# EHM and the spectrum of light mesons

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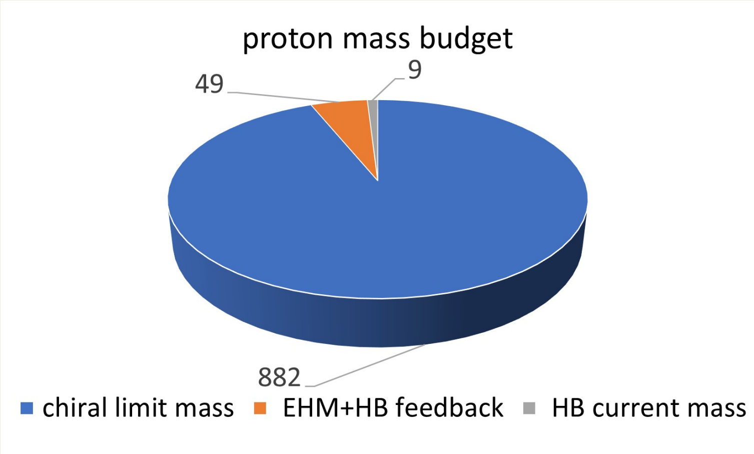
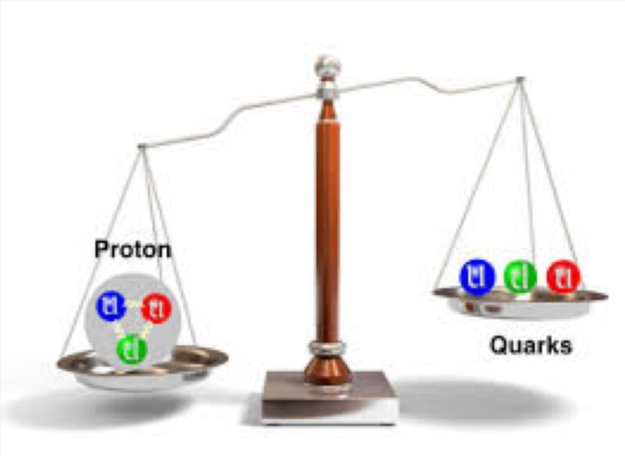
**Nankai University**

Perceiving the EHM through AMBER@CERN-V,
2021/04/30, online

# Mass budget




- chiral limit mass: absence of Higgs coupling(Emergent Hadronic Mass)
- HB current mass:  Higgs-boson effects
- EHM+HB feed back: interference between Emergent Hadronic Mass and HB current mass

**Absence of Higgs(in the chiral limit)**

✓ A very large fraction of the measured proton mass emerges as a consequence of the trace anomaly…by glue and the interactions between them;

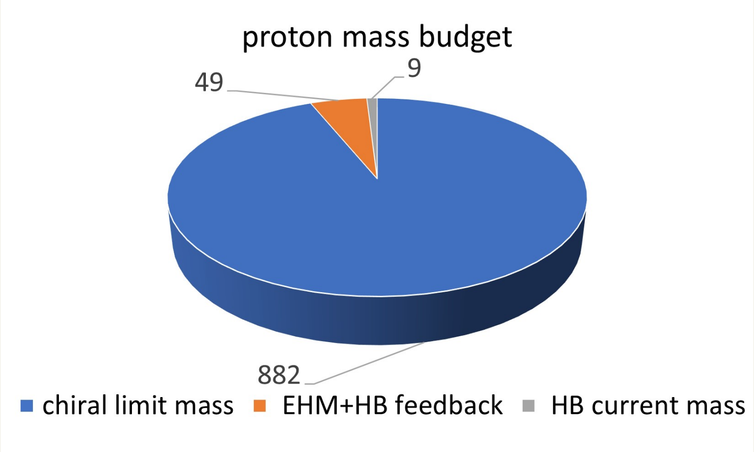
**Restoring Higgs boson couplings**

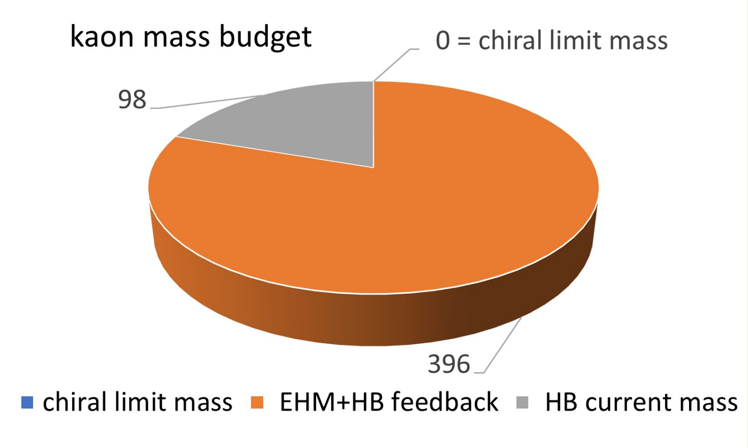
✓ Sum of hadron's valence-quark current masses……..0.01 $m_p$

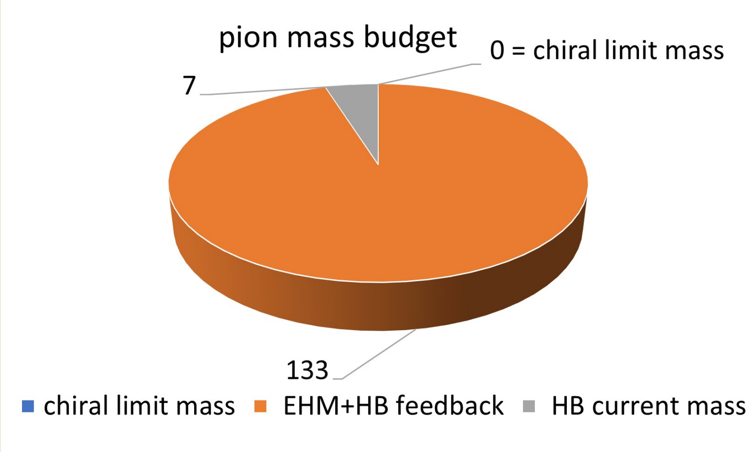
**Interference………………quark condensates☺**

