UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Illinois Center for Advanced Studies of the Universe

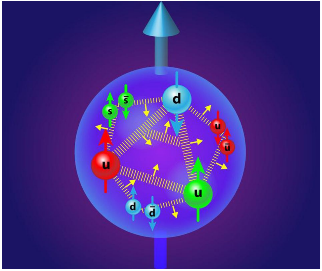
Quantum kinetic theory and the Wigner-function formalism

Enrico Speranza

Workshop on QGP Phenomenology - IPM

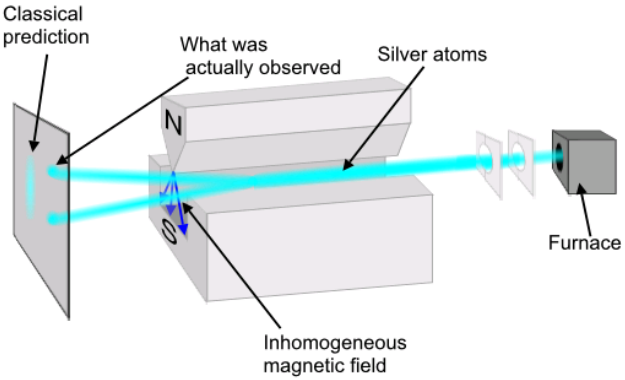
May 28, 2021

Quantum $\Longleftrightarrow$ Classical



Quantum spin

- Stern-Gerlach Experiment (1922)

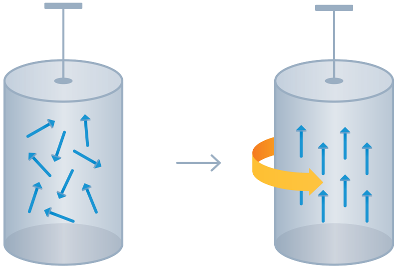


https://commons.wikimedia.org/w/index.php?curid=563880

- Two states: $|\uparrow\rangle$, $|\downarrow\rangle$
- Spin operator: $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$

Rotation and polarization

- Condensed matter: **Barnett effect**

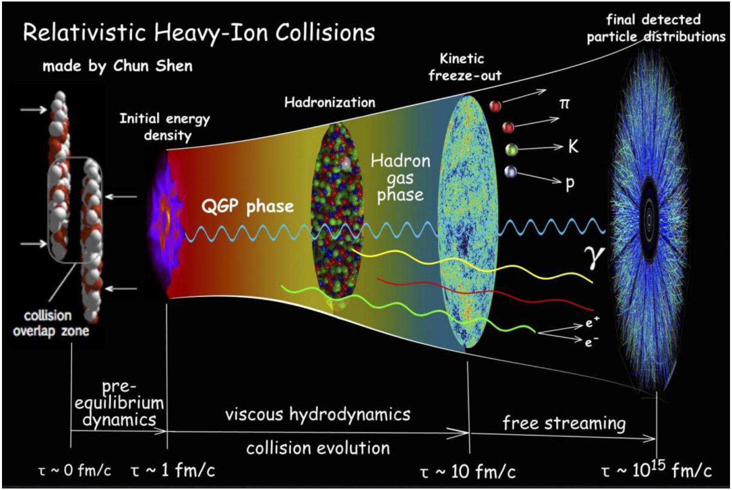


Ferromagnet gets magnetized when it rotates

Polarization effects through rotation in heavy-ion collisions? Yes!

Vorticity and spin physics in heavy-ion collisions

Relativistic heavy-ion collisions

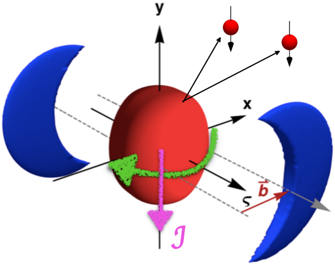


Picture by Chun Shen

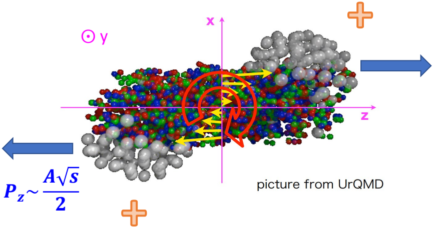
- Quark-gluon plasma $\Longrightarrow$ Early universe (up to 10 μs after Big Bang)
- Relativistic hydrodynamics is a powerful effective theory: $\partial_\mu T^{\mu\nu} = 0$

