Physical characteristics of glasma at very early times

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based on: **arXiv:2012.03042**
arXiv:2105:05327

Workshop on QGP Phenomenology
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Outline of the talk

1. Introduction

THEORY:

2. Nuclei before the collision - MV model
3. Strongly interacting matter after the collision
   - Glasma
   - Expansion in the proper time
   - Correlators

RESULTS:

4. Energy-momentum tensor
5. Pressure anisotropy
6. Azimuthal flow
7. Angular momentum of glasma

8. Summary and conclusions

Introduction & motivation

One of the biggest challenges in the dynamics of heavy-ion collisions is to understand the transition between early-time dynamics and hydrodynamics

early-time dynamics	hydrodynamics
- microscopic theory of non-Abelian gauge fields - out-of-equilibrium	- macroscopic effective theory based on universal conservation laws - close to equilibrium

Two possible strategies:

- unreasonable effectiveness of hydrodynamics: push hydrodynamics to the limits where it is not expected to be efficient
  (sensitivity to different initial conditions, attractor, different underlying theories)
- work out more constrained frameworks of the initial QGP evolution in quest of the onset of hydrodynamics
  (constrain numerical uncertainties, include appropriate sources of fluctuations, extend existing theoretical models, find new analytical tools)

Try to answer: how many unique features of fluid dynamics can be found in the initial state made of QCD quanta and to what extent QCD mimics hydrodynamics?

Caution required!

Some evidence that a cat is a fluid

Introduction & motivation

In this talk:

- analytical approach to the initial state
- purely classical

$\rightarrow$ allows for control over different approximations and sources of errors
$\rightarrow$ can be systematically extended
$\rightarrow$ no fluctuations of positions of nucleons $\rightarrow$ less detailed when compared, for example, to IP-glasma

Before the collision

Color Glass Condensate (CGC) - the effective theory to describe each nucleus in terms of QCD quanta

- MV model - a particular realization of CGC:
  - large x partons: valence quarks, color sources for gluon fields represented by the color density ρ
  - small x partons: due to large occupation numbers effectively represented by soft gluon fields $\beta^\mu(x)$
  - gluons are in the saturation regime controlled by the saturation scale Q_s
  - separation scale between small-x and large-x partons is fixed

  - instead of $\beta^\mu(x)$ potentials one can use $\mathbf{E}(x)$ and $\mathbf{B}(x)$ fields to describe a nucleus:
    - longitudinal components of chromodynamic fields vanish $E^z = B^z = 0$
    - transverse components of chromodynamic fields are non-zero and are perpendicular to each other

Before the collision

Description of the longitudinal physics (along the beam) separates from the transverse one (within the transverse area of the nuclei)
$\rightarrow$ light-cone coordinates $(x^+, x^-, \vec{x}_\perp)$ with $x^+ = \frac{t+z}{\sqrt{2}}$ and $x^- = \frac{t-z}{\sqrt{2}}$

Classical Yang-Mills equations (for an ion moving to the right)

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

color sources given by SU(N_c) four-current: $J^\mu(x^-, \vec{x}_\perp) = \delta^{\mu+}\rho(x^-, \vec{x}_\perp)$
gluon fields described by the strength tensor: $F^{\mu\nu} = \frac{i}{g}[D^\mu, D^\nu]$ with $D^\mu = \partial^\mu - ig\beta^\mu$

solutions to the equations:

$$\beta^-(x^-, \vec{x}_\perp) = 0 \qquad \beta^i(x^-, \vec{x}_\perp) = \theta(x^-)\beta^i(\vec{x}_\perp) \qquad \beta^i(\vec{x}_\perp) = \frac{i}{g}U(\vec{x}_\perp)\partial^i U^\dagger(\vec{x}_\perp)$$

$$U(\vec{x}_\perp) - \text{Wilson line}$$

After the collision

Glasma:

* glasma fields develop after the collision in the forward light-cone region
* valence quarks fly away
* highly energetic and anisotropic medium made of mostly gluon fields
* gluon fields obtained as solutions to classical Yang-Mills equations
* quarks appear as quantum corrections at NLO
* many approaches to study initial dynamics (mostly numerical)
* here: temporal evolution of glasma fields is obtained in the proper time expansion

