Bottomonium suppression in the QGP

From EFTs to non-unitary quantum evolution

Michael Strickland

Kent State University
Kent, OH USA

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo,
P. Vander Griend, and J.H. Weber, arXiv:2012.01240 and forthcoming

Workshop on QGP Phenomenology
May 27, 2021

KENT STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

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Bottomonium Suppression

- In a high temperature quark-gluon plasma we expect **weaker color binding** (Debye screening + asymptotic freedom)

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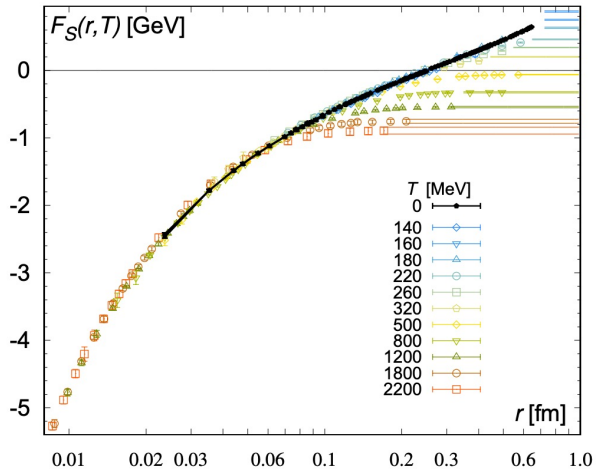
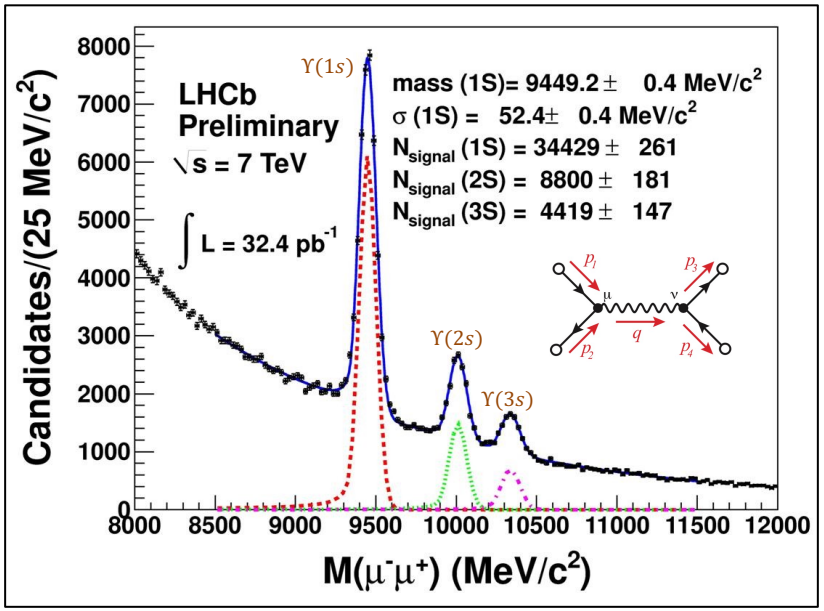
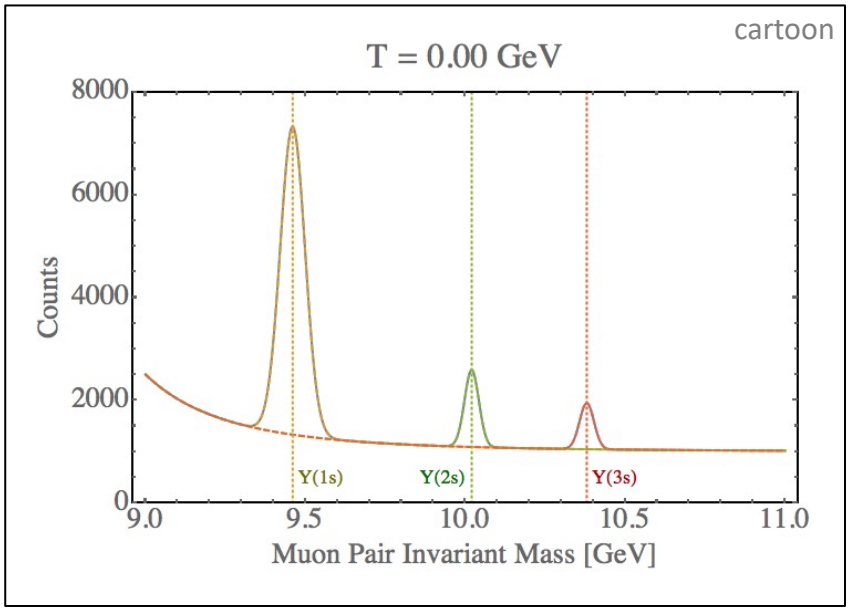
- Also, high energy plasma particles which slam into the bound states cause them to have shorter lifetimes → **larger spectral widths**

TUMQCD Collaboration, 1804.10600







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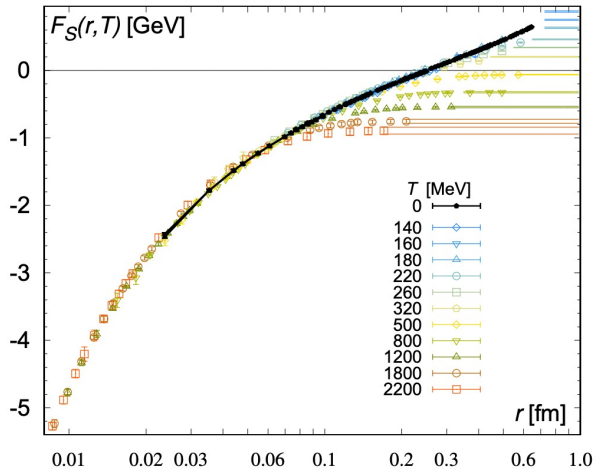

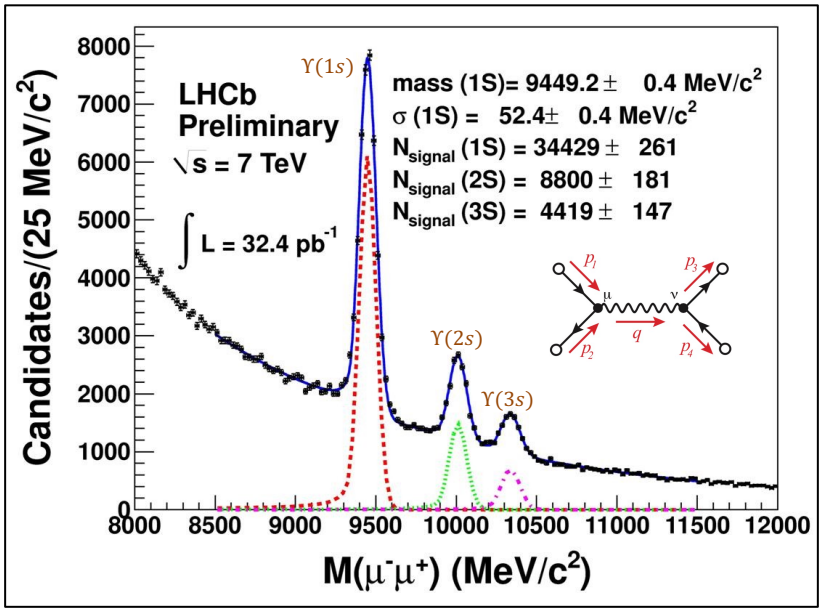

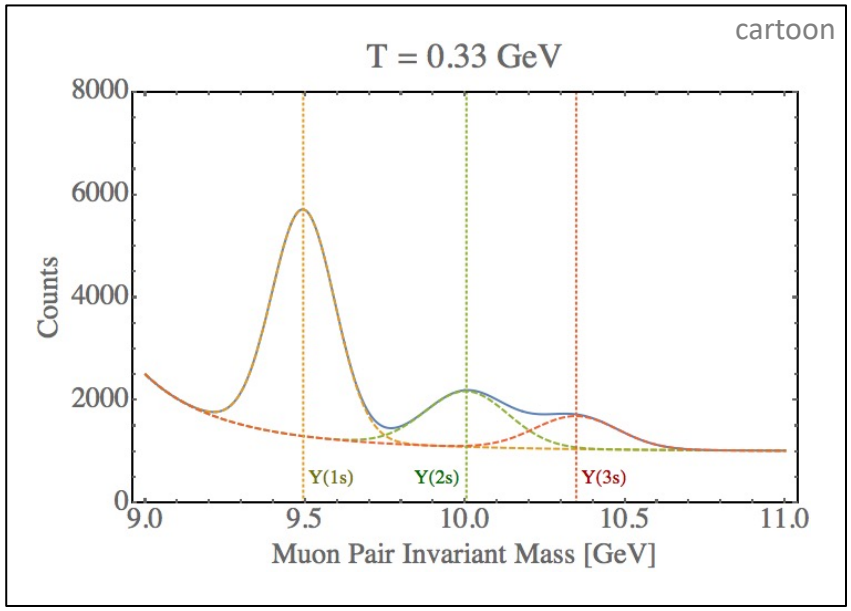

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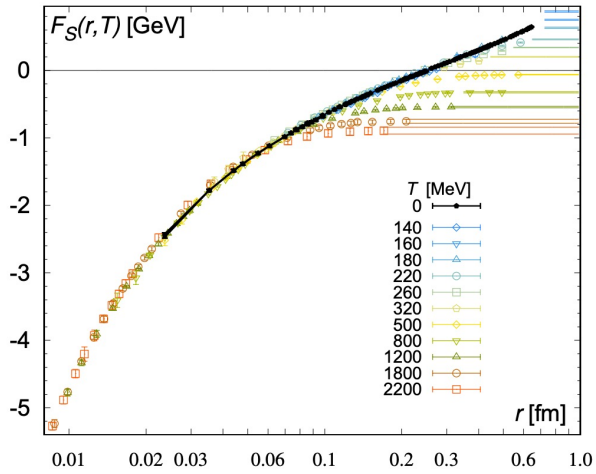