Heinz, Snellings, Ann. Rev. Nucl. Part. Sci. **63**, 123-151 (2013)

Noncentral heavy-ion collisions



picture from Florkowski, Ryblewski, Kumar, Prog. Part. Nucl. Phys. **108**, 103709 (2019)



picture from UrQMD

Large global angular momentum

$$J \sim \frac{A\sqrt{s}}{2}\, b \sim 10^5 \hbar$$

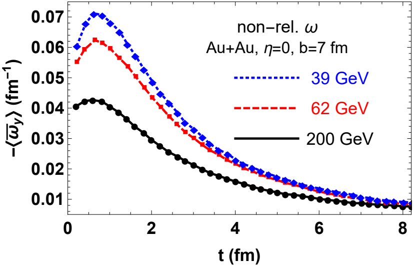
$\Rightarrow$ Vorticity of hot and dense matter $\Rightarrow$ particle polarization along vorticity

Vorticity

$$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{v}$$

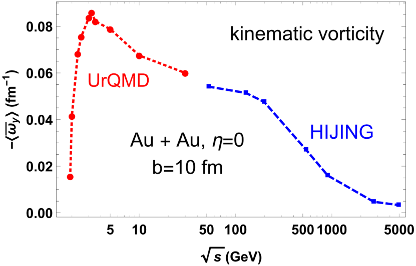
Vorticity in heavy-ion collisions

Time dependence



Jiang, Lin, Liao, PRC 94, 044910

Energy dependence



Deng, Huang, Ma, Zhang, PRC 101 064908
Deng, Huang, PRC 93 064907

see also, e.g., Huang, Liao, Wang, Xia, 2010.08937; Huang, 2002.07549; Becattini et al EPJC 75, 406; Csernai, Magas, Wang, PRC 87, 034906; Csernai, Wang, Bleicher, Stoecker, PRC 90, 021904; Ivanonv, Soldatov, PRC 95 054915

- Vorticity decreases at high energies
- Extremely high vorticity: $\omega_y \sim 10^{-2}\,\mathrm{fm}^{-1} \sim 10^{21}\,s^{-1}$

Spin-vorticity coupling

$$\text{Effective interaction} \sim -\hbar\,\vec{\sigma}\cdot\vec{\omega}$$

$$\sim \text{Quantum} \cdot \text{Classical}$$

$\vec{\sigma}$ - Particle spin, $\vec{\omega}$ - Medium rotation

- Thermodynamic equilibrium statistical operator $\rho = \exp\left(-\frac{\hbar\vec{\sigma}\cdot\vec{\omega}}{k_B T}\right)$

  T - Temperature, k_B - Boltzmann constant

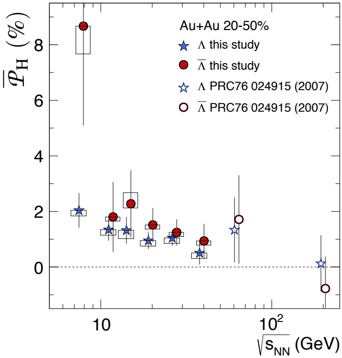
$$\vec{P} = \mathrm{Tr}(\rho\,\vec{\sigma}) \sim \frac{\hbar}{k_B T}\vec{\omega}$$

- $\Longrightarrow$ Λ-baryon polarization, ϕ and $K^{\star 0}$-mesons polarization (?)