Glasma

The ansatz of the gauge potential in the forward light-cone region:

$$\alpha^+(x) = x^+\alpha(\tau,\vec{x}_\perp) \qquad \alpha^-(x) = -x^-\alpha(\tau,\vec{x}_\perp) \qquad \alpha^i(x) = \alpha^i_\perp(\tau,\vec{x}_\perp)$$

Glasma fields $\alpha(\tau,\vec{x}_\perp)$ and $\alpha^i_\perp(\tau,\vec{x}_\perp)$:

- evolve in time parametrized by $\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$
- are boost-independent
- evolve according to source-less Yang-Mills equations:

$$\begin{aligned} \frac{1}{\tau}\partial_\tau \frac{1}{\tau}\partial_\tau \tau^2\alpha - [D^i,[D^i,\alpha]] &= 0 \\ ig\tau[\alpha,\partial_\tau\alpha] - \frac{1}{\tau}[D^i,\partial_\tau\alpha^i_\perp] &= 0 \\ \frac{1}{\tau}\partial_\tau\tau\partial_\tau\alpha^i_\perp - ig\tau^2[\alpha,[D^i,\alpha]] - [D^j,F^{ji}] &= 0 \end{aligned}$$

  - general solutions are not known
  - current dependence enters through boundary conditions, which connect different light-cone sectors

$$\alpha^i_\perp(\tau=0,\vec{x}_\perp) = \beta^i_1(\vec{x}_\perp) + \beta^i_2(\vec{x}_\perp) \qquad \alpha(\tau=0,\vec{x}_\perp) = -\frac{ig}{2}[\beta^i_1(\vec{x}_\perp),\beta^i_2(\vec{x}_\perp)]$$

  - initial chromodynamic fields are purely longitudinal

$$E_0 = ig[\beta^i_1,\beta^i_2] \qquad B_0 = ig\epsilon^{ij}[\beta^i_1,\beta^j_2]$$

Expansion in the proper time

An analytical approach to solve Yang-Mills equations proposed in:
Fries, Kapusta, Li, arXiv:0604054
Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

- glasma is a short-lived phase and decays before the system reaches equilibrium ($\tau < 1$ fm/c)
- proper time of such a system is small and can be treated as an expansion parameter of glasma fields:

$$\alpha^i_\perp(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha^i_{\perp(n)}(\vec{x}_\perp), \qquad \alpha(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{(n)}(\vec{x}_\perp)$$

and the chromodynamic fields:

$$\mathbf{E} = \mathbf{E}_{(0)} + \tau \mathbf{E}_{(1)} + \tau^2 \mathbf{E}_{(2)} + \dots \qquad \mathbf{B} = \mathbf{B}_{(0)} + \tau \mathbf{B}_{(1)} + \tau^2 \mathbf{B}_{(2)} + \dots$$

- the system of coupled Yang-Mills equations can be solved recursively to any order in τ
- 0th-rder coefficients are identified with boundary conditions
- solutions are written in terms of precollision potentials
- effective dimensionless parameter is $\tilde{\tau} = \tau Q_s$

Correlators of gauge potentials

- need for colour charge distributions which are not known
- average over colour sources assuming a Gaussian distribution of colour sources within each nucleus

$$\langle \rho_a(x^-, \vec{x}_\perp) \rho_b(y^-, \vec{y}_\perp) \rangle = g^2 \delta_{ab} \lambda(x^-, \vec{x}_\perp) \delta(x^- - y^-) \delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$\lambda(x^-, \vec{x}_\perp)$ - volume density of sources normalized as $\int dx^- \lambda(x^-, \vec{x}_\perp) = \mu(\vec{x}_\perp)$

- potentials of different nuclei are uncorrelated: $\langle \beta^i_{1a} \beta^j_{2b} \rangle = 0$

