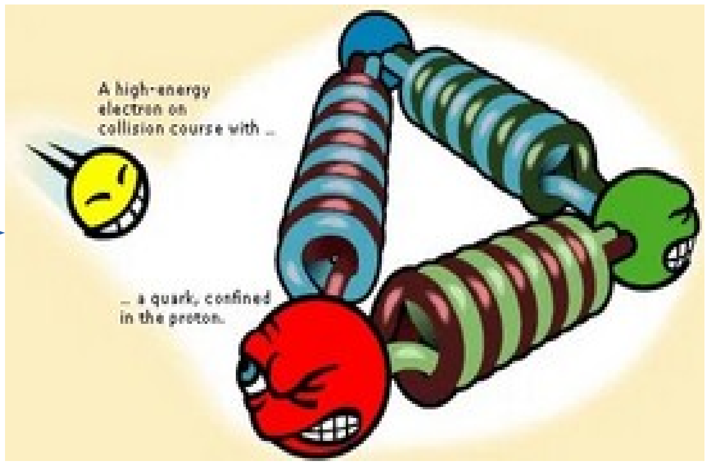
EHM via meson LFWFs II

J. Rodríguez-Quintero

QCD and hadron physics

- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

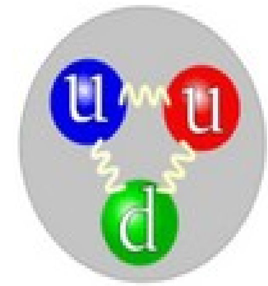
Gluons and quarks have never been seen isolated in nature; only colorless bound states (hadrons) have.



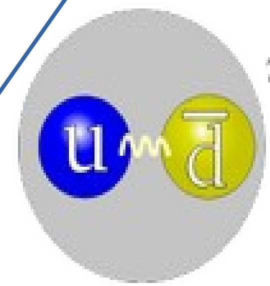
'Higgs' masses

$m_{u/d} \approx 0.004$ GeV

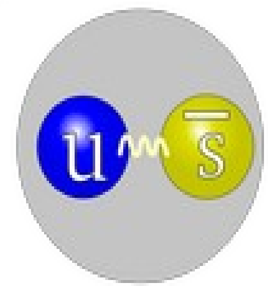
$m_s \approx 0.095$ GeV



$m_p \approx 0.940$ GeV



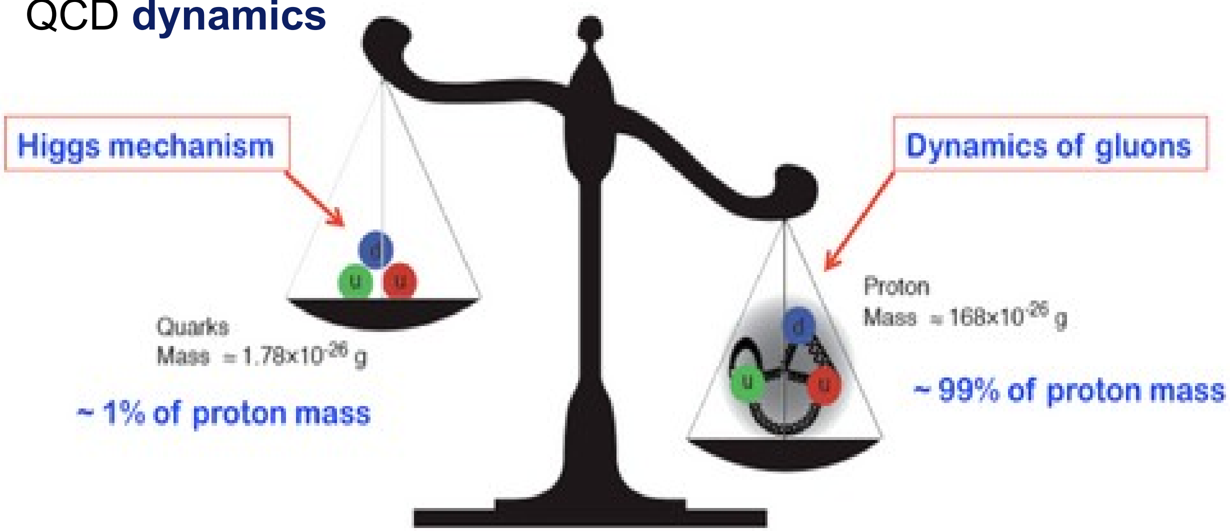
$m_\pi \approx 0.140$ GeV



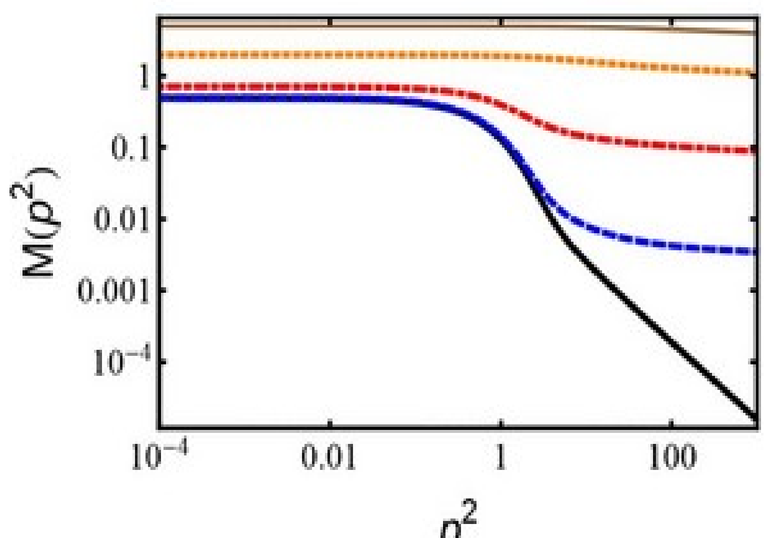
$m_K \approx 0.490$ GeV

Pions and **Kaons** emerge as QCD's (pseudo)-**Goldstone** bosons.

Emergence of hadron masses (EHM) from QCD **dynamics**

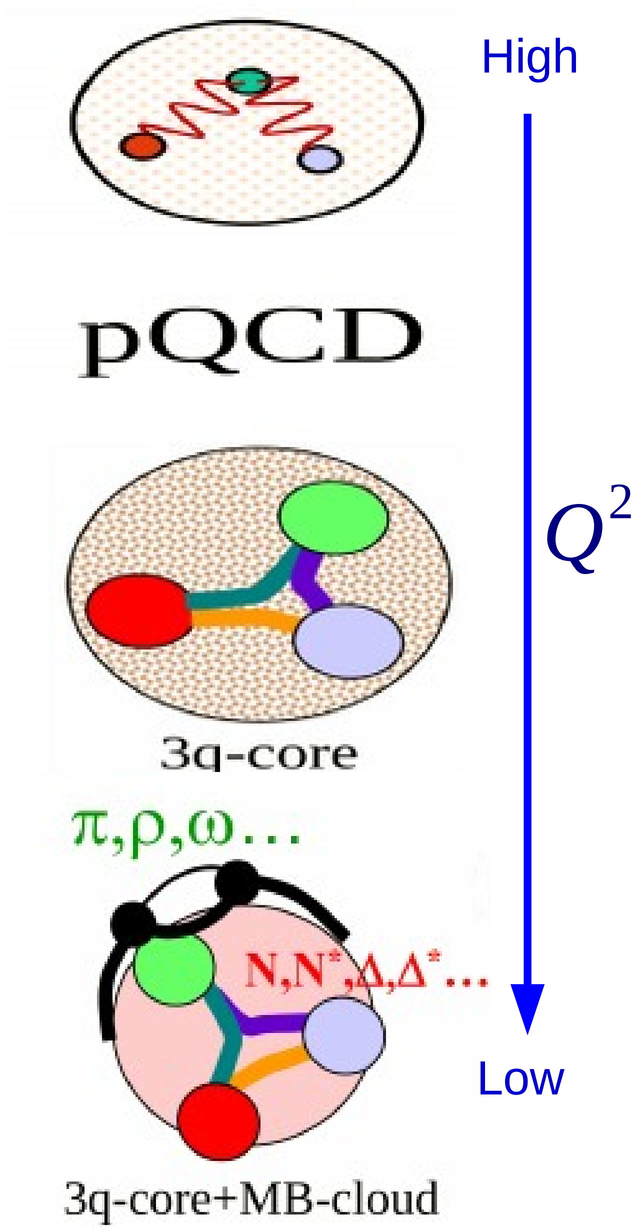


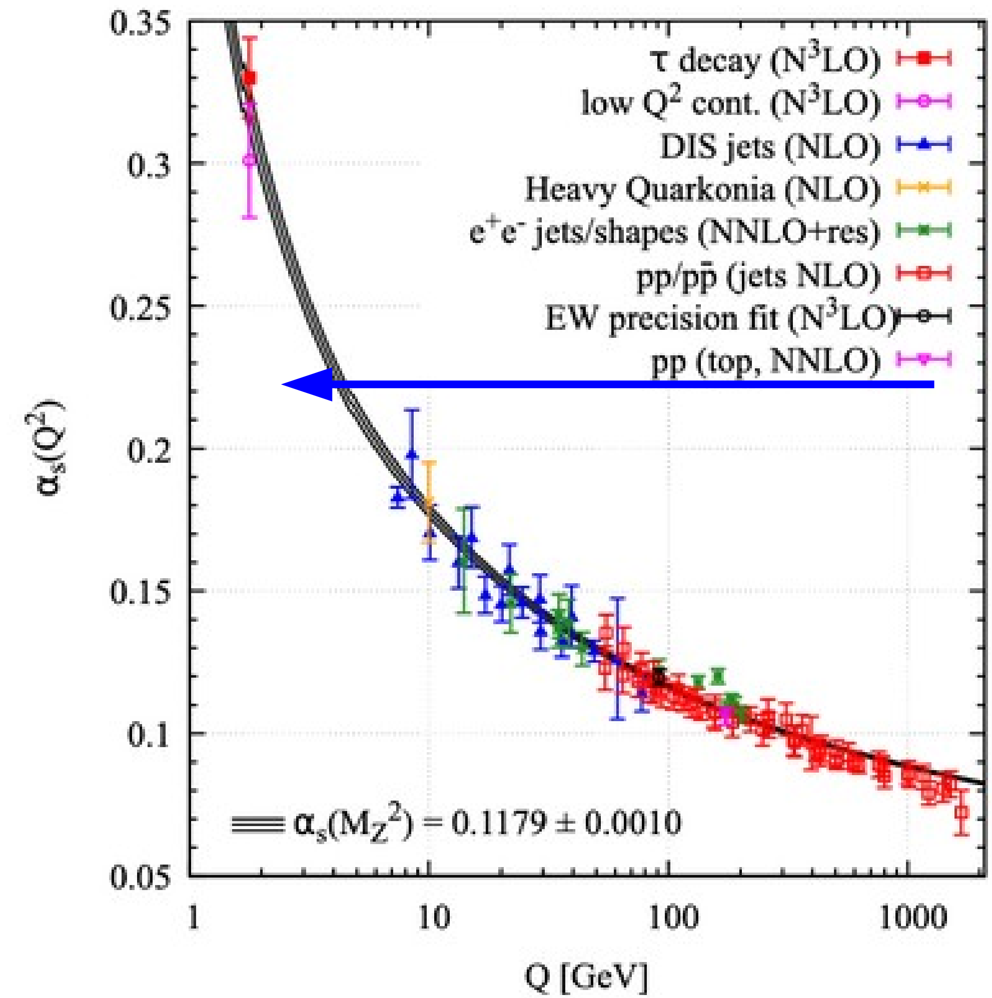
Dynamical Chiral Symmetry Breaking (**DCSB**)



DFs and QCD evolution

Emergent phenomena playing a dominant role in the real world dominated by the IR dynamics of QCD.





