Collective Behavior in Rarely-Interacting Systems

Time-Dependent Anisotropic Flow Coefficients

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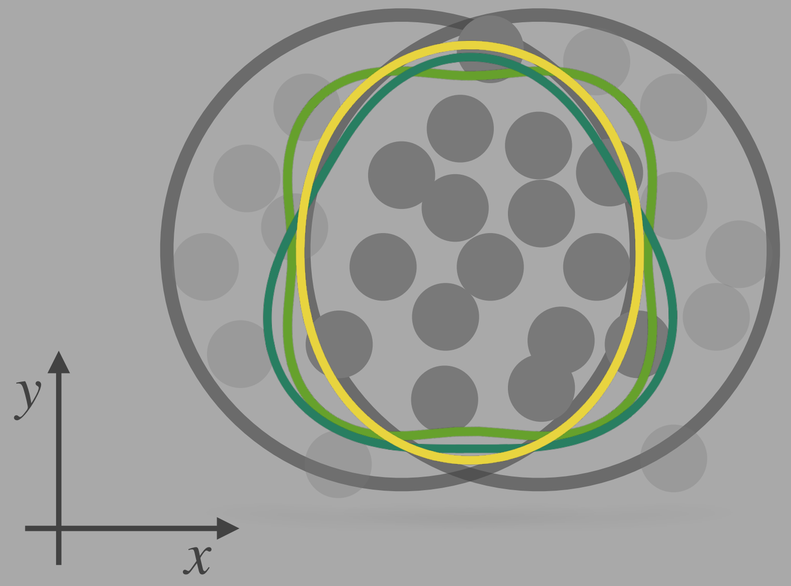
in collaboration with
Nicolas Borghini & Hendrik Roch

6 Khordad 1400 (May 27, 2021) Workshop on QGP Phenomenology Sharif University of Technology

Motivation

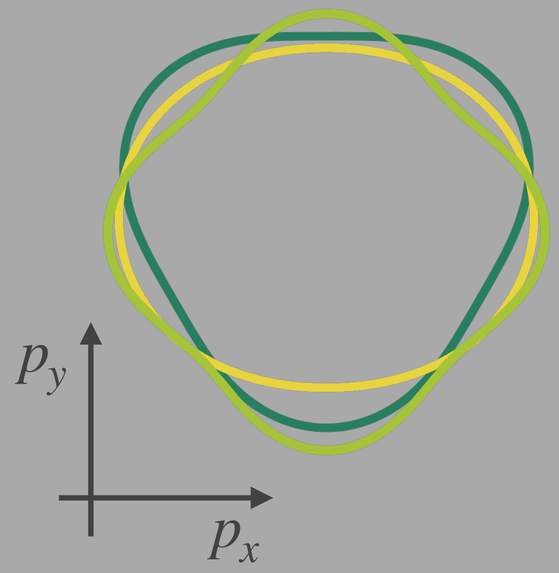
Link between Initial Eccentricities and Anisotropic Flow Coefficients

Initial state
Eccentricities
Position space



ε_2 ε_3 ε_4 . . .

Final state
Anisotropies
Momentum space



v_2

v_3

v_4

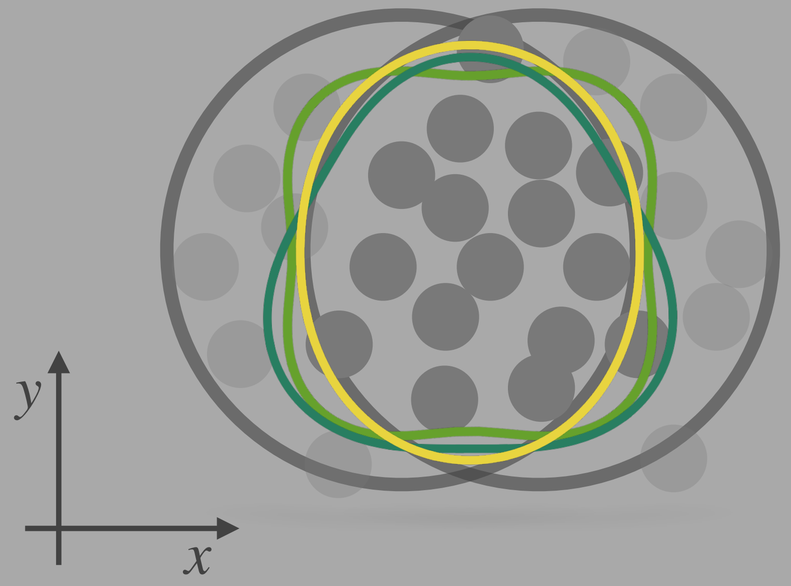
⋮

Link between Initial Eccentricities and Anisotropic Flow Coefficients

Initial state
Eccentricities
Position space

Theoretical description

Final state
Anisotropies
Momentum space



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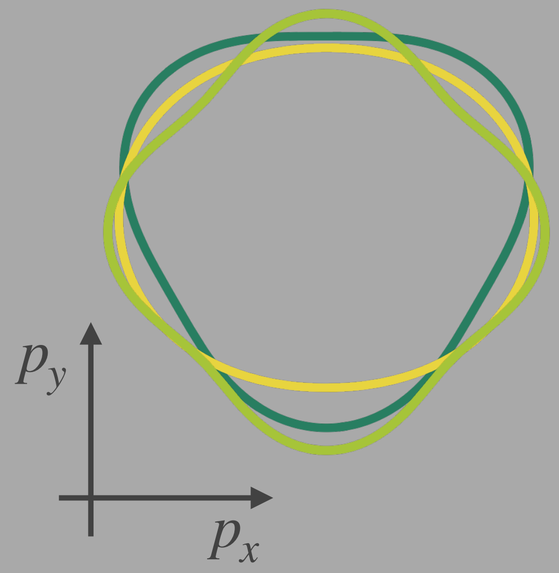

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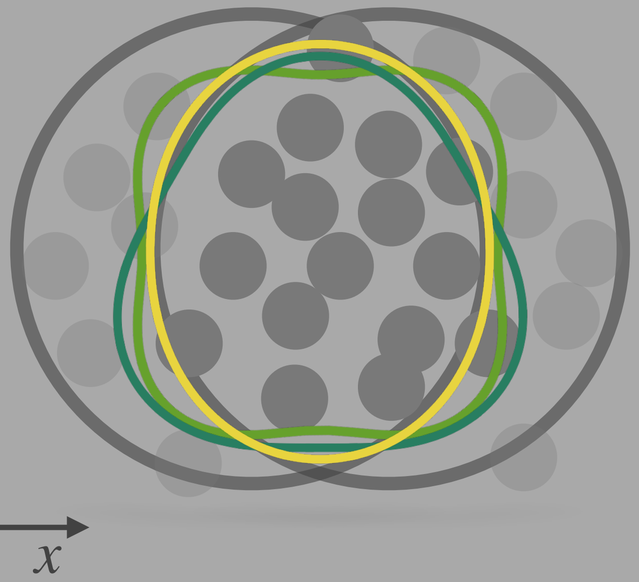

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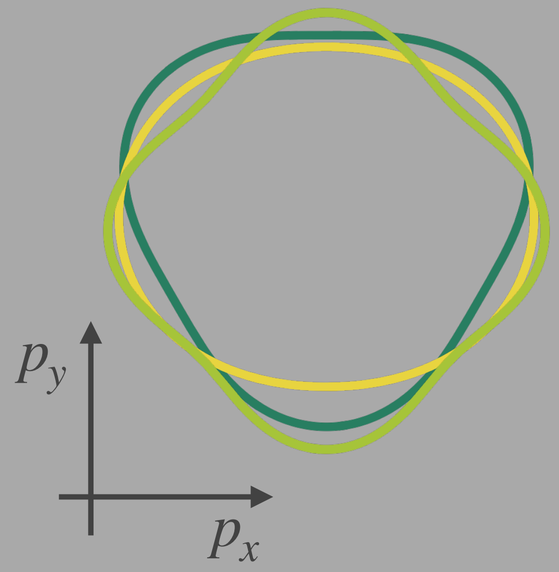
Hydrodynamics
Collective behavior
$N_{\text{resc}} \gg 1$