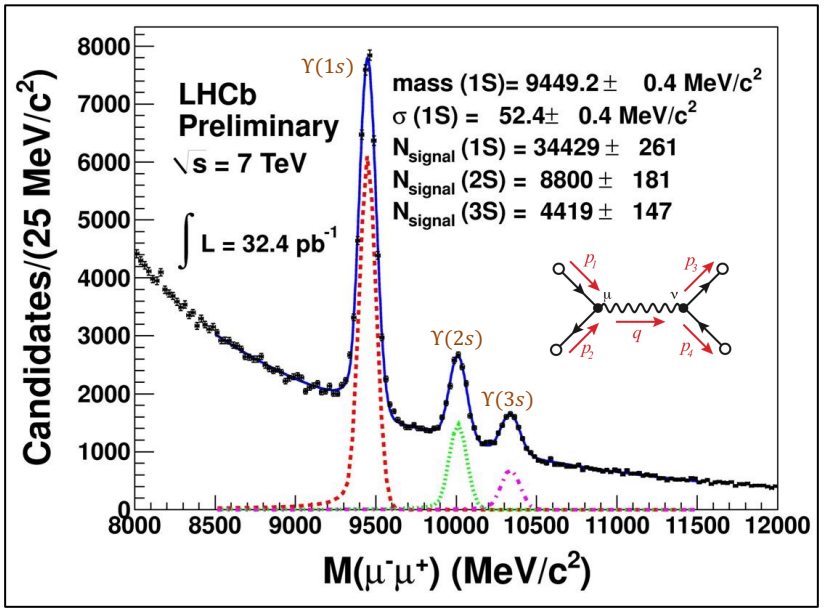

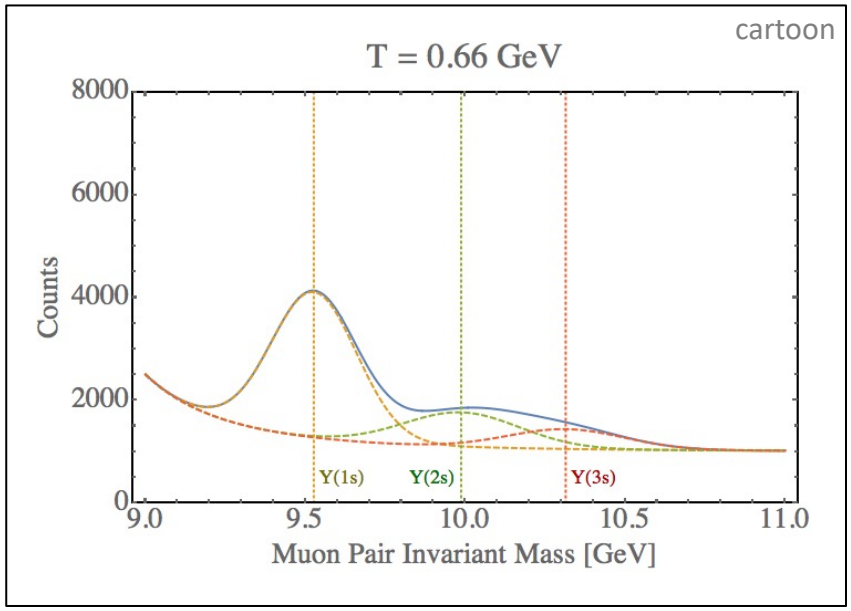

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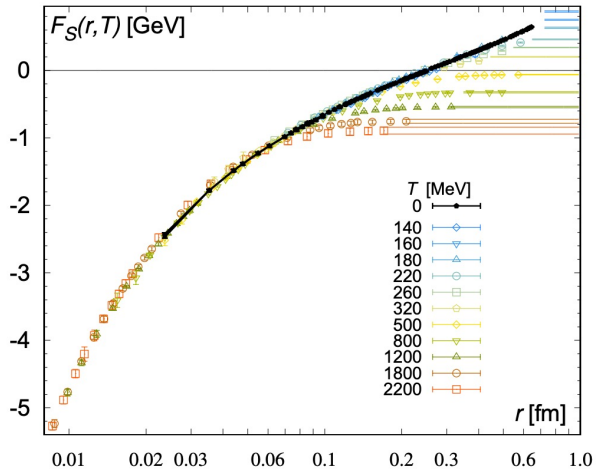
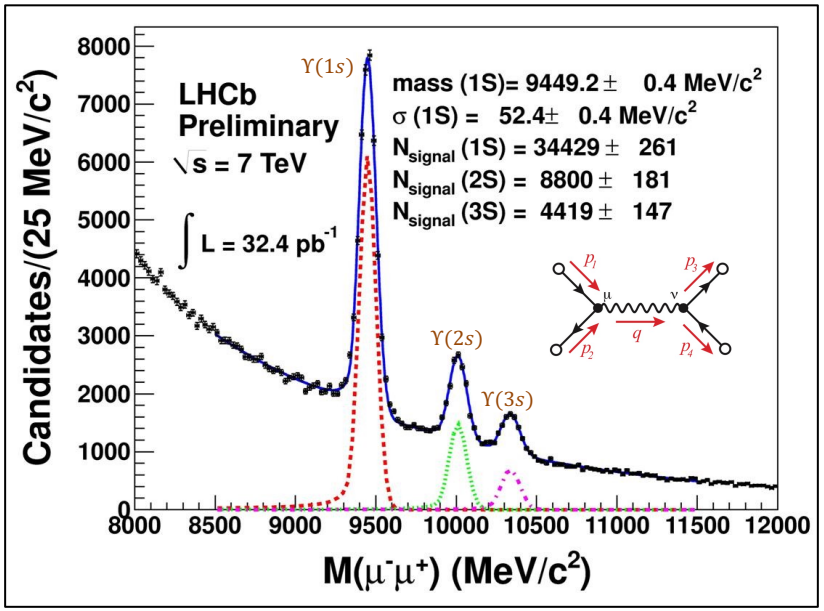
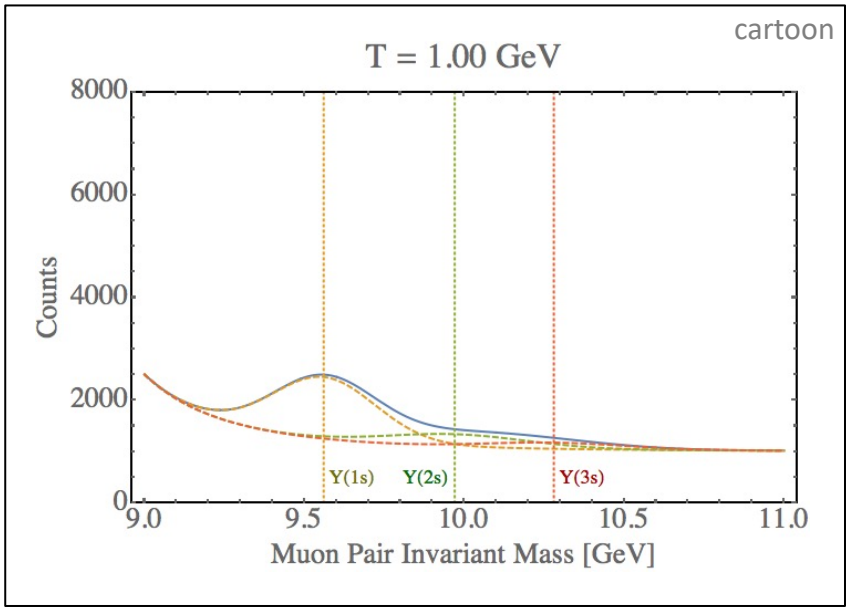


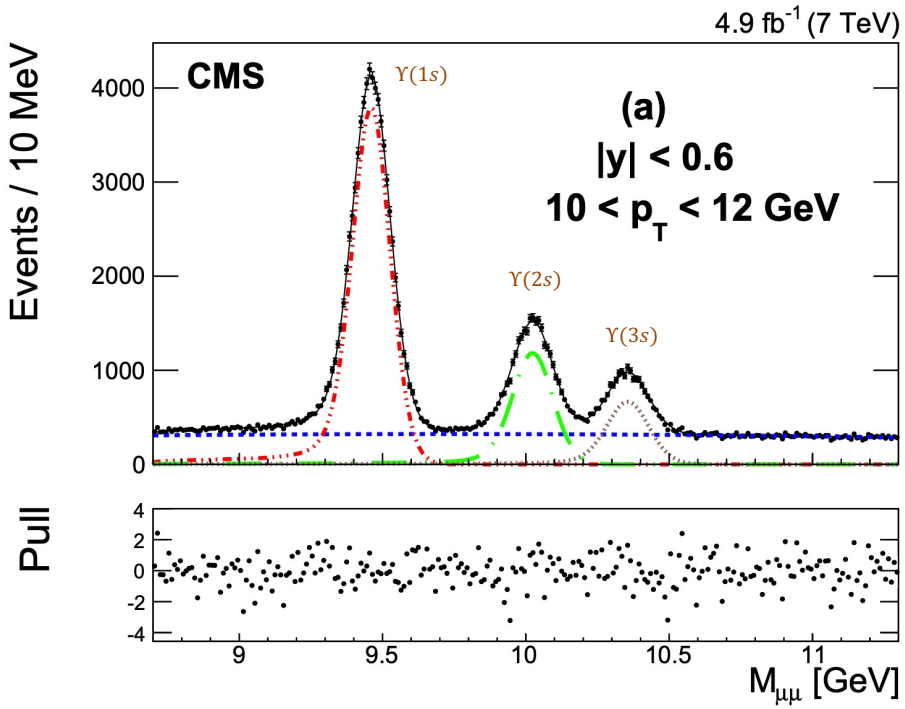




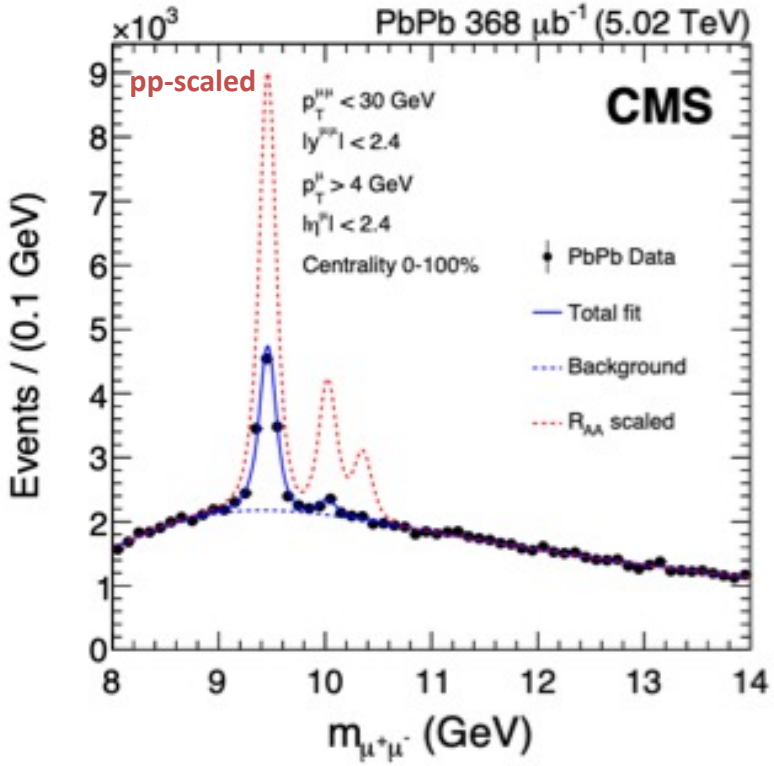
Experimental data – 5.02 TeV Dimuon Spectra

The **CMS, ALICE,** and **ATLAS** experiments have measured bottomonium production in both pp and Pb-Pb collisions. Here I show CMS results.

pp



PbPb



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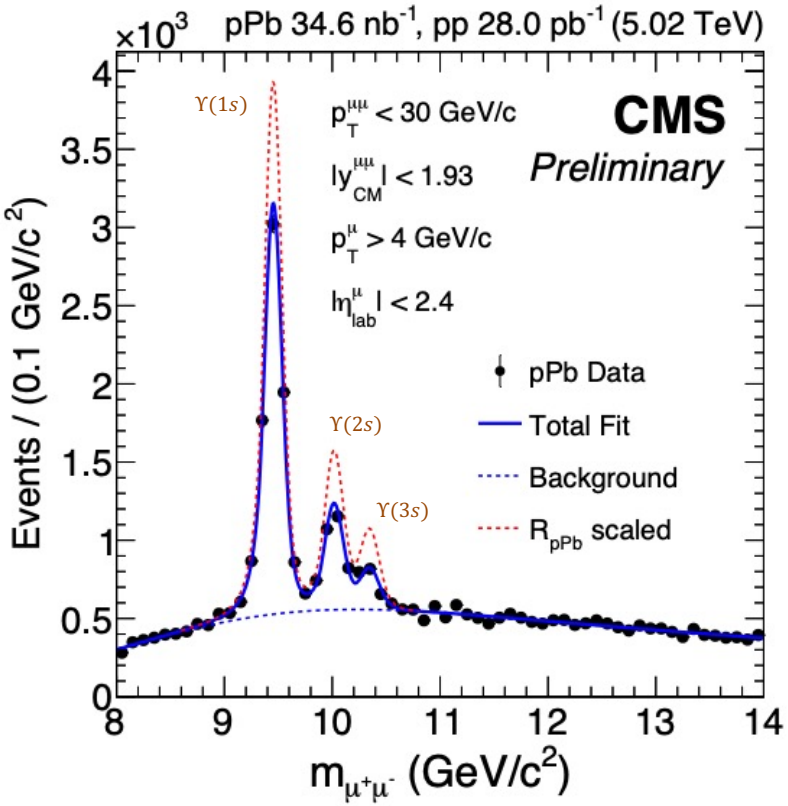
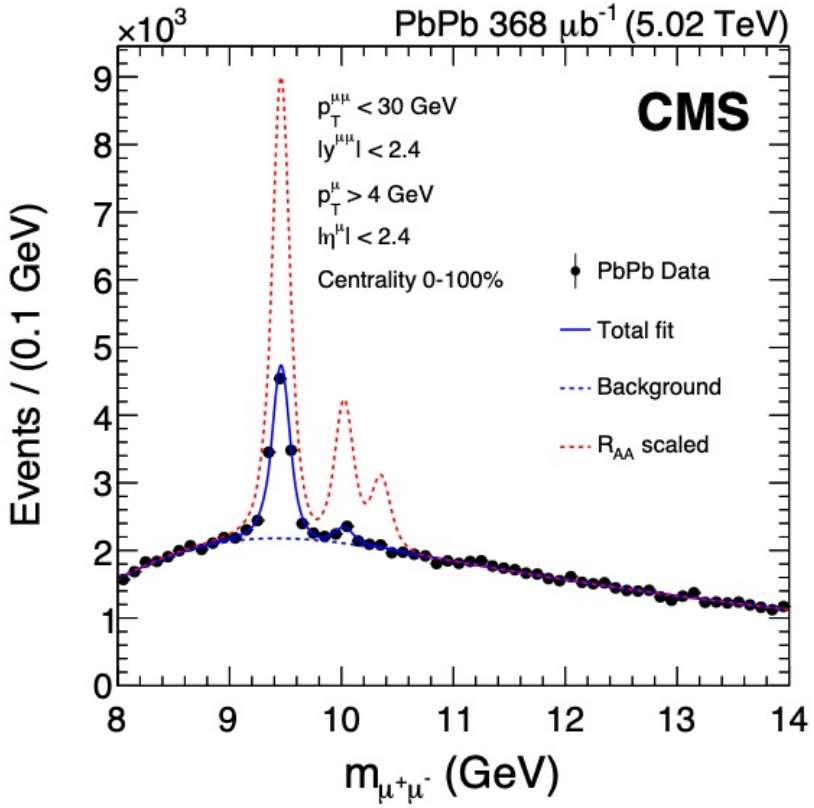
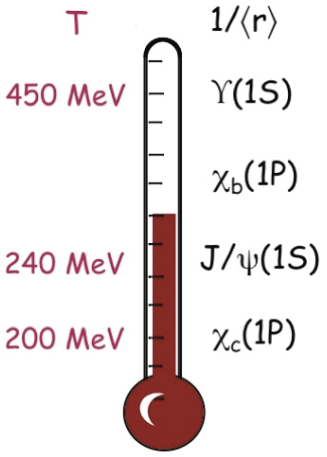


PbPb



Bottomonia are excellent probes of the QGP

- Can trust heavy quark effective theory more.
- Cold nuclear matter (CNM) effects in AA decrease with increasing quark mass.
- The masses of bottomonia (m ~ 10 GeV) are much higher than the temperature generated in HICs (T < 1 GeV) → bottomonium production dominated by initial hard scatterings.