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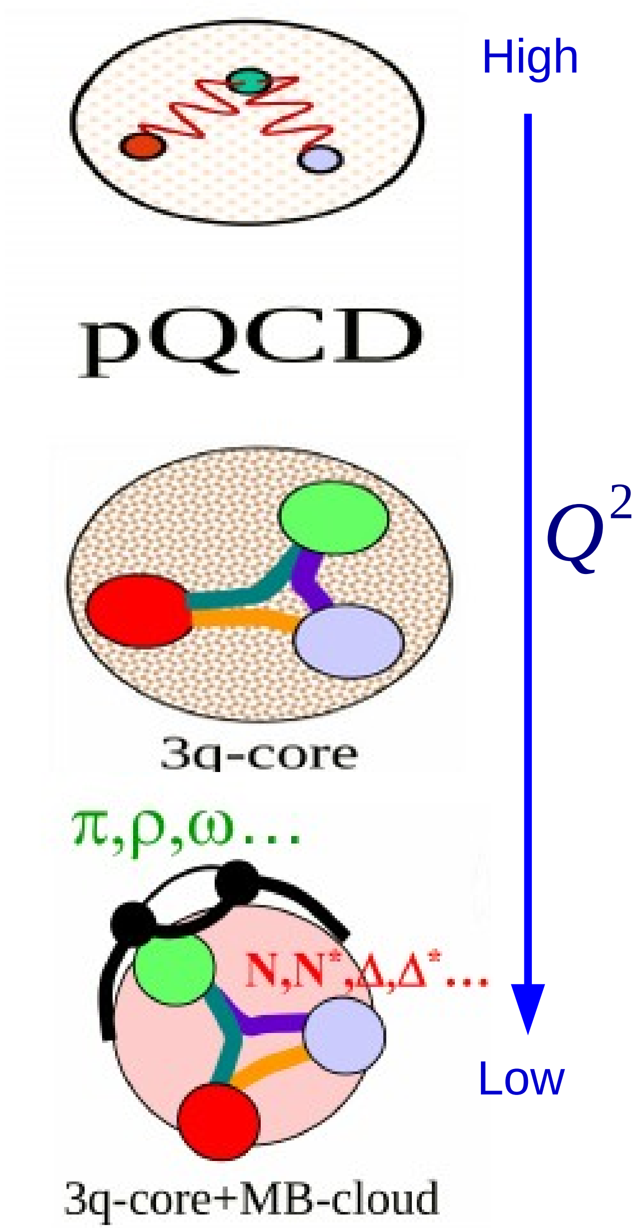
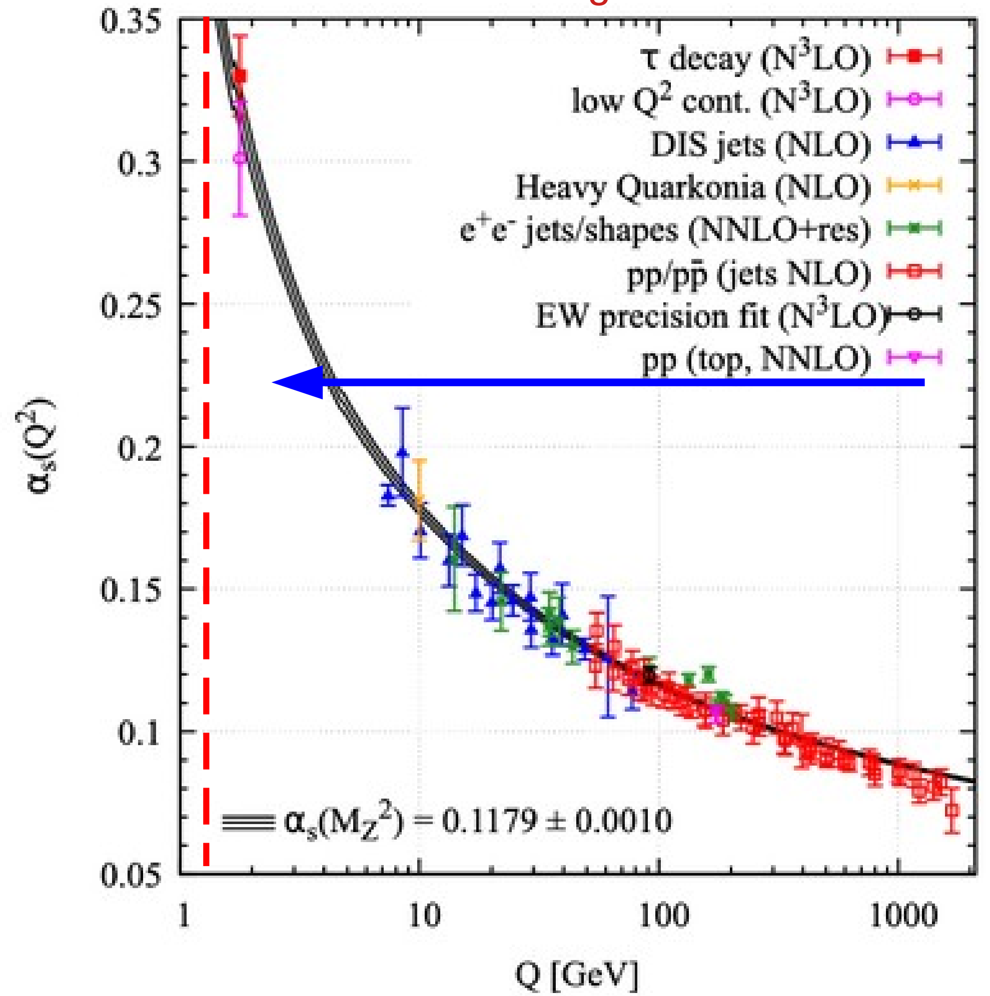
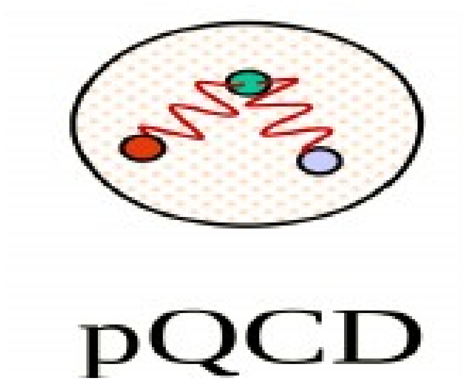
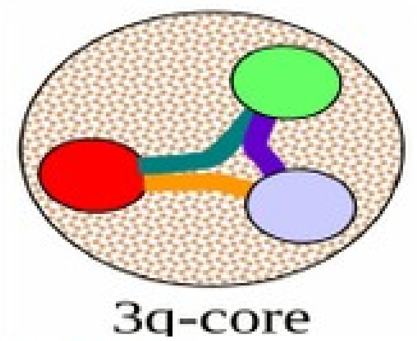


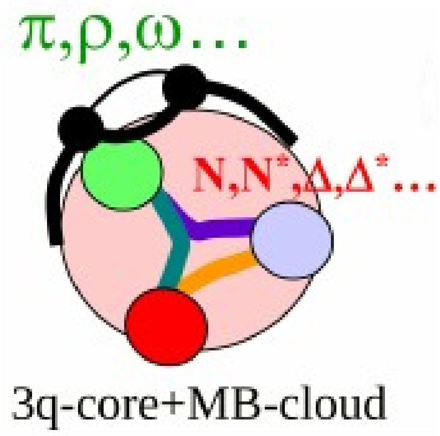
Landau pole is a perturbative artifact and a sensible description of the strong interaction should get rid of it.



DFs and QCD evolution







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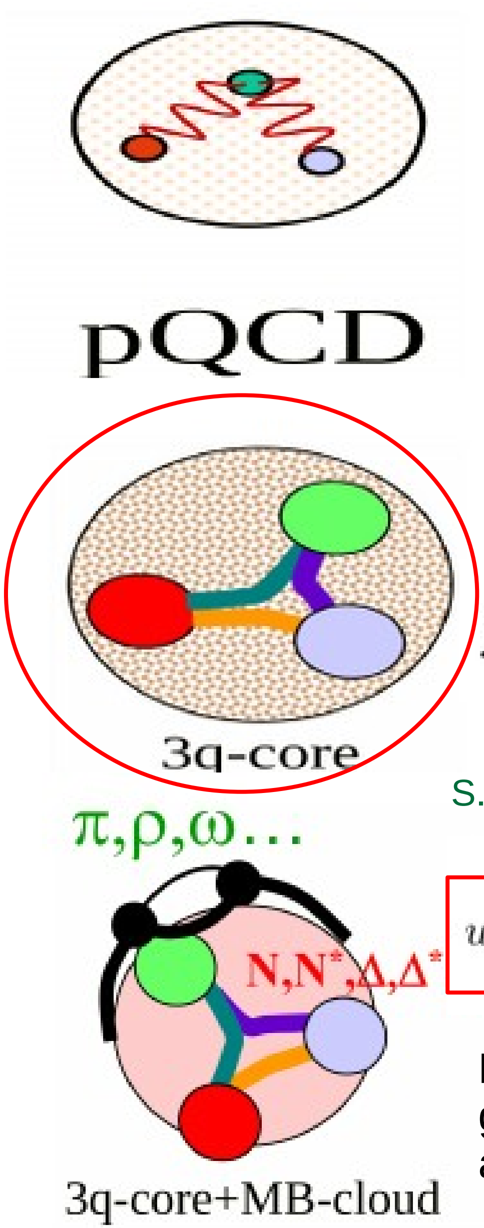


$$f_M\, \varphi_M^u(x;\zeta_H) = \frac{1}{16\pi^3}\int d^2k_\perp\, \psi_{M_u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H)$$

ζ_H : hadron scale

Factorization approximation:
S.-S. Xu et al,. Phys.Rev.D97094014(2018)

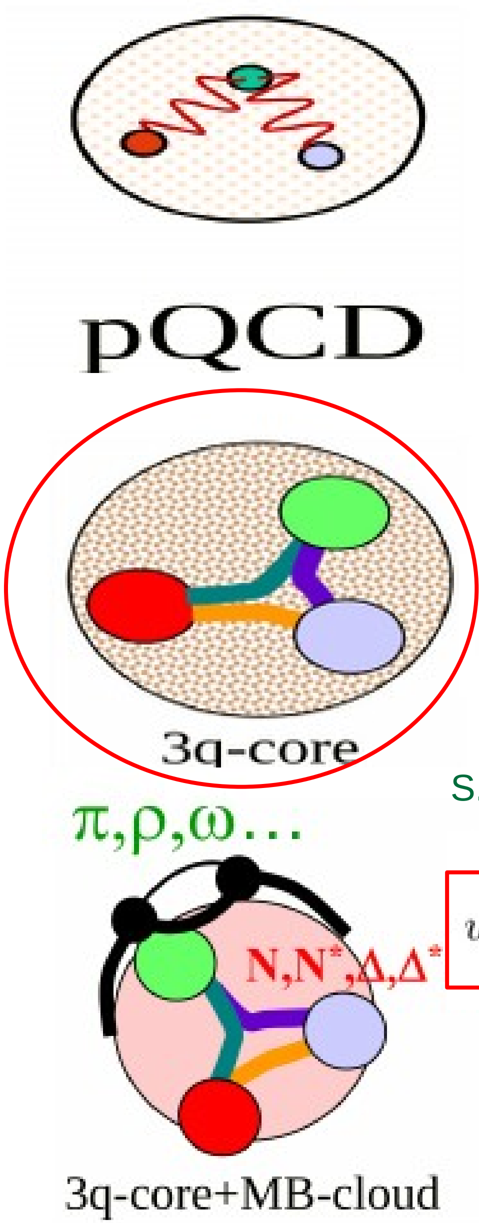
$$\psi_{M_u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H) = \varphi_M^u(x;\zeta_H)\psi_{M_u}^{\uparrow\downarrow}(k_\perp^2;\zeta_H)$$

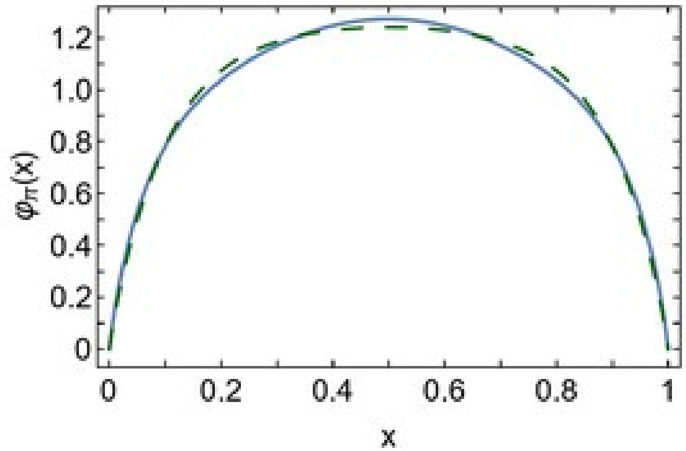
$$u^M(x;\zeta_H) = H_M^u(x,0,0;\zeta_H) \propto |\varphi_M^u(x;\zeta_H)|^2$$

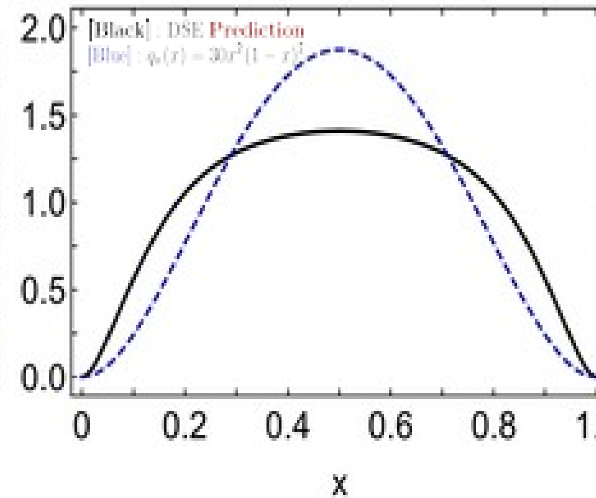
Direct connection between meson PDAs and PDFs at the hadronic scale, ζ_H, grounded on the *factorization approximation*, only valid for integrated quantities and not beyond ζ_H due to parton splitting effects.

Khépani's talk!

DFs and QCD evolution







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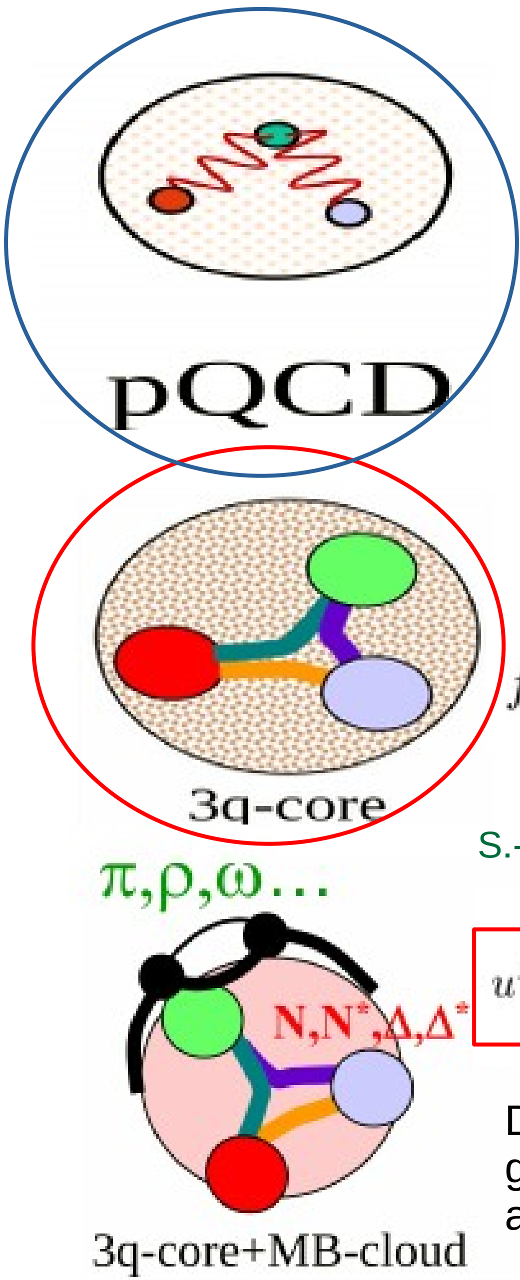

ζ_2 : "experimental" scale

Assumption: The QCD running from the hadronic scale up to the experimental one transmogrifies the quasi-particle into a full description incorporating glue and sea quark contributions.

$$f_M \, \varphi_M^u(x;\zeta_H) = \frac{1}{16\pi^3}\int d^2k_\perp \, \psi_{M_u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H)$$

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Khépani's talk!

