Theoretical aspects of Lambda polarization measurement

Radoslaw Ryblewski
The H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences, Kraków, Poland

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in collaboration with:

Wojciech Florkowski
Institute of Theoretical Physics, Jagiellonian University, Kraków, Poland

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Angular momentum in heavy-ion collisions

Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

$$\boldsymbol{L}_{\text{init}} \sim 10^5 \hbar$$

Part of the angular momentum can be transferred from the orbital to the spin part

$$\boldsymbol{J}_{\text{init}} = \boldsymbol{L}_{\text{init}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

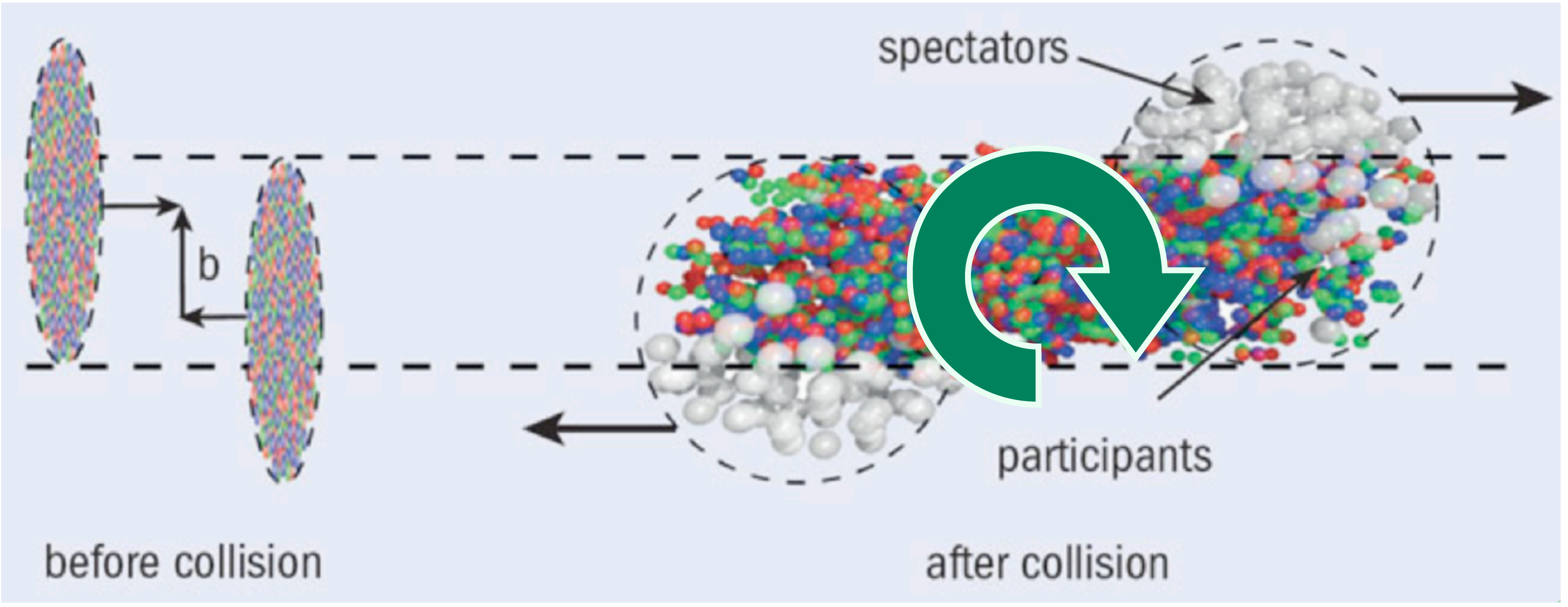
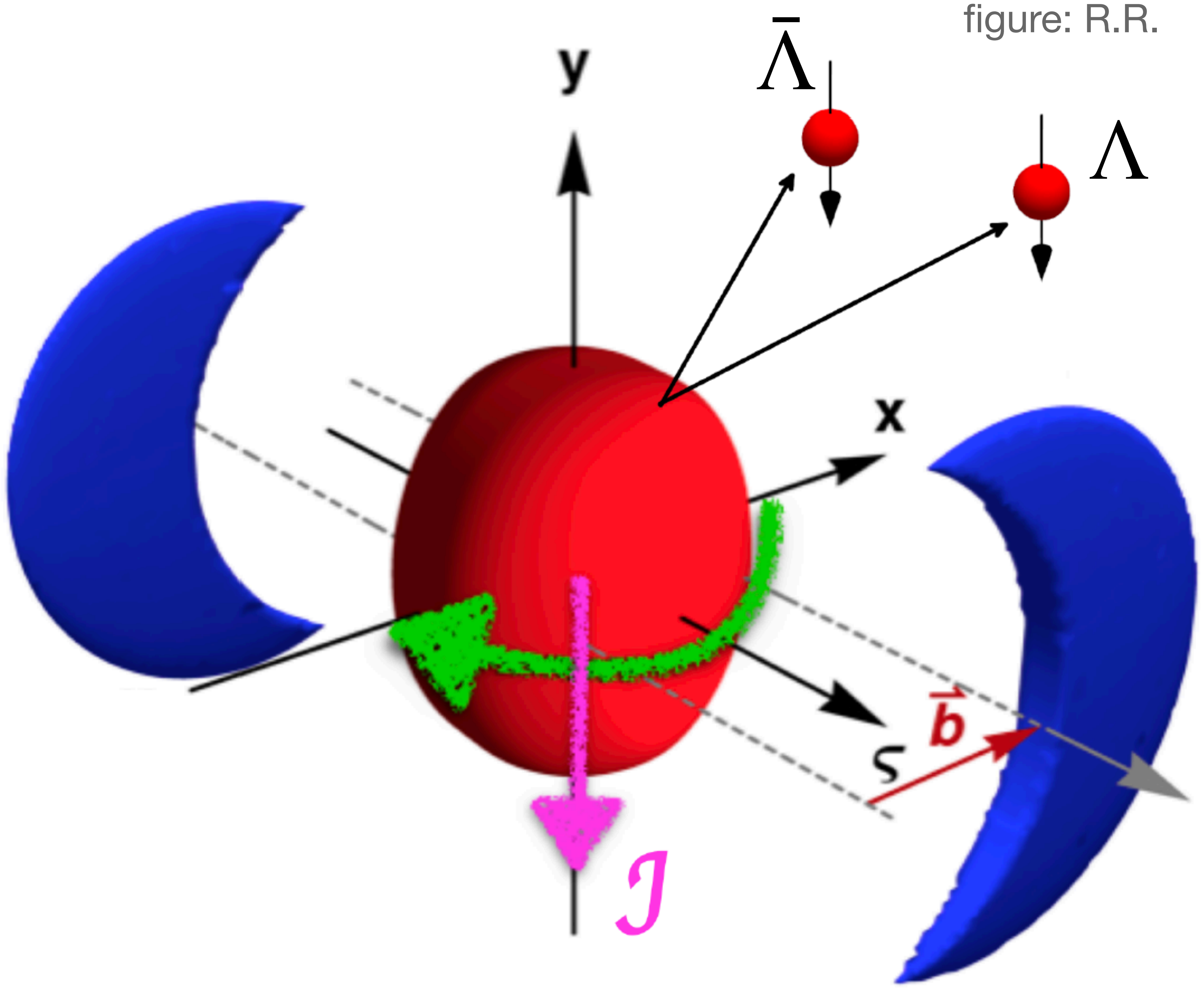


figure: M. Lisa, talk @ "Strangeness in Quark Matter 2016"

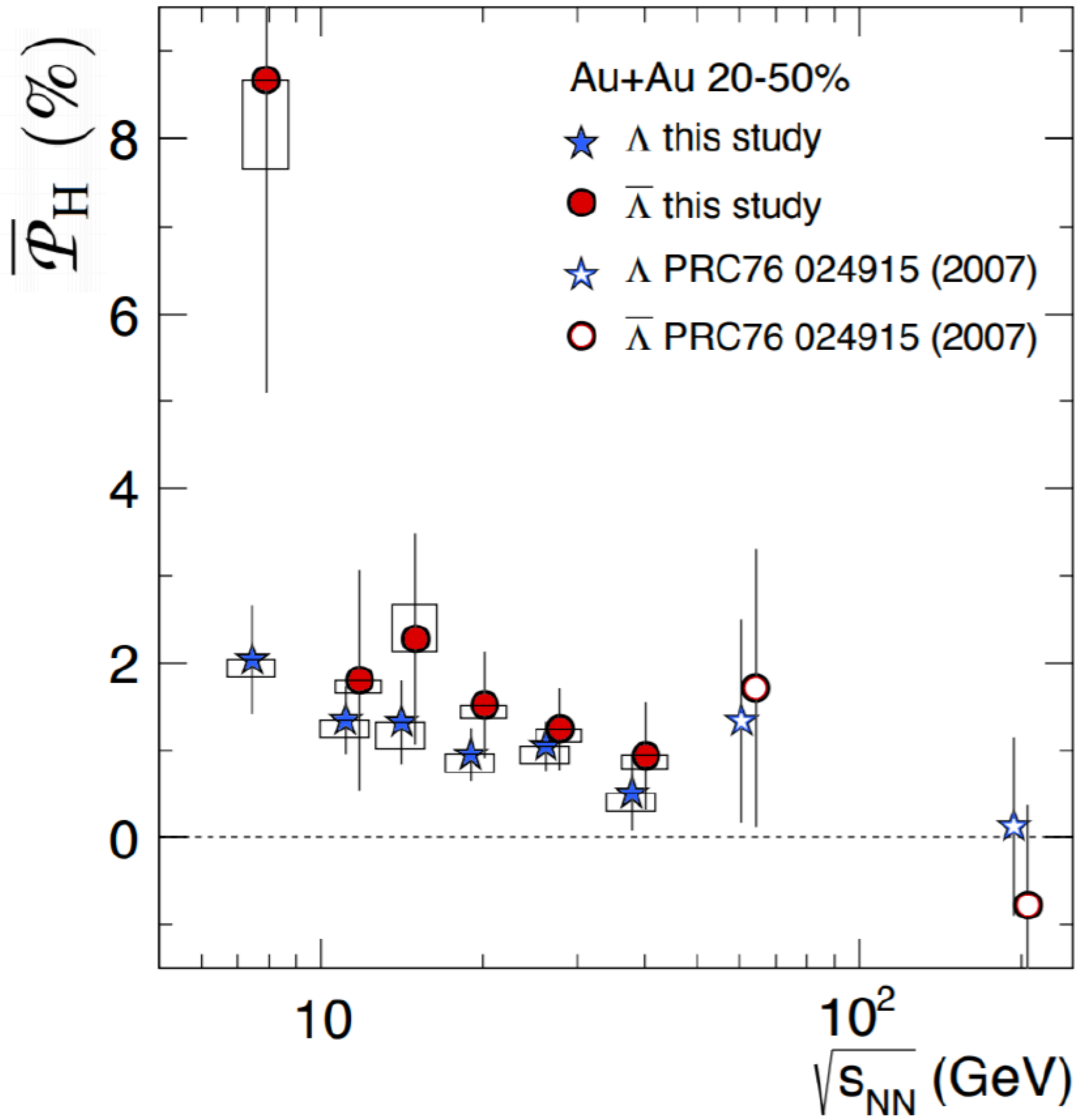
figure: R.R.