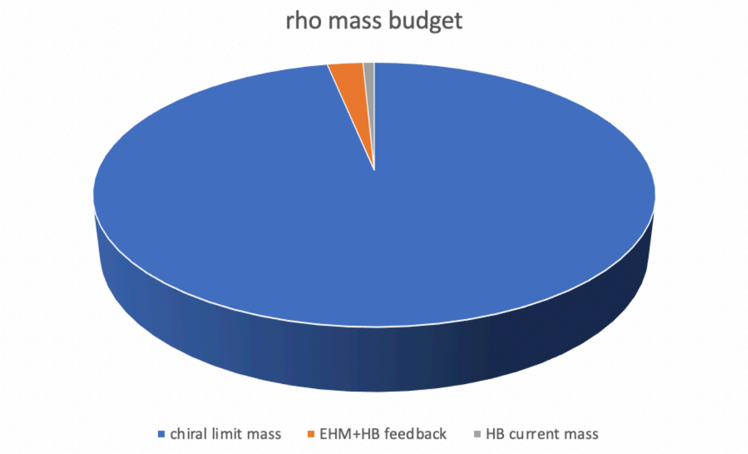
5% for proton

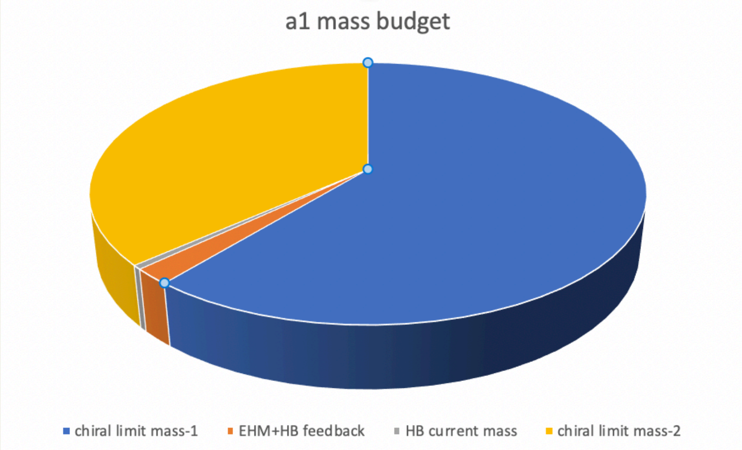
# Mass budget



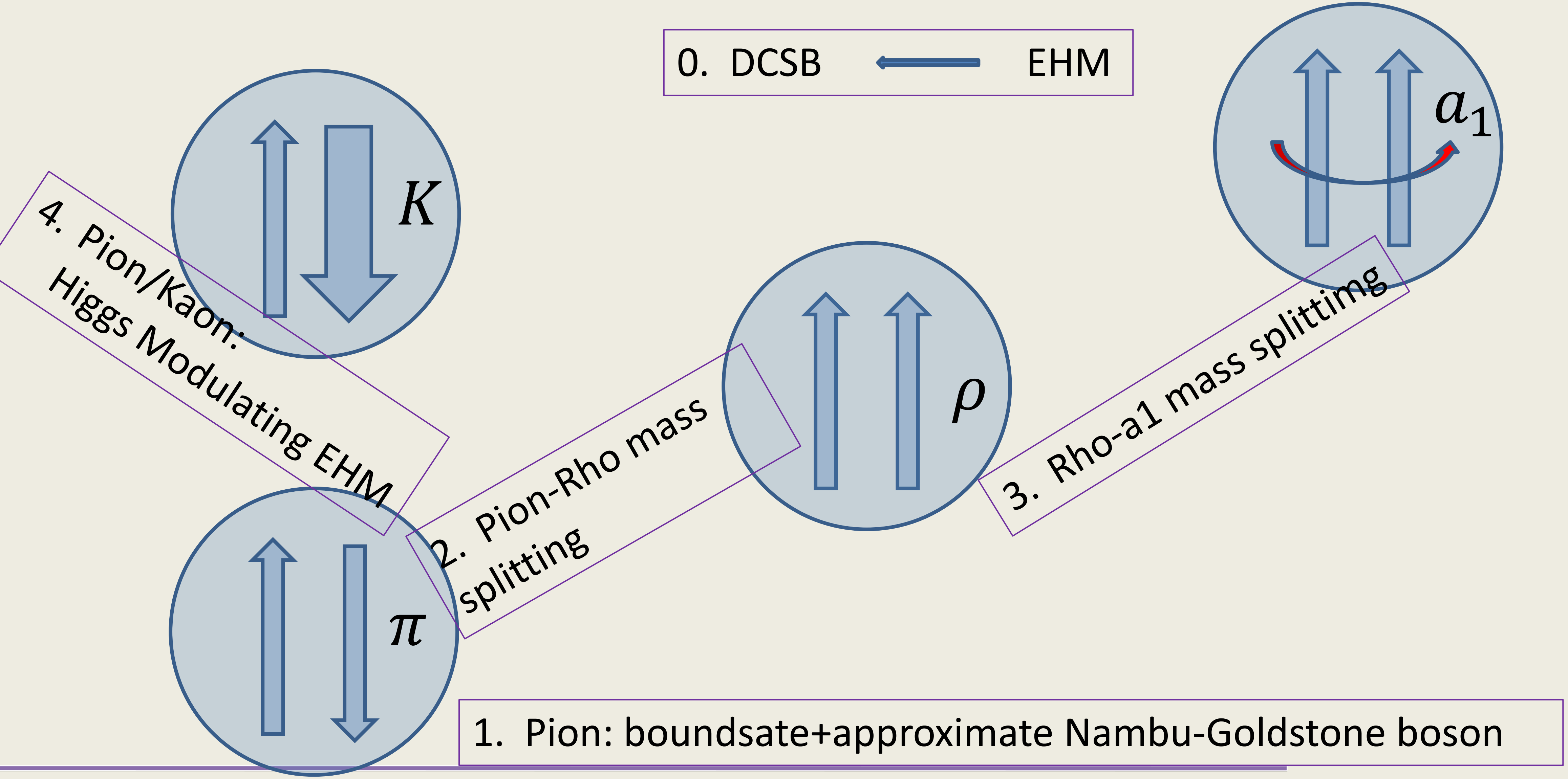




- Pion and Kaon masses are ZERO(NG mode associated with DCSB)
- Considering vector and axial-vector mesons mass budget is interesting and useful.
- More details of strong interaction encountered.

# Outline



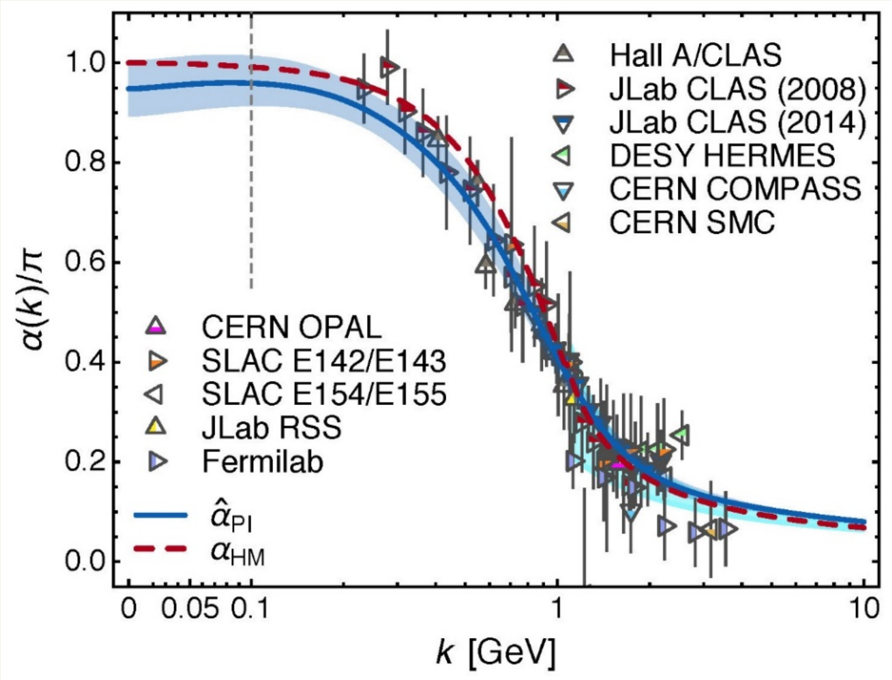
0. DCSB
EHM
K
4. Pion/Kaon: Higgs Modulating EHM
ρ
$a_1$
2. Pion-Rho mass splitting
3. Rho-a1 mass splittimg
π
1. Pion: boundsate+approximate Nambu-Goldstone boson

# Outline



0. DCSB ⟵ EHM

# PI running coupling of QCD



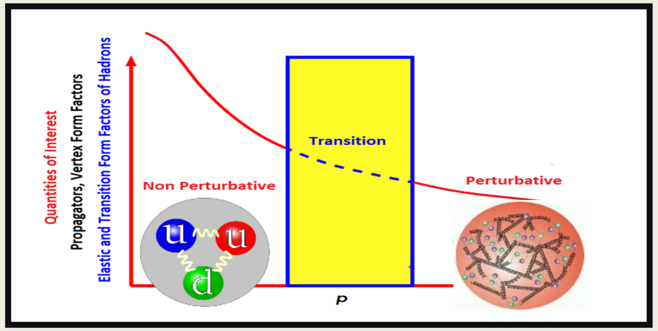

$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{QCD}}\right]}, \quad \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

Define a screening mass:

$$m_G := \mathcal{K}(k^2 = \Lambda^2_{QCD}) = 0.331\text{GeV}$$

The running coupling alters at $m_G$ so that modes with $k^2<m^2$ are screened from interactions and theory enters a practically conformal domain.

- Gluon/Quarks progressivley become more sorphisticated as experience grew with formulating and solving the quark gap equation and as computational methods and power improved for lattcie-regularised QCD.




*Valence Picture at Hadronic Scale*!

# In QCD: Gluons become massive!

PHYSICAL REVIEW VOLUME 125, NUMBER 1 JANUARY 1, 1962

**Gauge Invariance and Mass**

JULIAN SCHWINGER
*Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California*
(Received July 20, 1961)

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

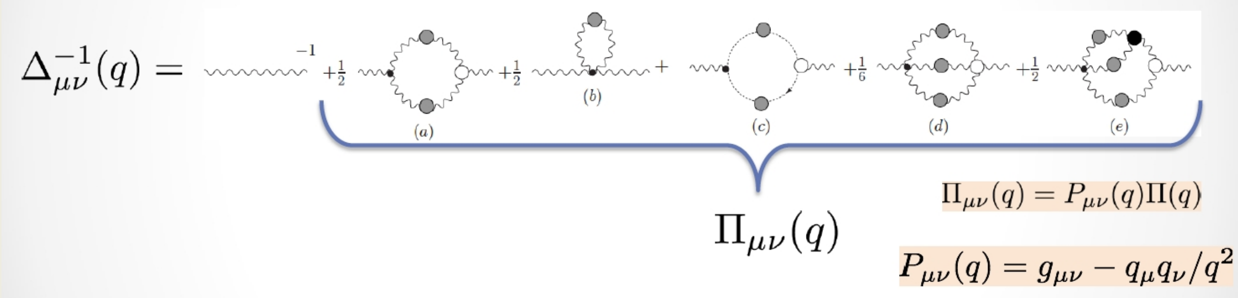
- Schwinger 1962

PHYSICAL REVIEW D VOLUME 26, NUMBER 6 15 SEPTEMBER 1982

**Dynamical mass generation in continuum quantum chromodynamics**

John M. Cornwall
*Department of Physics, University of California, Los Angeles, California 90024*
(Received 30 April 1982)

- Cornwall 1982

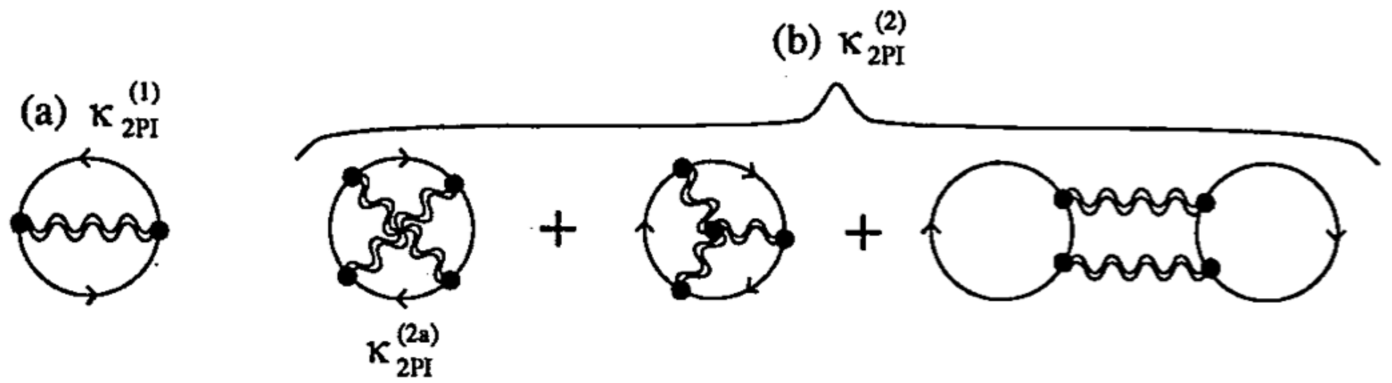



- Binosi & Papavassiliou

Phys. Rept. 479 (2009)1-152
Pinch Technique: Theory and Applications

# Systematics---CJT effective potential

$$\Gamma[S_F, A] = i\mathrm{TrLn}S_F - \mathrm{Tr}(i\not{D}S_F) + i^{-1}\mathcal{K}_{2PI}[S_F],$$




For Example

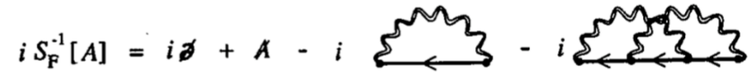



Fig. 10. SD equation using $\mathcal{K}_{2PI}=\mathcal{K}_{2PI}^{(1)}+\mathcal{K}_{2PI}^{(2a)}$.