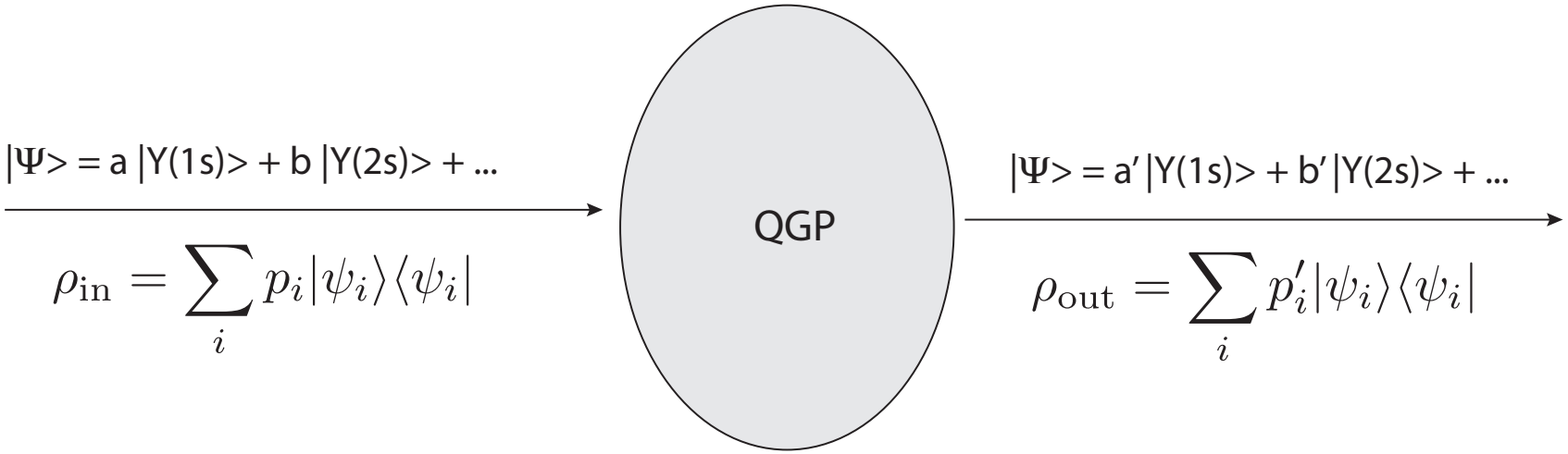


A. Mocsy, P. Petreczky, and MS, 1302.2180

- Both closed and open bottom production is quite rare in RHIC and LHC energy HICs → the probability for regeneration of bottomonia through statistical recombination is much smaller than for charm quarks; less model uncertainty.

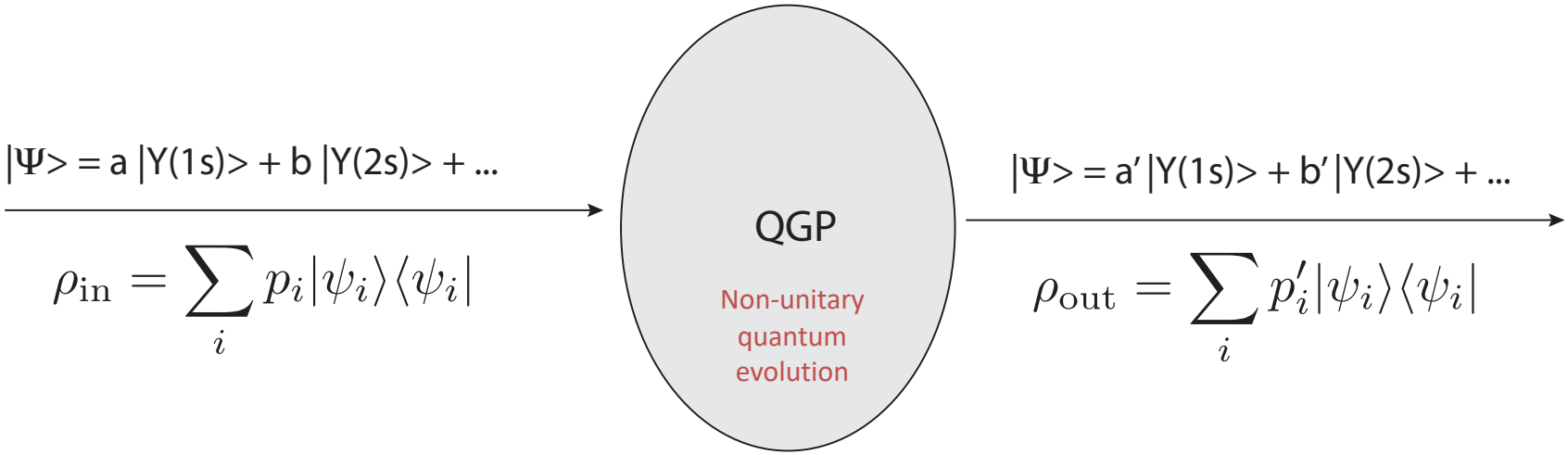
[see e.g. E. Emerick, X. Zhao, and R. Rapp, arXiv:1111.6537 and others]

Conceptual problem



- Bottomonium states have a large binding energy and are produced locally (hard processes) at early times in hard collisions (t < 1 fm/c).
- They then propagate through the plasma and interact with the medium.
- Bound states can break up and potentially re-form due to in-medium transitions induced by in-medium gluon absorption and emission.

Conceptual problem



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Heuristic understanding – Noisy QM

- Heavy quark bound states have an in-medium potential with both real and imaginary parts. This is related to the large in-medium width.
- How can we understand the emergence of the imaginary part in a simple manner before leaping into open quantum systems + pNRQCD?
- **Consider a non-relativistic bound state subject to a noisy potential**

$$H(\mathbf{r},t) = -\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) + \Theta(\mathbf{r},t) \qquad \Theta(\mathbf{r},t) = \theta\left(\mathbf{R}+\frac{\mathbf{r}}{2},t\right) - \theta\left(\mathbf{R}-\frac{\mathbf{r}}{2},t\right)$$

Noise due to environment (assumed here to be color neutral).

- Noise has zero mean, is uncorrelated in time, and has a spatial correlation function $D(\boldsymbol{r})$

$$\langle\theta(\mathbf{x},t)\rangle = 0 \qquad \langle\theta(\mathbf{x},t)\theta(\mathbf{x}',t')\rangle = D(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Note: The treatment presented here does not include possibility of color-charged noise, more on this coming…

Heuristic understanding – Noisy QM

- Expanding the time evolution operator up to $O(\Delta t^{3/2})$

$$e^{-i\Delta t\, H(\mathbf{r},t)} \simeq 1 - i\Delta t\, H(\mathbf{r},t) - \frac{1}{2}\left\{\Delta t\, H(\mathbf{r},t)\right\}^2 + \ldots$$

$$\approx 1 - i\Delta t\left[H(\mathbf{r},t) - \frac{i}{2}\Delta t\left\{\theta(\mathbf{x},t)^2 + \theta(\mathbf{x}',t)^2 - 2\,\theta(\mathbf{x},t)\theta(\mathbf{x}',t)\right\}\right]$$

- Now construct an effective Hamiltonian that is averaged over the noise

$$\langle \mathrm{H}_{\mathrm{eff}}(\mathbf{r},t)\rangle \simeq H(\mathbf{r},t) - \frac{i}{2}\Delta t\left\{\langle\theta(\mathbf{x},t)^2\rangle + \langle\theta(\mathbf{x}',t)^2\rangle - 2\,\langle\theta(\mathbf{x},t)\theta(\mathbf{x}',t)\rangle\right\}$$

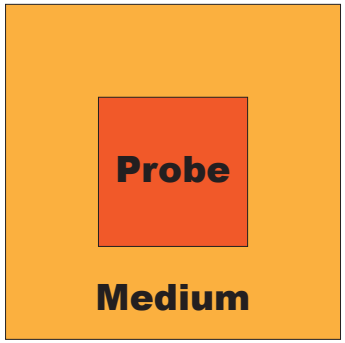
Imaginary part of the potential

$$\langle \mathrm{H}_{\mathrm{eff}}(\mathbf{r},t)\rangle = -\frac{\nabla^2_{\mathbf{r}}}{M} + V(\mathbf{r}) - i\left\{D(\mathbf{0}) - D(\mathbf{r})\right\}$$

$$\Im[V(r)] = D(\mathbf{r}) - D(\mathbf{0})$$

Imaginary part emerges through interference of wave function with itself when summing over environmental noise.

Open quantum system (OQS) approach



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm medium} + I_{\rm probe} \otimes H_{\rm medium} + H_{\rm int}$$

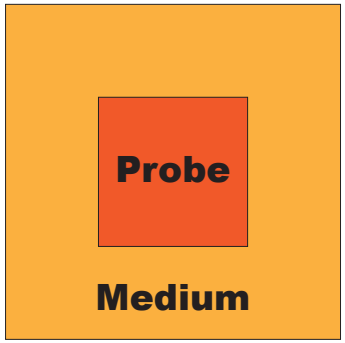
- Total density matrix

$$\rho_{\rm tot} = \sum_k \frac{1}{Z_{\rm tot}} e^{-E_k/T} |E_k\rangle\langle E_k| \longrightarrow \frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

- Reduced density matrix

$$\rho_{\rm probe} = {\rm Tr}_{\rm medium}[\rho_{\rm tot}] \longrightarrow$$ **Evolution equation?**

The Lindblad equation



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Separation of time scales
  - Medium relaxation time scale $\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/t_{\rm M}}$
  - Intrinsic probe time scale $t_P \sim \dfrac{1}{\omega_i - \omega_j}$
  - Probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}}$

Lindblad equation

$t_{\rm rel}\,,\, t_{\rm P} \gg t_{\rm M}$

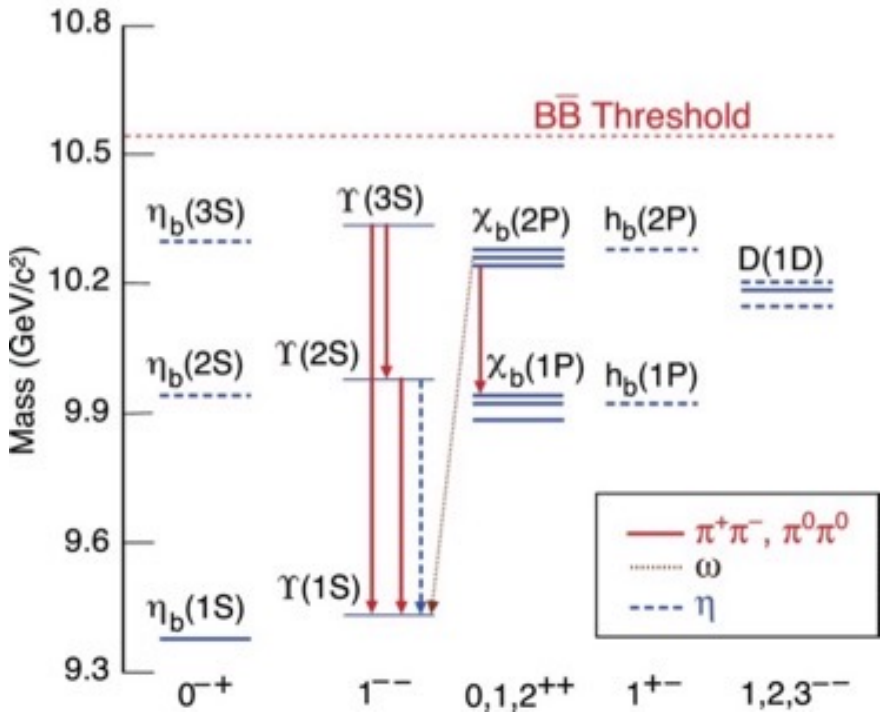
$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

- Trace preserving
- Completely positive
- In general, non-unitary evolution

G. Lindblad, Commun. Math. Phys. 48 (1976) 119
V. Gorini, et.al. J. Math. Phys. 17 (1976) 821

Bottomonium scales

- The mass scale is perturbative: $m_b \sim 5$ GeV
- The system is non-relativistic ($v \ll 1$), with $v_b \sim 0.1$.
- $\Delta_n E \sim mv^2$ and $\Delta_{\mathrm{fs}} E \sim mv^4$



Results of a non-relativistic potential model

State	Name	Exp. [92]	Model	Rel. Err.
1^1S_0	$\eta_b(1S)$	9.398 GeV	9.398 GeV	0.001%
1^3S_1	$\Upsilon(1S)$	9.461 GeV	9.461 GeV	0.004%
1^3P_0	$\chi_{b0}(1P)$	9.859 GeV	9.869 GeV	0.21%
1^3P_1	$\chi_{b1}(1P)$	9.893 GeV		
1^3P_2	$\chi_{b2}(1P)$	9.912 GeV		
1^1P_1	$h_b(1P)$	9.899 GeV		
2^1S_0	$\eta_b(2S)$	9.999 GeV	9.977 GeV	0.22%
2^3S_1	$\Upsilon(2S)$	10.002 GeV	9.999 GeV	0.03%
2^3P_0	$\chi_{b0}(2P)$	10.232 GeV	10.246 GeV	0.05%
2^3P_1	$\chi_{b1}(2P)$	10.255 GeV		
2^3P_2	$\chi_{b2}(2P)$	10.269 GeV		
2^1P_1	$h_b(2P)$	-		
3^1S_0	$\eta_b(3S)$	-	10.344 GeV	-
3^3S_1	$\Upsilon(3S)$	10.355 GeV	10.358 GeV	0.03%

J. Alford and MS, 1309.3003

Non-Relativistic QCD (NRQCD)

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{d_2}{m_Q^2}F^a_{\mu\nu}D^2F^{\mu\nu a} + d_g^3\frac{1}{m_Q^2}gf_{abc}F^a_{\mu\nu}F^{\mu b}_{\alpha}F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger\left(iD_0 + c_2\frac{\mathrm{D}^2}{2m_Q} + c_4\frac{\mathrm{D}^4}{8m_Q^3} + c_F g\frac{\sigma\mathrm{B}}{2m_Q} + c_D g\frac{\mathrm{DE-ED}}{8m_Q^2} + ic_S g\frac{\sigma(\mathrm{D}\times\mathrm{E}-\mathrm{E}\times\mathrm{D})}{8m_Q^2}\right)\psi$$

$$\mathcal{L}_\chi = c.c\ of\ \mathcal{L}_\psi$$

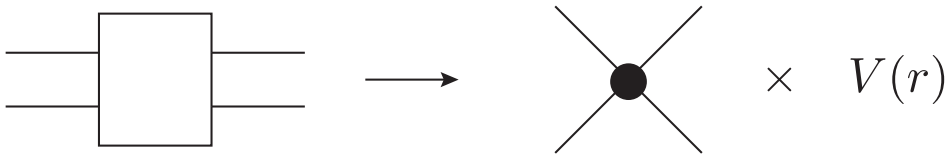
$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{f_1(^3S_1)}{m_Q^2}\psi^\dagger\sigma\chi\chi^\dagger\sigma\psi + \frac{f_8(^1S_0)}{m_Q^2}\psi^\dagger T^a\chi\chi^\dagger T^a\psi + \frac{f_8(^3S_1)}{m_Q^2}\psi^\dagger T^a\sigma\chi\chi^\dagger T^a\sigma\psi$$

- **Integrating out the scale m can be done perturbatively,** not affected by the presence of the medium since $m \gg \Lambda_{\mathrm{QCD}}, T$.
- **Hard gluons**, with energy and momentum of order m.
- **Soft gluons**, with energy and momentum of order mv.
- **Potential gluons**, with energy of order mv^2 and momentum of order mv.
- **Ultrasoft gluons**, with both energy and momentum of order mv^2

NRQCD → Potential NRQCD (pNRQCD)

Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

Degrees of freedom at scale $\frac{1}{r} = mv$ are integrated out



$$\times \quad V(r)$$

Power counting

$$r \sim \frac{1}{mv} \qquad t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\rm QCD}}$$

Gauge fields are multiple expanded

$$A(r, R, t) = A(R, t) + \mathbf{r} \cdot \nabla A(R, t) + \dots$$

Non-analytic behavior in r → matching coefficients V

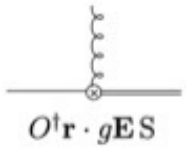
- Resulting degrees of freedom are singlet and octet states (see Lagrangian on next slide).
- Allows to obtain manifestly gauge-invariant results.
- Easier connection lattice QCD.
- If $1/r \gg T$ we can use this as a starting point.
- In other cases, the matching between NRQCD and pNRQCD will be modified.