  Voloshin, 0410089; Liang, Wang, PRL **94**, 102301; Betz, Gyulassy, Torrieri PRC **76**, 044901; Becattini, Piccinini, Rizzo, PRC **77**, 024906

Experimental observation - Global Λ polarization

- Polarization along global angular momentum

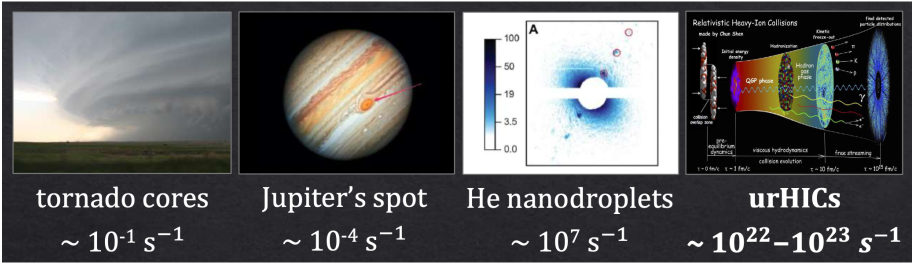


L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

- Weak decay: $\Lambda \to p + \pi^-$ angular distr.: $dN/d\cos\theta = \frac{1}{2}(1 + \alpha|\vec{\mathcal{P}}_H|\cos\theta)$
- Quark-gluon plasma is the "most vortical fluid ever observed"

$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 10^{21}\,\mathrm{s}^{-1}$$

The most vortical fluid ever observed



Polarization observable in heavy-ion collisions

- **Assumptions**:
  1. Local thermodynamic equilibrium of spin degrees of freedom
  2. Polarization determined at freeze-out - No spin dynamics during medium evolution

Becattini, Chandra, Del Zanna, Grossi, Annals. Phys. 338, 32 (2013)

- Relativistic spin vector at **local equilibrium**

$$\langle \hat{\Pi}_\mu(p) \rangle = -\frac{\hbar^2}{8m} \epsilon_{\mu\nu\alpha\beta} p^\nu \frac{\int d\Sigma_\lambda p^\lambda f(1-f) \varpi^{\alpha\beta}}{\int d\Sigma_\lambda p^\lambda f}$$

$\varpi^{\alpha\beta} = -\frac{1}{2}(\partial^\alpha \beta^\beta - \partial^\beta \beta^\alpha)$ - Thermal vorticity
$\beta^\mu = u^\mu / T$, u^μ - Fluid velocity, T - Temperature
p^μ - particle momentum
f - Fermi-Dirac distribution function
Σ_λ - Space-time hypersurface (Freeze-out)

- Note: Recently, extra terms at local equilibrium have been found

Liu, Yin, 2103.09200 (2021); Fu, Liu, Pang, Song, Yin, 2103.10403 (2021); Becattini, Buzzegoli, Palermo, 2103.10917, (2021); Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, 2103.14621 (2021)

What happens if spin is included as a dynamical variable in kinetic theory and hydrodynamics?

Spin as a dynamical variable

Goal: Relativistic hydrodynamics (classical) with spin (quantum) as dynamical variable

Florkowski, Friman, Jaiswal, ES, PRC **97**, no. 4, 041901 (2018)
Florkowski, Friman, Jaiswal, Ryblewski, ES, PRD **97**, no. 11, 116017 (2018)
Florkowski, Becattini, ES, Acta Phys. Polon. B **49**, 1409 (2018)
Becattini, Florkowski, ES, PLB **789**, 419 (2019)
Bhadury, Forkowski, Jaiswal, Kumar, Ryblewski, 2002.03937, 2008.10976 (2020)
Singh, Sophys, Ryblewski, PRD **103**, no.7, 074024 (2021)
ES, Weickgenannt, EPJA, 57, 155 (2021)

Starting point: Kinetic theory from quantum field theory

Weickgenannt, Sheng, ES, Wang, Rischke, PRD **100**, no. 5, 056018 (2019)
Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)
Sheng, Weickgenannt, ES, Rischke, Wang, 2103.10636 (2021)

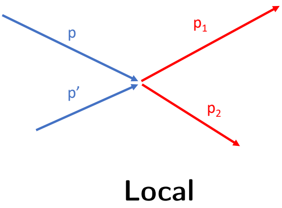
- Alternative approaches: Lagrangian formulation, entropy current, ...
  Montenegro, Tinti, Torrieri, PRD 96, 056012 (2017)
  Montenegro, Torrieri, 2004.10195 (2020)
  Hattori, Hongo, Huang, Matsuo, Taya, PLB795, 100 (2019)
  Garbiso, Kaminski, JHEP **12**, 112 (2020)
  Fukushima, Pu, 2010.01608 (2020)
  Li, Stephanov, Yee, 2011.12318 (2020)
  Gallegos, Gürsoy, Yarom, arXiv:2101.04759 (2020)

Our results

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)

- How do we describe the orbital-to-spin angular momentum conversion in kinetic theory?