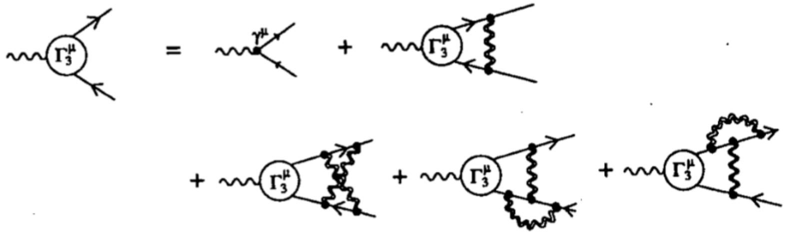



Fig. 11. BS equation for $\Gamma_3^\mu$ using $\mathcal{K}_{2PI}=\mathcal{K}_{2PI}^{(1)}+\mathcal{K}_{2PI}^{(2a)}$.

- 2PI[(1)] : Rainbow-Ladder

H.J.Munczek and A. M. Nemirovsky,PRD28(1983)181
P.Maris and P.C.Tandy, PRC60(1999)055214
Si-xue Qin, et al., PRC84(2011)042202
J.M.Cornwall, PRD83(2011)076001

- 2PI[(1)]+ 2PI[(2a)]

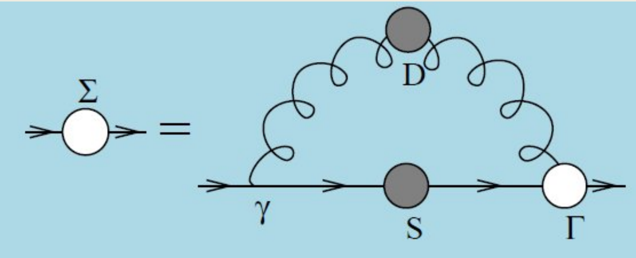
A. Bender, C. D. Roberts, L.Von Smmekal, PLB380(1996)7.

- 3PI

R. Williams, C. S. Fischer, W. Heupel, PRD93 (2016) 034026.

A long-standing way!!!

# Put Physics at Right Place: Minding the quark-gluon vertex




LC, and C.D.Roberts,PRL103(2009)081601, PRC85(2012)052201;
D. Binosi, et al., PLB742(20015) 183
Sixue Qin, C.D.Roberts, arXiv: 2009.13637

- Truncate quark-gluon vertex with DCSB-improvement ansatz;
- Performing the interaction from lattice QCD;
- Ward identity hold…guarantee proper current quark mass evolution;
- ACM generate quark mass and trigger DCSB.

$$\hat{\Gamma}_\nu(p,q) = \Gamma_\nu^{\rm BC}(p,q) + \Gamma_\nu^{\rm ACM}(p,q)$$

$$-i(p+q)_\mu \frac{B(p^2)-B(q^2)}{p^2-q^2} + \eta\sigma_{\mu\alpha}k_\alpha \frac{B(p^2)-B(q^2)}{p^2-q^2}\mathcal{H}(k^2)$$

DCSB improvement
Modulating spin-orbital splitting

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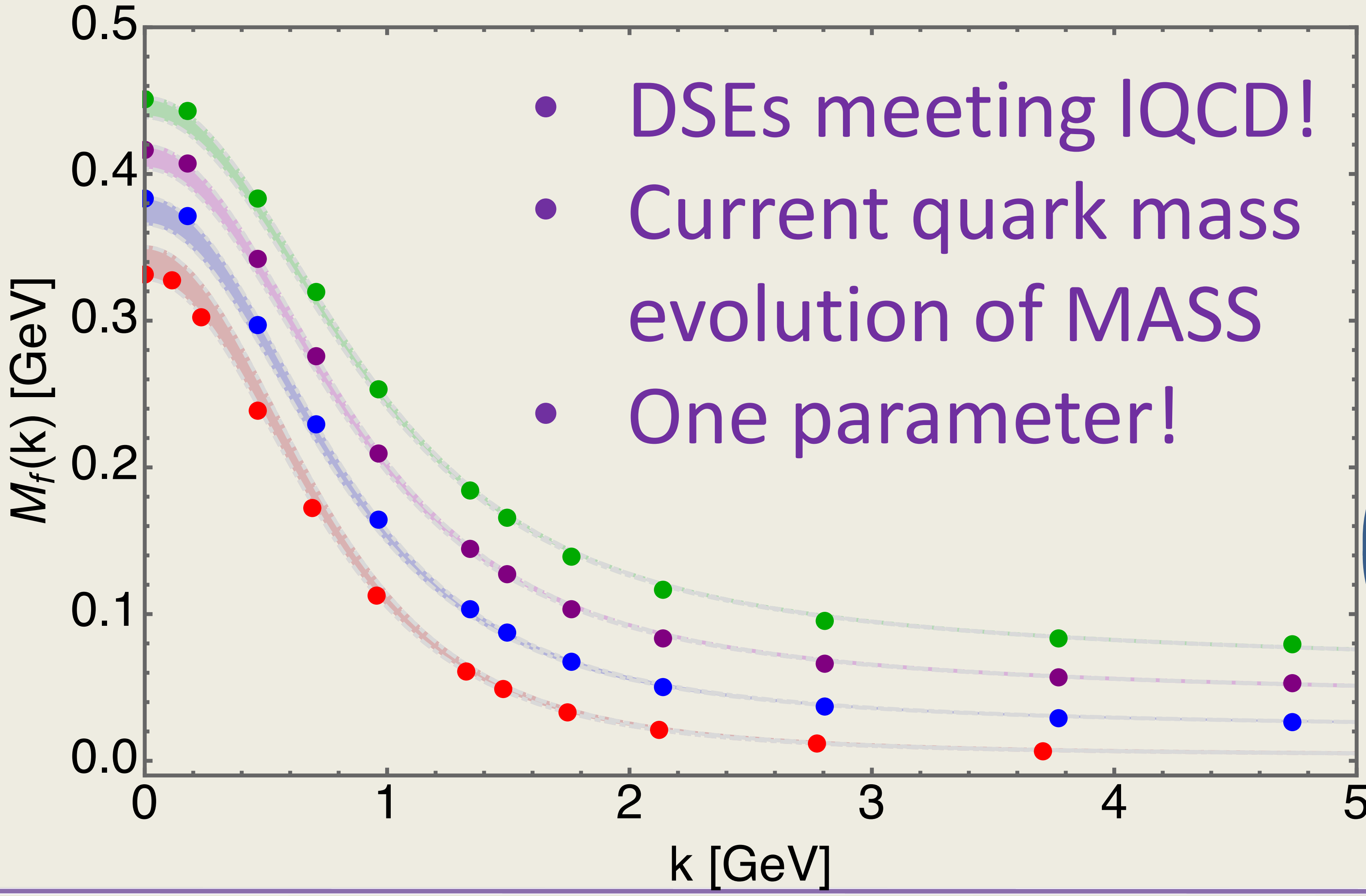
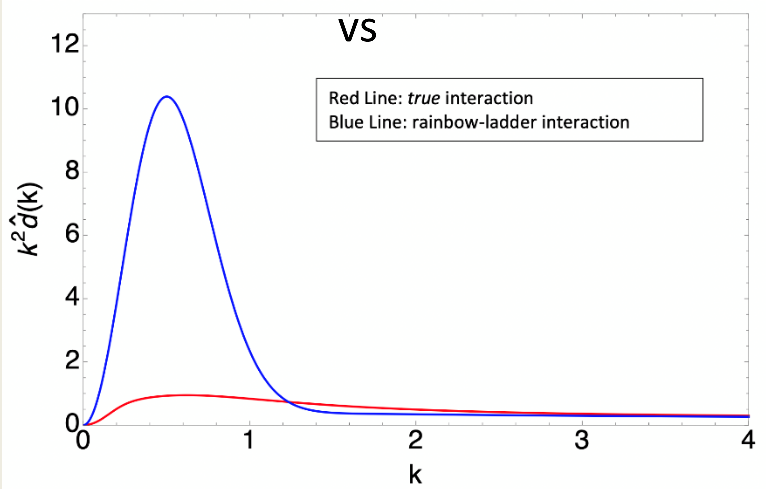
- DSEs meeting lQCD!
- Current quark mass evolution of MASS
- One parameter!



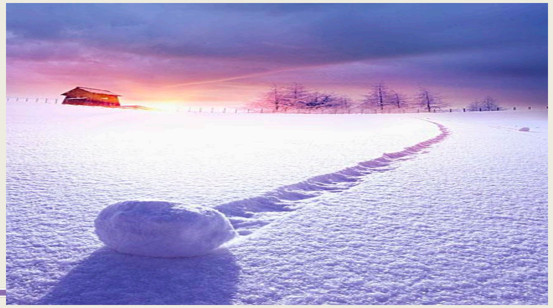

Rainbow-Ladder: input strong interaction by hand



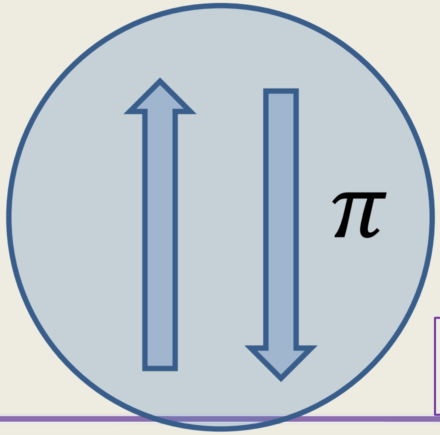

Mass scale introduced by PI coupling

Trigger quark mass

DCSB enhance quark-gluon coupling



# Outline




1. Pion: boundsate+approximate Nambu-Goldstone boson

# Symmetry constraints and Goldstone Boson------Model Independent!

Maris, Roberts and Tandy, **Phys. Lett. B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

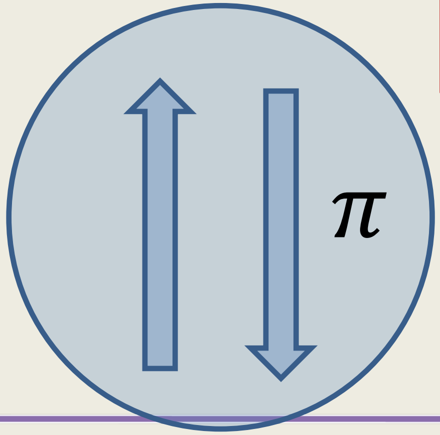
- Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2\frac{d^2B(k^2)}{d^2k^2} + \ldots$$

$$m_\pi \propto \sqrt{m}$$




- The symmetry-preserving improvement gurantee the Goldstone-boson character of PION;
- The symmetry-preserving improvement guarantee the cancelation of ATTRATIVE and REPULSIVE interaction in pseudoscalar and vector channels.