Basic building block - 2-point correlator

$$\delta_{ab} B^{ij}_n(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{w \to 0} \langle \beta^i_{n\,a}(x^\mp, \vec{x}_\perp) \beta^j_{n\,b}(y^\mp, \vec{y}_\perp) \rangle$$

$$B^{ij}_n(\vec{x}_\perp, \vec{y}_\perp) = \frac{2}{g^2 N_c \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp)} \left(\exp[\frac{g^4 N_c}{2} \, \tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp)] - 1 \right) \partial^i_x \partial^j_y \tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp)$$

$$\tilde{\Gamma}_n(\vec{x}_\perp, \vec{y}_\perp) = 2\tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp) - \tilde{\gamma}_n(\vec{x}_\perp, \vec{x}_\perp) - \tilde{\gamma}_n(\vec{y}_\perp, \vec{y}_\perp)$$

$$\tilde{\gamma}_n(\vec{x}_\perp, \vec{y}_\perp) = \int d^2 z_\perp \, \mu_n(\vec{z}_\perp) \, G(\vec{x}_\perp - \vec{z}_\perp) \, G(\vec{y}_\perp - \vec{z}_\perp)$$

$$G(\vec{x}_\perp) = \frac{1}{2\pi} K_0(m|\vec{x}_\perp|)$$

Correlators of gauge potentials

- **Wick's theorem:**
  - $\langle \beta_1^i \beta_1^j \, \beta_2^l \beta_2^m \beta_2^k \beta_2^r \rangle = \langle \beta_1^i \beta_1^j \rangle \left(\langle \beta_2^l \beta_2^m \rangle \langle \beta_2^k \beta_2^r \rangle + \langle \beta_2^l \beta_2^k \rangle \langle \beta_2^m \beta_2^r \rangle + \langle \beta_2^l \beta_2^r \rangle \langle \beta_2^k \beta_2^m \rangle \right)$
  - correlators of odd number of gauge fields vanish

- **charge density per unit transverse area**
  - $\bar{\mu} = g^{-4} Q_s^2$, where Q_s is the saturation scale (uniform nuclei)
  - μ given by the Woods-Saxon distribution (internal structure of nuclei)

$$\mu(\vec{x}_\perp) = \frac{\bar{\mu}}{2a \log(1 + e^{R_A/a})} \int_{-\infty}^{\infty} dz \frac{1}{1 + \exp\left[(\sqrt{(\vec{x}_\perp)^2 + z^2} - R_A)/a \right]} \, .$$

- R_A and a give the radius and skin thickness of the nucleus
- normalized so that $\mu(0) = \bar{\mu}$

- **IR regulator**

$m \sim \Lambda_{\mathrm{QCD}}$ - chosen so that because of confinement the effect of valence sources dies off at transverse length scales larger than $1/\Lambda_{\mathrm{QCD}}$

- **UV regulator**

Q_s - saturation scale

$$\lim_{r \to 0} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \delta^{ij} g^2 \frac{\bar{\mu}}{8\pi} \left(\ln \left(\frac{Q_s^2}{m^2} + 1 \right) - \frac{Q_s^2}{Q_s^2 + m^2} \right) \quad [+ \text{ gradients of } \mu]$$

Structure of the energy-momentum tensor

Correlators of gauge fields and the proper time expansion determine the structure of the energy-momentum tensor:

$$T^{\mu\nu} = 2\mathrm{Tr}\big[F^{\mu\lambda}F_{\lambda}{}^{\nu} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\big]$$

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

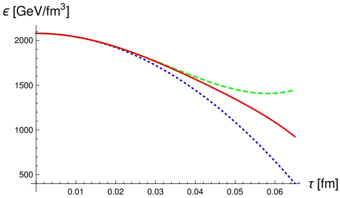
- the result is complicated and long and given in powers of τ up to τ^6 order

$$\mathcal{O}(T_{\mathrm{milne}}) = \begin{pmatrix} (0,2,4,6) & (1,3,5) & (1,3,5) & (1,3,5) \\ (1,3,5) & (-2,0,2,4) & (0,2,4) & (0,2,4) \\ (1,3,5) & (0,2,4) & (0,2,4,6) & (2,4,6) \\ (1,3,5) & (0,2,4) & (2,4,6) & (0,2,4,6) \end{pmatrix}.$$