Kinetic theory
Few collision regime
$N_{\text{resc}} < 1$



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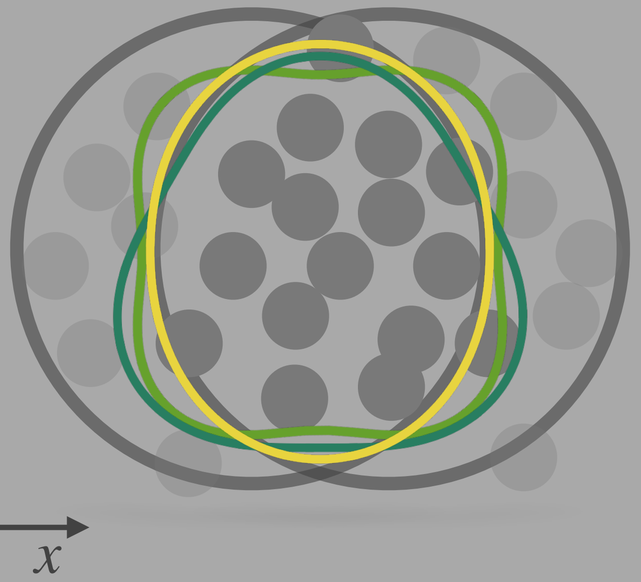

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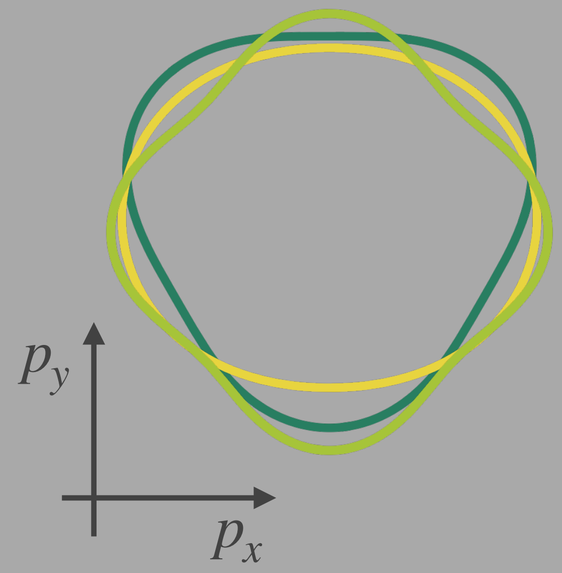
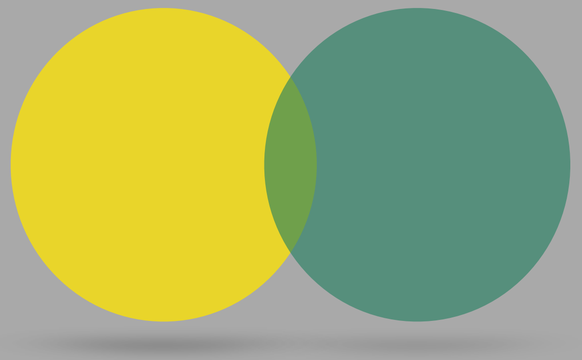


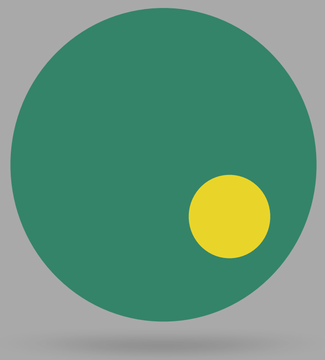
v_2
v_3
v_4
⋮

Reason for Kinetic Theory Approach

Small and dilute systems



ultra-peripheral
$A + A$



$A + p$

$p + p$

Leading Question

How do anisotropic flow coefficients v_n evolve in time with dependence on the various initial eccentricities ε_m?

Methods

Analytical Approach

to Calculate Time-Dependent v_n

Analytical model

Calculate time-dependent anisotropic flow coefficients

$$v_n\left(t, p_\perp\right) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}{\iint f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}$$

Analytical Approach

to Calculate Time-Dependent v_n

- Calculate time-dependent AFC

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Analytical model

↓

Loss term in Boltzmann equation

Use classical relativistic Boltzmann equation as equation of motion

$$\left[\partial_t + \mathbf{v} \cdot \nabla\right] f\left(t, \mathbf{x}, \mathbf{p}\right) = C_{coll}\left[f\left(t, \mathbf{x}, \mathbf{p}\right)\right]$$

with loss term of 2-to-2 elastic collision kernel

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to Calculate Time-Dependent v_n

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$$\mathcal{O}\left(N_{\text{resc}}\right) = 1$$

Few collision limit

Dependence on free-streaming distribution function

$$f^{(0)}\left(t, \mathbf{x}, \mathbf{p}\right) = f^{(0)}\left(0, \mathbf{x} - t\mathbf{v}, \mathbf{p}\right)$$

Analytical Approach

to Calculate Time-Dependent v_n

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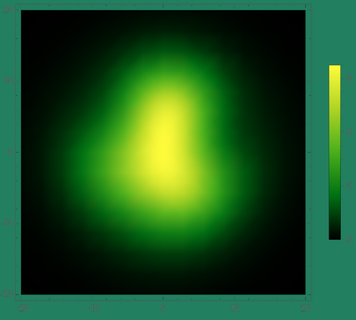
Analytical model

Loss term in Boltzmann equation

$$\mathcal{O}\left(N_{\text{resc}}\right) = 1$$

Initial distribution function (including initial eccentricities) as input for my calculation



$$- t\mathbf{v}, \mathbf{p}\Big)$$

Analytical Approach

to Calculate Time-Dependent v_n

- Calculate time-dependent AFC

$$\mathrm{v}_n\left(t, p_\perp\right) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}{\iint f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}$$

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- Few collision limit
- Dependence on free-streaming distribution function $f^{(0)}\left(t, \mathbf{x}, \mathbf{p}\right) = f^{(0)}\left(0, \mathbf{x} - t\mathbf{v}, \mathbf{p}\right)$
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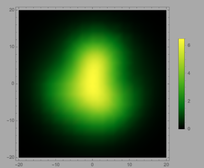

