GNN for Water Cherenkov Detector

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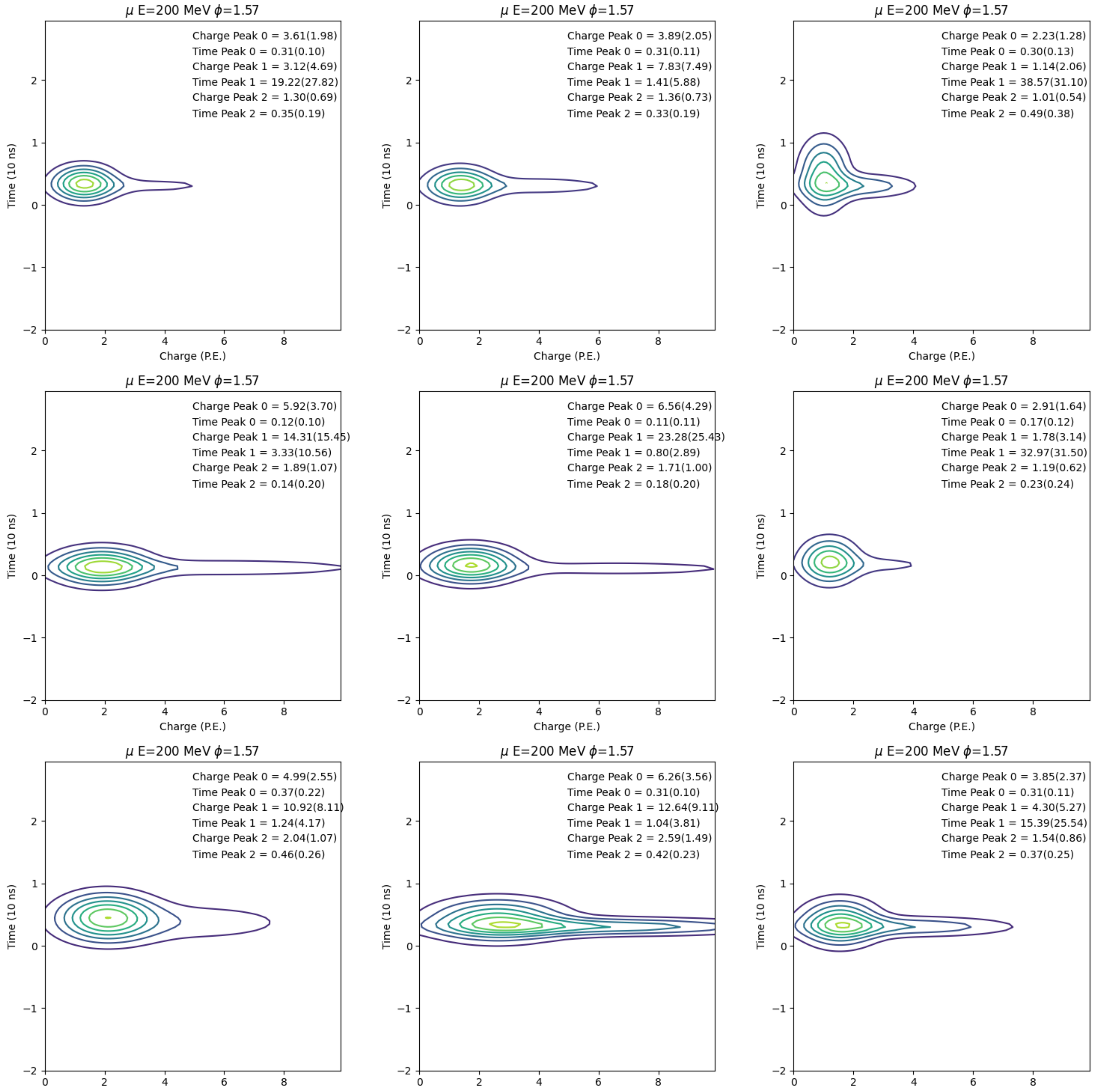