Emitted particles are expected to be globally polarized along the system's angular momentum in the center-of-mass (COM) frame

Measurement of Λ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ

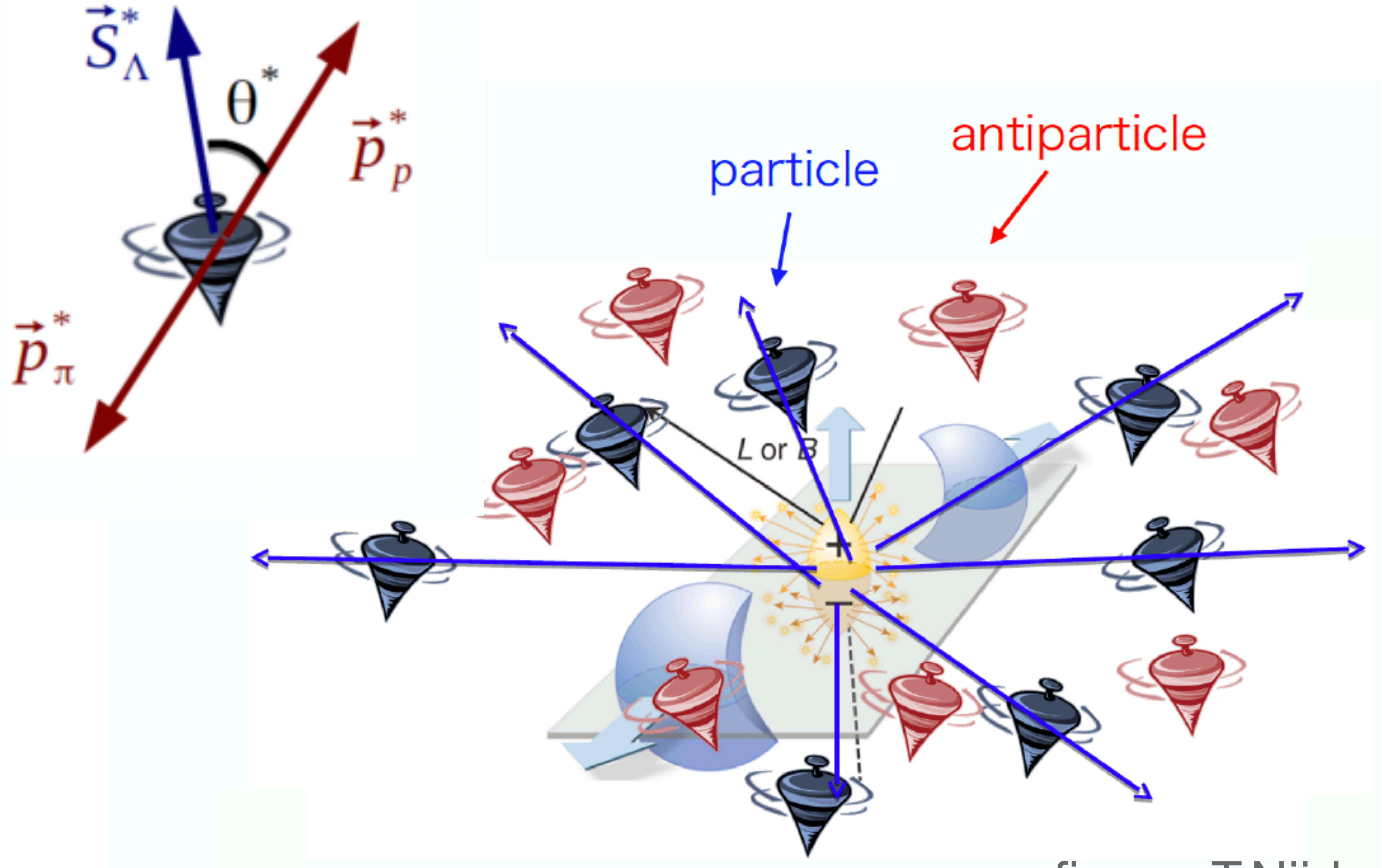


figure: T.Niida

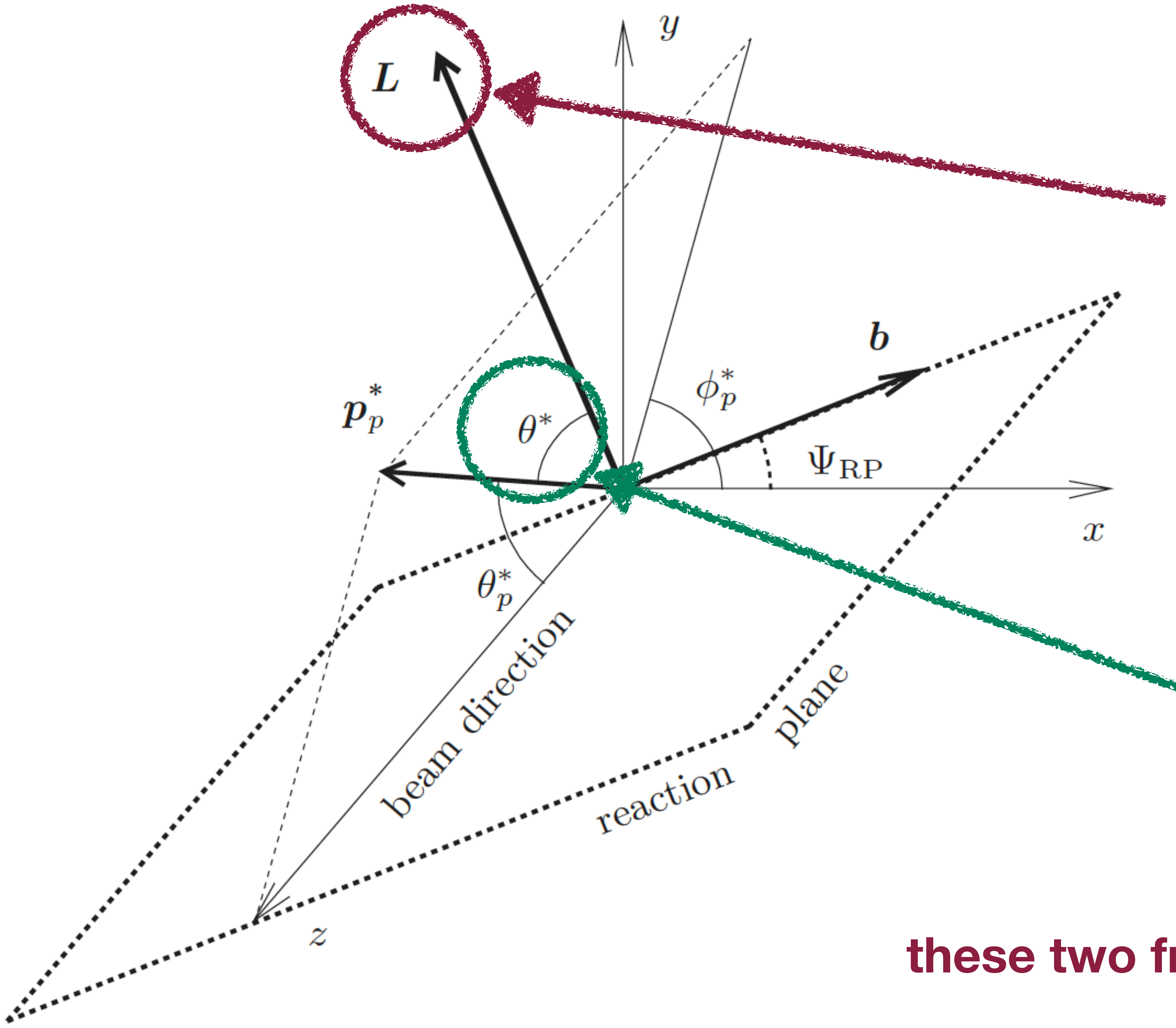
$$\boxed{\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_{\mathrm{H}} \mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}^*_{\mathbf{p}}\right)}$$

in the Λ rest frame

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

Measurement of Λ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions



as the outcome of the spin polarization experiments,
one cites the magnitude of the polarization
along a specific direction in the center-of-mass frame -
total angular momentum of the system L

to determine the magnitude of the polarization,
one studies distributions of momentum
components of protons
emitted in the weak decay of Lambdas
which are measured in their rest frame

