Strangeness Neutral Equation of State for QCD with a Critical Point

2103.08146; EPJ+ to appear

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QGP Pheno. | May 31, 2021

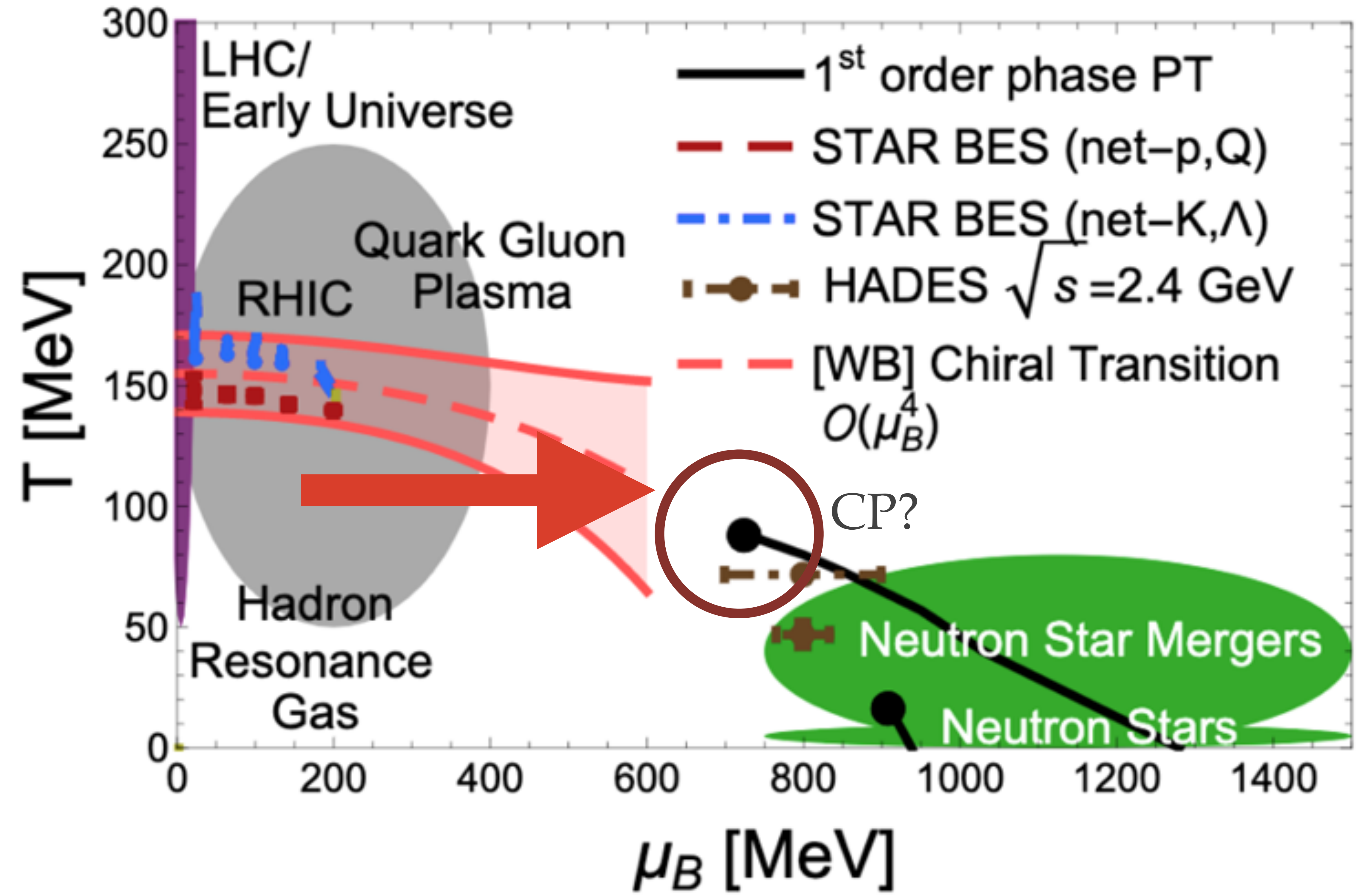
Illinois Center for Advanced Studies of the Universe

BEST COLLABORATION

Outline

1. (Brief) discussion of QCD phase diagram + experimental efforts
2. Constructing a family of Equations of State with a critical point
3. Enforcing charge density constraints
4. Implications to heavy-ion collisions

The QCD phase diagram – the one we don't know but love



Collision Energy

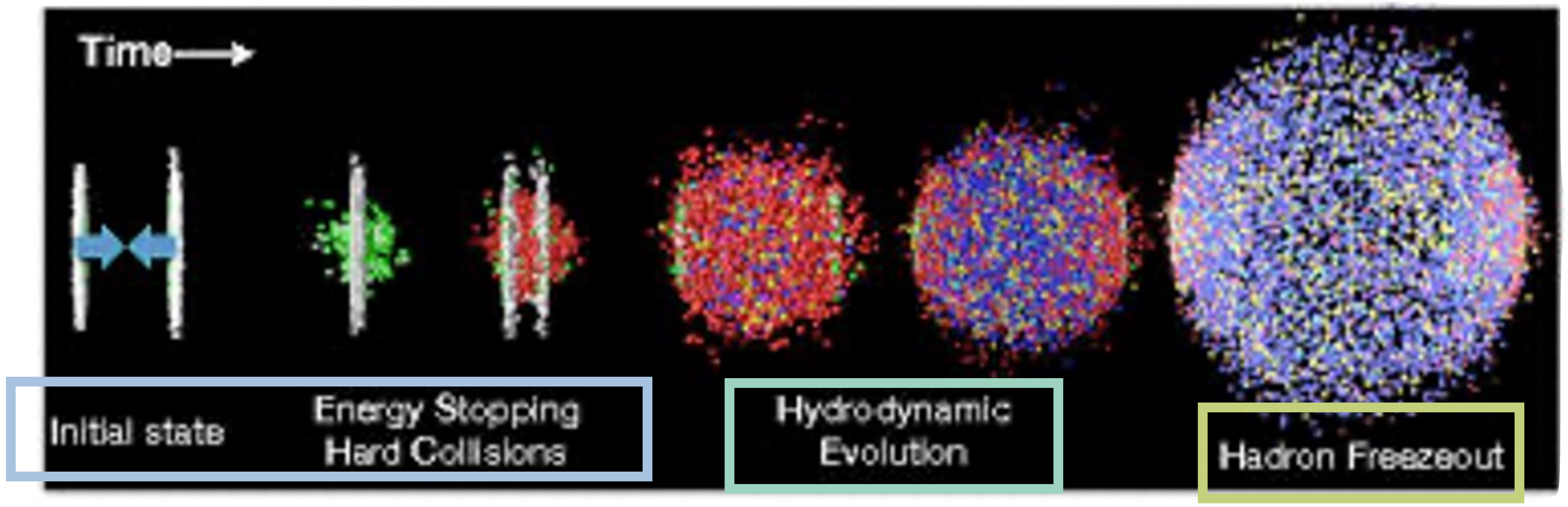
- Known with high precision at μ_B=0
  S. Borsanyi et al, JHEP (2018)
- Sign problem at finite μ_B
  M. Troyer and U.J. Wiese, Phys. Rev. Lett. (2005)
- Rich physics across different regimes.
- What are we probing in low beam energy heavy-ion collisions / fixed target experiments?

WB Phys.Rev.Lett. (2020); P. Alba et al Phys.Lett. (2014);
Bellwied et al arXiv:1805.00088; V. Dexheimer ariXiv:1708.08342;
Critelli et al, Phys.Rev. D96 (2017);
HADES Nature Phys. (2019); Nucl.Phys.A (2014)

QCD phase diagram – the critical point

- To-do list:
  - initial conditions + early dynamics
  - hydro evolution
  - hadronization and transport
  - Out of equilibrium effects?