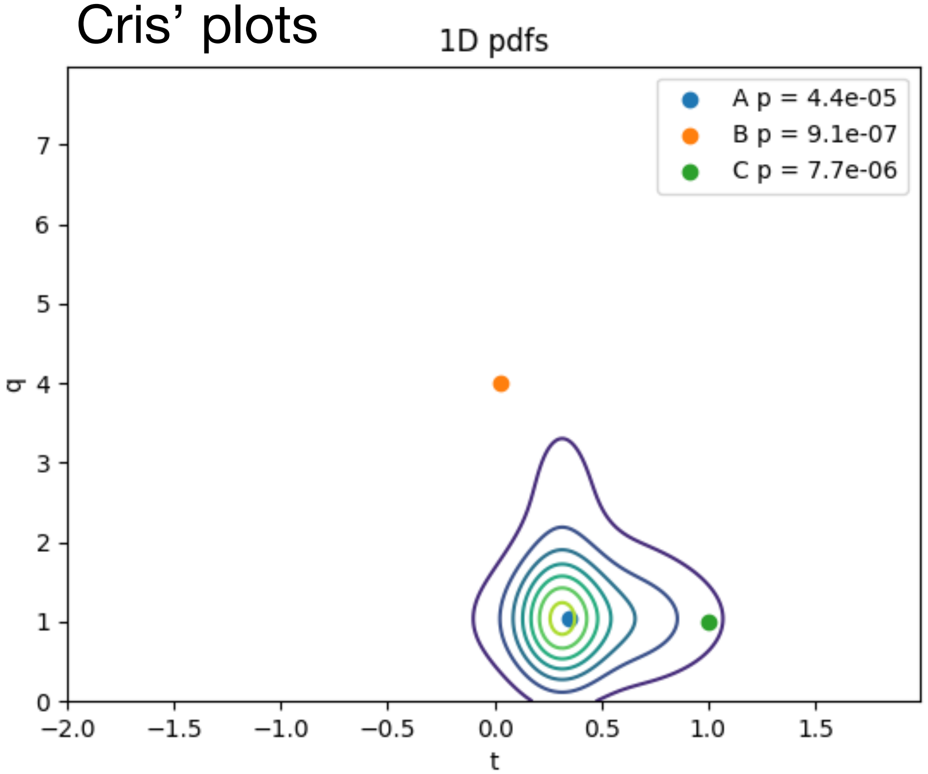
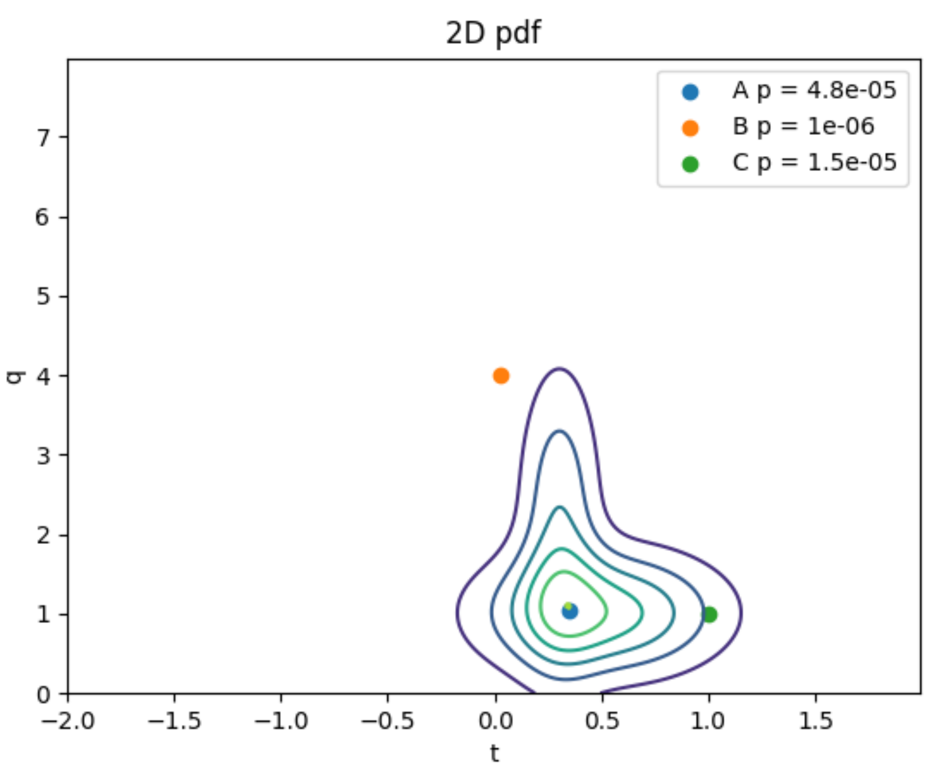
2D Charge-Time PDF of PMT 7787 - 3 Gaussian Peaks