$$\boxed{\frac{dN}{d\cos\theta^*} \sim 1 + \alpha_H P_H \cos\theta^*}$$

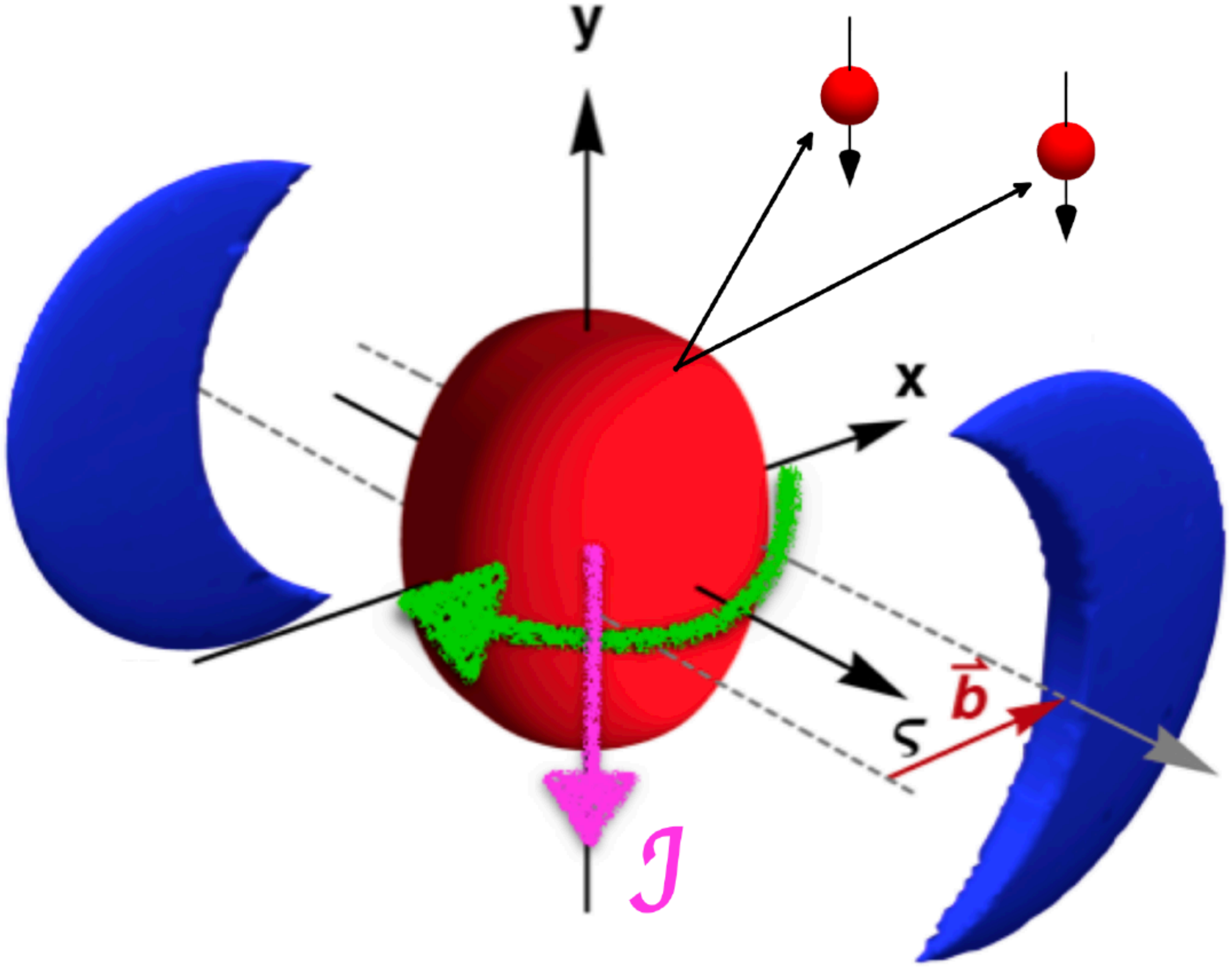
these two frames are linked by a non-trivial Lorentz transformation

interpretation of the results will depend on it!

figure: B. I. Abelev et al. (STAR) PRC 76, 024915 (2007)

Center-of-mass (COM) frame in heavy-ion collision

figure: R. Ryblewski



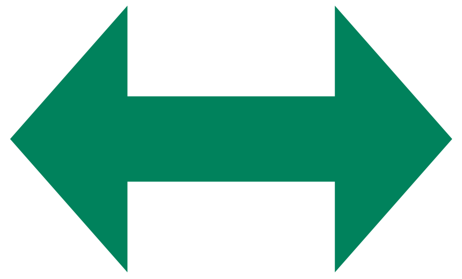
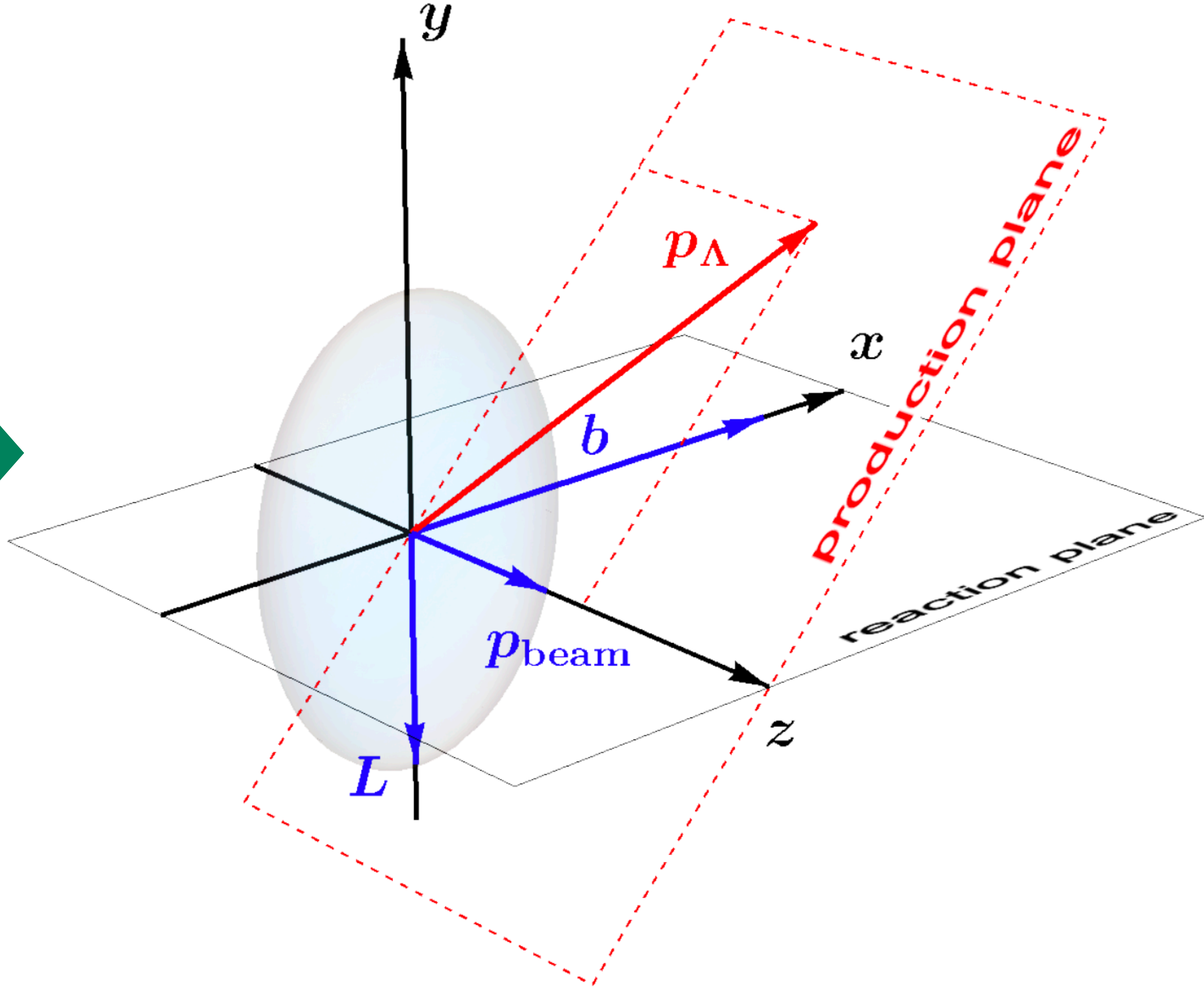


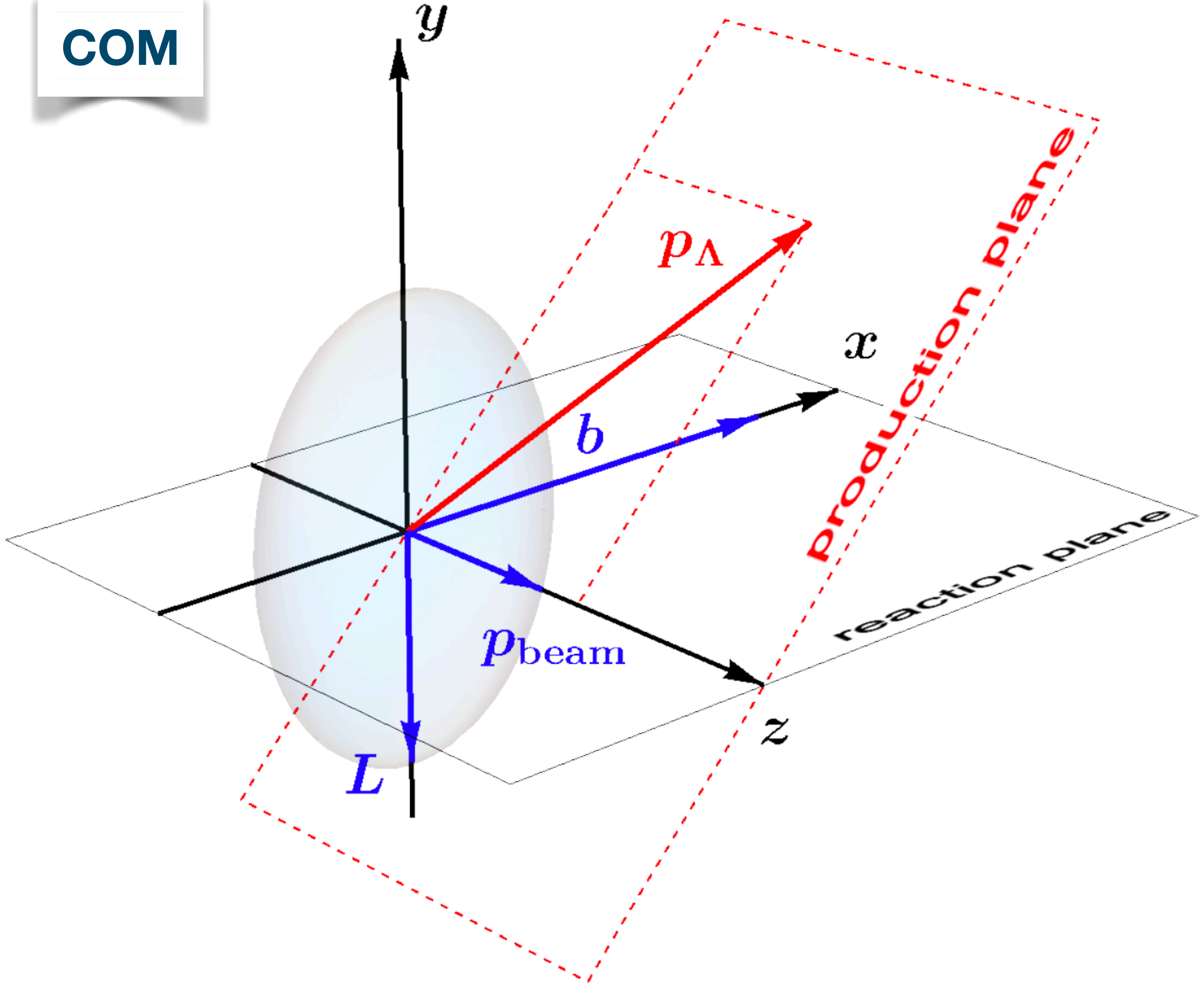
figure: W. Florkowski, R.R., 2102.02890 [hep-ph]