NRQCD → Potential NRQCD (pNRQCD)

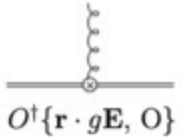
Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S + O^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_o \right) O \right\}$$

$$+ V_A \,\mathrm{Tr}\left\{ O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, S + S^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O \right\} \rightarrow$$



$$+ \frac{V_B}{2} \mathrm{Tr}\left\{ O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O + O^\dagger O\, \mathbf{r}\cdot g\mathbf{E} \right\} \rightarrow$$

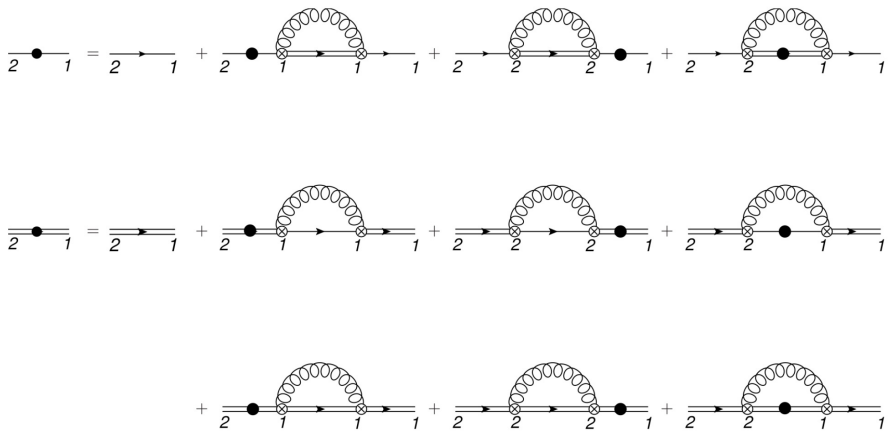


Singlet and octet potentials

$$V_s(r) = -C_F \frac{\alpha_s}{r}$$

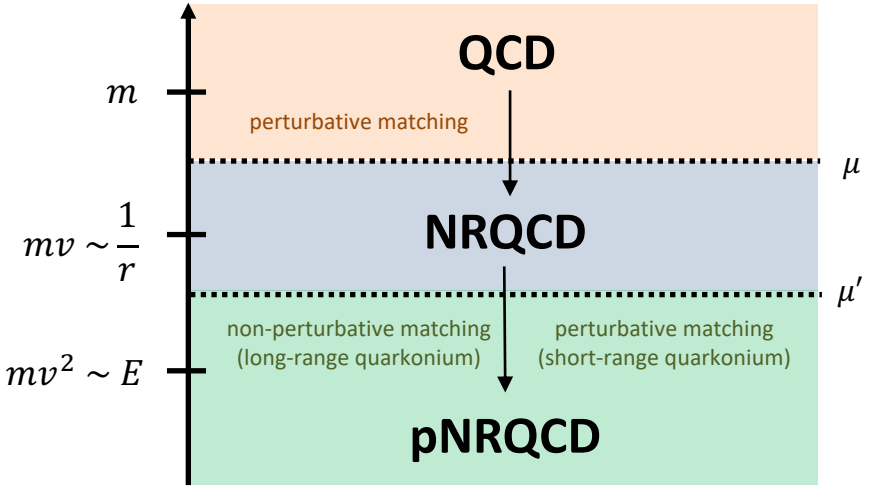
$$V_o(r) = \frac{\alpha_s}{2N_c r}$$

- Based on this Lagrangian, we can perform first-principles calculations.
- Right figure shows diagrams contributing to singlet and octet self-energies.
- These enter into the calculation of Lindblad/collapse/jump operators.



OQS + pNRQCD → Lindblad equation

- What are the relevant scales?
  - Temperature T
  - Bound state mass $m \gg T$
  - Bound state size $r \sim mv \sim a_0$ (Bohr radius)
  - Debye mass m_D
  - Binding energy $E \sim mv^2$



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  - Medium relaxation time scale $\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/t_{\rm M}} \to \frac{1}{T}$
  - Intrinsic probe time scale $t_P \sim \frac{1}{\omega_i - \omega_j} \to \frac{1}{E}$
  - Probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}} \to \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$ $\quad \Lambda = T, E$

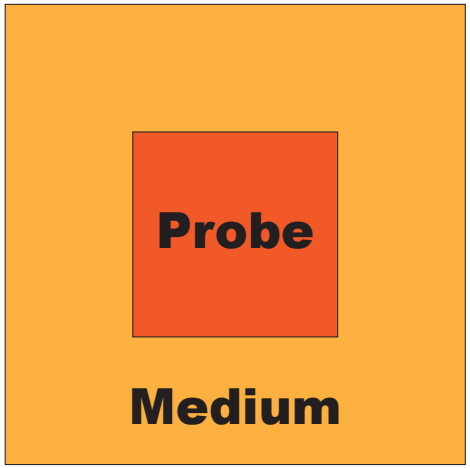
$$1/r \gg T \sim m_D \gg E \xrightarrow[t_{\rm rel}\,,\, t_{\rm P} \gg t_{\rm M}]{}$$

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

Lindblad equation

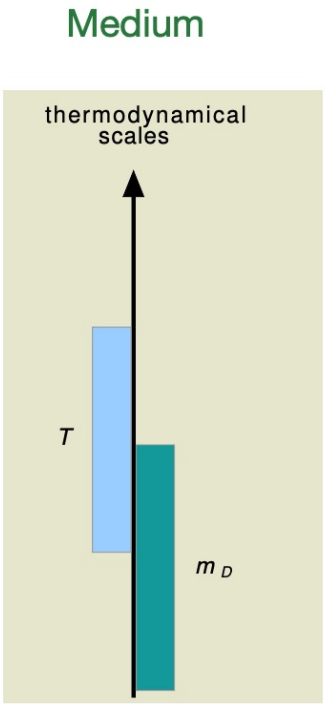
N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

OQS + pNRQCD – Relevant scaling



What are the relevant scales?

- Temperature T
- Bound state mass $m \gg T$
- Bound state size $r \sim mv \sim a_0$
- Debye mass m_D
- Binding energy $E \sim mv^2$



Strongly-coupled QGP

$$1/r \gg T \sim m_D \gg E$$

Lindblad equation

$$\rho_{\rm probe} = {\rm Tr}_{\rm medium}[\rho_{\rm tot}] \xrightarrow[1/r \gg T \sim m_D \gg E]{} \frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n \, \rho_{\rm probe} \, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

OQS + pNRQCD – Lindblad reorganization

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

- $H_{\rm probe}$ is a Hermitian operator (includes singlet and octet states)
- C_n are the **collapse (or jump) operators** (connect different internal states)
- Partial and **total decay widths** are

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

- Can reorganize Lindblad equation by defining

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$

← **Non-Hermitian effective Hamiltonian**

$$\longrightarrow \quad \frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\, \rho_{\rm probe}\, C_n^\dagger$$

OQS+pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H_{\rm probe} = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

mass shift

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

Total width → Im[V]

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$

Six collapse operators cover

- singlet → octet,
- octet → singlet
- octet → octet

$$\gamma \equiv \frac{g^2}{6\,N_c}\,{\rm Im}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\kappa \equiv \frac{g^2}{6\,N_c}\,{\rm Re}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

OQS+pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\,\rho_{\rm probe}\,C_n^{\dagger}$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Six collapse operators cover

- singlet → octet,
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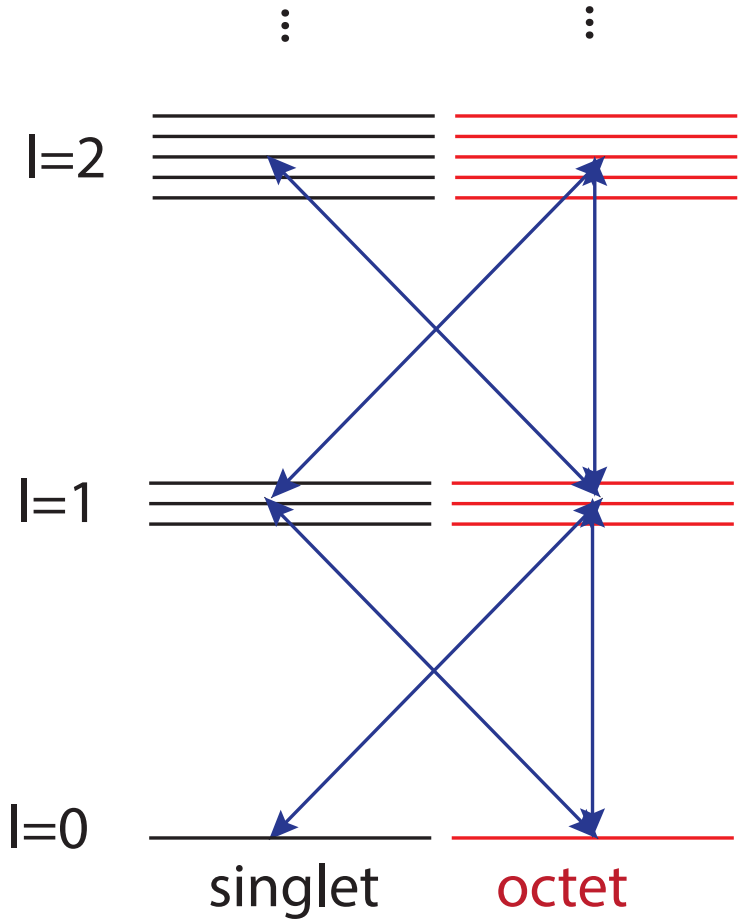


OQS+pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H_{\rm probe} = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Six collapse operators cover

- singlet → octet,
- octet → singlet
- octet → octet

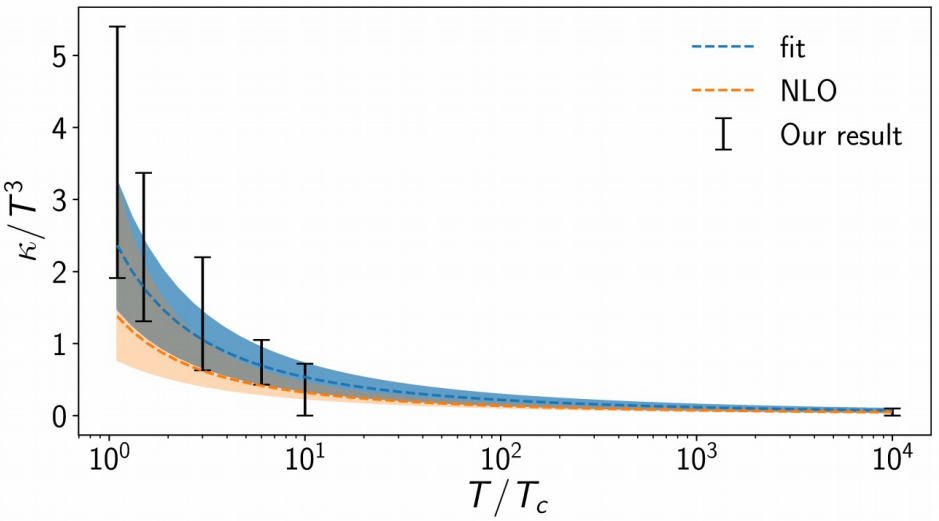
$$\gamma \equiv \frac{g^2}{6\,N_c}\,{\rm Im}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\kappa \equiv \frac{g^2}{6\,N_c}\,{\rm Re}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

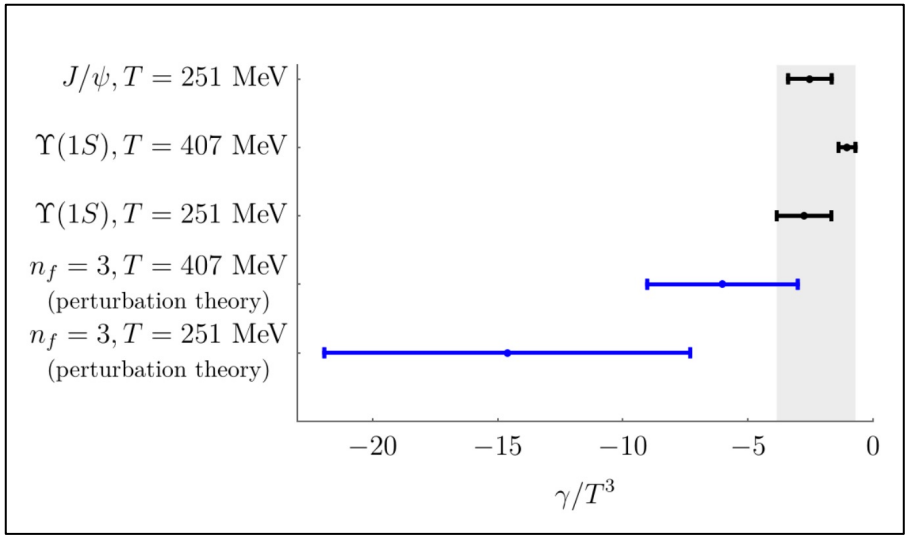
Values of $\hat{\kappa}$ and $\hat{\gamma}$ used

- We used NLO fits to recent lattice measurements of the heavy quark transport coefficient $\hat{\kappa} \equiv \kappa/T^3$. Note that this is related to the heavy quark diffusion constant D.
  - N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078

N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078



- The value of $\hat{\gamma} \equiv \gamma/T^3$ is less constrained, we vary it in the range -3.5 < $\hat{\gamma}$ < 0.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1711.04515.
  - N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.



