

# **Small $x$ prediction for forward charm production and implications for neutrino spectrum at FASER**

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# Outline

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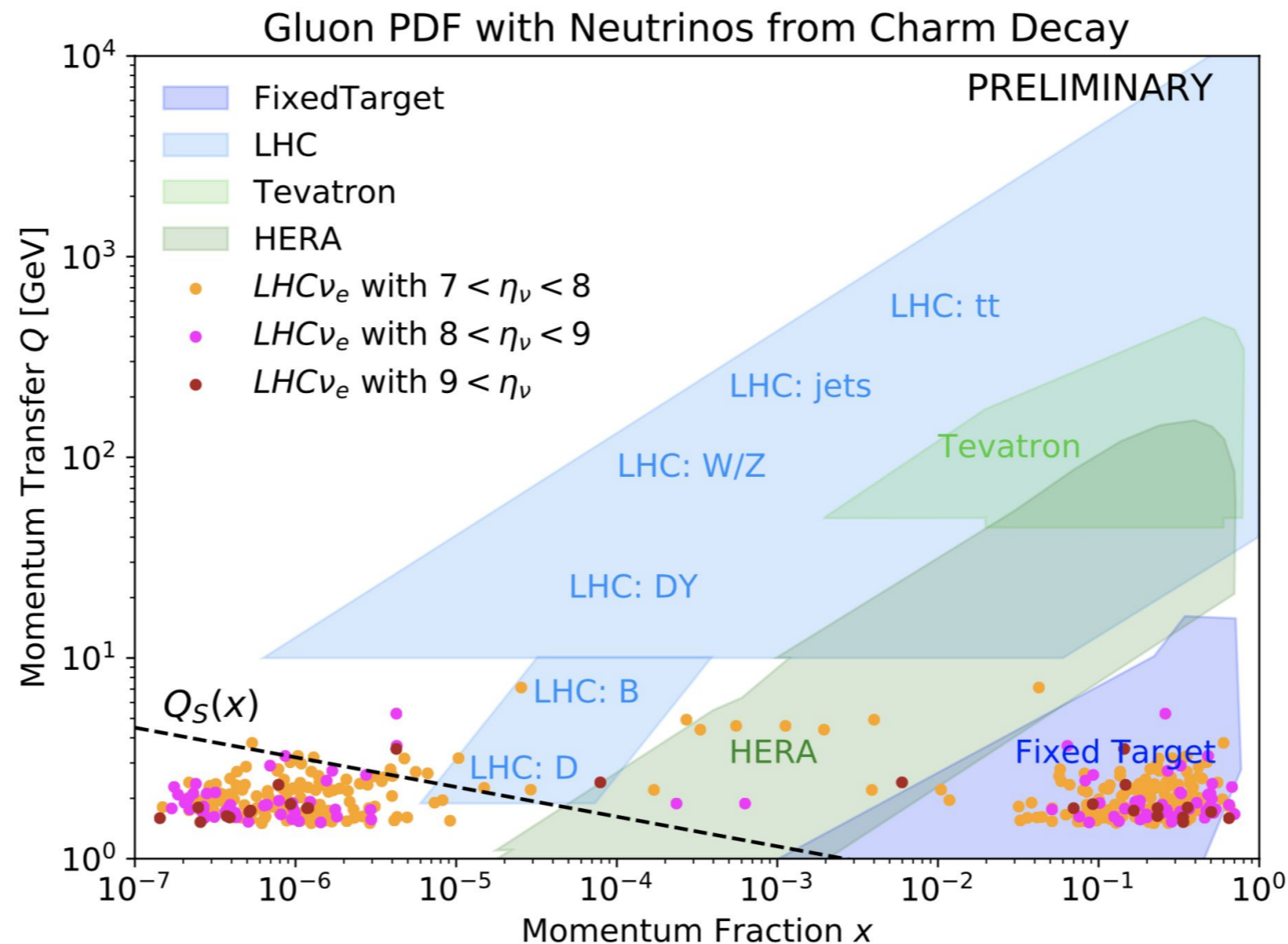
- Motivation for calculation with small  $x$  dynamics in the forward region
- Framework for *charm production*:  $k_T$  factorization + small  $x$  resummation with and without saturation
- Results: neutrino spectrum
- Other issues: fragmentation in the forward region
- Outlook

# Motivation

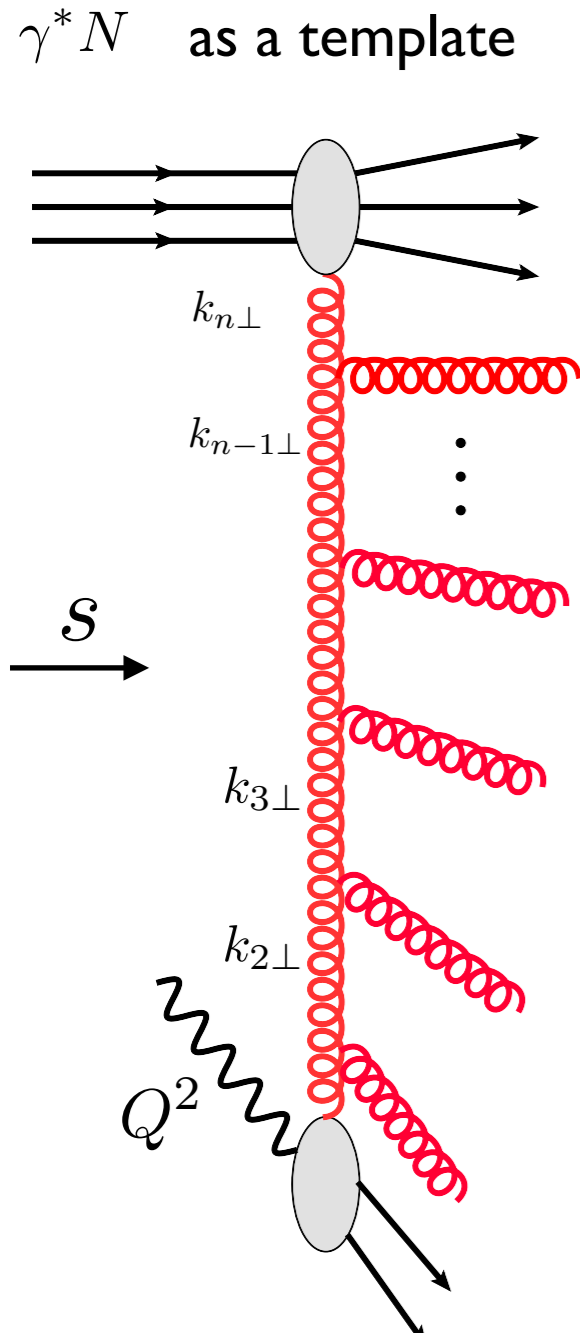
Small  $x$  region: BFKL Pomeron, saturation, diffraction

LHC and dedicated experiment FASER has potential to explore the extremely small  $x$

$(x, Q)$  phase space



# Collinear approach



Large parameter

$$Q^2 \rightarrow \infty$$

$S$  total energy is fixed

Probing small distances

Strong ordering in transverse momenta

$$Q^2 \gg k_{1\perp}^2 \gg k_{2\perp}^2 \gg k_{3\perp}^2 \cdots \gg k_{n\perp}^2$$