Analytical model

↓

Loss term in Boltzmann equation

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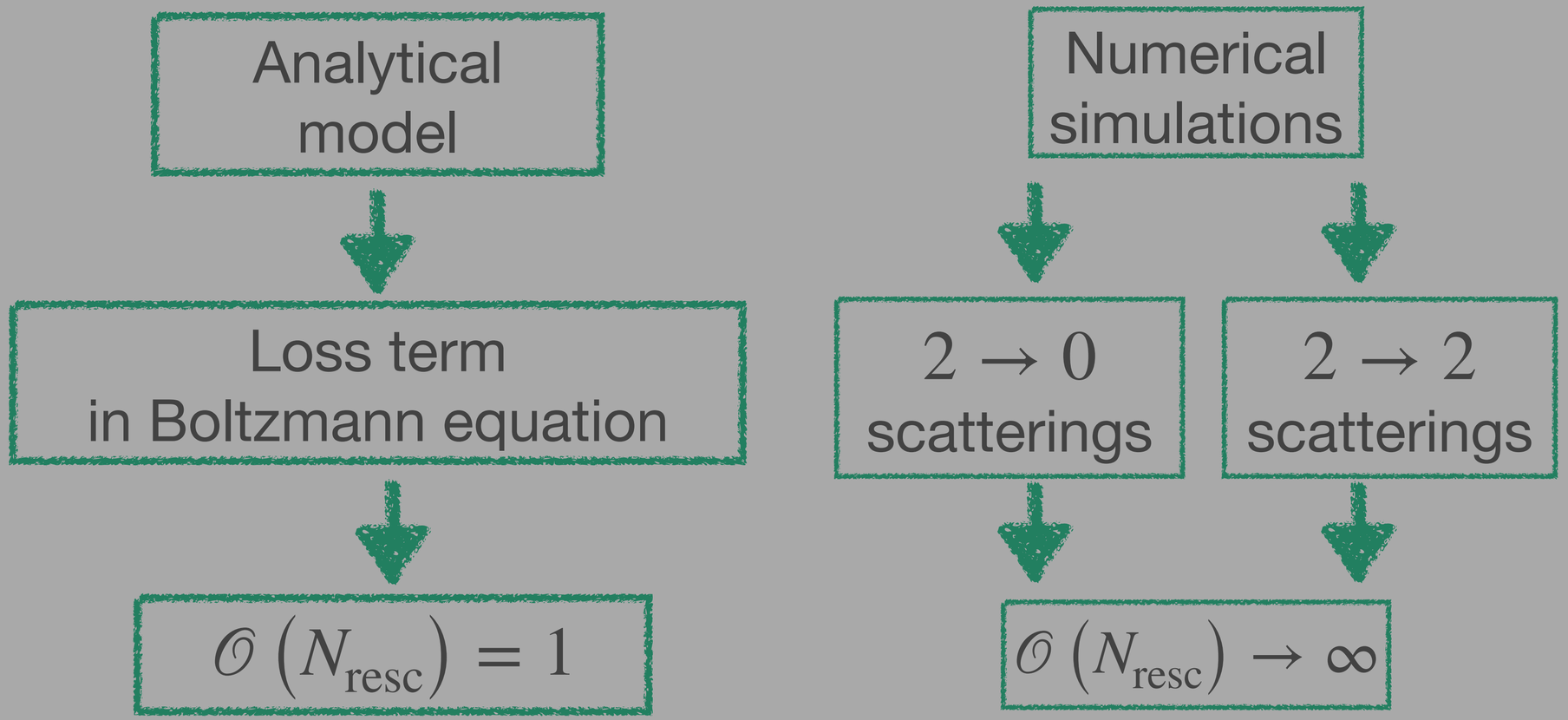
$\mathcal{O}\left(N_{\mathrm{resc}}\right) = 1$

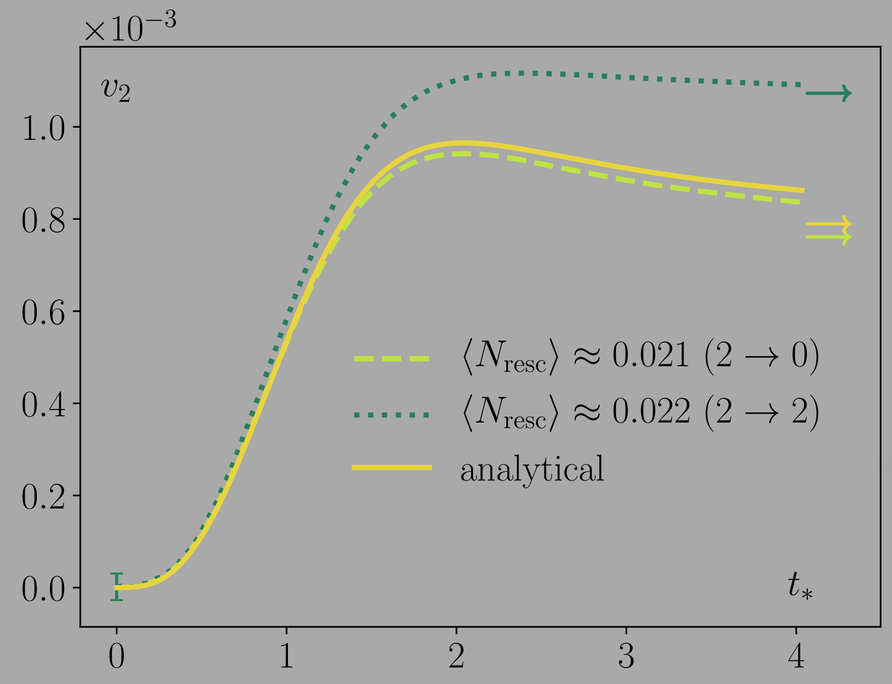
Our Approaches to Calculate v_n

- Kinetic theory
- 2 dimensional
- Massless particles



Results

Time-Dependent v_2



Loss term dominates signal for $N_{\mathrm{resc}} \lesssim 0.35$

Higher orders in N_{resc} can be neglected in few collision regime

Deviation increases with growing N_{resc}

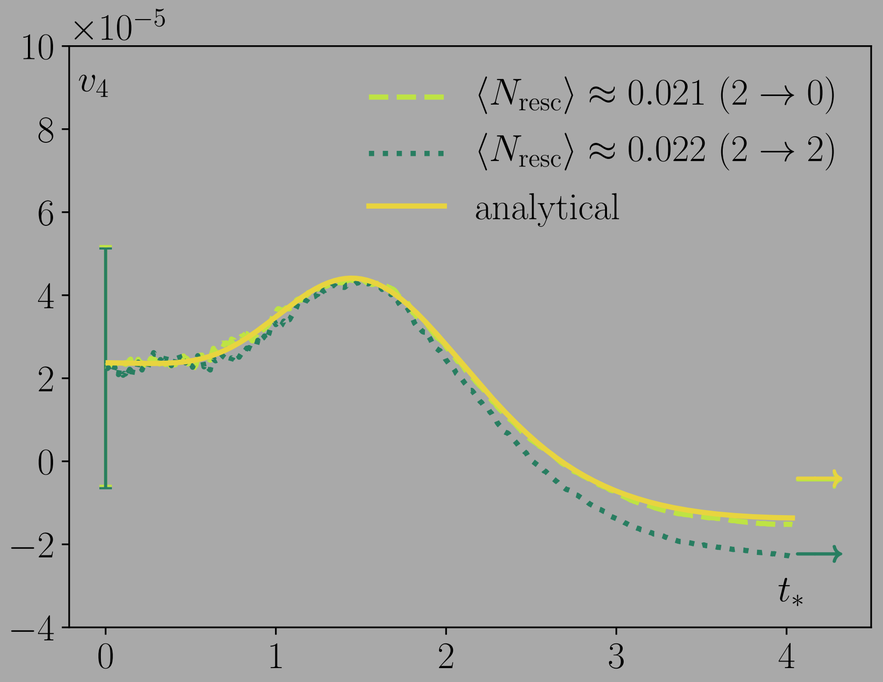
Good agreement for small N_{resc}

Expansion around small times $t \Rightarrow \mathrm{v}_2 \propto \varepsilon_2\, t^3$

- Analytical formula (with $t_* \equiv t/R$)

$$\mathrm{v}_2(t_*) = \frac{64\sqrt{\pi}\, N_{\mathrm{resc}}\, \varepsilon_2}{27\left(8 + 3\sqrt{2}\, \varepsilon_2^2\right)} e^{-\frac{2\, t_*^2}{3}} \left[-t_*\, I_0\left(\frac{2\, t_*^2}{3}\right) + \left(2\, t_* + \frac{3}{t_*}\right)\, I_1\left(\frac{2\, t_*^2}{3}\right) \right]$$

Time-Dependent $\mathrm{v}_4\left(\varepsilon_4\right)$



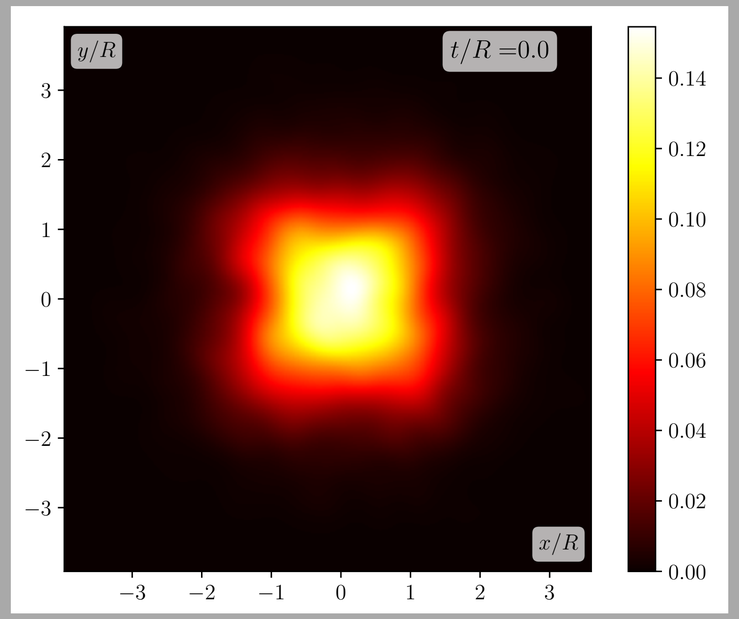
Good agreement for small N_{resc}

Expansion around small times $t \ \Rightarrow \mathrm{v}_4 \propto \varepsilon_4\, t^5$

