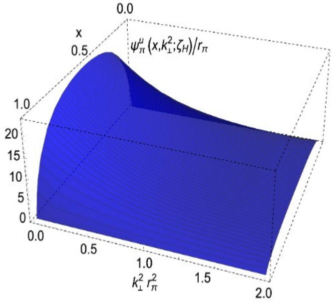
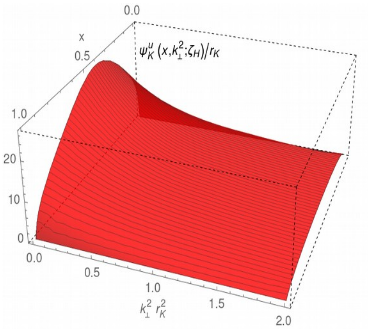
Instituto de Ciencias Nucleares UNAM

EHM via meson light-front wavefunctions

Khépani Raya Montaño





Lei Chang

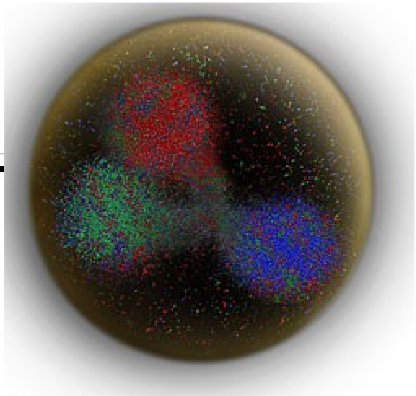
Craig D. Roberts

José Rodríguez Quintero...

Perceiving the EHM through AMBER@CERN

April 27 – April 30, 2021. CERN - Switzerland (online)

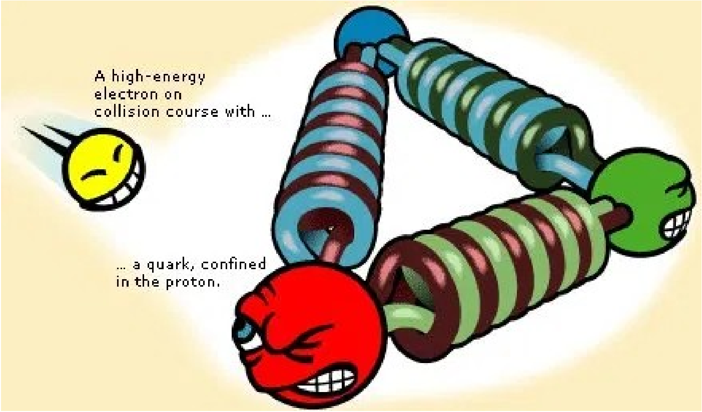
QCD and hadron physics

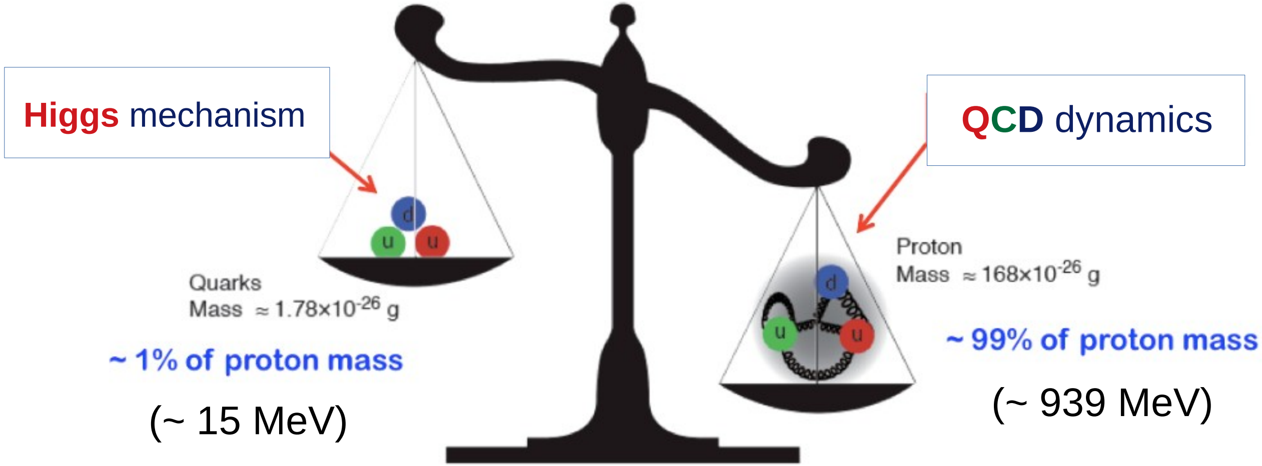


- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"

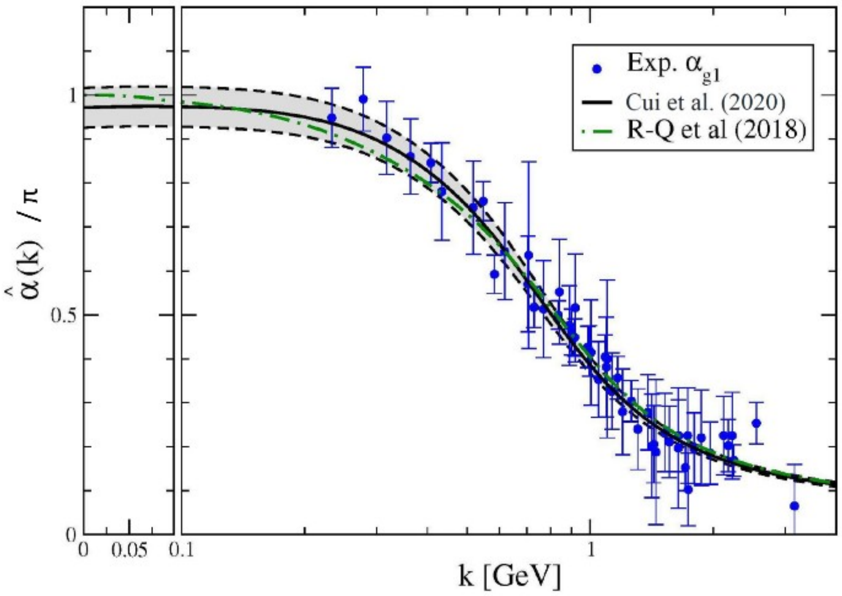
- Emergence of hadron masses (EHM) from QCD **dynamics**





QCD and hadron physics

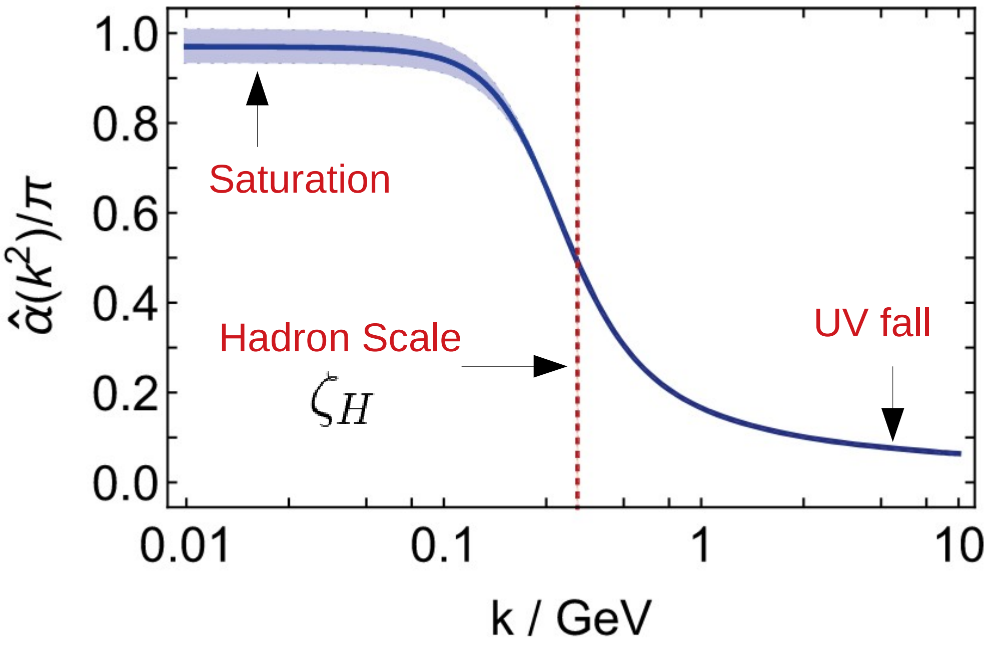
- These phenomena are tightly connected with **QCD's** peculiar **running coupling**.



Modern picture of **QCD** coupling.

Cui:2019dwv

J. R-Q's talk

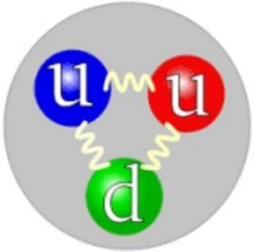


ζ_H: Fully **dressed valence** quarks express all hadron's properties

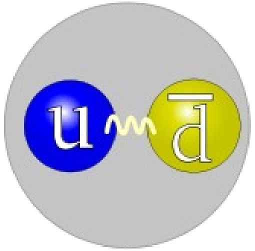
QCD and hadron physics

➤ **Pions** and **Kaons** emerge as QCD's (pseudo)-**Goldstone** bosons.

