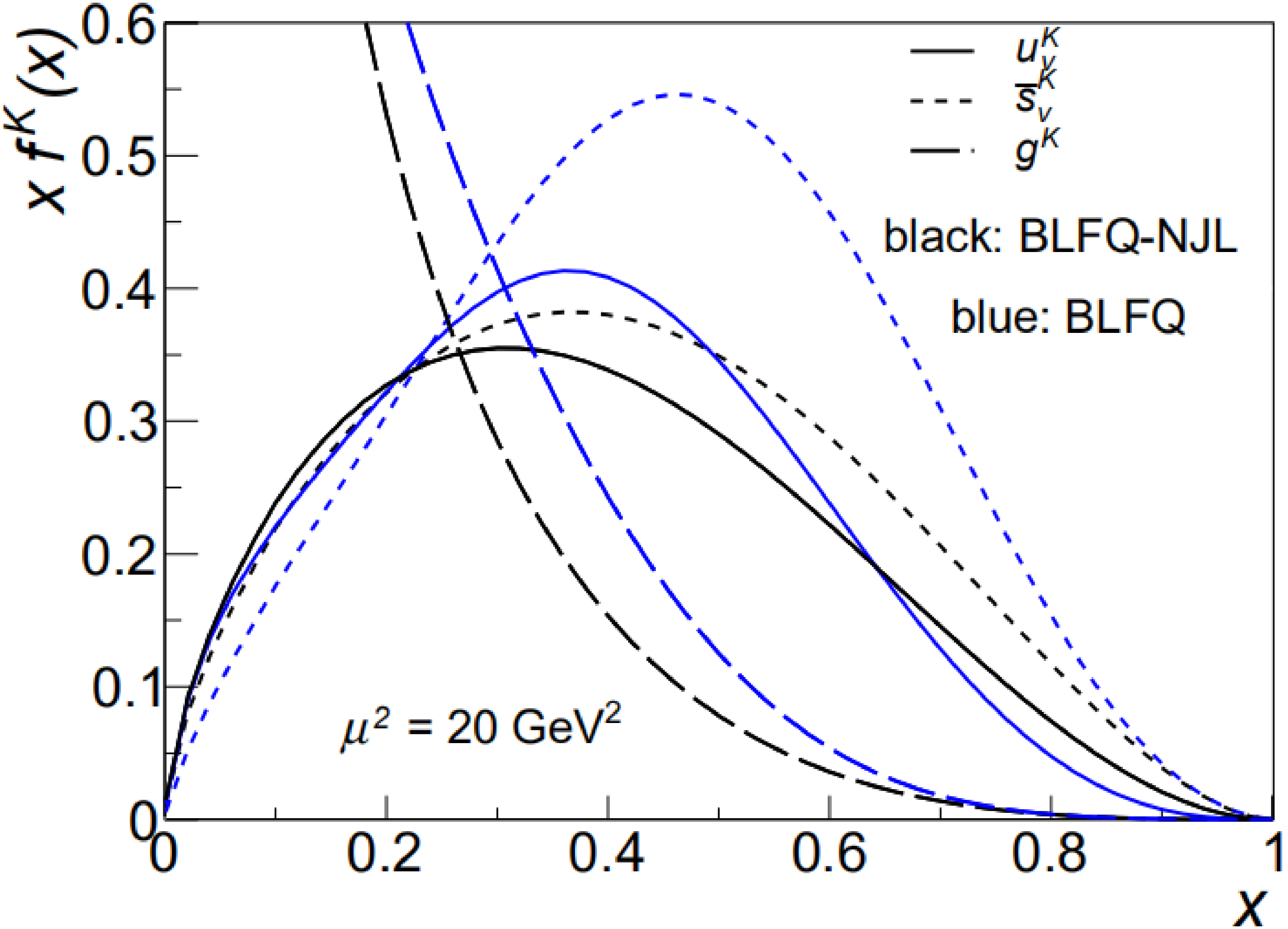


Preliminary

[Lan, et al, in progress]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



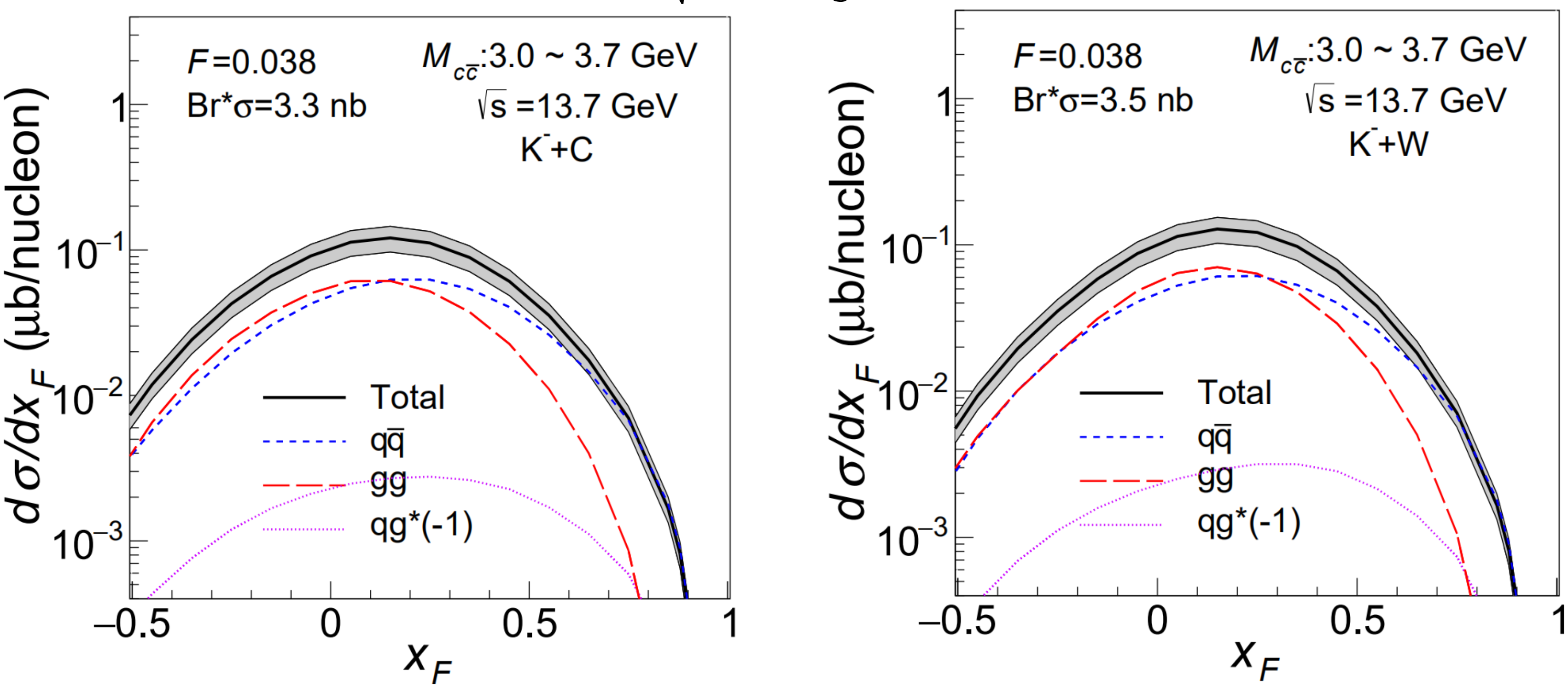
[Lan, et al, in progress]

J/ψ production cross section

[Wen-Chen Chang, et al, PRD 102 (2020) 054024];
[P. Nason, et al, NPB 303 (1988) 607];
[M. L. Mangano, et al, NPB 405 (1993) 507]

$K^{\pm}\ C(W) \rightarrow J/\psi\ X$ [nCTEQ 2015]

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{s\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{s}}} \hat{\sigma}_{ij}(s, M_{c\bar{c}}^2, \mu_R^2) f_i^{K^{\pm}}(x_1, \mu_F) f_j^N(x_2, \mu_F)$$