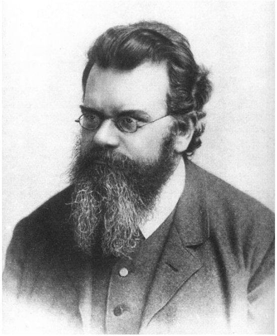
Nonlocal particle scatterings (finite impact parameter)



- And in hydrodynamics?

Antisymmetric part of energy-momentum tensor

Kinetic theory



Classical kinetic theory

- Distribution function

$$f(x, p)$$

- Boltzmann equation

$$p_\mu \partial^\mu f(x, p) = C[f(x, p)]$$

$C[f(x, p)]$ - Collision term

- Hydrodynamics from kinetic theory

$$T^{\mu\nu}(x) = \int d^4p \, p^\mu p^\nu f(x, p)$$

$$\partial_\mu T^{\mu\nu}(x) = \int d^4p \, p^\nu C[f(x, p)] = 0$$

How to formulate kinetic theory from quantum mechanics?

Wigner function - Quantum mechanics

"Quantum extension" of classical distribution function

$$W(x,p) = \int \frac{dy}{2\pi\hbar} e^{-\frac{i}{\hbar} p\cdot y} \psi^* \left(x + \frac{y}{2}\right) \psi \left(x - \frac{y}{2}\right)$$

Properties: $\int dp\, W(x,p) = |\psi(x)|^2, \qquad \int dx\, W(x,p) = |\psi(p)|^2$

Connected to probability!

- Expectation value of any operator $\hat{A}$

$$\langle \hat{A} \rangle = \int dx\, dp\, W(x,p) a(x,p)$$

- Schroedinger equation $\Longrightarrow$ Eqs. of motion $=$ Boltzmann eq.

$$\frac{\partial W(x,p)}{\partial t} + \frac{p}{m} \frac{\partial W(x,p)}{\partial x} = C$$

Wigner function - Quantum field theory

$$W_{\chi\sigma}(x,p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \left\langle : \bar{\Psi}_\sigma \left(x + \frac{y}{2}\right) \Psi_\chi \left(x - \frac{y}{2}\right) : \right\rangle$$

- Dirac equation $\Longrightarrow$ Equations of motion for Wigner function
  Elze, Gyulassy, Vasak, Ann. Phys. 173 (1987) 462
  de Groot, van Leeuwen, van Weert, Relativistic Kinetic Theory. Principles and Applications

$$\left[\gamma \cdot \left(p + i\frac{\hbar}{2}\partial\right) - m\right] W(x,p) = \hbar \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \left\langle : \rho \left(x - \frac{y}{2}\right) \bar{\psi} \left(x + \frac{y}{2}\right) : \right\rangle$$

$\rho = -(1/\hbar)\partial\mathcal{L}_I/\partial\bar{\psi}$, $\mathcal{L}_I$ = interaction Lagrangian

$\Longrightarrow$ Boltzmann equation $\Longrightarrow$ Kinetic theory

$$p_\mu \partial^\mu W_{\chi\sigma}(x,p) = C_{\chi\sigma}$$

$\hbar$-expansion

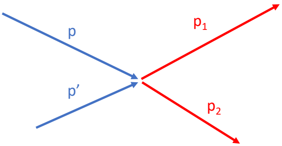
Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)

- Semiclassical expansion of Wigner function

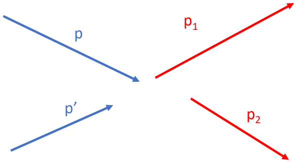
$$W = W^{(0)} + \hbar W^{(1)} + \mathcal{O}(\hbar^2)$$