T. Dore, Wed 18:00



Require Equation of State (EoS) as input, which must:

- **Include a critical point in the correct universality class**
- **Match what we already know from LQCD**

P. Parotto, DM et al. Phys. Rev. C (2020)

1. Expansion about μ_B=0 with $\mu_S = \mu_Q = 0$

J. Stafford, DM et al. EPJ+ to appear

2. Expansion about μ_B=0 with

$$\langle n_S \rangle = 0$$

$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$

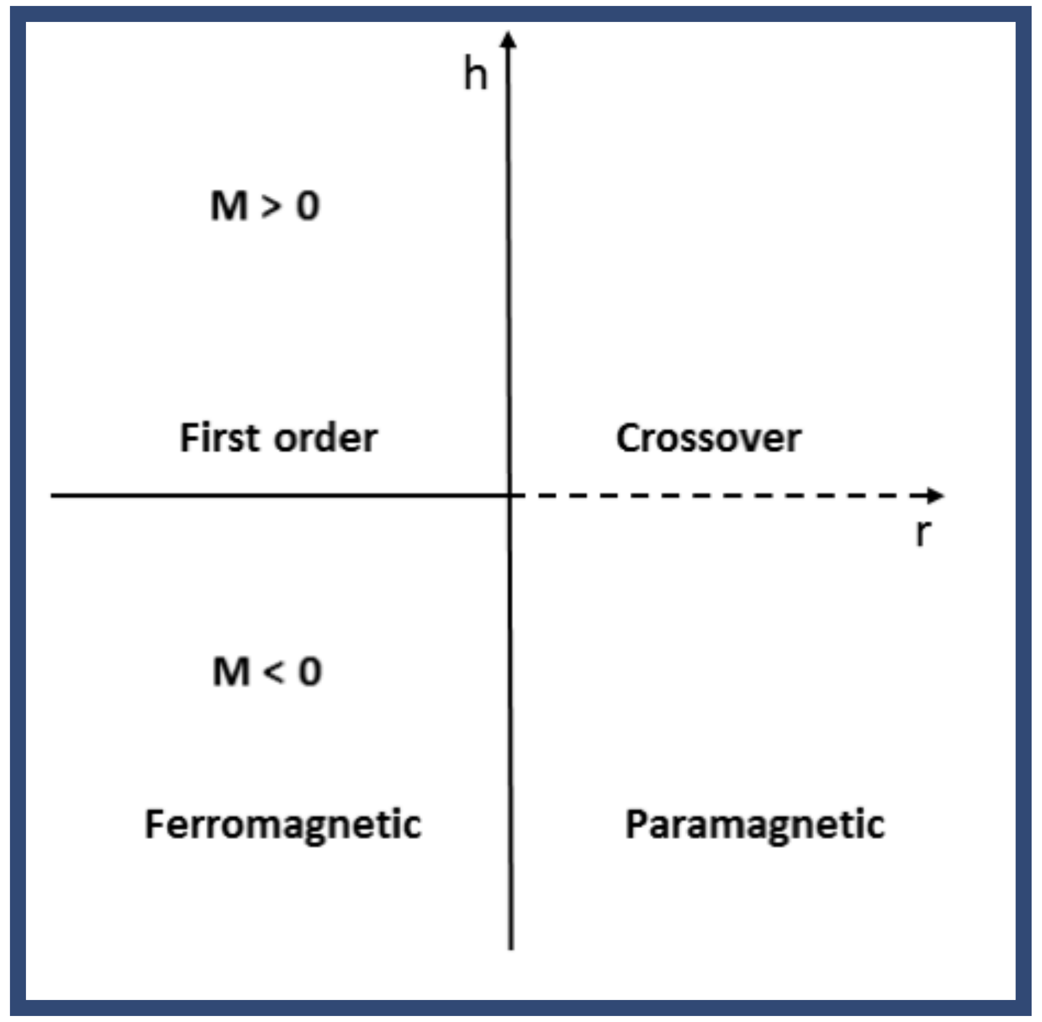
EoS – the recipe

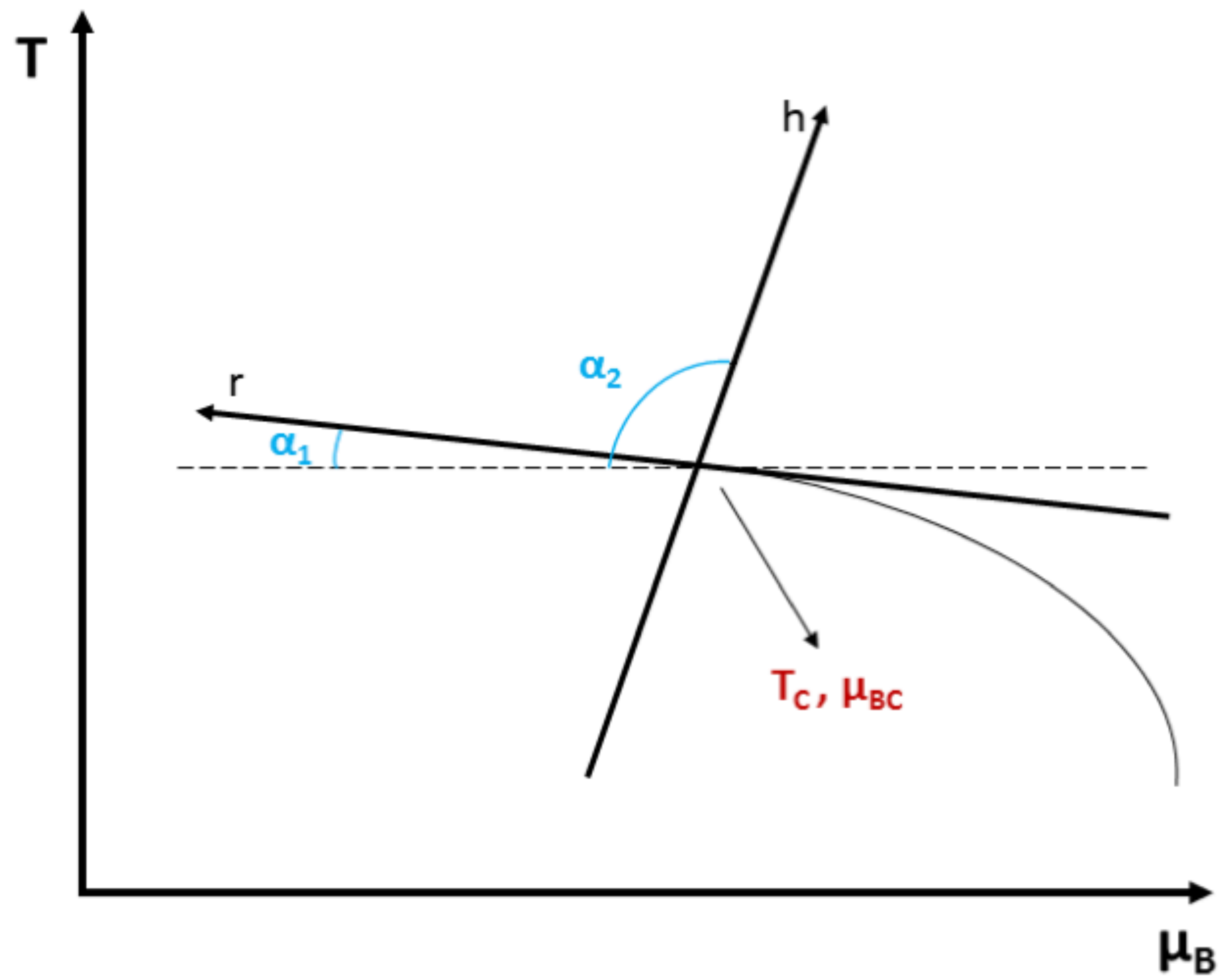
P. Parotto, DM et al. Phys. Rev. C (2020)

1. Critical point from correct universality class -> introduces new set of variables
2. Match new set of variables to QCD variables -> introduces free parameters
3. Use information from LQCD to reduce number of free parameters -> complete model
4. Obtain Taylor expansion coefficients from model
5. Reconstruct the full pressure from Taylor expansion
6. Merge with Hadron Resonance Gas regime (lower T)
7. Obtain thermodynamics from pressure

1. “Obtaining” a CP

- QCD expected to be in the same universality class as the 3D Ising model
- “Borrow” the critical region from Ising phase diagram





- Cannot be solved analytically, requires a non-universal parameterization (R, θ)

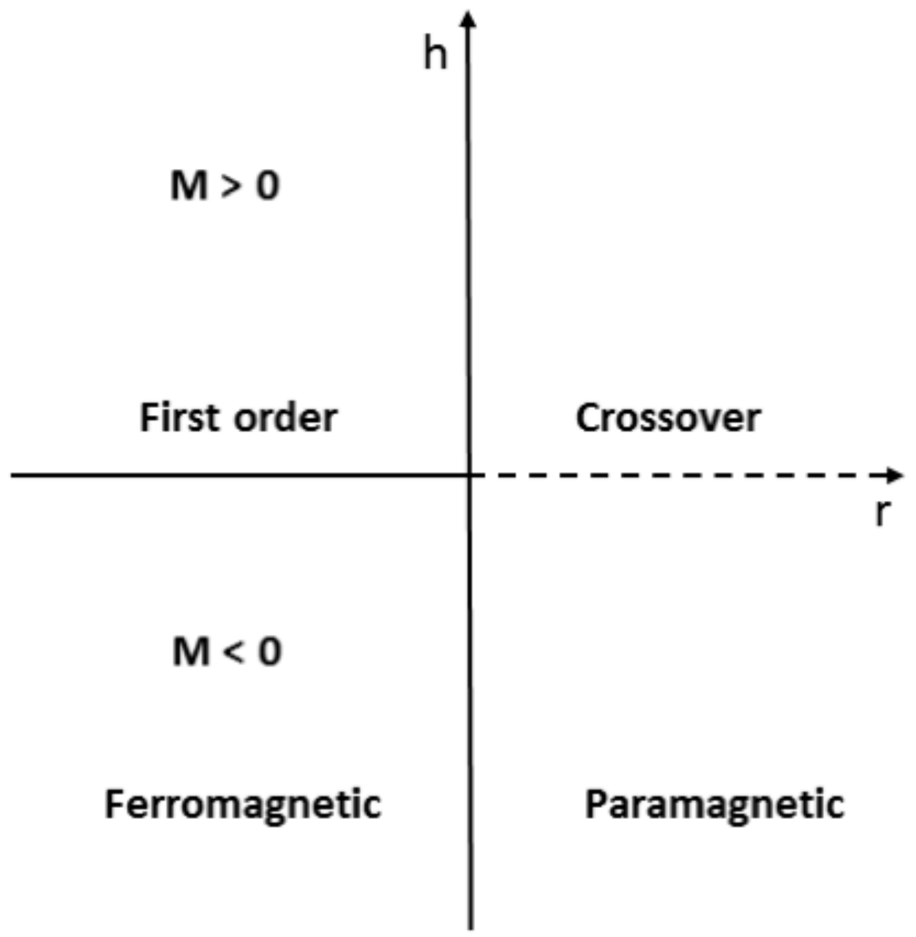
$$M = M_0 R^{\beta} \theta$$
$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$
$$r = R(1 - \theta^2)$$