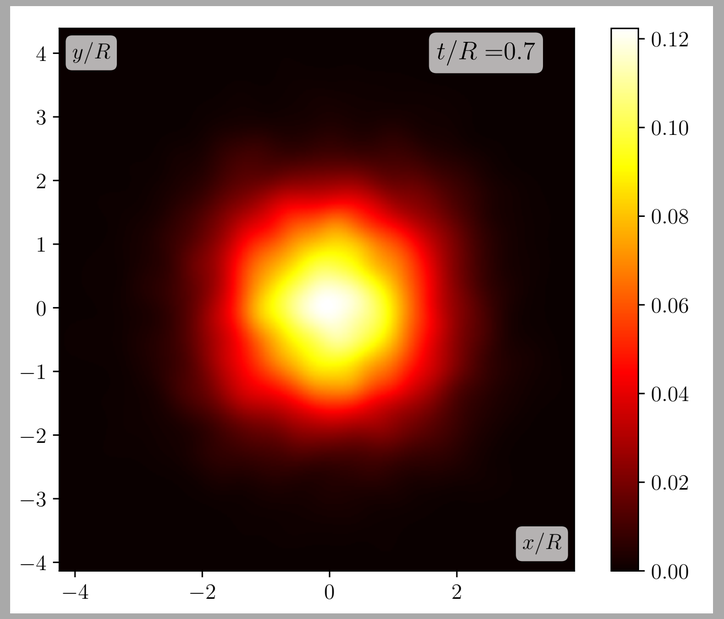
- Analytical formula (with $t_* \equiv t/R$)

$$\mathrm{v}_4\left(t_*\right) = \frac{2048\,\sqrt{\pi}\,N_{\mathrm{resc}}\,\varepsilon_4}{405\left(384+35\,\sqrt{2}\,\varepsilon_4^2\right)} e^{-\frac{2\,t_*^2}{3}} \left[-\left(5\,t_*^3+21\,t_*+\frac{54}{t_*}\right) I_0\left(\frac{2\,t_*^2}{3}\right)+\left(5\,t_*^3+24\,t_*+\frac{63}{t_*}+\frac{162}{t_*^3}\right) I_1\left(\frac{2\,t_*^2}{3}\right)\right]$$

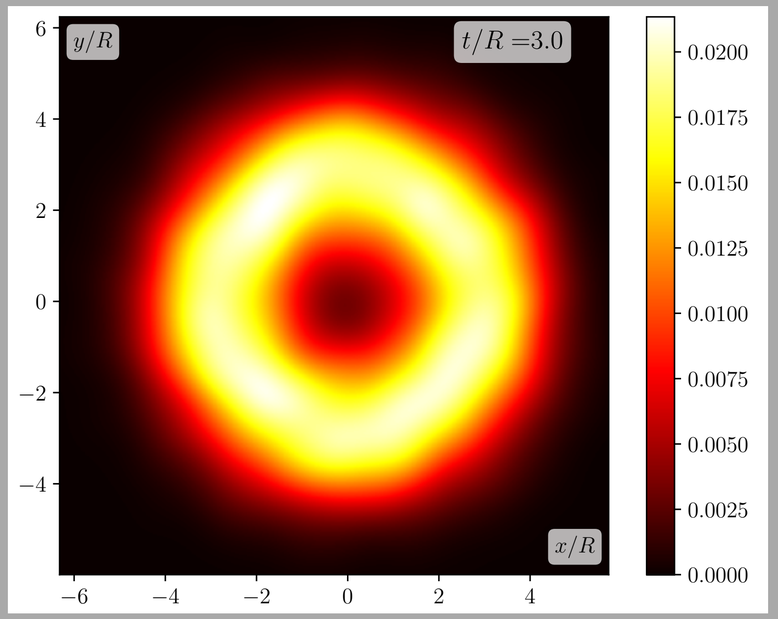
Density Plots from Numerical Simulations



- Initial eccentricity ε_4 only
- Bigger arrows ⇒ direction in which particles escape with higher probability



- $\pi/4$-rotated high density region ⇒ locally $\varepsilon_4 < 0$
- Dilute regions ⇒ small contribution to v_4



- More diluted system
- Denser regions still with $\varepsilon_4 < 0$

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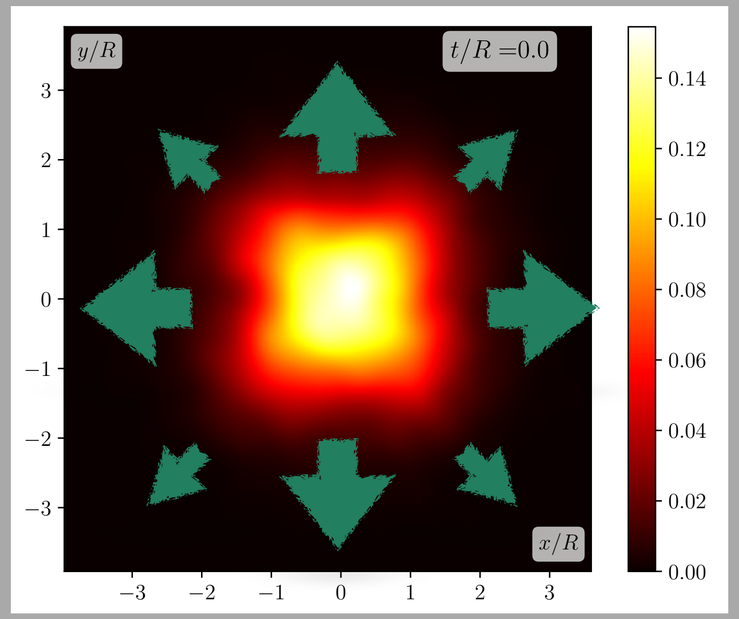

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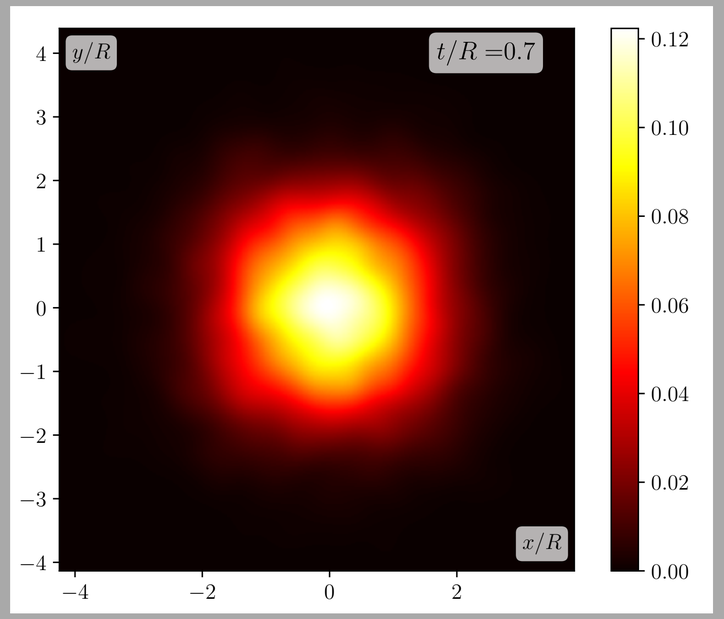