➔ Their study is crucial to understand the **EHM** and the ***hadron structure***.



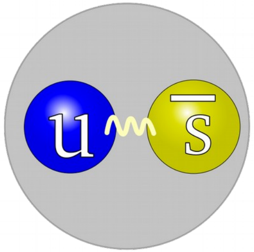
$m_p \approx 0.940\ \text{GeV}$



$m_\pi \approx 0.140\ \text{GeV}$

- Dominated by **QCD** dynamics

  Simultaneously explains the mass of the proton and the *masslessness* of the pion



$m_K \approx 0.490\ \text{GeV}$

'Higgs' masses

$m_{u/d} \approx 0.004\ \text{GeV}$

$m_s \approx 0.095\ \text{GeV}$

- Interplay between **Higgs** and **strong** mass generating mechanisms.

Light-front wave function (LFWF)

“One ring to rule them all”

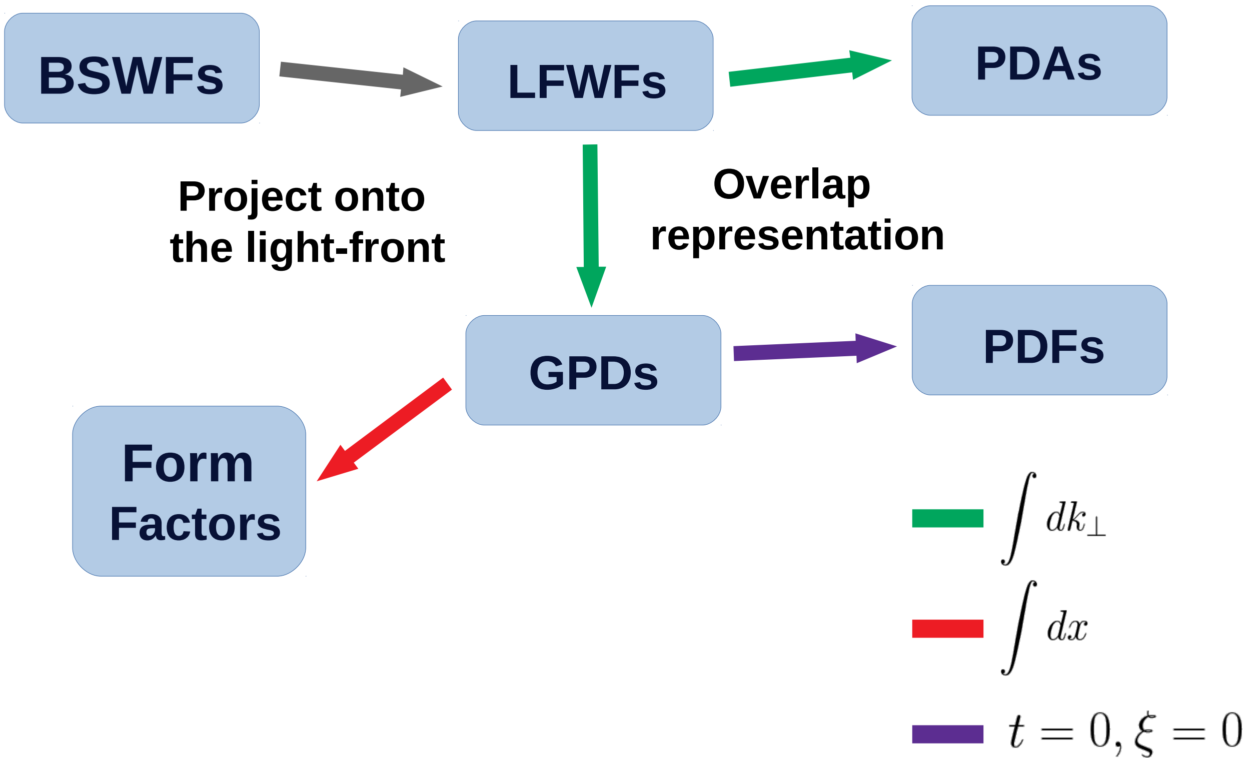
$$\psi_{\rm M}^{q}\left(x,k_{\perp}^{2}\right)=\mathrm{tr}\int_{dk_{\parallel}}\delta_{n}^{x}(k_{\rm M})\gamma_{5}\gamma\cdot n\,\chi_{\rm M}(k_{-},P)$$

Bethe-Salpeter wave function

- Yields a **variety** of **distributions**.

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and kaon structure.



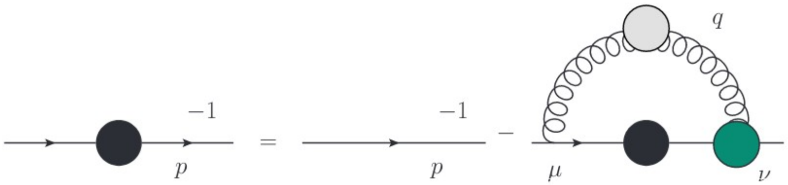
The idea:
Compute ***everything*** from the **LFWF.**

LFWF approach

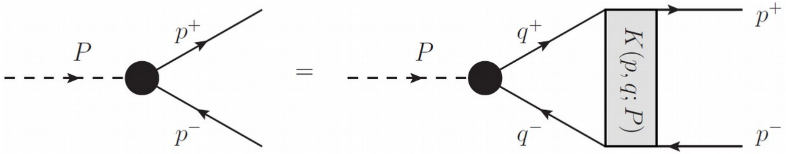
$$\psi^q_{\mathrm{M}}\left(x,k_\perp^2\right) = \mathrm{tr}\int_{dk_\parallel} \delta_n^x(k_{\mathrm{M}})\gamma_5\gamma\cdot n\,\chi_{\mathrm{M}}(k_-,P)$$

- **Goal:** get a **broad picture** of the pion and kaon structure.

BSWFs → **LFWFs**



Quark DSE



Meson BSE

Truncation required!

The idea:
Compute ***everything*** from the **LFWF.**

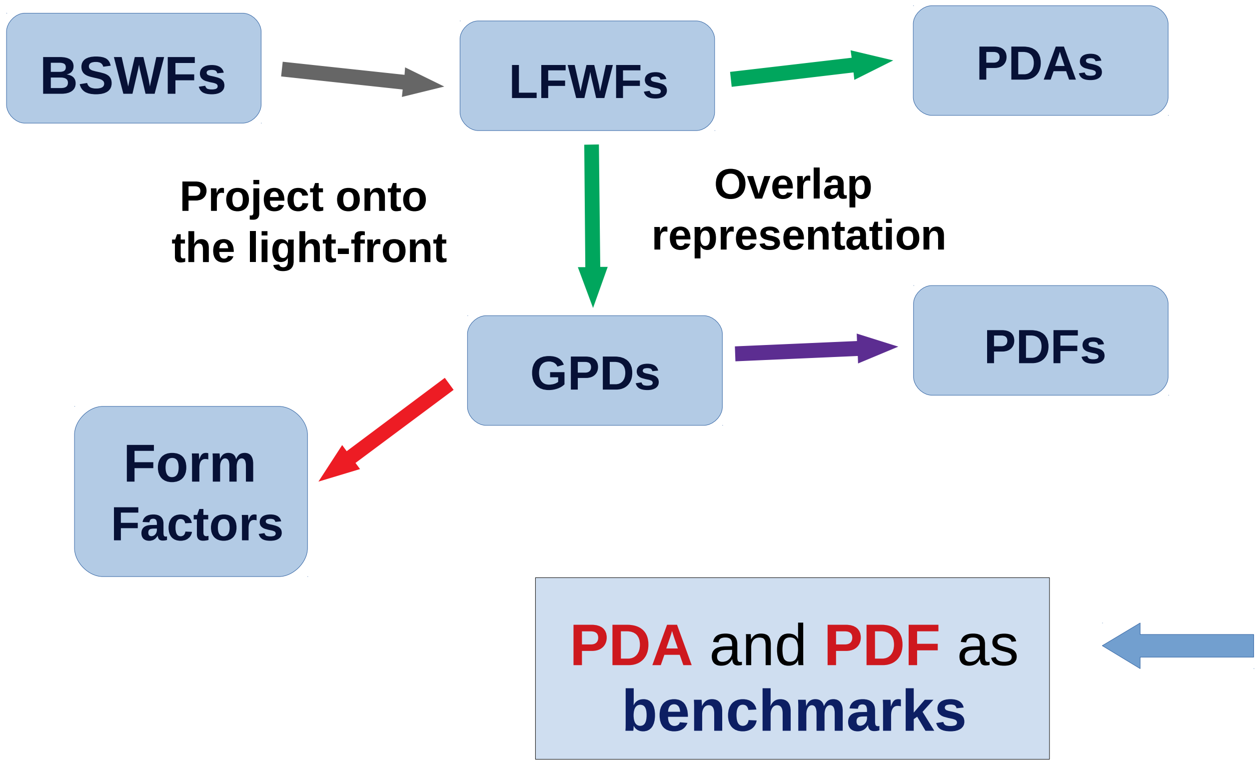
The inputs:
Solutions from quark **DSE** and meson **BSE**.

- Numerically challenging, but doable
- Already on the market: PDAs, PDFs, Form factors...