herein, lets assume that the reaction plane angle in the laboratory (LAB) frame can be well measured hence $\Psi_{\rm RP} = 0$

Transformation to Λ rest frame

COM



canonical boost

$$\mathcal{L}^{\mu}{}_{\nu}\left(-\boldsymbol{v}_\Lambda\right) = \begin{bmatrix} \frac{E_\Lambda}{m_\Lambda} & -\frac{p^1_\Lambda}{m_\Lambda} & -\frac{p^2_\Lambda}{m_\Lambda} & -\frac{p^3_\Lambda}{m_\Lambda} \\ -\frac{p^1_\Lambda}{m_\Lambda} & 1+\alpha p^1_\Lambda p^1_\Lambda & \alpha p^1_\Lambda p^2_\Lambda & \alpha p^1_\Lambda p^3_\Lambda \\ -\frac{p^2_\Lambda}{m_\Lambda} & \alpha p^2_\Lambda p^1_\Lambda & 1+\alpha p^2_\Lambda p^2_\Lambda & \alpha p^2_\Lambda p^3_\Lambda \\ -\frac{p^3_\Lambda}{m_\Lambda} & \alpha p^3_\Lambda p^1_\Lambda & \alpha p^3_\Lambda p^2_\Lambda & 1+\alpha p^3_\Lambda p^3_\Lambda \end{bmatrix}$$

$$\alpha \equiv 1/\left(m_\Lambda\left(E_\Lambda + m_\Lambda\right)\right)$$

COM

$$p^\mu = \left(E, p^1, p^2, p^3\right)$$

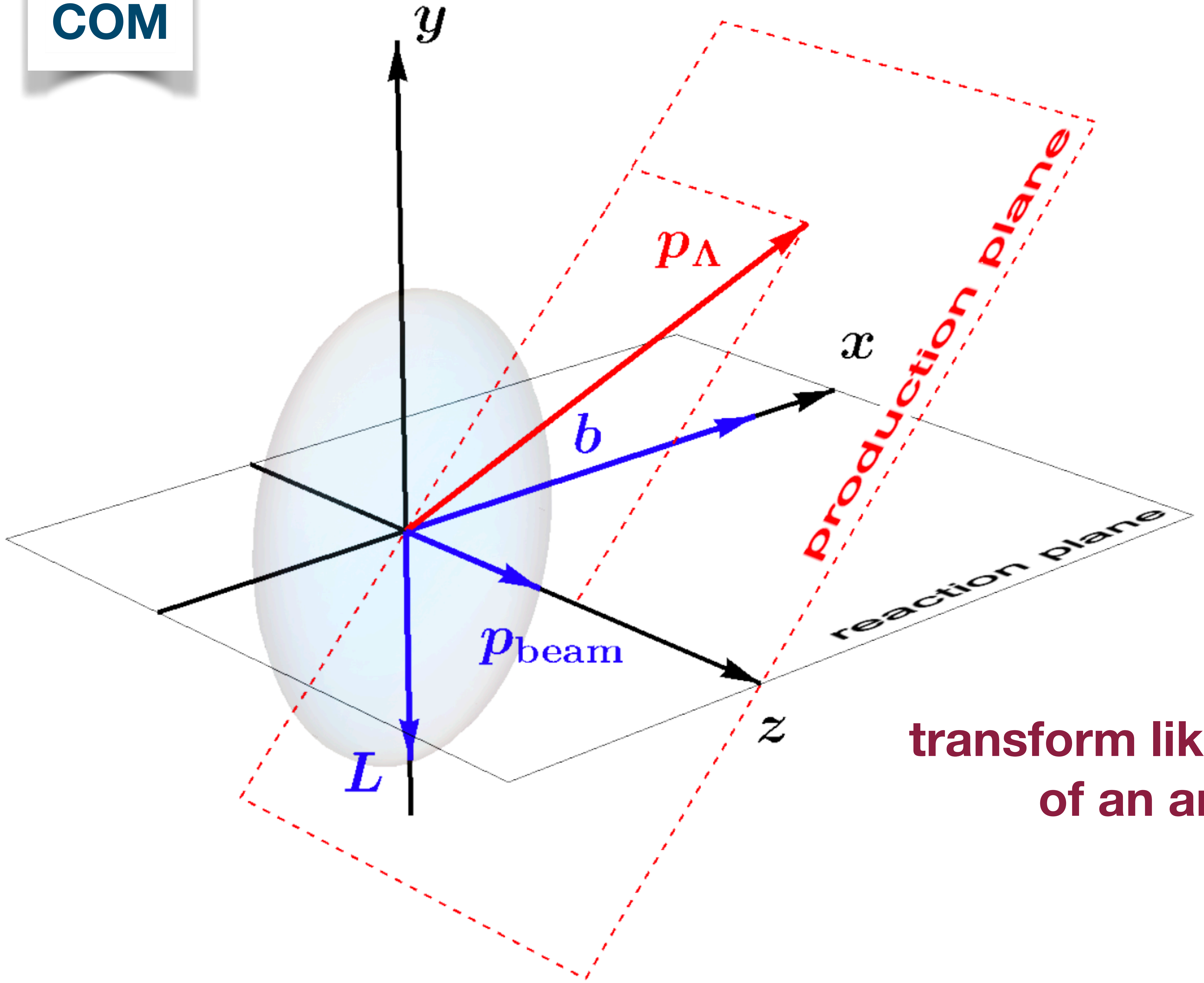
$$\boxed{\mathcal{L}^{\mu}{}_{\nu}\left(-\boldsymbol{v}_\Lambda\right)}$$

$S'(\boldsymbol{p}_\Lambda)$

$$p'^\mu_\Lambda = \left(m_\Lambda, 0, 0, 0\right)$$

Transformation of system's angular momentum

COM



COM

$$J^{\mu\nu} = \boxed{L^{\mu\nu}} + S^{\mu\nu}$$

$$L^k = -\frac{1}{2}\epsilon^{kij}L^{ij} \qquad K^i = -L^{0i} = 0$$

$$\boxed{\mathcal{L}^{\mu}{}_{\nu}\,(-\boldsymbol{v}_\Lambda)}$$

$S'(p_\Lambda)$

$$\boldsymbol{L}' = \gamma_\Lambda \boldsymbol{L} - \frac{\gamma_\Lambda^2}{\gamma_\Lambda + 1}\boxed{\boldsymbol{v}_\Lambda(\boldsymbol{v}_\Lambda \cdot \boldsymbol{L})}$$

