

Beam Instrumentation & Diagnostics Part 1

CAS Introduction to Accelerator Physics

Chavannes de Bogis, 29th of September 2021

Peter Forck

Gesellschaft für Schwerionenforschung (GSI)

p.forck@gsi.de

Beam Instrumentation: Functionality of devices & basic applications

Beam Diagnostics: Usage of devices for complex measurements

Diagnostics is the 'sensory organs' for the beam in the real environment.

(Referring to lecture by Volker Ziemann: 'Detecting imperfections to enable corrections')

Different demands lead to different installations:

- Quick, non-destructive measurements leading to a single number or simple plots
Used as a check for online information. Reliable technologies have to be used
Example: Current measurement by transformers
- Complex instruments for severe malfunctions, accelerator commissioning & development
The instrumentation might be destructive and complex
Example: Emittance determination, chromaticity measurement

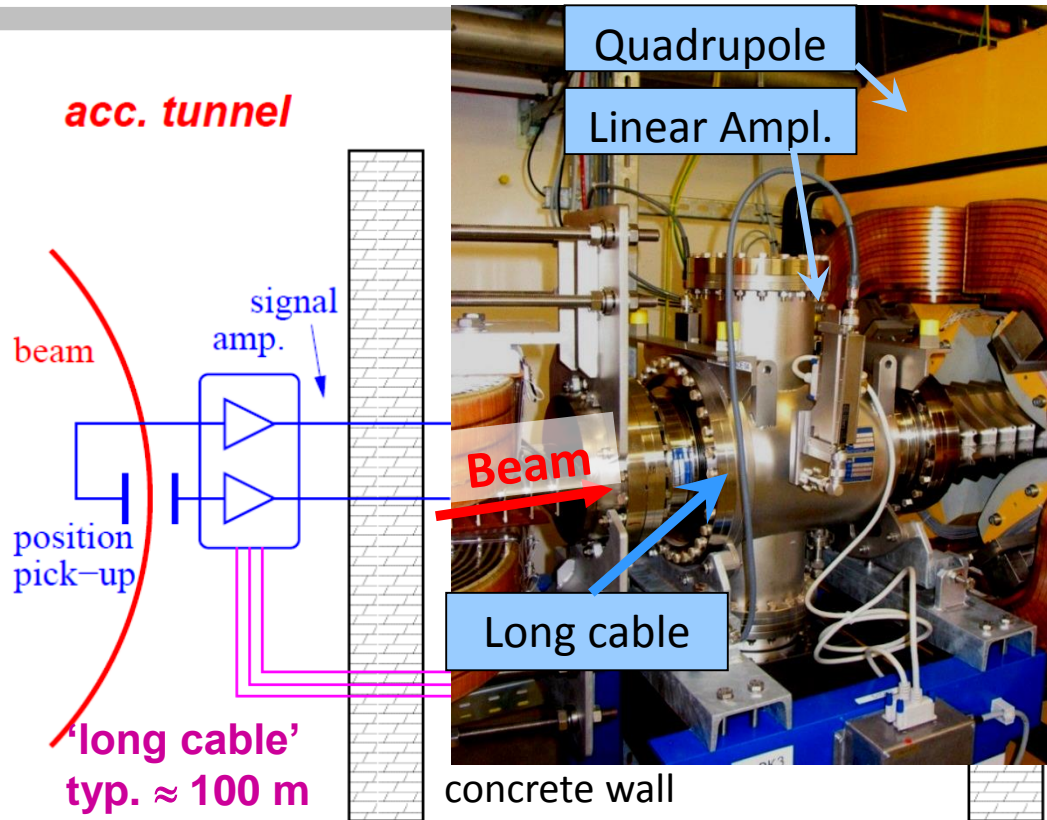
General usage of beam instrumentation:

- Monitoring of beam parameters for operation, beam alignment & accelerator development
- Instruments for automatic, active beam control
Example: Closed orbit feedback at synchrotrons using position measurement by BPMs

Non-invasive (= 'non-intercepting' or 'non-destructive') methods are preferred:

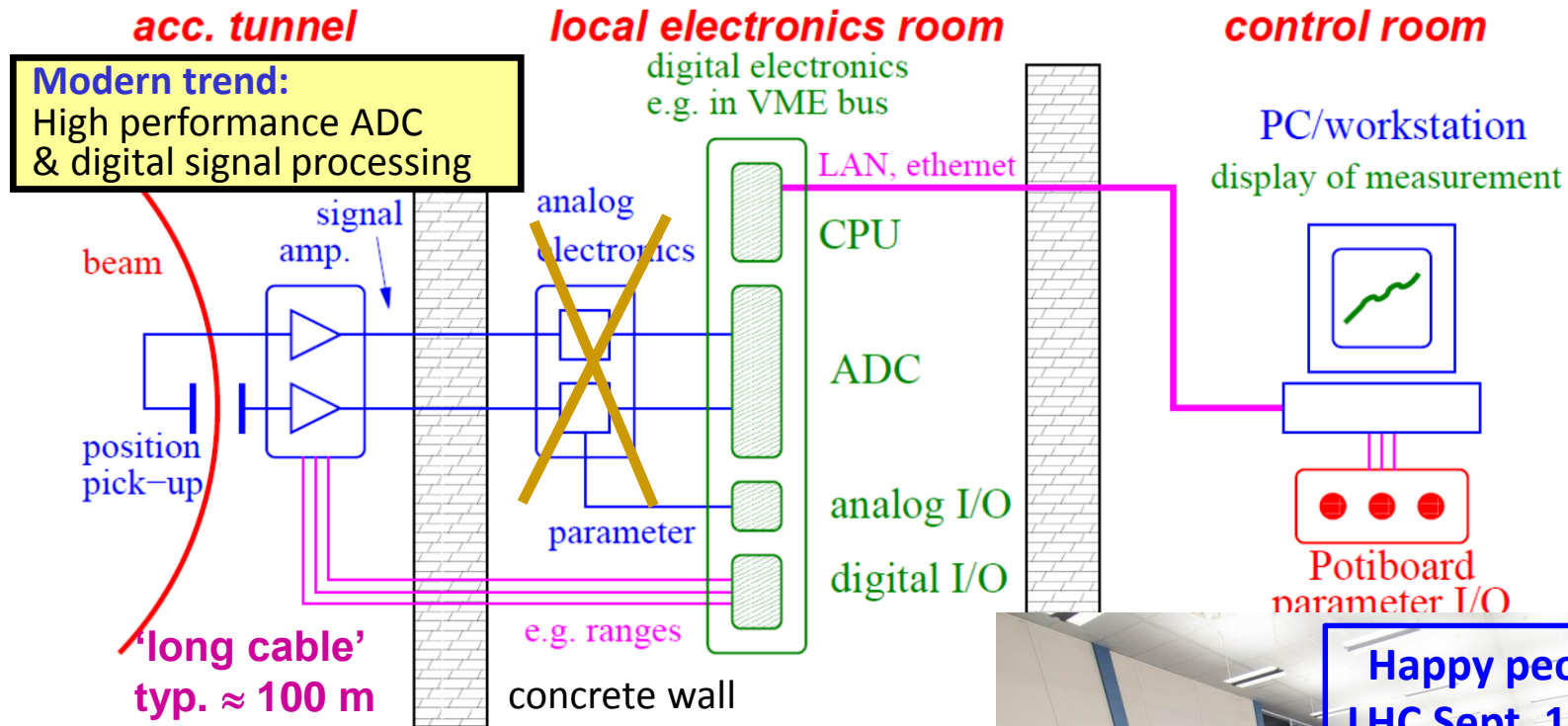
- The beam is not influenced \Rightarrow the **same** beam can be measured at several locations
- The instrument is not destroyed due to high beam power

Typical Installation of a Beam Instrument

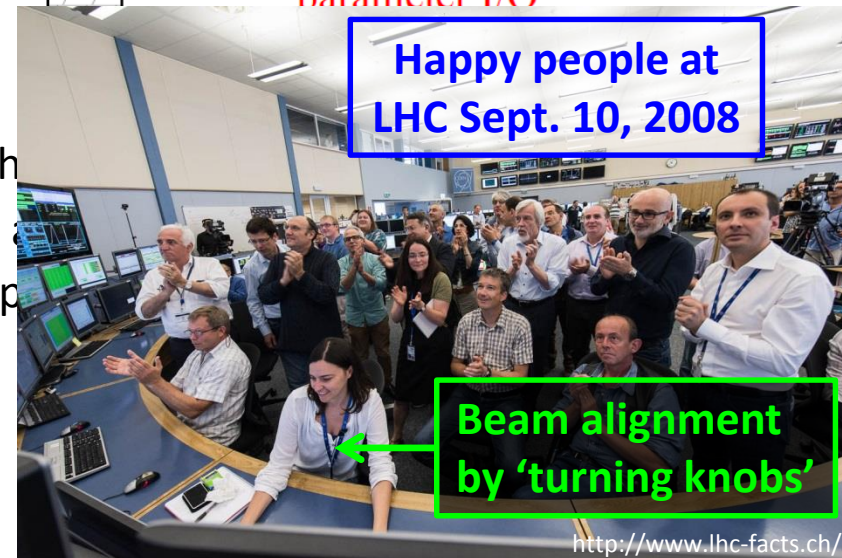


- Accelerator tunnel:** {
- action of the beam to the detector
 - low noise pre-amplifier and first signal shaping
- Local electronics room:** {
- analog treatment, partly combining other parameters
 - digitalization, data bus systems (GPIB, VME, cPCI, μ TCA...)