K. Raya *et al.*,
arXiv: 1911.12941 [nucl-th]

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and kaon structure.



The idea:
Compute ***everything*** from the **LFWF.**

The inputs:
Solutions from quark **DSE** and meson **BSE**.

The alternative inputs:
Model **BSWF** from realistic DSE ***predictions***.

LFWF: Nakanishi model

- A Nakanishi-like representation for the **BSWF**:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \; \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \,,$$

1: Matrix structure:

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \,,$$

Equivalent to considering the **leading** Bethe-Salpeter amplitude:

(from a total of 4)

$$\boxed{\Gamma_{\mathrm{M}}(q; P) = i\gamma_5 E_{\mathrm{M}}(q; P)}$$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

- A Nakanishi-like representation for the BSWF:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega\ \underbrace{\rho_K(\omega)}_{2}\underbrace{\mathcal{D}(k; P_K)}_{3}\ ,$$

1: Matrix structure (leading BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}]\ ,$$

2: Spectral weight: Tightly connected with the meson properties.

3: Denominators: $\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2)\Delta((k - P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2)\ ,$

where: $\Delta(s,t) = [s+t]^{-1},\ \hat{\Delta}(s,t) = t\Delta(s,t)\ .$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

- Recall the expression for the **LFWF**:

$$\psi_{\mathrm{M}}^{q}\left(x,k_{\perp}^{2}\right)=\mathrm{tr}\int_{dk_{\|}}\delta_{n}^{x}(k_{\mathrm{M}})\gamma_{5}\gamma\cdot n\,\chi_{\mathrm{M}}(k_{-},P)$$

$$<x>_{\mathrm{M}}^{q}:=\int_{0}^{1}dx\,x^{m}\psi_{\mathrm{M}}^{q}(x,k_{\perp}^{2})$$

- Algebraic manipulations yield:

+ Uniqueness of Mellin moments

$$\boxed{\Rightarrow\psi_{\mathrm{M}}^{q}(x,k_{\perp})\sim\int dw\,\rho_{\mathrm{M}}(w)\cdots}$$

✓ Compactness of this result is a merit of the AM.

- Thus, $\rho_{\mathrm{M}}(w)$ determines the profiles of, *e.g.* **PDA** and **PDF**: (it also works the other way around)

$$f_{\mathrm{M}}\phi_{\mathrm{M}}^{q}(x;\zeta_{H})=\int\frac{d^{2}k_{\perp}}{16\pi^{3}}\psi_{\mathrm{M}}^{q}(x,k_{\perp};\zeta_{H})$$

$$q_{\mathrm{M}}(x;\zeta_{H})=\int\frac{d^{2}k_{\perp}}{16\pi^{3}}|\psi_{\mathrm{M}}^{q}(x,k_{\perp};\zeta_{H})|^{2}$$

LFWF: Nakanishi model

- More **explicitly**:

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = 12\left[M_q(1-x)+M_{\bar{h}}x\right] X_{\mathrm{P}}(x;\sigma_\perp^2)$$

$$\sigma_\perp = k_\perp^2 + \Omega_{\mathrm{P}}^2$$

$$X_{\mathrm{M}}(x;\sigma_\perp^2) = \left[\int_{-1}^{1-2x} dw \int_{1+\frac{2x}{w-1}}^{1} dv + \int_{1-2x}^{1} dw \int_{\frac{w-1+2x}{w+1}}^{1} dv\right] \frac{\rho_{\mathrm{M}}(w)}{n_{\mathrm{M}}} \frac{\Lambda_{\mathrm{M}}^2}{\sigma_\perp^2}$$

$$\begin{aligned}\Omega_{\mathrm{M}}^2 &= vM_q^2 + (1-v)\Lambda_{\mathrm{P}}^2 \\ &\quad +(M_{\bar{h}}^2 - M_q^2)\left(x - \tfrac{1}{2}[1-w][1-v]\right) \\ &\quad +(x[x-1] + \tfrac{1}{4}[1-v][1-w^2])\, m_{\mathrm{M}}^2\end{aligned}$$

- Model **parameters**:

P	m_{P}	M_u	M_h	Λ_{P}	b_0^{P}	ω_0^{P}	v_{P}
π	0.14	0.31	M_u	M_u	0.275	1.23	0
K	0.49	0.31	$1.2M_u$	$3M_s$	0.1	0.625	0.41

$$\rho_{\mathsf{P}}(\omega) = \frac{1+\omega\, v_{\mathsf{P}}}{2a_{\mathsf{P}} b_0^{\mathsf{P}}}\left[\mathrm{sech}^2\left(\frac{\omega-\omega_0^{\mathsf{P}}}{2b_0^{\mathsf{P}}}\right) + \mathrm{sech}^2\left(\frac{\omega+\omega_0^{\mathsf{P}}}{2b_0^{\mathsf{P}}}\right)\right]$$

Chiral limit / Factorized model

- In the **chiral limit**, the **Nakanishi model** reduces to:

$$\psi^q_{\rm M}(x,k_\perp^2;\zeta_H) \sim \tilde{f}(k_\perp)\phi^q_{\rm M}(x;\zeta_H) \sim f(k_\perp)[q_{\rm M}(x;\zeta_H)]^{1/2}$$

"Factorized model"

$$[\phi^q_{\rm M}(x;\zeta_H)]^2 \sim q_{\rm M}(x;\zeta_H)$$

- Sensible assumption as long as:

$$m_{\rm M}^2 \approx 0 \qquad M_{\bar{h}}^2 - M_q^2 \approx 0$$

(meson mass) (antiquark – quark masses)

→ Produces **identical** results as Nakanishi model for pion

- Therefore:

$$\psi^q_{\rm M}(x,k_\perp^2;\zeta_H) = [q^{\rm M}(x;\zeta_H)]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

Single parameter!

No need to determine the spectral weight !

Factorized/chiral limit models

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^{2}}\right]$$

"Chiral M1"

Implies an **extra power** of $1/k_\perp^2$

- Can be **improved** as follows:

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

"Chiral M2"

Equivalent to considering the <u>two</u> **most dominant BSAs**:

$$\Gamma_{\mathrm{M}}(q;P) = \gamma_5\left[iE_{\mathrm{M}}(q;P)+\gamma\cdot PF_{\mathrm{M}}(q;P)\right]$$

Factorized/chiral limit models

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = [q^{\mathrm{M}}(x;\zeta_H)]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

"Chiral M1"

- We can also consider a **"Gaussian model"**:

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = [q^{\mathrm{M}}(x;\zeta_H)]^{1/2}\left(\frac{32\pi^2 r_{\mathrm{M}^2}}{\chi_{\mathrm{M}}^2(\zeta_H)}\right)^{1/2}\exp\left[-\frac{r_{\mathrm{M}}^2 k_\perp^2}{2\chi_{\mathrm{M}}^2(\zeta_H)}\right]$$

$$\chi_{\mathrm{M}}^2(\zeta_H) = <x^2>_{\zeta_H}^{\bar{h}} + \frac{1}{2}(1-c)<x^2>_{\zeta_H}^{\bar{h}}$$

Asymmetry factor $\sim M_{\bar{h}}^2 - M_u^2$

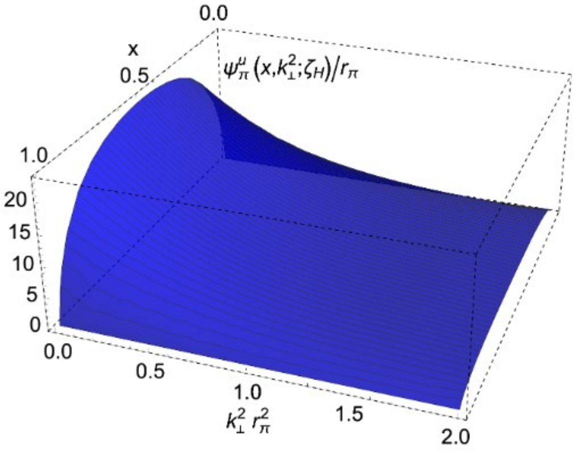
- **One parameter** to determine **both** models:
  - Either M_q or r_π

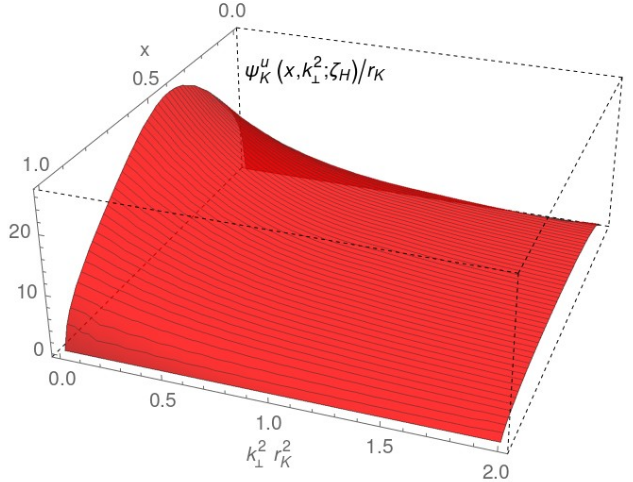
(charge radius)