$$\gamma_\Lambda = E_\Lambda / m_\Lambda$$

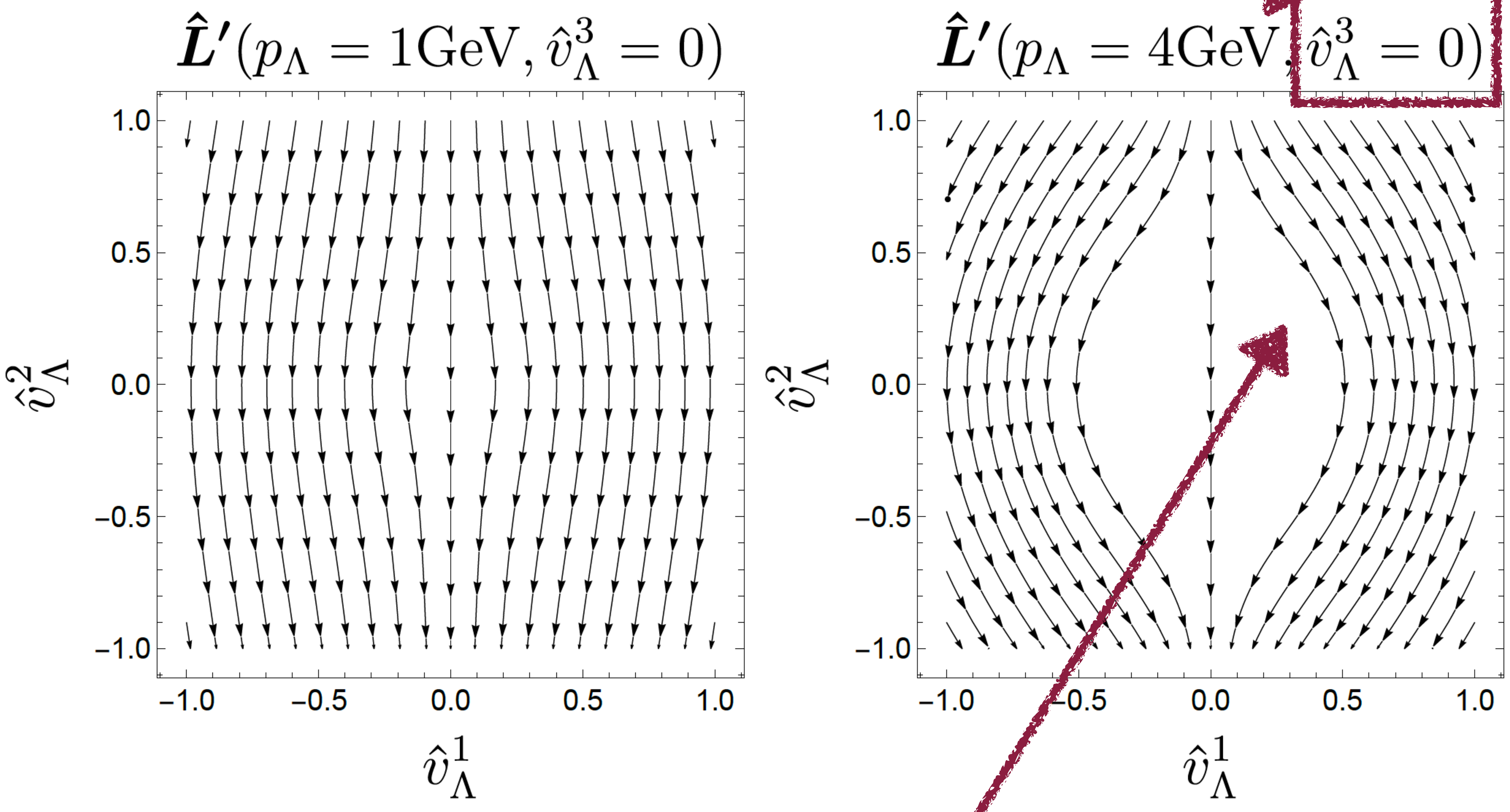
transform like the spatial components of an antisymmetric tensor

relativistic correction

Λ sees direction different from y!

Transformation of system's angular momentum

midrapidity Λ's



change of the system's angular momentum direction
due to relativistic effects

COM

$$\hat{\boldsymbol{L}} = \frac{\boldsymbol{L}}{L} = (0,-1,0)$$

$$\hat{\boldsymbol{K}} = 0$$

$\mathcal{L}^\mu{}_\nu(-\boldsymbol{v}_\Lambda)$

$S'(\boldsymbol{p}_\Lambda)$

$$\hat{L}'^1 = \left(1-(v^2_\Lambda)^2\right)^{-1/2} \frac{\gamma_\Lambda}{\gamma_\Lambda+1} v^1_\Lambda v^2_\Lambda,$$

$$\hat{L}'^2 = \left(1-(v^2_\Lambda)^2\right)^{-1/2} \left(\frac{\gamma_\Lambda}{\gamma_\Lambda+1} v^2_\Lambda v^2_\Lambda - 1\right)$$

$$\hat{L}'^3 = \left(1-(v^2_\Lambda)^2\right)^{-1/2} \frac{\gamma_\Lambda}{\gamma_\Lambda+1} v^3_\Lambda v^2_\Lambda.$$

$$\hat{\boldsymbol{K}}' = \frac{(v^3_\Lambda, 0, -v^1_\Lambda)}{\sqrt{(v^1_\Lambda)^2+(v^3_\Lambda)^2}}$$

Transformation of system's angular momentum

midrapidity Λ's

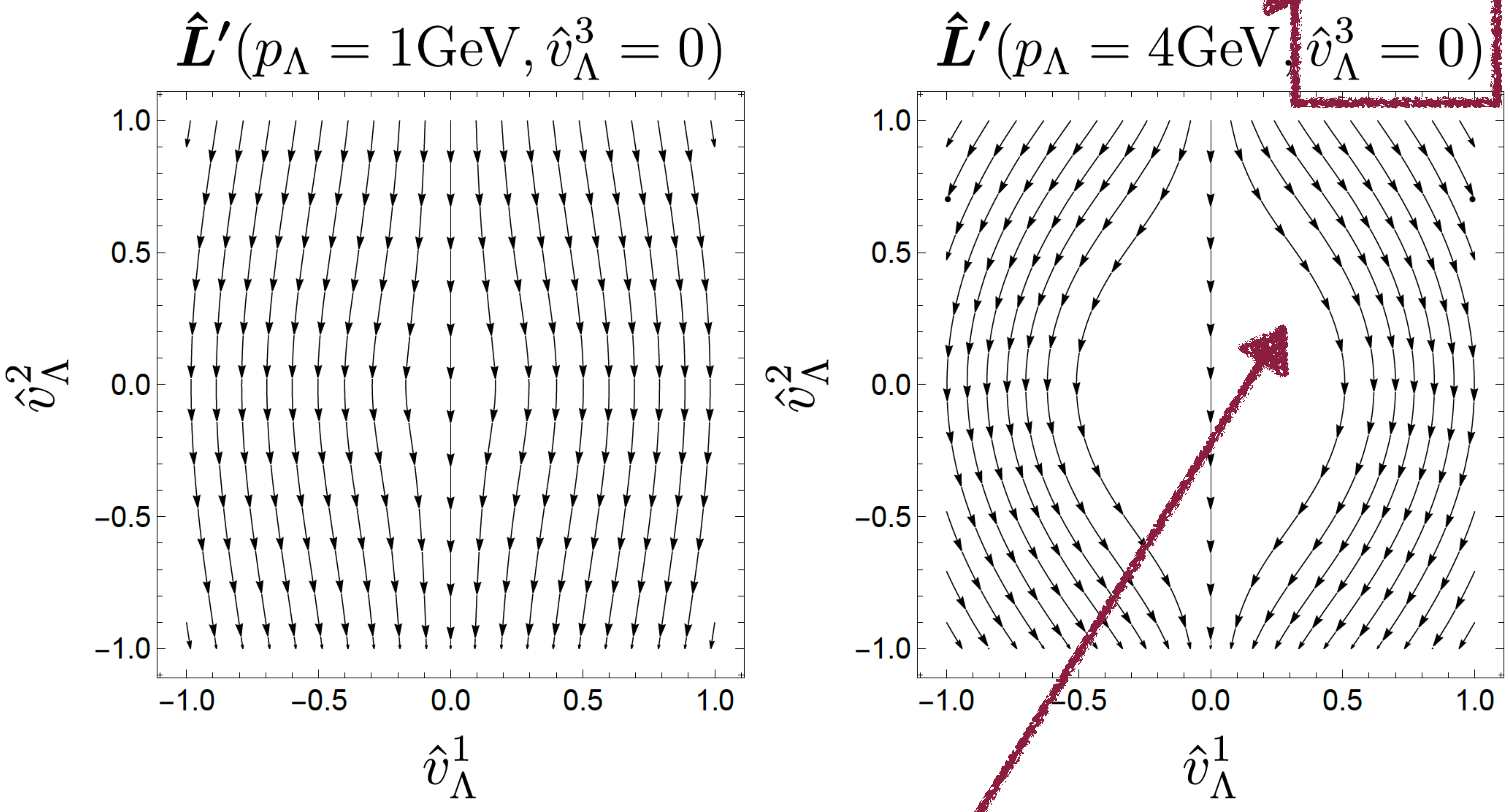
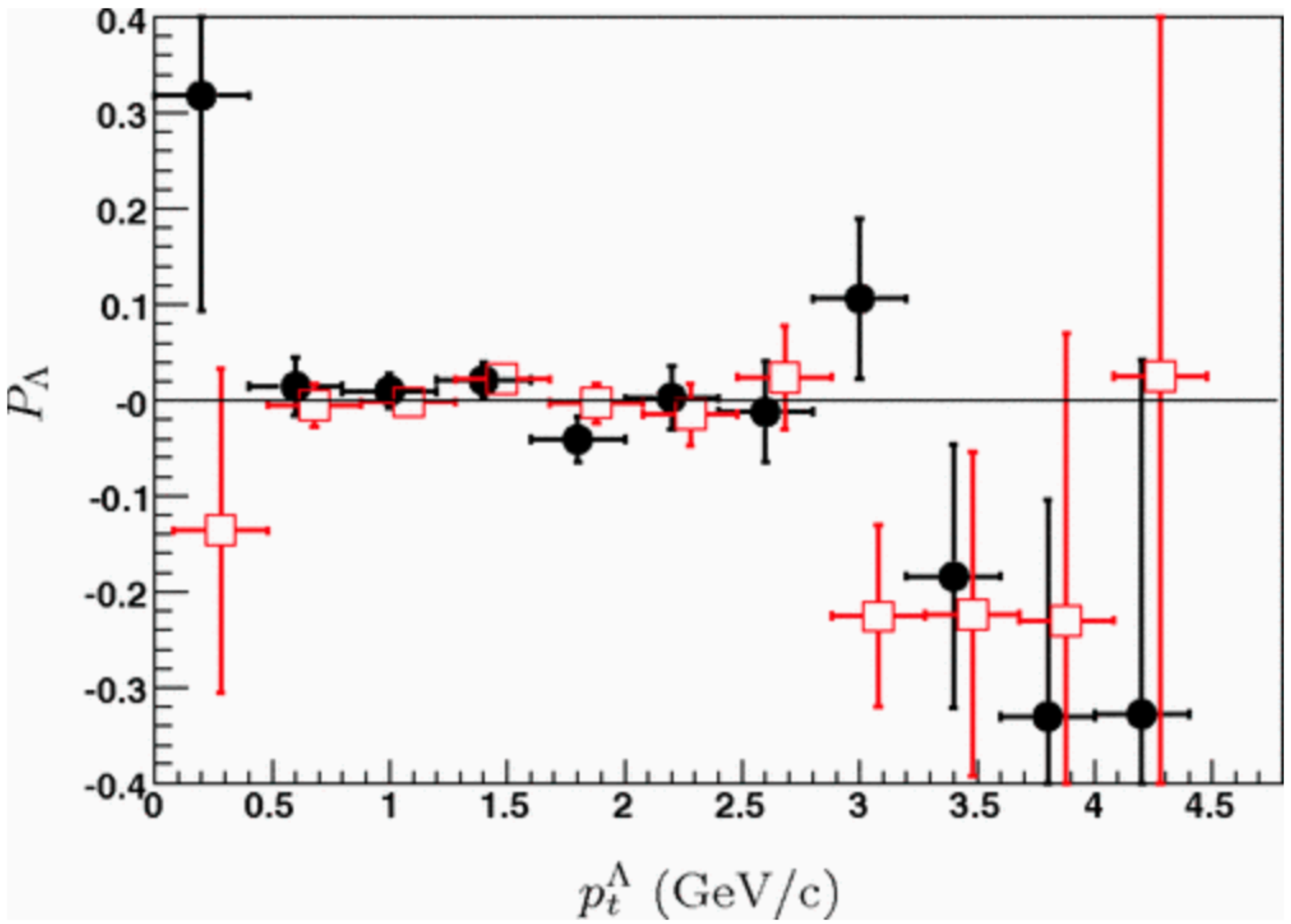


