Emergence of Hadron Mass (EHM) as Revealed by the Spectra of Baryons with Heavy Quarks

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Regarding the empirical hadron spectrum

☞ There are five flavors of quarks which live long enough to produce measurable bound states: $f \in \{u, d, s, c, b\}$.

Due to the large weak decay rate, $t \to bW^+$, it is not expected that the top quark appears as a constituent in bound states.

☞ The empirical hadron spectrum is rich, even without including the array of recently discovered exotic states.

P. Zyla, et al., Review of Particle Physics, PTEP 2020 (2020) 083C01.
N. Brambilla, et al., Phys. Rept. 873 (2020) 1.

☞ Contemporary theory still predicts more states than have been observed:

- Specially true when one goes beyond systems comprised of light $\{u, d\}$-quarks.
- Even for ground-states in the J^P-channels accessible to constituent quark models.

S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189.
B. Silvestre-Brac, Few-Body Syst. 20 (1996) 1-25.

☞ The challenge of finding such *missing* bound states has been accepted, with an array of dedicated experiments underway, at facilities worldwide.

B-factories (BaBar, Belle, CLEO), τ-charm facilities (CLEO-c, BES), hadron-hadron (CDF, D0, COMPASS/AMBER, LHCb, ATLAS, CMS) and lepton-hadron colliders (JLab, EIC, EicC).

Regarding calculations of hadron spectrum

☞ Lattice-regularised quantum chromodynamics (lQCD) provides the most direct connection with the standard model of particle physics and many separate efforts are underway.

- Some of the successes and challenges have been identified.
- Recent spectrum calculations have been reported.

A. Bazavov, et al., Rev. Mod. Phys. 82 (2010) 1349.
R.A. Briceño, et al., Rev. Mod. Phys. 90 (2018) 025001.
R.G. Edwards, et al., Phys. Rev. D84 (2011) 074508.
H. Bahtiyar, et al., Phys. Rev. D102 (2020) 054513.

☞ Dyson-Schwinger equations (DSEs), a collection of coupled integral equations that provide for a symmetry-preserving treatment of the continuum bound-state problem, have also been widely employed to compute hadron spectra and interactions.

- The limitations and capacities of the approach have been identified.
- The breadth and quality of the description have been reported.

C.D. Roberts and A.G. Williams, Prog. Part. Nucl. Phys. 33 (1994) 477.
R. Alkofer and L. von Smekal, Phys. Rept. 353 (2001) 281
G. Eichmann, et al., Prog. Part. Nucl. Phys. 91 (2016) 1.
S.-X Qin and C.D. Roberts, Chin. Phys. Lett. 37 (2020) 121201

Both demand the use of large arrays of cutting-edge computers

A largely algebraic approach:
Symmetry-preserving regularisation of a vector $\otimes$ vector contact interaction.

☞ We are going to exploit the fact that the mass of any given hadron is an integrated (long-wavelength) quantity and thus not very sensitive to details of the system's wave function.

☞ Deliver a unified, symmetry-preserving description of all positive and negative-parity ground-states of mesons and baryons, accounting properly for DCSB.

☞ Enable insights into features of these systems that are obscured in approaches that rely heavily on computer resources.

☞ Connect to quantum chromodynamics, when considered judiciously, because it preserves many qualities of the leading-order truncation of QCD's DSEs.

Note: Quark models have also been widely employed in the calculation of hadron spectra; however, owing to complications to faithfully express DCSB, quark models are most reliable for baryons and mesons containing only heavy quarks.

Series of papers establishes strengths and limitations

- Masses of Light and Heavy Mesons and Baryons: A Unified Picture, L.X. Gutiérrez-Guerrero, A. Bashir, et al., Phys. Rev. D100 (2019) 114032.
- Spectrum of fully-heavy tetraquarks from a diquark+antidiquark perspective, M.A. Bedolla and J. Ferretti, et al., Eur. Phys. J. C80 (2020) 1004.
- Parity partners in the baryon resonance spectrum, Y. Lu, C. Chen, C.D. Roberts, et al., Phys. Rev. C96 (2017) 015208.
- Spectrum of hadrons with strangeness, C. Chen, L. Chang, C.D. Roberts, S. Wang and D.J. Wilson, Few Body Syst. 53 (2012) 293.
- π- and ρ-mesons, and their diquark partners, from a contact interaction, H.L.L. Roberts, A. Bashir, et al., Phys. Rev. C83 (2011) 065206.
- Masses of ground and excited-state hadrons, H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts, Few Body Syst. 51 (2011) 1.

The results presented herein are based on:

- *Masses of ground-state mesons and baryons, including those with heavy quarks,* **Pei-Lin Yin**, *et al., Phys. Rev. D100 (2019) 034008.*
- *Masses of positive- and negative-parity hadron ground-states, including those with heavy quarks,* **Pei-Lin Yin**, *et al., Eur. Phys. J. C81 (2021) 327.*

- In Rainbow-Ladder (RL) truncation, the quark-quark scattering kernel can be written as $(k = p_1 - p_1' = p_2' - p_2)$:

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'}\,,$$

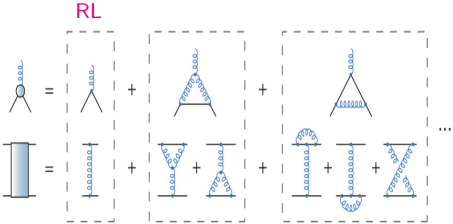
$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k)\,,$$

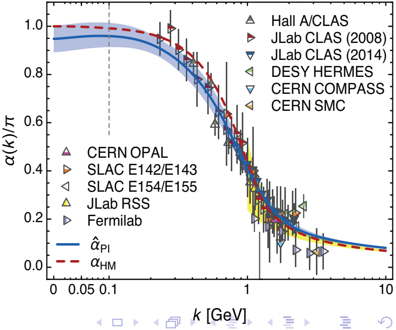
  where $T_{\mu\nu}(k) = \delta_{\mu\nu} - k_\mu k_\nu/k^2$, and our Euclidean metric and Dirac-matrix conventions are specified elsewhere.