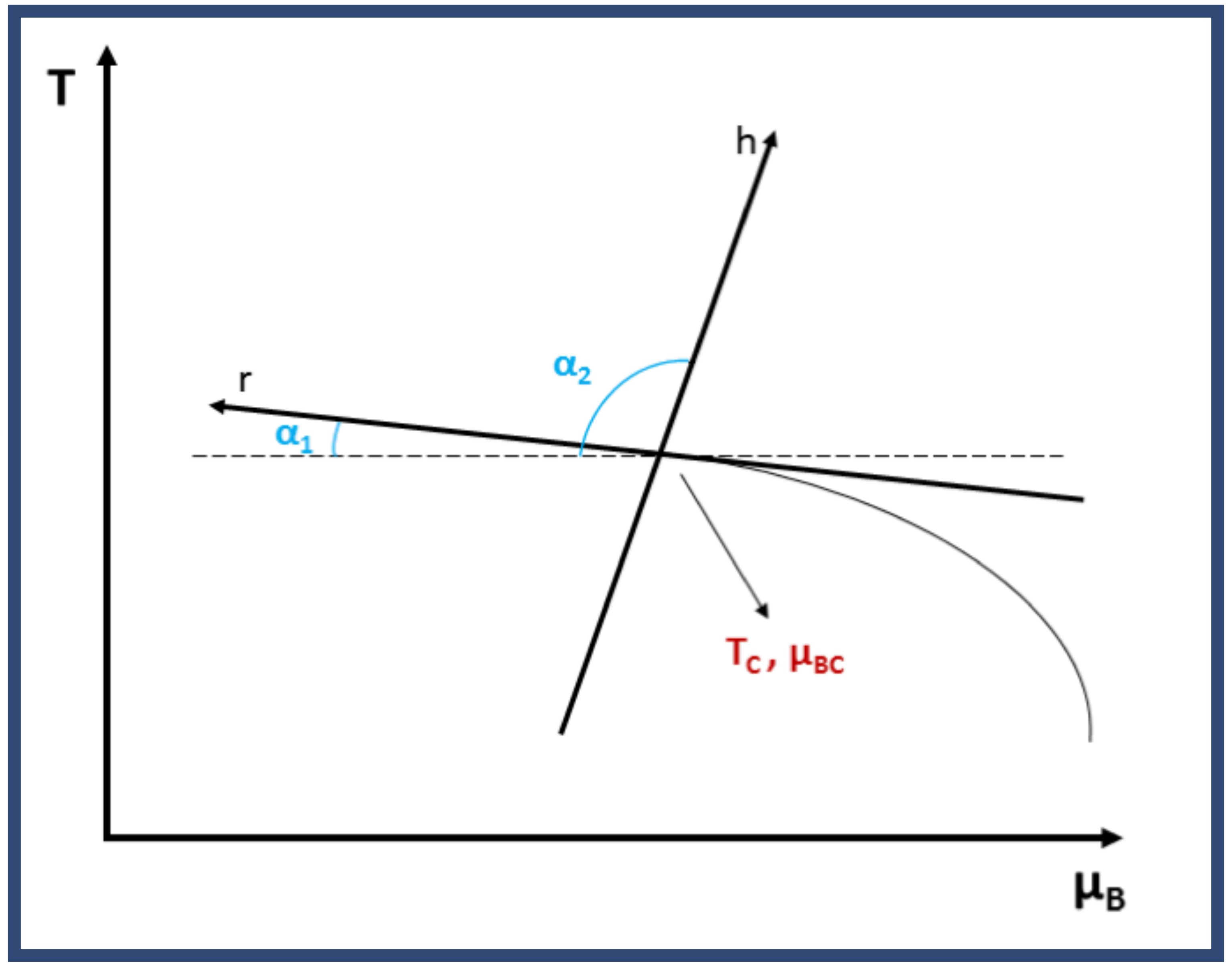
$$M_0 \simeq 0.605 \quad h_0 \simeq 0.364$$
$$\tilde{h}(\theta) = \theta(1 + a\theta^2 + b\theta^4)$$
$$\beta \simeq 0.326, \quad \delta \simeq 4.80$$

C. Nonaka, M. Asakawa, Phys. Rev C (2005)

2. Mapping Ising → QCD

- Size and shape of critical region are **unknown features of QCD**
- Mapping the Ising phase diagram to the QCD one introduces free parameters





$$(\mathbf{r},\mathbf{h}) \longleftrightarrow (\mathbf{T},\mu_\mathbf{B}): \quad \frac{T-\mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B-\mu_{\mathbf{BC}}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

- position of CP: (T_C, μ_{BC})
- angular parameters: α_1, α_2
- scaling parameters: w, ρ

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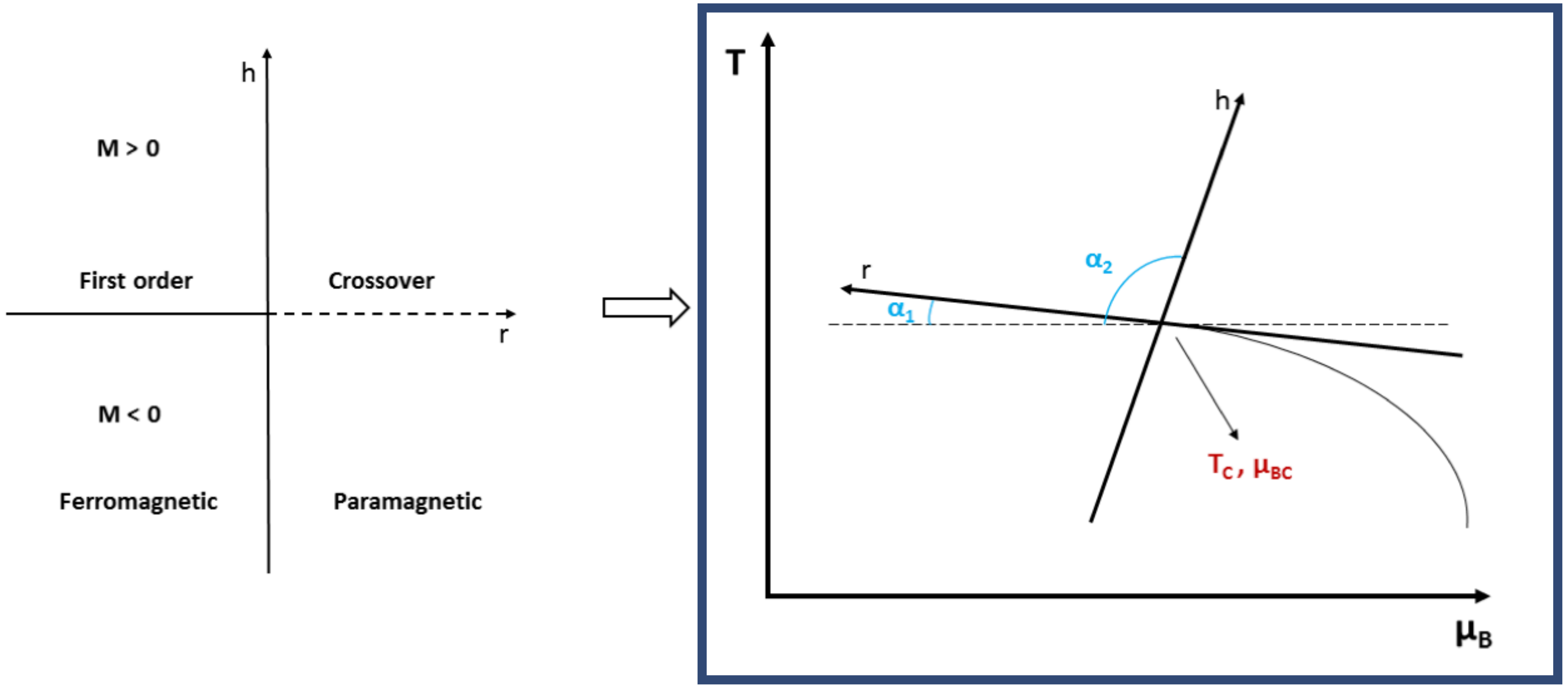

CP in correct universality class ✓

$$(\mathbf{r}, \mathbf{h}) \longleftrightarrow (\mathbf{T}, \mu_\mathbf{B}) : \quad \frac{T - \mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho \,\sin\alpha_1 + h\,\sin\alpha_2)$$