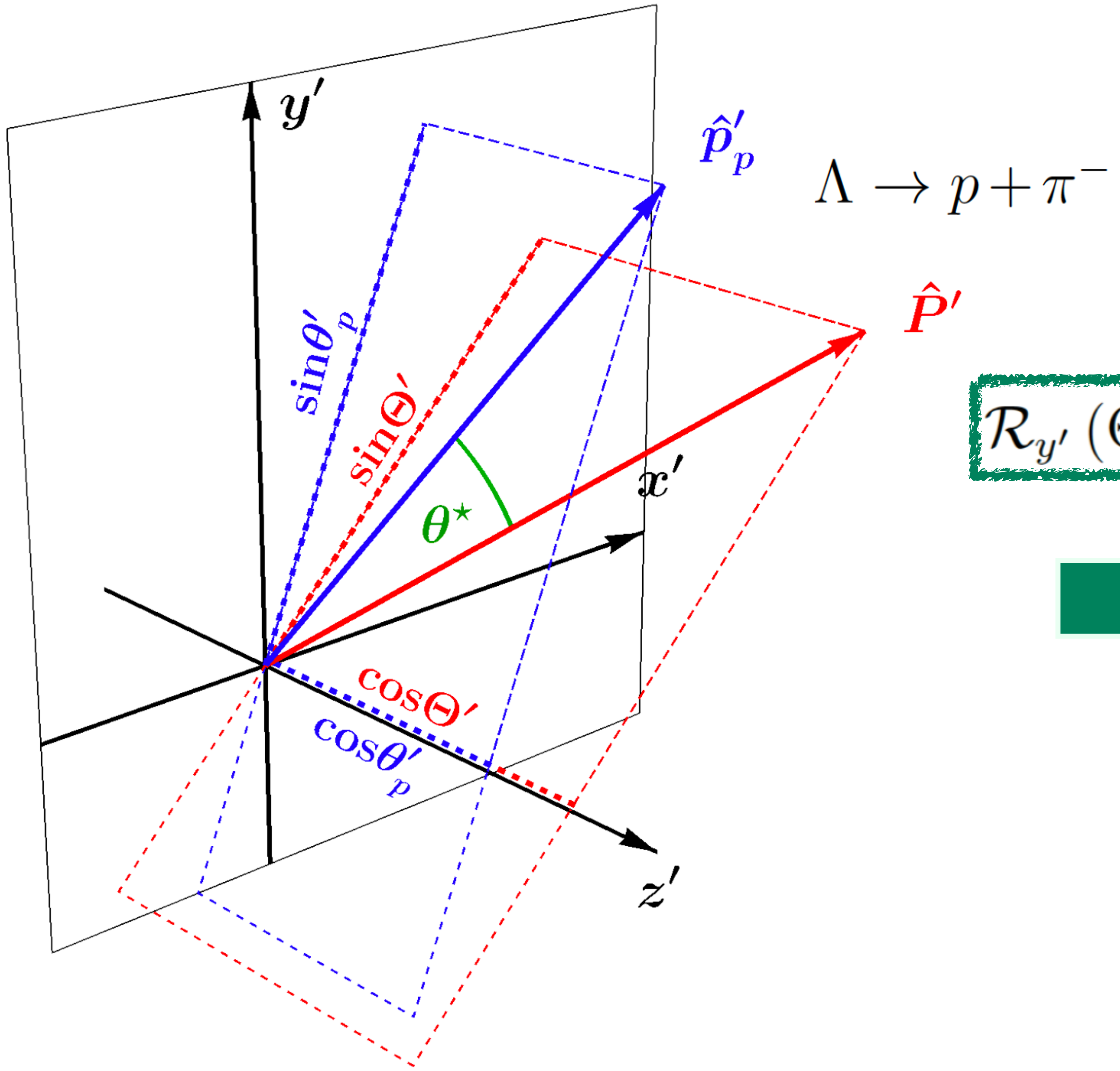
figure: B. I. Abelev et al. (STAR) PRC 76, 024915 (2007)



change of the system's angular momentum direction
due to relativistic effects

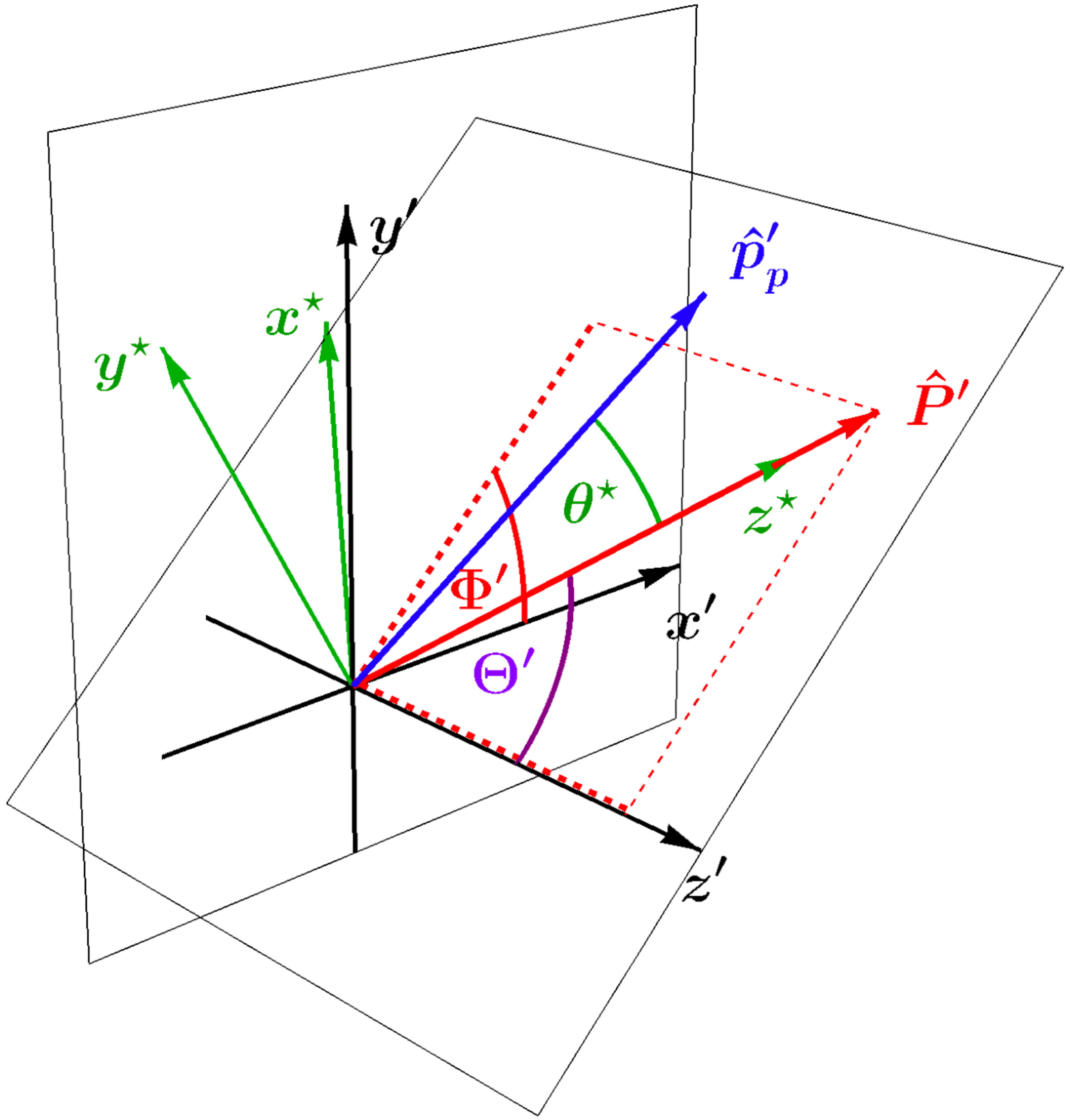
Λ weak decay: $S'(p_\Lambda)$ vs $S^*(p_\Lambda)$ rest frames

$S'(p_\Lambda)$



$$\boxed{\mathcal{R}_{y'}\left(\Theta'\right)\mathcal{R}_{z'}\left(\Phi'\right)}$$