QCD evolution

DGLAP leading-order evolution of forward (and non-skewed) GPDs:

$$\left\{\zeta^2\frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) \;-\; \frac{\alpha(\zeta^2)}{4\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\mathrm{NS}}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\mathrm{S}}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{\mathrm{NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\mathrm{S}}(y,t;\zeta)\end{pmatrix} \;=\; 0$$

$$\mathbf{P}^{\mathrm{S}}\left(\frac{x}{y}\right) = \begin{pmatrix} P_{qq}^{\mathrm{S}}\left(\frac{x}{y}\right) & 2n_f P_{qg}^{\mathrm{S}}\left(\frac{x}{y}\right) \\ P_{gq}^{\mathrm{S}}\left(\frac{x}{y}\right) & P_{gg}^{\mathrm{S}}\left(\frac{x}{y}\right)\end{pmatrix}$$

$$\mathbf{H}_\pi^{\mathrm{S}}(y,t;\zeta) = \begin{pmatrix} H_\pi^{\mathrm{S},+}(y,t;\zeta) \\ \frac{1}{x}H_\pi^{g}(y,t;\zeta)\end{pmatrix}$$

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Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

QCD evolution

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$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\boxed{\alpha(\zeta^2)}}{4\pi}\underbrace{\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}}_{\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)}\right\}\begin{pmatrix} \langle x^n\rangle_{\mathrm{NS}}(\zeta) \\ \langle x^n\rangle_{\mathrm{S}}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

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$$\gamma_{AB}^{(n)} = -\int_0^1 dx \, x^n P_{AB}^C(x)$$

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Implication 1: valence-quark PDF

$$\underset{q=u,\bar{d}}{\langle x^n(\zeta_f)\rangle_q} = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_0, \zeta_f) \right) \langle x^n(\zeta_0) \rangle_q$$

$$S(\zeta_0, \zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\mathrm{QCD}})}^{2\ln(\zeta_f/\Lambda_{\mathrm{QCD}})} dt \, \alpha(t)$$

$$t = \ln \frac{\zeta^2}{\Lambda_{\mathrm{QCD}}^2}$$

QCD evolution

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$$q = u, \bar{d}$$

This ratio encodes the information of the charge

$$S(\zeta_0, \zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

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QCD evolution

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This ratio encodes the information of the charge

Use Isospin symmetry:

$$\langle x(\zeta_H) \rangle_u = \langle x(\zeta_H) \rangle_{\bar{d}} = 1/2$$

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This ratio encodes the information of the charge

Use Isospin symmetry:

$$\langle x(\zeta_H) \rangle_u = \langle x(\zeta_H) \rangle_{\bar{d}} = 1/2$$

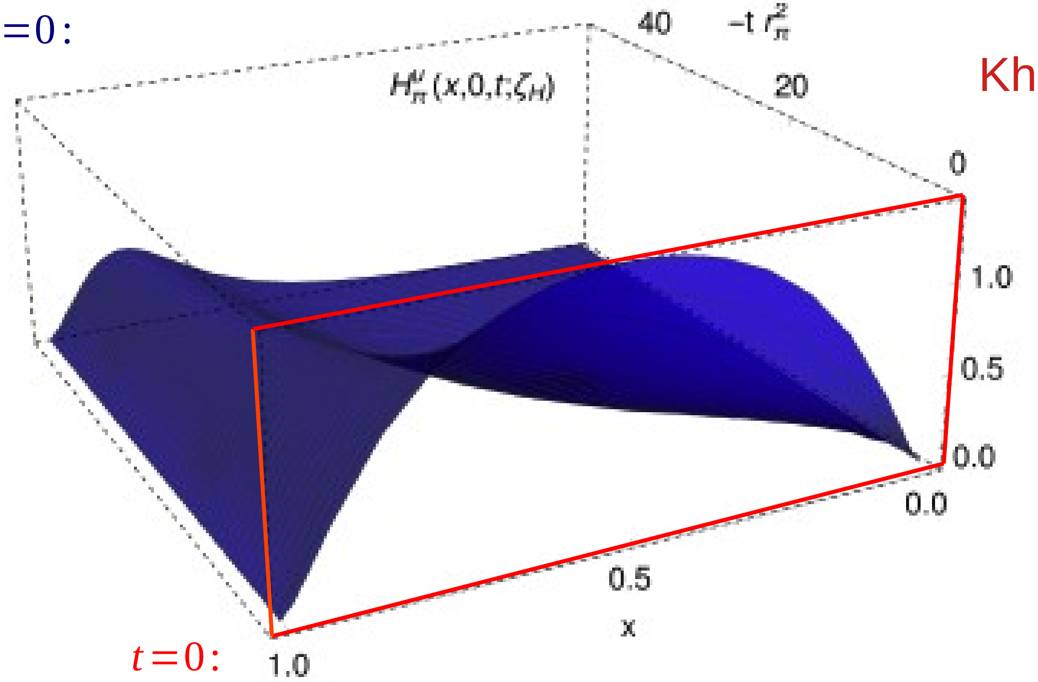
Then, after deriving the GPD from the LFWF, taking its forward limit and computing "all" the Mellin moments, only one input is needed for their being evolved up and used for the PDF reconstruction.

Evolved PDF from LFWFs

Pion GPD: $$H^{\pi}_{M}{}^{u}(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\,\psi^{\pi\,\uparrow\downarrow *}_{Mu}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp + \frac{\mathbf{1}-\mathbf{x}}{\mathbf{1}-\xi}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi^{\pi\,\uparrow\downarrow}_{Mu}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp - \frac{\mathbf{1}-\mathbf{x}}{\mathbf{1}+\xi}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$$

Valence-quark overlap GPD and forward PDF limit

$\xi = 0:$



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Khépani's talk!

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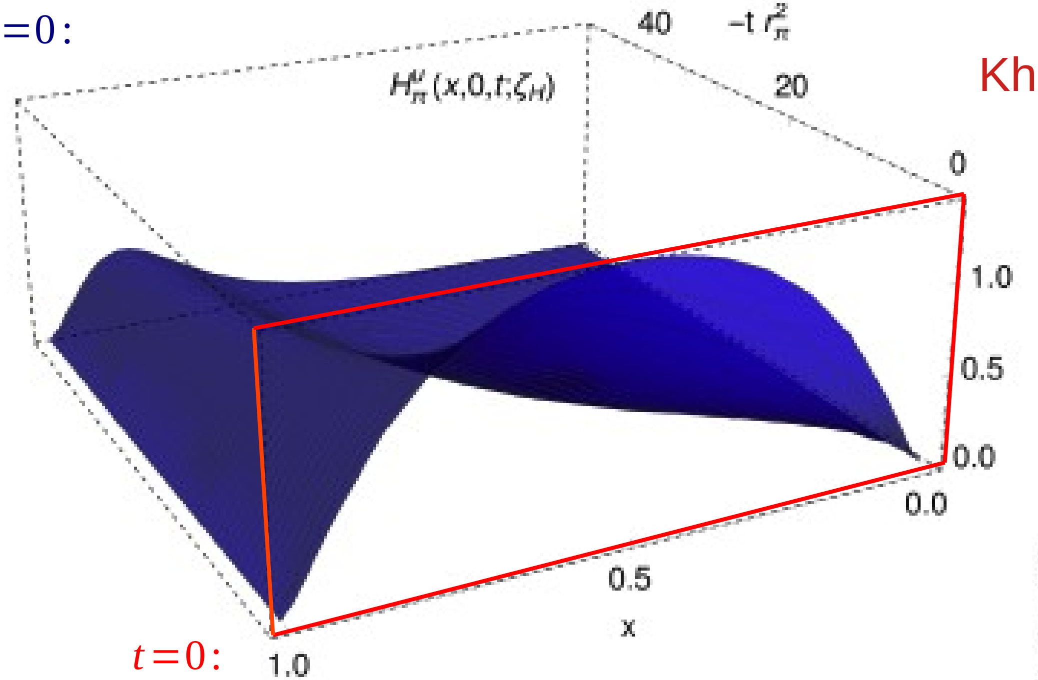

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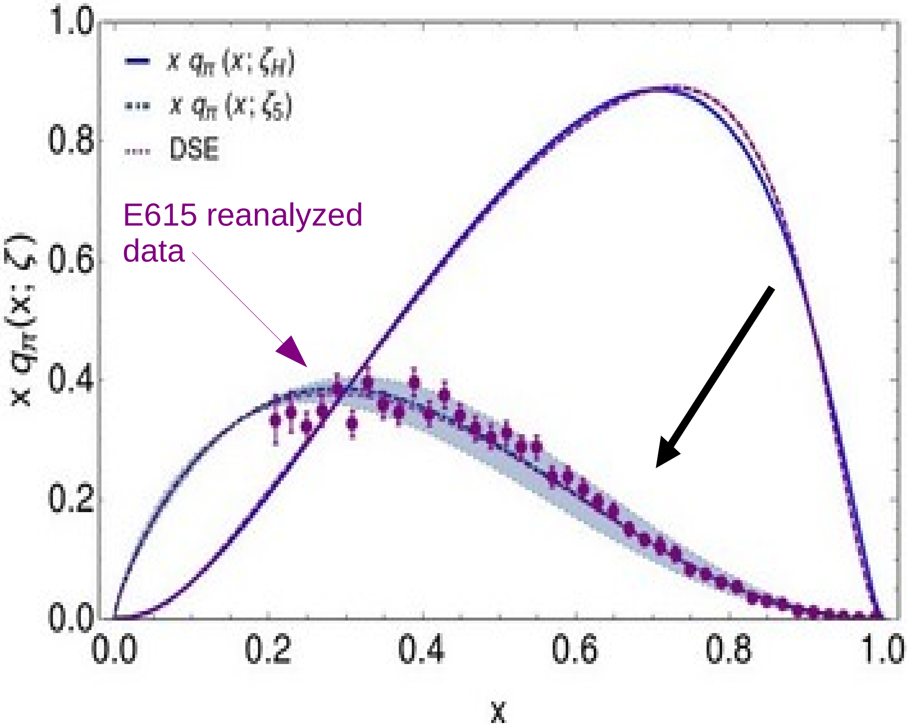

Lattice:

ζ_5	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [55]	0.18(3)	0.064(10)	0.030(5)
Herein	$\boxed{0.20(2)}$	0.074(10)	0.035(6)

R.S. Sufian et al., arXiv:2001.04960

Glue and sea

DGLAP ~~leading-order~~ evolution of forward (and non-skewed) GPDs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \boxed{\alpha(\zeta^2)}\frac{}{4\pi}\underbrace{\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}}_{}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f = 4)$

$$\langle 2x(\zeta_f)\rangle_q = \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d};$$

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Momentum sum rule:

$$\boxed{\langle 2x(\zeta_f)\rangle_q+\langle x(\zeta_f)\rangle_{\rm sea}+\langle x(\zeta_f)\rangle_g = 1}$$

$$\left.\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar d;\\
\langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar q}-(\langle x(\zeta_f)\rangle_u+\langle x(\zeta_f)\rangle_{\bar d}),\\
&= \frac{3}{7}+\frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4}-\langle 2x(\zeta_f)\rangle_u\\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}\right\}$$

Glue and sea

DGLAP ~~leading-order~~ evolution of forward (and non-skewed) GPDs:

$$\left\{\zeta^2\frac{d}{d\zeta^2}\mathbb{1}+\boxed{\frac{\alpha(\zeta^2)}{4\pi}}\begin{pmatrix}\gamma_{qq}^{(n)} & 0 & 0\\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)}\\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)}\end{pmatrix}\right\}\begin{pmatrix}\langle x^n\rangle_{\rm NS}(\zeta)\\ \langle x^n\rangle_{\rm S}(\zeta)\\ \langle x^n\rangle_g(\zeta)\end{pmatrix}=0$$

$$\gamma_{AB}^{(n)}=-\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f=4)$

Momentum sum rule:

$$\boxed{\langle 2x(\zeta_f)\rangle_q+\langle x(\zeta_f)\rangle_{\rm sea}+\langle x(\zeta_f)\rangle_g=1}$$

$$\begin{aligned}\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar d;\\ \langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar q}-(\langle x(\zeta_f)\rangle_u+\langle x(\zeta_f)\rangle_{\bar d}),\\ &= \boxed{\frac{3}{7}}+\frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4}-\langle 2x(\zeta_f)\rangle_u\\ \langle x(\zeta_f)\rangle_g &= \boxed{\frac{4}{7}}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right);\end{aligned}$$

$\zeta_f/\zeta_H\to\infty$

A textbook result:
G. Altarelli, Phys. Rep. 81, 1 (1982)

Glue and sea

DGLAP ~~leading-order~~ evolution of forward (and non-skewed) GPDs:

$$\left\{\zeta^2\frac{d}{d\zeta^2}\mathbb{1}+\frac{\boxed{\alpha(\zeta^2)}}{4\pi}\underbrace{\begin{pmatrix}\gamma_{qq}^{(n)} & 0 & 0\\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)}\\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)}\end{pmatrix}}_{\gamma_{AB}^{(n)} \;=\; -\int_0^1 dx\, x^n P_{AB}^C(x)}\right\}\begin{pmatrix}\langle x^n\rangle_{\rm NS}(\zeta)\\ \langle x^n\rangle_{\rm S}(\zeta)\\ \langle x^n\rangle_g(\zeta)\end{pmatrix}=0$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f=4)$

$$\begin{aligned}\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar{d};\\ \langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}}-\left(\langle x(\zeta_f)\rangle_u+\langle x(\zeta_f)\rangle_{\bar{d}}\right),\\ &= \frac{3}{7}+\frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4}-\langle 2x(\zeta_f)\rangle_u\\ \langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right);\end{aligned}$$

ζ_5	$\langle 2x\rangle_q^\pi$	$\langle x\rangle_g^\pi$	$\langle x\rangle_{\rm sea}^\pi$
Ref.[55]	0.412(36)	0.449(19)	0.138(17)
Herein	$\boxed{0.40(4)}$	0.45(2)	0.14(2)