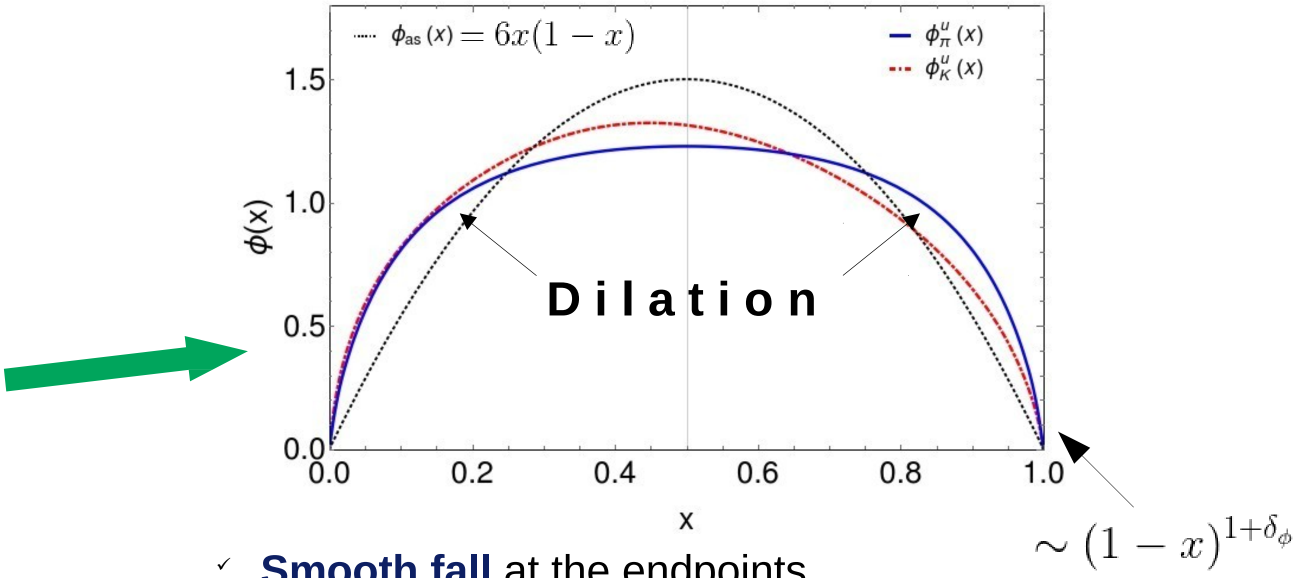
- Unless specified otherwise, **Nakanishi model** results will be shown.
- By construction, **PDA** and **PDF** are the **same** in any presented model.
- In general, **Chiral M1** ≈ **Nakanishi** (for pion)

LFWFs and PDAs

$$f_{\mathrm{M}}\phi^q_{\mathrm{M}}(x;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\psi^q_{\mathrm{M}}(x,k_\perp;\zeta_H)$$







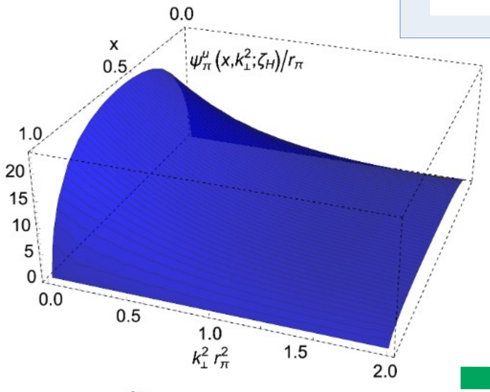
- **Smooth fall** at the endpoints
- **Broad** and concave functions of ***x***
  - Consequence of **DCSB**
- **Higgs** induced asymmetry for **Kaon**:
  - Moduled by the difference M_s - M_u

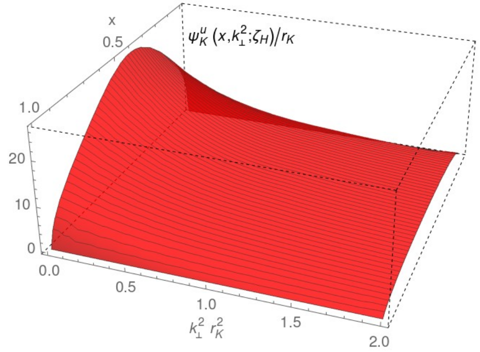
LFWFs and GPDs

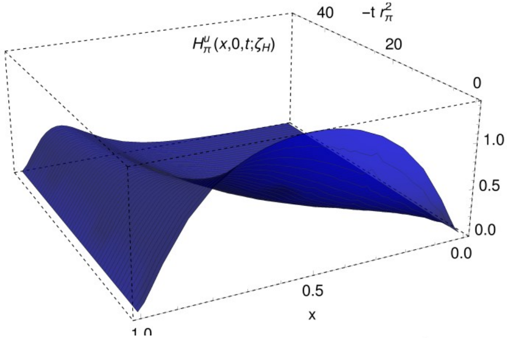
LFWFs → **GPDs**

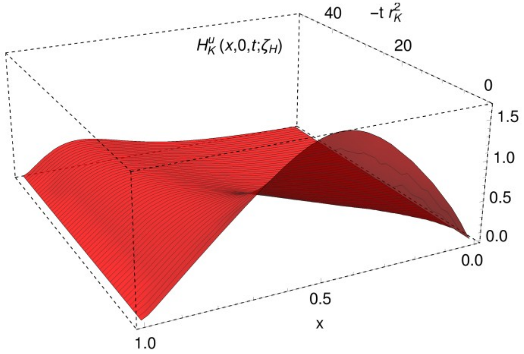
- In the **overlap representation**, the valence-quark **GPD** reads as:

$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$









- ✓ ***Valid*** in the **DGLAP** region
- ✓ ***Positivity*** fulfilled
- ✓ Can be ***extended*** to the **ERBL** region $|x| \leq \xi$

J. Manuel's talk

- ✓ ***Analytic*** in our factorized models.

GPD: factorized model

$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\psi_{\mathrm{M}}^{q*}\left(x^-,(\mathbf{k}_\perp^-)^2\right)\psi_{\mathrm{M}}^{q}\left(x^+,(\mathbf{k}_\perp^+)^2\right)$$

Overlap representation

Factorized **LFWF**

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\widetilde{\psi}_{\mathrm{M}}(k_\perp^2;\zeta_H)$$

PDF controls (mostly) the *x*-dependence

$$H_{\mathrm{M}}^{q}(x,\xi,t;\zeta_H) = \theta(x_-)\left[q^{\mathrm{M}}(x_-;\zeta_H)q^{\mathrm{M}}(x_+;\zeta_H)\right]^{1/2}\Phi_{\mathrm{M}}(z;\zeta_H)$$

$$\Phi_{\mathrm{M}}(z;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\widetilde{\psi}_{\mathrm{M}}(k_\perp^2;\zeta_H)\widetilde{\psi}_{\mathrm{M}}\left((k_\perp - s_\perp)^2;\zeta_H\right)$$

t-dependence of **GPD** is **contained** herein

$$x_\pm = \frac{x\pm\xi}{1\pm\xi}$$

$$z = s_\perp^2 = \frac{-t(1-x)^2}{1-\xi^2}$$

GPD: factorized model

$$\psi_{\mathrm{M}}^{q}(x, k_\perp^2; \zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2} \left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2} + \gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

"Chiral M2"

Chiral M2 *factorized* model produces**:**

$$\Phi_{\mathrm{M}}(z;\zeta_H) = \frac{1}{1+\gamma\sqrt{3}+\gamma^2}\left(\Phi_{\mathrm{M}}^{(A)}(z;\zeta_H) + \gamma\sqrt{3}\Phi_{\mathrm{M}}^{(AB)}(z;\zeta_H) + \gamma^2\Phi_{\mathrm{M}}^{(B)}(z;\zeta_H)\right)$$

$$\Phi_{\mathrm{M}}^{(A)}(z;\zeta_H) = \frac{6M^6}{(z+4M^2)^3}\left(10 + \frac{z}{M^2} + \frac{8(z+M^2)}{z}\left[\sqrt{\frac{z+4M^2}{z}}\,\mathrm{atanh}\left(\sqrt{\frac{z}{z+4M^2}}\right) - 1\right]\right)$$

GPD: factorized model

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

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$$\Phi_{\mathrm{M}}^{(AB)}(z;\zeta_H) = \frac{8M^4}{(z+4M^2)^2}\left(1+\frac{z}{4M^2}+\sqrt{\frac{z+4M^2}{z}}\,\mathrm{atanh}\left(\sqrt{\frac{z}{z+4M^2}}\right)\right)$$

GPD: factorized model

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = [q^{\mathrm{M}}(x;\zeta_H)]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\,\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

"Chiral M2"

Chiral M2 ***factorized*** model produces**:**

$$\Phi_{\mathrm{M}}(z;\zeta_H) = \frac{1}{1+\gamma\sqrt{3}+\gamma^2}\left(\Phi_{\mathrm{M}}^{(A)}(z;\zeta_H)+\gamma\sqrt{3}\Phi_{\mathrm{M}}^{(AB)}(z;\zeta_H)+\gamma^2\Phi_{\mathrm{M}}^{(B)}(z;\zeta_H)\right)$$

Observations:

- ɣ=0 recovers **Chiral M1**
- **Chiral M1** ≈ **Nakanishi** (for pion)
- **Gaussian** model also gives an *algebraic* **GPD** Zhang:2021mtn