$S^*(p_\Lambda)$



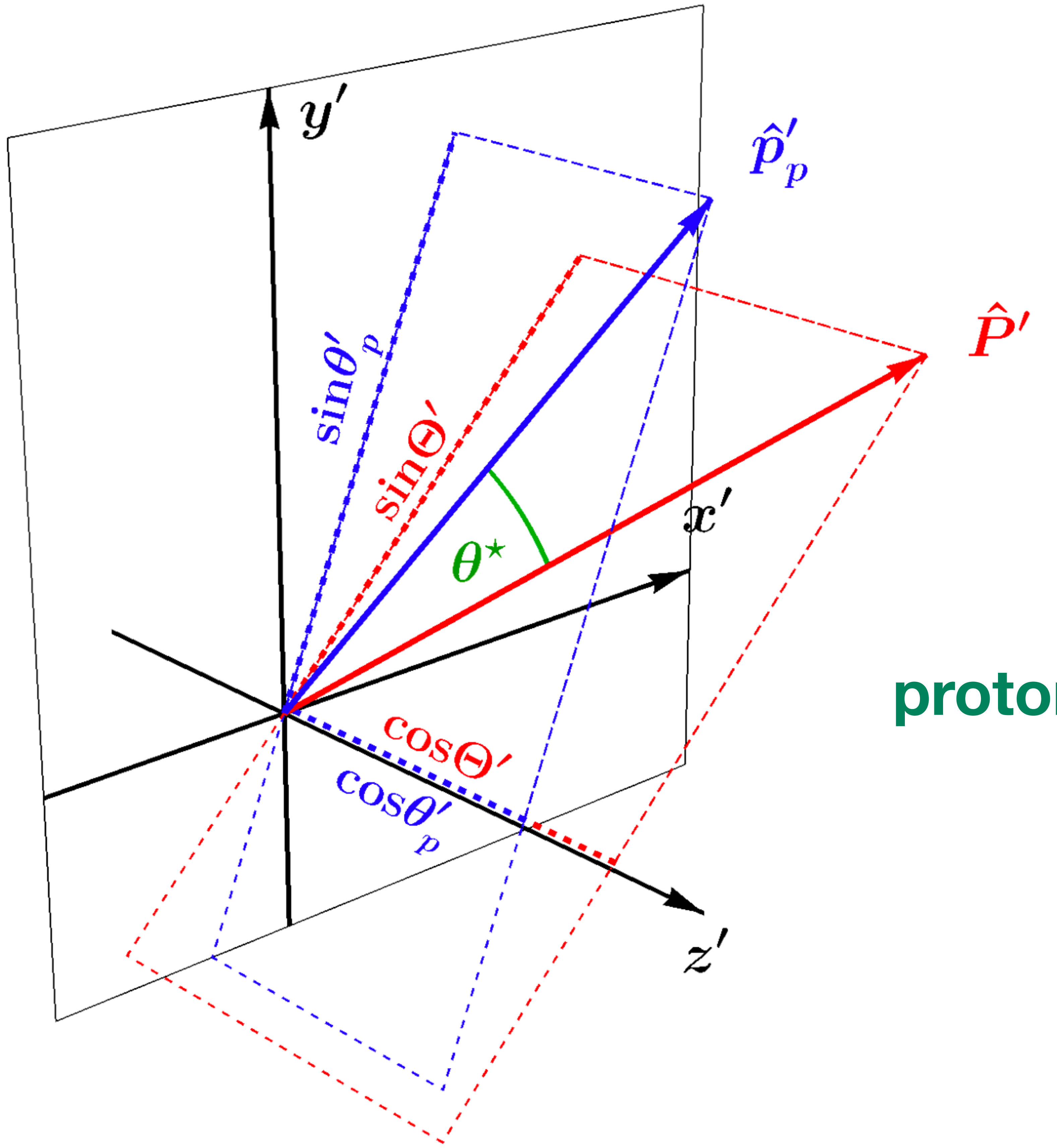
$$\hat{\boldsymbol{p}}'_p = \left(\sin\theta'_p\cos\phi'_p, \sin\theta'_p\sin\phi'_p, \cos\theta'_p\right)$$

$$\hat{\boldsymbol{P}}' = \left(\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta'\right)$$

$$\hat{\boldsymbol{P}}^* = \mathcal{R}_{y'}\left(\Theta'\right)\mathcal{R}_{z'}\left(\Phi'\right)\hat{\boldsymbol{P}}' = (0, 0, 1).$$

Λ weak decay: $S'(p_\Lambda)$ vs $S^*(p_\Lambda)$ rest frames

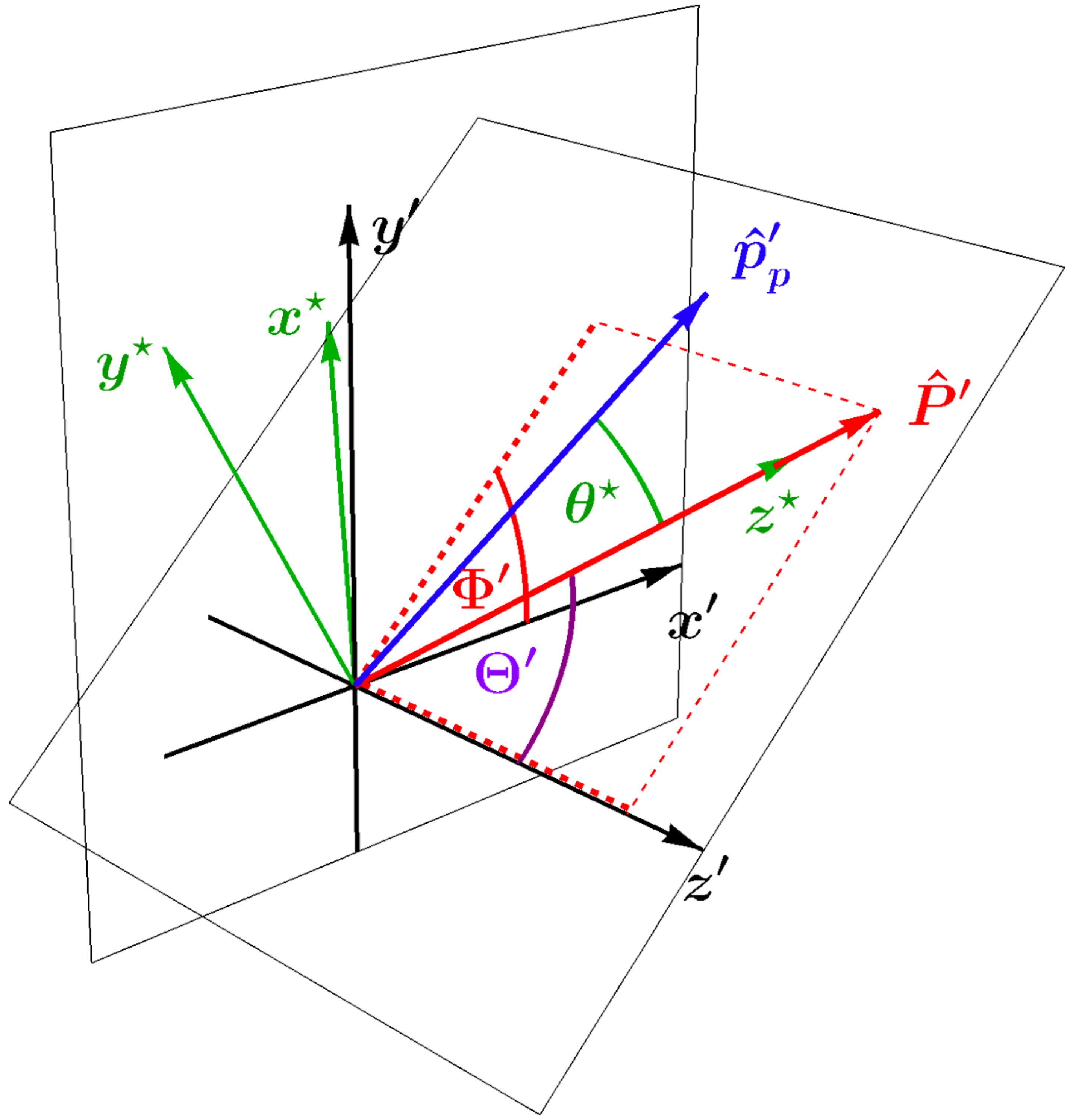
$S'(p_\Lambda)$



$$\hat{\boldsymbol{p}}'_p = \left(\sin\theta'_p \cos\phi'_p, \sin\theta'_p \sin\phi'_p, \cos\theta'_p\right)$$

$$\hat{\boldsymbol{P}}' = \left(\sin\Theta' \cos\Phi', \sin\Theta' \sin\Phi', \cos\Theta'\right)$$

$S^*(p_\Lambda)$



proton momentum in $S^*(p_\Lambda)$

$$\begin{aligned}
\hat{p}^*_{p,x} &= \cos(\Phi' - \phi'_p)\sin\theta'_p \cos\Theta' - \cos\theta'_p \sin\Theta' \equiv \sin\theta^* \cos\phi^*, \\
\hat{p}^*_{p,y} &= -\sin(\Phi' - \phi'_p)\sin\theta'_p \equiv \sin\theta^* \sin\phi^*, \\
\hat{p}^*_{p,z} &= \cos(\Phi' - \phi'_p)\sin\theta'_p \sin\Theta' + \cos\theta'_p \cos\Theta' \equiv \cos\theta^*.
\end{aligned}$$

$$\hat{\boldsymbol{P}}^* = \mathcal{R}_{y'}(\Theta')\,\mathcal{R}_{z'}(\Phi')\,\hat{\boldsymbol{P}}' = (0, 0, 1).$$

Λ weak decay law

$S^*(\boldsymbol{p}_\Lambda)$

$$\boxed{\frac{dN_p^{\rm pol}}{d\Omega^*} = \frac{1}{4\pi}\left(1+\alpha_\Lambda \boldsymbol{P}^*\cdot\hat{\boldsymbol{p}}_p^*\right)}$$

$$\hat{\boldsymbol{P}}'\cdot\hat{\boldsymbol{p}}_p' = \hat{\boldsymbol{P}}^*\cdot\hat{\boldsymbol{p}}_p^* = \cos\theta^*$$