- Semiclassical and **nonlocal** expansion of collision kernel

$$C = C^{(0)}_{\text{local}} + \hbar C^{(1)}_{\text{local}} + \hbar C^{(1)}_{\text{nonlocal}} + \mathcal{O}(\hbar^2)$$



Local



Nonlocal

Spin in phase space

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)

- Account for spin dynamics enlarge phase space
  J. Zamanian, M. Marklund, and G. Brodin, NJP 12, 043019 (2010)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)
- Introduce new phase-space variable $\mathfrak{s}^\mu$

$$W_{\chi\sigma}(x,p) \to \mathfrak{f}(x,p,\mathfrak{s})$$

- Boltzmann equation

$$p \cdot \partial\, \mathfrak{f}(x,p,\mathfrak{s}) = m\, \mathfrak{C}[\mathfrak{f}]$$

All dynamics in one scalar equation!

Nonlocal collisions

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)

$$\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_{\text{local}}[\mathfrak{f}] + \hbar\, \mathfrak{C}_{\text{nonlocal}}[\mathfrak{f}]$$

- Long calculation $\Longrightarrow$ Intuitive result in low-density approximation:

$$\mathfrak{C}[\mathfrak{f}] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \,\mathcal{W}\, [\mathfrak{f}(x+\Delta_1, p_1, \mathfrak{s}_1)\mathfrak{f}(x+\Delta_2, p_2, \mathfrak{s}_2) - \mathfrak{f}(x+\Delta, p, \mathfrak{s})\mathfrak{f}(x+\Delta', p', \mathfrak{s}')]$$
$$+ \int d\Gamma_2\, dS_1(p)\, \mathfrak{W}\, \mathfrak{f}(x+\Delta_1, p, \mathfrak{s}_1)\mathfrak{f}(x+\Delta_2, p_2, \mathfrak{s}_2)$$

$d\Gamma \equiv d^4p\, dS(p)$

- Structure: Momentum and spin exchange + Spin exchange only
- Nonlocal Collisions $\Longrightarrow$ Displacement $\Delta \sim \mathcal{O}(\hbar) \sim \mathcal{O}(\partial)$
- $\mathcal{W}$, $\mathfrak{W}$ vacuum transition probabilities, depend on phase-space spins

Equilibrium distribution function

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021)

- **Equilibrium condition:** $\mathfrak{C}[\mathfrak{f}] = 0$
- Ansatz for distribution function
  F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$\mathfrak{f}_{eq}(x,p,\mathfrak{s}) \propto \exp\left[\underbrace{-\beta(x)\cdot p}_{\text{Energy-momentum}} \quad \underbrace{+\frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma_{\mathfrak{s}}^{\mu\nu}}_{\text{Total angular momentum}} \right]\delta(p^2 - M^2)$$

  - M - mass (possibly modified by interactions)
  - $\beta^\mu = u^\mu/T$, Spin potential $\Omega^{\mu\nu}$
  - Spin-dipole-moment tensor $\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha \mathfrak{s}_\beta$

- Insert into $\mathfrak{C}[\mathfrak{f}]$ and expand up to $\mathcal{O}(\hbar)$

Condition for $\mathfrak{C}[\mathfrak{f}] = 0 \Longrightarrow$ **Global equilibrium**

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0 \qquad \Omega_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) = \text{const.}$$

System gets polarized through rotations!

Equilibrium distribution function

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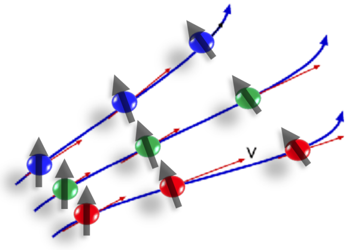
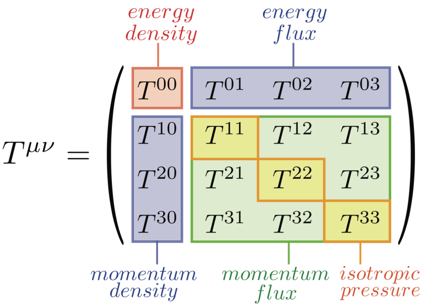
Spin hydrodynamics