Typical Installation of a Beam Instrument



- Accelerator tunnel:** {
- action of the beam to the instrument
 - low noise pre-amplifier
- Local electronics room:** {
- analog treatment, parameter
 - digitalization, data transfer
- Control room:** {
- visualization and storage
 - parameter setting



<http://www.lhc-facts.ch/>

The ordering of the subjects is oriented by the beam quantities:

Part 1 of the lecture on electro-magnetic monitors:

- Current measurement
- Beam position monitors for bunched beams

Part 2 of the lecture on transverse and longitudinal diagnostics:

- Profile measurement
- Transverse emittance measure
- Measurement of longitudinal parameters

Lecture on Machine Protection System on Thursday:

- Beam loss detection as one subject

Instruments could be different for:

- Transfer lines with single pass \leftrightarrow synchrotrons with multi-pass
- Electrons are (nearly) always relativistic \leftrightarrow protons are at the beginning non-relativistic

Remark:

Most instrumentation is installed outside of rf-cavities to prevent for signal disturbance

The beam current and its time structure the basic quantity of the beam:

- It is the first check of the accelerator functionality
- It has to be determined in an absolute manner
- Important for transmission measurement and to prevent beam losses.

Different devices are used:

- **Transformers:** Measurement of the beam's **magnetic field**
 - Non-destructive
 - No dependence on beam type and energy
 - They have lower detection threshold.
- **Faraday cups:** Measurement of the beam's **electrical charges**

Magnetic field of the beam and the ideal Transformer

➤ Beam current of N_{part} charges with velocity β

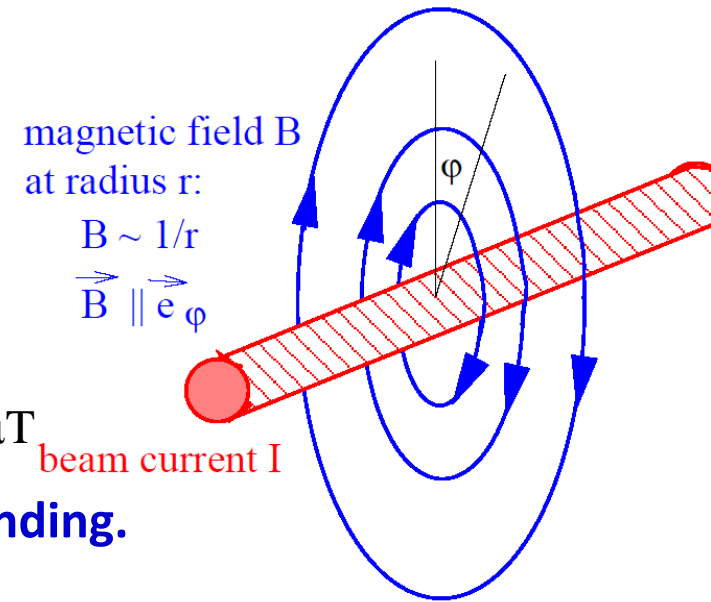
$$I_{beam} = qe \cdot \frac{N_{part}}{t} = qe \cdot \beta c \cdot \frac{N_{part}}{l}$$

➤ cylindrical symmetry

→ only azimuthal component

$$\vec{B} = \mu_0 \frac{I_{beam}}{2\pi r} \cdot \vec{e}_\phi$$

Example: $I = 1\mu A$, $r = 10cm \Rightarrow B_{beam} = 2pT$, earth $B_{earth} = 50\mu T$



Idea: Beam as primary winding and sense by sec. winding.

⇒ Loaded current transformer

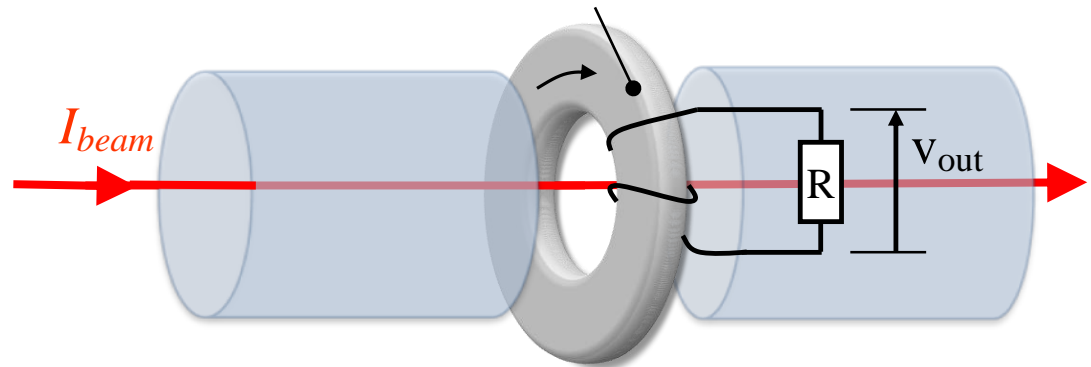
$$I_1/I_2 = N_2/N_1 \Rightarrow I_{sec} = 1/N \cdot I_{beam}$$

Inductance of a torus of μ_r

$$L = \frac{\mu_0 \mu_r}{2\pi} \cdot l N^2 \cdot \ln \frac{r_{out}}{r_{in}}$$

➤ Goal of torus: Large inductance **L** **and** guiding of field lines.

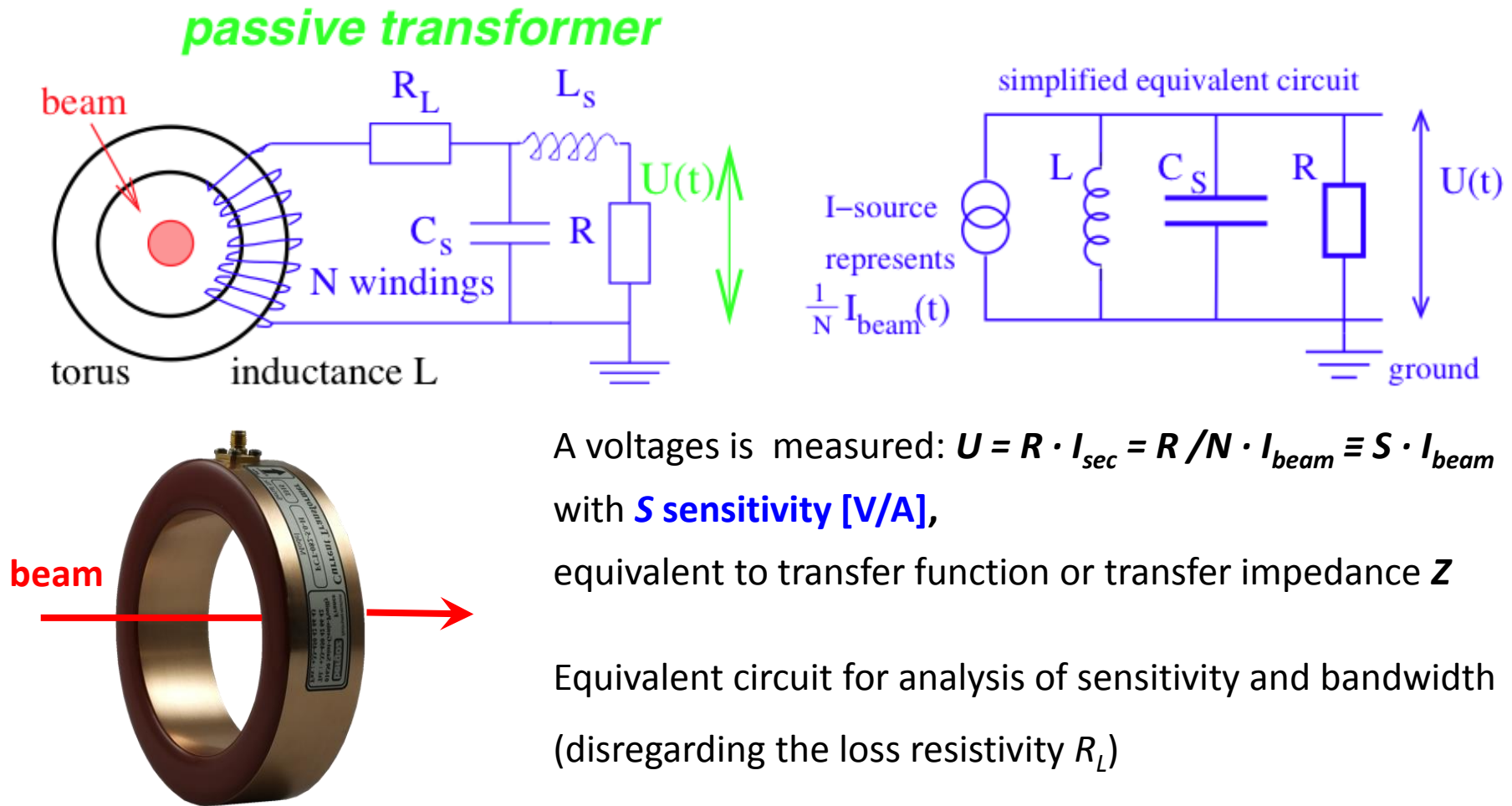
Torus to guide the magnetic field



Definition: $U = L \cdot dI/dt$

Fast Current Transformer FCT (or Passive Transformer)

Simplified electrical circuit of a passively loaded transformer:



Response of the Passive Transformer: Rise and Droop Time

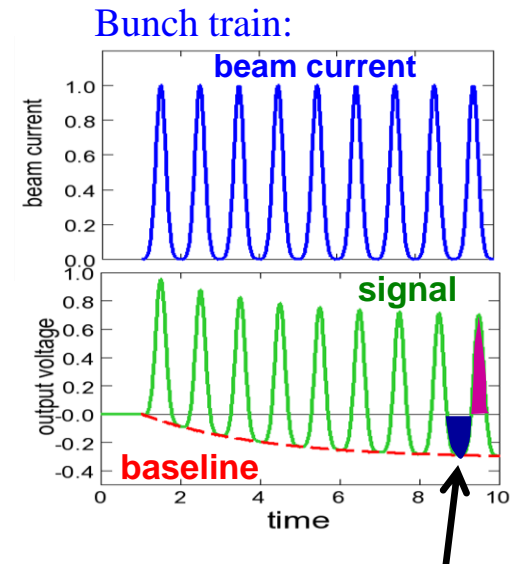
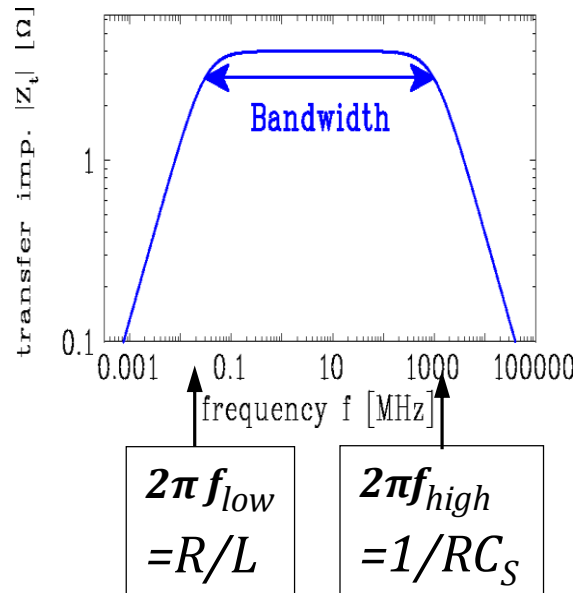
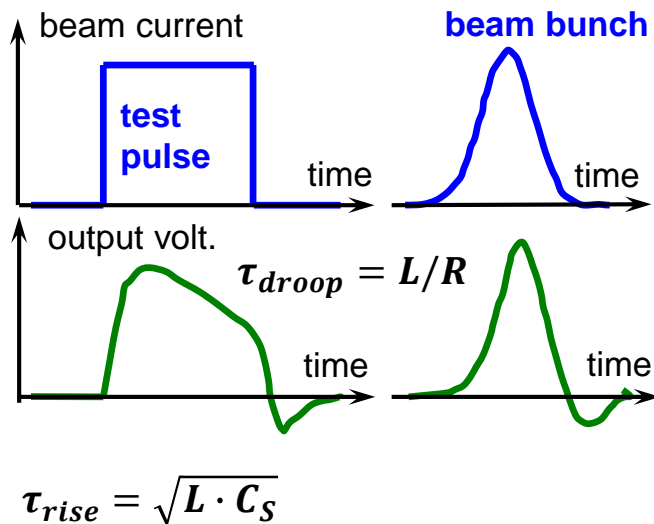
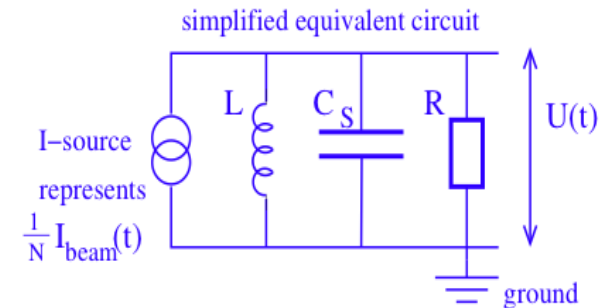
Time domain description:

Droop time: $\tau_{droop} = 1/(2\pi f_{low}) = L/R$

Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = RC_S$ (ideal without cables)

Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = \sqrt{L_S C_S}$ (with cables)

R_L : loss resistivity, R : for measuring.



Baseline: $U_{base} \propto 1 - \exp(-t/\tau_{droop})$
positive & negative areas are equal

Example for Fast Current Transformer

For bunch beams e.g. during accel. in a synchrotron
typical bandwidth of $2\text{ kHz} < f < 1\text{ GHz}$

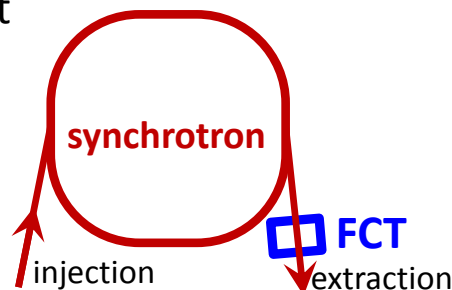
$\Leftrightarrow 10\text{ ns} < t_{\text{bunch}} < 1\text{ }\mu\text{s}$ is well suited

Example: GSI Fast Current Transformer **FCT**:

Inner / outer radius	70 / 90 mm
Permeability	$\mu_r \approx 10^5$ for $f < 100\text{ kHz}$ $\mu_r \propto 1/f$ above
Windings	10
Sensitivity	4 V/A for $R = 50\text{ }\Omega$
Droop time $\tau_{\text{droop}} = L/R$	0.2 ms
Rise time $\tau_{\text{rise}} = \sqrt{L_S C_S}$	1 ns
Bandwidth	2 kHz ... 500 MHz

Numerous application e.g.:

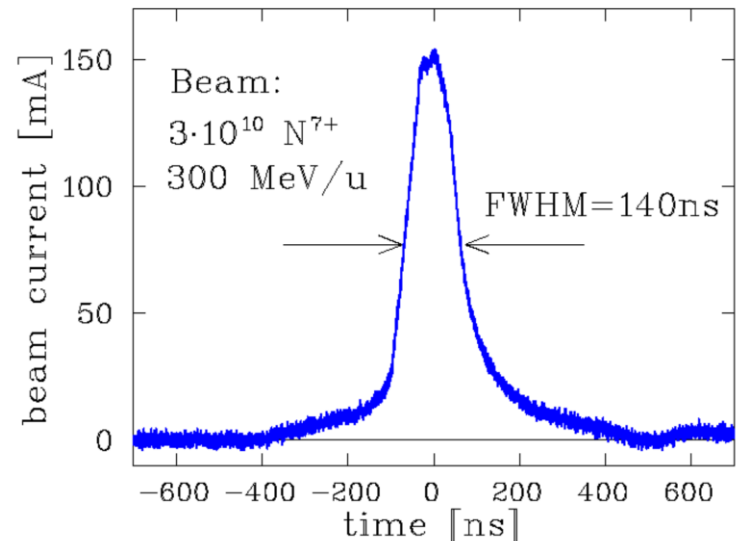
- Transmission optimization
- Bunch shape measurement
- Input for synchronization of 'beam phase'



From
Company Bergoz



Fast extraction from GSI synchrotron:



Example for Fast Current Transformer

For bunch beams e.g. during accel. in a synchrotron
typical bandwidth of $2 \text{ kHz} < f < 1 \text{ GHz}$

$\Leftrightarrow 10 \text{ ns} < t_{\text{bunch}} < 1 \text{ } \mu\text{s}$ is well suited

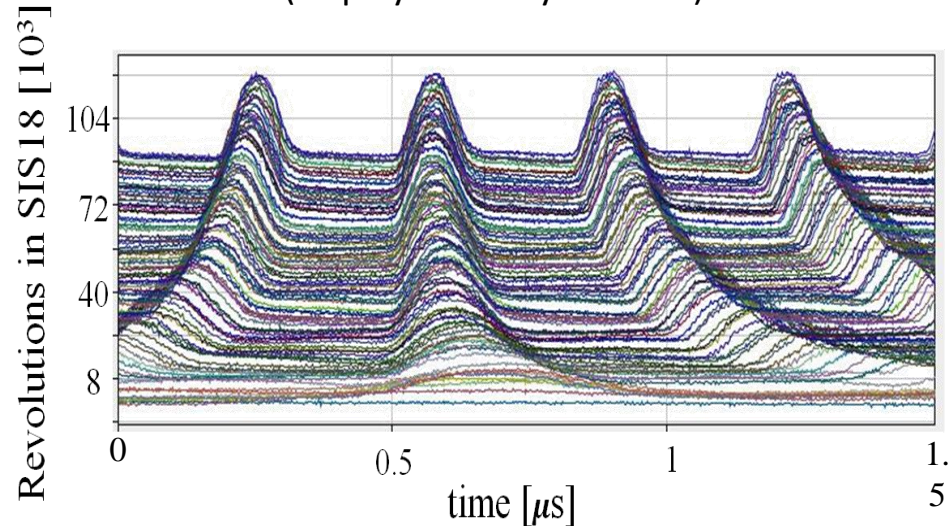
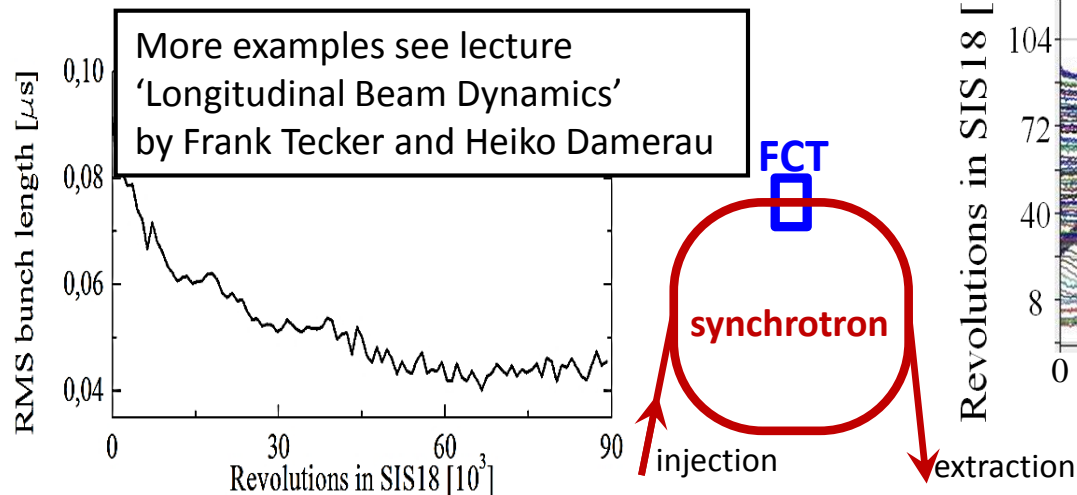
Example GSI type:

Inner / outer radius	70 / 90 mm
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Windings	10
Sensitivity	4 V/A for $R = 50 \text{ } \Omega$
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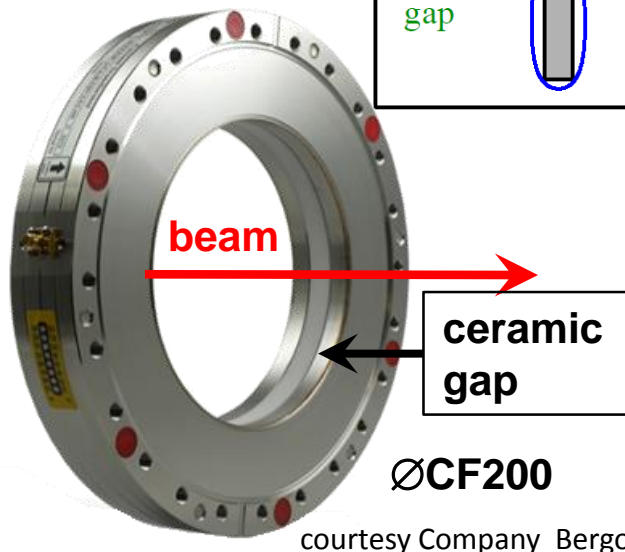
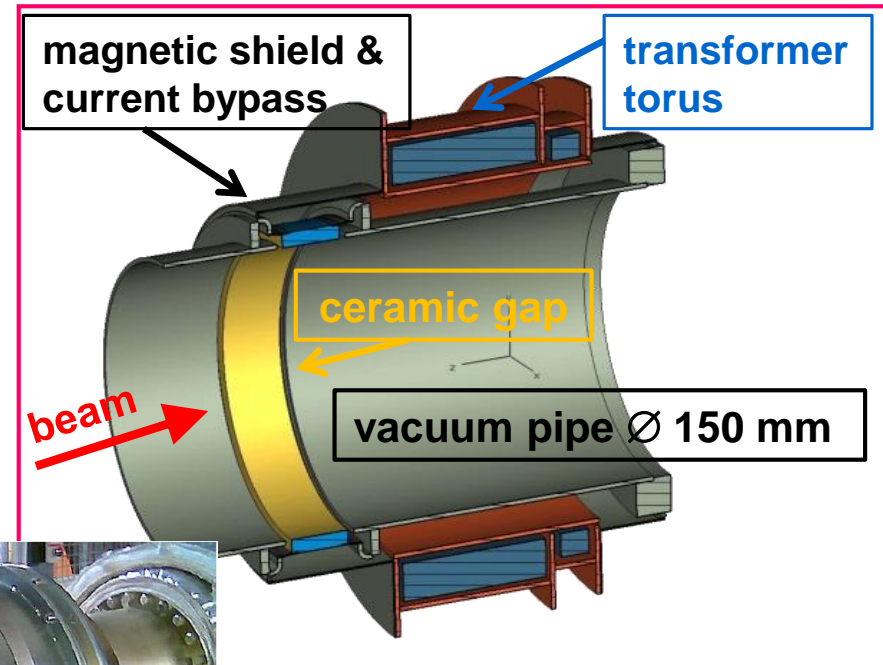
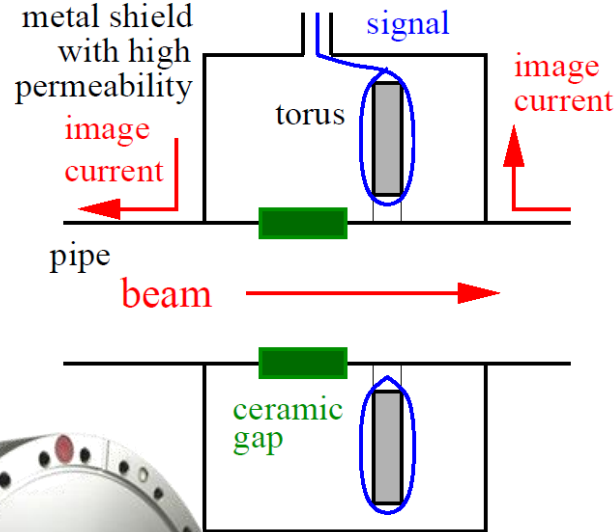
Example: U^{73+} from 11 MeV/u ($\beta = 15 \%$) to 350 MeV/u within 300 ms (displayed every 0.15 ms)



Shielding of a Transformer

Task of the shield:

- The image current of the walls have to be bypassed by a gap and a metal housing.
- This housing uses μ -metal and acts as a shield of external B-field
(remember: $I_{beam} = 1 \mu A$, $r = 10 \text{ cm} \Rightarrow B_{beam} = 2 \text{ pT}$, earth field $B_{earth} = 50 \mu T$)



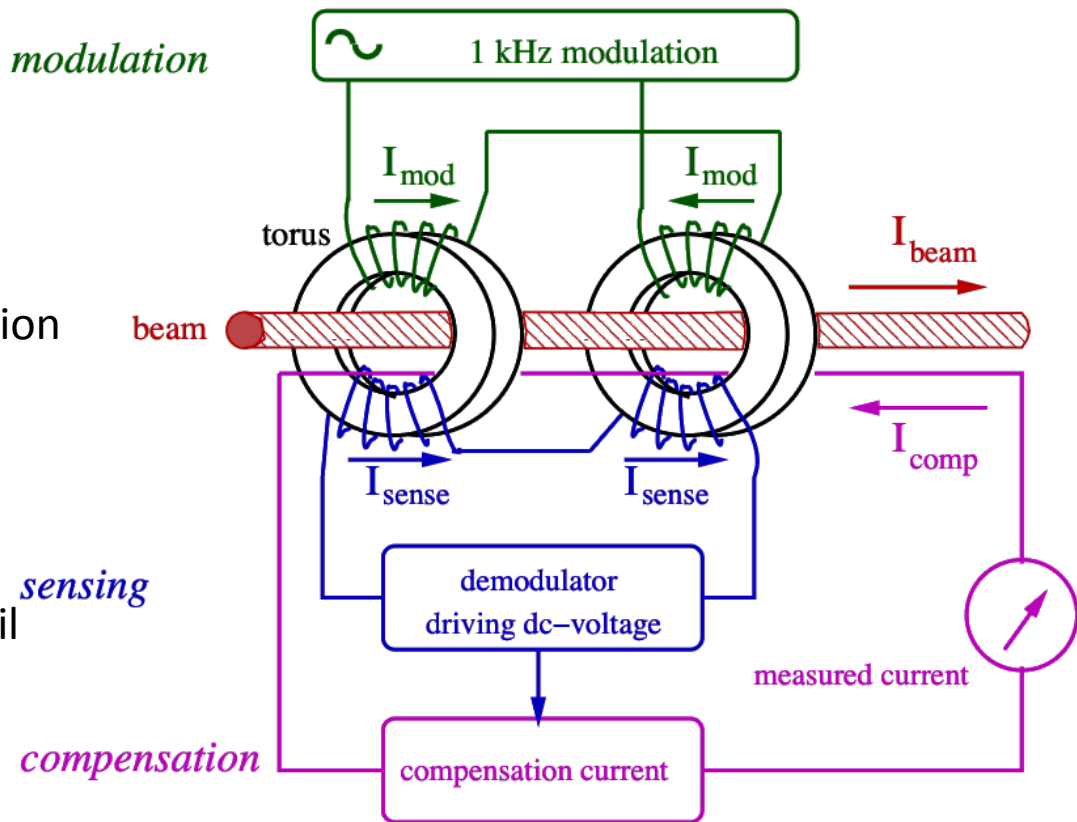
The dc Transformer

How to measure the DC current? The current transformer discussed sees only B-flux *changes*.
 The DC Current Transformer (DCCT) → look at the magnetic saturation of two torii.