Resummation of large logarithms

$$\int_{\mu_0^2}^{Q^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2} g^2 \int_{\mu_0^2}^{k_{1\perp}^2} \frac{dk_{2\perp}^2}{k_{2\perp}^2} g^2 \int_{\mu_0^2}^{k_{2\perp}^2} \frac{dk_{3\perp}^2}{k_{3\perp}^2} g^2 \cdots \int_{\mu_0^2}^{k_{n-1\perp}^2} \frac{dk_{n\perp}^2}{k_{n\perp}^2} g^2 \simeq \left( g^2 \log \frac{Q^2}{\mu_0^2} \right)^n$$

# Collinear approach

DGLAP evolution equation for parton densities

$$\frac{\partial f_i(x, Q^2)}{\partial \log(Q^2)} = \sum_j \int_x^1 \frac{dz}{z} P_{j \rightarrow i}(z) f_j\left(\frac{x}{z}, Q^2\right)$$

Splitting functions calculated perturbatively

$$P_{j \rightarrow i}(z) = \alpha_s P_{j \rightarrow i}^{(LO)}(z) + \alpha_s^2 P_{j \rightarrow i}^{(NLO)}(z) + \alpha_s^3 P_{j \rightarrow i}^{(NNLO)}(z) + \dots$$

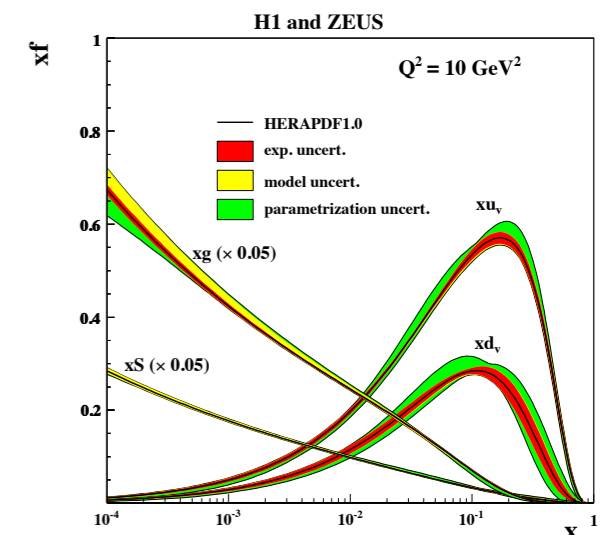
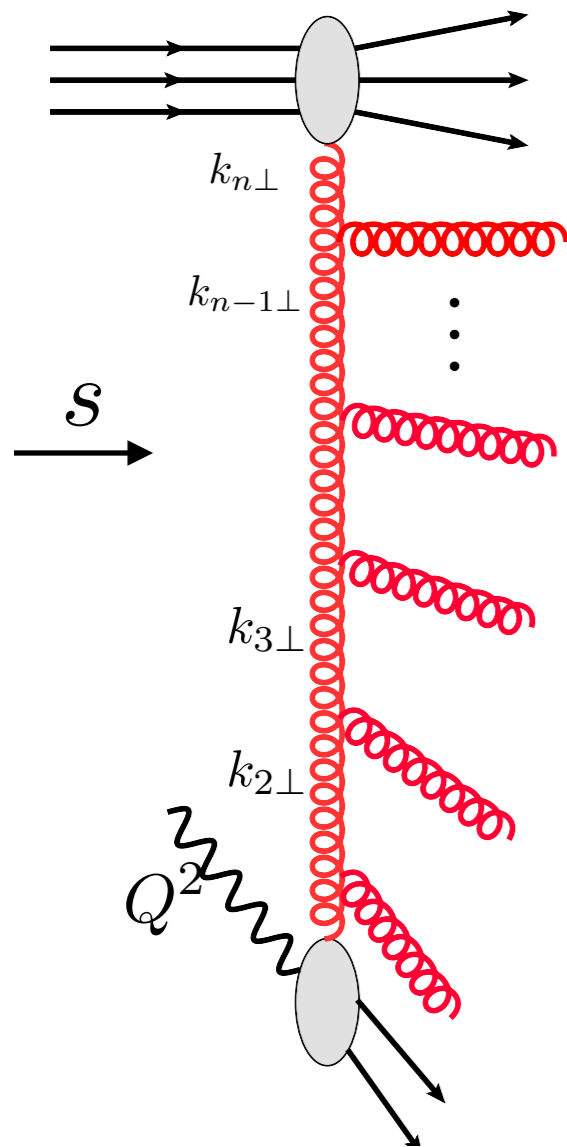
Parton densities: distributions in longitudinal momenta at a given scale

$$f_i(x, Q^2)$$

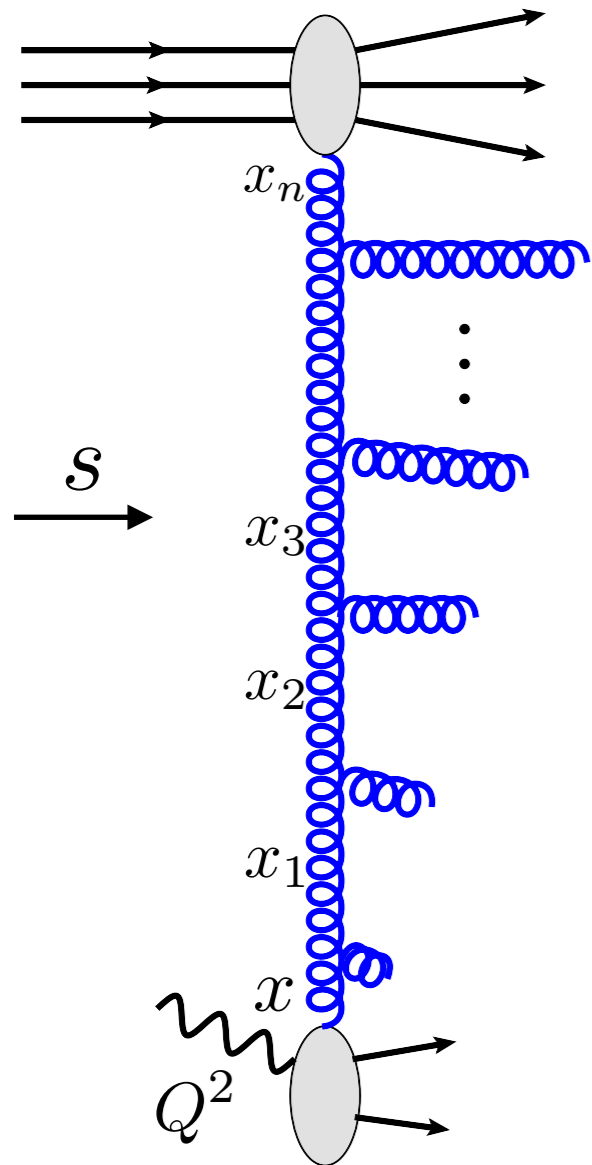
Collinear factorization of the cross section

$$d\sigma(x, Q^2) = \sum_i f_i \otimes d\hat{\sigma}^i + \mathcal{O}(\Lambda^2/Q^2)$$