- Energy-momentum tensor: $T^{\mu\nu} = \langle \hat{T}^{\mu\nu} \rangle$



picture from Rezzolla, Zanotti, Relativistic Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

Canonical energy-momentum and spin tensor

Action $\Rightarrow$ Poincaré symmetry $\Rightarrow$ Noether's th. $\Rightarrow$ Conservation laws

- **Conservation of energy and momentum:**
  Canonical energy-momentum tensor $\hat{T}^{\mu\nu}_C(x)$

$$\partial_\mu \hat{T}^{\mu\nu}_C(x) = 0$$

- **Conservation of total angular momentum:**
  Canonical total angular momentum tensor ("orbital"+"spin")

$$\hat{J}^{\lambda,\mu\nu}_C(x) = x^\mu \hat{T}^{\lambda\nu}_C(x) - x^\nu \hat{T}^{\lambda\mu}_C(x) + \hat{S}^{\lambda,\mu\nu}_C(x)$$

$$\partial_\lambda \hat{J}^{\lambda,\mu\nu}_C(x) = 0 \Longrightarrow \partial_\lambda \hat{S}^{\lambda,\mu\nu}_C(x) = \hat{T}^{\nu\mu}_C(x) - \hat{T}^{\mu\nu}_C(x)$$

Pseudo-gauge transformations

ES, Weickgenannt, EPJA, 57, 155 (2021) (Review)

Densities are not uniquely defined $\Longrightarrow$ Relocalization

F. W. Hehl, Rep. Mat. Phys. 9, 55 (1976)

$$\hat{T}'^{\mu\nu}(x) = \hat{T}_C^{\mu\nu}(x) + \frac{1}{2}\partial_\lambda \left[\hat{\Phi}^{\lambda,\,\mu\nu}(x) + \hat{\Phi}^{\mu,\,\nu\lambda}(x) + \hat{\Phi}^{\nu,\,\mu\lambda}(x)\right]$$

$$\hat{S}'^{\lambda,\,\mu\nu} = \hat{S}_C^{\lambda,\,\mu\nu}(x) - \hat{\Phi}^{\lambda,\,\mu\nu}(x) + \partial_\rho \hat{Z}^{\mu\nu,\lambda\rho}(x)$$

$$\hat{\Phi}^{\lambda,\,\mu\nu} = -\hat{\Phi}^{\lambda,\,\nu\mu},\ \hat{Z}^{\mu\nu,\lambda\rho} = -\hat{Z}^{\nu\mu,\lambda\rho} = -\hat{Z}^{\mu\nu,\rho\lambda}$$

- Global charges are left invariant
- Conservation laws $\partial_\mu \hat{T}'^{\mu\nu} = 0\ ,\ \partial_\lambda \hat{S}'^{\lambda,\mu\nu} = \hat{T}'^{\nu\mu} - \hat{T}'^{\mu\nu}$

See Nora Weickgenannt's talk

Spin hydrodynamics

ES, Weickgenannt, EPJA, 57, 155 (2021) (Review)

- Hydrodynamic densities from quantum field theory

$$T^{\mu\nu} = \langle \hat{T}^{\mu\nu} \rangle \qquad S^{\lambda,\mu\nu} = \langle \hat{S}^{\lambda,\mu\nu} \rangle$$

- **10 hydro eqs.**: 4 Energy-momentum + 6 Total angular momentum

$$\partial_\mu T^{\mu\nu} = 0 \qquad \hbar\, \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- **10 unknowns**: $\beta^\mu = u^\mu / T$ and $\Omega^{\mu\nu}$

- $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ are NOT uniquely defined
  $\Longrightarrow$ Pseudo-gauge transformations
  - Canonical - **Problem:** Total spin is not a tensor for free fields
  - **Solution**: Hilgevoord, Wouthuysen, NP 40 (1963) 1

Spin hydrodynamics

ES, Weickgenannt, EPJA, 57, 155 (2021) (Review)

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Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021);
ES, Weickgenannt, EPJA, 57, 155 (2021) (Review)

- Equations of motion from kinetic theory

$$\partial_\mu T^{\mu\nu}_{\mathrm{HW}} = \int d\Gamma\, p^\nu\, \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar\, \partial_\lambda S^{\lambda,\mu\nu}_{\mathrm{HW}} = \int d\Gamma\, \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathfrak{s}}\, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{\mathrm{HW}}$$