N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.

How can one numerically solve these equations?

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\,\rho_{\rm probe}\,C_n^{\dagger}$$

- Each block of the density matrix in color space can be decomposed into orbital angular momentum blockwise.
- Upon truncating in angular momentum ($l \leq l_{max}$) one can reduce both the singlet and octet blocks of the reduced density matrix to size $(l_{max}+1)^2$.
- One can then discretize the radial wavefunction (N = # of lattice points) and evolve the reduced density matrix using standard differential equation and matrix solvers gives $\sim N^2(l_{max}+1)^2$ matrix size.
- **Need to describe bound and unbound states with highly localized initial wave function, so the box must be large and have small lattice spacing → large N and large l_{max}.**
- As N and l_{max} become large, **the computation becomes very challenging**.
- **Need a better/faster method which we can easily parallelize.**

A parallelizable approach: Quantum trajectories

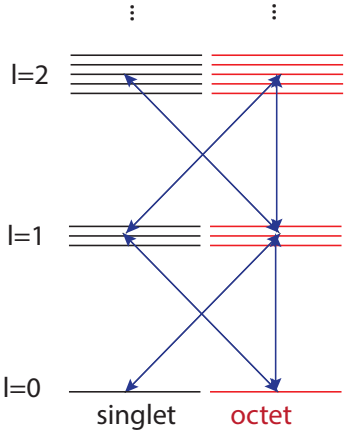
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2012.01240

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\,\rho_{\rm probe}\,C_n^{\dagger}$$

Non-unitary "no jump" evolution

Can treat this "quantum jump" term stochastically

- Can be reduced to the solution of a large set of "quantum trajectories" in which we solve a 1D Schrödinger equation with a **non-Hermitian Hamiltonian $H_{\rm eff}$,** subject to **stochastic quantum jumps**.
- The evolution with the non-Hermitian $H_{\rm eff}$ preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode transitions between different color/angular momentum states (subject to selection rules).



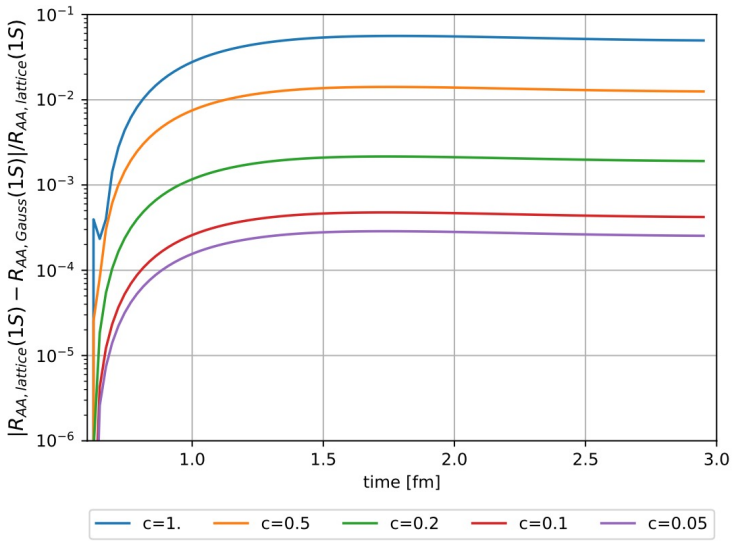
- For each **physical trajectory** (path through the QGP) we average over a large set of **independent quantum trajectories → Embarrassingly parallel**
- **Added benefit: Can describe all angular momentum states (no cutoff) .**

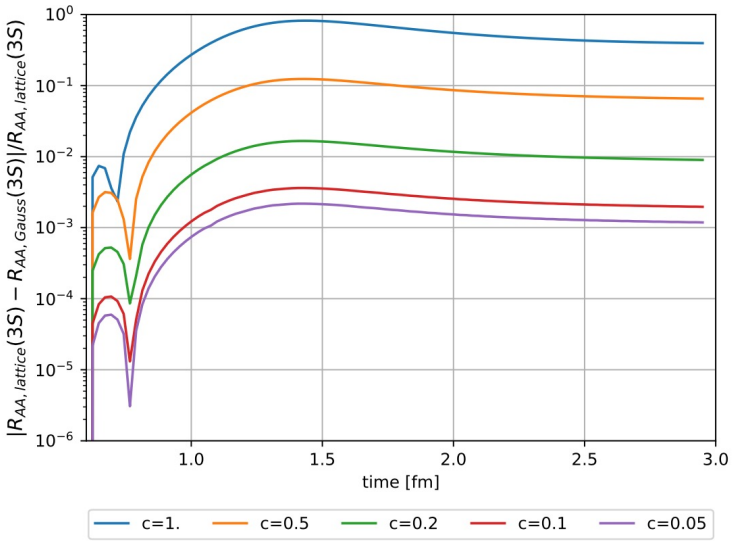
Initial bottomonium wavefunction

- We took the initial wavefunction to be given by a smeared delta function (local production due to large mass, $\Delta \sim 1/M$) of the form

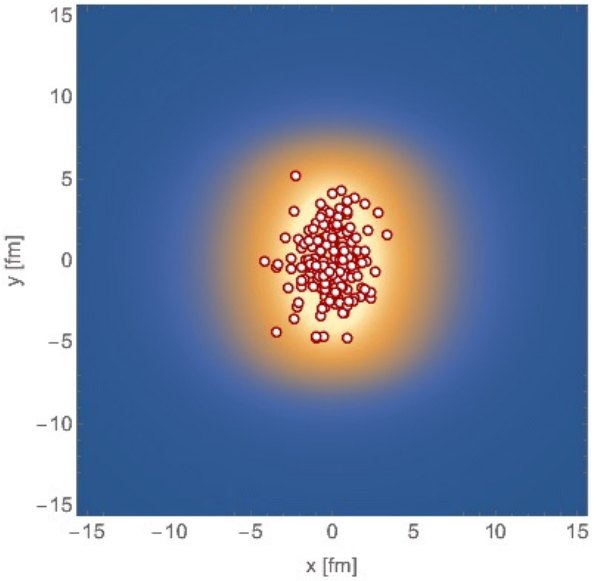
$$u_\ell(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2)$$

- For a given l, the **initial state is a quantum linear superposition** of the eigenstates of H.
- **Includes both bound and unbound states.**
- We took $\Delta = 0.2\ a_0$ which reproduces results obtained with a true delta to within 1%.





Computing survival probabilities with QTraj



Survival probability

$$SP(n,l) = \frac{|\langle n,l|\psi(t_f)\rangle|^2}{|\langle n,l|\psi(t_0)\rangle|^2}$$

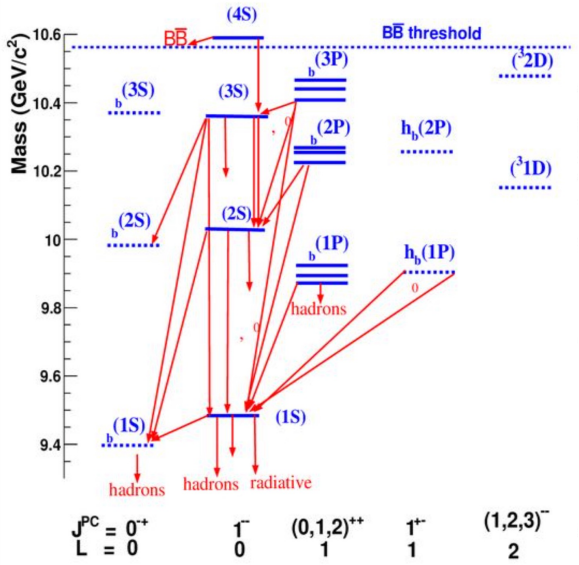
- Used $N = 4096$ points
- $L = 108\, a_0$
- $\Delta t = 2 \times 10^{-4}$ fm

- We sampled bottomonium production points and transverse momentum using **Monte-Carlo sampling**.
- Temperature evolution provided **by 3+1D anisotropic hydrodynamics** (good description of identified hadron spectra and anisotropic flow, see backup slides).
- We solved the **real-time 3D Schrödinger equation with a complex potential and stochastically sampled jumps** → Lindblad equation.
- We then solved for the **survival probability** of S- and P-wave states (see box to the left).

Feed-down implementation

$$\vec{N}_{\text{observed}} = F\vec{N}_{\text{direct}}$$

$$F = \begin{pmatrix} 1 & 0.2645 & 0.0194 & 0.352 & 0.18 & 0.0657 & 0.0038 & 0.1153 & 0.077 \\ 0 & 1 & 0 & 0 & 0 & 0.106 & 0.0138 & 0.181 & 0.089 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0091 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0051 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

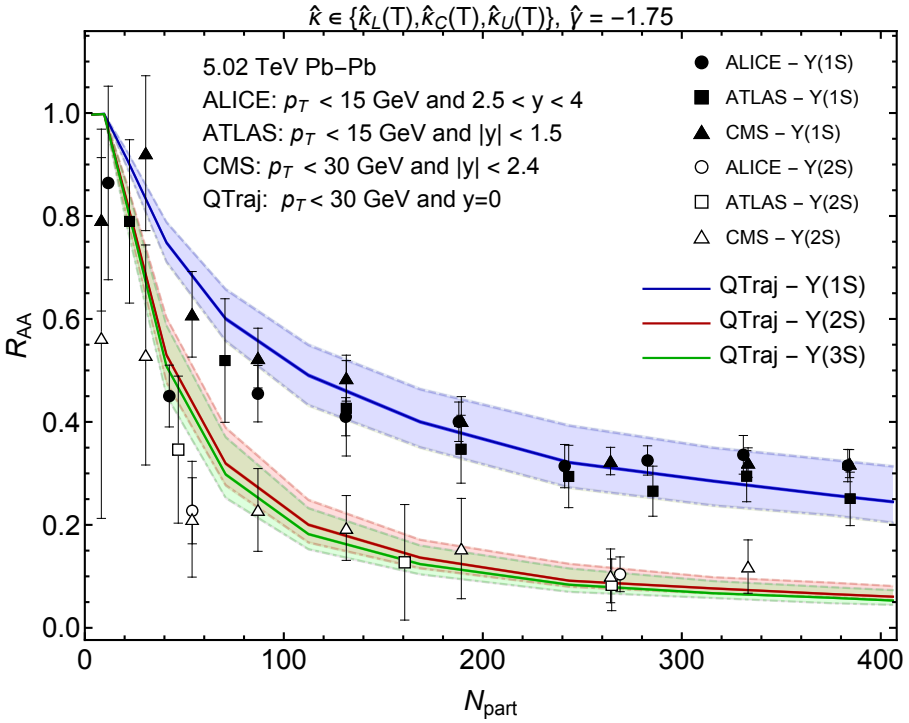


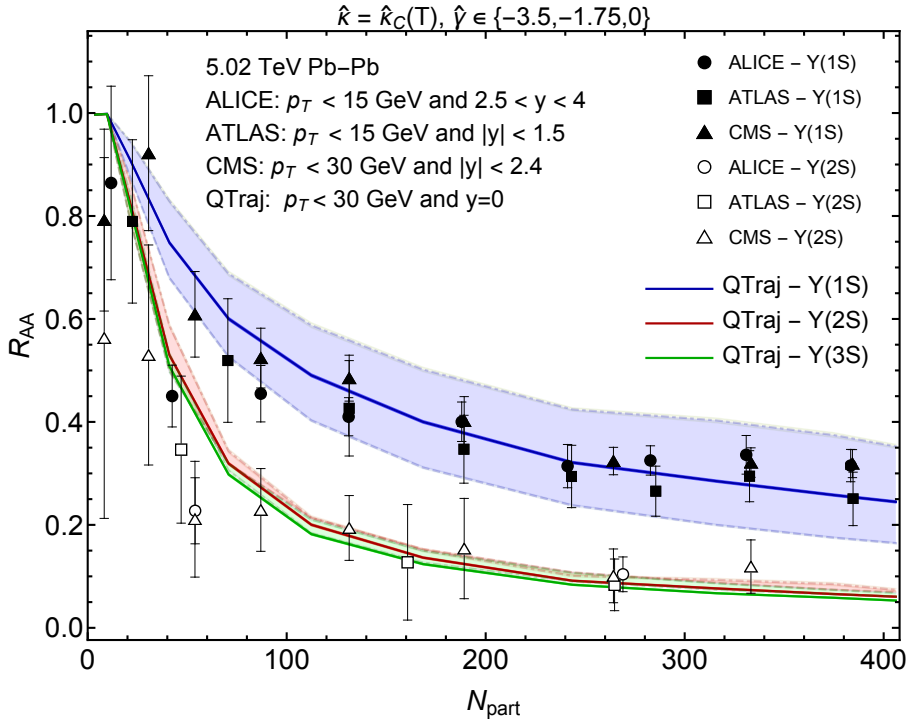
- N_{direct} corresponds to (N_{1S}, N_{2S}, N_{1P} x 3, N_{3S}, N_{2P} x 3, N_{2D})^T where, e.g., N_{1S} is the final number of Y(1S) states that can decay in the dilepton channel.
- N_{direct} can be obtained using $\langle N_{bin}(b) \rangle * \sigma_{direct} *$ (Survival probability)
- After feed down, we then normalize to by the pp collision result scaled to AA → R_{AA}.

$$R^i_{AA}(c) = \frac{\left(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}}\right)^i}{\sigma^i_{\text{exp}}}$$