R.S. Sufian et al., arXiv:2001.04960

Glue and sea

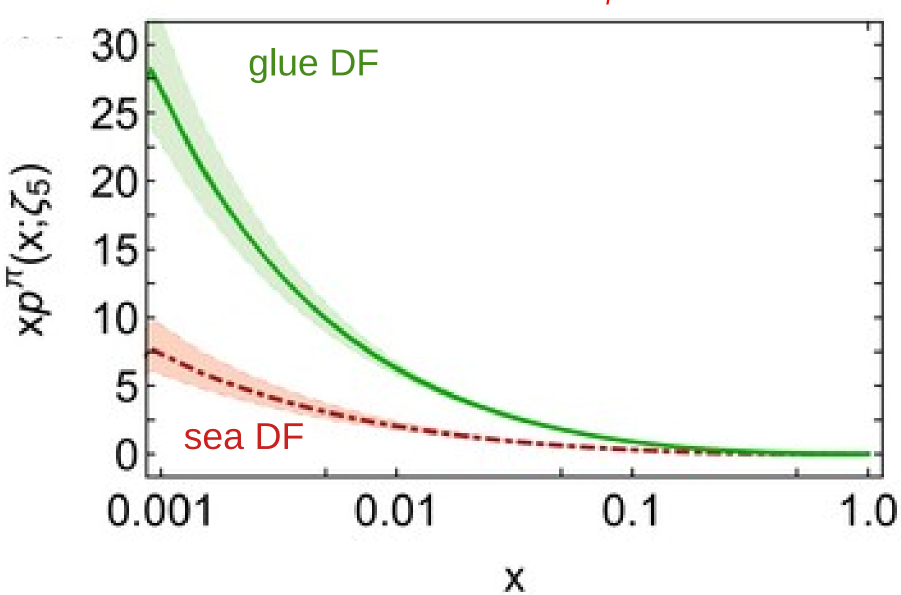
DGLAP ~~leading-order~~ evolution of forward (and non-skewed) GPDs:

$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \mathbb{1} + \frac{\boxed{\alpha(\zeta^2)}}{4\pi} \underbrace{\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f \gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}} \right\} \begin{pmatrix} \langle x^n \rangle_{\rm NS}(\zeta) \\ \langle x^n \rangle_{\rm S}(\zeta) \\ \langle x^n \rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = - \int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f = 4)$. Compute all the moments a reconstruct



(or, equivalently, evolve with the integral equations)

ζ_5	$\langle 2x \rangle_q^\pi$	$\langle x \rangle_g^\pi$	$\langle x \rangle_{\rm sea}^\pi$
Ref.[55]	0.412(36)	0.449(19)	0.138(17)
Herein	$\boxed{0.40(4)}$	0.45(2)	0.14(2)

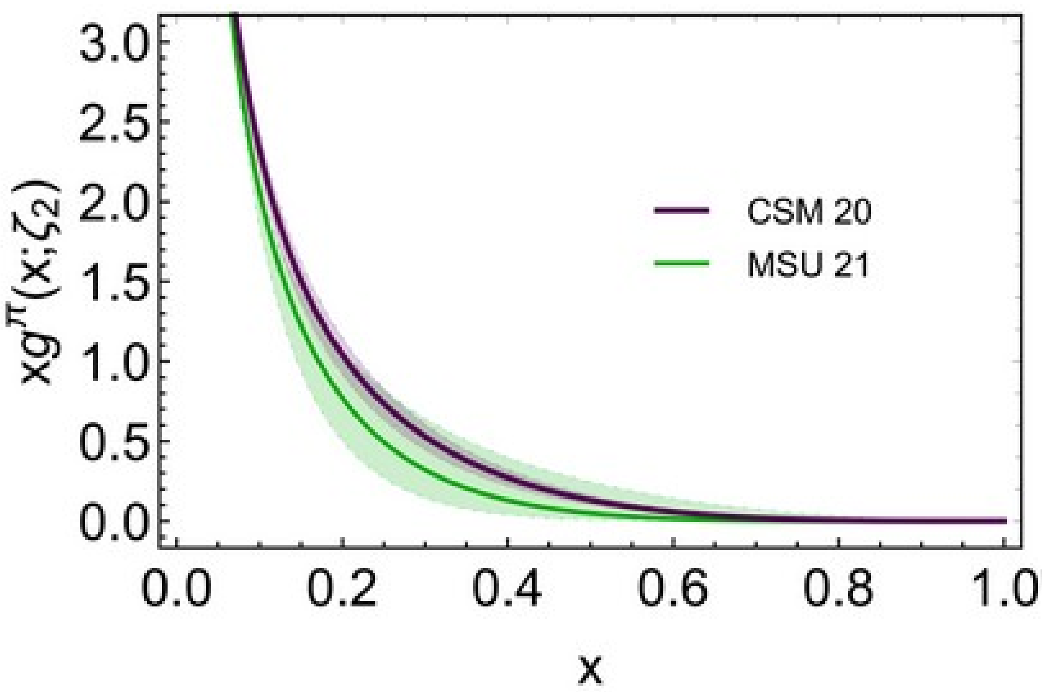
R.S. Sufian et al., arXiv:2001.04960

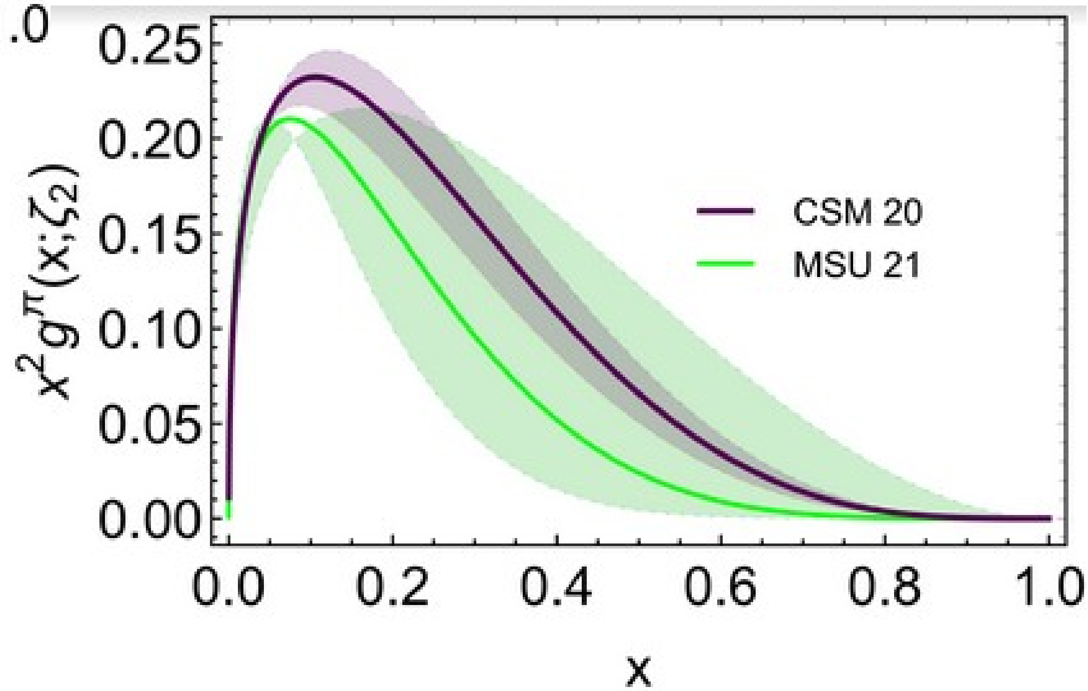
Glue and sea

Cf. Craig's talk on Tuesday!

Focus on glue DF and compare on the domain $x \in (0.1, 1)$ with recent lattice MSU results:
[Z. Fan and H-W. Lin, arXiv:2104.06372]

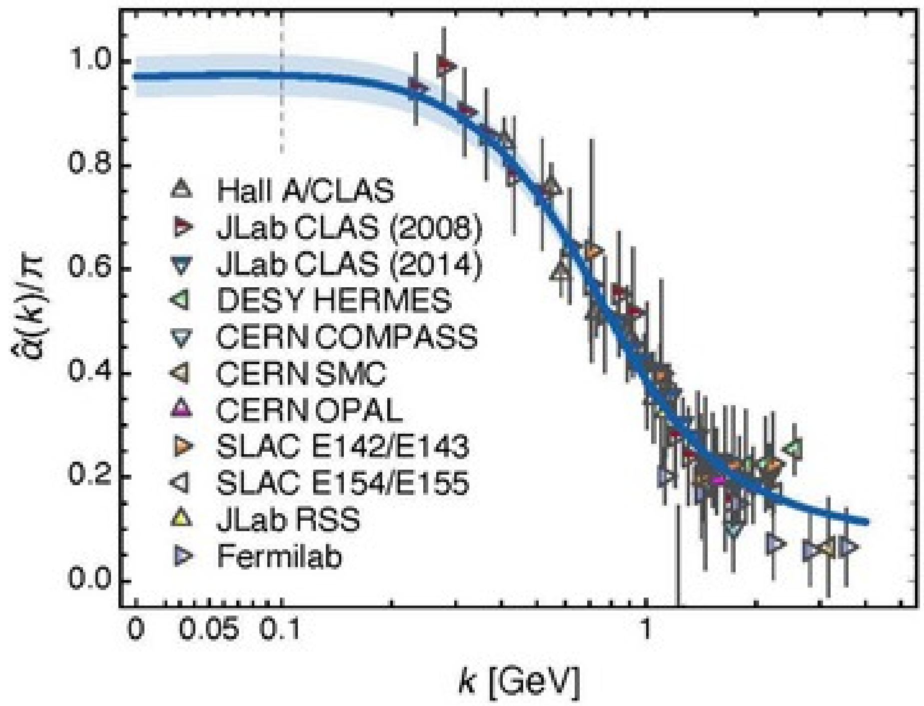
Highlight: pion's glue DF is obtained (via all-orders QCD evolution with an effective charge) from the valence-quark DF computed at the hadronic scale from our LFWFs (or, equivalentely, from a direct evaluation of Mellin moments in DSE/BSE [M. Ding et al, CPC44(2020)3,031002]).





Excellent agreement!
So far, QCD evolution is just encoded by the valence-quark momentum fraction at the final scale: $\langle 2x(\zeta_f)\rangle_q$

QCD effective charge



Modern continuum & lattice QCD analysis in the gauge sector delivers an analogue "Gell-Mann-Low" running charge, from which one obtains a process-independent, parameter-free prediction for the low-momentum saturation

- No landau pole
- Below a given mass scale, the interaction become scale-independent and QCD practically conformal again (as in the lagrangian).

Then, we define:

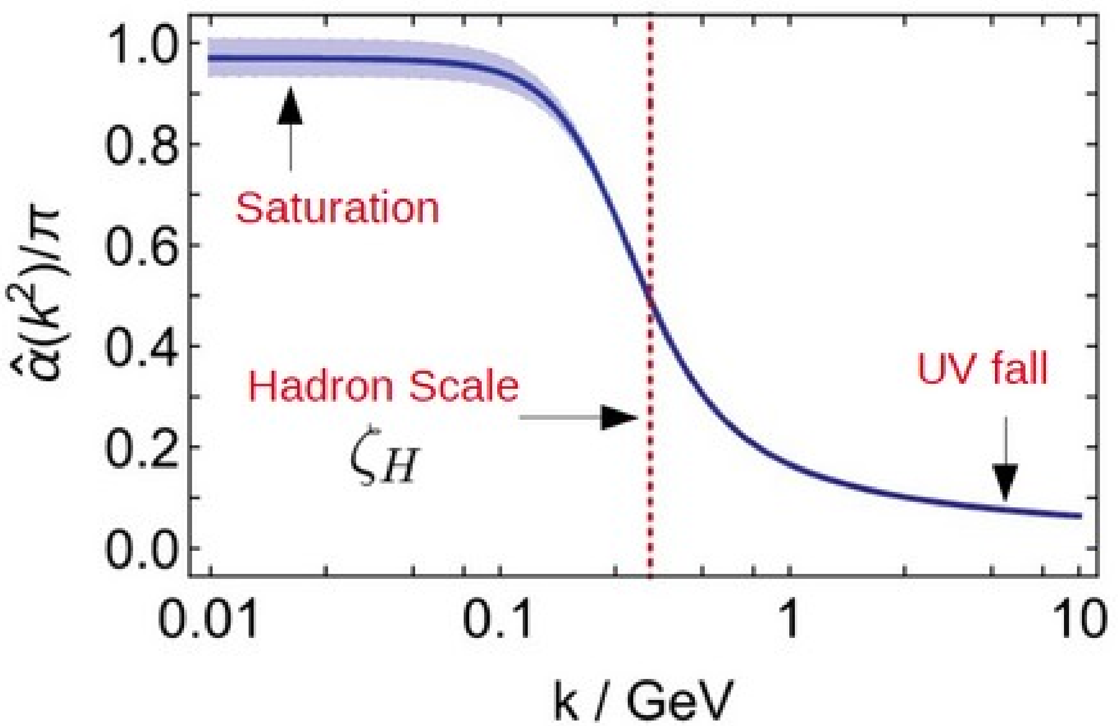
$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}\,;\ \alpha(0) = 0.97(4)$$

where

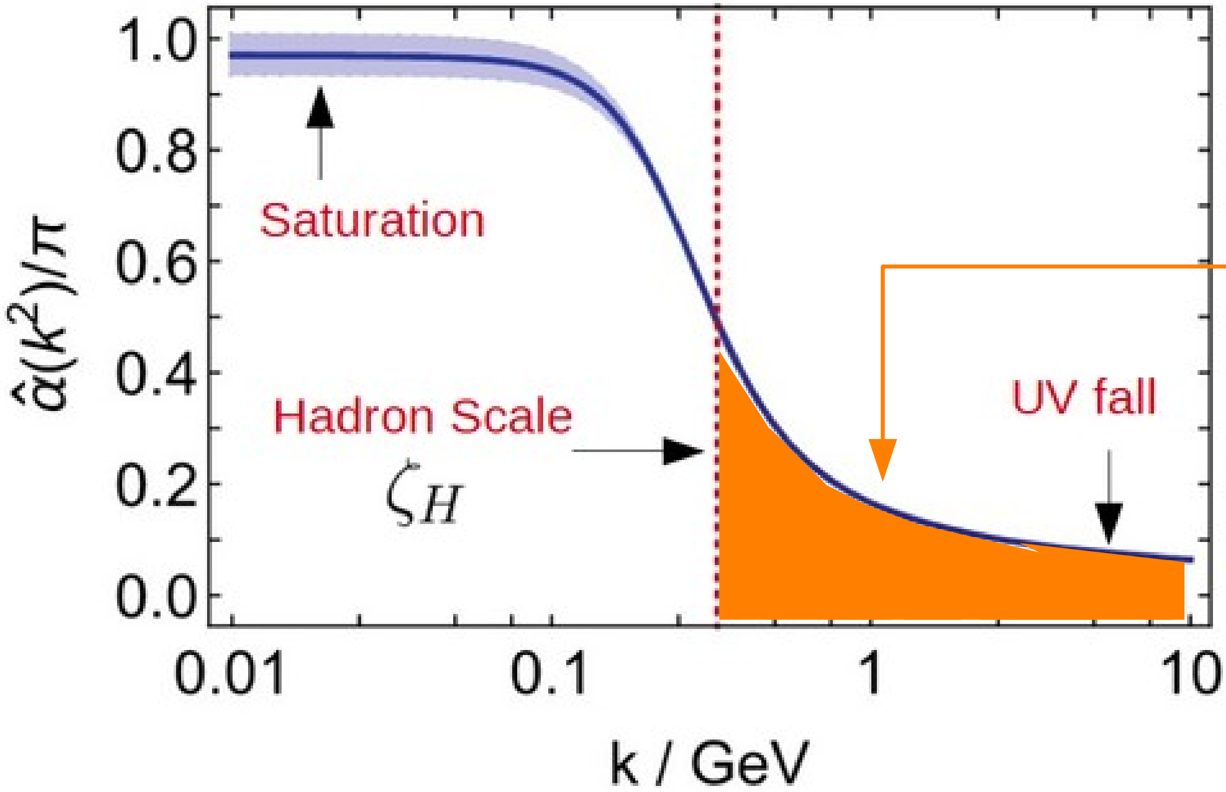
$$\mathcal{M}(k^2 = \Lambda^2_{\rm QCD}) := m_G = 0.331(2)\ \text{GeV}$$

defines the screening mass and an associated wavelength, such that larger gluon modes decouple.