$\Sigma^{\mu\nu}_{\mathfrak{s}} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$

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- Energy-momentum conserved in a collision

Spin-dipole $\Sigma^{\mu\nu}_{\mathfrak{s}}$ not conserved in nonlocal collisions $\Longrightarrow T^{[\nu\mu]}_{\mathrm{HW}} \neq 0$
$\Longrightarrow$ Conversion between spin and orbital angular momentum

Spin hydro with Hilgevoord-Wouthuysen currents

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$\Longrightarrow$ Conversion between spin and orbital angular momentum

- $T^{[\nu\mu]}_{\mathrm{HW}} = 0$: (i) for local collisions (**spin** is collisional invariant)
(ii) in global equilibrium ($\mathfrak{C}[\mathfrak{f}] = 0$)

- Nonlocal collisions away from global equilibrium $\Longrightarrow$ Dissipative dynamics

Fluid gets polarized through rotation!

Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, PRL, 2005.01506 (2020), 2103.04896 (2021);
ES, Weickgenannt, EPJA, 57, 155 (2021) (Review)

- Equations of motion from kinetic theory

$$\partial_\mu T^{\mu\nu}_{\rm HW} = \int d\Gamma\, p^\nu\, \mathfrak{C}[\mathfrak{f}] = 0$$
$$\hbar\, \partial_\lambda S^{\lambda,\mu\nu}_{\rm HW} = \int d\Gamma\, \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathfrak{s}}\, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{\rm HW}$$

$\Sigma^{\mu\nu}_{\mathfrak{s}} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$

- Energy-momentum conserved in a collision

Spin-dipole $\Sigma^{\mu\nu}_{\mathfrak{s}}$ not conserved in nonlocal collisions $\Longrightarrow T^{[\nu\mu]}_{\rm HW} \neq 0$
$\Longrightarrow$ Conversion between spin and orbital angular momentum

- $T^{[\nu\mu]}_{\rm HW} = 0$: (i) for local collisions (**spin** is collisional invariant)
  (ii) in global equilibrium ($\mathfrak{C}[\mathfrak{f}] = 0$)
- Nonlocal collisions away from global equilibrium $\Longrightarrow$ Dissipative dynamics

Fluid gets polarized through rotation!

- What is the meaning of local equilibrium with nonlocal collisions?

Scales

- Knudsen number

$$\mathrm{Kn} \equiv \frac{\ell_{\mathrm{mfp}}}{L_{\mathrm{hydro}}} \ll 1$$

- Deviations from equilibrium $\Longrightarrow$ Dissipative effects

$$\frac{\delta f}{f_{\mathrm{eq}}} \sim \mathcal{O}(\mathrm{Kn})$$

- Δ - New scale associated to spin effects and nonlocality

$$\kappa \equiv \frac{\Delta}{L_{\mathrm{hydro}}} \sim \frac{\hbar f^{(1)}}{f^{(0)}}$$

- If $\Delta \lesssim \ell_{\mathrm{mfp}}$

$$\kappa \lesssim \mathrm{Kn}$$

Spin effects $\Longrightarrow$ No standard notion of local equilibrium!

Outlook - Corrections to polarization formula

$$\langle \hat{\Pi}_\mu(p) \rangle = -\frac{\hbar^2}{8m}\epsilon_{\mu\nu\alpha\beta}p^\nu \frac{\int d\Sigma_\lambda p^\lambda f(1-f)\varpi^{\alpha\beta}}{\int d\Sigma_\lambda p^\lambda f} \quad + \quad ???$$

- Use quantum kinetic theory + spin hydrodynamics to find nonequilibrium corrections

Conclusions

- Vorticity and spin polarization are inherently connected
- New era where spin degrees of freedom must be taken into account in kinetic theory/hydrodynamics - New experimental observables in heavy-ion collisions
- Quantum field theory calculations suggest that spin hydrodynamics is always dissipative in the presence of nonlocal particle collisions
- Spin brings new conceptual problems for the emergence of hydrodynamics