- The defining quantity is $\tilde{\mathcal{G}}$; and following two decades of study, much has been learnt about its pointwise behaviour using a combination of continuum and lattice methods in QCD.

- The qualitative conclusion is that owing to the dynamical generation of a gluon mass-scale in QCD, $\tilde{\mathcal{G}}$ saturates at infrared momenta; hence, one may write:

$$\tilde{\mathcal{G}}(k^2) \overset{k^2\simeq 0}{=} \frac{4\pi\alpha_{IR}}{m_G^2}\,.$$





Two-body scattering kernel (II)

- In QCD, $m_G \approx 0.5\,\text{GeV}$, $\alpha_{\rm IR} \approx \pi$. We keep this value of m_G in developing the contact-interaction for use in RL truncation, but reduce α_{IR} to a parameter.

  D. Binosi, et al., Phys. Lett. B742 (2015) 183.
  D. Binosi, et al., Phys. Rev. D96 (2017) 054026.

- Since a contact interaction (CI) cannot support relative momentum between bound-state constituents, we simplify the tensor structure and define:

$$\mathcal{K}^{\rm CI}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \frac{4\pi\alpha_{IR}}{m_G^2}[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\mu]_{\alpha_2\alpha_2'}\,.$$

- Infrared and ultraviolet cut-offs:
  - Any use of equation above will require imposition of an ultraviolet regularisation scheme, which should be symmetry-preserving.

    $\rightarrow$ Since the theory is not renormalisable, the associated mass-scale(s), $\Lambda_{\rm uv}$, will be additional physical parameter(s).

    $\rightarrow$ Interpret them as an upper bound on the momentum domain within which the properties of the associated system are effectively momentum-independent.
  - Since chiral symmetry is dynamically broken in our approach, ensuring the absence of infrared divergences, $\Lambda_{\rm ir}$ is not a necessary part of the contact-interaction's definition.

    $\rightarrow$ By excising momenta less-than $\Lambda_{\rm ir}$, one achieves a rudimentary expression of confinement via elimination of quark production thresholds.

- In RL truncation, using the quark-quark interaction above, the dressed-quark gap equation takes the following form:

$$S_f^{-1}(p) = i\gamma\cdot p + m_f + \frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu\,,$$

where m_f is the quark's current-mass.

- The integral is quadratically divergent; but when it is regularised in a Poincaré-invariant manner, the solution is

$$S_f^{-1}(p) = i\gamma\cdot p + M_f\,,$$

where M_f is momentum-independent and determined by

$$M_f = m_f + M_f\frac{4\alpha_{\rm IR}}{3\pi m_G^2}\left[\int_0^\infty ds\, s\,\frac{1}{s+M_f^2}\right]_{\rm reg}\,.$$

C. Chen, et al., Few Body Syst. 53 (2012) 293.

- Note that regularisation in a Poincaré-invariant manner means:

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\,{\rm e}^{-\tau(s+M^2)}$$
$$\to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\,{\rm e}^{-\tau(s+M^2)} = \frac{{\rm e}^{-(s+M^2)\tau_{\rm uv}^2}-{\rm e}^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}\,,$$

where $\tau_{\rm ir,uv} = 1/\Lambda_{\rm ir,uv}$.

D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B388 (1996) 154.

- The Bethe-Salpeter amplitude for a pseudoscalar meson constituted from a valence f-quark and valence g-antiquark has the following restricted form

$$\Gamma_{0^-}(Q) = \gamma_5 \left[iE_{0^-} + \frac{1}{2M_R}\gamma \cdot Q F_{0^-} \right] ,$$

Q is the bound-state's total momentum: $Q^2 = -m_{0^-}^2$, m_{0^-} is the meson's mass; and $M_R = M_f M_g/[M_f + M_g]$, with $M_{f,g}$ being the dressed-quark masses.

- The amplitude is determined by the following equation:

$$\Gamma_{0^-}(Q) = -\frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q+Q)\Gamma_{0^-}(Q)S_g(q)\gamma_\mu \, .$$

- Using the symmetry-preserving regularisation scheme, one arrives at the following explicit form of the Bethe-Salpeter equation (BSE):

$$\left[\begin{array}{c} E_{0^-}(Q) \\ F_{0^-}(Q) \end{array} \right] = \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \left[\begin{array}{cc} \mathcal{K}_{EE}^{0^-} & \mathcal{K}_{EF}^{0^-} \\ \mathcal{K}_{FE}^{0^-} & \mathcal{K}_{FF}^{0^-} \end{array} \right] \left[\begin{array}{c} E_{0^-}(Q) \\ F_{0^-}(Q) \end{array} \right] .$$

L. X. Gutiérrez-Guerrero, et al., Phys. Rev. C81 (2010) 065202.
C. Chen, et al., Phys. Rev. C87 (2013) 045207.

- We allow $\Lambda_{\rm uv}^{0^-}$ to vary with the meson's mass and fix the associated coupling by requiring

$$\alpha_{\rm IR}(\Lambda_{\rm uv}^{0^-})[\Lambda_{\rm uv}^{0^-}]^2 \ln \frac{\Lambda_{\rm uv}^{0^-}}{\Lambda_{\rm ir}} = \alpha_{\rm IR}(\Lambda_{\rm uv}^{\pi})[\Lambda_{\rm uv}^{\pi}]^2 \ln \frac{\Lambda_{\rm uv}^{\pi}}{\Lambda_{\rm ir}} \,.$$

Any increase in the momentum-space extent of a hadron wave function must be accompanied by a commensurate decrease in the effective coupling between the constituents so as to avoid critical over-binding.