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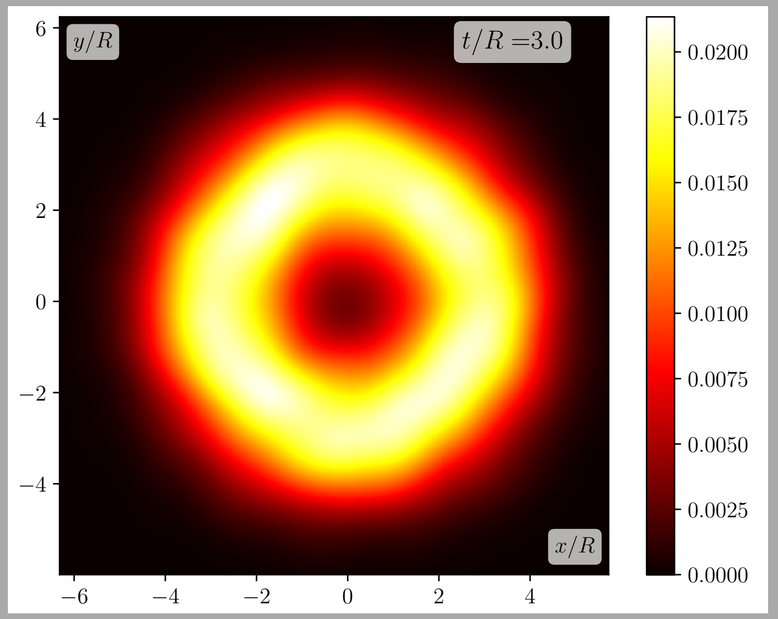

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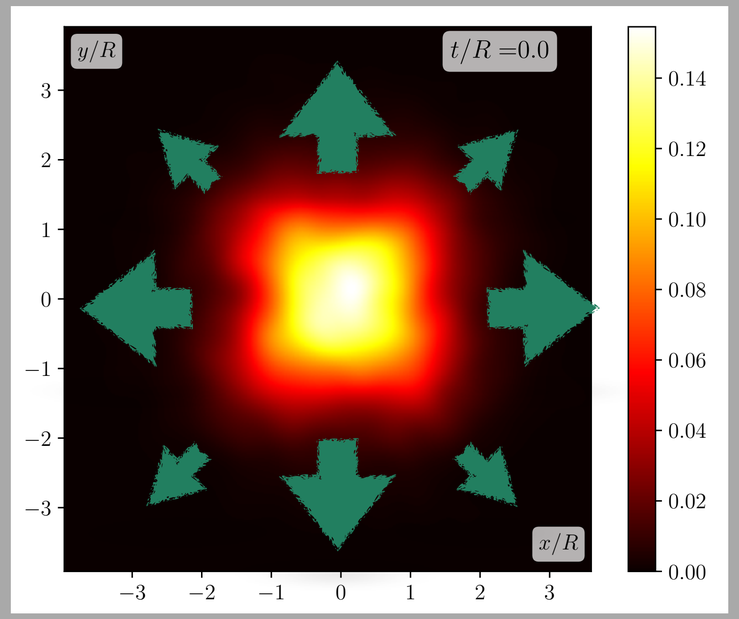

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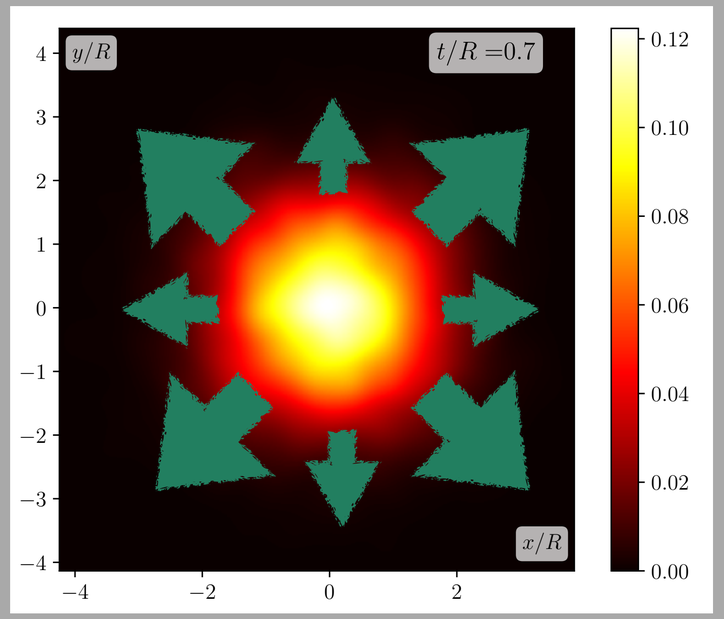

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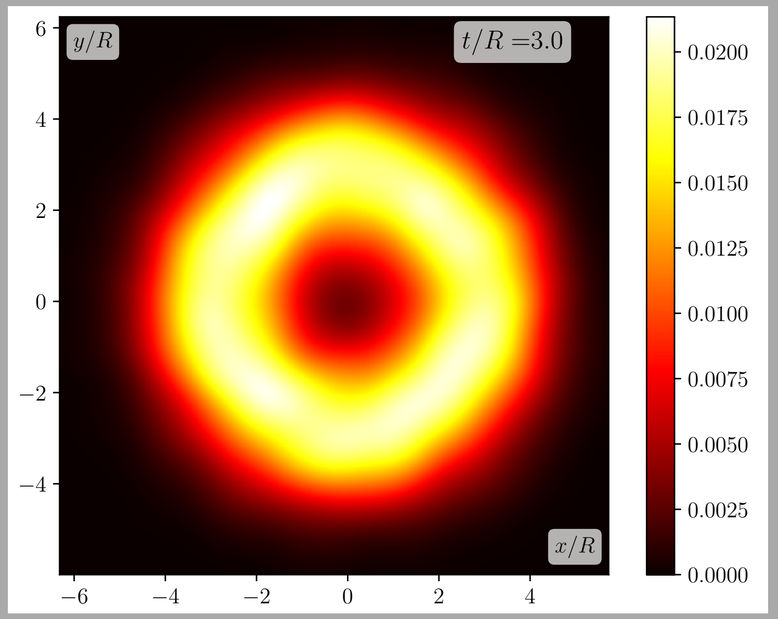

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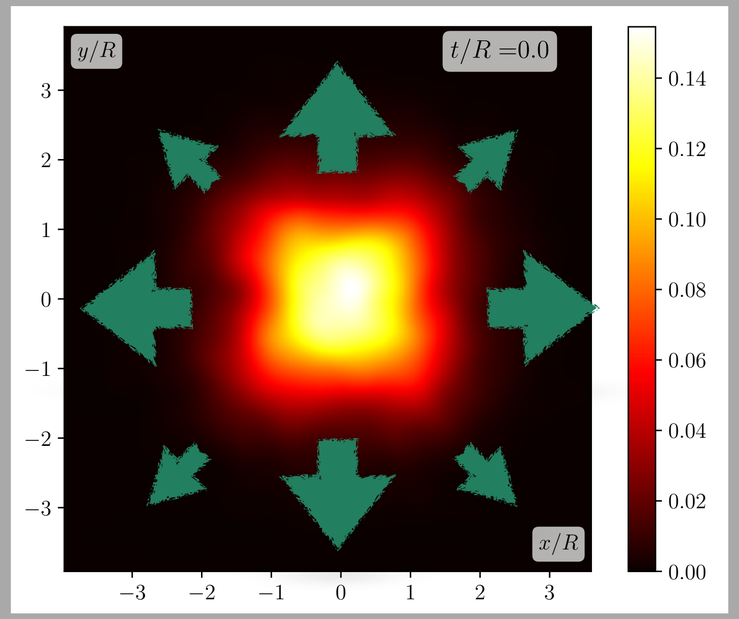