$d\hat{\sigma}^i$  partonic cross section, calculable perturbatively



# High energy limit



Large parameter

$$s \rightarrow \infty$$

High energy or Regge limit

$$s \gg Q^2 \gg \Lambda^2$$

$Q^2$  fixed, perturbative

Light cone proton momentum

$$p^+ = p^0 + p^z$$

$$k_i^+ = x_i p^+$$

Strong ordering in longitudinal momenta

$$x \ll x_1 \ll x_2 \ll \dots \ll x_n$$

Perturbative coupling but large logarithm

$$\bar{\alpha}_s \ll 1$$

$$\ln \frac{1}{x} \simeq \ln \frac{s}{Q^2} \gg 1$$

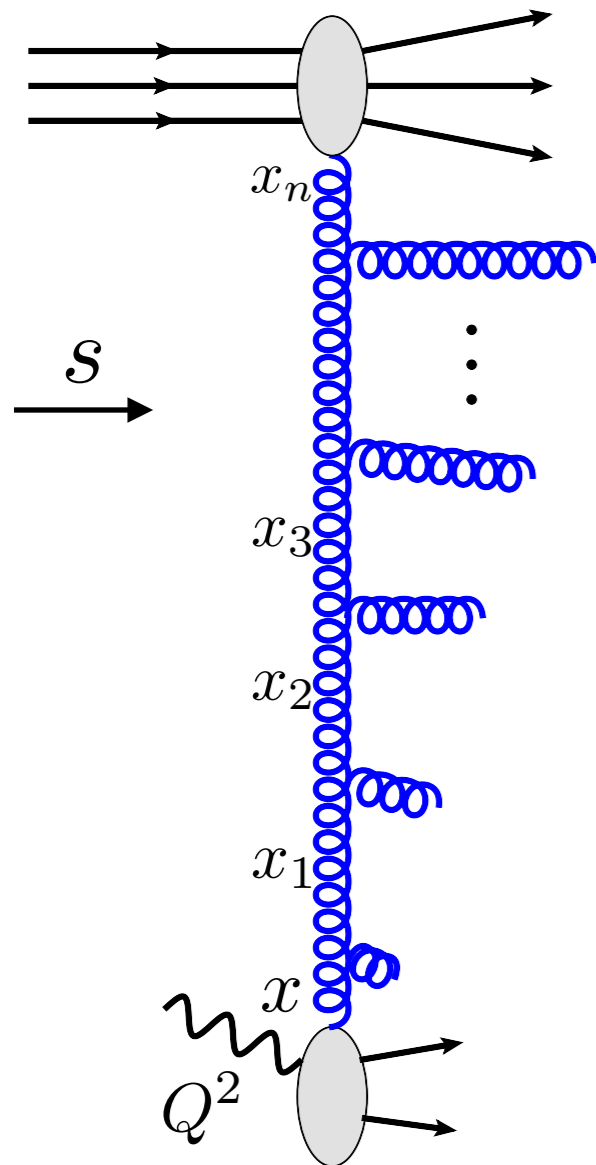
Large logarithms

$$\frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dz}{z} = \frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} = \bar{\alpha}_s \ln \frac{1}{x}$$

Leading logarithmic resummation

$$\left( \bar{\alpha}_s \ln \frac{1}{x} \right)^n \quad \left( \bar{\alpha}_s \ln \frac{s}{s_0} \right)^n$$

# High energy limit



compare with DGLAP-collinear approach

Resummation performed by Bethe-Salpeter type equation  
BFKL evolution equation

$$\frac{\partial \mathcal{F}_g(x, k_T)}{\partial \ln 1/x} = \int d^2 k'_T \mathcal{K}(k_T, k'_T) \mathcal{F}_g(x, k'_T)$$

Branching kernel (perturbative expansion)

$$\mathcal{K} = \bar{\alpha}_s \mathcal{K}^{LLx} + \bar{\alpha}_s^2 \mathcal{K}^{NLLx} + \bar{\alpha}_s^3 \mathcal{K}^{NNLLx} + \dots$$

QCD

N=4 SYM

Unintegrated, (transverse momentum dependent) gluon density

$\mathcal{F}_g(x, k_T)$

$$\frac{\partial f_i(x, Q^2)}{\partial \log(Q^2)} = \sum_j \int_x^1 \frac{dz}{z} P_{j \rightarrow i}(z) f_j\left(\frac{x}{z}, Q^2\right)$$

# Resummation

- Problems with convergence of small  $x$  expansion: NLL terms large
- Resummation need to be performed:
- Combining DGLAP and BFKL
- Kinematical constraints, momentum sum rule

*Altarelli, Ball, Forte  
Kwiecinski  
Ciafaloni, Colferai, Salam, AS  
Thorne, White; Sabio-Vera*

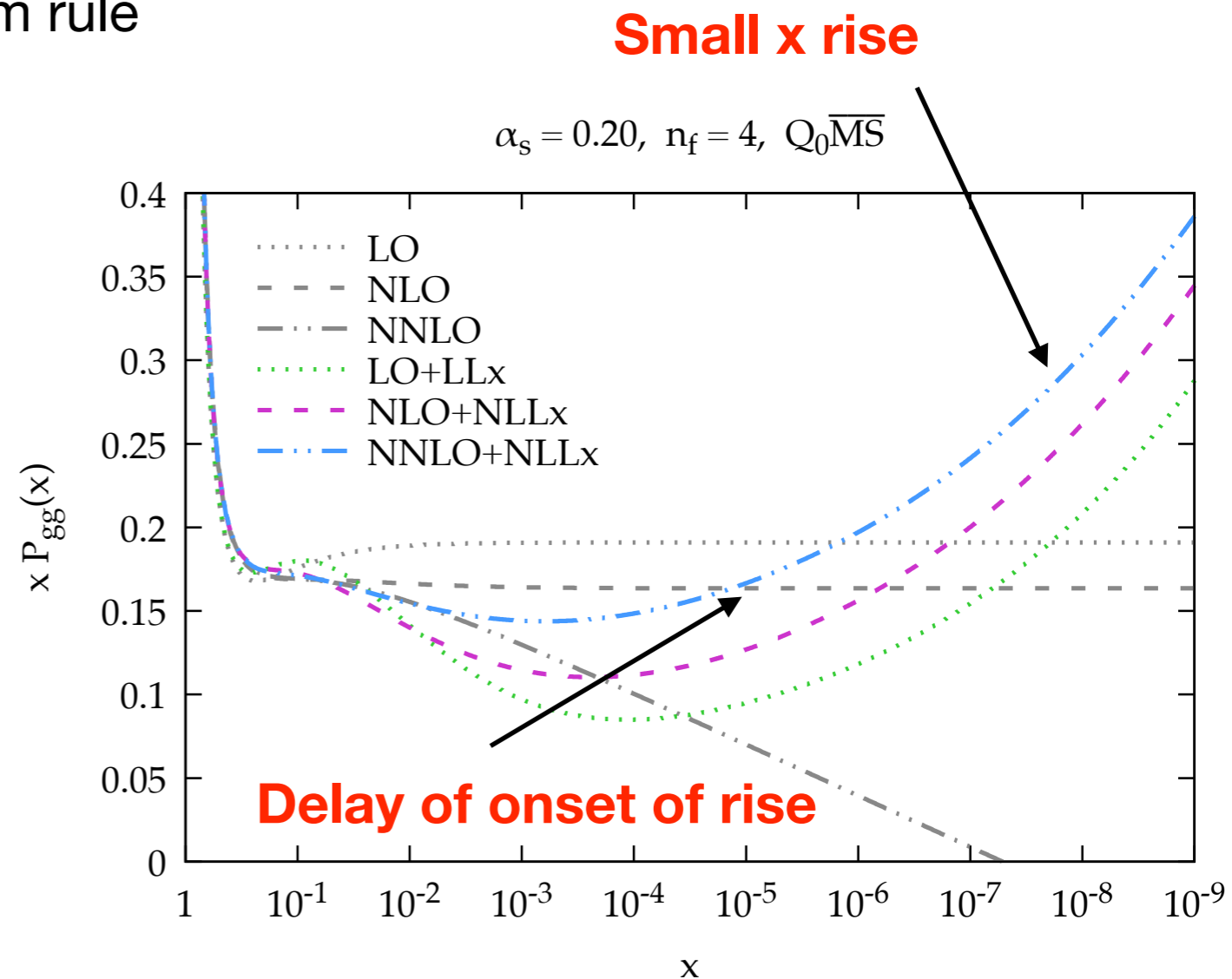
DGLAP will fail at small  $x$ , at some point.  
Question is when exactly?