OQS + pNRQCD predictions for R_{AA} vs N_{part}

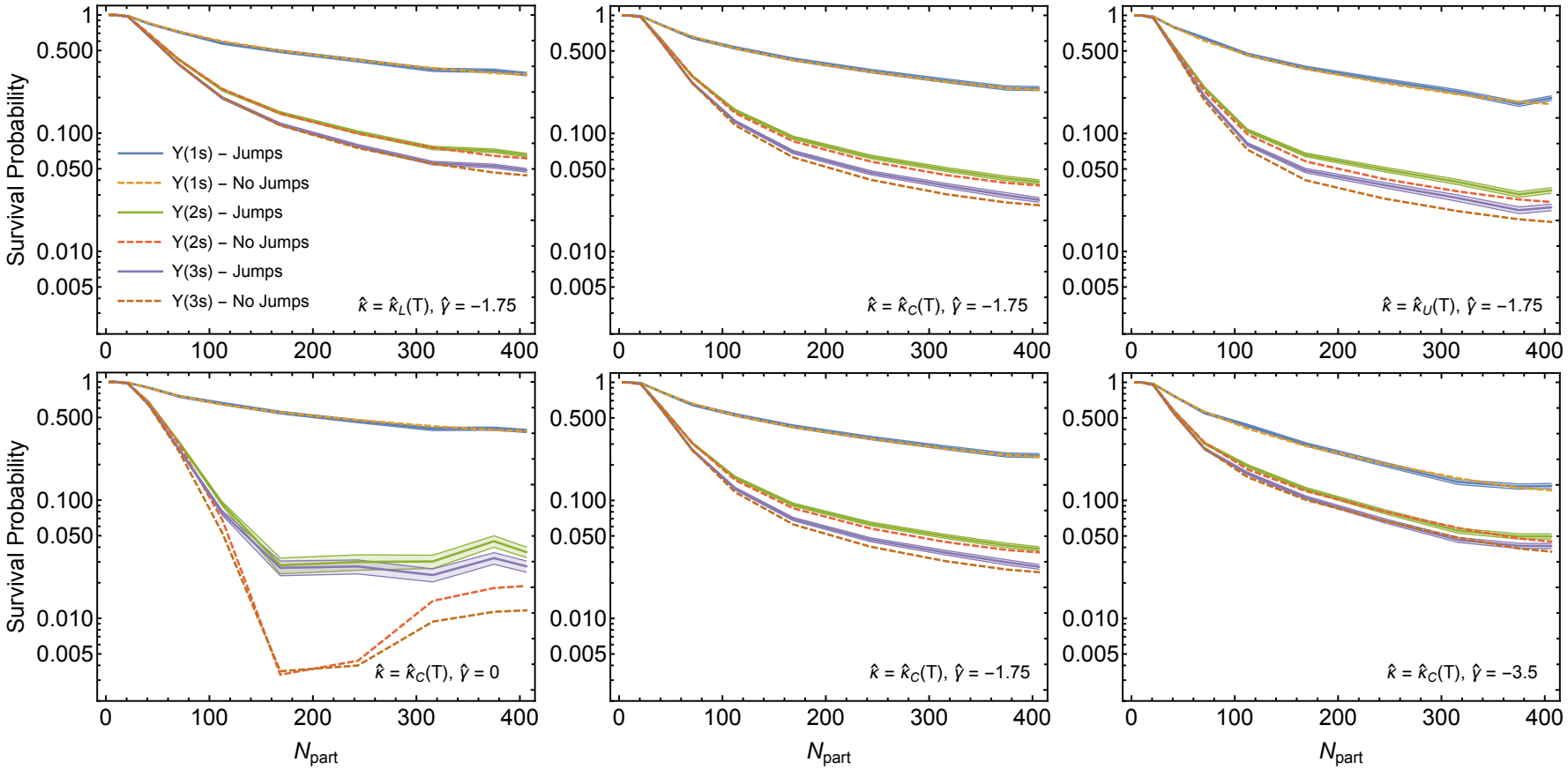
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming





- **Left panel**: Result including feed down, when varying $\hat{\kappa}$ over the theoretical uncertainty.
- **Right panel**: Result including feed down, when varying $\hat{\gamma}$ over the theoretical uncertainty
- Statistical uncertainty associated with average over quantum trajectories is on the order of the line width.

Effects of quantum jumps

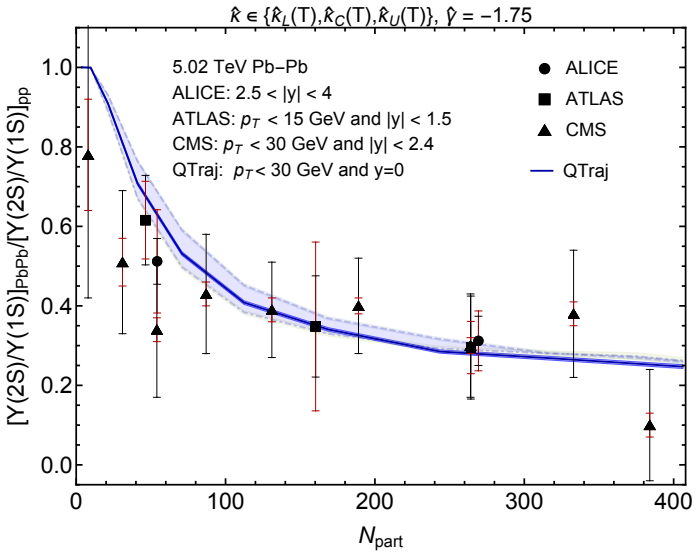


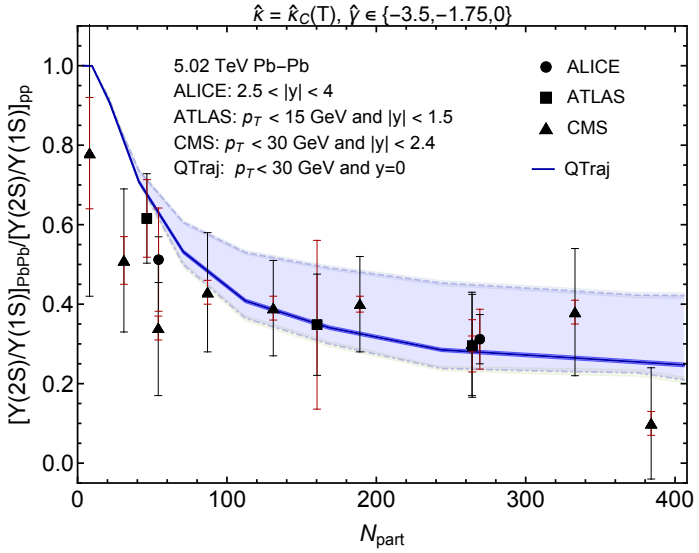
- **Solid lines show result with jumps**
- **Dashed lines show result without jumps (H_{eff} evolution)**

2S/1S and 3S/1S double ratios

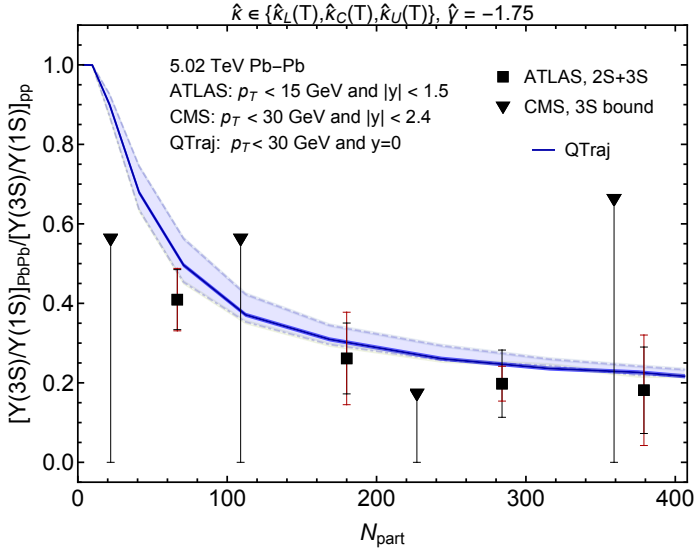
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