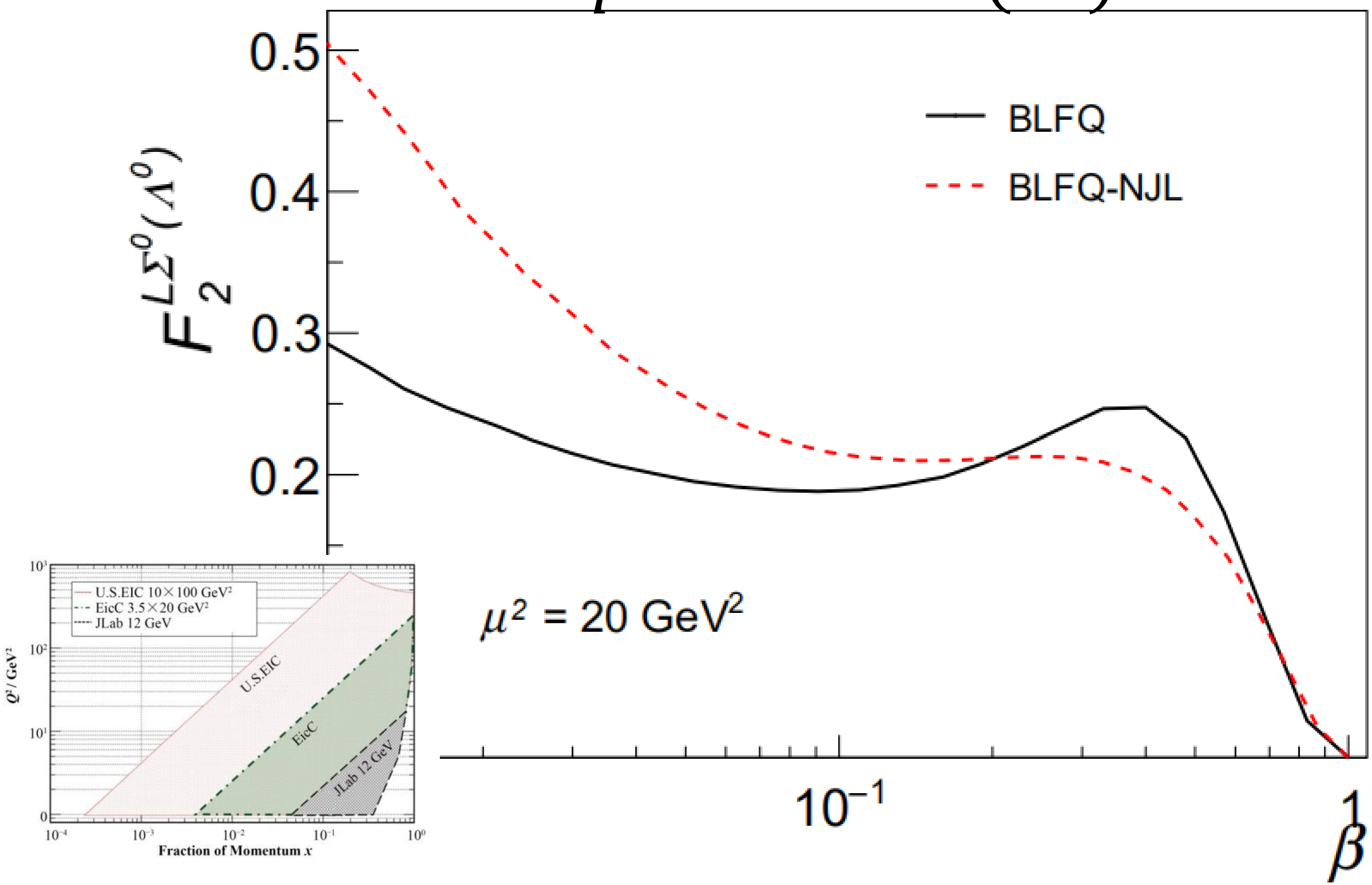


- F: set $Br * \sigma = 3.3$ nb from J/ψ, K^- +C 100 GeV
- Band: F with ±20%

[Lan, et al, in progress]

Kaon Structure function

$$ep \longrightarrow e'\, X\, \Sigma^0(\Lambda^0)$$



EIC ?

- Difference in structure function at small x

Conclusions

- Lightfront Hamiltonian approach: mass spectrum ⟺ structure
- Compared to NJL interaction, dynamical gluon in light meson:
  - ✓ Explains the properties of exotic state $\pi_1(1400)$
  - ✓ Improves endpoint behavior in PDF/PDA
  - ✓ Generates more gluon at moderate x/less gluon at small x
  - ✓ Improves π structure function at small x

- Systematically expandable by including higher Fock sectors

$$|\mathrm{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \cdots$$

Thank you !

Questions/suggestions: xbzhao@impcas.ac.cn