Splitting function: NNLO has large  
negative term which dominates at small  $x$ .

Resummation stabilizes the result

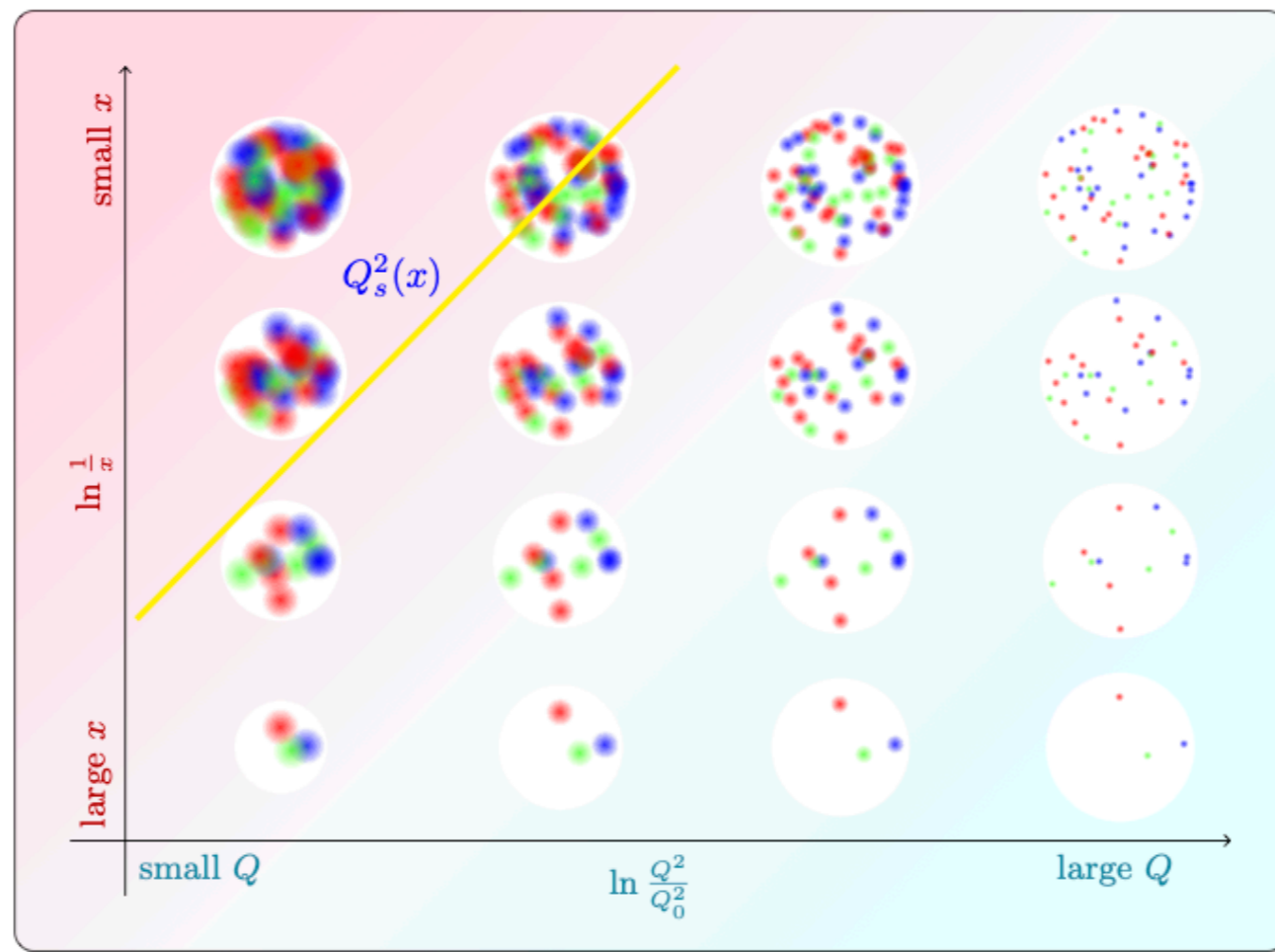
Improves the description of the F2 data

*Ball, Bertoni, Bonvini, Marzani, Rojo, Rottoli*





# Parton saturation



At small  $x$  another effect possible: gluon saturation

High gluon density can lead to increase of recombination effects. Unitarity.

QCD at high energy : dynamically generated saturation scale

# DGLAP+BFKL+saturation : example

Kutak,Sapeta; based on model by Kwiecinski, Martin,AS

$$\phi_p(x, k^2) = \phi_p^{(0)}(x, k^2)$$

BFKL+kinematical  
constraint

$$+ \frac{\alpha_s(k^2) N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dl^2}{l^2} \left\{ \frac{l^2 \phi_p\left(\frac{x}{z}, l^2\right) \theta\left(\frac{k^2}{z} - l^2\right) - k^2 \phi_p\left(\frac{x}{z}, k^2\right)}{|l^2 - k^2|} + \frac{k^2 \phi_p\left(\frac{x}{z}, k^2\right)}{|4l^4 + k^4|^{\frac{1}{2}}}\right\}$$

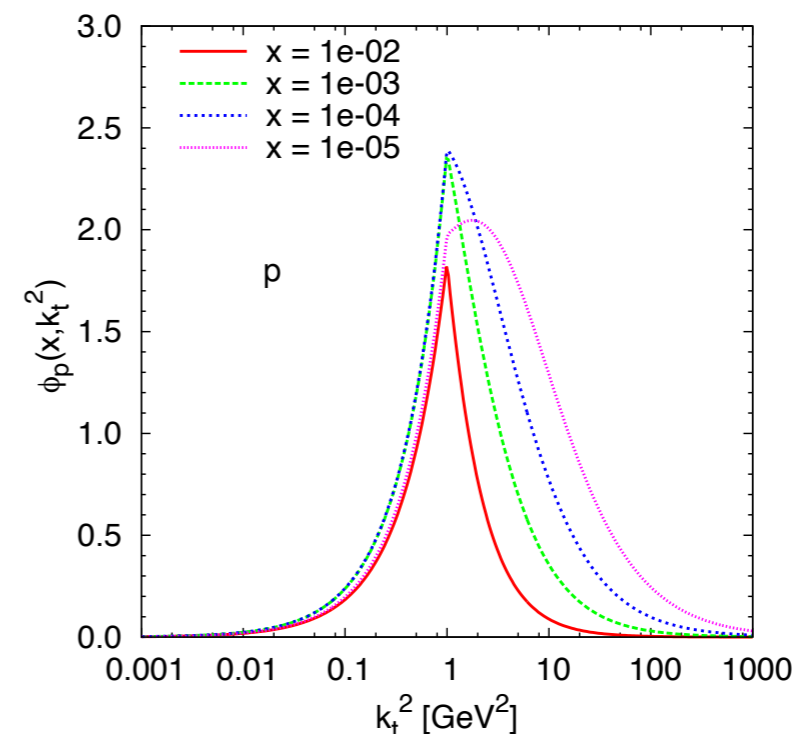
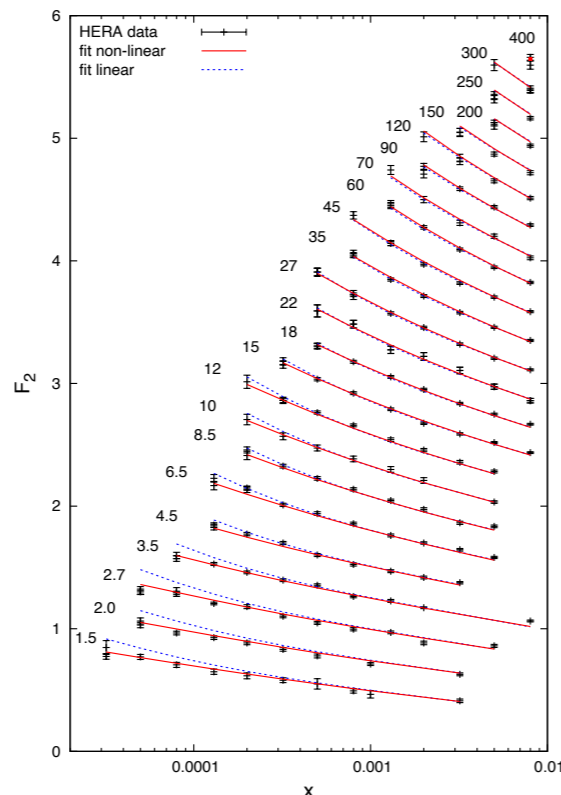
DGLAP splitting function  
(nonsingular part)