Then, we identify: $\zeta_H := m_G(1 \pm 0.1)$



QCD effective charge

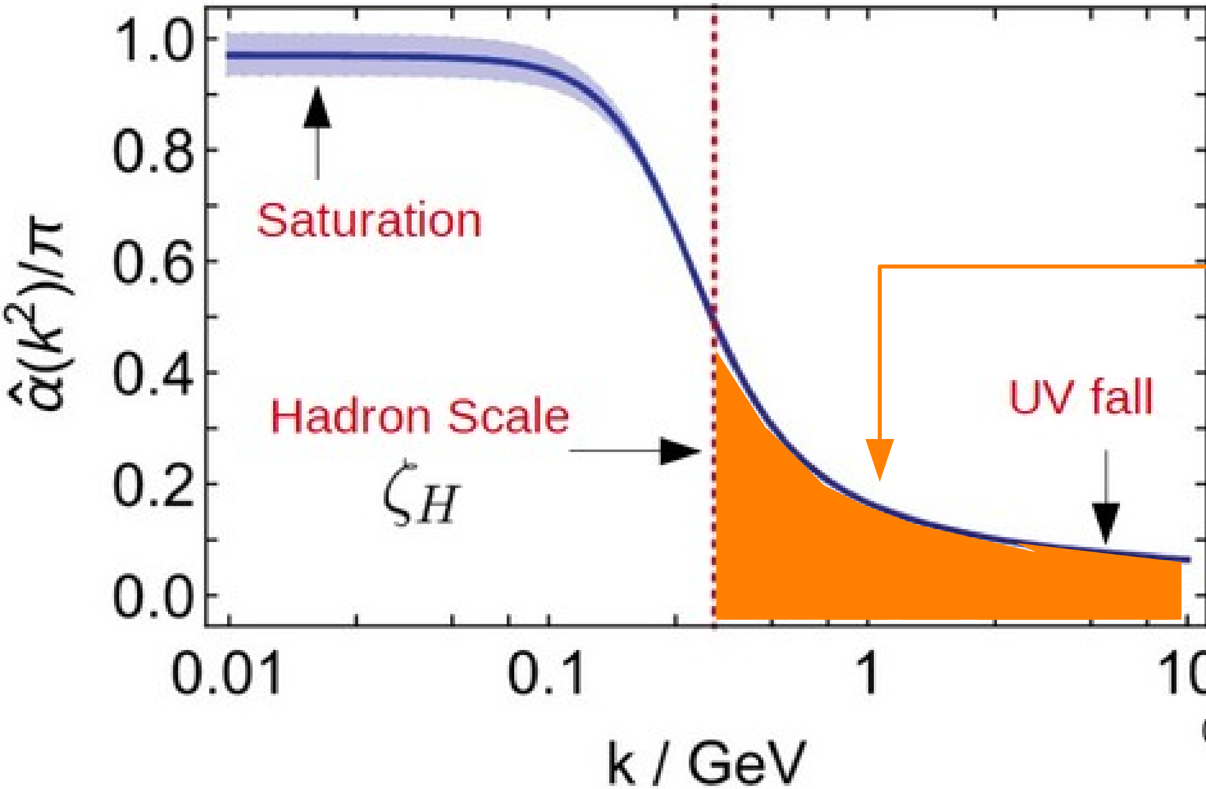


The strength of the charge defines de input for the evolution

$$S(\zeta_H, \zeta_f) = \int_{2\ln(\zeta_H/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \widehat{\alpha}(t)$$

$$\langle x(\zeta_5)\rangle_q^\pi = \frac{1}{2}\exp\left(-\frac{8}{9\pi}S(\zeta_H, \zeta_5)\right) = 0.20(2)$$

QCD effective charge

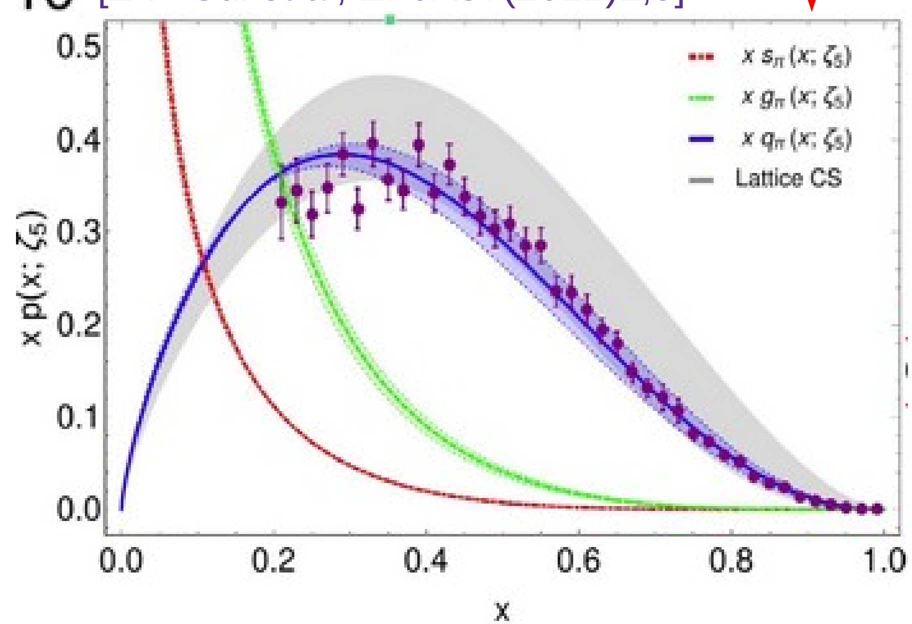


The strength of the charge defines de input for the evolution

$$S(\zeta_H, \zeta_f) = \int_{2\ln(\zeta_H/\Lambda_{\mathrm{QCD}})}^{2\ln(\zeta_f/\Lambda_{\mathrm{QCD}})} dt\, \widehat{\alpha}(t)$$

$$\langle x(\zeta_5)\rangle_q^\pi = \frac{1}{2}\exp\left(-\frac{8}{9\pi} S(\zeta_H, \zeta_5)\right) = \boxed{0.20(2)}$$

[Z-F. Cui et al, EPJC80(2020)11,1064]
[Z-F. Cui et al, EPJA57(2021)1,5]



Then, the glue, valence- and sea-quark DFs can be predicted, with no tuned parameter, on the ground of the effective charge definition, from the LFWF (or, equivalentely, from a symmetry-preserving DSE/BSE computation of the valence-quarks Mellin moments

[M. Ding et al, CPC44(2020)3,031002]

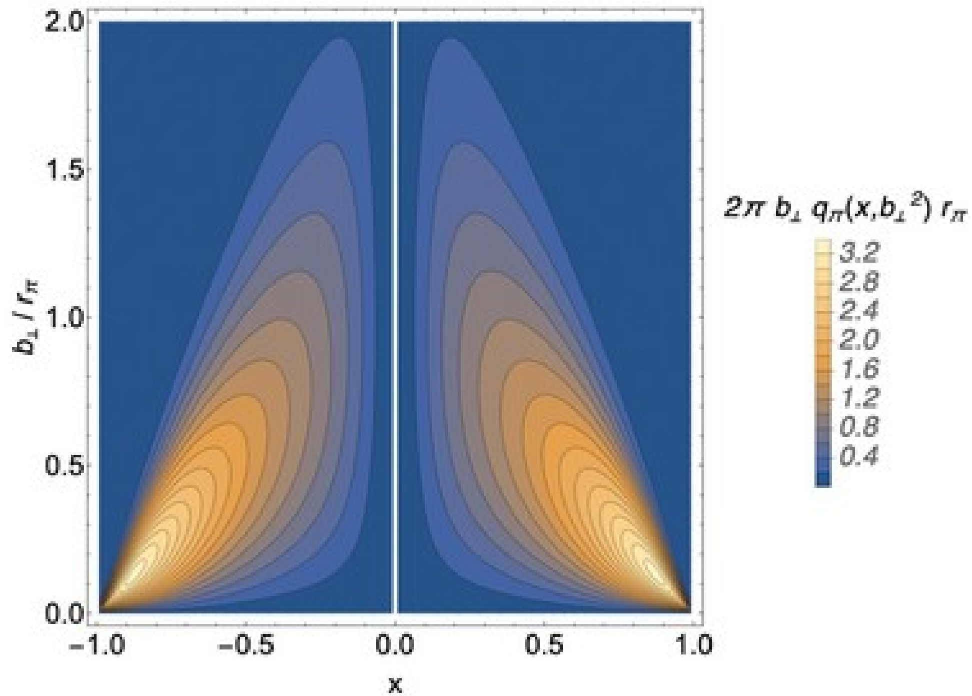
GPDs from LFWFs

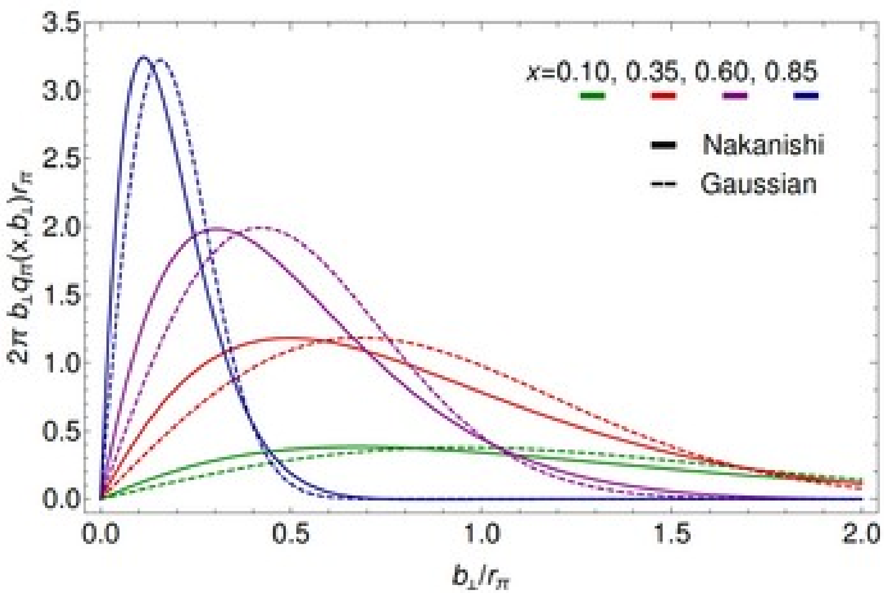
Pion IPD GPD:

$$u^{\pi}(x, b_\perp^2;\zeta_H) = \int_0^\infty \frac{d\Delta_\perp}{2\pi}\Delta_\perp J_0(b_\perp \Delta_\perp)\boxed{H_\pi^u(x,\xi,t;\zeta_H)|_{\xi=0}}$$

The probability of finding the pion's u-quark (x>0) or d-antiquark (x<0) at a distance $b_\perp$ away from the CoTM peaks up at a small but non-zero value and at |x| near 1.

This probability density at x=cte. peaks around a maximum at non-zero $b_\perp$; the larger is x, the smaller $b_\perp$ and the narrower the distribution.
The larger is the momentum fraction carried by the parton, the more it bears on the CoTM definition.





