Beam Characterization & Emittance Measurement by the Upstream TOFs

Some numbers and thoughts on how to improve resolution and calculate errors in Stage 1

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What's new in TOF beam characterization?

1. Stability of the calibration, and the $e^{\pm}$ peak – tomorrow
2. Impact of mis-aligned, rotated TOFs – tomorrow
3. "Drift chamber" measurement of x and y
4. What's the error on x_1'?
5. What's the error on the optical parameters?

Useful geometry numbers

Corners A, B, C, D of each TOF surveyed

TOF centre planes defined by $\hat{n} \cdot (\vec{r} - \vec{r}_0) = 0$

$\hat{n}$ is the normal, obtained from $\vec{AC} \times \vec{BD}$

$\vec{r}_0$ is the centre of the detector, deduced from $\vec{AC} + \vec{BD}$

Unrotated TOF would sit in $z = z_0$ where $\hat{n} = \hat{z}$

Dihedral angle given by $\cos\alpha = \hat{n}_1 \cdot \hat{n}_2$

TOF0 normal $= (-0.467, -0.509, 99.998)$, $\alpha = 0.396$ deg

TOF1 normal $= (-0.927, -1.423, 99.986)$, $\alpha = 0.974$ deg

Middles of the detector (from the survey recon)

TOF0 $\vec{r}_0 = (1.8, 2.6, 5293.8)$ mm

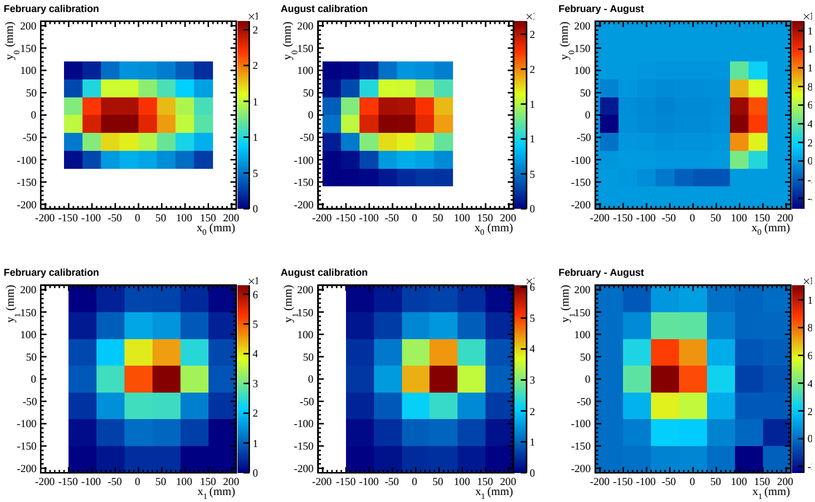
TOF1 $\vec{r}_0 = (30.2, -17.6, 12999.0)$ mm

Intersection with design orbit at $z_{\mathrm{design}} = \hat{n} \cdot \vec{r}_0 / \hat{n}_z$

TOF0 $z_{\mathrm{design}} = 5293.7$ mm

TOF1 $z_{\mathrm{design}} = 12999.0$ mm

Pixel coverage of the February and August calibrations



Use the February calibration due to its superior pixel coverage

Drift chamber measurement of x and y

Timing resolution of a station

$$t_{\mathrm{PMT}} \sim \mathrm{N}(0, 100\ \mathrm{ps})$$

$$t = \frac{t_{\mathrm{A}} + t_{\mathrm{B}} + t_{\mathrm{C}} + t_{\mathrm{D}}}{4}$$

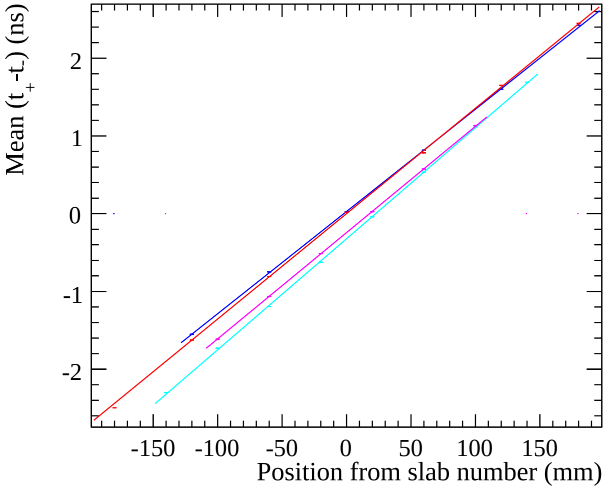
$$\sigma_t = \frac{\sigma_{\mathrm{PMT}}}{2}$$