$$+ \frac{\alpha_s(k^2)}{2\pi k^2} \int_x^1 dz \left[ \left( P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_0^2}^{k^2} dl^2 \phi_p\left(\frac{x}{z}, l^2\right) + z P_{gq}(z) \Sigma\left(\frac{x}{z}, k^2\right) \right]$$

Nonlinear term

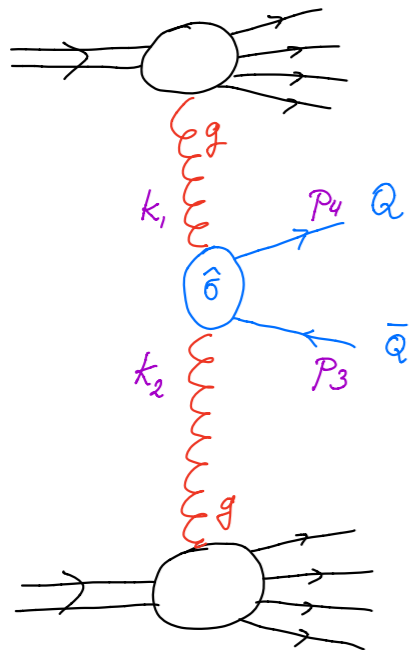
$$- \frac{2\alpha_s^2(k^2)}{R^2} \left[ \left( \int_{k^2}^{\infty} \frac{dl^2}{l^2} \phi_p(x, l^2) \right)^2 + \phi_p(x, k^2) \int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln\left(\frac{l^2}{k^2}\right) \phi_p(x, l^2) \right],$$

Good description  
of the data on F2



# $k_T$ factorization for heavy quark production

Catani, Ciafaloni, Hautmann;  
Collins, Ellis



Forward region:

$$x_1 \gg x_2$$

$$k_1 = (p_1^+, 0, \mathbf{k}_{1T})$$

$$p_1^+ = x_1 P^+$$

Off-shell gluons

$$k_2 = (0, p_2^-, \mathbf{k}_{2T})$$

$$p_2^- = x_2 \tilde{P}^-$$

$$k_1^2 = -\mathbf{k}_{1T}^2$$

$$p_1^+ p_2^- = x_1 x_2 s$$

$$k_2^2 = -\mathbf{k}_{2T}^2$$

Unintegrated gluon density

$$\sigma(m^2, s) = \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int d^2 k_{1T} F(x_1, \mathbf{k}_{1T}) \int d^2 k_{2T} F(x_2, \mathbf{k}_{2T}) \hat{\sigma}(x_1, x_2, \mathbf{k}_{1T}, \mathbf{k}_{2T}, m)$$

Off-shell xsection

Take hybrid (on-shell – off-shell) approach:

$$k_{1T} \rightarrow 0, k_{2T} \neq 0$$

$$k_1 = (p_1^+, 0, 0)$$

$$\sigma(m^2, s) = \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} x g(x_1, \mu^2) \int d^2 k_{2T} F(x_2, \mathbf{k}_{2T}) \hat{\sigma}(x_1, x_2, \mathbf{k}_{1T} \rightarrow 0, \mathbf{k}_{2T}, m)$$

Collinear gluon density

# Setup

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$k_T$  factorization (based on Catani-Ciafaloni-Hautmann formula) for charm with unintegrated gluon from Kutak-Sapeta (the same formalism used previously (*BEJKRSS*) for evaluation of prompt neutrino flux in IceCube ; other models for unintegrated gluon still to be considered)