- Our analysis yields

$$\Lambda_{\rm uv}(s=m_{0^-}^2) \stackrel{m_{0^-} \geq m_K}{=} 0.83 \ln[2.79 + s/(4.66\Lambda_{\rm ir})^2] \,;$$

and

$$\alpha_{\rm IR}(s) \stackrel{m_{0^-} \geq m_K}{=} \frac{0.047\,\alpha_{\rm IR}(m_K^2)}{\ln[1.04 + s/(21.77\Lambda_{\rm ir})^2]} \,.$$

The origin of this outcome: The decay-constant integral diverges logarithmically with increasing $\Lambda_{\rm uv}$; and $\alpha_{\rm IR}$ flows to compensate for analogous behaviour in the Bethe-Salpeter kernel and thereby maintain the given meson's mass.

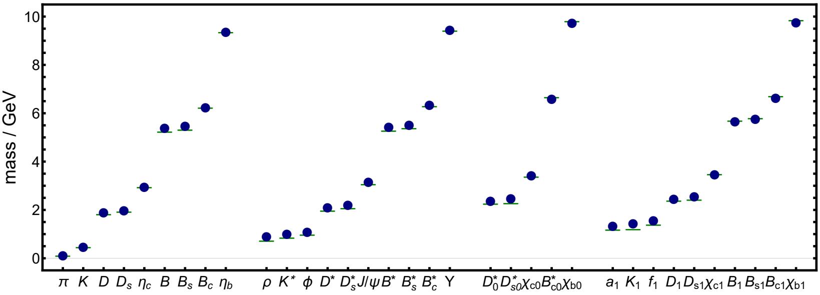
Note that similar approaches were adopted in M.A. Bedolla, et al., Phys. Rev. D92 (2015) 054031, F.E. Serna, et al., Phys. Rev. D96 (2017) 014013.

☞ Couplings, ultraviolet cutoffs and current-quark masses that provide a good description of pseudoscalar meson properties, along with the dressed-quark masses and selected pseudoscalar meson properties they produce; all obtained with $m_G = 0.5$ GeV, $\Lambda_{\rm ir} = 0.24$ GeV. (Dimensioned quantities in GeV.)

quark	$\alpha_{\rm IR}/\pi$	$\Lambda_{\rm uv}$	m	M	m_{0^-}	f_{0^-}
$l = u/d$	0.36	0.91	0.007	0.37	0.14	0.10
s	0.36	0.91	0.17	0.53	0.50	0.11
c	0.054	1.88	1.24	1.60	2.98	0.24
b	0.012	3.50	4.66	4.83	9.40	0.41

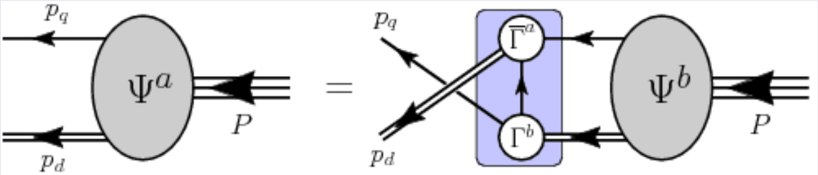
☞ Empirically:

- $m_\pi = 0.14,\ f_\pi = 0.092$;
- $m_K = 0.50,\ f_K = 0.11$;
- $m_{\eta_c} = 2.98,\ f_{\eta_c} = 0.24$;
- $m_{\eta_b} = 9.40,\ f_{\eta_b} = f_\Upsilon [f_{\eta_c}/f_{J/\psi}] = 0.41(2)$.



☞ Observations:

- Good estimates for masses of positive and negative parity ground-state flavour SU(5) mesons; the difference between theory and experiment is only 5(6)%.
- The computed masses neatly follow a pattern prescribed by equal spacing rules (ESRs) whose origin is both DCSB- and Higgs-mechanisms of mass generation.
- EHM introduces spin-orbit corrections to RL-truncation and generate the empirical splitting between parity partners and radial excitations.
- The F_{0^-} (pseudovector) component of each pseudoscalar meson is nonzero and quantitatively important: 15(6)% of the E_{0^-} (pseudoscalar) piece.

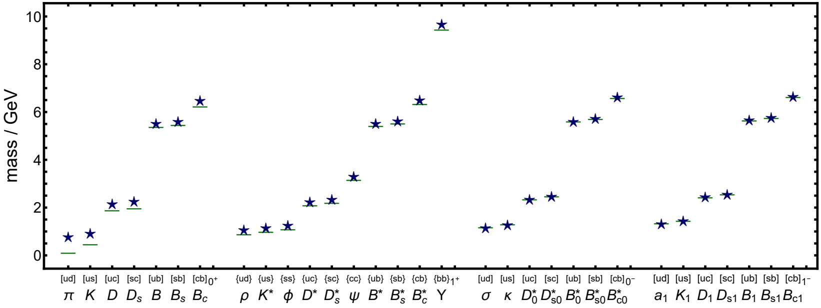


We use a quark+diquark approximation to the Faddeev equation when calculating the spectrum of flavour-SU(5) ground-state baryons with $J^P = 1/2^{\pm}$, $3/2^{\pm}$.

- Diquarks described herein are fully dynamical and appear in a Faddeev kernel which requires their continual breakup and reformation. Consequently, they are vastly different from the static, pointlike diquarks introduced originally in an attempt to solve the so-called "missing resonance" problem.
  M. Yu. Barabanov, Prog. Part. Nucl. Phys. 116 (2021) 103835.