Position resolution of a slab

$$t_{\mathrm{A}} - t_{\mathrm{C}} = \frac{2x}{c}$$

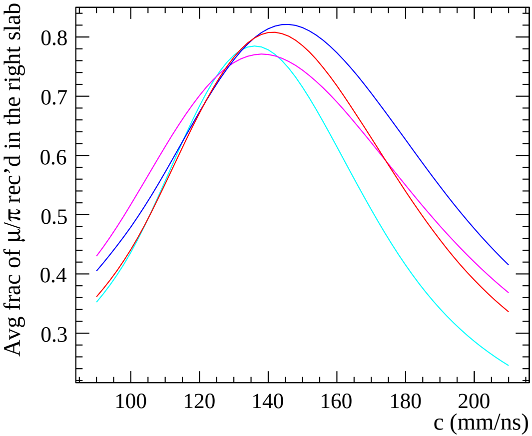
$$\sigma_x = \frac{c\sigma_{\mathrm{PMT}}}{\sqrt{2}}$$

Correlation between $t_+ - t_-$ and slab number



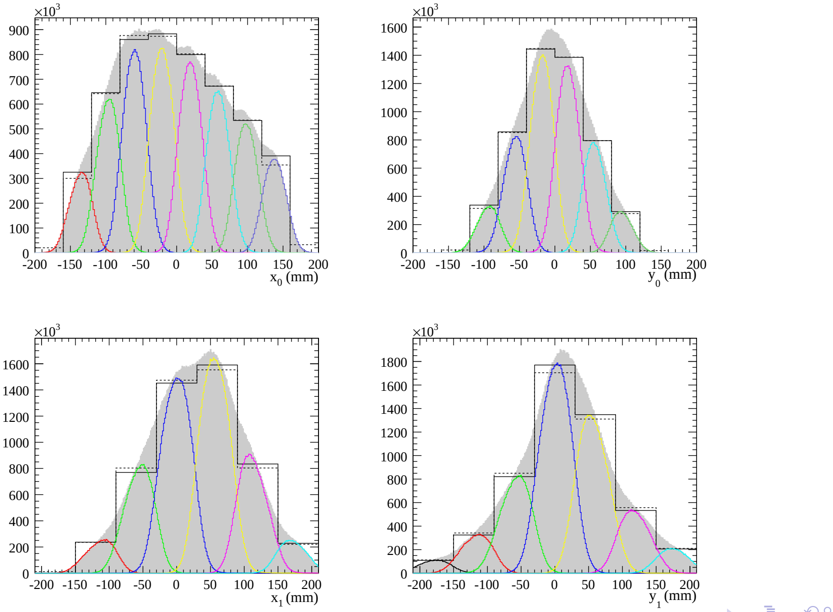
c given by 2/gradient = 140.00, 146.33, 152.05, 147.53 mm/ns.

Minimize average number of impossible reconstruction



c = 136, 138, 146, 142 mm/ns
Choose 140.5 mm/ns.

TOF0 x and y



×10^3
900
800
700
600
500
400
300
200
100
0
-200 -150 -100 -50 0 50 100 150 200
x0 (mm)
×10^3
1600
1400
1200
1000
800
600
400
200
0
-200 -150 -100 -50 0 50 100 150 200
y0 (mm)
×10^3
1600
1400
1200
1000
800
600
400
200
0
-200 -150 -100 -50 0 50 100 150 200
x1 (mm)
×10^3
1800
1600
1400
1200
1000
800
600
400
200
0
-200 -150 -100 -50 0 50 100 150 200
y1 (mm)

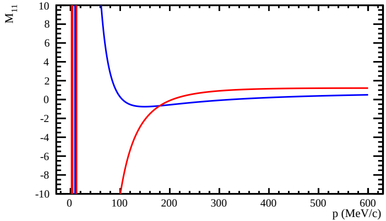
Reconstruction technique

The angles (x_0', x_1') are implied by the positions (x_0, x_1):