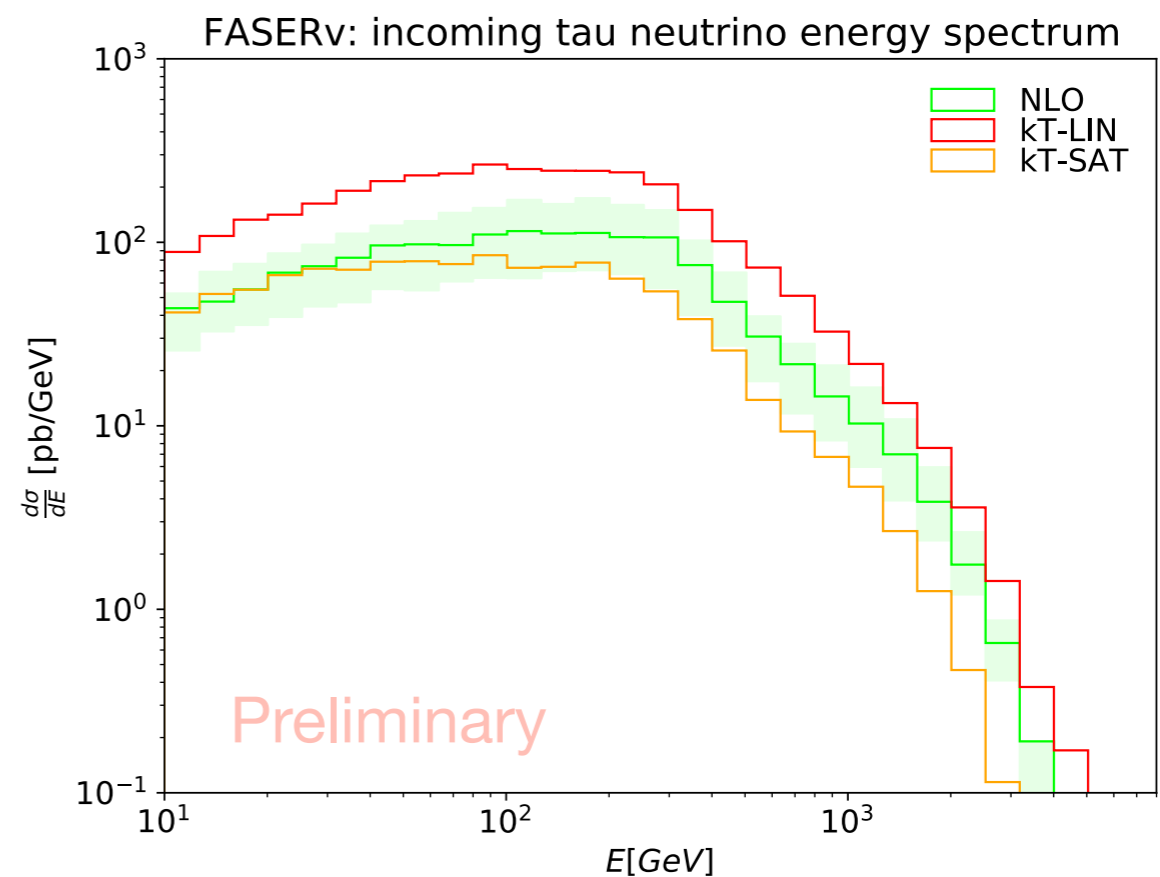
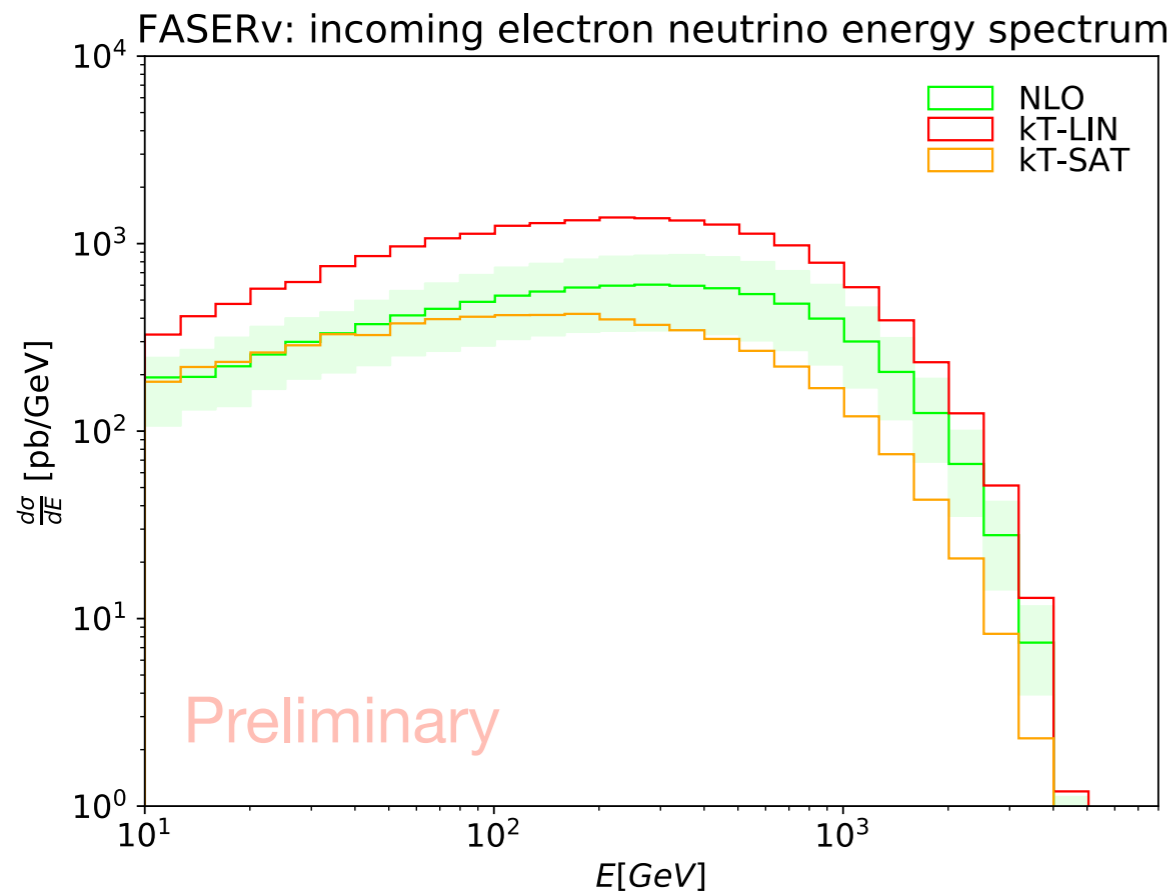
Different fragmentation scenarios tested for D mesons: ex. Peterson function vs PYTHIA

Compare with NLO and MC generators and cross check with collider data on D mesons