- the energy-momentum tensor is gauge-invariant, divergence-less, traceless and symmetric
- due to symmetries only 6 components are independent

Energy density

- energy density evolution in τ at $\eta = 0$ for uniform $\bar{\mu}$
- including higher order terms allows for testing the convergence of the proper time expansion (blue=τ^2, green=τ^4, red=τ^6)



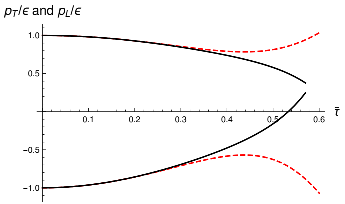
We analysed various energy density profiles for different geometries of the collision and different parameters of Woods-Saxon distribution
$\rightarrow$ **energy density is a smooth function in time and space**

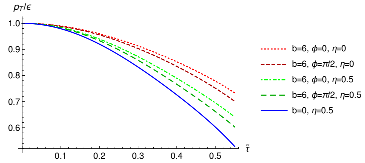
Pressure

- transverse and longitudinal pressure components (in equilibrium $p_T/\mathcal{E} = p_L/\mathcal{E} = 1/3$)

$$\frac{p_L}{\mathcal{E}} = \frac{T^{11}_{\rm mink}}{T^{00}_{\rm mink}} \qquad \frac{p_T}{\mathcal{E}} = \frac{1}{2}\frac{(T^{22}_{\rm mink} + T^{33}_{\rm mink})}{T^{00}_{\rm mink}} .$$

- pressure components as a function of $\tilde{\tau}$ for uniform $\bar{\mu}$ - left $\tilde{\tau} = \tau Q_s$ with $Q_s = 2$ GeV, red=τ^4, black=τ^6
- transverse pressure as a function of $\tilde{\tau}$ for Woods-Saxon distribution - right (dependence on impact parameter, rapidity and azimuthal angle studied)



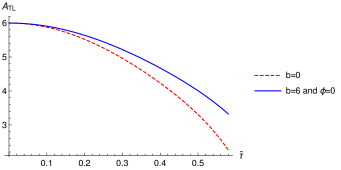


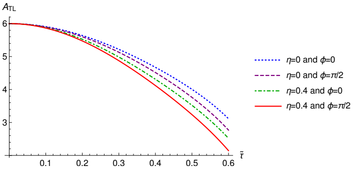
→ **proper time expansion works reasonably well for times $\tilde{\tau} \sim 0.5$ (or $\tau \sim 0.05$ fm)**
→ **transverse pressure is sensitive to the geometry of the collision**
→ **dependence on azimuthal angle and rapidity emerges → anisotropies**

Anisotropy of p_L and p_T

- anisotropy of the transverse and longitudinal pressure ($A_{TL} = 6$ at $\tau = 0$ and $A_{TL} = 0$ in equilibrated plasma)

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L}$$





→ approach to equilibrium faster for central collisions
→ approach to equilibrium faster at space points in the reaction plane than perpendicular to it
→ approach to equilibrium faster for larger rapidities

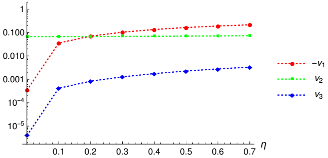
Azimuthal flow

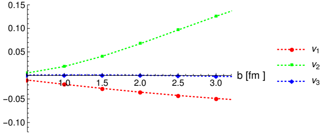
- Fourier coefficients of the momentum azimuthal flow

$$v_n = \int_0^{2\pi} d\phi \, \cos(n\phi) \, P(\phi)$$

distribution $P(\phi)$ defined as: $P(\phi) \equiv \frac{1}{\Omega} \int d^2\vec{x}_\perp \; \delta(\phi - \varphi(\vec{x}_\perp)) \; W(\vec{x}_\perp)$ with $W(\vec{x}_\perp) \equiv \sqrt{(T^{0x}(\vec{x}_\perp))^2 + (T^{0y}(\vec{x}_\perp))^2}$ and $\varphi(\vec{x}_\perp) = \cos^{-1}\left(\frac{T^{0x}(\vec{x}_\perp)}{W(\vec{x}_\perp)}\right)$