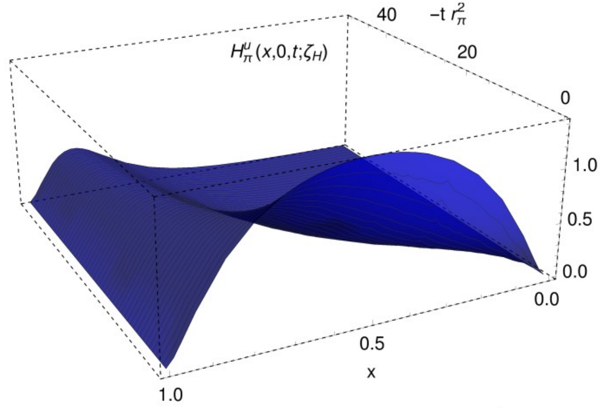
$$\Phi_{\mathrm{M}}^{(B)}(z;\zeta_H) = \frac{4M^2}{z+4M^2}\,\frac{\operatorname{atanh}\left(\sqrt{\frac{z}{z+4M^2}}\right)}{\sqrt{\frac{z}{z+4M^2}}}$$

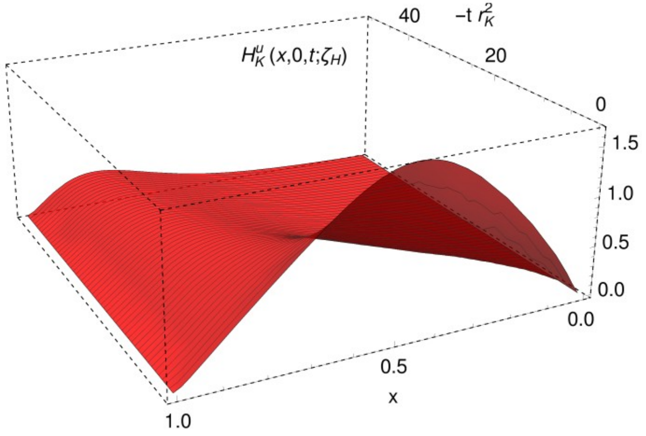
LFWFs and PDFs

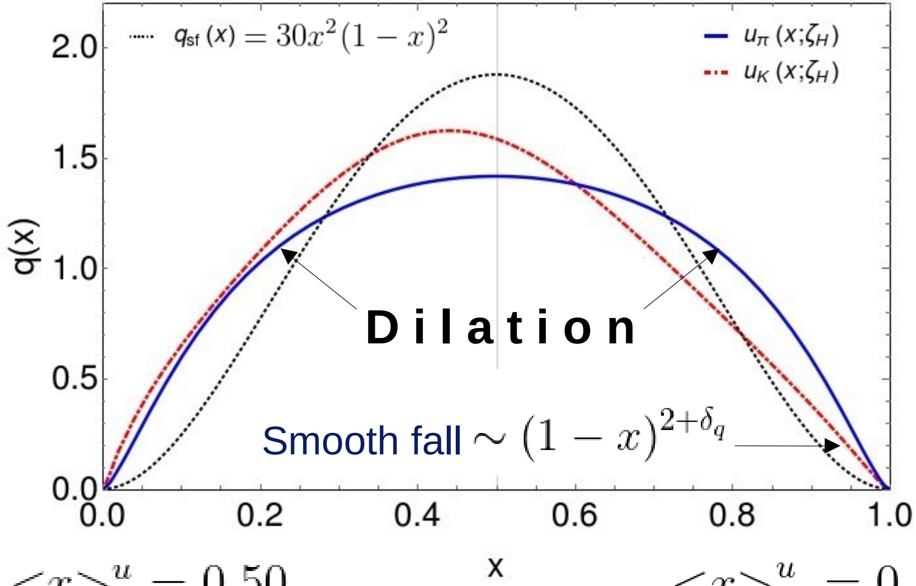
GPD → PDF

- The **PDF** is obtained from the **forward limit** of the **GPD**.

$$q(x) = H(x,0,0)$$





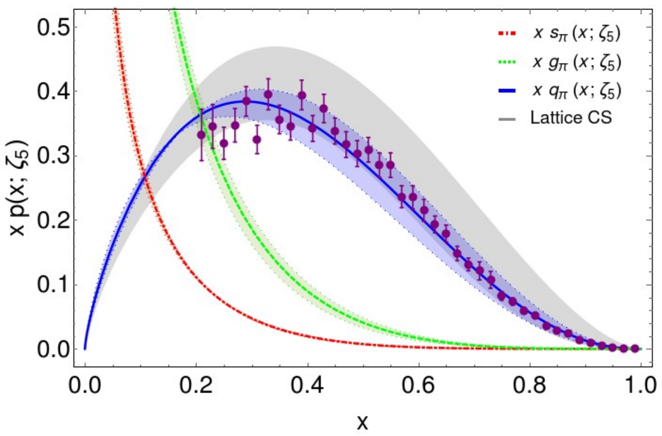


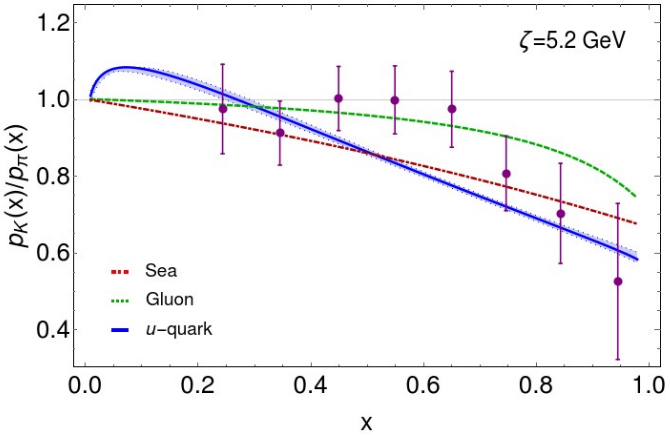
$<x>^u_\pi = 0.50$ $\qquad$ $<x>^u_K = 0.47$

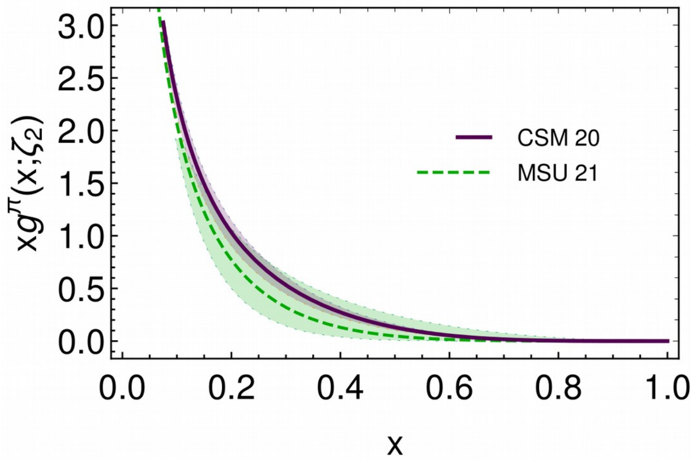
- **ζ_H:** meson properties determined by the fully-dressed valence-quarks.
- **Broad** + **Higgs**-induced *asymmetry*

Evolved PDFs

GPD → PDF







- Same, **not tuned**, initial scale for evolution
- **Determined** from **QCD** PI effective charge.

$$\zeta_H = 0.331\ \text{GeV}$$

- In **agreement** with:
  - ✓ **ASV** analysis Aicher:2010cb
  - ✓ **Lattice** CS Sufian:2020vzb Sufian:2019bol
  - ✓ **DSEs** Cui:2020tdf

- **Gluon** in pion:
  - ✓ **Lattice** MSU Fan:2021bcr

$$<\mathbf{x}>_\pi^{\text{val}} = 0.41(4)$$

$$<\mathbf{x}>_K^{\text{val}} = 0.43(4)$$

J. R-Q's talk

Electromagnetic FFs

GPD → FFs

- **Electromagnetic** form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F^q_M(-t=\Delta^2) = \int_{-1}^{1} dx\; H^q_M(x,\xi,t)$$