# Wave Function------Model dependent!



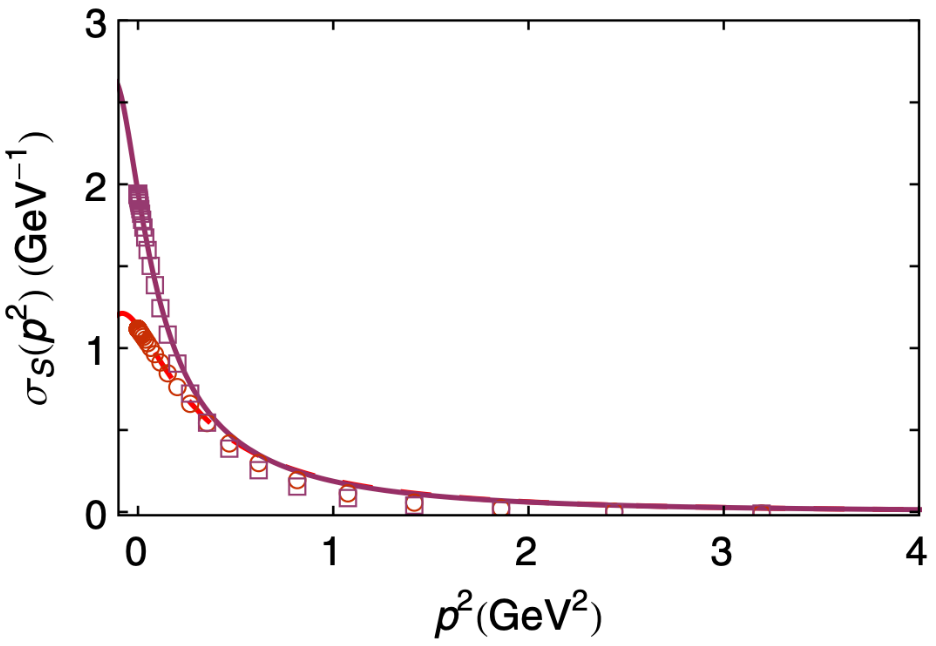



FIG. 1 (color online). $\sigma_S(p^2)$ in Eq. (4a)—RL kernel: solution (open circles) and interpolation function (long-dashed curve); and DB kernel: solution (open squares) and interpolation function (solid curve). In the chiral limit at large $p^2$, $\sigma_S(p^2) \sim 1/p^4$.

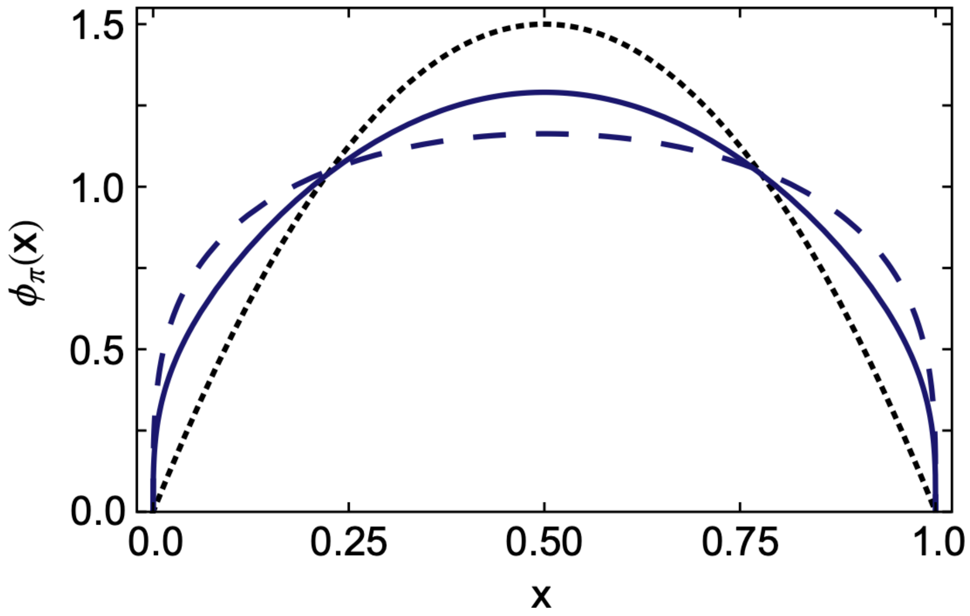



FIG. 2 (color online). Computed distribution amplitude at $\zeta = 2$ GeV. Curves: solid, DCSB-improved kernel (DB); dashed, rainbow ladder (RL); and dotted, asymptotic distribution.

Lei Chang, I. C. Cloët, J. J. Cobos-Martinez, C. D. Roberts, S. M. Schmidt, and P. C. Tandy
Phys. Rev. Lett. **110**, 132001 (2013)

# Gluon distribution in Pion------DB prediction

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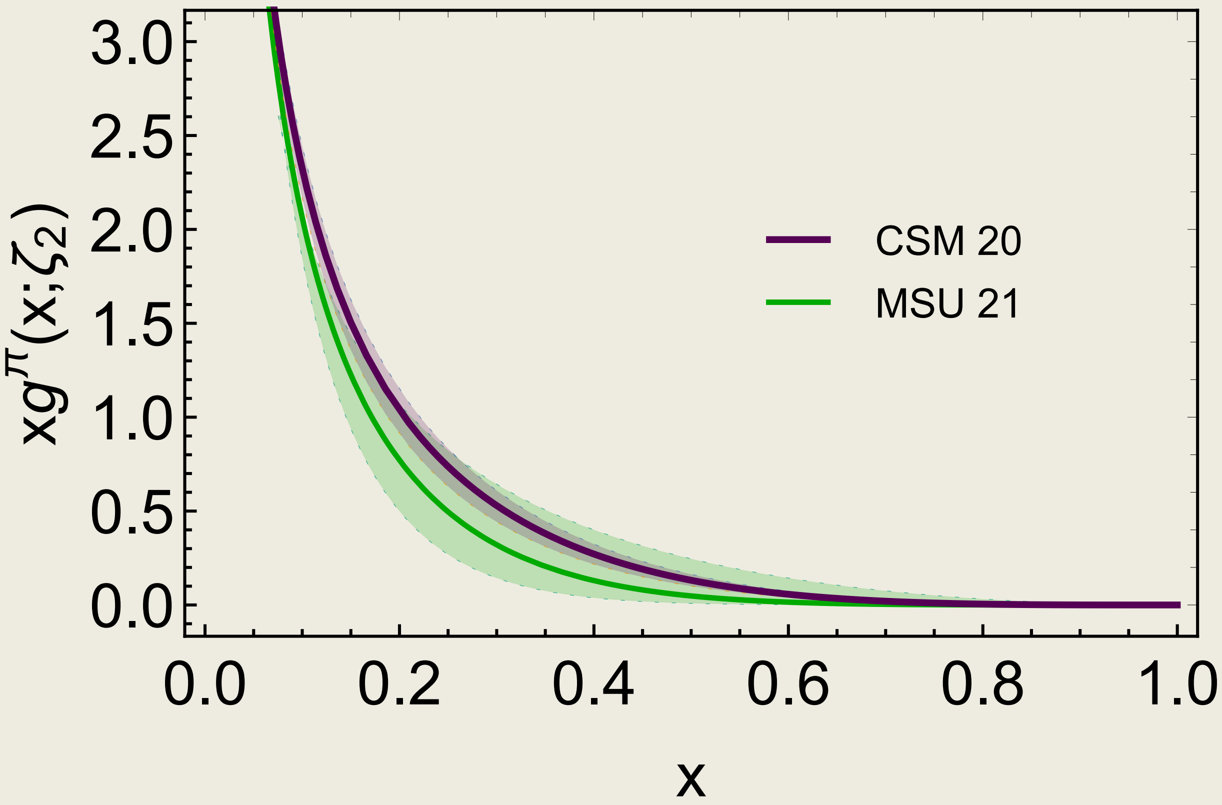


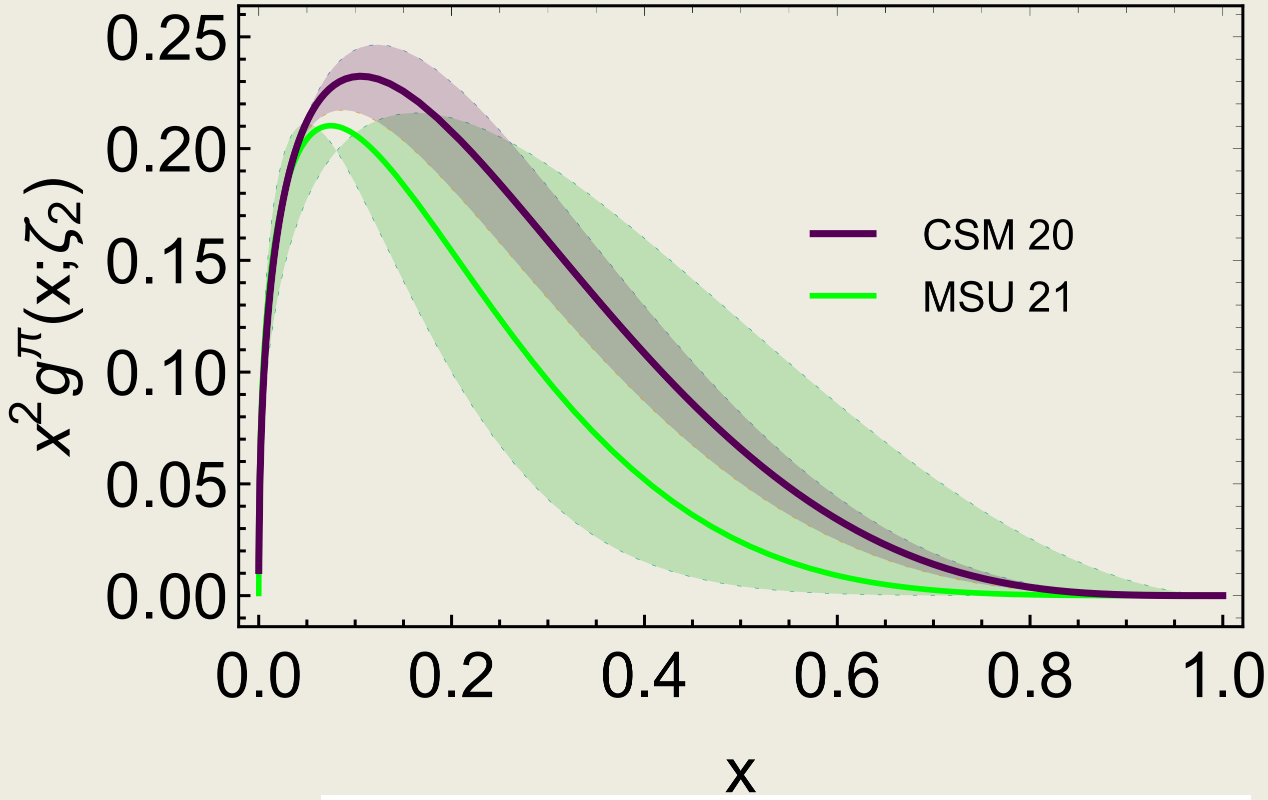


Craig Roberts's plot!
Zhouyou Fan and Huey-Wen Lin, arXiv:2104.06372.