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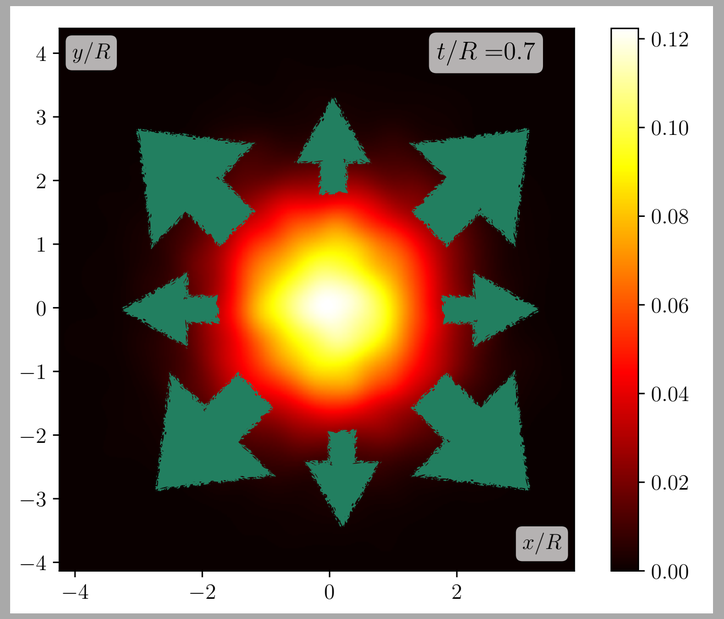

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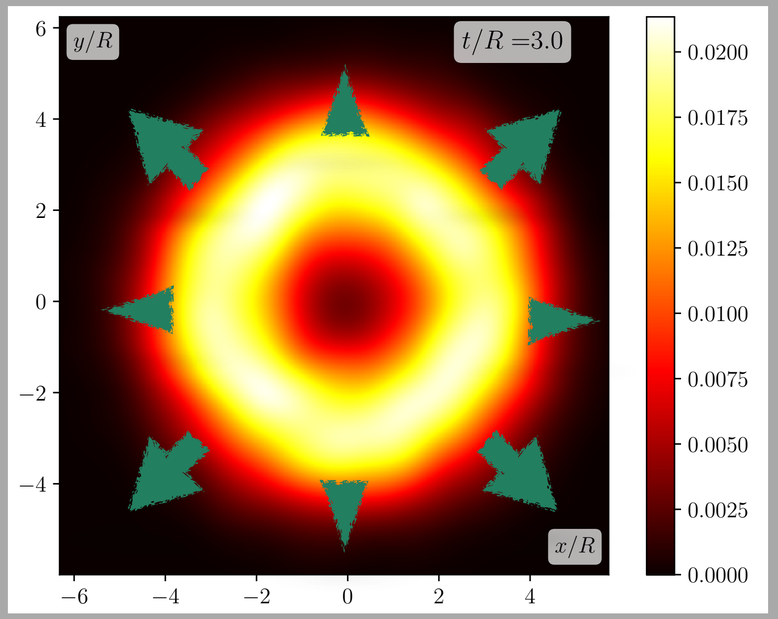

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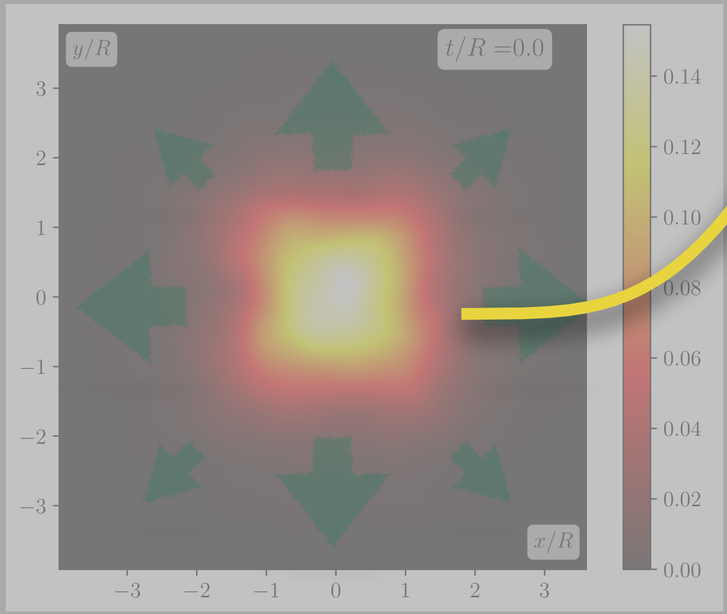

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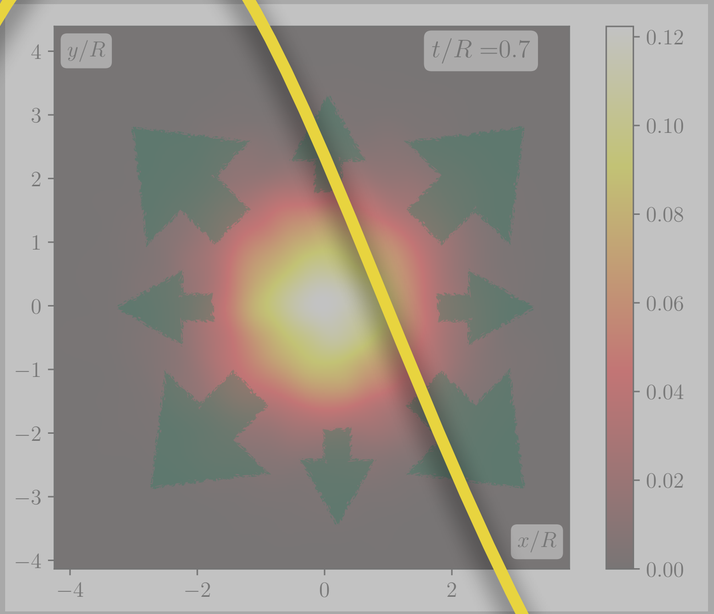

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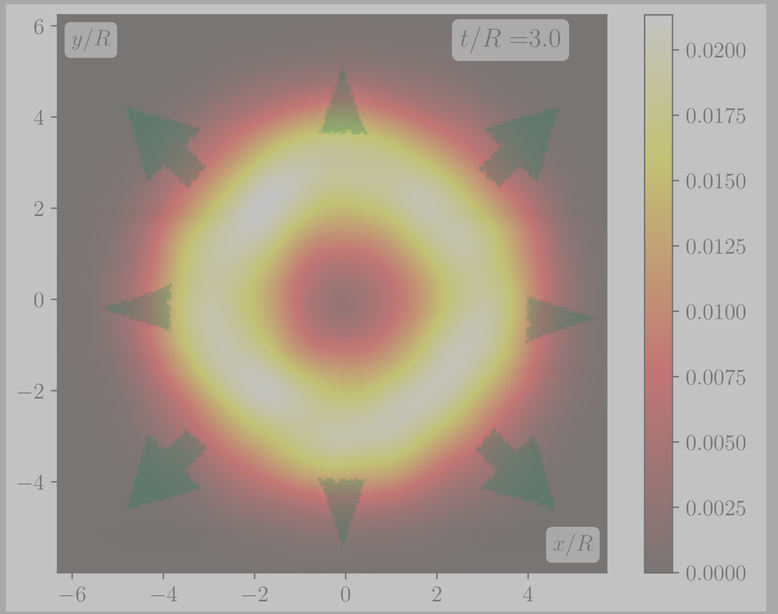