Can safely take **ξ = 0**

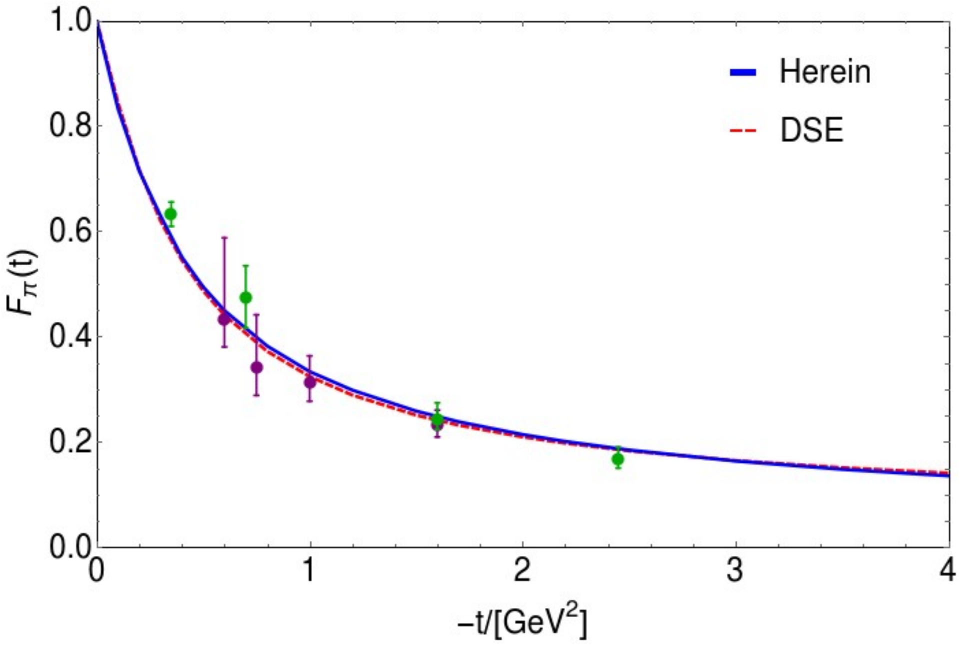
"Polinomiality"

$$F_M(\Delta^2) = e_u F^u_M(\Delta^2) + e_{\bar{f}} F^{\bar{f}}_M(\Delta^2)$$

Weighed by electric charges

→ **Isospin symmetry**

$$\Rightarrow\quad F_{\pi^+}(-t) = F^u_{\pi^+}(-t)$$



Data: G.M. Huber *et al*. PRC 78 (2008) 045202

DSE: L. Chang *et al*. PRL 111 (2013) 14, 141802

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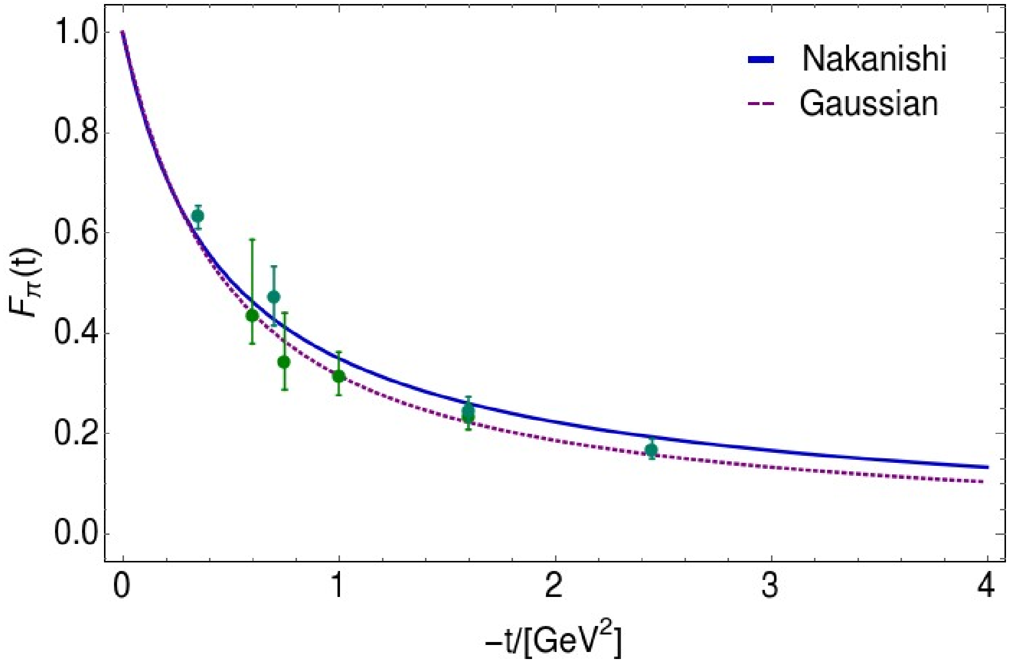

Data: G.M. Huber *et al*. PRC 78 (2008) 045202

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Electromagnetic FFs

GPD → FFs

- **Electromagnetic** form factor: pion models

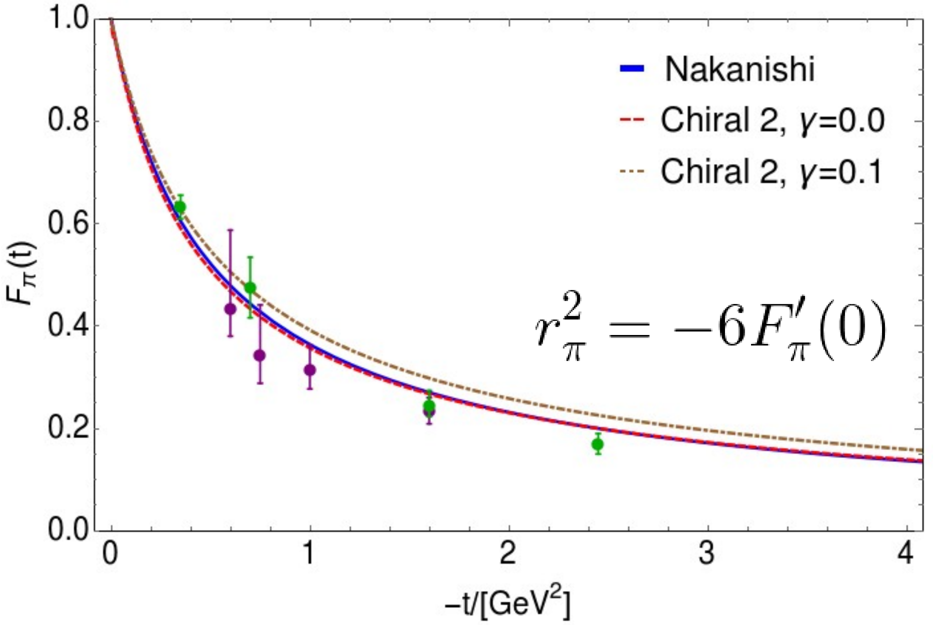
$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

- In the **chiral limit M2**:

$$\frac{1+\frac{5\sqrt{3}}{9}\gamma+\frac{5}{18}\gamma^2}{1+\gamma\sqrt{3}+\gamma^2} = \frac{5}{18}\frac{M^2 r_\pi^2}{\langle x^2\rangle_u^{\zeta_H}}$$

- For $M_q \simeq 0.3\ \mathrm{GeV}$ and $\gamma \simeq 0.1$

$$\Rightarrow \boxed{r_\pi \simeq 0.66\ \mathrm{fm}}$$



"Chiral M2"

$i\gamma_5 E_\pi(k)$ $\qquad$ $\gamma_5\gamma\cdot P\, F_\pi(k)$

$$\psi_M^q(x,k_\perp^2;\zeta_H) = \left[q^M(x;\zeta_H)\right]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

Gravitational FFs

GPD → FFs

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

  - ✓ Directly obtained if **ξ = 0**
  - ✓ Only **DGLAP** GPD is required
  - ✗ **ERBL** GPD needed

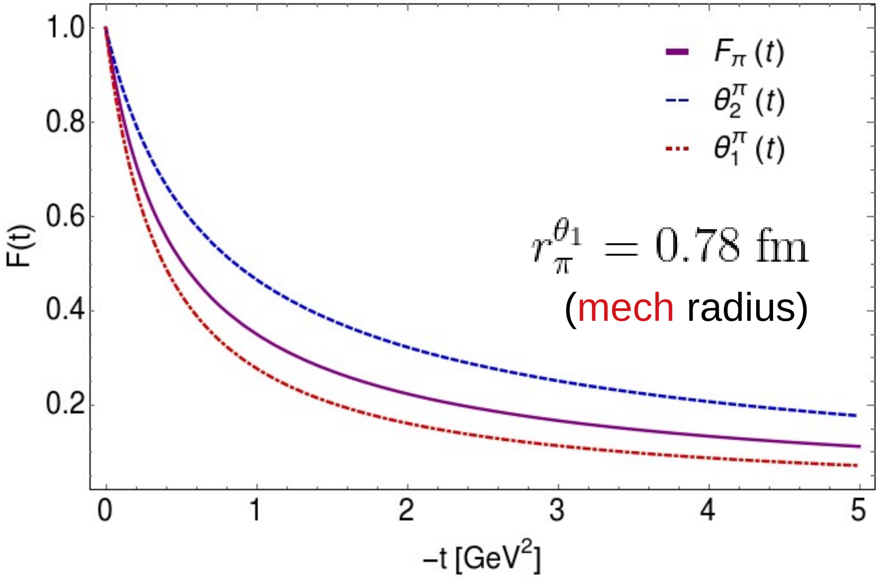
J. Manuel's talk

- Sophisticated techniques exist. Chouika:2017dhe
- But a sound expression can be constructed:

$$\theta_1^{\mathsf{P}_q}(\Delta^2) = c_1^{\mathsf{P}_q}\theta_2^{\mathsf{P}_q}(\Delta^2) + \int_{-1}^{1} dx\, x\left[H_{\mathsf{P}}^q(x,1,0)P_{M_q}(\Delta^2) - H_{\mathsf{P}}^q(x,1,-\Delta^2)\right]$$

"Soft pion theorem"

Zhang:2021mtn



$r_\pi^{\theta_1} = 0.78$ fm (mech radius)