Compute neutrino flux at FASERnu

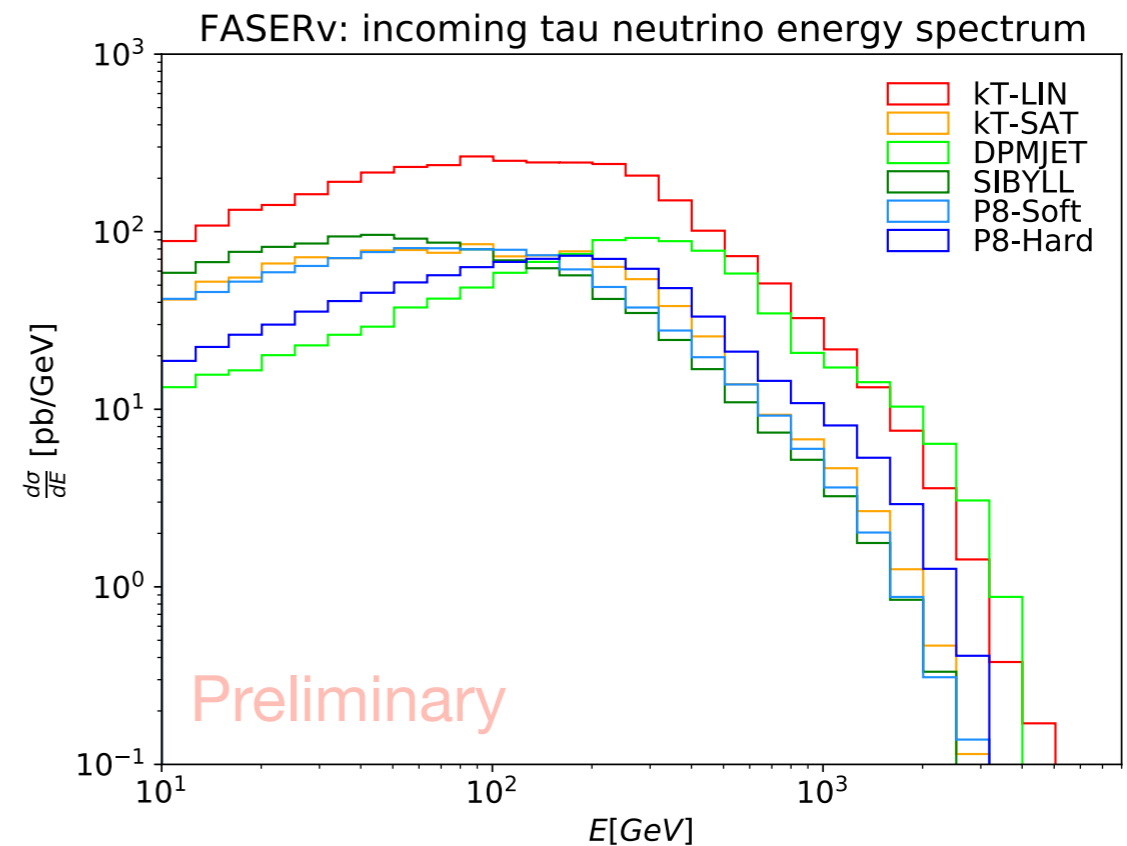
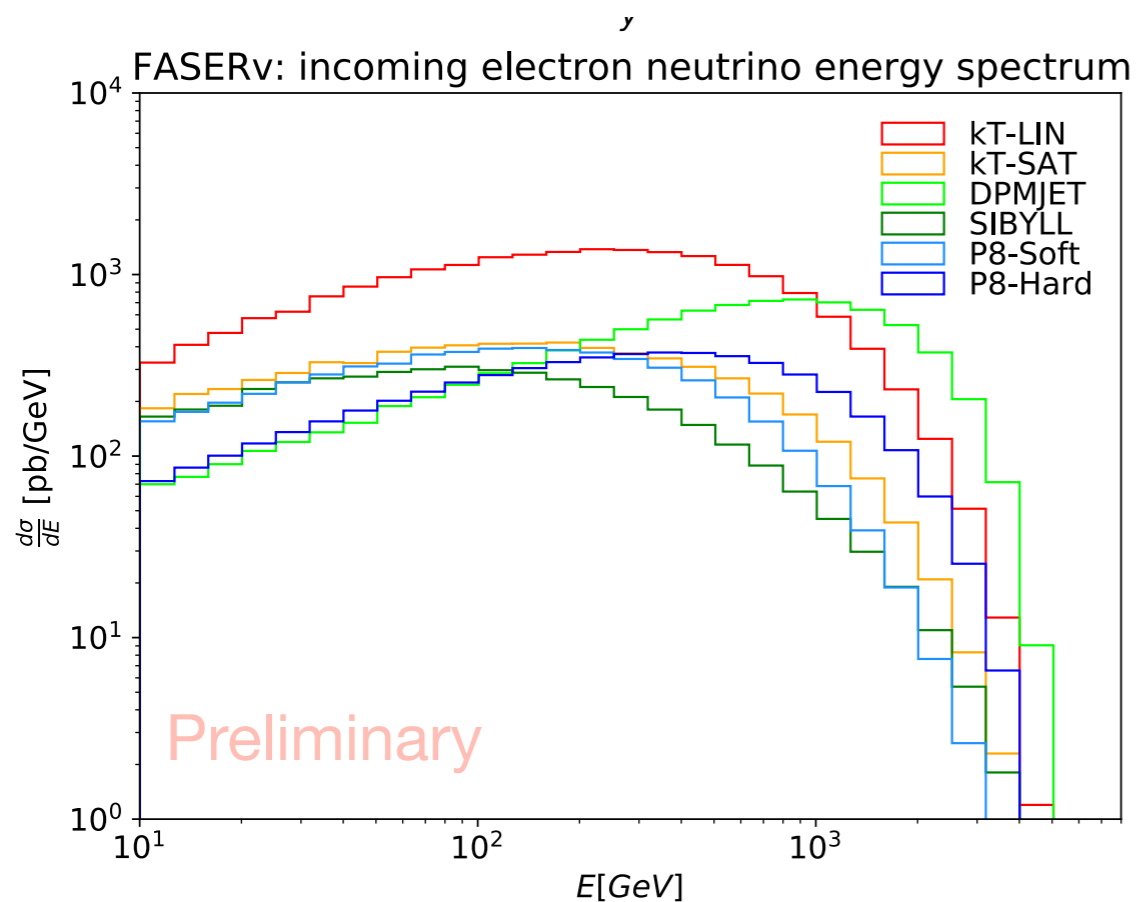
*Note : all the results are preliminary.*

# Neutrino spectrum



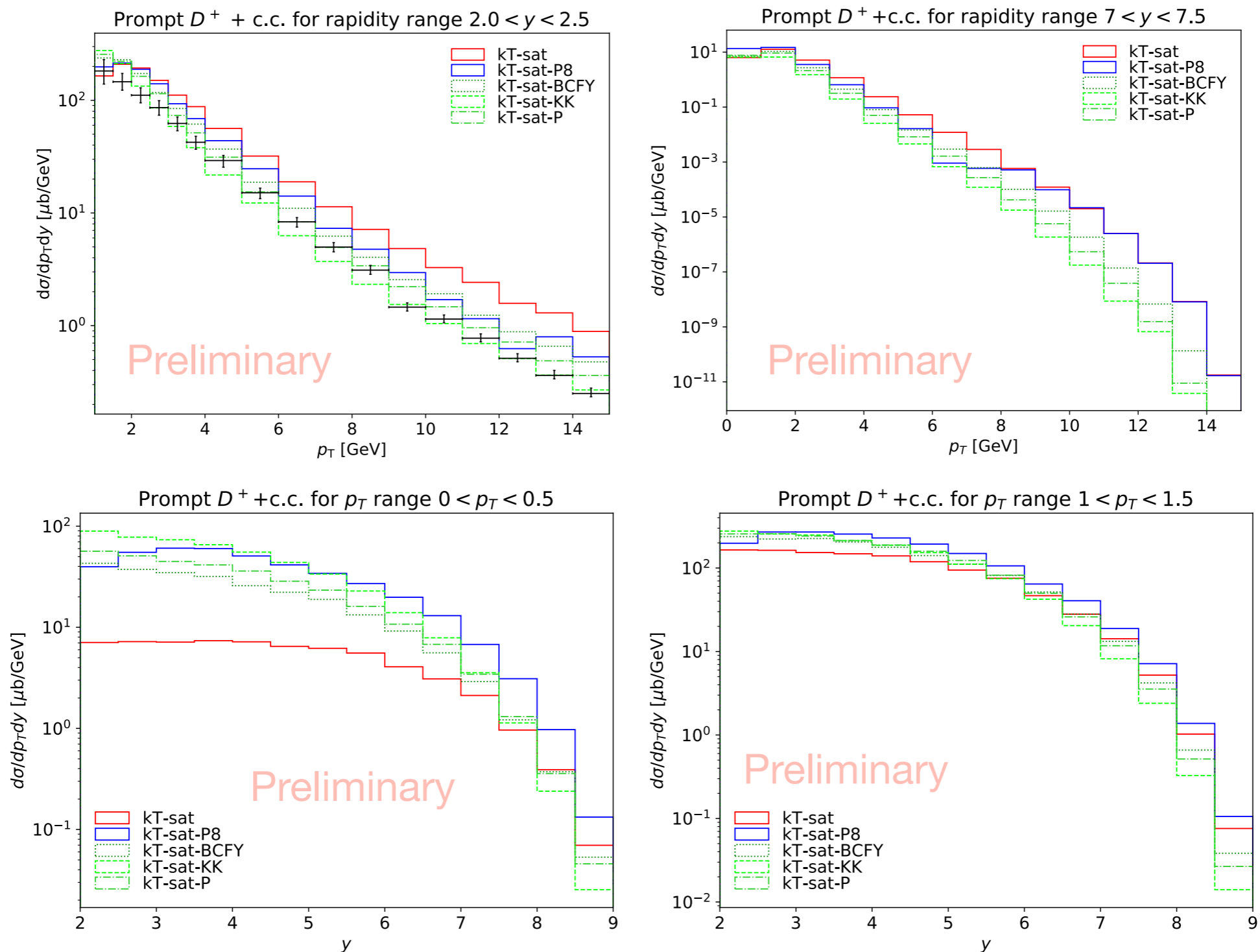
Range of uncertainties for the NLO scale variation of the same order as the small  $x$  calculation i.e. difference between linear and nonlinear (in the forward region)