Comparing 2x1D and 2D Fits

Cris' plots





Peak	0	1	2
Coefficient	0.2395	0.2920	0.4685
Charge Mean	2.23	1.14	1.01
Charge Sigma	1.28	2.06	0.54
Time Mean	0.30	38.57	0.49
Time Sigma	0.13	31.10	0.38

Fixing Correlation Implementation

2. Say that

$\boxed{\textbf{This can be interesting?}}$

$$X_1 = Z_1 + Z_c$$
$$X_2 = Z_2 + Z_c$$

Last time showed:

where $Z_i \sim \mathcal{N}(0, e^{O_i})$ are "private information" and $Z_c \sim \mathcal{N}(0, e^{O_3})$ is a "shared" gaussian information. Then we will have that

$$v_1 = e^{O_1} + e^{O_3}$$
$$v_2 = e^{O_2} + e^{O_3}$$
$$\rho = \frac{e^{O_3}}{\sqrt{v_1 \cdot v_2}}$$

But in this case, the correlation can only be positive!
Instead:

$$f_{\mathbf{X}}(x_1, \ldots, x_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^k|\boldsymbol{\Sigma}|}}$$

$$\Sigma^{-1} = A^{\mathrm{T}}A = \begin{pmatrix} \alpha_{11} & 0 & 0 & 0 \\ \alpha_{12} & \alpha_{22} & 0 & 0 \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & 0 \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix} \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ 0 & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ 0 & 0 & \alpha_{33} & \alpha_{34} \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$$

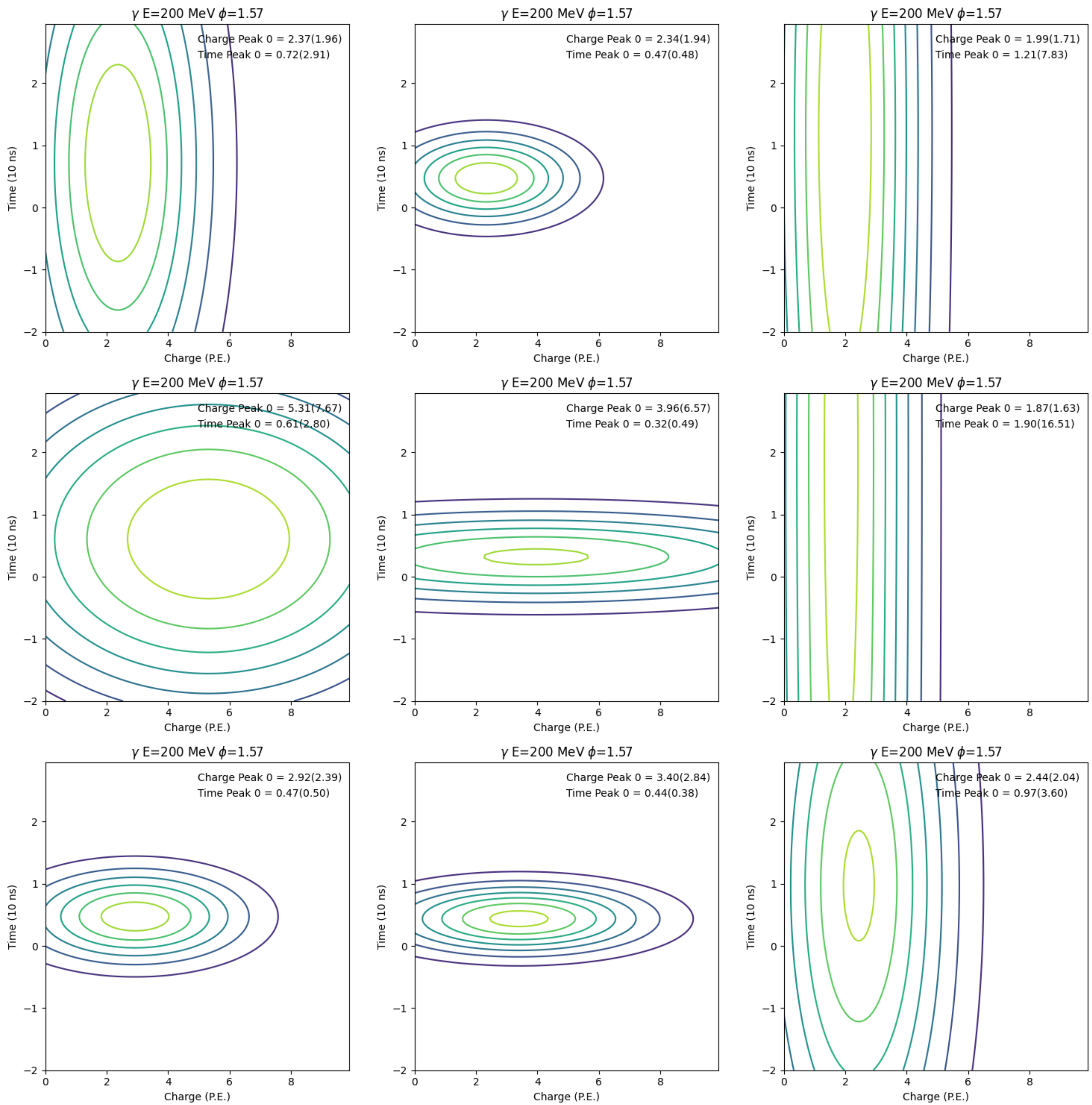
The upper-triangular matrix has to be positive definite, which imposes restriction on the network parameter output. Specifically:

1. The max value has to be on diagonal.
2. The average of diagonal elements must be larger than the off-diagonal term.

- The determinant can be written as the square product of all diagonal elements
- NN output 1/variance instead of variance since the cholesky decomposition is applied to the inverted matrix
- Avoided (1-rho) in the denominator

Back Ups

2D Charge-Time PDF of PMT 7787 - 1 Gaussian Peak



2D Charge-Time PDF of PMT 7787 - 2 Gaussian Peaks