$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{QCD}}\right]}, \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

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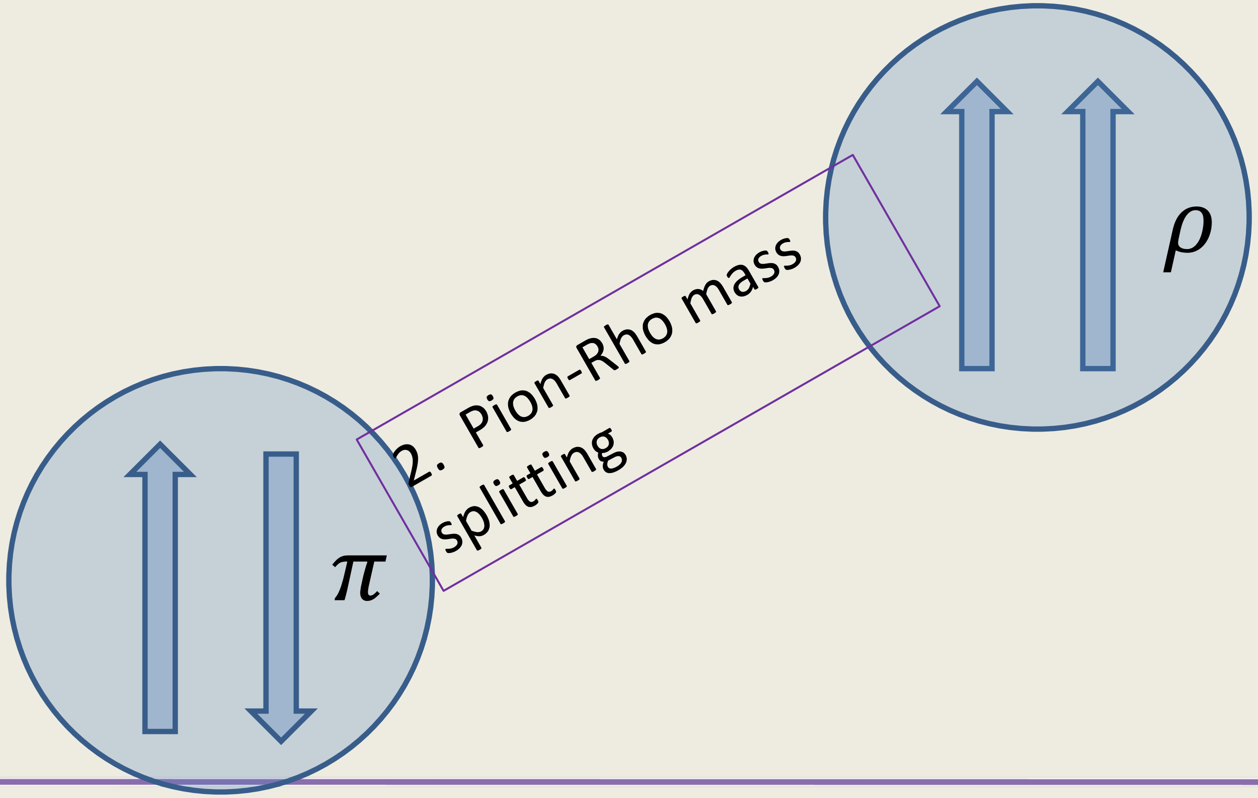
# Outline



# Pion-rho mass splitting

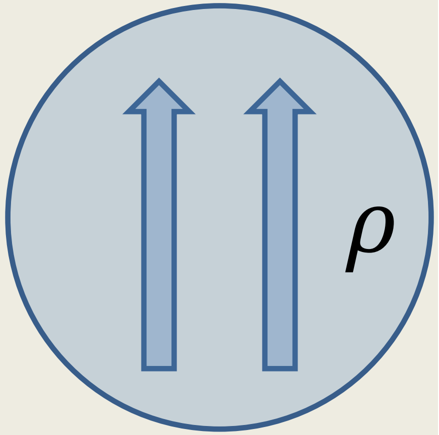
$$\gamma_\mu \gamma_\nu \rightarrow \sigma_i \sigma_j \rightarrow \begin{cases} +1 \; for \; S = 1 \\ -3 \; for \; S = 0 \end{cases}$$

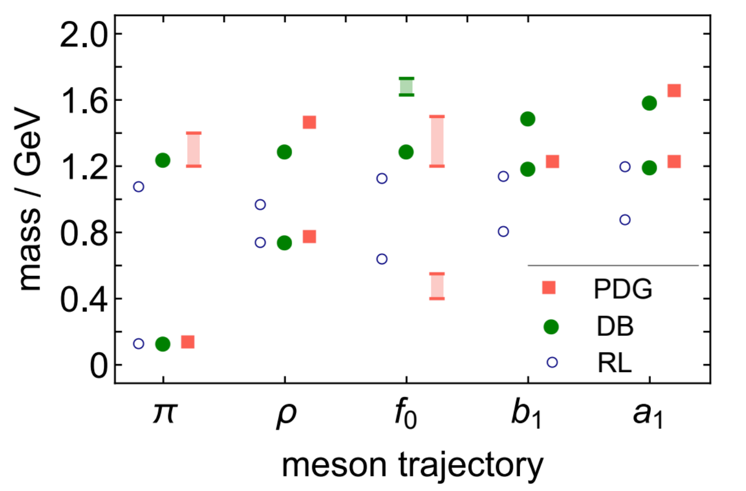
| | This work | Expt. | RL Padé | RL direct |
|---|---|---|---|---|
| $m_\pi$ | 0.138 | 0.138 | 0.138 | 0.137 |
| $m_\rho$ | $0.84 \pm 0.03$ | 0.777 | 0.754 | 0.758 |
| $m_\sigma$ | $1.13 \pm 0.01$ | 0.4–1.2 | 0.645 | 0.645 |
| $m_{a_1}$ | $1.28 \pm 0.01$ | $1.24 \pm 0.04$ | 0.938 | 0.927 |
| $m_{b_1}$ | $1.24 \pm 0.10$ | $1.21 \pm 0.02$ | 0.904 | 0.912 |
| $m_{a_1} - m_\rho$ | $0.44 \pm 0.04$ | $0.46 \pm 0.04$ | 0.18 | 0.17 |
| $m_{b_1} - m_\rho$ | $0.40 \pm 0.14$ | $0.43 \pm 0.02$ | 0.15 | 0.15 |

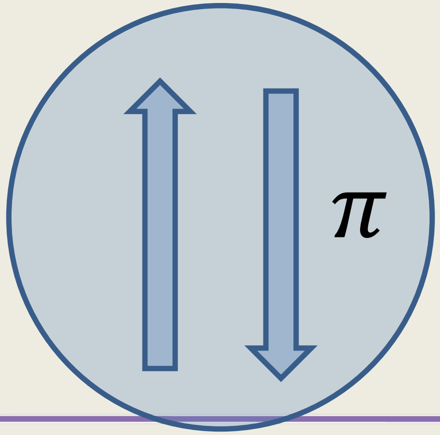
Lei Chang, C.D.Roberts, PRC85(2012)052201(R)

Sixue Qin, C.D.Roberts, arXiv: 2009.13637

- Scalar and tensor part of quark-gluon vertex bring abundant types of interaction




- Nonperturbative Symmetry-Preserving truncation of scattering kernle ensure that the cancelation of ATTRATIVE and REPULSIVE interaction in pseudoscalar and vector channels.

# Pion-rho mass splitting

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Lei Chang, C.D.Roberts, PRC85(2012)052201(R)

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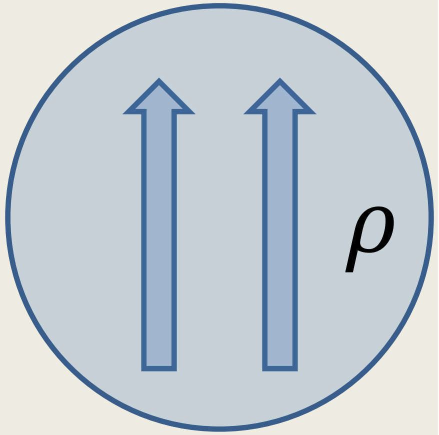
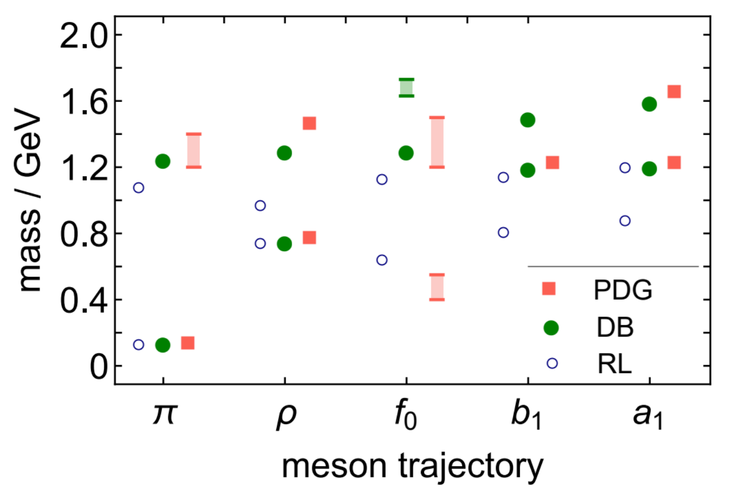
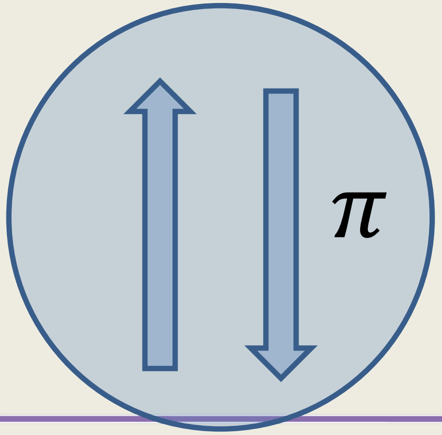



Chiral Quark Condensate at 1GeV

$(0.241\text{GeV})^3$ Roberts and Maris, arXiv: 97008029.