- One must calculate the masses and amplitudes for all diquark correlations that can exist in $J^P = 1/2^{\pm}, 3/2^{\pm}$ baryons: flavour-$\overline{10}$ scalar; flavour-15 pseudovector; flavour-$\overline{10}$ pseudoscalar; and flavour-$\overline{10}$ vector. A $\bar{3}_c$ flavour-15 vector-diquark is generally possible, but is not supported by a RL-like treatment of the CI.
  Y. Lu, et al., Phys.Rev.C 96 (2017) 015208,
  C. Chen, et al., Phys. Rev. D97 (2018) 034016.

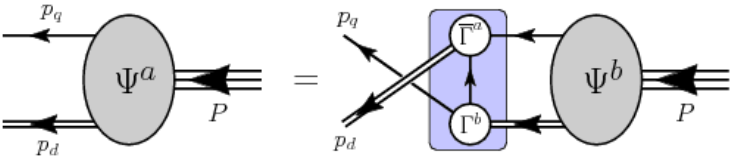
- A straightforward exercise because the RL BSE for a J^P diquark is generated from that for a J^{-P} meson by simply multiplying the meson kernel by a factor of 1/2.



☞ Observations:

- Since valence-quarks within a diquark are less tightly correlated than in mesons, spin-orbit repulsion in diquarks should be weaker than it is in mesons.
- The scalar diquark is always lighter than the pseudovector combination, followed by the pseudoscalar and then the vector diquarks.
- The level ordering of diquark correlations is precisely the same as that for mesons and thus the computed masses neatly follow a pattern prescribed by ESRs.
- Scalar and pseudovector diquarks are heavier than their partner mesons. This ordering is reversed for pseudoscalar and vector diquark correlations.

☞ The Faddeev equation:



can be written in the following form:

$$\Psi_{(fg)h}(P) = \left\{ g_{DB}^{\pi_\Psi \pi_d} \Gamma_{(gh)}(k_{(gh)}) S_g^T(q_g) g_{DB}^{\pi_\Psi \pi_d} \bar{\Gamma}_{(fg)}(-p_{(fg)}) \right\} \times$$
$$\times\, S_f(k_f) \Delta_{(gh)}(k_{(gh)}) \Psi_{(gh)f}(P)\,,$$

where: the first line on the right-hand-side, express the exchange kernel; a sum over all contributing diquark correlations is implicit; $k_{(gh)} = -k + (2/3)P$, $q_g = p_{(fg)} - k_f$, $p_{(fg)} = -p + (2/3)P$, $k_f = k + P/3$; $\bar{\Gamma}(Q) = C^\dagger \Gamma(Q)^{\mathrm{T}} C$.

☞ The complete amplitude for the bound-state is

$$\Psi = \Psi_{(gh)f} + \Psi_{(hf)g} + \Psi_{(fg)h}\,.$$

- The kernel introduces binding through diquark breakup and reformation via exchange of a dressed-quark. Within the CI, the quark exchanged between the diquarks is represented as

$$S_g^T(k) = \frac{g_B^2}{M_g} \,,$$

making the Faddeev amplitudes momentum-independent.

- The ground-state positive-parity octet baryons are primarily constituted from like-parity diquarks, with negligible contributions from negative-parity correlations.

- It is found, too, that their parity partners are also dominated by positive-parity diquark correlations; hence, too light. Like mesons and diquarks, the missing element was identified as too little spin-orbit repulsion generated by RL-like kernels. EHM dictates to use:

$$g_{DB}^{\pi_\Psi \pi_d} = \begin{cases} 1.0 & \pi_\Psi = \pi_d \,, \\ 0.1 & \pi_\Psi = -\pi_d \,. \end{cases}$$

and then negative-parity octet baryons are primarily constituted from negative-parity diquarks.

- $S_f(k_f)$ and $\Delta_{gh}(k_{(gh)})$ are quark and diquark propagators, respectively.

☞ Regarding $J^P = 1/2^{\pm}$ baryons, one has

$$\Psi^{\pm}(P) = \psi^{\pm}(P)u(P)\,,$$

where the positive energy spinor satisfies the Dirac equation and $\psi^{\pm}(P)$ is a sum of the following Dirac structures:

$$\begin{aligned}
\mathcal{S}^{\pm} &= \mathcal{G}^{\pm}(s^{\pm}\,\mathrm{I}_{\mathrm{D}}),\\
\mathcal{A}^{\pm}_{\mu} &= \mathcal{G}^{\pm}(a_1^{\pm}\,i\gamma_5\gamma_\mu + a_2^{\pm}\gamma_5\hat{P}_\mu),\\
\mathcal{P}^{\pm} &= \mathcal{G}^{\pm}(p^{\pm}\,i\gamma_5),\\
\mathcal{V}^{\pm}_{\mu} &= \mathcal{G}^{\pm}(v_1^{\pm}\,i\gamma_\mu + v_2^{\pm}\mathrm{I}_{\mathrm{D}}\hat{P}_\mu),
\end{aligned}$$

where $\mathcal{G}^{+(-)} = \mathrm{I}_{\mathrm{D}}(\gamma_5)$. Faddeev equation dynamics determines the values of the coefficients: $\{s^{\pm}, a_{1,2}, p^{\pm}, v_{1,2}\}$, each of which is a vector in flavour space.