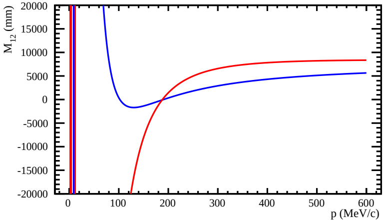
$$\begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \frac{1}{M_{12}} \begin{pmatrix} -M_{11} & 1 \\ -1 & M_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

6–200, -, M0 optics

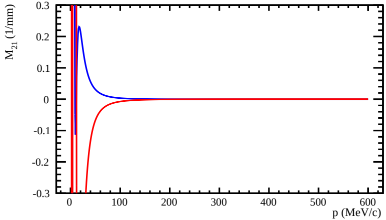
(1,1)



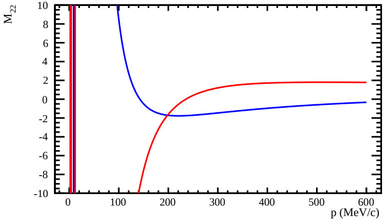
(1,2)



(2,1)



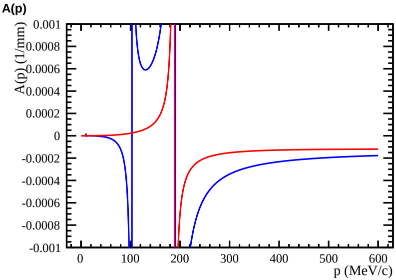
(2,2)

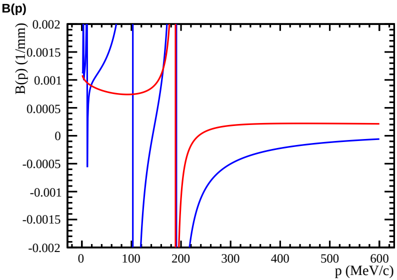


6–200, -, M0 optics

u is the position on TOF0
v is the position on TOF1

$$x_1' = A(p)u + B(p)v$$





Trace space error matrix

$$\sigma_{v'v'} = A_0^2 \sigma_{uu} + B_0^2 \sigma_{vv}$$

$$\sigma_{vv'} = B_0^2 \sigma_{vv}$$

- Error on momentum is negligible
- Occluded momentum bites
- Error ellipses
- Unbiased estimate of true covariance matrix

A dodgy back of the envelope calculation

$$\epsilon = \sqrt{D}$$
$$\sigma_\epsilon = \frac{\sigma_D}{2\epsilon}$$

$$\begin{aligned} D &= \left(\sum_i v_i^2\right)\left(\sum_j v_j'^2\right) - \left(\sum_i v_i v_i'\right)^2 \\ &= \left(\sum_i v_i^2\right)\left(\sum_j \left(A_j u_j + B_j v_j\right)'^2\right) - \left(\sum_i v_i \left(A_i u_i + B_i v_i\right)\right)^2 \end{aligned}$$

Error on emittance 2

Consider the k-th particle:

$$\frac{\partial D}{\partial u_k} = \frac{2}{N}\left(A_k v'_k \langle v^2\rangle - A_k u_k \langle v v'\rangle\right)$$

$$\frac{\partial D}{\partial v_k} = \frac{2}{N}\left(B_k v'_k \langle v^2\rangle + v_k \langle v'^2\rangle - \left(v'_k + A_k u_k\right)\langle v v'\rangle\right)$$

$$\sigma_{DD} = \sum_k \left[\left(\frac{\partial D}{\partial u_k}\right)^2 \sigma_{u_k u_k} + \left(\frac{\partial D}{\partial v_k}\right)^2 \sigma_{v_k v_k}\right]$$

Conclusion

1. Error ellipses in (x_1, x_1') space depend on p_z: there are unreconstructable regions
2. x from $t_+ - t_-$ does not improve resolution much
3. But does result in uncorrelated errors
4. Propose to apply this analysis to the Stage 1 data and summarize results on Thursday
5. New versions of `TofTrace` and `OnlineReconstruction` will be uploaded soon
6. MICE note 317 "Momentum measurement by the upstream TOFs" is now published