Factorized gaussian ansatz:

$$q^{\pi}(x, b_\perp^2;\zeta_H) = \frac{\gamma_\pi(\zeta_H)}{\pi r_\pi^2}\frac{q^{\pi}(|x|;\zeta_H)}{(1-|x|)^2}\exp\left(-\frac{\gamma_\pi(\zeta_H)}{(1-|x|)^2}\frac{b_\perp^2}{r_\pi^2}\right)$$

$$\gamma_\pi(\zeta_H) = \frac{3\langle x^2\rangle_u^{\zeta_H}}{2} \qquad (q=u[x\geq 0], d[x\leq 0])$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG:\ r_\pi = 0.659(8)\,fm \qquad DSE:\ r_\pi = 0.69\,fm\,[Nakanishi]$

GPDs from LFWFs

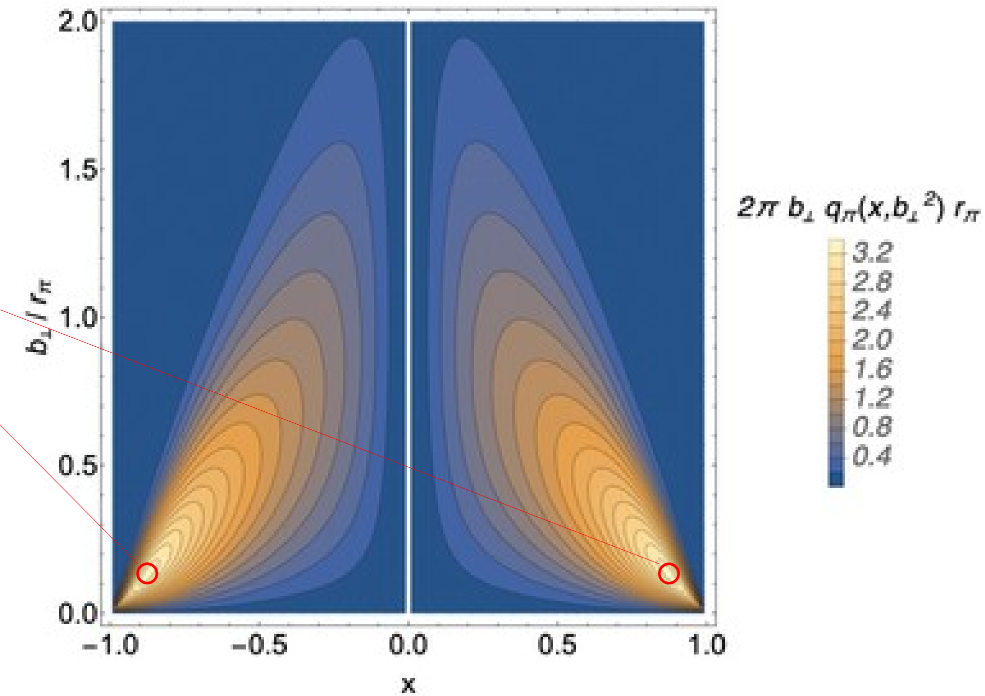
Pion IPD GPD:

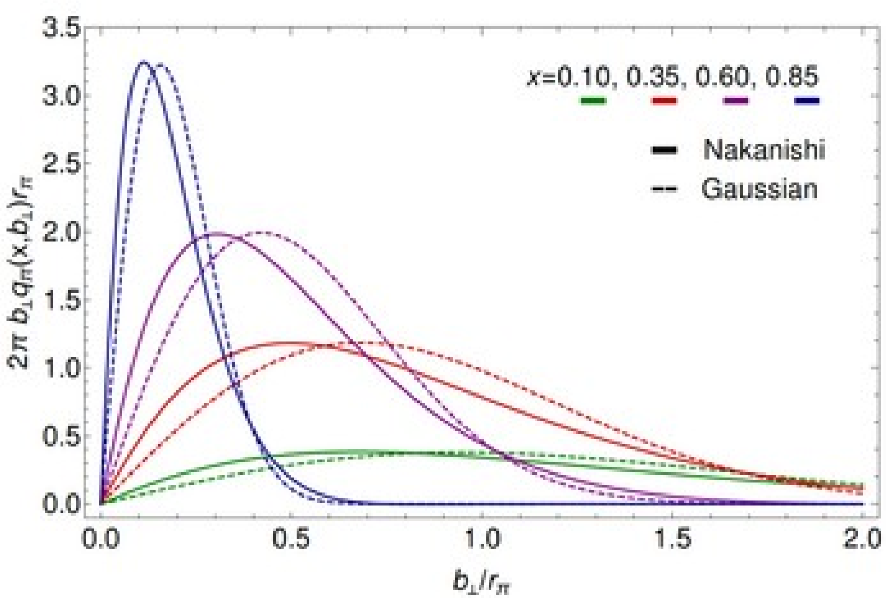
$$u^{\pi'}(x, b_\perp^2; \zeta_H) = \int_0^\infty \frac{d\Delta_\perp}{2\pi} \Delta_\perp J_0(b_\perp \Delta_\perp)\; H^u_{\pi'}(x,\xi,t;\zeta_H)|_{\xi=0}$$

The probability of finding the pion's u-quark (x>0) or d-antiquark (x<0) at a distance $b_\perp$ away from the CoTM peaks up at a small but non-zero value and at |x| near 1.

$(|x|, b_\perp/r_\pi)=(0.91,0.065)$

This probability density at x=cte. peaks around a maximum at non-zero $b_\perp$; the larger is x, the smaller $b_\perp$ and the narrower the distribution.
The larger is the momentum fraction carried by the parton, the more it bears on the CoTM definition.





Factorized gaussian ansatz:

$$q^{\pi}(x, b_\perp^2; \zeta_H) = \frac{\gamma_{\pi'}(\zeta_H)}{\pi r_\pi^2}\, \frac{q^{\pi'}(|x|;\zeta_H)}{(1-|x|)^2} \exp\left(-\frac{\gamma_\pi(\zeta_H)}{(1-|x|)^2}\frac{b_\perp^2}{r_\pi^2}\right)$$

$$\gamma_{\pi'}(\zeta_H) = \frac{3\langle x^2\rangle_u^{\zeta_H}}{2} \qquad (q=u[x\geq 0], d[x\leq 0])$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG:\ r_\pi = 0.659(8)\, fm \qquad DSE:\ r_\pi = 0.69\, fm\, [Nakanishi]$

GPDs from LFWFs

Kaon IPD GPD:

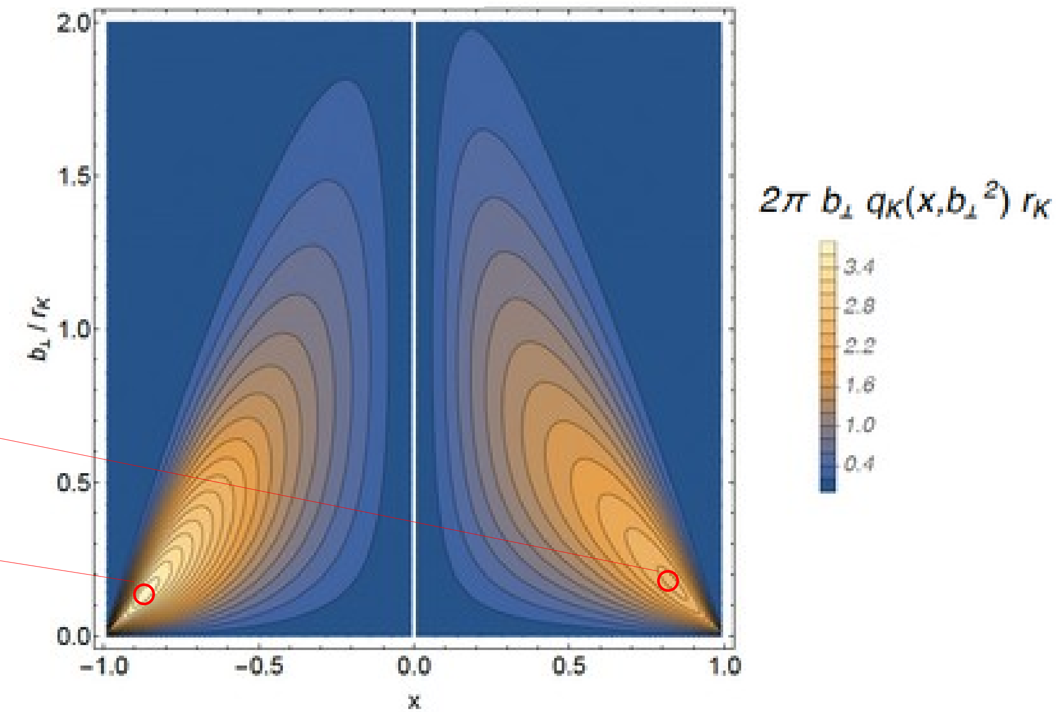
$$u^{K}(x, b_\perp^2; \zeta_H) = \int_0^\infty \frac{d\Delta_\perp}{2\pi} \Delta_\perp J_0(b_\perp \Delta_\perp)\; H_K^u(x,\xi,t;\zeta_H)|_{\xi=0}$$

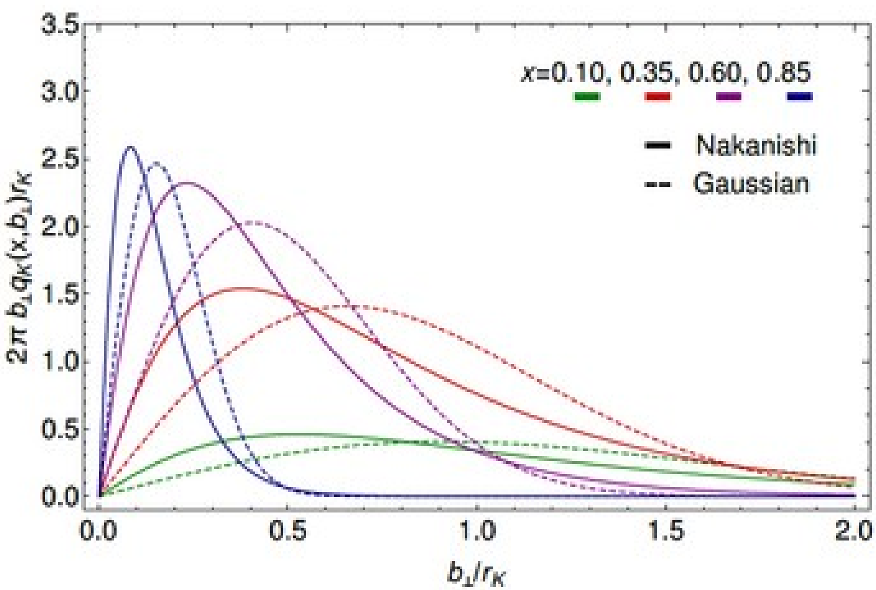
Gaussian LFWF

The flavor asymmetry is made manifest by the comparison of u-quark (x>0) and s-antiquark (x<0) probability densities: the heavier parton, carrying a larger momentum fraction, is more probably found close to the CoTM, to the definition of which it contributes more than the lighter.

$(|x|, b_\perp/r_K) = (0.84, 0.17)$

$(|x|, b_\perp/r_K) = (0.87, 0.13)$





Factorized gaussian ansatz:

$$q^K(x, b_\perp^2; \zeta_H) = \frac{\gamma_K(\zeta_H)}{\pi r_K^2} \frac{q^K(|x|;\zeta_H)}{(1-|x|)^2} \exp\left(-\frac{\gamma_K(\zeta_H)}{(1-|x|)^2}\frac{b_\perp^2}{r_K^2}\right)$$

$$\gamma_K(\zeta_H) = \langle x^2\rangle_{\bar{s}}^{\zeta_H} + \frac{1+\delta}{2}\langle x^2\rangle_u^{\zeta_H} \qquad (q = u[x \geq 0], s[x \leq 0])$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG: \; r_K = 0.560(31)\, fm \qquad DSE: \; r_K = 0.56\, fm\, [Nakanishi]$

GPDs from LFWFs: evolution

Pion IPD GPD:

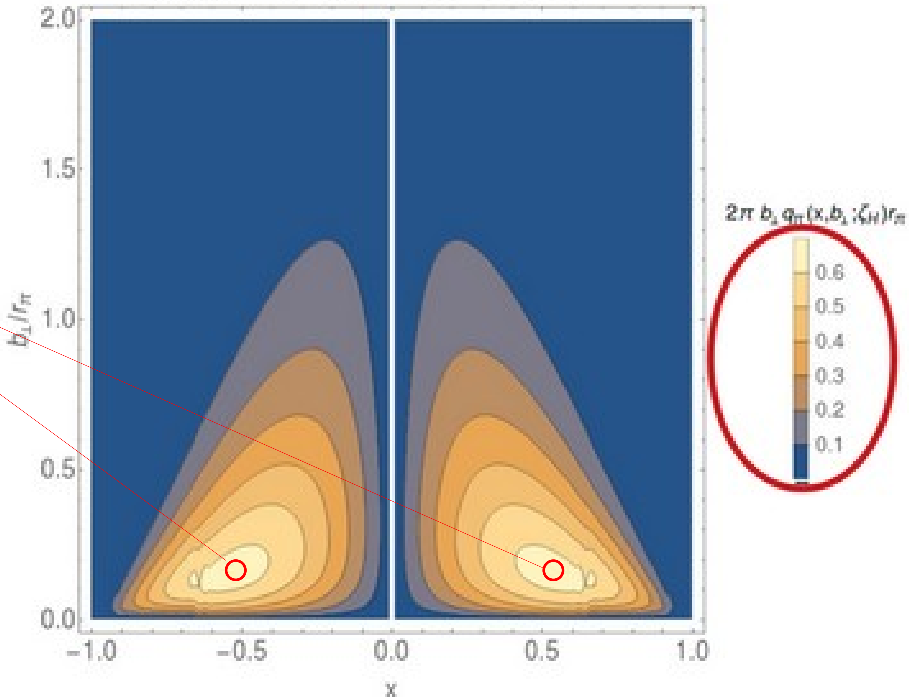
$$u^{\pi'}(x, b_\perp^2; \zeta_H) = \int_0^\infty \frac{d\Delta_\perp}{2\pi} \Delta_\perp J_0(b_\perp \Delta_\perp)\ H_{\pi'}^u(x,\xi,t; \boxed{\zeta})|_{\xi=0}$$

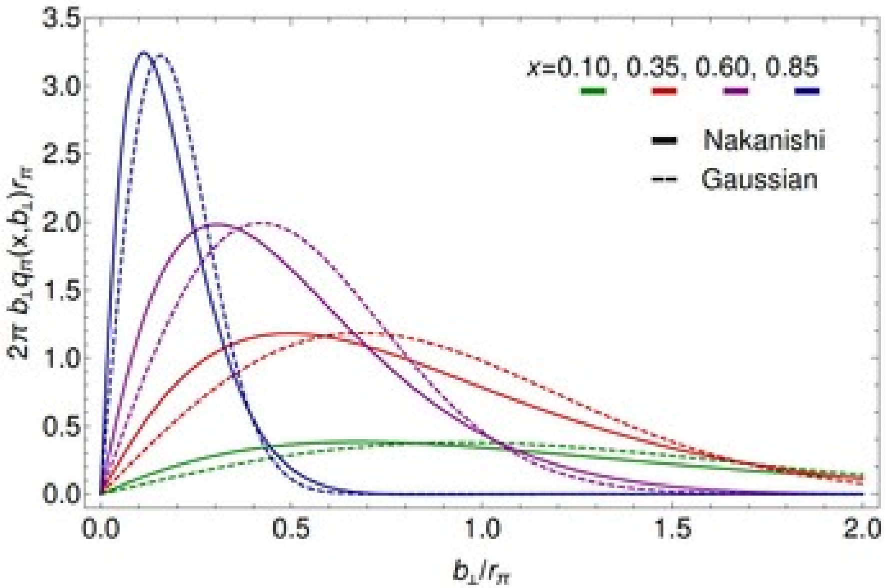
$$\zeta_H \to \zeta = 2\ \text{GeV}$$

The probability of finding the pion's u-quark (x>0) or d-antiquark (x<0) at a distance $b_\perp$ away from the CoTM peaks up at a small but non-zero value and at |x| near 1.