☞ Regarding $J^P = 3/2^{\pm}$ baryons, one has

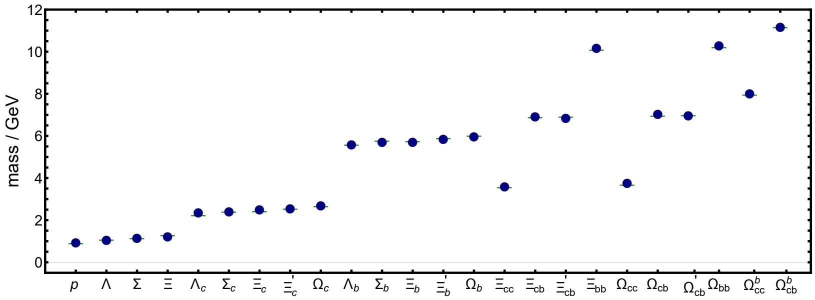
$$\Psi^{\pm}_{\mu\nu}(P) = \psi^{\pm}_{\mu\nu}(P)u_\nu(P)\,,$$

where $u_\nu(P)$ is a Rarita-Schwinger spinor; and

$$\psi^{\pm}_{\mu\nu} = \mathcal{G}^{\pm}(f^{\pm}\,\mathrm{I}_{\mathrm{D}}\delta_{\mu\nu})\,,$$

where, again, the coefficients $f^{\pm}$ carry flavour labels appropriate to the baryon.

Ground-state flavour-SU(5) $J^P = 1/2^+$ baryons (I)



☞ Observations:

- The computed masses are compared with empirical or lQCD values and the mean absolute-relative-difference is 1.3(1.3)%.
- We obtained a 5.2(2.8)% difference when compared with an ab-initio calculation of the Faddeev equation in a RL truncation, eschewing a quark+diquark approx.
- This indicates that we have implemented a phenomenologically efficacious CI formulation and the validity of the equal spacing rules (ESRs).

Ground-state flavour-SU(5) $J^P = 1/2^+$ baryons (II)

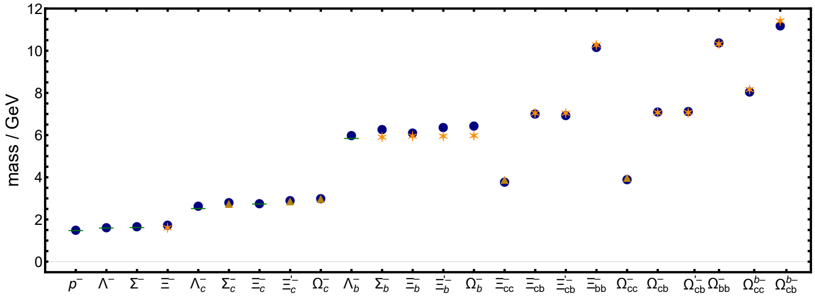
	M^{CI}	M^e	M^l	M^3	s^{r_1}	s^{r_2}	$a_1^{r_2}$	$a_2^{r_2}$	$a_1^{r_3}$	$a_2^{r_3}$	p^{r_4}	p^{r_5}	$v_1^{r_5}$	$v_2^{r_5}$	$v_1^{r_6}$	$v_2^{r_6}$	$v_1^{r_7}$	$v_2^{r_7}$	dom. corr.
N	0.98	0.94	0.94	0.95	0.88		-0.38	-0.06	0.27	0.05	-0.02		-0.02	0					$u[ud]_{0^+}$
Λ	1.10	1.12	1.11	1.11	0.66	0.59			-0.45	-0.08	-0.02	-0.03			-0.01	0	-0.01	0	$s[ud]_{0^+}$
Σ	1.20	1.19	1.17	1.11	0.84		-0.45	0.13	0.26	0.01	-0.01		-0.02	0					$u[us]_{0^+}$
Ξ	1.27	1.32	1.32	1.28	0.89		-0.31	0.04	0.33	0.05	-0.03		-0.02	0.01					$s[us]_{0^+}$
Λ_c	2.40	2.29	2.25	2.18	0.18	0.86			-0.35	-0.33	-0.02	-0.05			-0.02	0.03	-0.02	0.01	$d[uc]_{0^+} - u[dc]_{0^+}$
Σ_c	2.45	2.45	2.47	2.18	0.46		-0.20	0.86	0.09	0.06	0.02		0	0					$c\{uu\}_{1^+}$
Ξ_c	2.55	2.47	2.43	2.35	0.18	0.84			-0.36	-0.35	-0.02	-0.06			-0.02	0.03	-0.02	0.01	$s[uc]_{0^+} - u[sc]_{0^+}$
Ξ_c'	2.59	2.58	2.57	2.35	0.48		-0.21	0.85	0.09	0.06	0.02		0	0					$c\{us\}_{1^+}$
Ω_c	2.73	2.70	2.68	2.52	0.50		-0.21	0.83	0.09	0.06	0.01		0	0					$c\{ss\}_{1^+}$
Λ_b	5.62	5.62	5.63	5.39	0.10	0.93			-0.31	-0.13	0	0.03			-0.01	0.04	-0.01	-0.01	$d[ub]_{0^+} - u[db]_{0^+}$
Σ_b	5.75	5.81	5.86	5.39	0.33		-0.12	0.93	0.04	0.10	0.01		0	0					$b\{uu\}_{1^+}$
Ξ_b	5.75	5.79	5.77	5.56	0.11	0.94			-0.31	-0.13	0	-0.03			-0.01	0.03	-0.01	0	$s[ub]_{0^+} - u[sb]_{0^+}$
Ξ_b'	5.88	5.94	5.93	5.56	0.37		-0.11	0.92	0.05	0.12	0.01		0	0					$b\{us\}_{1^+}$
Ω_b	6.00	6.05	6.06	5.73	0.42		-0.11	0.89	0.05	0.14	0.02		0	-0.01					$b\{ss\}_{1^+}$
Ξ_{cc}	3.64	3.62	3.61	3.42	0.88		-0.34	0.30	0.10	0.06	-0.05		-0.03	0.03					$c[uc]_{0^+}$
Ξ_{cb}	6.97		6.94	6.63	0.07	-0.77			0.62	-0.01	0.12	0.01			0.02	-0.01	0.01	0	$u[cb]_{0^+}$
Ξ_{cb}'	6.89		6.96	6.63	0.89		-0.33	-0.14	0.13	0.25	-0.06		-0.03	0.02					$b[uc]_{0^+} + c[ub]_{0^+}$
Ξ_{bb}	10.22		10.14	9.84	0.85		-0.36	0.37	0.04	0.06	-0.06		-0.05	0.04					$b[ub]_{0^+}$
Ω_{cc}	3.79		3.74	3.59	0.86		-0.35	0.33	0.15	0.07	-0.05		-0.03	0.03					$c[sc]_{0^+}$
Ω_{cb}	7.08		7.00	6.80	0.05	-0.84			0.50	-0.19	0.08	0.01			0.01	0	0.01	0	$s[cb]_{0^+}$
Ω_{cb}'	7.01		7.03	6.80	0.83		-0.32	-0.21	0.18	0.36	-0.06		-0.03	0.02					$b[sc]_{0^+} + c[sb]_{0^+}$
Ω_{bb}	10.33		10.27	10.01	0.83		-0.38	0.39	0.07	0.09	-0.07		-0.05	0.04					$b[sb]_{0^+}$
Ω_{ccb}	8.04		8.01	7.87	0.40		-0.17	0.89	0.07	0.08	0.01		0	0					$b\{cc\}_{1^+}$
Ω_{cbb}	11.22		11.20	11.08	0.80		-0.39	0.40	0.16	0.14	-0.05		-0.03	0.03					$b[cb]_{0^+}$