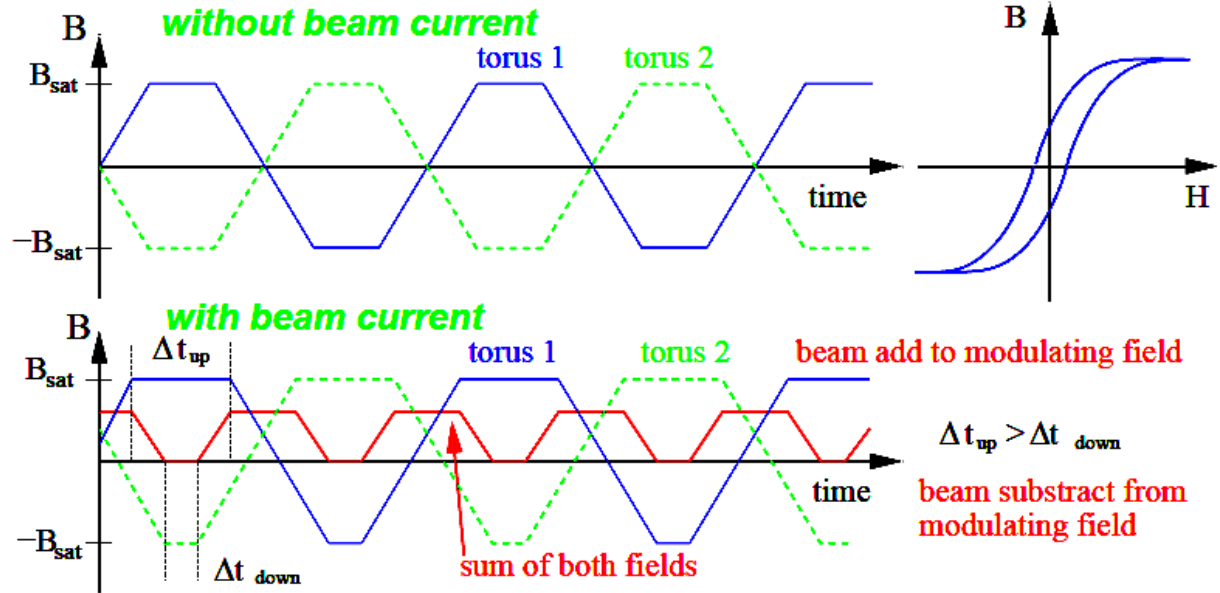
Depictive statement:

A single transformer needs varying beam. The trick is to ‘switch two transformers’!

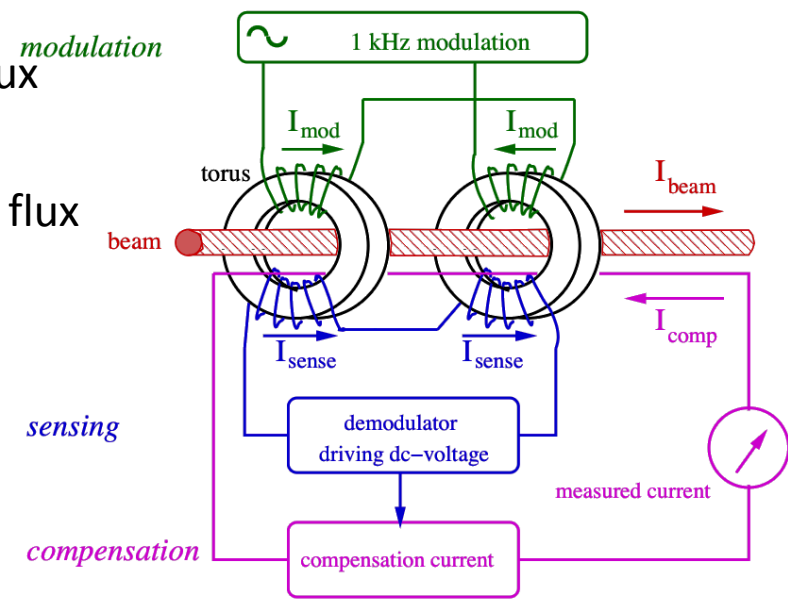
- **Modulation** of the primary windings forces both torii into saturation twice per cycle
- **Sense windings** measure the modulation signal and cancel each other.
- But with the I_{beam} , the saturation is shifted and I_{sense} is not zero
- **Compensation current** adjustable until I_{sense} is zero once again



The dc Transformer



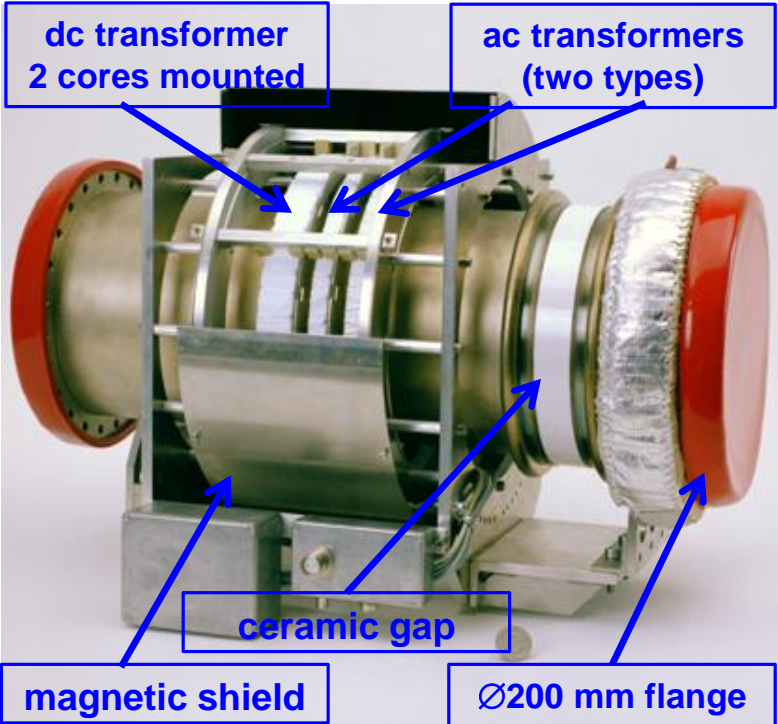
- **Modulation without beam:**
typically about 9 kHz to saturation → **no** net flux
 - **Modulation with beam:**
saturation is reached at different times, → net flux
 - **Net flux:** double frequency than modulation
 - **Feedback:** Current fed to compensation winding
for larger sensitivity
 - **Two magnetic cores:** Must be very similar.
- Remark: Same principle used for power suppliers



The dc Transformer Realization

Example: The DCCT at GSI synchrotron

Torus radii	$r_i = 135 \text{ mm}$ $r_o = 145 \text{ mm}$
Torus thickness	$d = 10 \text{ mm}$
Torus permeability	$\mu_r = 10^5$
Saturation inductance	$B_{\text{sat}} = 0.6 \text{ T}$
Number of windings	16 for modulation & sensing 12 for feedback
Resolution	$I_{\text{min}}^{\text{beam}} = 2 \text{ }\mu\text{A}$
Bandwidth	$\Delta f = \text{dc} \dots 20 \text{ kHz}$
Rise time constant	$\tau_{\text{rise}} = 10 \text{ }\mu\text{s}$
Temperature drift	$1.5 \text{ }\mu\text{A}/^\circ\text{C}$



Measurement with a dc Transformer

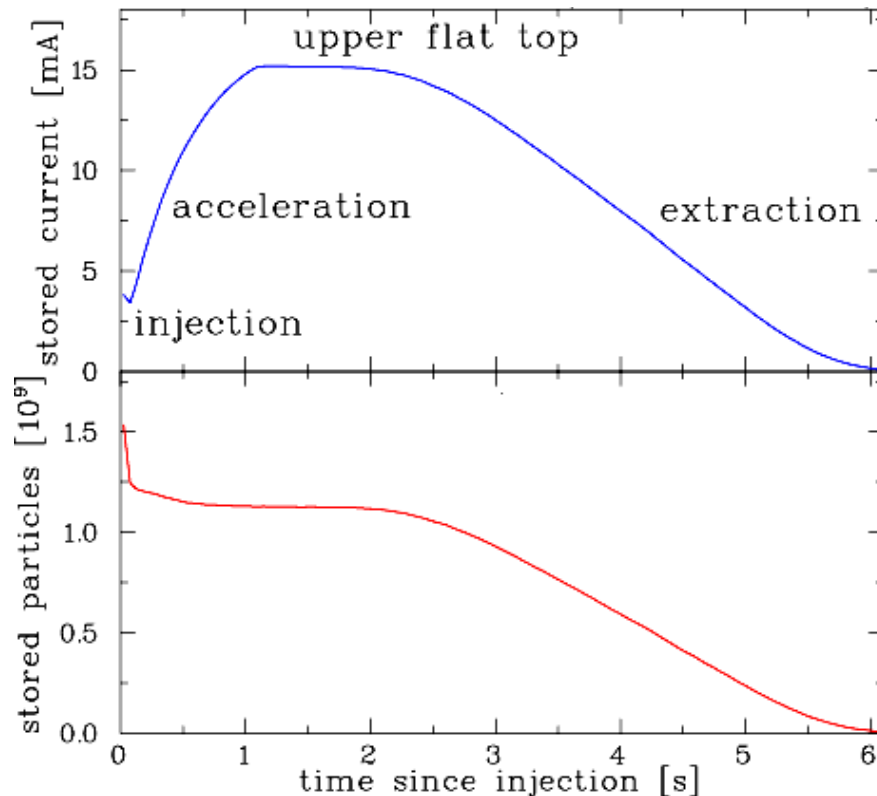
Application for dc transformer:

⇒ Observation of beam behavior with typ. 20 μ s time resolution → the basic operation tool

Example: The DCCT at GSI synchrotron

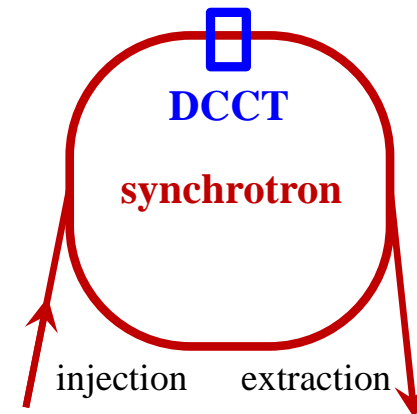
U^{73+} accelerated from

11.4 MeV/u ($\beta = 15.5\%$) to 750 MeV/u ($\beta = 84\%$)



Important parameter:

- Detection threshold: $\approx 1 \mu A$
(= resolution)
- Bandwidth: $\Delta f = \text{dc to } 20 \text{ kHz}$
- Rise-time: $t_{rise} = 20 \mu s$
- Temperature drift: $1.5 \mu A/^{\circ}C$
⇒ compensation required.