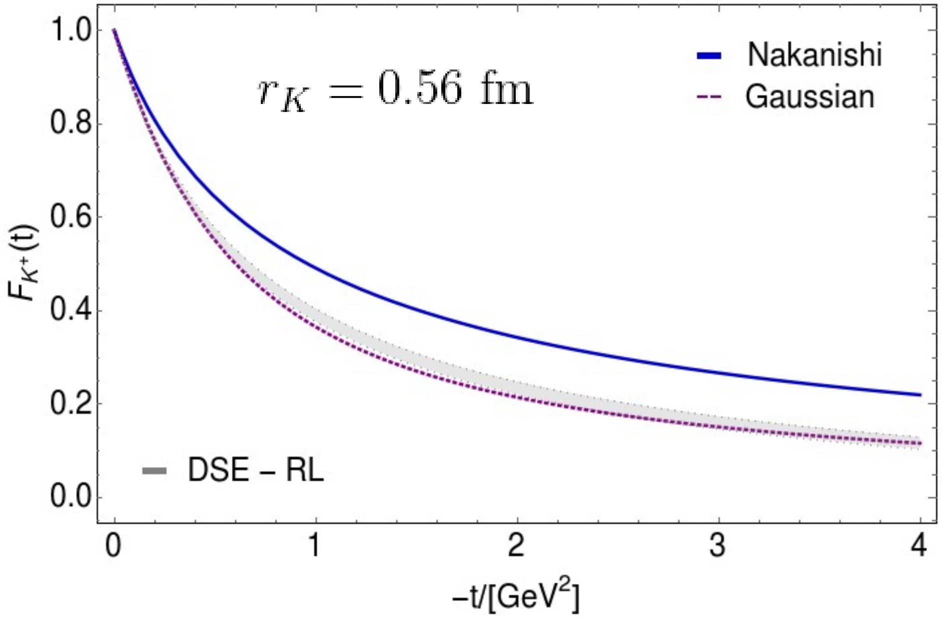
$r_\pi^{E} = 0.68$ fm , $r_\pi^{\theta_2} = 0.56$ fm

(charge radius) (mass radius)

Electromagnetic FFs

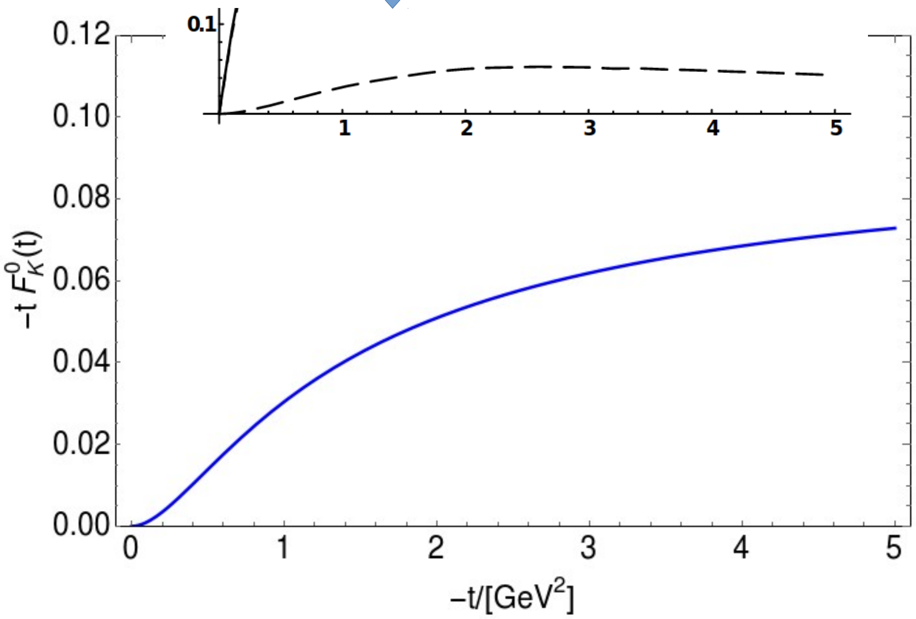
GPD → FFs

- **Electromagnetic** form factor: **charged** and **neutral** kaon



DSE - K⁺: `Gao:2017mmp, Eichmann:2019bqf`

DSE - K⁰: `Burden:1995ve`



Kaon is more **compressed**

$$r_K^j \approx 0.85\ r_\pi^j$$

j = *mech, charge, mass*

On the Radii

GPD → FFs

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

- The ***ordering*** of **radii**: (in fm)

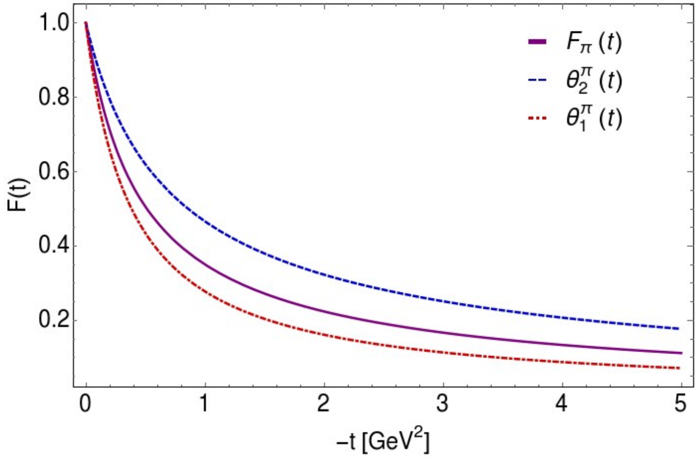
$$r_\pi^{\theta_1} = 0.78 > r_\pi^{E} = 0.68 > r_\pi^{\theta_2} = 0.55$$

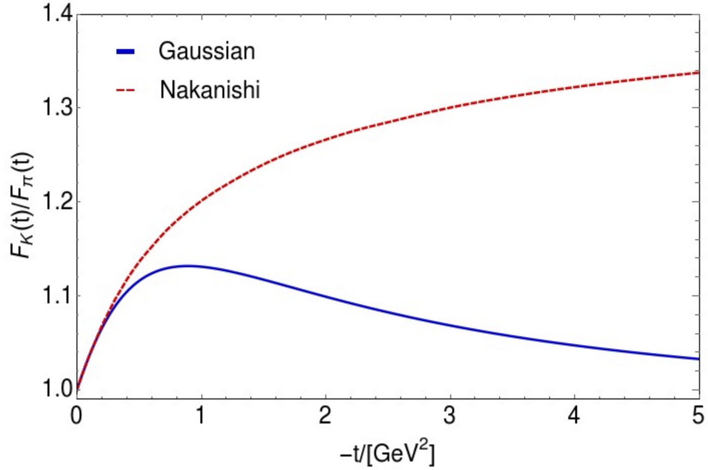
(mech) (charge) (mass)

For **Kaon**:

$$r_K^j \approx 0.85\, r_\pi^j$$

j = *mech, charge, mass*





On the Radii

GPD → FFs

$$H_{\mathrm{M}}^{q}(x,\xi,t;\zeta_H) = \theta(x_-)\left[q^{\mathrm{M}}(x_-;\zeta_H)q^{\mathrm{M}}(x_+;\zeta_H)\right]^{1/2}\Phi_{\mathrm{M}}(z;\zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial^n z}\Phi_{\mathsf{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = \frac{1}{\langle x^{2n}\rangle_{\bar{h}}^{\zeta_{\mathcal{H}}}}\frac{d^n F_{\mathsf{P}}^{u}(\Delta^2)}{d(\Delta^2)^n}\Big|_{\Delta^2=0}$$

PDF moments

Derivatives of **EFF**

$$\frac{\partial}{\partial z}\Phi_{\mathsf{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = -\frac{r_{\mathsf{P}}^2}{4\chi_{\mathsf{P}}^2(\zeta_{\mathcal{H}})},$$

$$\frac{\partial}{\partial z}\Phi_{\mathsf{P}}^{\bar{h}}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = (1-d_{\mathsf{P}})\frac{\partial}{\partial z}\Phi_{\mathsf{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0}$$

Asymmetry term = **0** for **pion**

- Therefore, the **mass *radius***:

$$r_{\mathsf{P}_u}^{\theta_2 2} = \frac{3r_{\mathsf{P}}^2}{2\chi_{\mathsf{P}}^2}\langle x^2(1-x)\rangle_{\mathsf{P}_{\bar{h}}},$$

$$r_{\mathsf{P}_{\bar{h}}}^{\theta_2 2} = \frac{3r_{\mathsf{P}}^2}{2\chi_{\mathsf{P}}^2}(1-d_{\mathsf{P}})\langle x^2(1-x)\rangle_{\mathsf{P}_u}$$

$$\boxed{\left(\frac{r_\pi^{\theta_2}}{r_\pi^{E}}\right)^2 = \frac{<x^2(1-x)>_{\zeta_H}^{q}}{<x^2>_{\zeta_H}^{q}} \approx \left(\frac{4}{5}\right)^2}$$

Determined from **PDF moments!**

On the Radii

GPD → FFs

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

- The ***ordering*** of **radii**: (in fm)

$$r_\pi^{\theta_1} = 0.78 > r_\pi^{E} = 0.68 > r_\pi^{\theta_1} = 0.56$$

(mech) (charge) (mass)

$$r_K^j \approx 0.85\, r_\pi^j$$

j = *charge, mass, mech*.