☞ Observations:

- The lightest participating diquark correlation usually defines the most important component of a baryon's Faddeev amplitude and this is true even if a pseudovector diquark is the lightest channel available.

- Light-diquark dominance may be overcome in flavour channels for which the bound-state's spin-flavour structure and the quark-exchange character of the kernel lead dynamically to a preference for mixed-flavour correlations.

Ground-state flavour-SU(5) $J^P = 1/2^-$ baryons (I)



☞ Observations:

- The computed masses are compared with empirical or lQCD values and the mean absolute-relative-difference is 1.4(1.1)%.
- We obtained a 3.7(3.5)% difference when compared with an ab-initio calculation of the Faddeev equation in a RL truncation, eschewing a quark+diquark approx.
- This level of quantitative success suggests that some credibility be given to the qualitative conclusions that follow from our CI analysis.

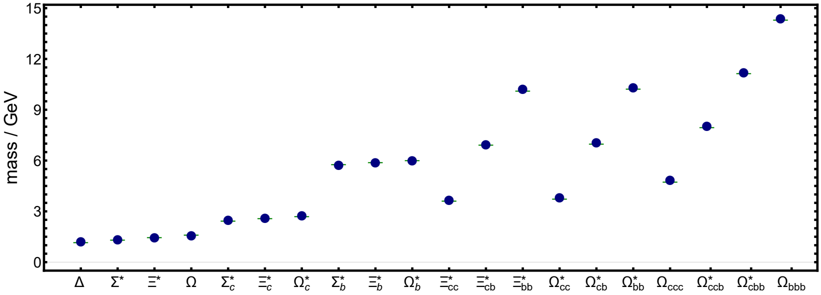
Ground-state flavour-SU(5) $J^P = 1/2^-$ baryons (II)