$\boxed{(|x|, b_\perp/r_\pi) = (0.53, 0.065)}$

The peaks clearly broaden, the maximum clearly decreases and drifts towards lower values of the momentum fraction, implying that the dressed quasi-particles share the momentum with the "interacting cloud", losing identity!





Factorized gaussian ansatz:

$$q^{\pi}(x, b_\perp^2; \zeta_H) = \frac{\gamma_{\pi'}(\zeta_H)}{\pi r_\pi^2} \frac{q^{\pi'}(|x|; \zeta_H)}{(1-|x|)^2} \exp\left(-\frac{\gamma_\pi(\zeta_H)}{(1-|x|)^2} \frac{b_\perp^2}{r_\pi^2}\right)$$

$$\gamma_{\pi'}(\zeta_H) = \frac{3\langle x^2\rangle_u^{\zeta_H}}{2} \qquad (q = u[x \geq 0], d[x \leq 0])$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG:\ r_\pi = 0.659(8)\,fm \qquad DSE:\ r_\pi = 0.69\,fm\,[Nakanishi]$

Summary

"One ring to rule them all"

Very much relevant information can be derived from the BSA (solving the bound-state problem in QFT), via the LFWF in the meson sector: DAs, DFs, GPDs, form factors…

The problem can be approached by deriving the low-Fock space LFWF, at a scale where DAs and PDFs can be tightly related, and where the dressed quasi-particles are the relevant degrees of freedom in describing the hadron.

The impact-parameter GPDs can be then obtained and displayed, thus featuring the spatial distribution of the partons inside the mesons.

QCD evolution take the PDFs and the non-skewed GPDs from the hadronic scale to the empirical one. Assuming that an effective charge is defined such that the leading-logarithm kernel gives an all-orders forward (DGLAP) evolution, closed algebraic results can be derived for the momentum fractions and glue, valence- and sea-quars DFs are obtained and remarkably agree with the experiment and lattice QCD.

Backslides

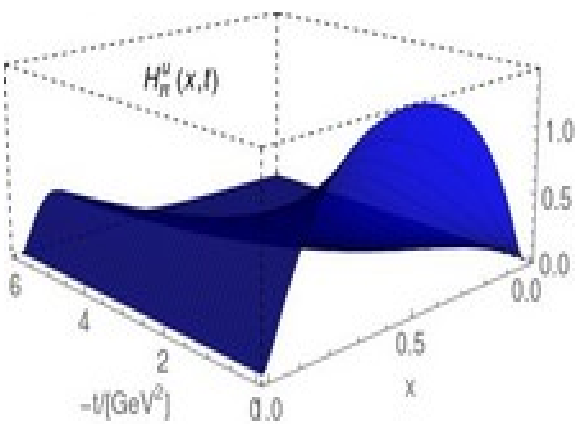
QCD evolution

Evolved GPDs:

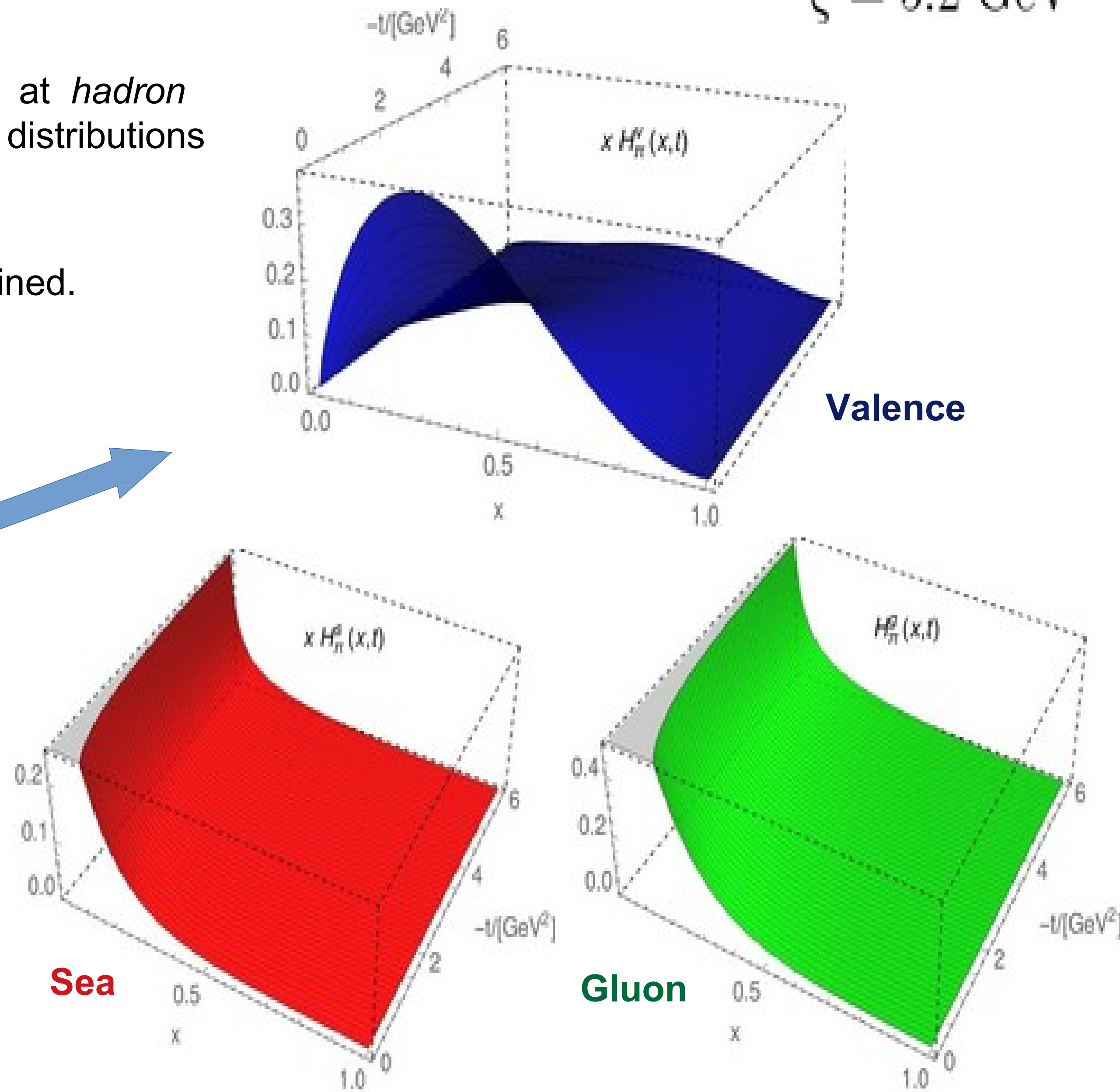
- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.

- Thus **gluon** and **sea** **GPDs** are obtained.

$\zeta = 5.2\ \mathrm{GeV}$



$\zeta_H = 0.331\ \mathrm{GeV}$



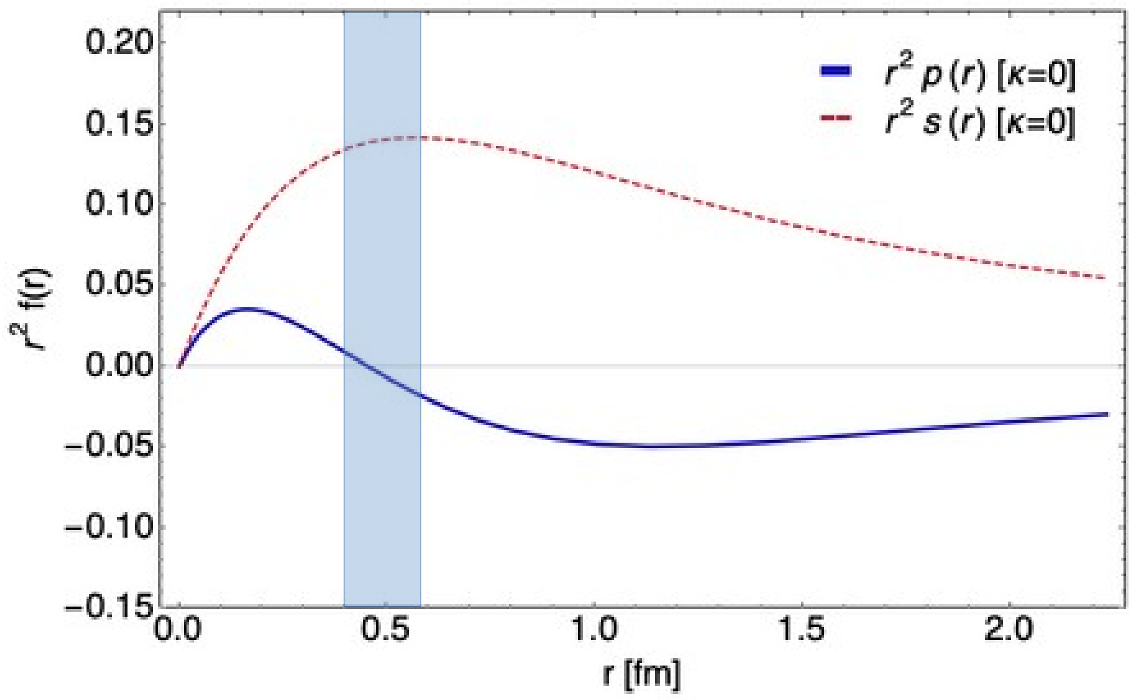
Very preliminary results: 2-d FT pressure distributions

$$p(r) = \frac{1}{3}\int \frac{d^2\Delta}{(2\pi)^2}\frac{e^{i\Delta\cdot r}}{2E(\Delta)}[\Delta^2\theta_1(\Delta)] = \frac{1}{3(2\pi)^2}\int_0^\infty d\Delta \frac{\Delta}{2E(\Delta)}[\Delta^2\theta_1(\Delta)]\int_0^{2\pi} d\phi\, e^{i\Delta r\cos(\phi)}$$

$$\Rightarrow\ p(r) = \frac{1}{6\pi}\int_0^\infty d\Delta \frac{\Delta}{2E(\Delta)}[\Delta^2\theta_1(\Delta)]J_0(\Delta r) \qquad (1)$$

$$s(r) = -\frac{3}{4}\int \frac{d^2\Delta}{(2\pi)^2}\frac{e^{i\Delta\cdot r}}{2E(\Delta)}P_2(\hat{\Delta}\cdot\hat{r})[\Delta^2\theta_1(\Delta)] = -\frac{3}{4(2\pi)^2}\int_0^\infty d\Delta\frac{\Delta}{2E(\Delta)}[\Delta^2\theta_1(\Delta)]\left(2\pi J_0(\Delta r) - \frac{3\pi J_1(\Delta r)}{\Delta r}\right)$$

$$\Rightarrow\ s(r) = -\frac{3}{4}p(r) + \frac{9}{16\pi r}\int_0^\infty d\Delta\frac{\Delta}{2E(\Delta)}[\Delta^2\theta_1(\Delta)]\frac{J_1(\Delta r)}{\Delta} \qquad (2)$$



QCD evolution

Evolved GFFs:

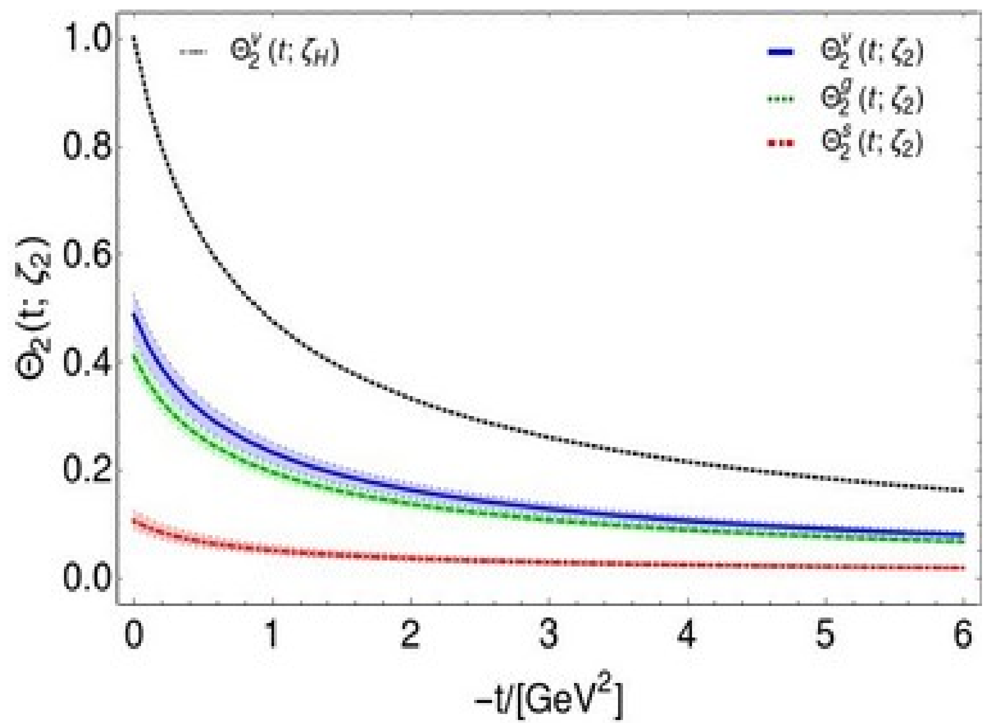
$\zeta = 2$ GeV

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.

- Thus **gluon** and **sea** **GPDs** are obtained.

- The *forward* limit corresponds to the **PDFs**.