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- position of CP: (T_C, μ_{BC})
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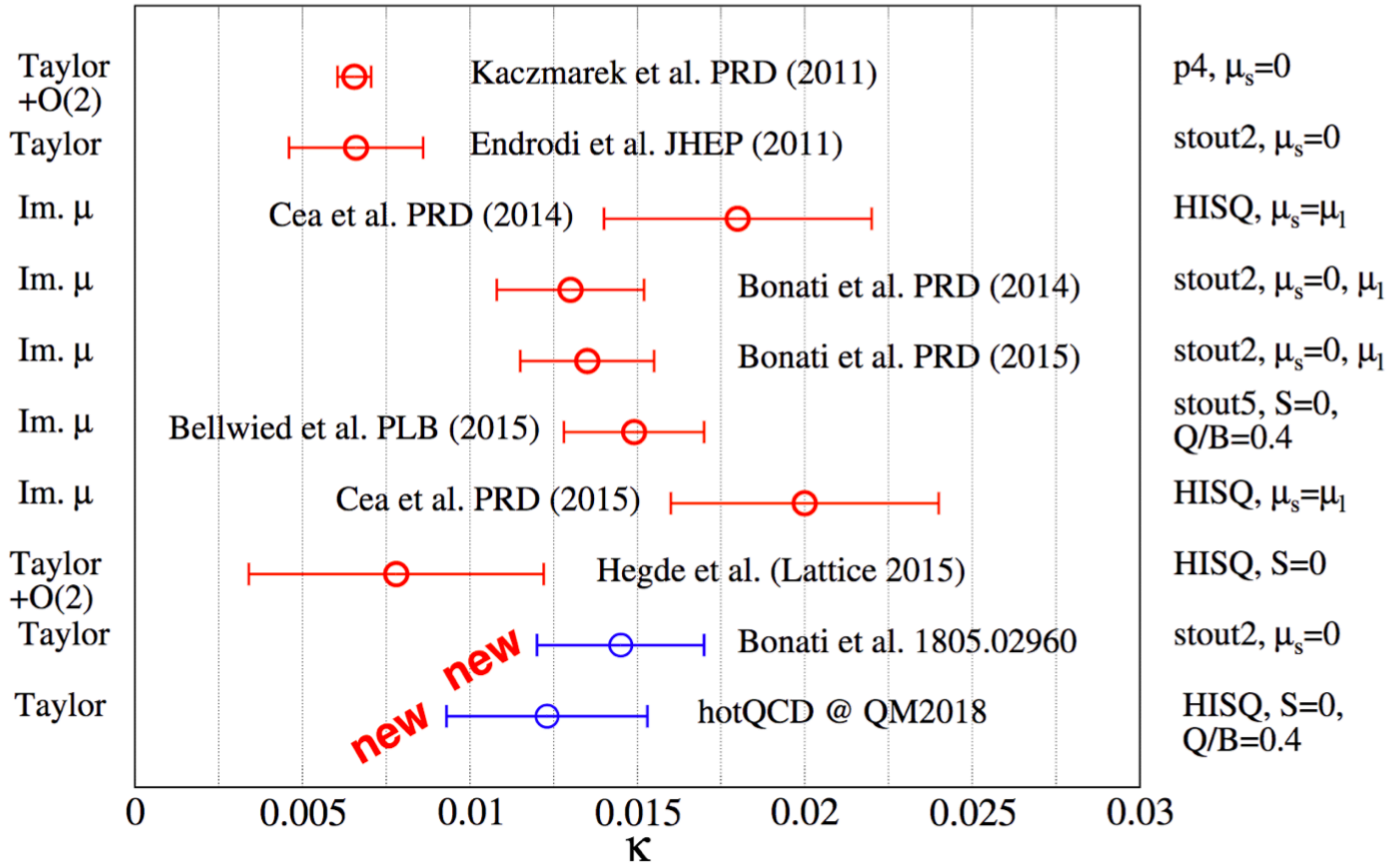
3. Reducing number of free parameters

- Assume transition line follows a parabola with curvature κ



C. Bonati et al. 1807.10026

$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4);$$

- Several calculations, largely consistent within error

- Use $\kappa = -0.0149$ from Bellwied et al.

- From geometry of parabola

$$\alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

6 parameters $\rightarrow$ 4 parameters

4 & 5. Build Taylor expansion

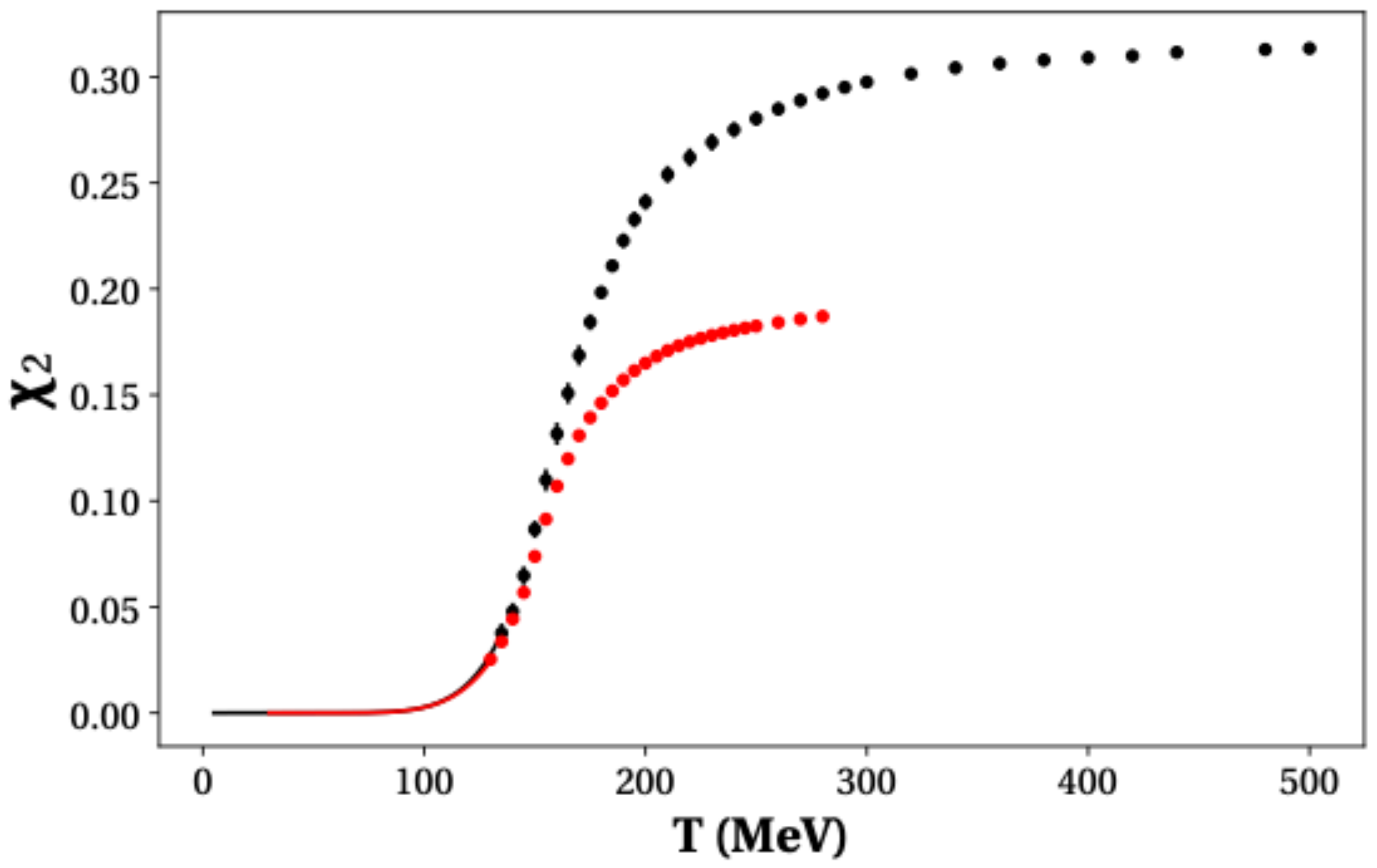
- Pressure given by Taylor expansion around $\mu_B = 0$