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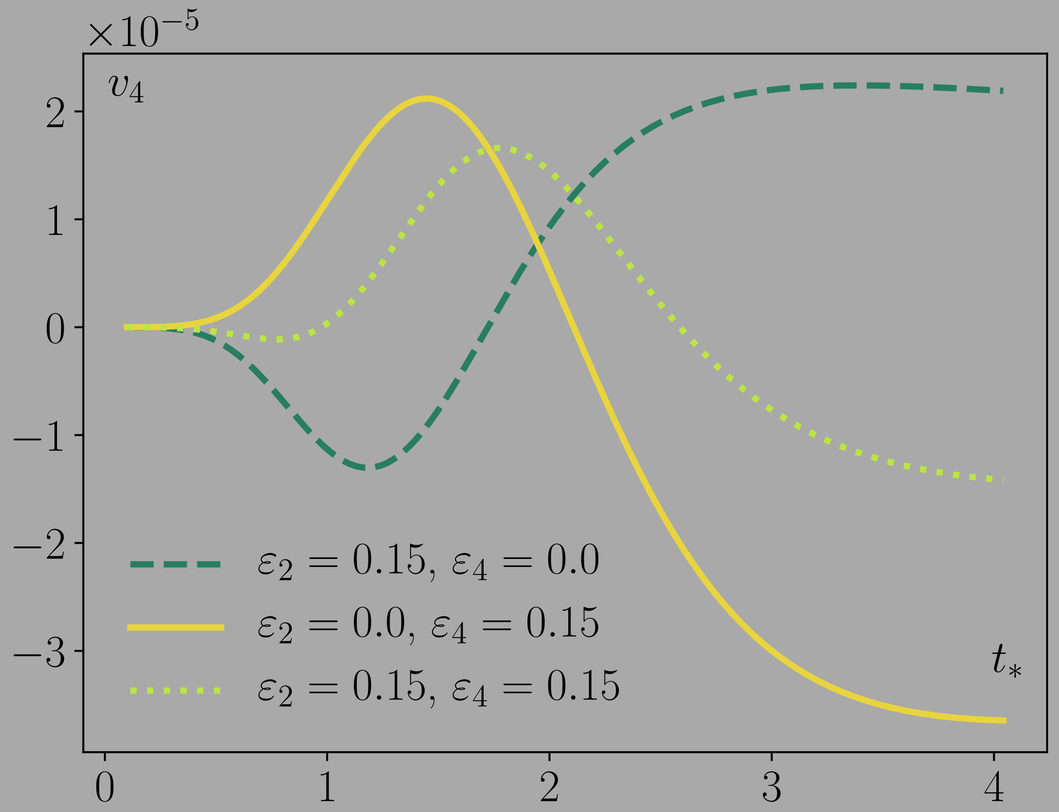
$\mathrm{v}_4\,(t_*)$

Time-Dependent $\mathrm{v}_4\left(\varepsilon_2, \varepsilon_4\right)$



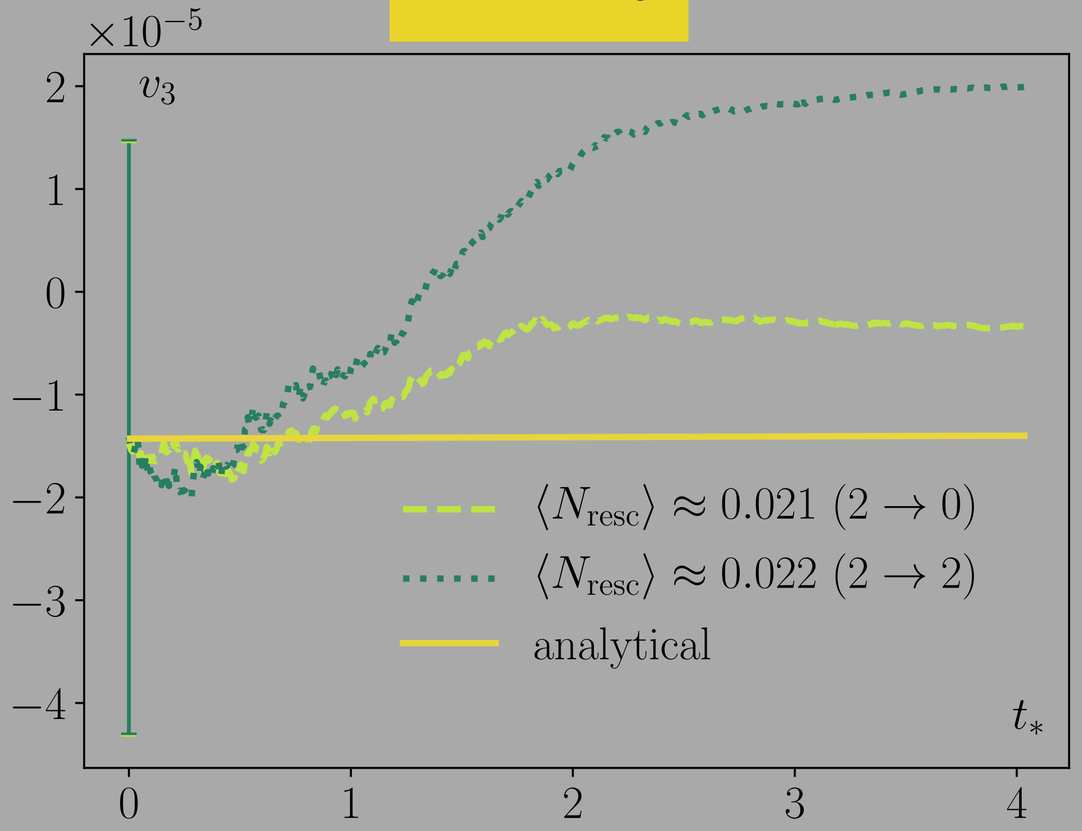
Two contributions from ε_4 and ε_2^2 to v_4

Oscillating v_4 with dependence on time t

Contributions compensate each other (opposite trend)