➤ **Transformers:** Measurement of the beam's **magnetic field**

Non-destructive

No dependence on beam type and energy

They have lower detection threshold.

➤ **Faraday cups:** Measurement of the beam's **electrical charges**

They are destructive

For low energies only

Low currents can be determined.

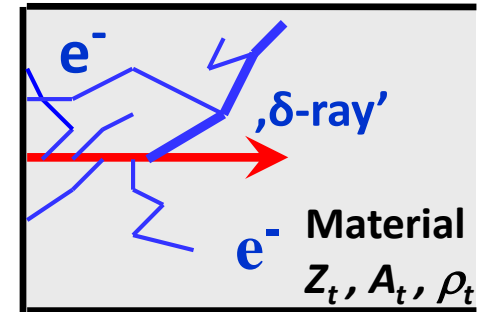
Bethe-Bloch formula: $-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \cdot \frac{Z_t}{A_t} \rho_t \cdot Z_p^2 \cdot \frac{1}{\beta^2} \left(\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 \cdot W_{max}}{I^2} - \beta^2 \right)$

(simplest formulation)

Semi-classical approach:

- Projectiles of mass **M** collide with free electrons of mass **m**
- If **M** >> **m** then the relative energy transfer is low
⇒ many collisions required many electrons participate
proportional to target electron density $n_e = \frac{Z_t}{A_t} \rho_t$

beam, charge Z_p
mass **M**



- ⇒ low straggling for the heavy projectile i.e. 'straight trajectory'
- If projectile velocity $\beta \approx 1$ low relative energy change of projectile (γ is Lorentz factor)
- I is mean ionization potential including kinematic corrections $I \approx Z_t \cdot 10 \text{ eV}$ for most metals
- Strong dependence on projectile charge Z_p as $\frac{dE}{dx} \propto Z_p^2$

Constants: N_A Avogadro number, r_e classical e^- radius, m_e electron mass, c velocity of light

Maximum energy transfer from projectile **M** to electron m_e : $W_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$

Bethe-Bloch formula: $-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \cdot \frac{Z_t}{A_t} \rho_t \cdot Z_p^2 \cdot \frac{1}{\beta^2} \left(\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 \cdot W_{max}}{I^2} - \beta^2 \right)$
(simplest formulation)

Range:

$$R = \int_0^{E_{max}} \left(\frac{dE}{dx} \right)^{-1} dE$$

with approx. scaling $R \propto E_{max}^{1.75}$

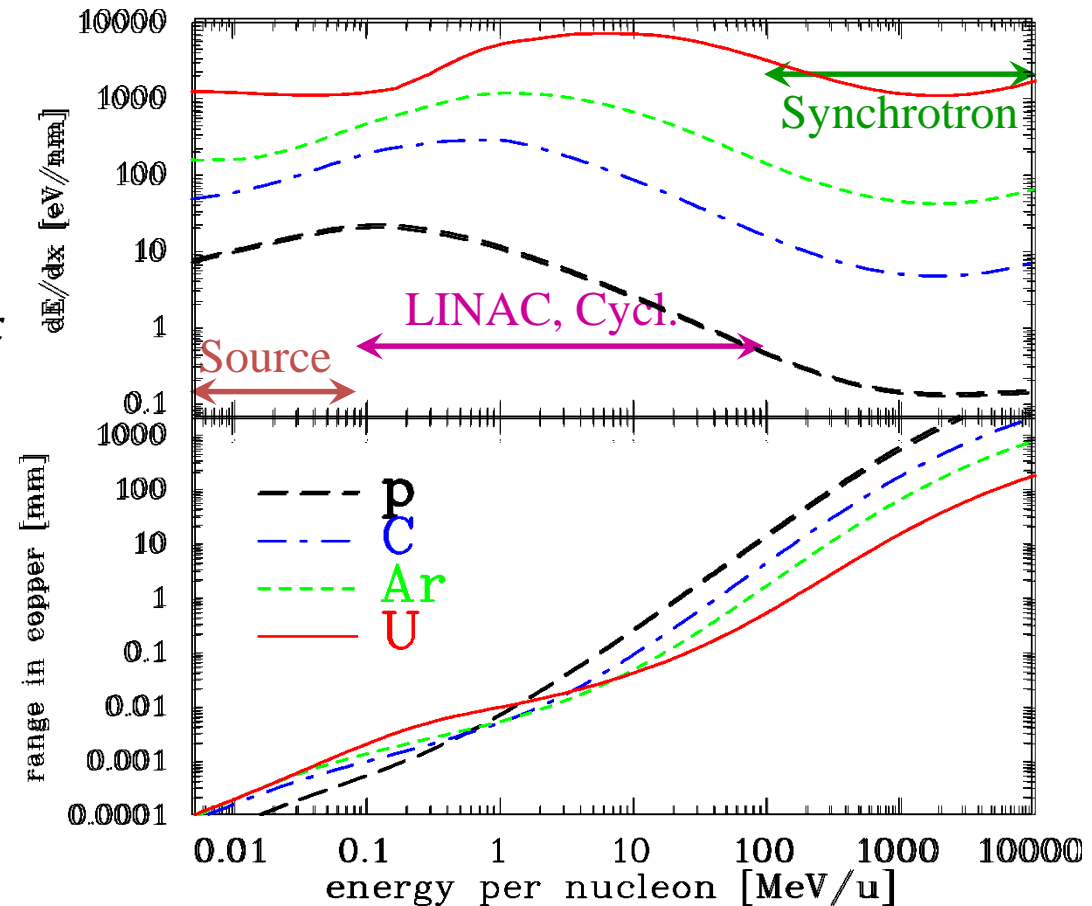
Numerical calculation for **ions**

with semi-empirical model e.g. SRIM

Main modification $Z_P \rightarrow Z_p^{eff}(E_{kin})$

\Rightarrow Cups only for

$E_{kin} < 100 \text{ MeV/u}$ due to $R < 10 \text{ mm}$



Approximation e.g. $Z_p^{eff} \approx Z_p \left[1 - \exp \left(-Z_p^{-2/3} c\beta / V_{Bohr} \right) \right]$

Secondary Electron Emission caused by Ion Impact

Energy loss of ions in metals close to a surface:

Closed collision with large energy transfer: \rightarrow fast e^- with $E_{kin} > 100$ eV

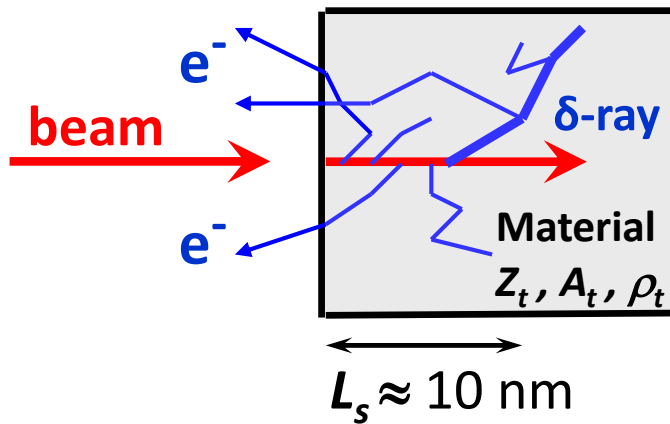
Distant collision with low energy transfer \rightarrow slow e^- with $E_{kin} \leq 10$ eV

\rightarrow 'diffusion' & scattering with other e^- : scattering length $L_s \approx 1 - 10$ nm

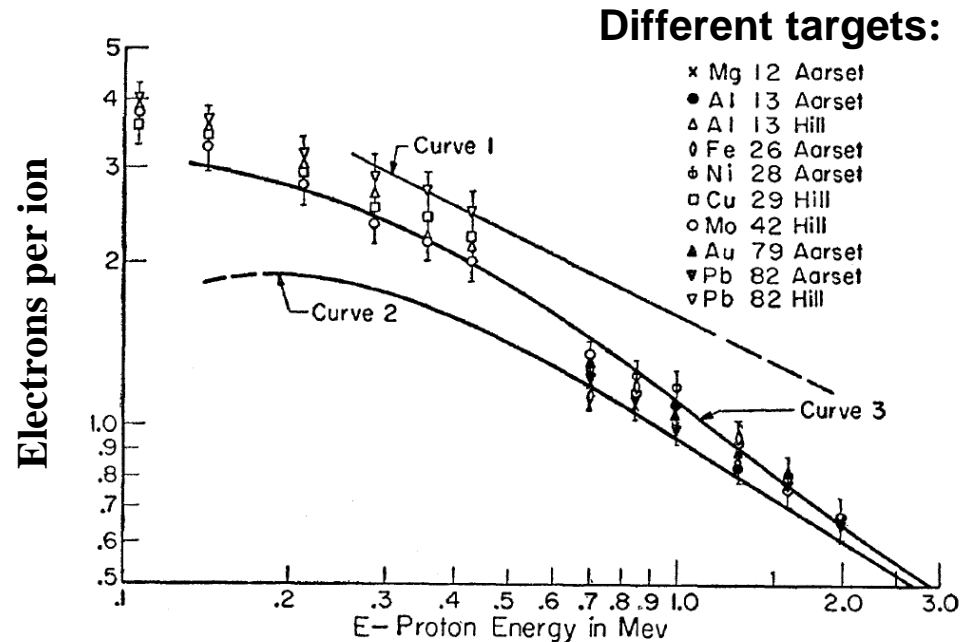
\rightarrow at surface ≈ 90 % probability for escape