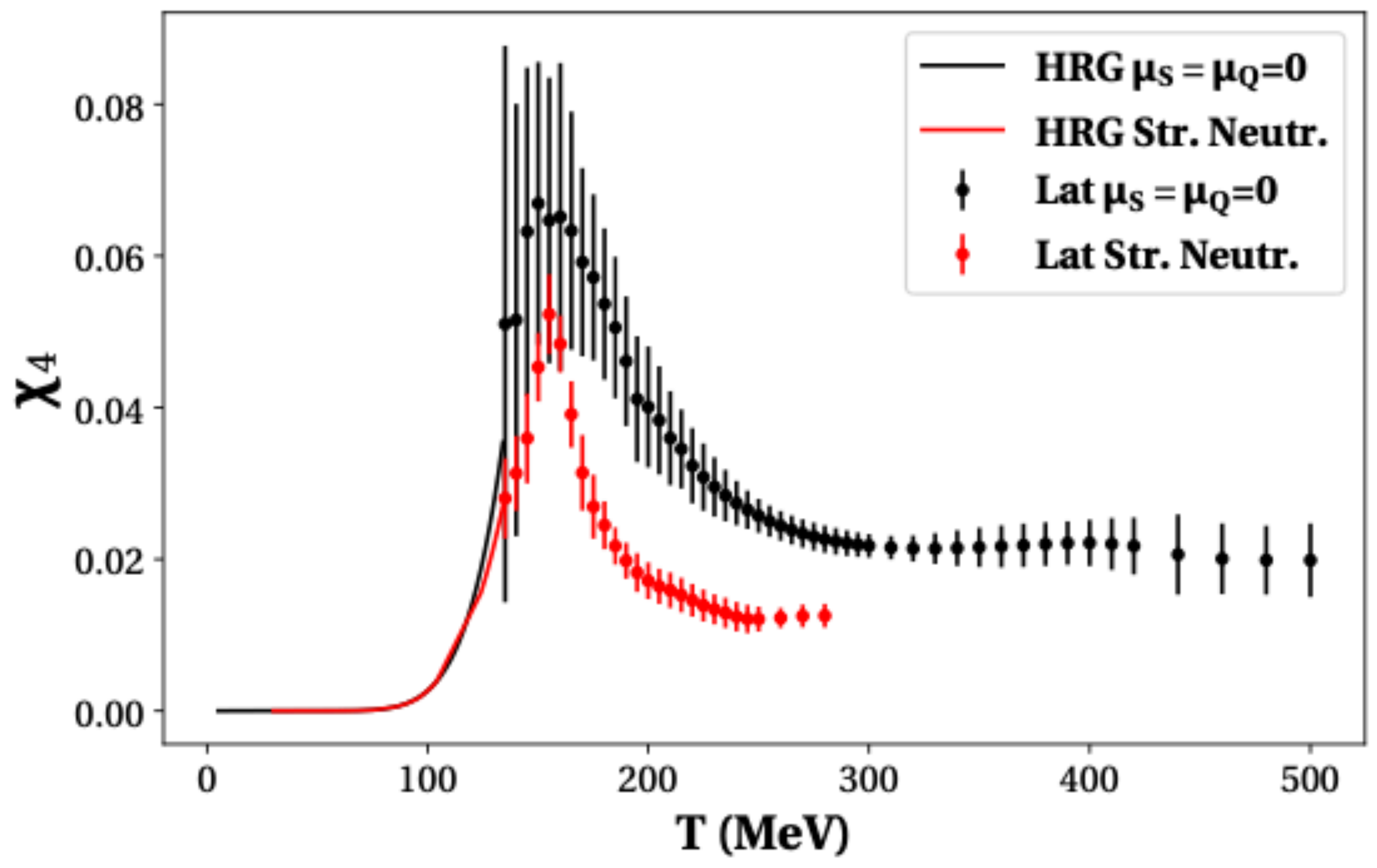
$$P(T,\mu_B) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

$$c_n(T) = \frac{1}{n!} \frac{\partial^n P/T^4}{\partial(\mu_B/T)^n}\bigg|_{\mu_B=0} = \frac{1}{n!}\chi_n(T).$$

$$T^4 c_n^{\mathrm{LAT}}(T) = T^4 c_n^{\mathrm{Non-Ising}}(T) + c_n^{\mathrm{Ising}}(T)$$

- Coefficients are proportional to the baryon susceptibilities → directly from LQCD & 3D Ising





- Matching to LQCD
- At this level, we can enforce (or not) correct **charge density constraints.**

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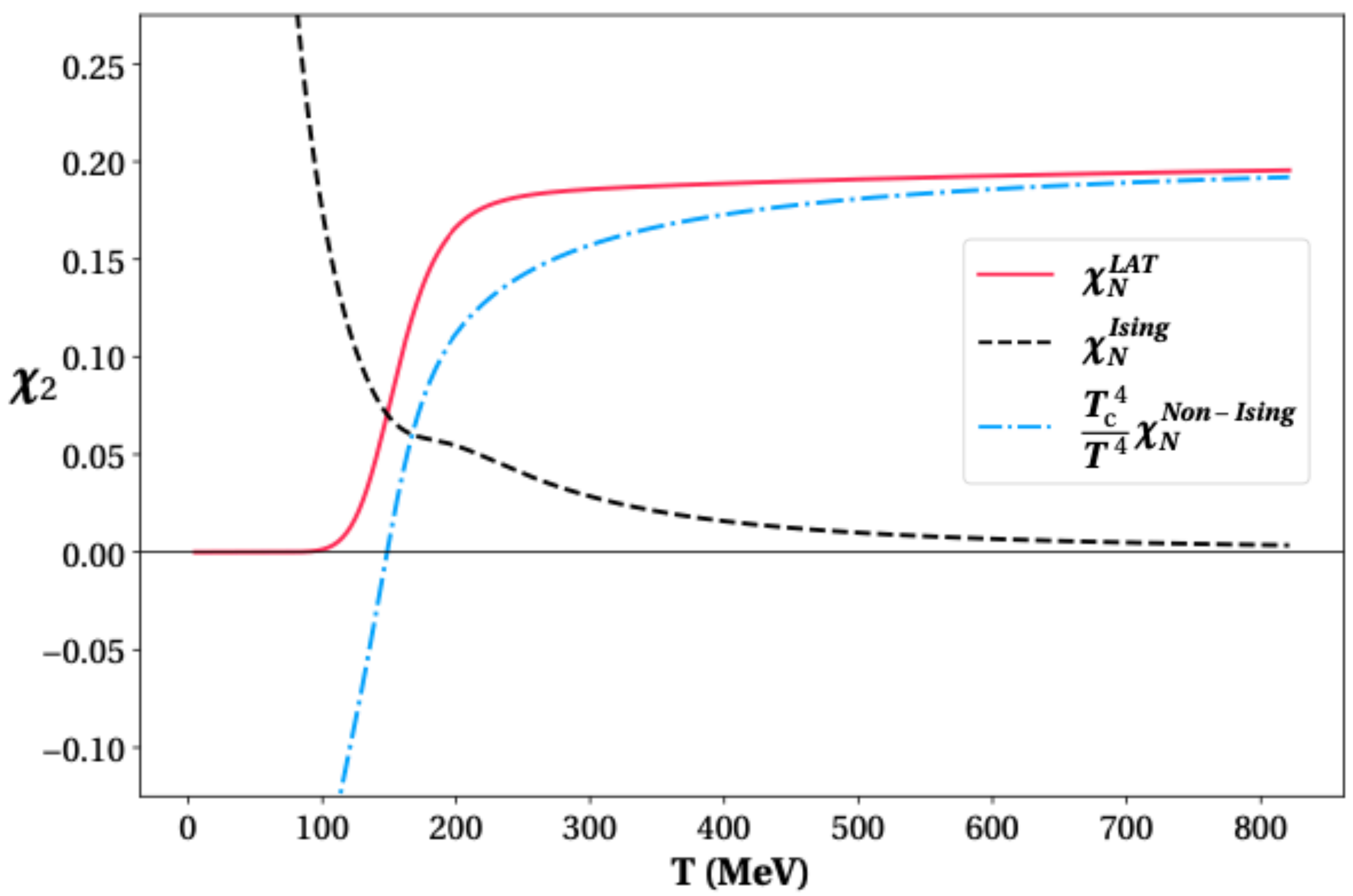
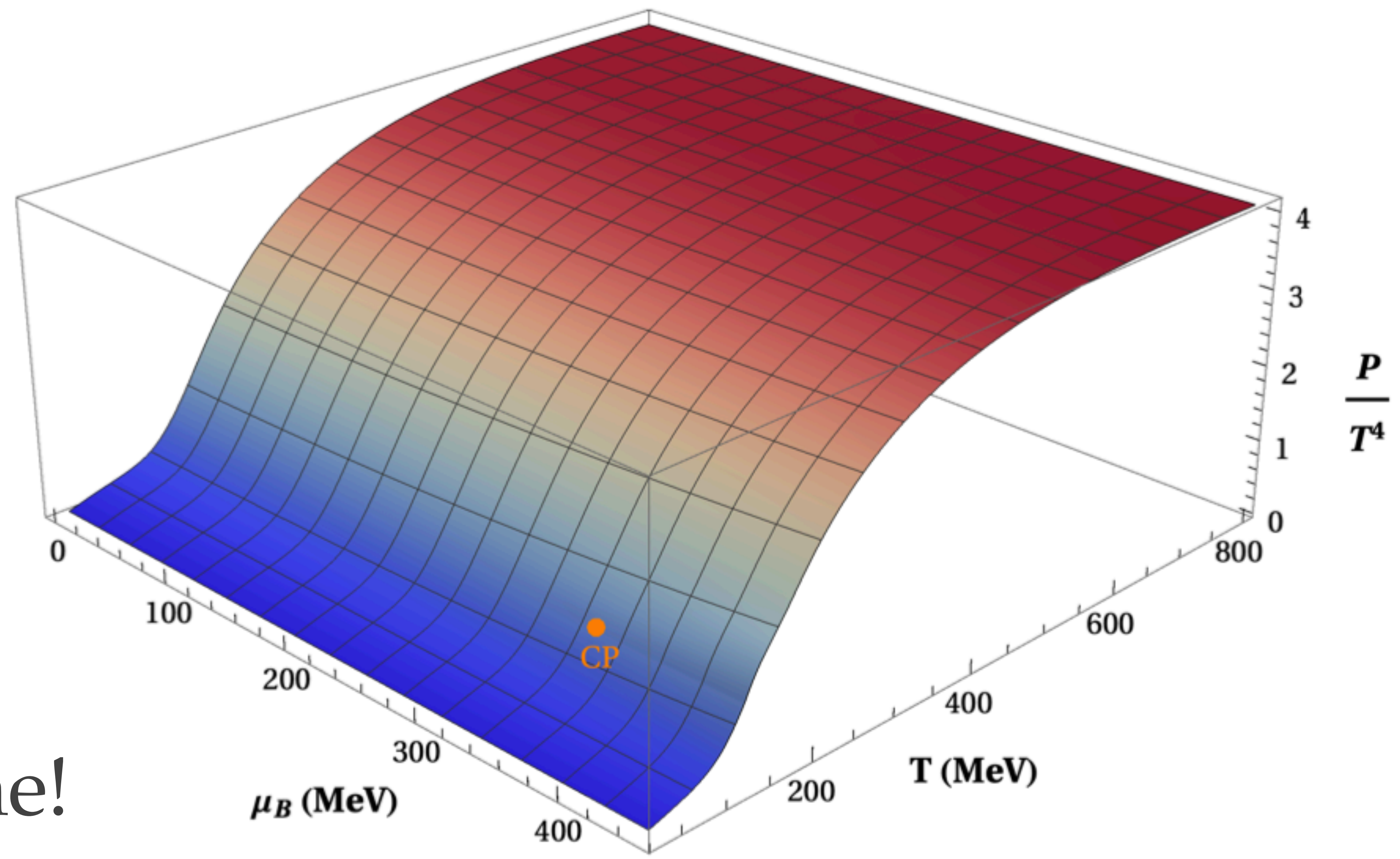