	$M^{\rm CI}$	M^e	M^l	M^3	s^{r_1}	s^{r_2}	$a_1^{r_2}$	$a_2^{r_2}$	$a_1^{r_3}$	$a_2^{r_3}$	p^{r_4}	p^{r_5}	$v_1^{r_5}$	$v_2^{r_5}$	$v_1^{r_6}$	$v_2^{r_6}$	$v_1^{r_7}$	$v_2^{r_7}$	dom. corr.
N	1.55	1.54		1.54	0.35		0.04	0	-0.03	0	-0.92		-0.06	0.18					$u[ud]_{0^-}$
Λ	1.65	1.67		1.58	0.27	0.24			0.03	0	-0.66	-0.62			-0.06	0.14	-0.05	0.13	$s[ud]_{0^-}$
Σ	1.70	1.68		1.58	0.46		0.17	-0.13	-0.08	0.06	-0.85		0.06	0.13					$u[us]_{0^-}$
Ξ	1.78			1.62	0.39		0.04	-0.01	-0.04	0.02	-0.90		-0.05	0.19					$s[us]_{0^-}$
Λ_c	2.67	2.59	2.67	2.67	0.07	0.45			0.06	-0.03	-0.21	-0.83			-0.09	0.14	0.03	0.20	$d[uc]_{0^-} - u[dc]_{0^-}$
Σ_c	2.84		2.81	2.67	0.13		0.14	-0.19	0.08	-0.24	-0.17		-0.27	0.88					$u[uc]_{1^-}$
Ξ_c	2.79	2.79	2.77	2.71	0.07	0.51			0.09	-0.05	-0.20	-0.78			-0.06	0.07	0.09	0.25	$s[uc]_{0^-} - u[sc]_{0^-}$
Ξ_c'	2.94		2.93	2.71	0.15		0.13	-0.19	0.08	-0.24	-0.16		-0.27	0.87					$s[uc]_{1^-} + u[sc]_{1^-}$
Ω_c	3.03		3.04	2.74	0.18		0.13	-0.19	0.08	-0.25	-0.13		-0.27	0.87					$s[sc]_{1^-}$
Λ_b	6.03	5.91		5.92	0	-0.22			0.12	-0.03	0	0.15			0.14	-0.72	0.44	0.43	$b[ud]_{1^-}$
Σ_b	6.32			5.92	0.01		0	0	-0.05	0.25	-0.01		0.24	-0.94					$u[ub]_{1^-}$
Ξ_b	6.15			5.96	0	-0.21			0.13	-0.04	0	0.14			0.13	-0.67	0.47	0.48	$b[us]_{1^-}$
Ξ_b'	6.40			5.96	0.01		0	0	-0.05	0.26	-0.03		0.22	-0.94					$s[ub]_{1^-} + u[sb]_{1^-}$
Ω_b	6.49			5.99	0.02		0	0	-0.05	0.27	-0.06		0.21	-0.94					$s[sb]_{1^-}$
Ξ_{cc}	3.81		3.93	3.79	0.54		0.09	-0.07	-0.02	0.02	-0.83		0.03	0.10					$c[uc]_{0^-}$
Ξ_{cb}	7.04			7.04	0.01	-0.19			-0.29	0.21	0.51	-0.29			-0.24	-0.64	-0.19	-0.01	$b[uc]_{1^-} - c[ub]_{1^-}$
Ξ_{cb}'	6.98			7.04	0.48		0.11	-0.03	-0.05	0.02	-0.75		0.16	0.40					$b[uc]_{0^-} + c[ub]_{0^-}$
Ξ_{bb}	10.22			10.29	0.62		0.15	-0.14	-0.01	0.01	-0.74		0.15	-0.04					$b[ub]_{0^-}$
Ω_{cc}	3.93		4.04	3.83	0.66		0.16	-0.15	-0.05	0.04	-0.70		0.17	-0.02					$c[sc]_{0^-}$
Ω_{cb}	7.15			7.08	0.01	-0.28			-0.23	0.17	0.68	-0.31			-0.11	-0.48	-0.20	-0.02	$b[sc]_{0^-} - c[sb]_{0^-}$
Ω_{cb}'	7.16			7.08	0.53		0.14	-0.05	-0.08	0.04	-0.69		0.21	0.40					$b[sc]_{0^-} + c[sb]_{0^-}$
Ω_{bb}	10.42			10.33	0		0	0	-0.01	0.01	0.01		0.72	-0.69					$b[sb]_{1^-}$
Ω_{ccb}	8.10			8.16	0.24		0.12	-0.15	0.08	-0.34	-0.02		-0.21	0.86					$c[cb]_{1^-}$
Ω_{cbb}	11.23			11.41	0.01		-0.09	0.33	-0.07	0.14	0.54		0.56	-0.52					$b[cb]_{1^-}$

☞ Observations:

They follow the pattern of the $J^P = 1/2^+$ amplitudes in almost all cases: typically, the lightest like-parity diquark correlation dominates; however, such a dominance is sometimes overcome by a mixed-flavour correlation.

Ground-state flavour-SU(5) $J^P = 3/2^+$ baryons (I)



☞ Observations:

- The computed masses are compared with empirical or lQCD values and the mean absolute-relative-difference is 1.0(0.8)%.
- We obtained a 2.6(1.6)% difference when compared with an ab-initio calculation of the Faddeev equation in a RL truncation, eschewing a quark+diquark approx.
- This also indicates the utility of our CI formulation and the validity of the equal spacing rules (ESRs).

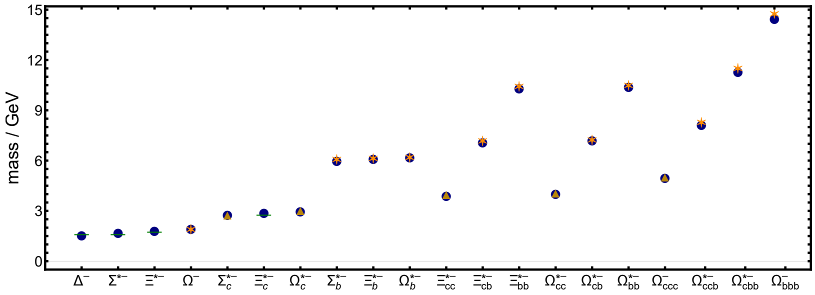
Ground-state flavour-SU(5) $J^P = 3/2^+$ baryons (II)

	M^{CI}	M^e	M^l	M^3	a^{r_1}	a^{r_2}	dom. corr.
Δ	1.27	1.23	1.23	1.21	1		$u\{uu\}_{1^+}$
Σ^*	1.39	1.38	1.40	1.36	0.61	0.79	$u\{us\}_{1^+}$
Ξ^*	1.51	1.53	1.56	1.52	0.85	0.52	$s\{us\}_{1^+}$
Ω	1.63	1.67	1.67	1.67	1		$s\{ss\}_{1^+}$
Σ_c^*	2.54	2.52	2.55	2.39	0.63	0.78	$u\{uc\}_{1^+}$
Ξ_c^*	2.67	2.65	2.65	2.55	0.61	0.79	$s\{uc\}_{1^+} + u\{sc\}_{1^+}$
Ω_c^*	2.80	2.77	2.76	2.70	0.59	0.81	$s\{sc\}_{1^+}$
Σ_b^*	5.79	5.83	5.88	5.60	0.66	0.75	$u\{ub\}_{1^+}$
Ξ_b^*	5.92	5.95	5.96	5.75	0.61	0.79	$s\{ub\}_{1^+} + u\{sb\}_{1^+}$
Ω_b^*	6.04		6.09	5.90	0.55	0.84	$s\{sb\}_{1^+}$
Ξ_{cc}^*	3.73		3.69	3.58	0.98	0.20	$c\{uc\}_{1^+}$
Ξ_{cb}^*	7.00		6.99	6.78	0.97	0.24	$b\{uc\}_{1^+} + c\{ub\}_{1^+}$
Ξ_{bb}^*	10.26		10.18	9.98	0.99	0.08	$b\{ub\}_{1^+}$
Ω_{cc}^*	3.88		3.82	3.73	0.96	0.28	$c\{sc\}_{1^+}$
Ω_{cb}^*	7.12		7.06	6.93	0.94	0.35	$b\{sc\}_{1^+} + c\{sb\}_{1^+}$
Ω_{bb}^*	10.37		10.31	10.14	0.99	0.12	$b\{sb\}_{1^+}$
Ω_{ccc}	4.90		4.80	4.76	1		$c\{cc\}_{1^+}$
Ω_{ccb}^*	8.08		8.04	7.96	0.62	0.79	$c\{cb\}_{1^+}$
Ω_{cbb}^*	11.26		11.23	11.17	0.96	0.28	$b\{cb\}_{1^+}$
Ω_{bbb}	14.45		14.37	14.37	1		$b\{bb\}_{1^+}$