$S'(\boldsymbol{p}_\Lambda)$

$$\frac{dN_p^{\rm pol}}{d\Omega'} = \frac{1}{4\pi}\left[1+\alpha_\Lambda P\boxed{\left(\cos(\Phi'-\phi_p')\sin\theta_p'\sin\Theta' + \cos\theta_p'\cos\Theta'\right)}\right]$$

$$\begin{aligned}
\langle \hat{p}_{p,x}'\rangle &= \int \boxed{\left(\frac{dN_p^{\rm pol}}{d\Omega'}\right)} (\sin\theta_p')^2\cos\phi_p'\,d\theta_p'\,d\phi_p' = \frac{1}{3}P'\alpha_\Lambda\sin\Theta'\cos\Phi',\\
\langle \hat{p}_{p,y}'\rangle &= \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right) (\sin\theta_p')^2\sin\phi_p'\,d\theta_p'\,d\phi_p' = \frac{1}{3}P'\alpha_\Lambda\sin\Theta'\sin\Phi',\\
\langle \hat{p}_{p,z}'\rangle &= \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right) \sin\theta_p'\cos\theta_p'\,d\theta_p'\,d\phi_p' = \frac{1}{3}P'\alpha_\Lambda\cos\Theta'.
\end{aligned}$$

the magnitude and direction of the polarization can be directly obtained from the averaged values of the three momentum components measured in $S'(\boldsymbol{p}_\Lambda)$

$$\boldsymbol{P}' = P'\left(\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta'\right) = \frac{3}{\alpha_\Lambda}\left(\langle\hat{p}_{p,x}'\rangle, \langle\hat{p}_{p,y}'\rangle, \langle\hat{p}_{p,z}'\rangle\right)$$

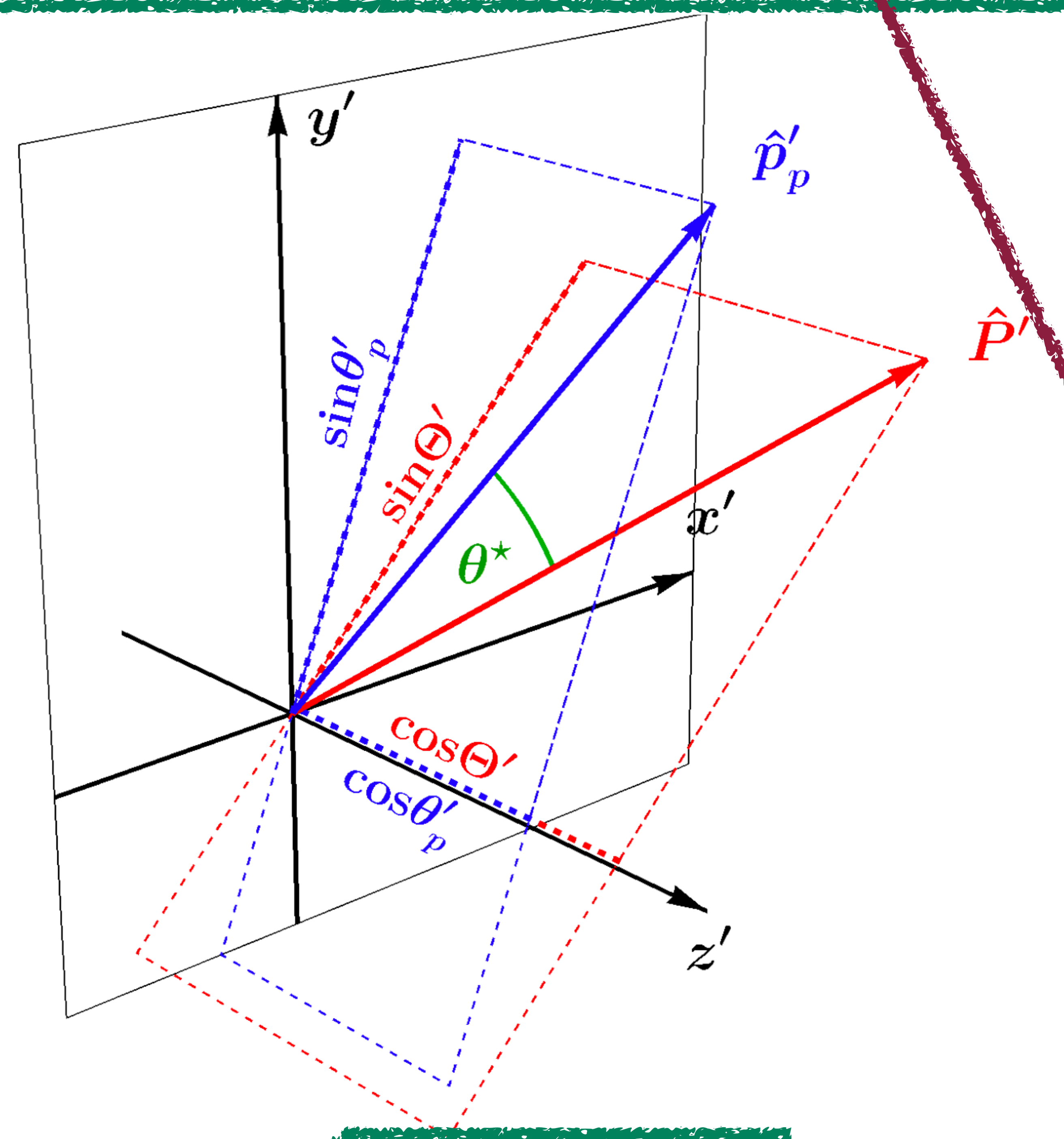
Interpretation of the Λ polarisation measurement

$$\langle \cos\phi'_p \rangle = \int \left(\frac{dN_p^{\mathrm{pol}}}{d\Omega'} \right) \sin\theta'_p \cos\phi'_p \, d\theta'_p \, d\phi'_p = \frac{\pi\alpha_\Lambda}{8} P' \sin\Theta' \cos\Phi',$$

$$\boxed{\langle \sin\phi'_p \rangle = \int \left(\frac{dN_p^{\mathrm{pol}}}{d\Omega'} \right) \sin\theta'_p \sin\phi'_p \, d\theta'_p \, d\phi'_p = \frac{\pi\alpha_\Lambda}{8} P' \sin\Theta' \sin\Phi'.}$$

$$\boxed{P_H = P' \sin\Theta' \sin\Phi'}$$

$$\boxed{P_H = \frac{8}{\pi\alpha_\Lambda} \langle \sin\phi'_p \rangle}$$



$$\hat{\boldsymbol{P}}' = (\sin\Theta' \cos\Phi', \boxed{\sin\Theta' \sin\Phi'}, \cos\Theta')$$

- "y" component of the polarization three-vector measured in the Lambda rest frame

  not the component of the polarization along the total angular momentum vector, as the y directions is COM and ΛRF differ

- it is tempting to measure also mean $\langle \cos\phi'_p \rangle$
- ratio would give information about the angle Φ'

Correlation with global angular momentum

$$\boldsymbol{P}' = P'\left(\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta'\right) = \frac{3}{\alpha_\Lambda}\left(\langle\hat{p}'_{p,x}\rangle, \langle\hat{p}'_{p,y}\rangle, \langle\hat{p}'_{p,z}\rangle\right)$$

direction of the total angular momentum
that is “seen” by the spin of the decaying that has three-momentum in COM

$$\hat{\boldsymbol{L}}' = \left(1-(\boldsymbol{v}_\Lambda\cdot\hat{\boldsymbol{L}})^2\right)^{-1/2}\left(\hat{\boldsymbol{L}} - \frac{\gamma_\Lambda}{\gamma_\Lambda+1}\boldsymbol{v}_\Lambda(\boldsymbol{v}_\Lambda\cdot\hat{\boldsymbol{L}})\right)$$

projection of the polarization along the direction of the total angular momentum

$$\boxed{\hat{\boldsymbol{L}}'\cdot\boldsymbol{P}' = \left(1-(\boldsymbol{v}_\Lambda\cdot\hat{\boldsymbol{L}})^2\right)^{-1/2}\left(\hat{\boldsymbol{L}}\cdot\boldsymbol{P}' - \frac{\gamma_\Lambda}{\gamma_\Lambda+1}\boldsymbol{v}_\Lambda\cdot\boldsymbol{P}'\;\;\boldsymbol{v}_\Lambda\cdot\hat{\boldsymbol{L}}\right)}$$

In the case of spin polarization of Lambdas, a reference frame where all particles are at rest does not exist, since the analyzed Lambdas have usually different momenta in COM

The advantage of $\hat{L}'\cdot P'$ compared to $\hat{L}\cdot P'$ is that the spin polarization of each Λ irrespectively of its three-momentum in COM, is projected on the same physical axis corresponding to L in COM

Numerical estimates

assuming

$$\boldsymbol{P}' = P'\hat{\boldsymbol{L}}$$

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = P'\left(1 - v_2^2\right)^{-1/2}\left(1 - \frac{v_2^2}{1+\sqrt{1-v^2}}\right) \equiv P' F_P(\boldsymbol{v})$$

$$v = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

average value for Lambdas in momentum range (m,n) GeV

$$\boxed{\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{m-n} = P' \frac{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega \, F_P(\boldsymbol{v}) F_T(v)}{\int_{v_{(m)}}^{v_{(n)}} dv \int d\Omega \, F_T(v)}}$$

$$F_T(v) = N\left[\exp\left(\frac{m_\lambda}{T_{\rm eff}\sqrt{1-v^2}}\right) + 1\right]^{-1}$$

$$v_{(n)} = \tanh\left[\sinh^{-1}\left(\frac{n\,\mathrm{GeV}}{m_\Lambda}\right)\right]$$

$$T_{\rm eff} = 150 \;\; \mathrm{MeV}$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{2-3} = 0.97 \;\; P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{3-4} = 0.94 \;\; P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{4-5} = 0.92 \;\; P'$$

$$\langle \hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' \rangle_{5-6} = 0.90 \;\; P'$$

relativistic effects studied in this work may reach 10% for the most energetic Lambdas studied at STAR

Summary

we have discussed the interpretation of the recent measurements of the spin polarization in relativistic heavy-ion collisions

we have shown that the appropriate interpretation of the relation between the spin direction (measured in the ΛRF) and the total angular momentum of the system (measured in the COM frame) requires that the direction of the angular momentum is boosted to the ΛRF

we have given the necessary formula that may be used to average the measured polarization of Lambdas with different momenta in the COM frame

Thank you for your attention!