6. Merge with HRG at lower T

- At lower T, HRG is a suitable description of the system*
  - Smooth merging via hyperbolic tangent switching function

$$\frac{P_{\text{Final}}(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)}{T^4}\frac{1}{2}\left[1+\tanh\left(\frac{T-T'(\mu_B)}{\Delta T}\right)\right]$$
$$+\frac{P_{HRG}(T,\mu_B)}{T^4}\frac{1}{2}\left[1-\tanh\left(\frac{T-T'(\mu_B)}{\Delta T}\right)\right]$$



Done!

* Charge constraints must be consistent

7. Thermodynamics!

- All other thermodynamic quantities can be obtained from **derivatives of the pressure + thermodynamic relations**

$$\frac{n_B}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial \mu_B}\right)_T$$

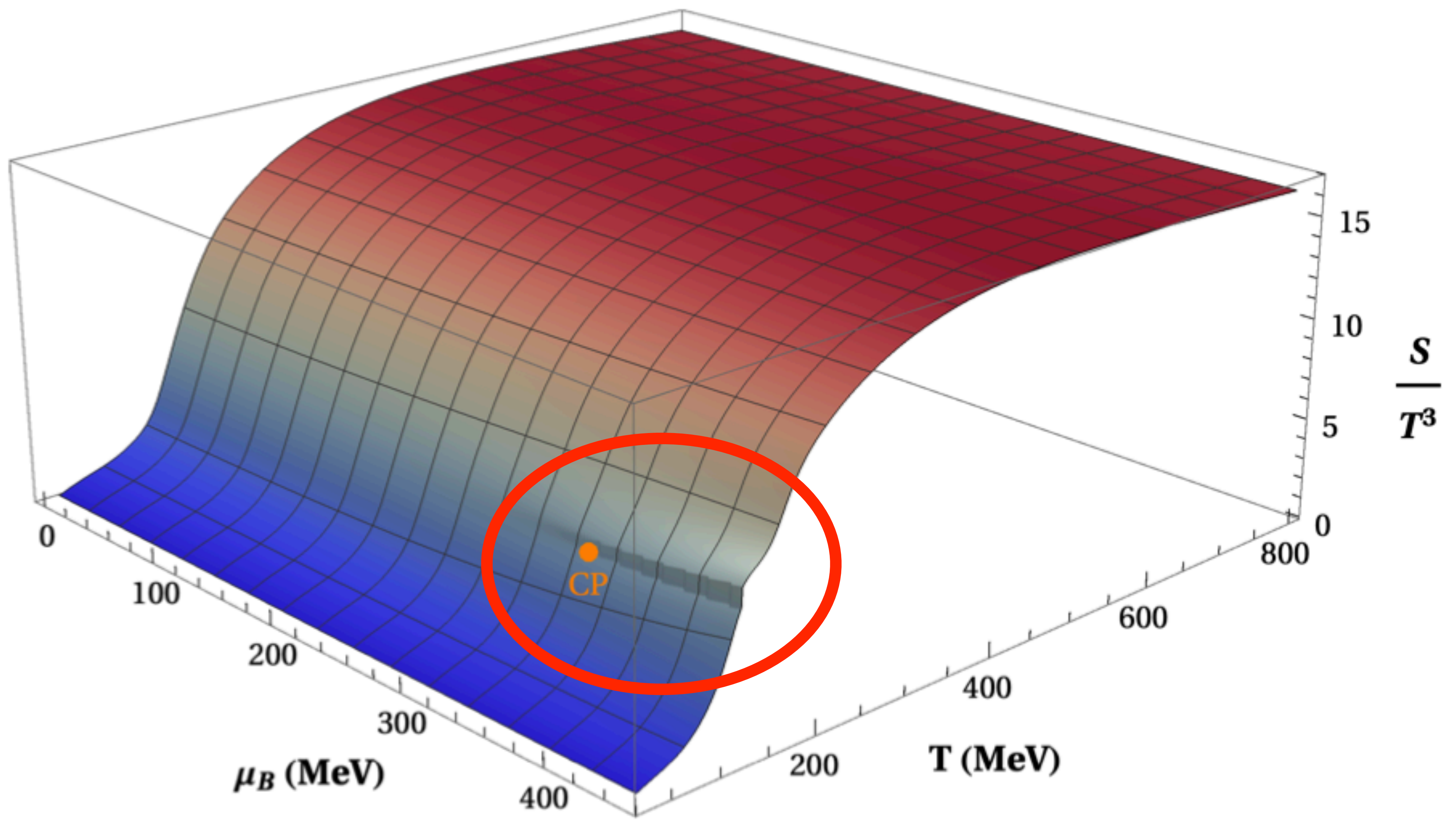
$$\boxed{\frac{S}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial T}\right)_{\mu_B}}$$

$$\frac{\chi_2^B}{T^2} = \frac{1}{T^2}\left(\frac{\partial^2 P}{\partial \mu_B^2}\right)_T$$

$$\frac{\epsilon}{T^4} = \frac{S}{T^3} - \frac{P}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3}$$