☞ Observations:

- Fully dynamical nature of the diquarks and the character of the Faddeev kernel work together to ensure a continual reshuffling of the dressed-quarks within the diquark correlations.
- Consequently, in all cases involving more than one quark flavor, the diquark combination with maximal flavour shuffling is favored.

Ground-state flavour-SU(5) $J^P = 3/2^-$ baryons (I)



☞ Observations:

- The $J^P = 3/2^-$ baryons can only contain (symmetric) positive parity axial vector diquarks because the CI only supports (antisymmetric) negative-parity diquarks.
- The computed masses are compared with empirical or lQCD values and the mean absolute-relative-difference is 2.2(1.4)%.
- We obtained a 1.4(1.9)% difference when compared with an ab-initio calculation of the Faddeev equation in a RL truncation, eschewing a quark+diquark approx.

	M^{CI}	M^e	M^l	M^3	a^{r_1}	a^{r_2}	dom. corr.
Δ	1.59	1.67		1.73	1		$u\{uu\}_{1^+}$
Σ^*	1.72	1.66		1.79	0.72	0.69	$s\{uu\}_{1^+}$
Ξ^*	1.84	1.82		1.84	0.91	0.42	$s\{us\}_{1^+}$
Ω	1.95			1.90	1		$s\{ss\}_{1^+}$
Σ_c^*	2.80		2.80	2.83	0.85	0.53	$c\{uu\}_{1^+}$
Ξ_c^*	2.91	2.82	2.80	2.89	0.74	0.67	$c\{us\}_{1^+}$
Ω_c^*	3.01		3.07	2.94	0.62	0.79	$s\{sc\}_{1^+}$
Σ_b^*	6.03			6.07	0.84	0.54	$b\{uu\}_{1^+}$
Ξ_b^*	6.13			6.13	0.69	0.72	$s\{ub\}_{1^+} + u\{sb\}_{1^+}$
Ω_b^*	6.23			6.19	0.52	0.86	$s\{sb\}_{1^+}$
Ξ_{cc}^*	3.93		4.01	3.93	0.98	0.18	$c\{uc\}_{1^+}$
Ξ_{cb}^*	7.13			7.18	0.96	0.28	$b\{uc\}_{1^+} + c\{ub\}_{1^+}$
Ξ_{bb}^*	10.35			10.42	0.99	0.08	$b\{ub\}_{1^+}$
Ω_{cc}^*	4.04		4.12	3.99	0.96	0.26	$c\{sc\}_{1^+}$
Ω_{cb}^*	7.23			7.23	0.92	0.39	$b\{sc\}_{1^+} + c\{sb\}_{1^+}$
Ω_{bb}^*	10.44			10.48	0.99	0.11	$b\{sb\}_{1^+}$
Ω_{ccc}	5.01		5.08	5.03	1		$c\{cc\}_{1^+}$
Ω_{ccb}^*	8.17			8.28	0.63	0.77	$c\{cb\}_{1^+}$
Ω_{cbb}^*	11.32			11.52	0.96	0.28	$b\{cb\}_{1^+}$
Ω_{bbb}	14.49			14.77	1		$b\{bb\}_{1^+}$

☞ Observations:

- Similar to $J^P = 3/2^+$ states except for Σ^*, Σ_c^*, Σ_b^*, in which a competition between the lightest diquark correlation and the heavy-light diquark exists.
- The former case wins in the negative-parity systems, whereas the latter case is dominant in the positive-parity systems.
- This ordering reverses in the $3/2^-$ states as $g_{DB}^{\pi_{3/2^-}\pi_{1^+}}$ is reduced from unity.

Summary

A confining, symmetry-preserving treatment of a vector $\otimes$ vector contact interaction has been used to compute spectra of ground-state $J^P = 0^\pm, 1^\pm$ $(f\bar{g})$ mesons and $J^P = 1/2^\pm, 3/2^\pm$ (fgh) baryons, where $f, g, h \in \{u, d, s, c, b\}$.

- The calculated meson masses agree well with experiment: the mean-relative difference for 38 states is 5(6)%. Expressing effects tied to the emergence of hadronic mass (EHM) was crucial to achieving this level of agreement.
- A quark+diquark approximation to the baryon Faddeev equation was used herein. Its formulation required the calculation of masses and correlation strengths for all 38 distinct participating diquarks.
- Every possible three-quark $1/2^\pm, 3/2^\pm$ ground-state baryon is realised. 34 states are already known empirically and lQCD provides another 30. The mean-relative difference between CI prediction and experiment/lQCD mass is 1.4(1.2)%.

A primary merit of the framework employed herein is its simplicity, enabling all analyses and calculations to be completed algebraically.

Using 12 parameters that implements EHM-induced effects associated with scale-dependent interaction, spin-orbit repulsion and meson-cloud, 126 hadron masses are predicted.