$(0.236\text{GeV})^3$ Maris and Tandy, arXiv: 9905056.

$(0.251\text{GeV})^3$ Qin, et al., arXiv: 1108.0603.

$(0.271\text{GeV})^3$ DB., herein

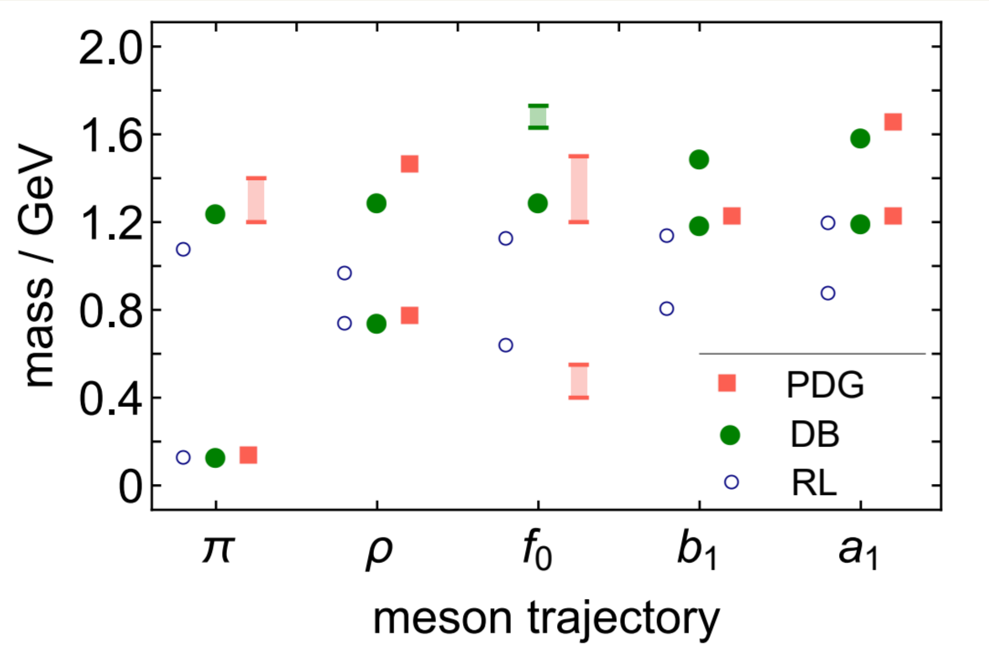
Mass splitting v.s Condensate

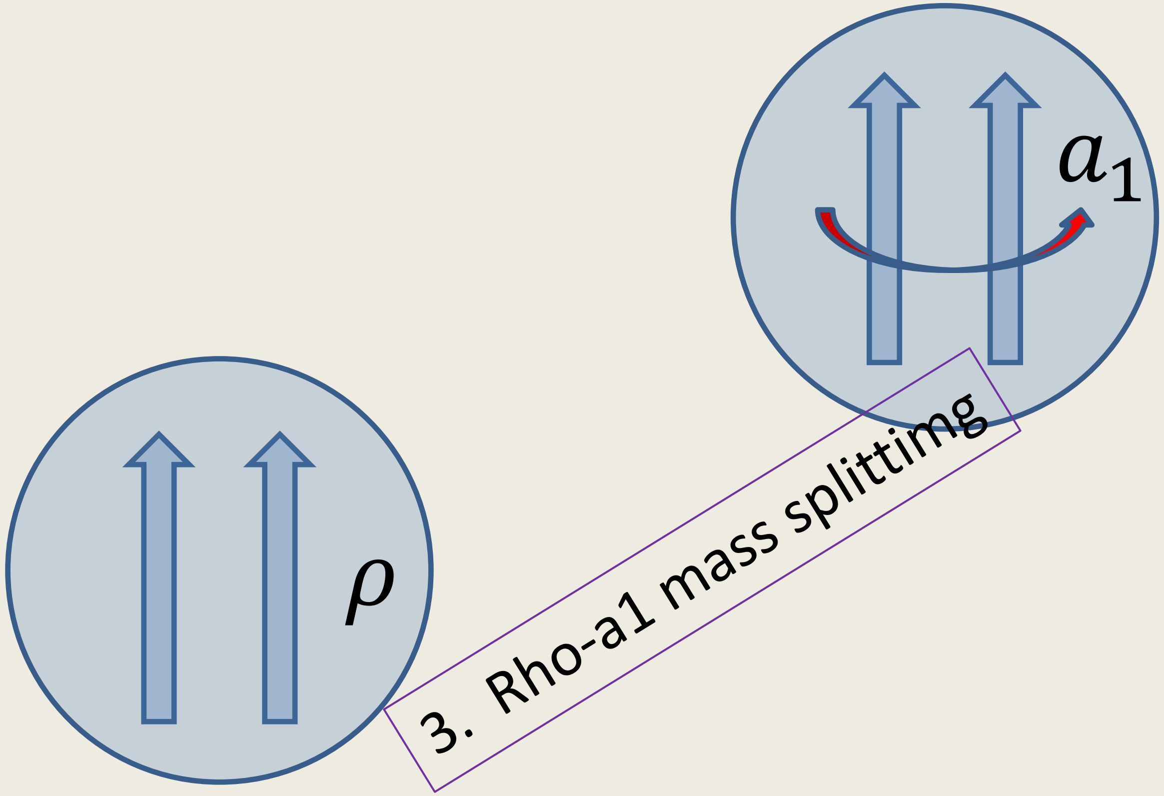
Interaction details does not matter!

M. Blank, A. Krassnigg and A. Maas arXiv:1007.3901

# Outline




2.0
1.6
1.2
0.8
0.4
0
mass / GeV
PDG
DB
RL
$\pi$
$\rho$
$f_0$
$b_1$
$a_1$
meson trajectory


$a_1$
$\rho$
3. Rho-a1 mass splittimg

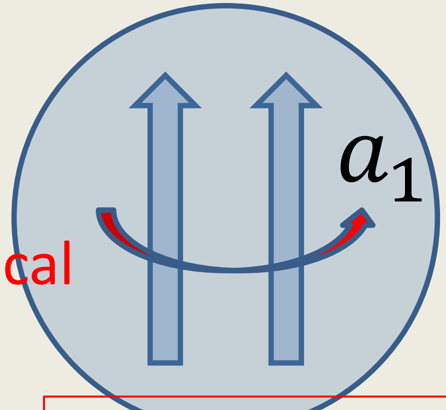
# DB improved kernel enhance mass splitting

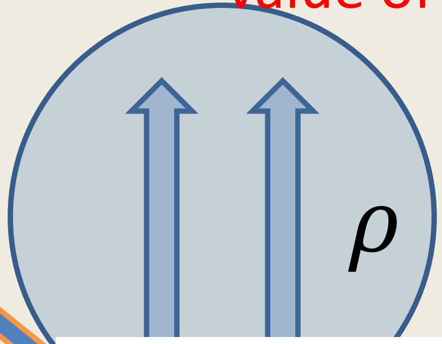
Sixue Qin, C.D.Roberts, arXiv: 2009.13637
R. Williams, C. S. Fischer, W. Heupel, PRD93(2016)034026
Lei Chang, C.D.Roberts, PRC85(2012)052201(R)

$$\Gamma_\nu(q,k) = \gamma_\nu + \tau_\nu(l=k-q)\,,$$
$$\tau_\nu(l) = \sigma_{\rho\nu} l_\rho \kappa(l^2)\,,\ \kappa(l^2) = \frac{\eta}{\omega}\, \mathrm{e}^{-l^2/\omega^2}\,.$$

- ACM does matter!
- Enhance the spin-orbital splitting!