$$c_S^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_{S/n_B}$$

- First order derivatives display a kink near CP



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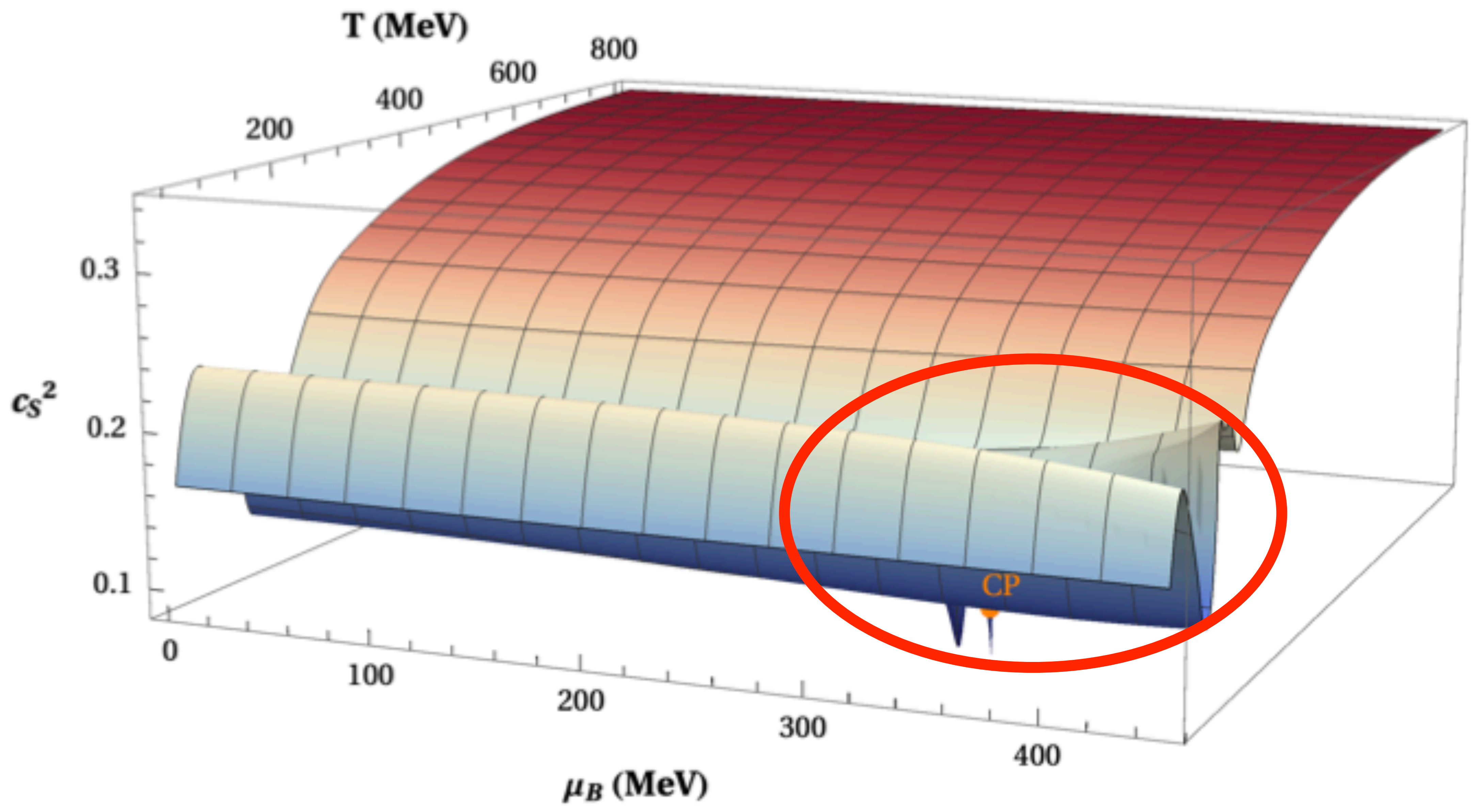
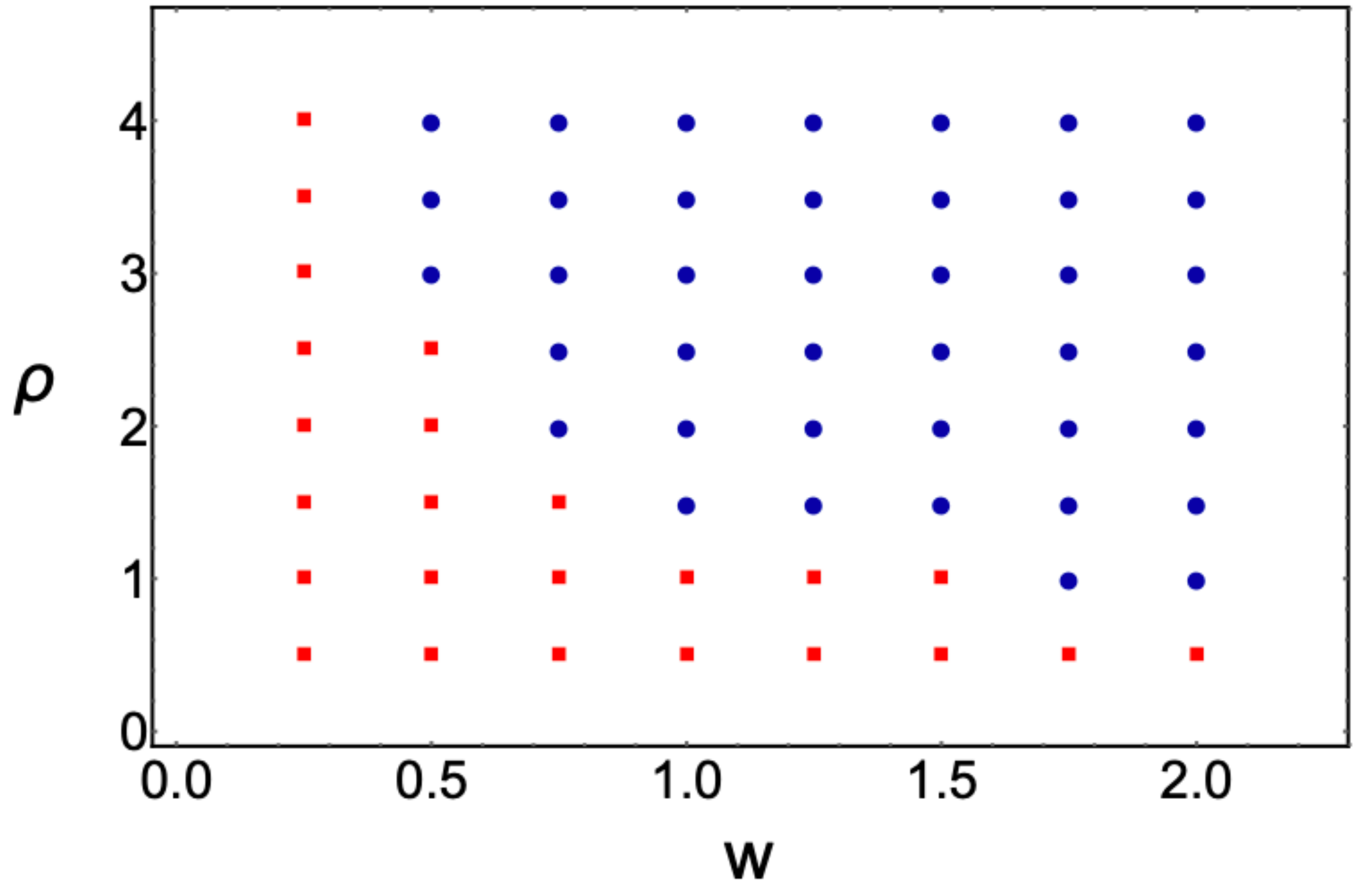
- Effect is enhanced in second-order derivatives



8*. Explore parameter space

- Ising → QCD map introduces 4 free parameters
- **Thermodynamic stability is not guaranteed**