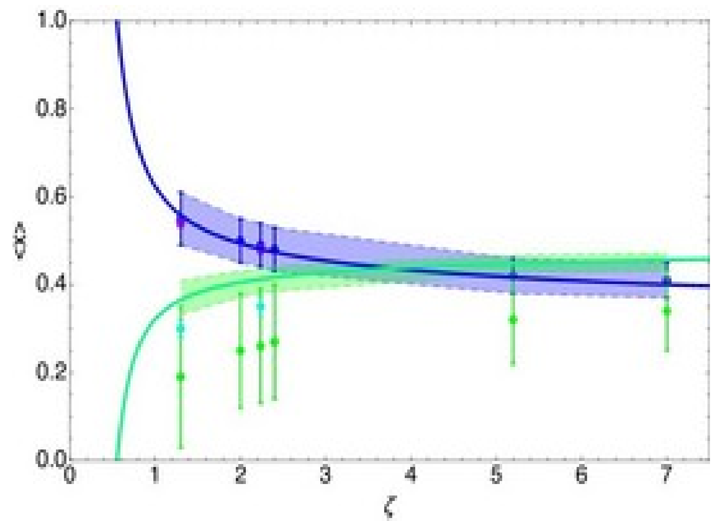
- **GFF** $\theta_2(t)$ comes from the *off-forward* <x>.

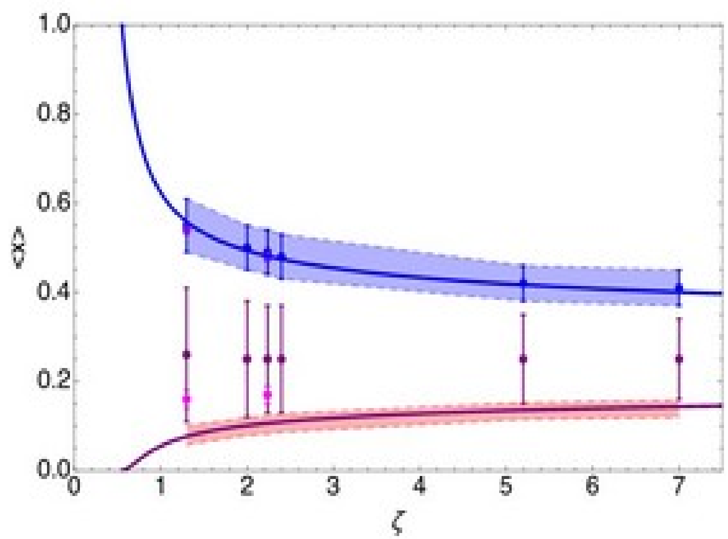


QCD evolution

Evolved GFFs:

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
- **GFF** $\theta_2(t)$ comes from the *off-forward* <x>
- One can also test the evolution with the scale, for instance for the momentum fraction



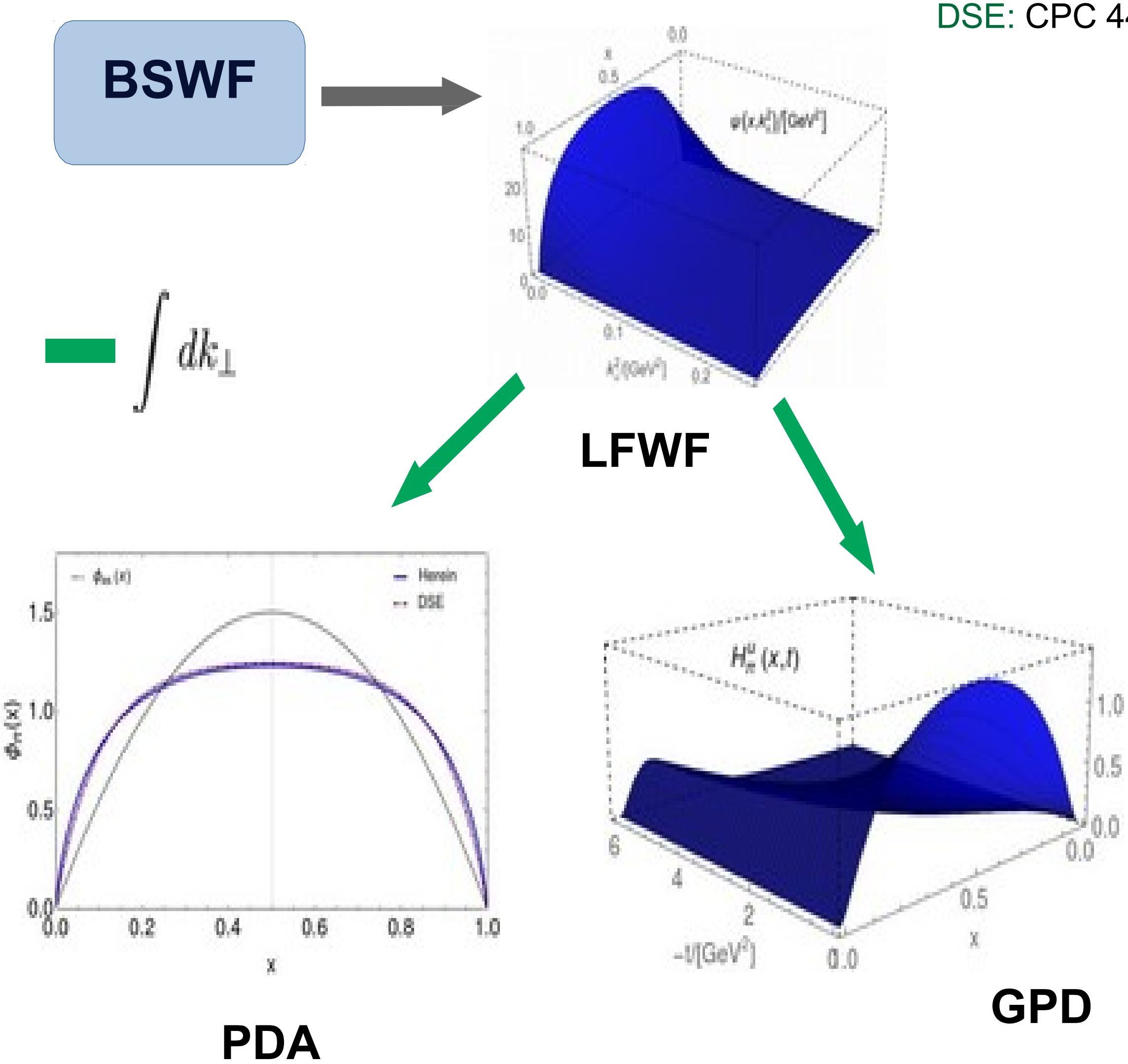


I. Novikov et al, arXiv: 2002.02902 "**xFitter**"

Summary: Pion

➢ Using our **DSE prediction** of pion PDF as **benchmark**, we modeled the pion **BSWF**.

DSE: CPC 44 (2020) no.3, 031002, PRD 101 (2020) no.5, 054014



➢ **Consistent** features of the **PDA**:

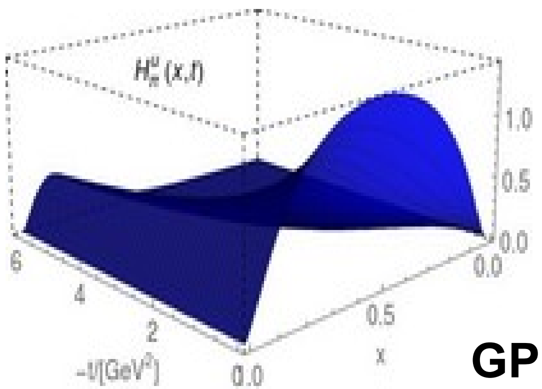
- ✔ Broad and concave at real world scales.
- ✔ Correct endpoint behavior.
- ✔ Agreement with **Lattice** and **DSE** results.

➢ The valence **GPD** is obtained from the **overlap** representation.

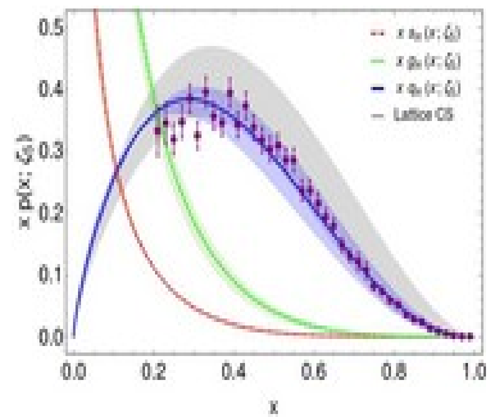
- ✔ Limited to the **DGLAP** region.
- ✔ **Gluon** and **sea** obtained from *evolution* equations.
- ✔ **Extension** to ERBL region is possible.

(but insufficient)

Summary: Pion



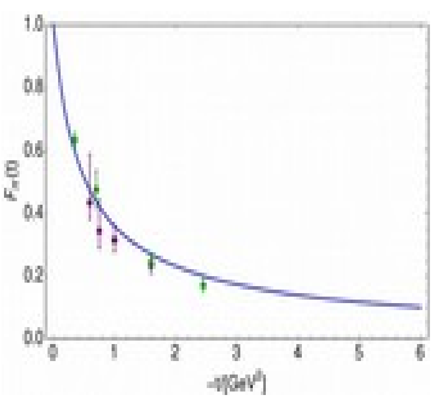
GPD



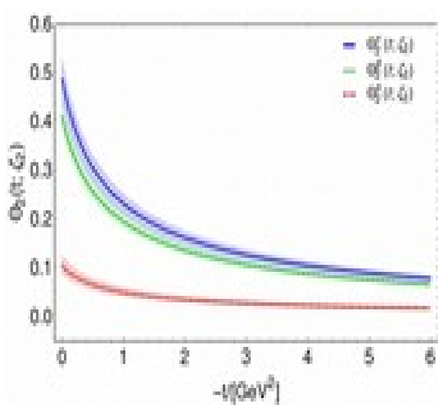
PDF

— $\int dx$

— $t = 0, \xi = 0$



EFF



GFF

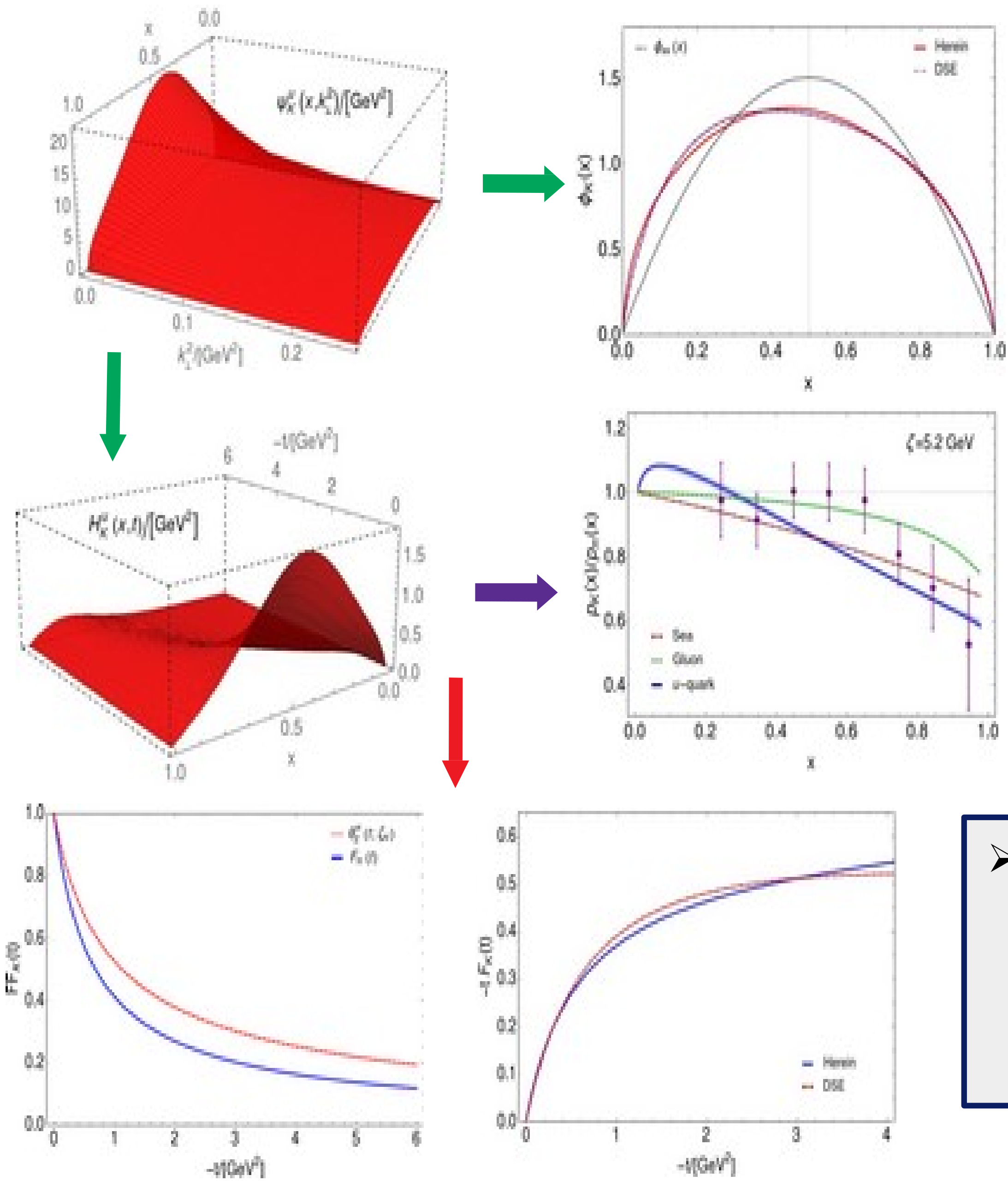
- ➢ Connection **PDF** with **DSE predictions** implies:
  - ✔ Keen agreement with reanalyzed data.
  - ✔ Large-x behavior as predicted by **pQCD**.
  - ✔ Compatible with novel **Lattice** results.

- ➢ **EFF** ***consistent*** with empirical data.
  - ✔ One can trust the off-forward quantities.
  - ✗ ERBL region + D-term **needed.**

- ➢ Intimate ***connection*** with the **running coupling**:
  - ✔ **PI** effective charge → effective coupling for **evolution**.
  - ✔ Specific ***definition*** of the hadron scale.

➔Both **LFWF** and **GPD** are **promising candidates** to be the real objects.

Summary: Kaon



- Connection with **DSE predictions** implies:
  - ✔ **Qualitiative** features of the distributions are properly captured.
  - ✔ *Large-x* behavior of the **PDA** and **PDF** as predicted by **pQCD**.
  - ✔ **K/pi PDF** ratio in agreement with data.

We still need new experiments !!!

✔ Computed **Gluon** and **Sea** Kaon **PDFs**

the **GPDs** are available too !!!

- Next steps:
  - ✔ **Impact** parameter distributions
  - ✔ **Transverse** momentum distributions **(TMDs)**