Produce the empirical value of $m_{a1}$-$m_{rho}$



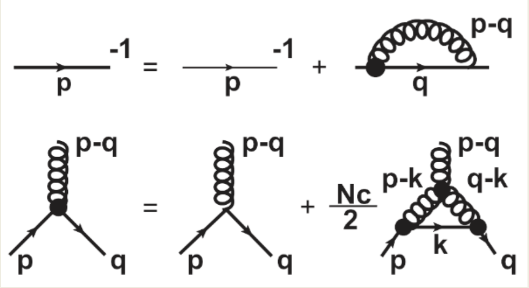



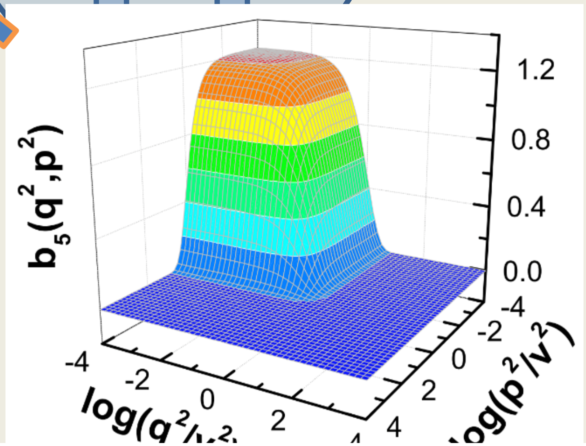

Quarks carry charge

Rotation generate magnetic field

Affect magnetic moment

Spin-obital

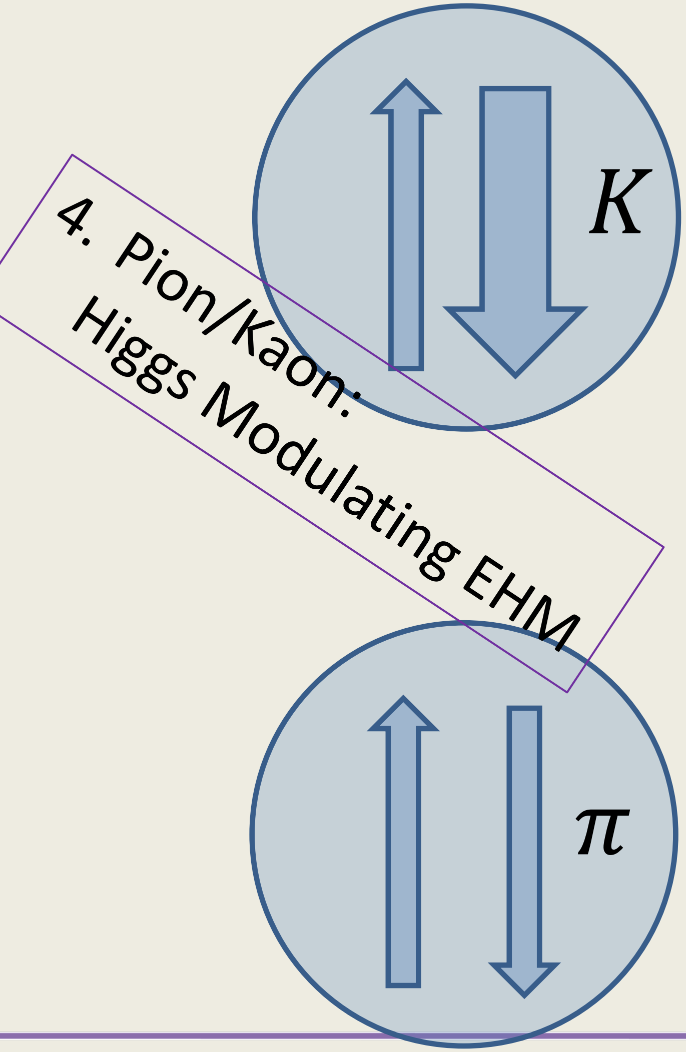



Can Tang, Fei Gao, Yu-xin Liu, PRD100(2019)056001

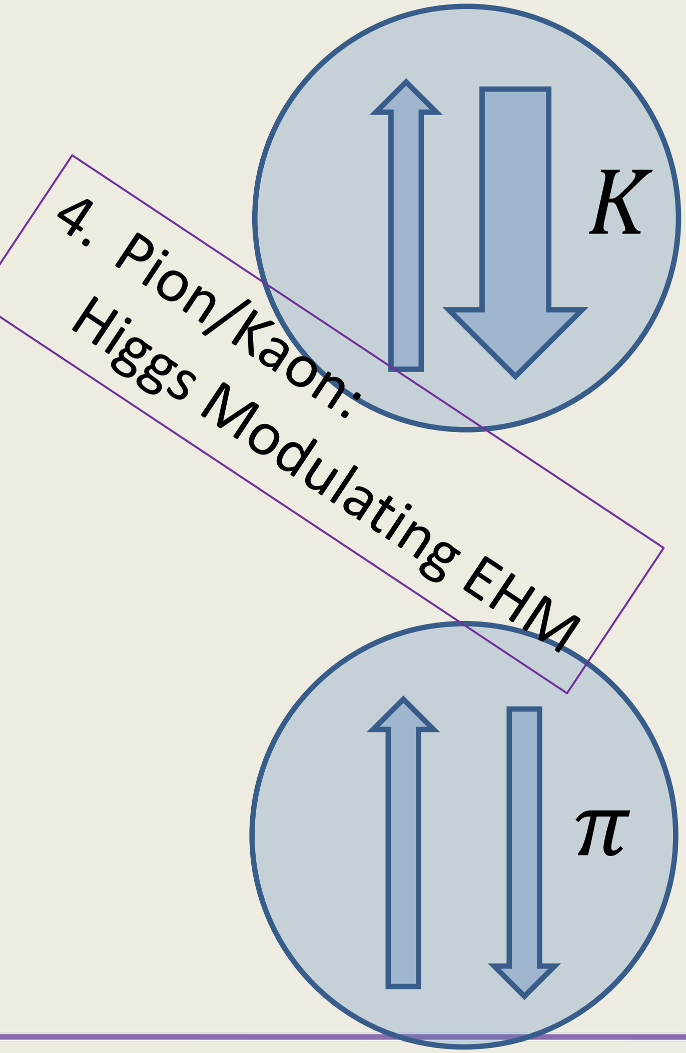
$$-i(p+q)_\mu \frac{B(p^2)-B(q^2)}{p^2-q^2} + \eta\sigma_{\mu\alpha}k_\alpha \frac{B(p^2)-B(q^2)}{p^2-q^2}\mathcal{H}(k^2)$$

# Outline

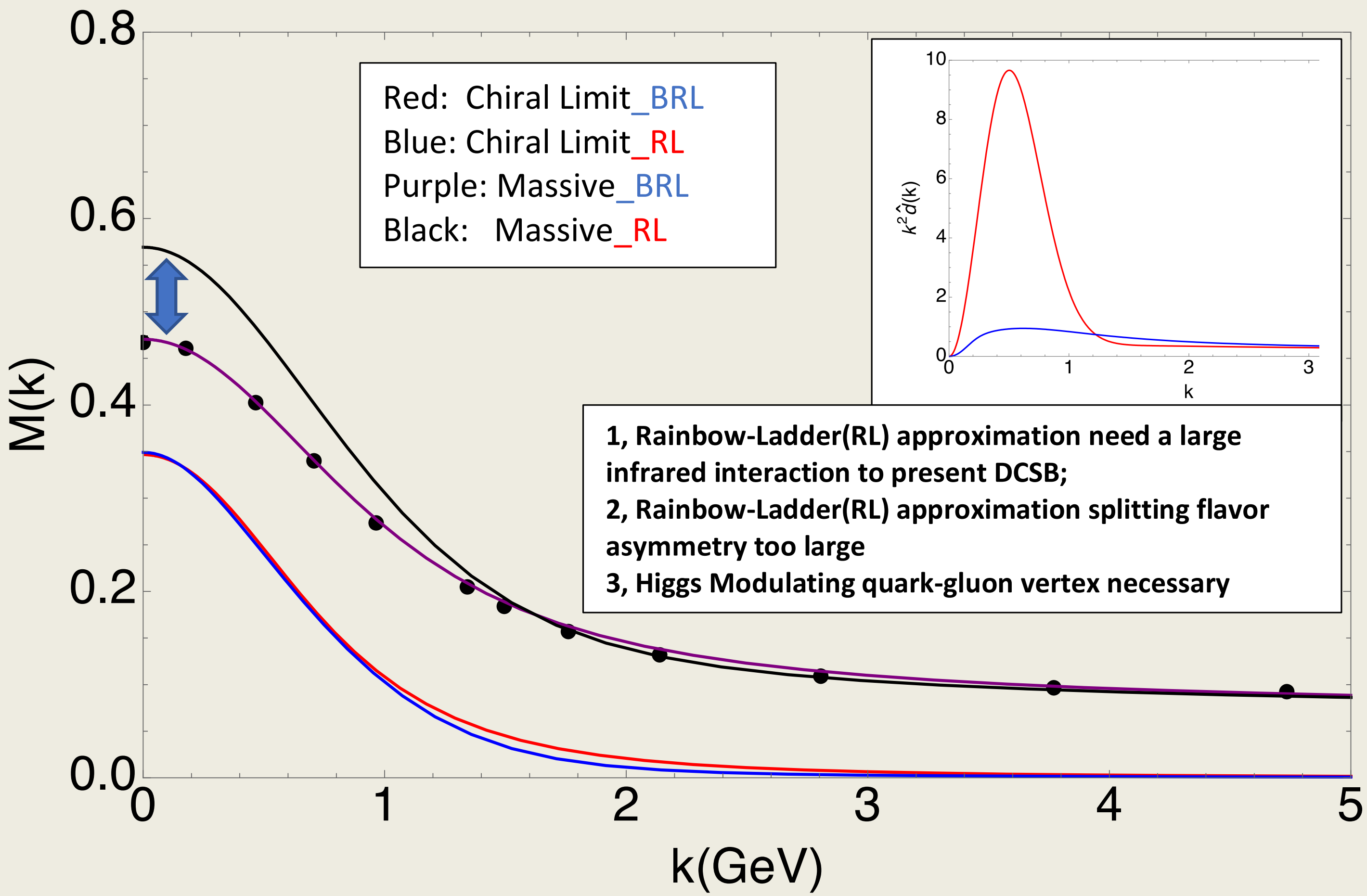



up to strange…current mass evolution

# Benefits




- Ward identity hold…guarantee proper current quark mass evolution;