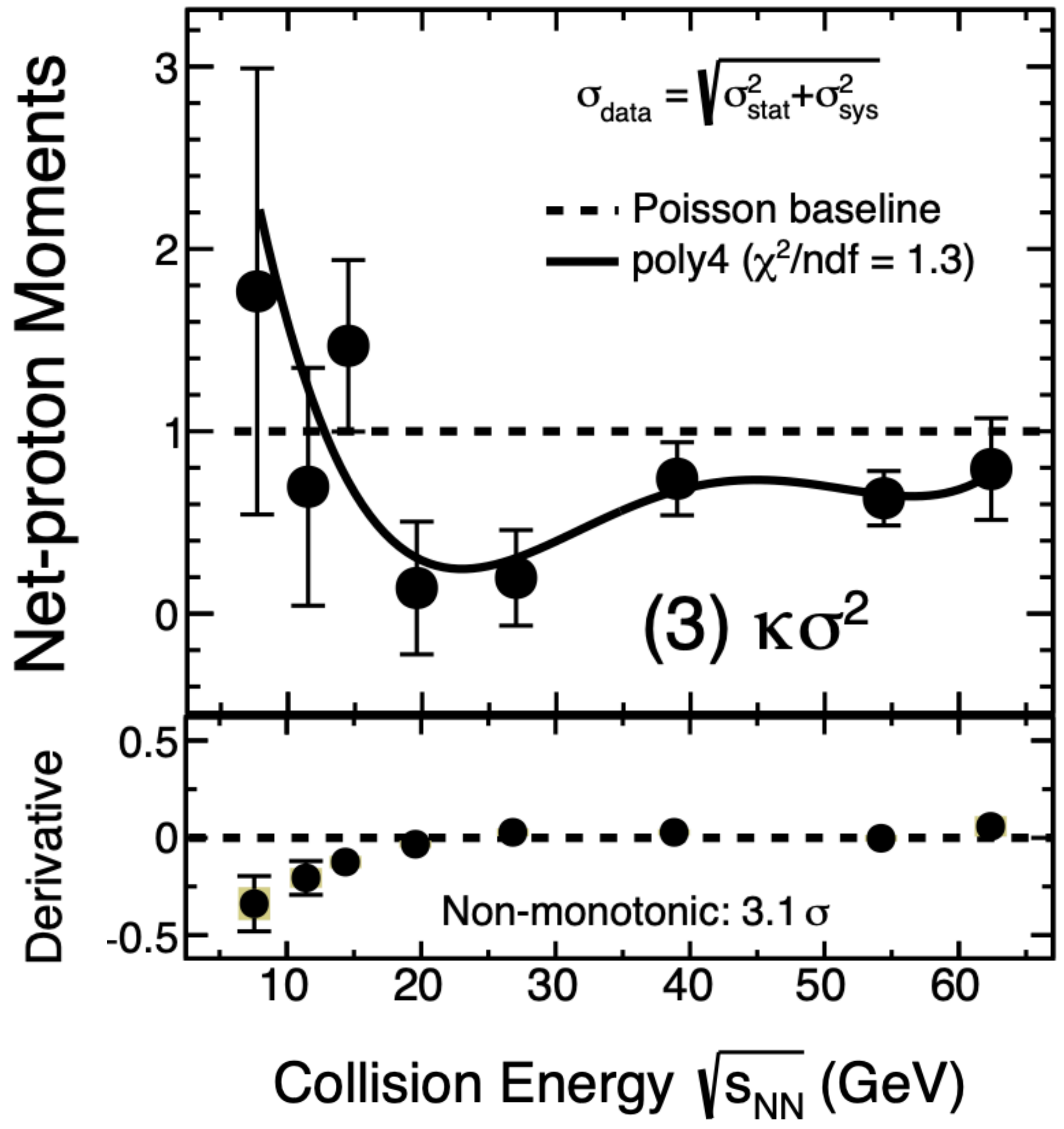
Fix CP at

$T_C = 143.5, \quad \mu_{BC} = 350$ MeV

stable / unstable

- Imposes constraints on size and shape of the critical region

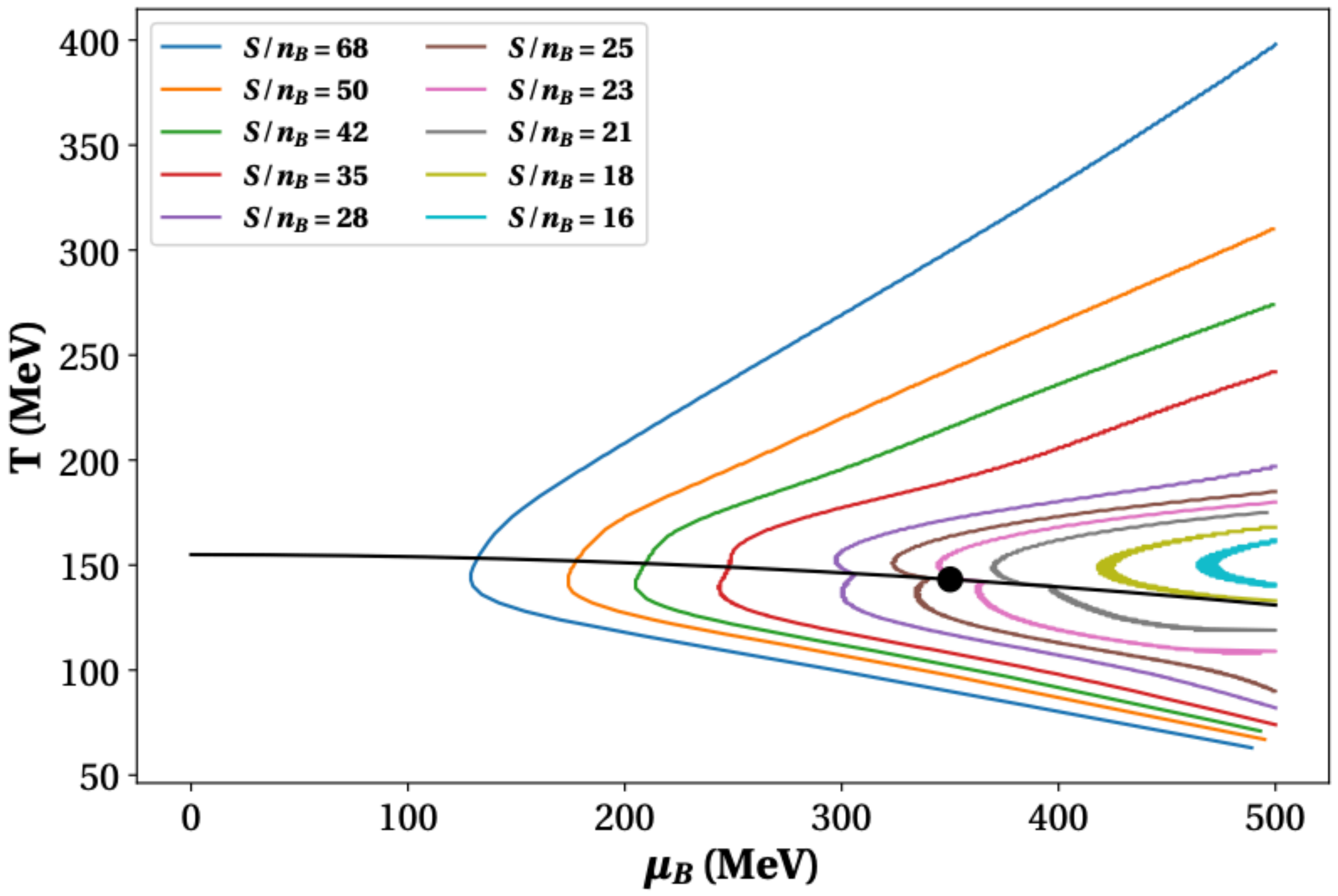
Central Au+Au - STAR (2021): 2101.12413

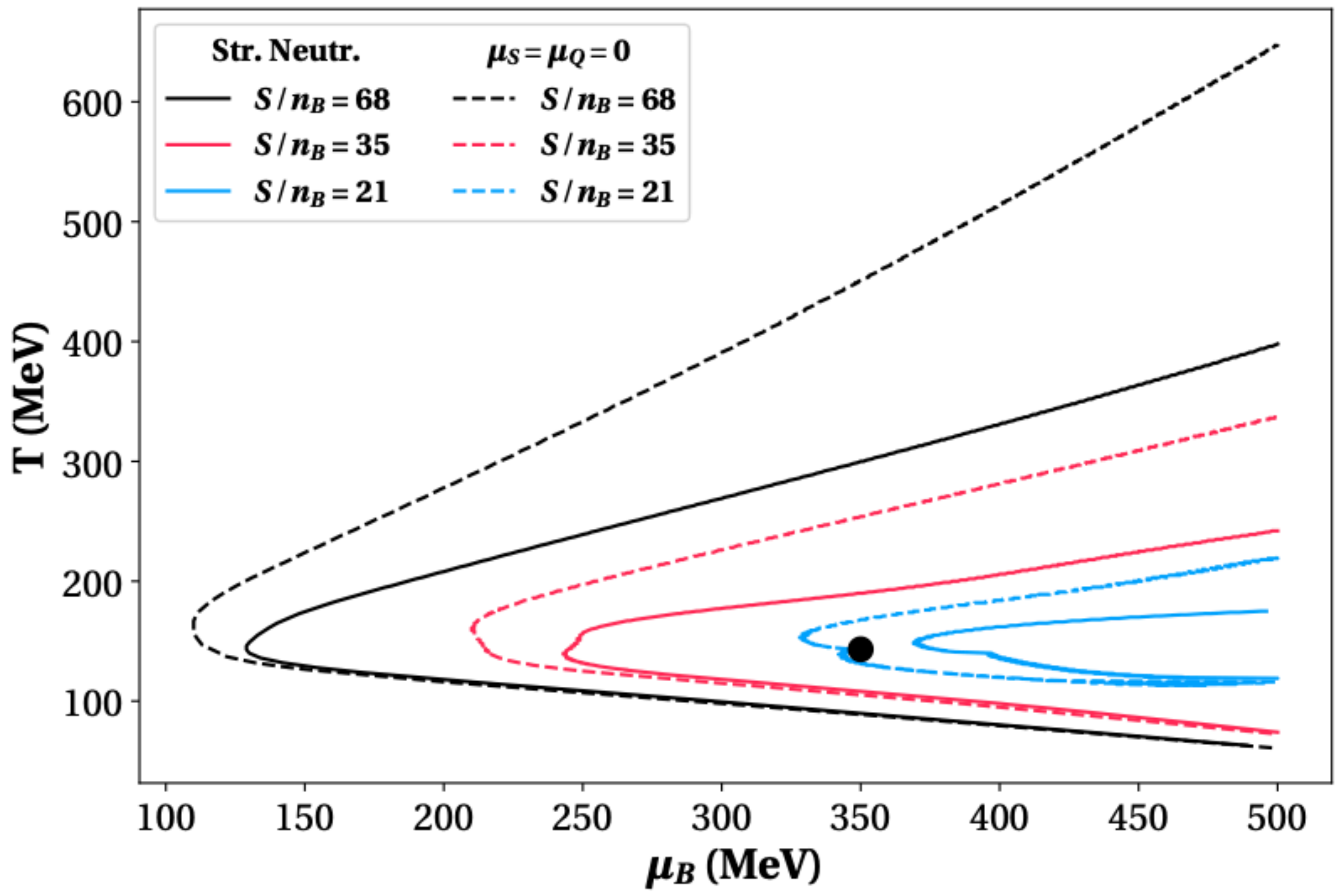


Relevant for experimental searches of CP (e.g. BES-II)

Isentropic paths

- Paths probed by the system in the absence of dissipation





- Strangeness neutrality plays important role

Conclusions and considerations

- This procedure guarantees:
  - i) strangeness neutrality and fixed baryon-number-to-electric charge ratio
  - ii) matches LCQD at $\mu_B = 0$
  - ii) CP in correct universality class
- Ready to be implemented in hydro calculations
- Current study: **equilibrium** properties of QCD EoS. HIC are dynamical systems, **need EbE relativistic viscous BSQ hydro evolution + critical fluctuations + hadronic transport.**