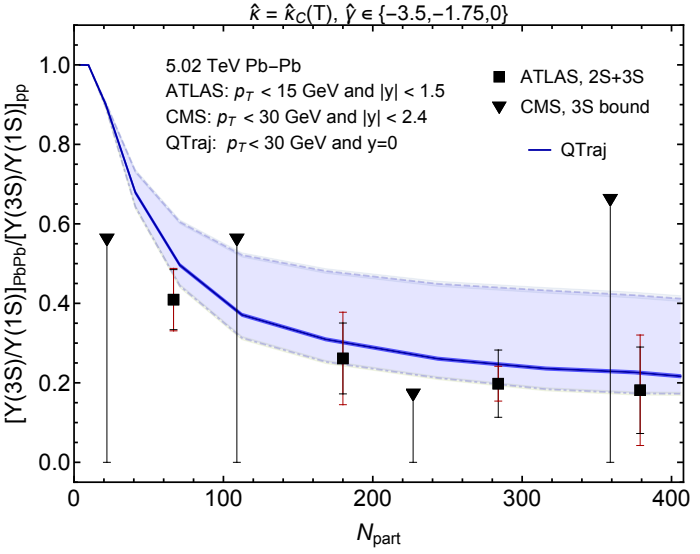
2S/1S





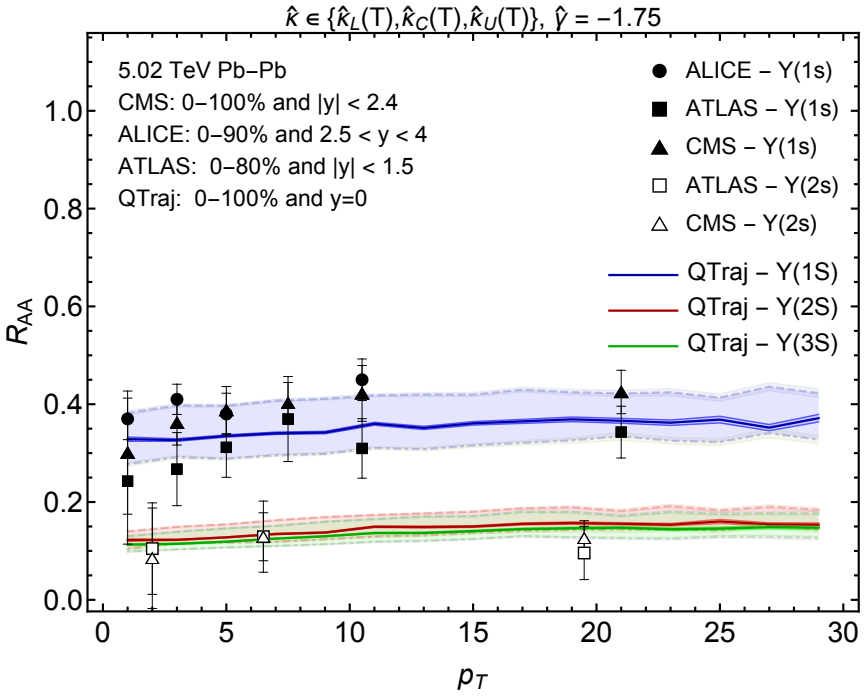
3S/1S

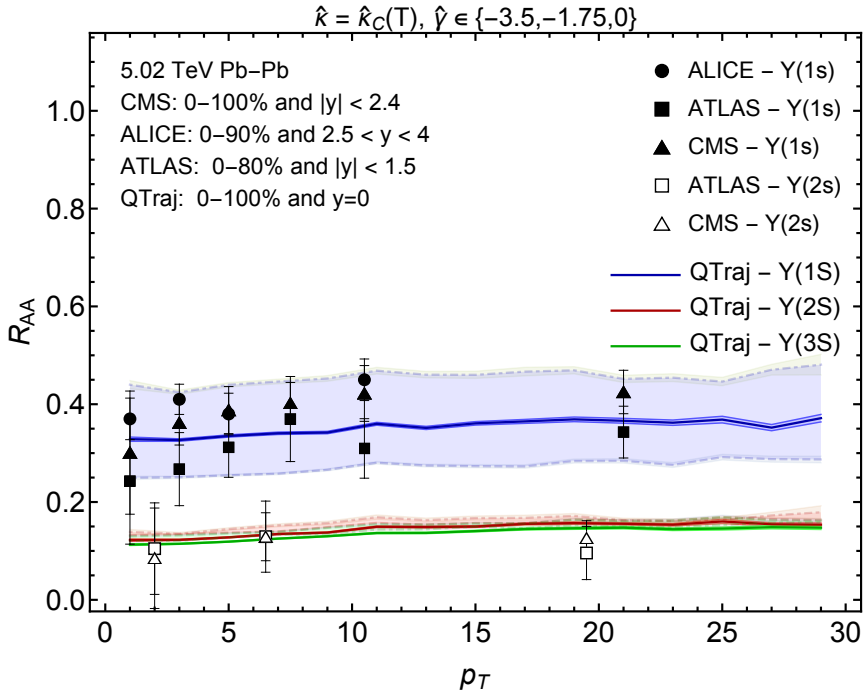




R_{AA} vs transverse momentum

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

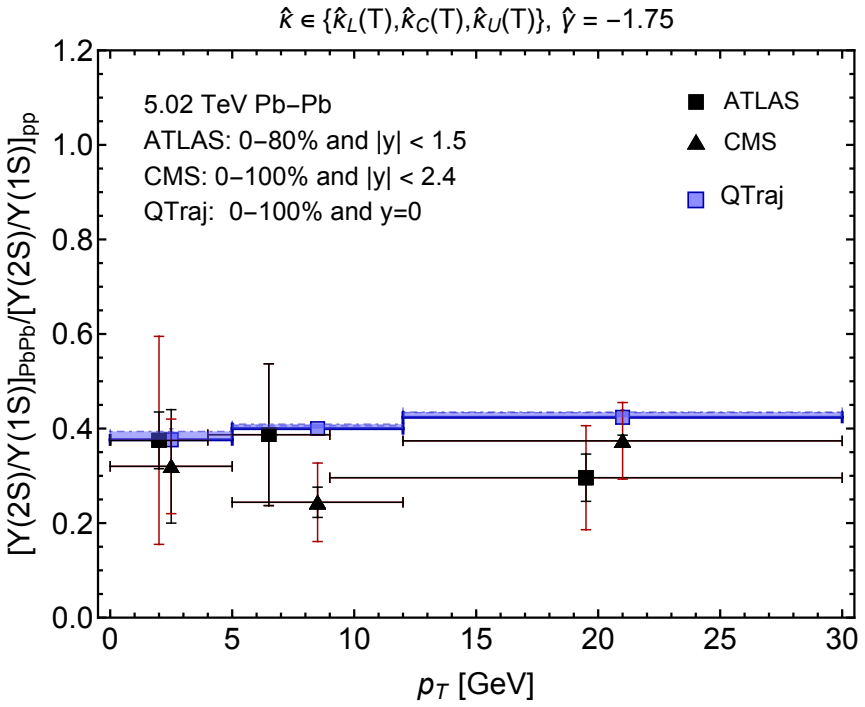


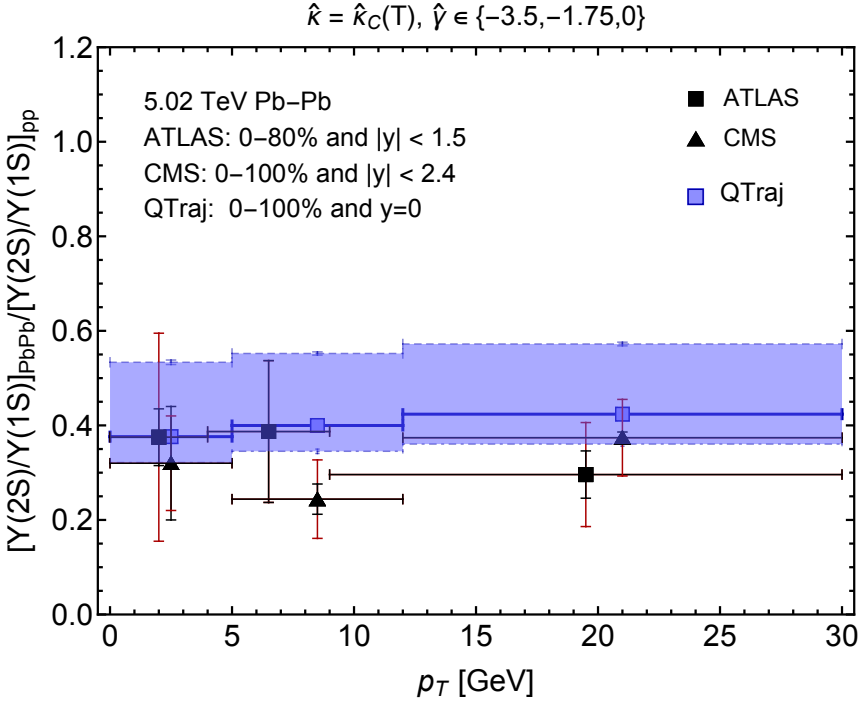


- QTraj predictions consistent with experimental observations.
- Very flat. Small decrease comes from longer time spent in the QGP.
- Once again, larger variation from variation of $\hat{\gamma}$.

2S/1S ratio vs transverse momentum

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming



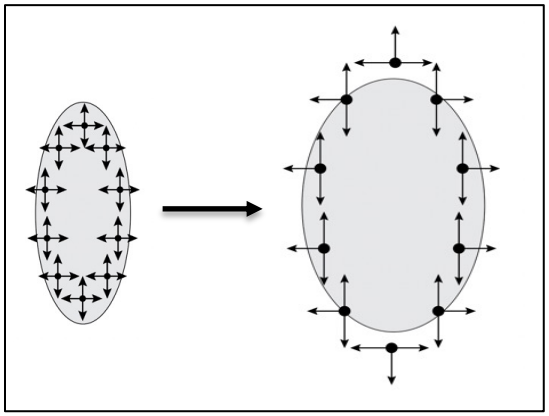


- Result does not depend on choice of κ, however, we see larger variation when varying γ; **value of $\gamma = -3.5$ has tension with data**

- **This offers some hope to constrain this transport coefficient from 2S/1S double ratio data.**

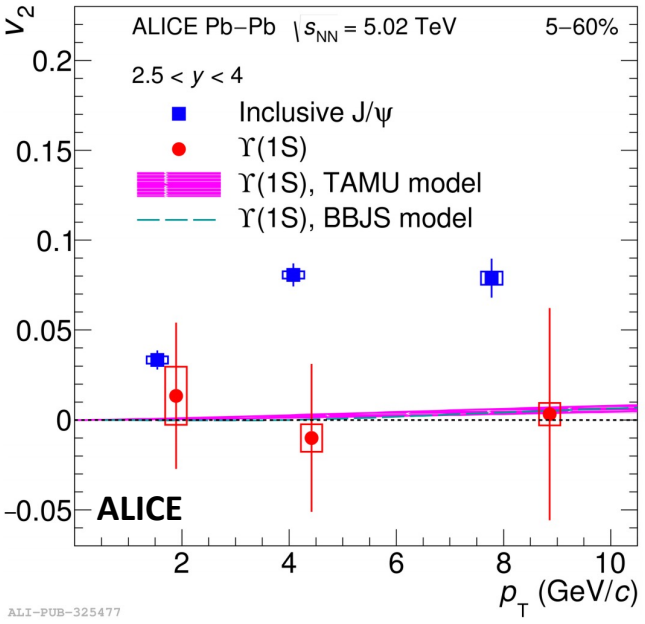
Momentum-space anisotropies

4d flow tomography



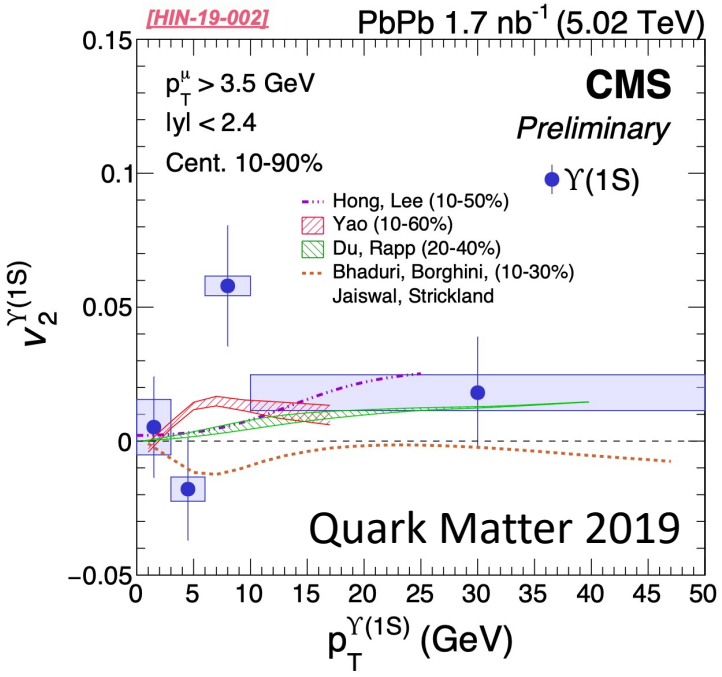
- Bottomonium probably doesn't flow in the "collective flow" sense.
- However, there can be momentum-space anisotropies induced by path-length differences in suppression along the short and long sides of the QGP.

Cern Courier, Fall 2019



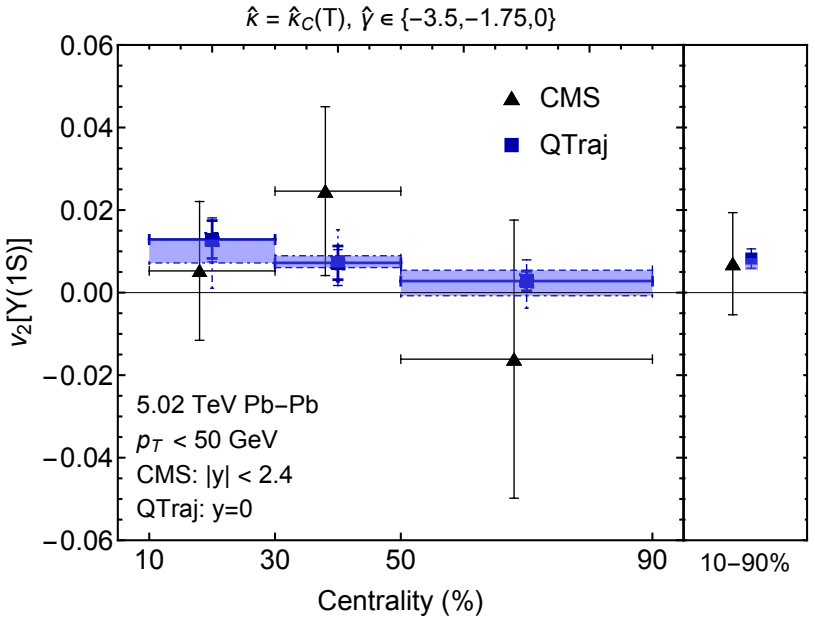
TAMU: Phys. Rev. C 96, (2017) 054901
BBJS: arXiv:1809.06235