Time-Dependent v_3

Preliminary

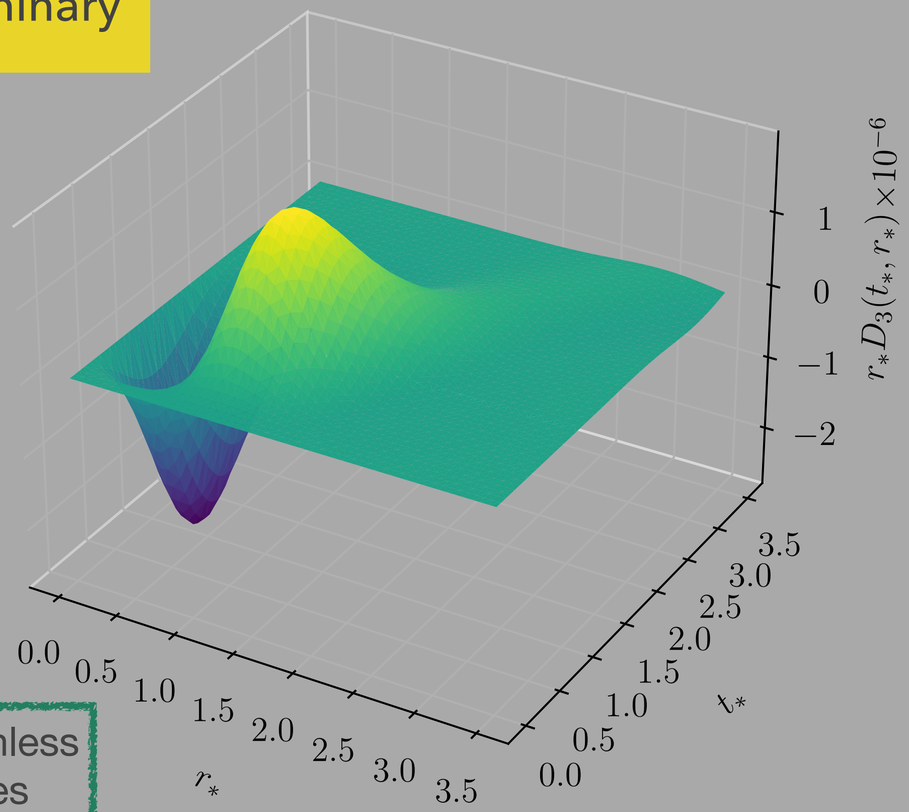


No significant v_3 signal from analytical approach

Higher orders in $N_{\rm resc}$ needed

Origin of v_3

Preliminary



Dimensionless variables

$t_* = t/R$

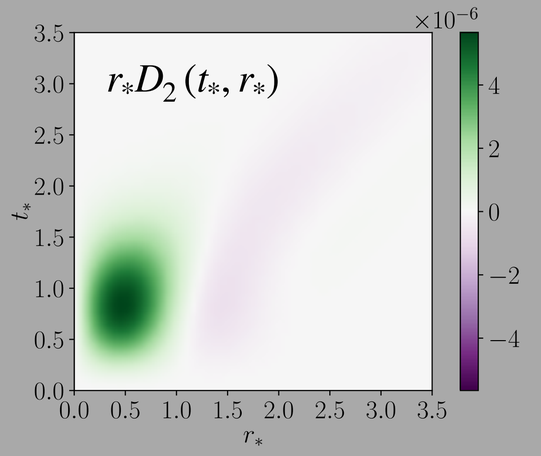
$r_* = r/R$

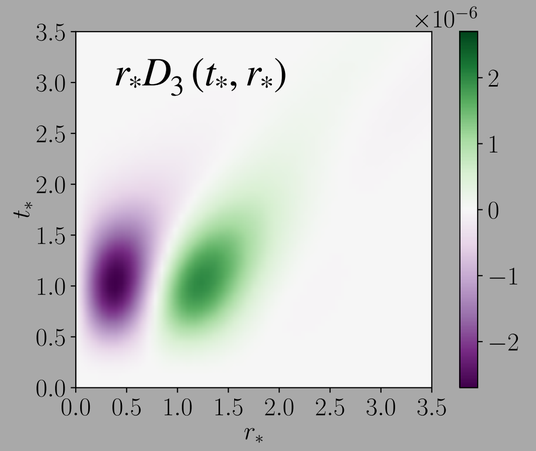
- Differential distribution $r_* D_3\,(t_*, r_*)$ of triangular flow v_3 arising from collision kernel C_{coll}
- Negative triangular flow response generated at small radii $r_* \lesssim 1$
- Positive triangular flow response generated at $r_* \gtrsim 1$
- Cancellations between these two regions
- Vanishing response v_3

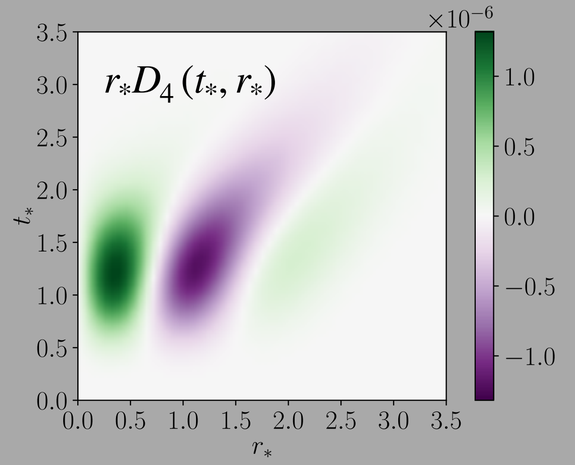
Kurkela, Mazeliauskas and Törnkvist arXiv:2104.08179

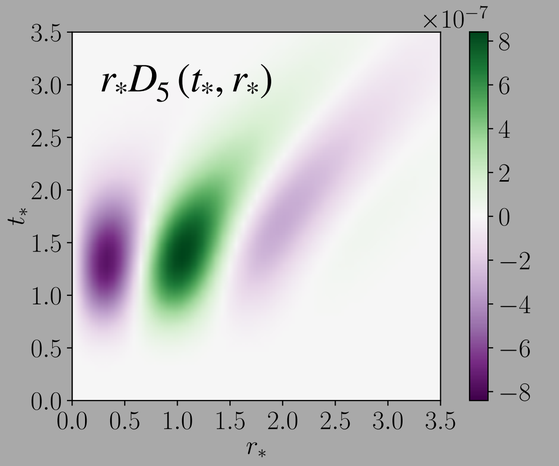
Origin of Flow

to Calculate Time-Dependent $\mathrm{v}_n\left(\varepsilon_n\right)$









- Positive odd flow response generated at $r_* \gtrsim 1$
- Negative odd flow response generated at small radii $r_* \lesssim 1$
- Vice versa for even flow response
- Additional, small contributions for higher flow harmonics at lager r_*

Preliminary

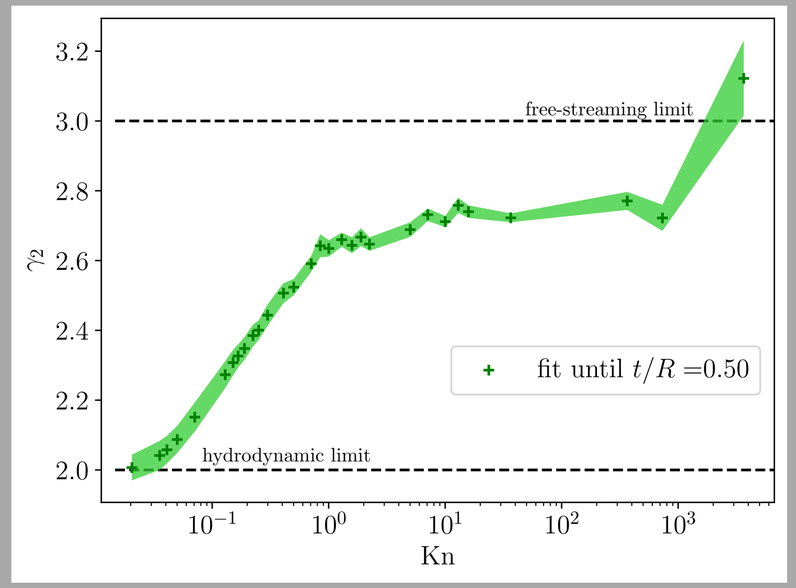
Few Collision Regime vs. Hydrodynamics

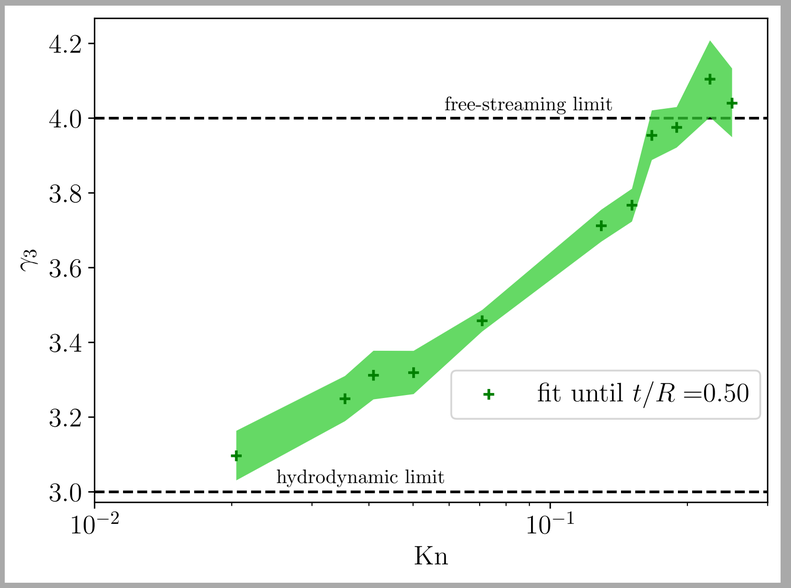
Time-Dependent v_n from Numerical Simulations

- Fit function $v_n(t_*, \mathrm{Kn}) = \beta_n \cdot t_*^{\gamma_n}$
- Time-dependence for different Knudsen numbers Kn

$$v_n(t_*) \propto \begin{cases} t_*^n, & \text{hydro limit} \\ t_*^{n+1}, & \text{f.s. limit} \end{cases}$$

Preliminary





Conclusion

In few collision regime:

- Loss term in Boltzmann eq. dominates the v_{2n} signal at $\mathcal{O}\left(N_{\text{resc}}\right)$
- Higher orders in N_{resc} are needed to create a significant v_{2n+1}
- Rediscovered scaling behavior $v_n(t) \propto t^{n+1}$
- HYDRO: Rediscovered scaling behavior $v_n(t) \propto t^n$
- Possible compensation of $v_n(t)$ signal by linear and non-linear terms

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