$$f_K/f_\pi \approx 1.2 \approx M_s(0)/M_{\overline{ud}}(0).$$

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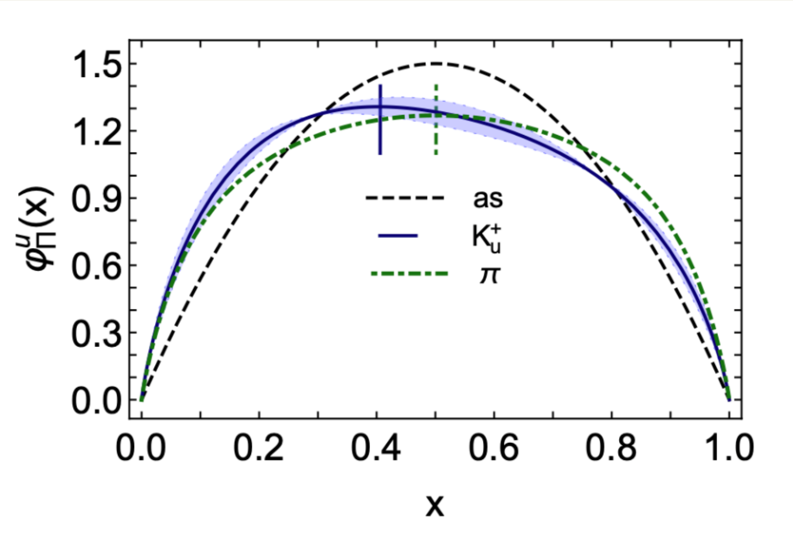
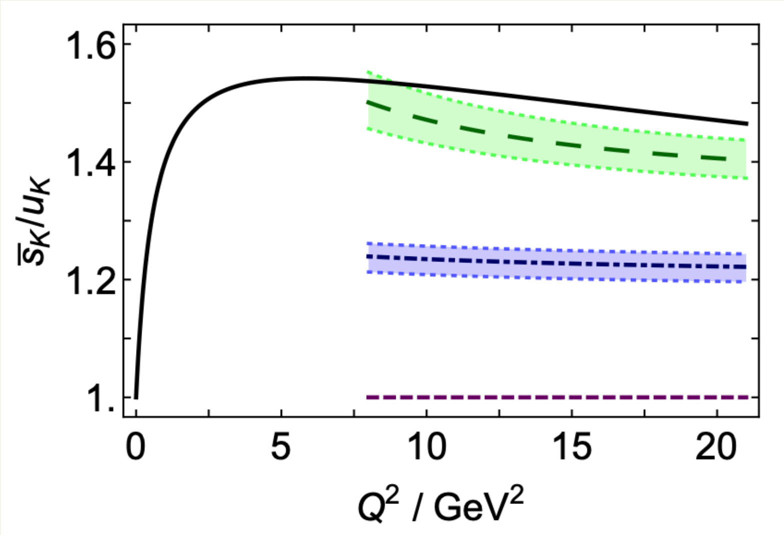
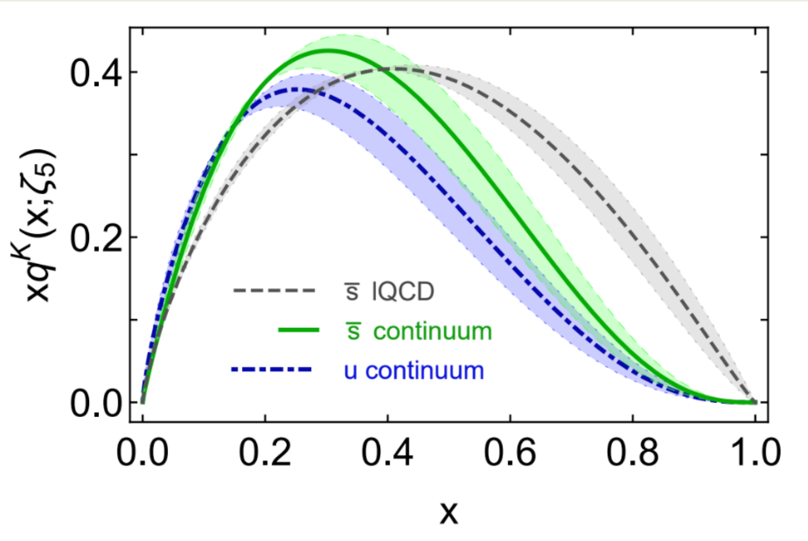



PDA




EMFF



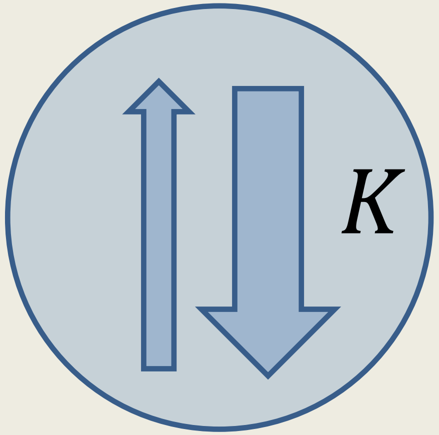

PDF

- Peak shifted to x=0.4, 20% to the left
- With increasing current mass of the heavier quark the distortion of this DA becomes more pronounced and its peak location moves toward x=0.

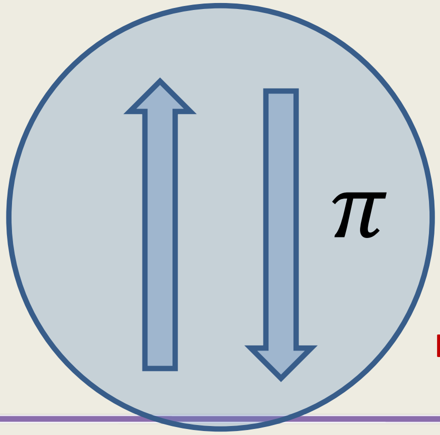
- The ratio is unity at $Q^2$=0, owing to current conservation
- pQCD predicts Unity on $\Lambda_{QCD}^2/Q^2 \sim 0$
- Between these limits, a peak value of roughly 1.5 at $Q^2 \sim 6GeV^2 (\frac{f_k^2}{f_\pi^2} \approx 1.4)$

- $\frac{\langle x\bar{s}\rangle^K}{\langle x\bar{u}\rangle^K} = 1.18(1)$ vs $\frac{\langle x\bar{s}\rangle^K}{\langle x\bar{u}\rangle^K} = 1.38(7)$
- It may reasonably to anticipated that future refinements of lQCD setups, algorithms and analyses will move lattice and continuum DFs closer together

# Thanks for your attention

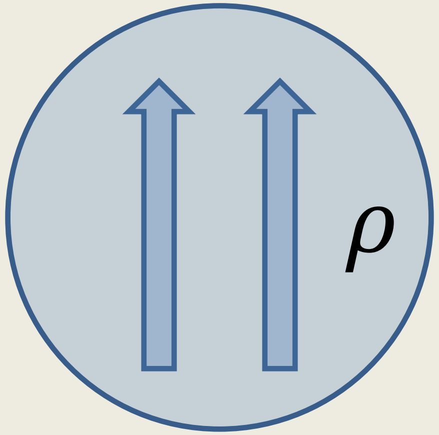



hyperon

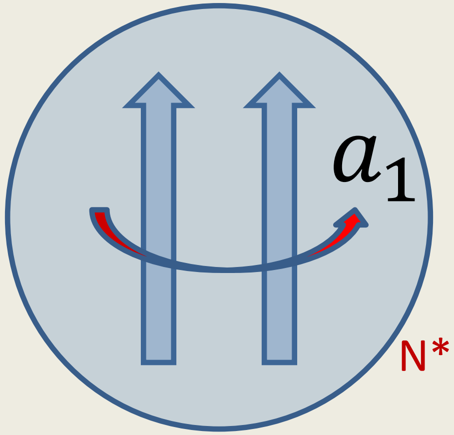



nucleon

- DCSB is illustrated and measured by chiral quark condenate which is almost truncation and model independent.
- Pion-rho mass splitting can be expressed by condensate.




Delta




N*(1535)

- Rho-a1 mass splitting is due to the ACM which is a result of DCSB.
- The proper currrent quark mass evolution in the quark-gluon vertex guarantee correct description of pion-kaon difference.

- pion/rho → nucleon/delta
  pion/kaon → nucleon/hyperon
  rho/a1 → nucleon/N*(1535)

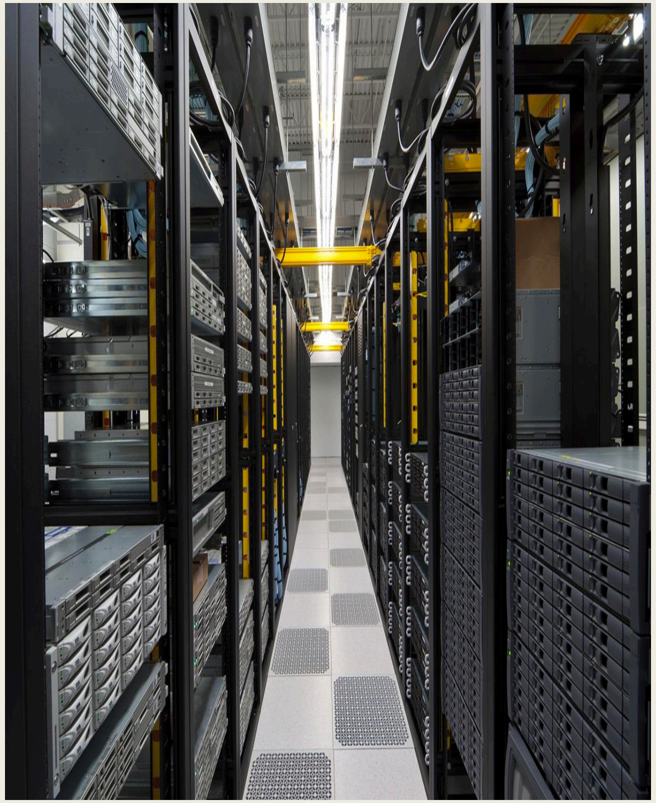
NJU…see Jorge's talk

## By machine (3PI)

TABLE I. Meson masses and pion decay constant in GeV as calculated in rainbow-ladder (RL) [68], the 2PI effective action at 3-loop (2PI-3L) [39] and in the 3PI effective action at 3-loop (3PI-3L) truncation as detailed here, compared to values from the Particle Data Group (PDG) [69]. Results affixed with † are fitted values.

| | RL | 2PI-3L | 3PI-3L | PDG |
|---|---|---|---|---|
| $0^{-+}$ ($\pi$) | $0.14^{\dagger}$ | $0.14^{\dagger}$ | $0.14^{\dagger}$ | 0.14 |
| $0^{++}$ ($\sigma$) | 0.64 | 0.52 | 1.1(1) | 0.48(8) |
| $1^{--}$ ($\rho$) | 0.74 | 0.77 | 0.74 | 0.78 |
| $1^{++}$ ($a_1$) | 0.97 | 0.96 | 1.3(1) | 1.23(4) |
| $1^{+-}$ ($b_1$) | 0.85 | 1.1 | 1.3(1) | 1.23 |
| $f_\pi$ | $0.092^{\dagger}$ | 0.103 | 0.105 | 0.092 |

R. Williams, C. S. Fischer, W. Heupel, PRD93(2016)034026.



Computers！ ！ ！