# Neutrino spectrum

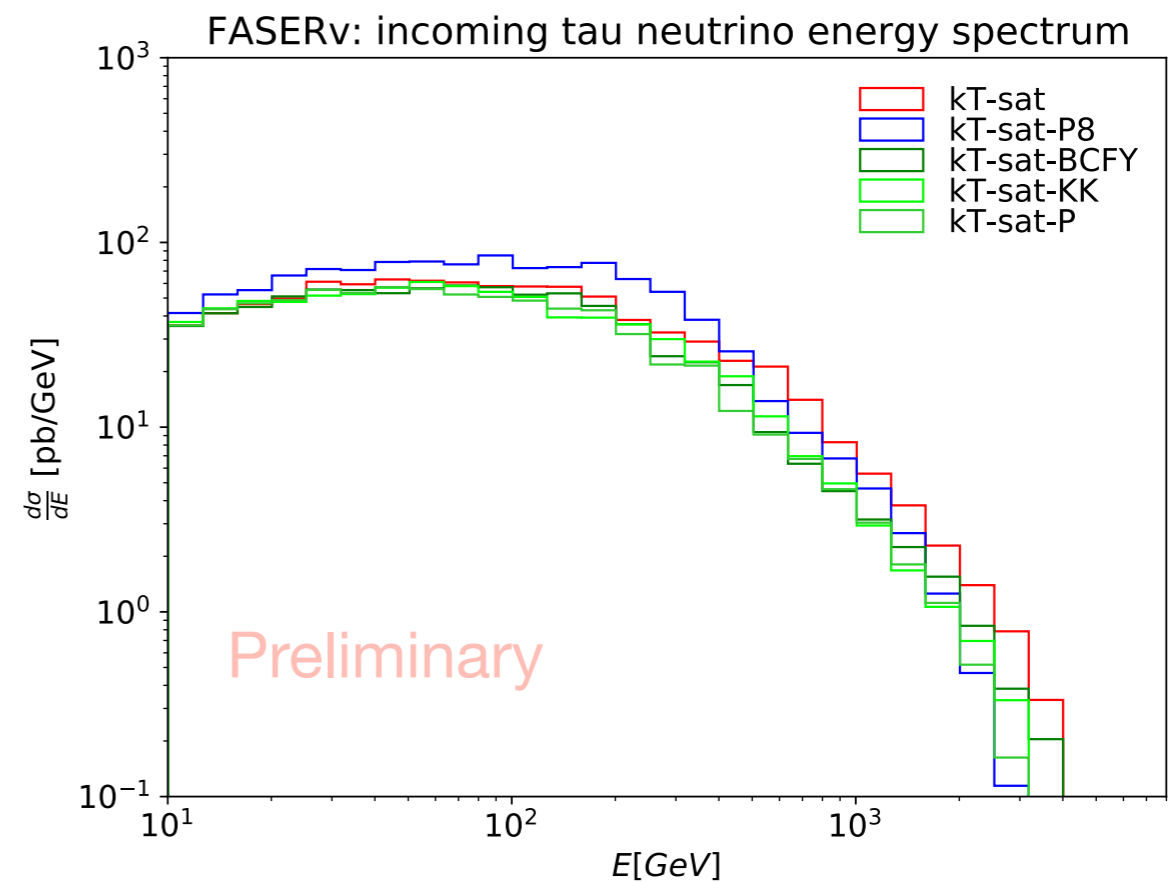
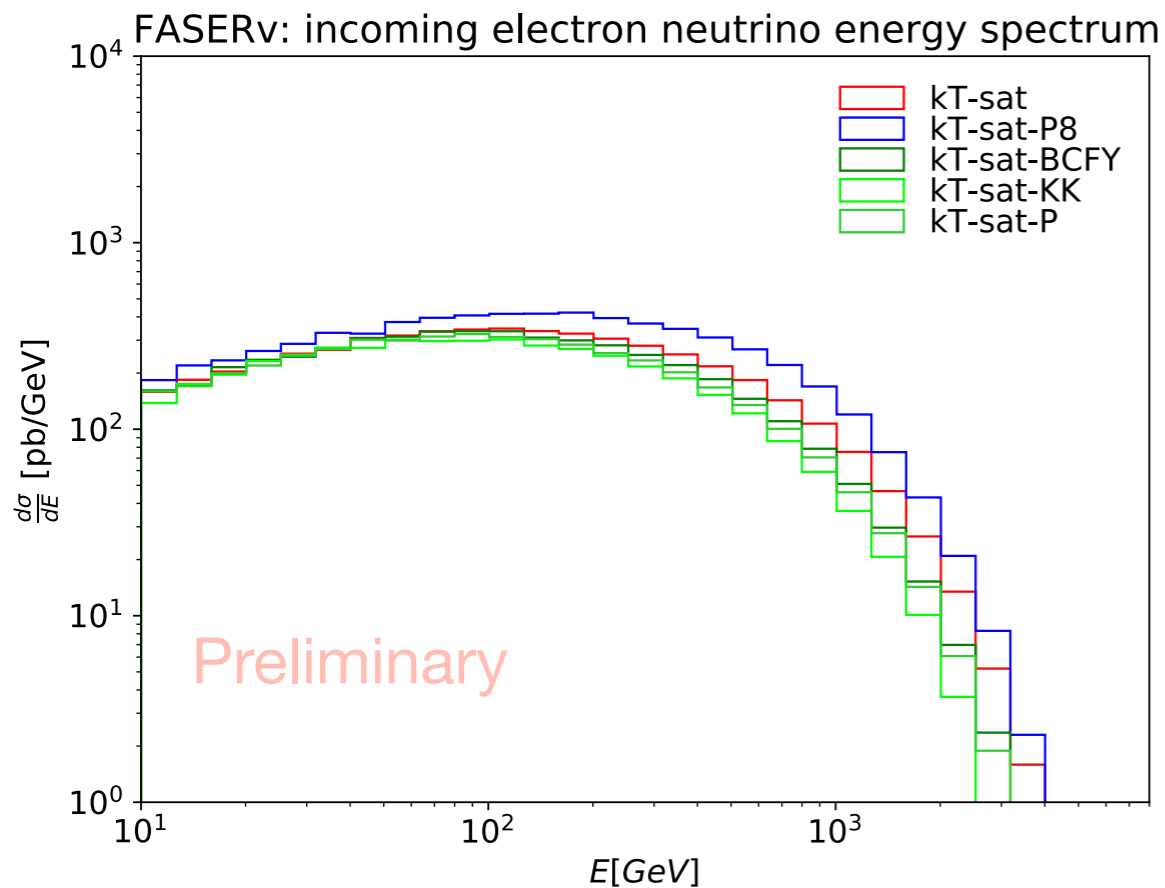


Significant differences in the energy spectrum from different generators

# Impact of fragmentation



# Impact of fragmentation



Significant differences in the energy spectrum due to the fragmentation  
Does factorization hold for fragmentation in the very forward region ??



# Summary

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- With increasing energies small  $x$  resummation becomes necessary
- FASER sensitive to very forward region in particle production
- Small  $x$  dynamics important: differences between linear and non-linear scenarios large.
- Sensitivity to fragmentation scenarios: is standard fragmentation applicable in the forward region in hadron-hadron collision environment ? Applicability of standard factorization?