- Mean-squared **transverse extent**:

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2\,,\ \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \boxed{\frac{2}{3}} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi$$

Algebraic derivation!

Merely from the definitions of charge radius and Impact Parameter Space **GPD**:

$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

J. R-Q's talk

Pressure distributions

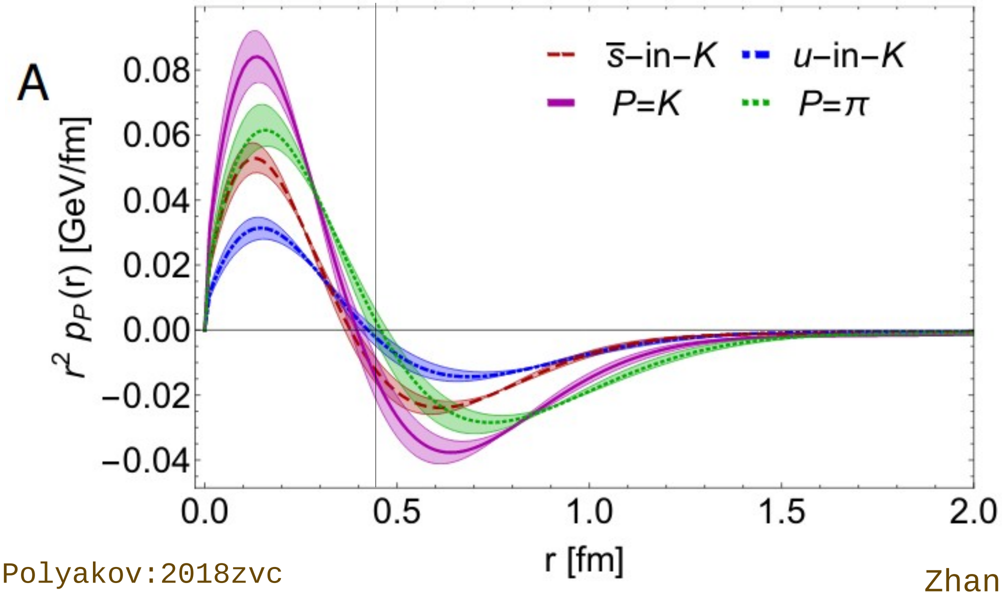
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta\, \frac{\Delta}{2E(\Delta)}\, \sin(\Delta r)[\Delta^2\theta_1^{K_u}(\Delta^2)]\,,$$

$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta\, \frac{\Delta^2}{2E(\Delta)}\, j_2(\Delta r)\,[\Delta^2\theta_1^{K_u}(\Delta^2)]\,,$$

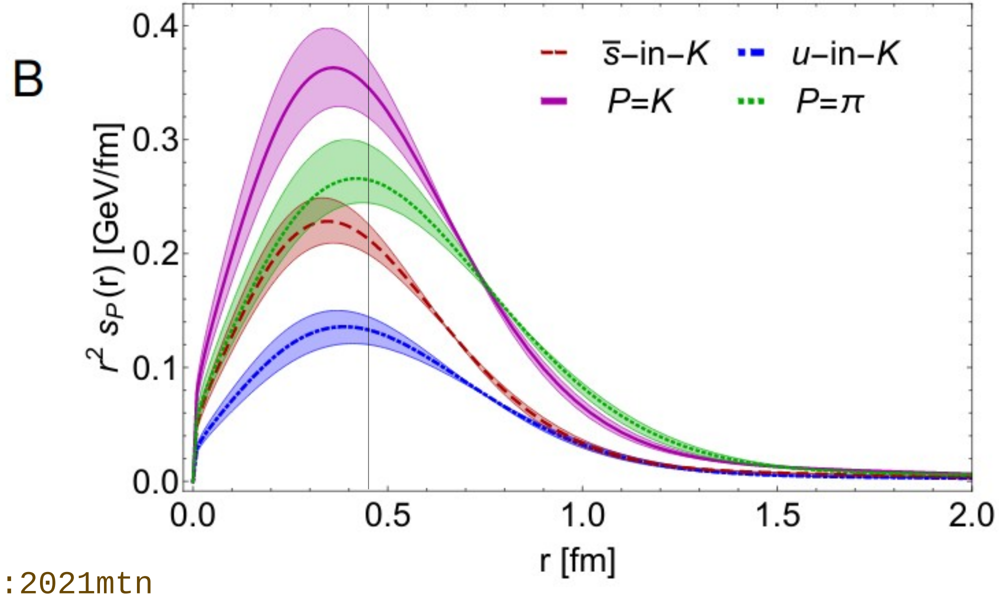
"Pressure" Quark attraction/repulsion

CONFINEMENT

"Shear" *Deformation* QCD forces

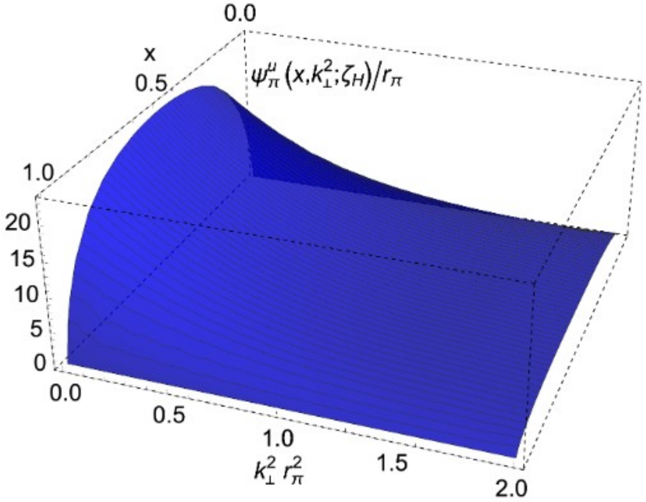


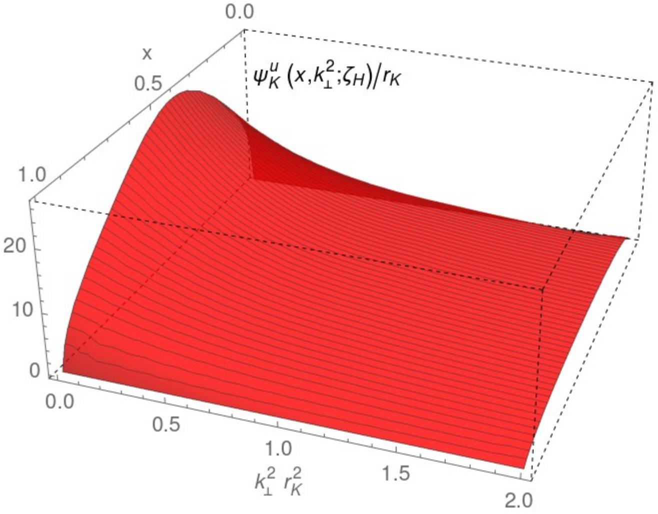
Polyakov:2018zvc

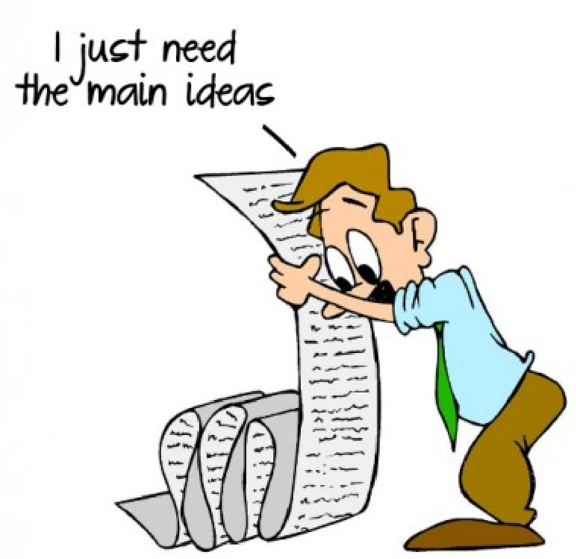


Zhang:2021mtn

Summary and Highlights







Summary

- Focusing on the **pion** and **kaon**, we discussed a variety of **parton distributions**:

Light-front **wave function**
(encodes many properties)

Parton distribution amplitudes
(momentum distribution)

Generalized parton distributions
(leads to many other objects)

Impact parameter space GPD
(spatial distributions)

- ✓ Parton distribution functions
  (momentum distribution, valence-sea-gluon)
- ✓ Electromagnetic form factors
  (charge and momentum distributions)
- ✓ Gravitational form factors
  (mass distribution, mechanical properties)

J. R-Q's talk

Pressures and distributions
(spatial distribution, mechanical properties)

Zhang:2021mtn

Highlights

- **QCD's EHM** produce **broad π-K** distributions.
- **Interplay** between **QCD** and **Higgs** mass generation:
  - ➔ Slightly *skewed* kaon distributions.
- The **ordering** of **radii**: $r_\pi^{\theta_1} > r_\pi^{E} > r_\pi^{\theta_2}$
  - ➔ Kaon is more compressed: $r_K^j \approx 0.85\, r_\pi^j$
- **Gluon** and **sea** generated through **evolution**.

J. R-Q's talk

- **Factorized** models:
  - ✔ Insightful
  - ✔ Analytical
  - ✔ Adequate for pion

Realistic **PDF** + r_{M}^{E} is all we need.