- Fourier coefficients v_1, v_2 and v_3 calculated as a function of rapidity (at fixed $b = 2$ fm) and as a function of impact parameter (at fixed $\eta = 0.1$)





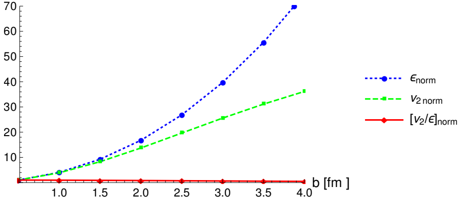
$\rightarrow$ **symmetries: n-odd coefficients are rapidity odd and n-even coefficients are rapidity even** (we know the reaction plane and we do not include fluctuations in the positions of participants)
$\rightarrow$ v_2 **and** v_3 **are of the same order as experimental values**
$\rightarrow$ $|v_1|$ **is bigger than expected**

Eccentricity and elliptic flow coefficient

- eccentricity - spatial deviations from azimuthal symmetry

$$\varepsilon_n = -\frac{\int d^2\vec{R}|\vec{R}|\cos(n\phi)\mathcal{E}(\vec{R})}{\int d^2\vec{R}|\vec{R}|\mathcal{E}(\vec{R})} \qquad \phi = \tan^{-1}(R_y/R_x)$$

- calculated as a function of the impact parameter at $\tau = 0.04$ fm and $\eta = 0$



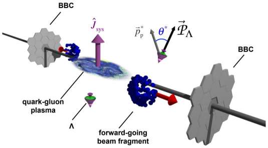
$\rightarrow$ **correlation of eccentricity ϵ_2 and v_2 is treated as a indication of onset of hydrodynamic behaviour**

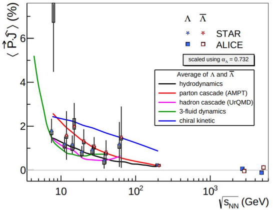
The expected role of angular momentum in HIC

- large angular momentum is expected to be generated in non-central collisions

STAR Collaboration, Nature 548, 62 (2017)

Becattini and Lisa, Ann.Rev.Nucl.Part.Sci. 70 (2020) 395-423





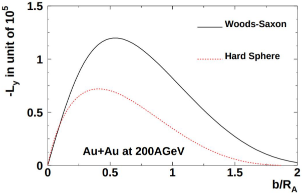
- consequences:
  - ∗ spin-orbit coupling leads to alignment of spins to the direction of the angular momentum → polarization of hyperons and vector mesons
  - ∗ QGP is rapidly-rotating (vortical) system
- many attempts to formulate hydrodynamics with spin

Experimental observations:

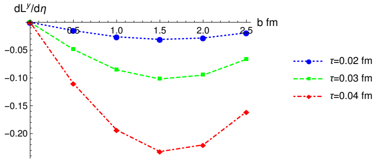
* at RHIC energies polarization of a few percent is seen in non-central AA collisions
* at LHC energies polarization is not observed at all

Angular momentum of glasma

- angular momentum at RHIC energies
  Gao et al, Phys. Rev C 77, 044902 (2008)



- our result: angular momentum as a function of the impact parameter



- the shape and the position of the peak similar
- the result at RHIC energies $\sim 10^5$ bigger than our results
- most of the momentum of the incoming nuclei is NOT transmitted to the glasma
- small angular momentum of the glasma $\rightarrow$ no polarization effect at highest collision energies

Summary and conclusions

- Glasma dynamics studied in the proper time expansion
- Convergence of the proper time expansion tested
- Many physical characteristics of glasma dynamics calculated

- Proper time expansion can be trusted to about $\tau = 0.05$ fm; glasma moves towards equilibrium within this time
- Onset of hydrodynamic-like behaviour in the glasma phase
  - Fourier coefficients of the azimuthal flow relatively large
  - Sizeable correlation of the eccentricity and elliptic flow coefficient
- Angular momentum of glasma is found to be small
  - Need to study the transmission of the momentum from incoming nuclei to the interaction region
  - Glasma is not a rapidly rotating system
  - No polarization effect - in agreement with experimental observations for LHC energies