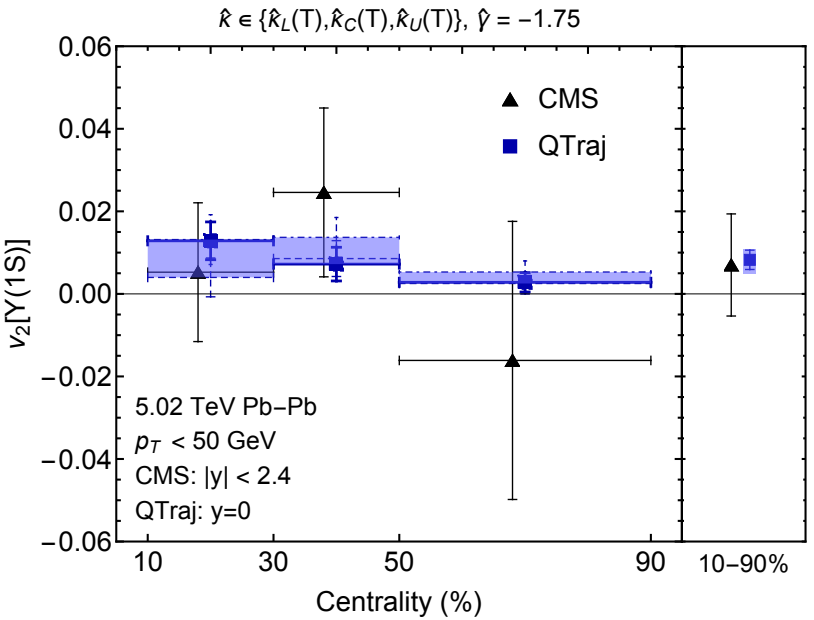
ALICE Collaboration: arXiv:1907.03169

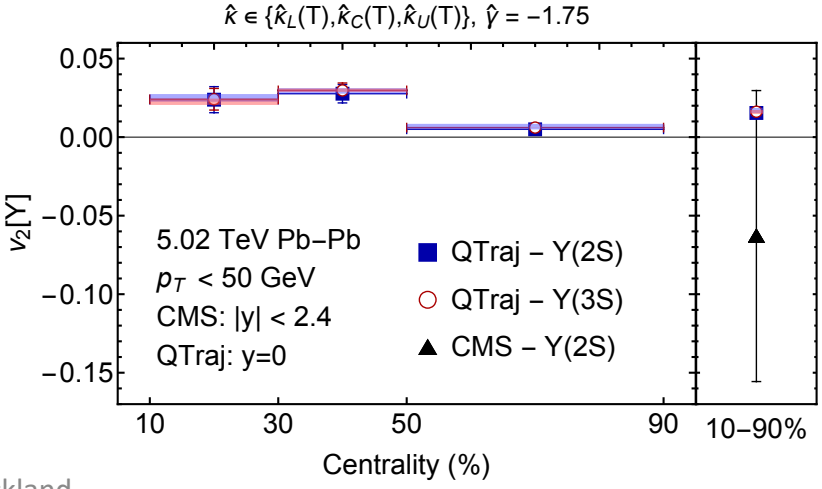


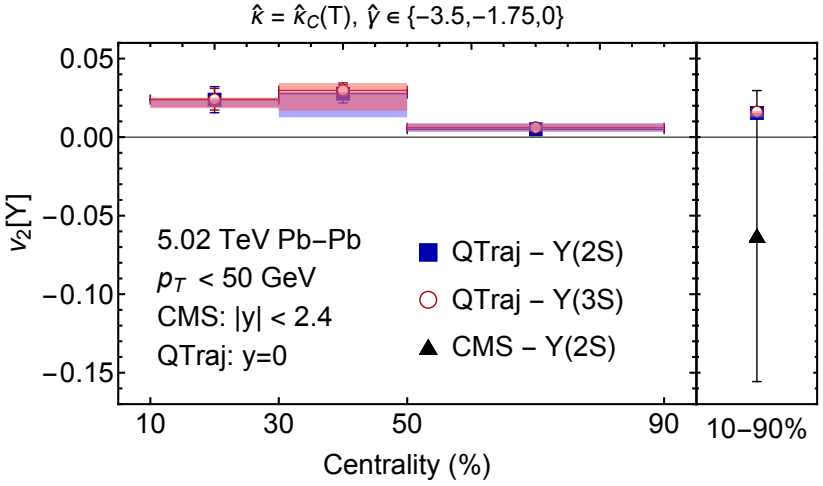
Momentum-space anisotropies

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming



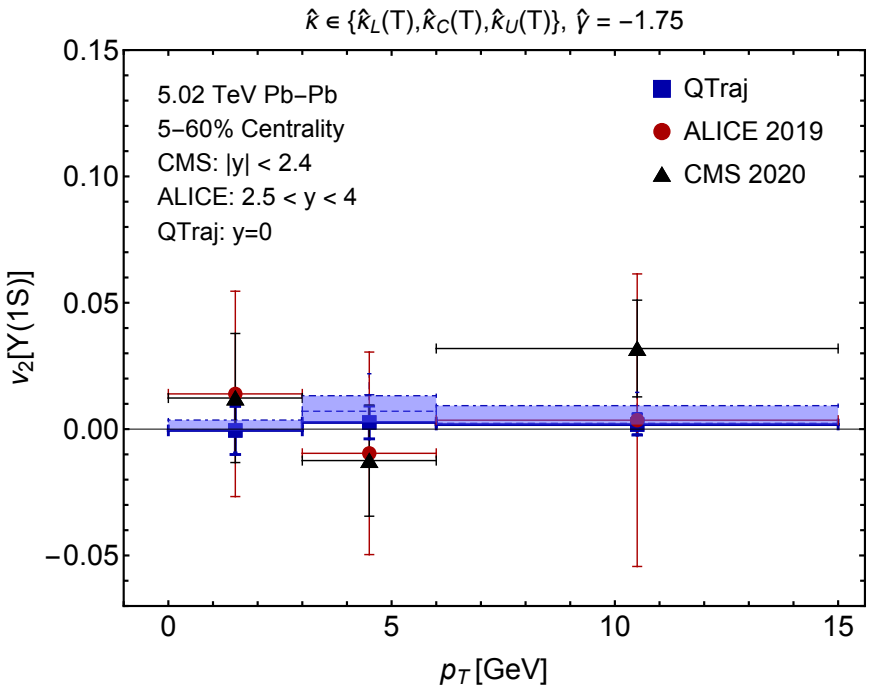


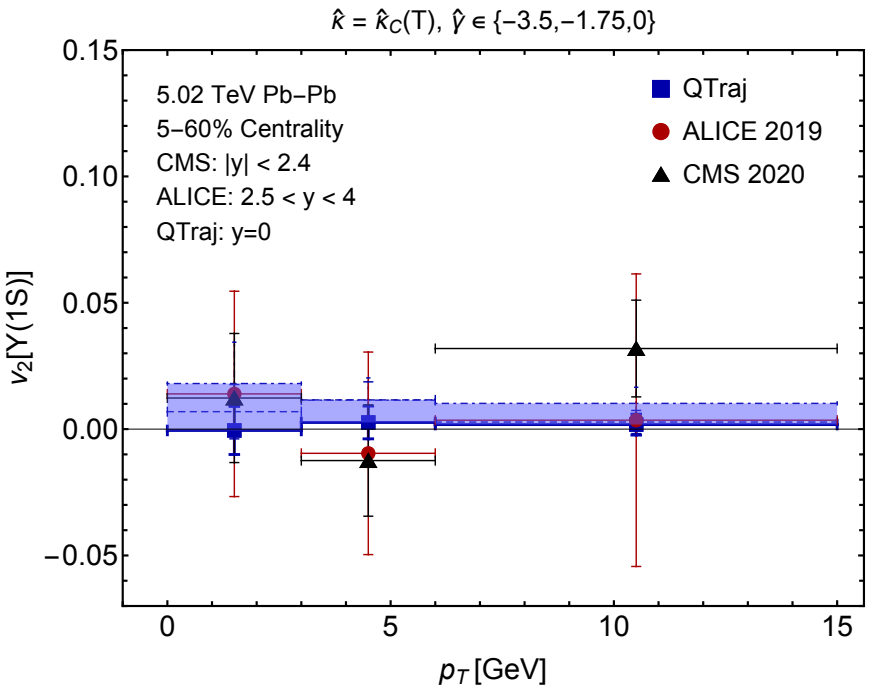




Momentum-space anisotropies

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming





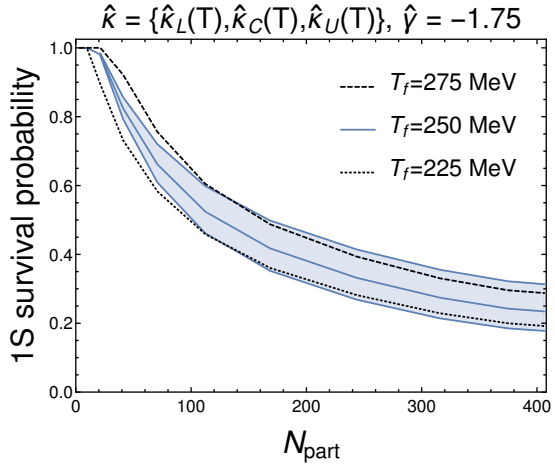
- Y(1S) v_2 due to path length differences in suppression is small.
- **Qtraj predicts $|v_2[\Upsilon(1s)]| < 0.02$ at all p_T.**
- Magnitude is consistent with prior works.
- Data have large uncertainties, hopefully more statistics in the future.

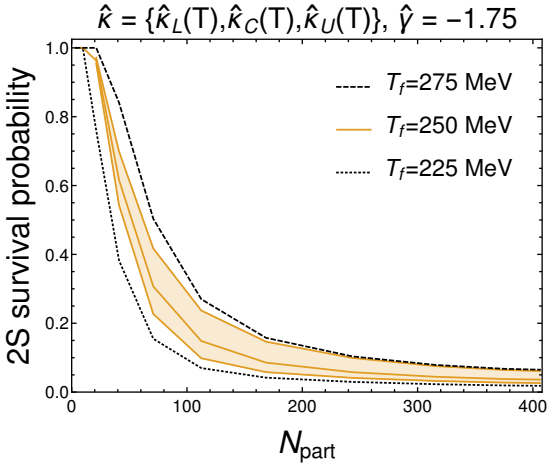
Conclusions and Outlook

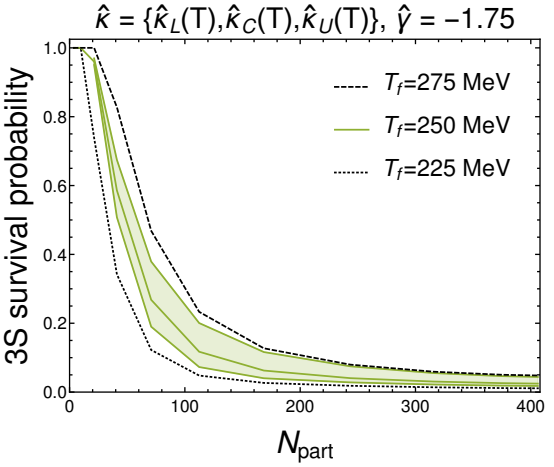
- Lots of recent progress using the open quantum system approach.
- OQS + pNRQCD works very well to describe the suppression vs $N_{\rm part}$ and p_T, double ratios, and "flow" seen at LHC.
- **First full 3D quantum and non-abelian treatment within OQS.**
- Transport coefficients used were **constrained by independent lattice measurements.**
- Demonstrated that Upsilon R_{AA} and double ratios can be used to provide **experimental constraints on these transport coefficients**.
- The **quantum trajectory algorithm** (implemented in QTraj) allowed us to **include effect of quantum jumps** between color and angular momentum states in a **computationally scalable manner**.
- One outstanding issue is the transition to low-temperature bottomonium dynamics ($T < 200 - 250$ MeV). Different ordering of scales, $T \lesssim E$ → **work in progress**.

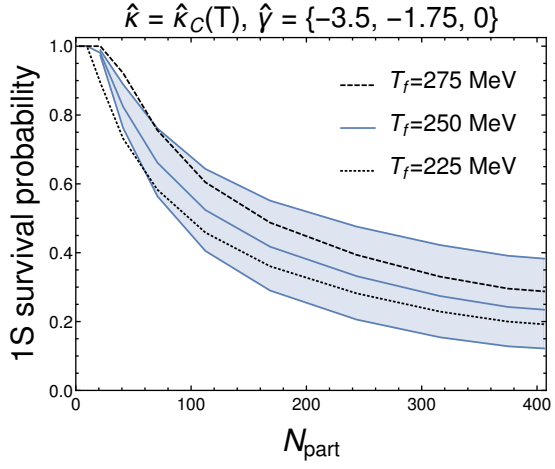
Additional slides

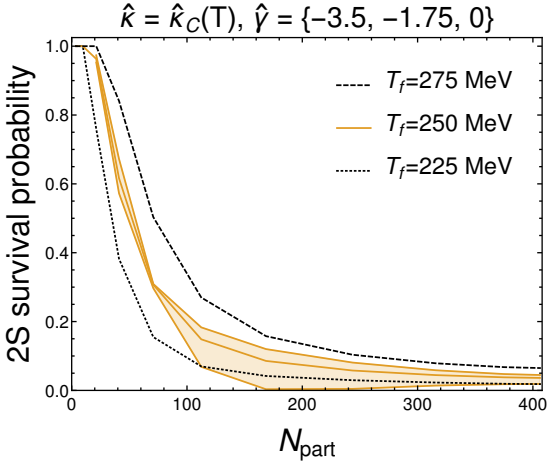
Dependence on T_f

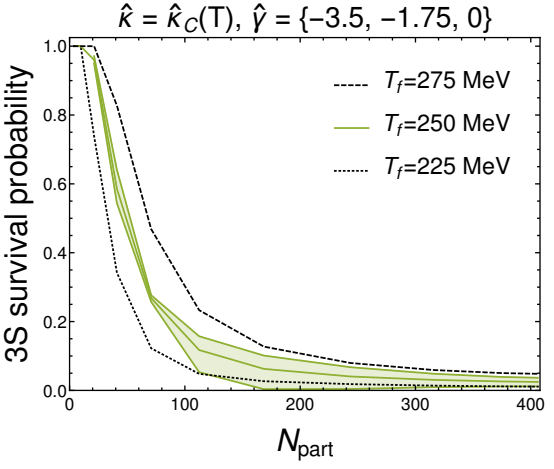




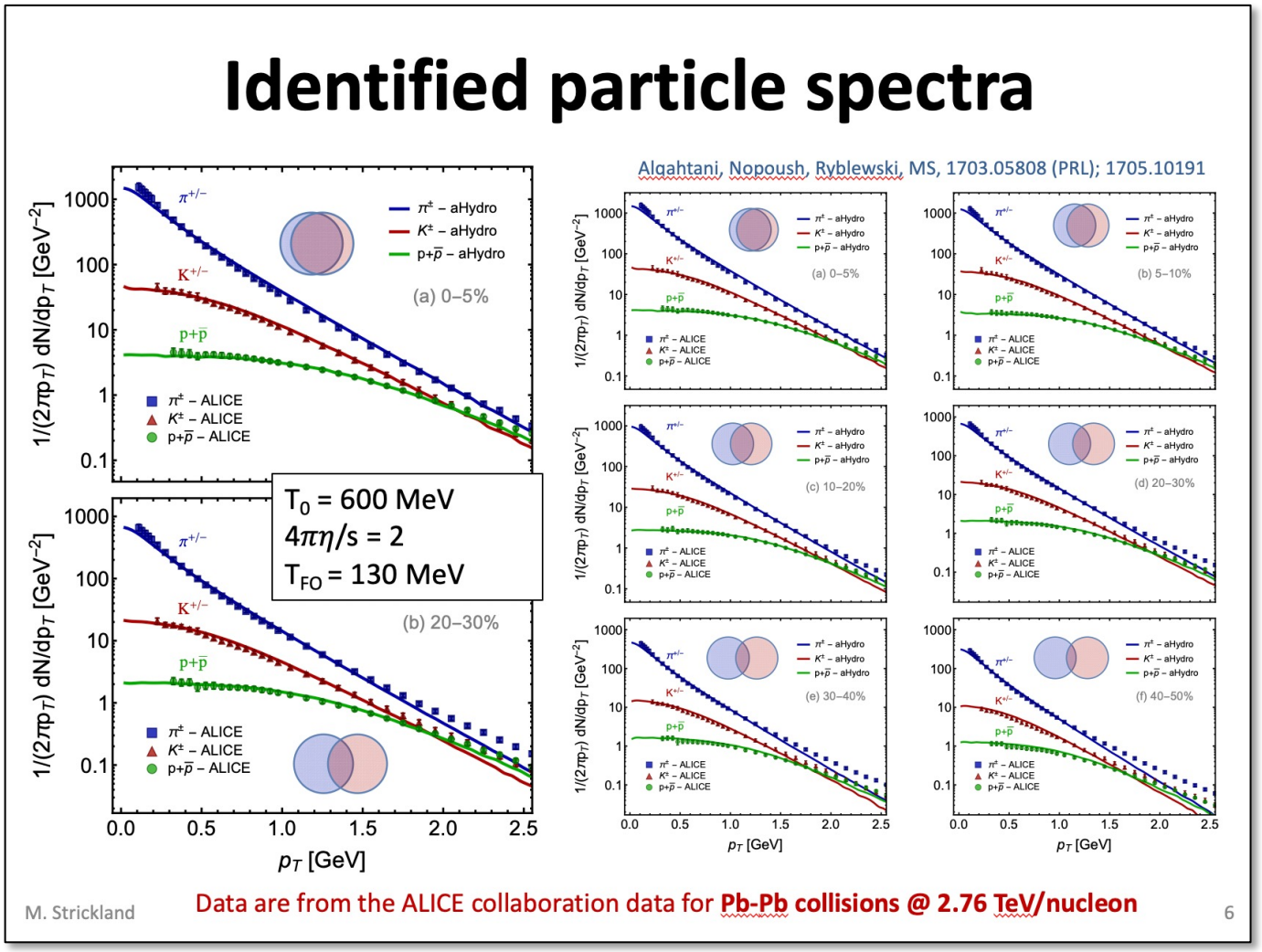








3+1D hydrodynamical background



- We use a 3+1D dissipative code for the hydro background (quasiparticle anisotropic hydrodynamics)
- Has been tuned to RHIC and LHC heavy ion collisions
- Reproduces spectra, multiplicities, identified elliptic flow of light hadrons, HBT radii, etc.

For 5.02 TeV, T_0 = 630 MeV @ t_0 = 0.25 fm/c