Secondary **electron yield** and energy distribution comparable for all metals!

$$\Rightarrow Y = \text{const.} * dE/dx \quad (\text{Sternglass formula})$$

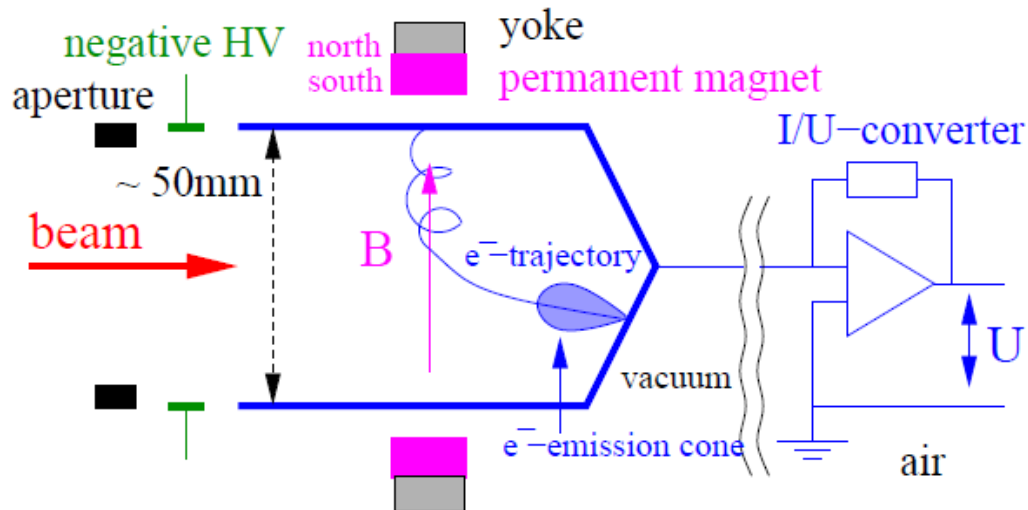


E.J. Sternglass, Phys. Rev. 108, 1 (1957)

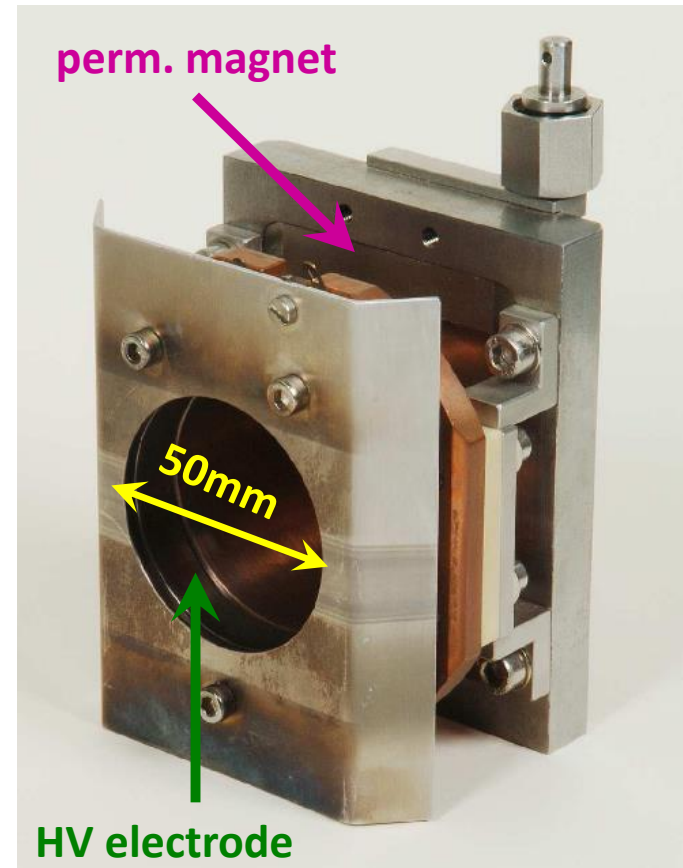


Faraday Cups for Beam Charge Measurement

The beam particles are collected inside a metal cup
 \Rightarrow The beam's charge are recorded as a function of time.



The cup is moved in the beam pass
 \rightarrow destructive device



Currents down to 10 pA with bandwidth of 100 Hz!

To prevent for secondary electrons leaving the cup

Magnetic field:

The central field is $B \approx 10 \text{ mT} \Rightarrow r_c = \frac{mB}{e} \cdot v_{\perp} \approx 1 \text{ mm}$.

or Electric field: Potential barrier at the cup entrance $U \approx 1 \text{ kV}$.

Realization of a Faraday Cup at GSI LINAC

The Cup is moved into the beam pass.

Faraday Cup
 $\varnothing 60$ mm

vacuum flange
 here $\varnothing 150$ mm

bellow
compression
for movement

pneumatic
drive

beam

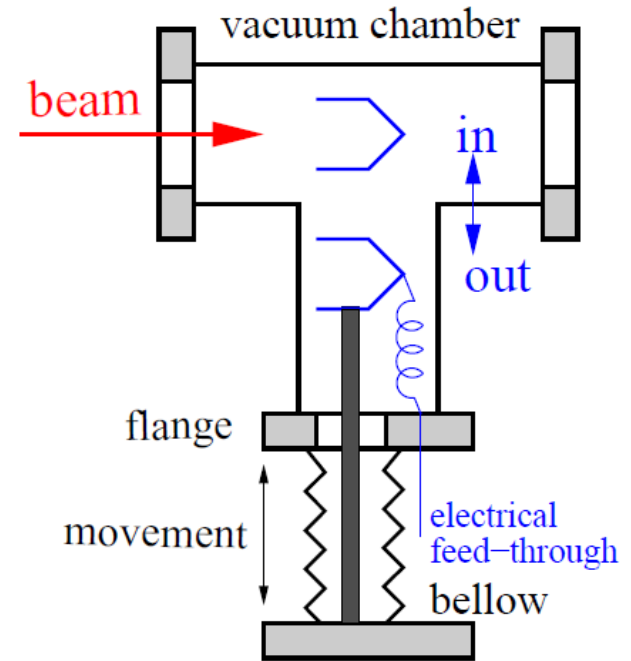
≈ 1 m

source

Cup: beam stopped

RFQ

LINAC



Transformer: → measurement of the beam's magnetic field

- Magnetic field is guided by a high μ toroid
- **Types:** FCT → large bandwidth, $I_{min} \approx 30 \mu\text{A}$, BW = 10 kHz ... 500 MHz
[ACT : $I_{min} \approx 0.3 \mu\text{A}$, BW = 10 Hz 1 MHz, used at proton LINACs]
DCCT: two toroids + modulation, $I_{min} \approx 1 \mu\text{A}$, BW = dc ... 20 kHz
- non-destructive, used for all beams

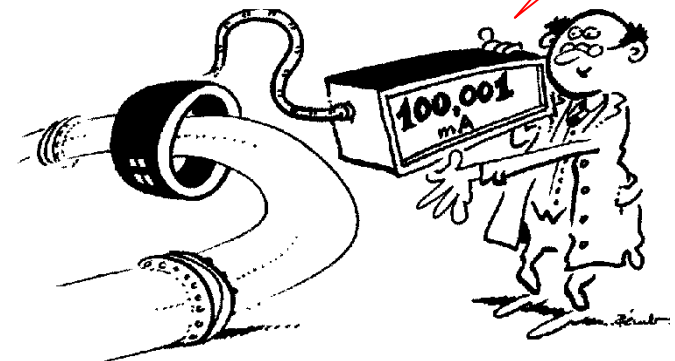
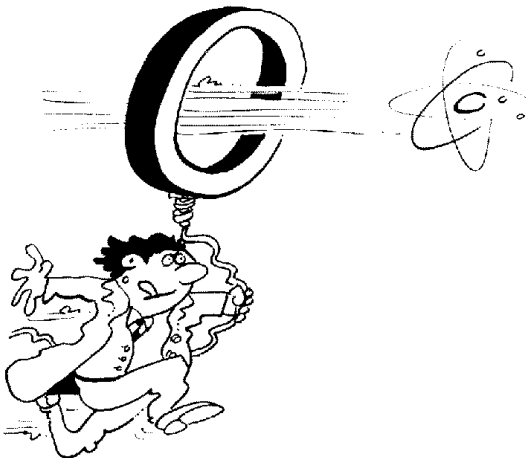
Faraday cup: → measurement of beam's charge,

- low threshold by I/U-converter: $I_{beam} > 10 \text{ pA}$
- totally destructive, used for low energy beams only

Fast Transformer FCT

Active transformer ACT

DC transformer DCCT



Company Bergoz

Outline:

- Signal generation → transfer impedance
- Capacitive *button* BPM for high frequencies
- Capacitive *linear-cut* BPM for low frequencies
- Electronics for position evaluation
- BPMs for measurement
- Summary

A Beam Position Monitor is an non-destructive device for bunched beams

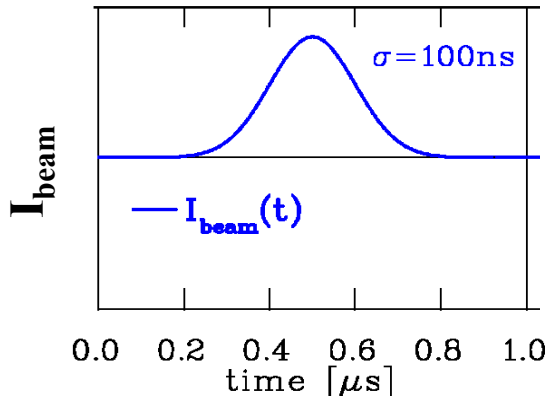
It delivers information about the transverse center of the beam:

- **Trajectory:** Position of an individual bunch within a transfer line or synchrotron
- **Closed orbit:** Central orbit averaged over a period much longer than a betatron oscillation
- **Single bunch position:** Determination of parameters like tune, chromaticity, β -function

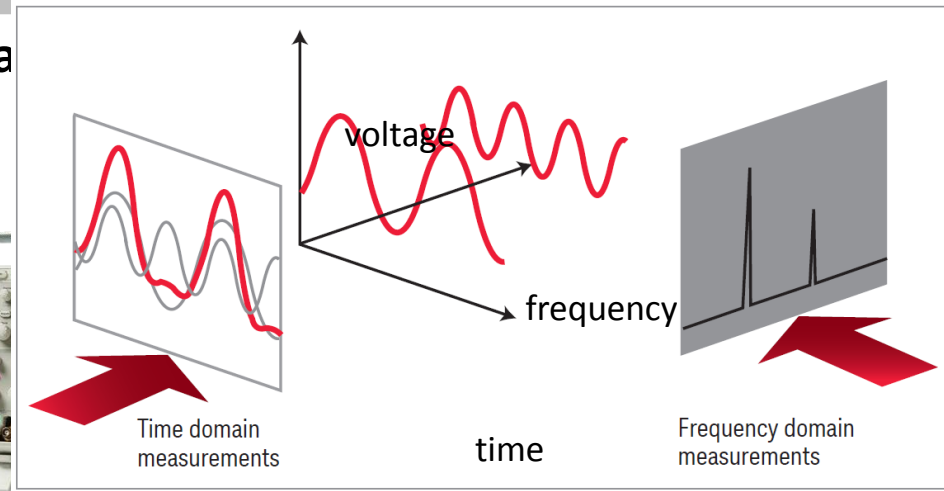
Remarks: - BPMs have a low cut-off frequency \Leftrightarrow dc-beam behavior can't be monitored
- The abbreviation **BPM** and pick-up **PU** are synonyms

Time Domain ↔ Frequency Domain

Time domain: Recording of a voltage as a function of time

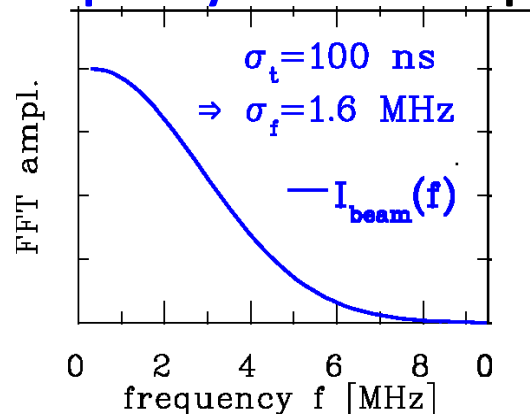


Instrument:
Oscilloscope



Frequency domain: Displaying of a voltage as a function of frequency:

courtesy company Keysight



Instrument:
Spectrum Analyzer



Fourier Transformation:

- Contains amplitude & phase
- The **same** information is displayed differently

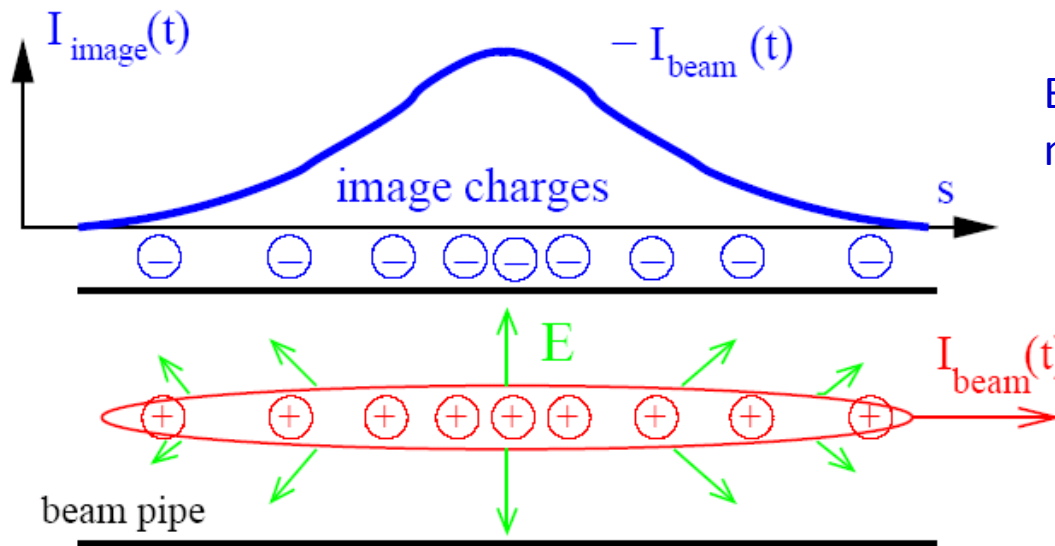
Law of Convolution: For a convolution in time: $f(t) = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t - \tau) d\tau$

$$\Rightarrow \hat{f}(\omega) = \hat{f}_1(\omega) \cdot \hat{f}_2(\omega) \Leftrightarrow \text{convolution be expressed as multiplication of FTs}$$

See lecture 'Time and Frequency Domain Signals' by Hermann Schmickler

Pick-Ups for bunched Beams

The image current at the beam pipe is monitored on a high frequency basis
i.e. the ac-part given by the bunched beam.



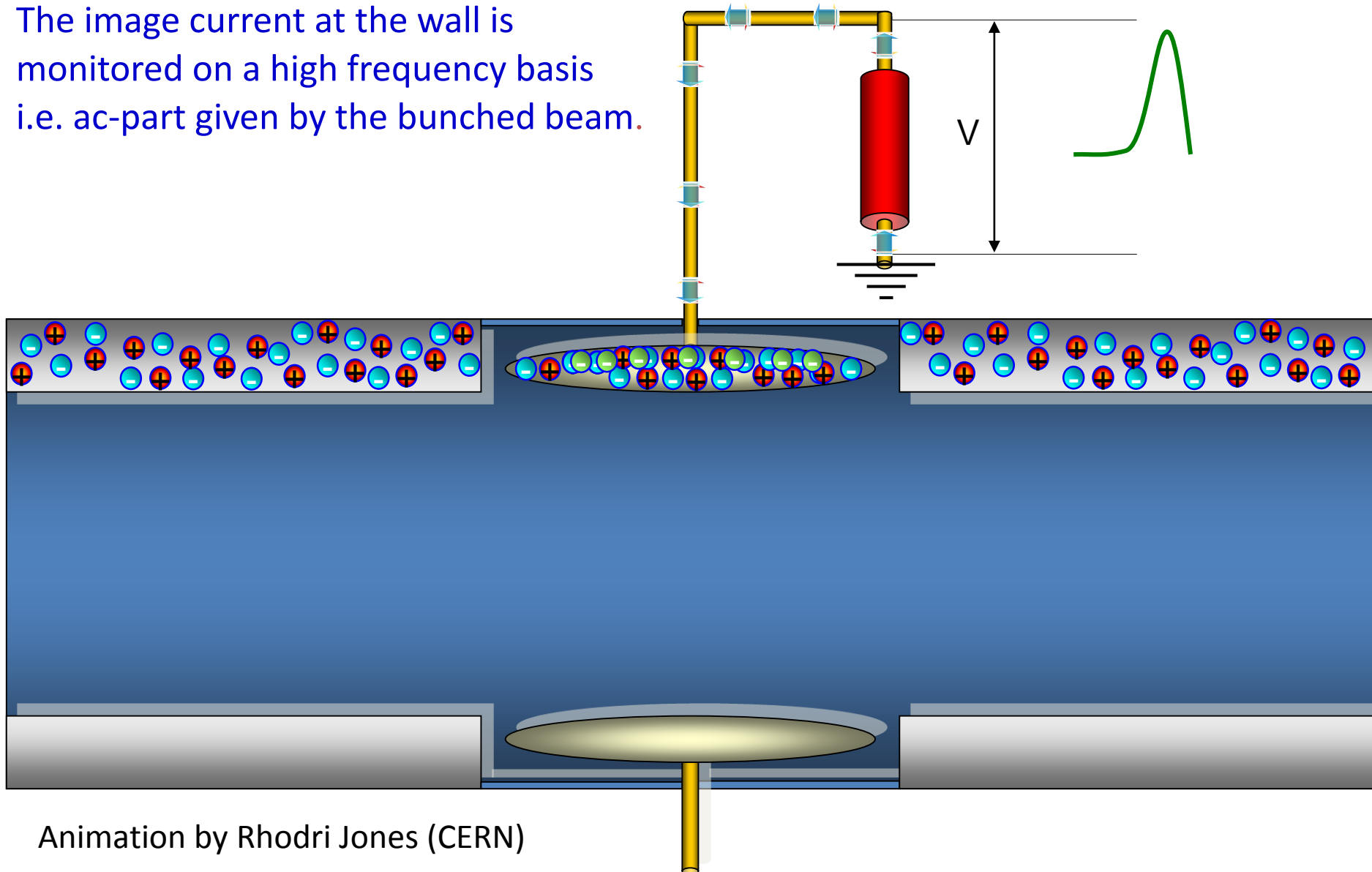
Beam Position Monitor **BPM** is the most frequently used instrument!

For relativistic velocities,
the electric field is transversal:

$$E_{\perp,lab}(t) = \gamma \cdot E_{\perp,rest}(t')$$

Principle of Signal Generation of a BPMs, centered Beam

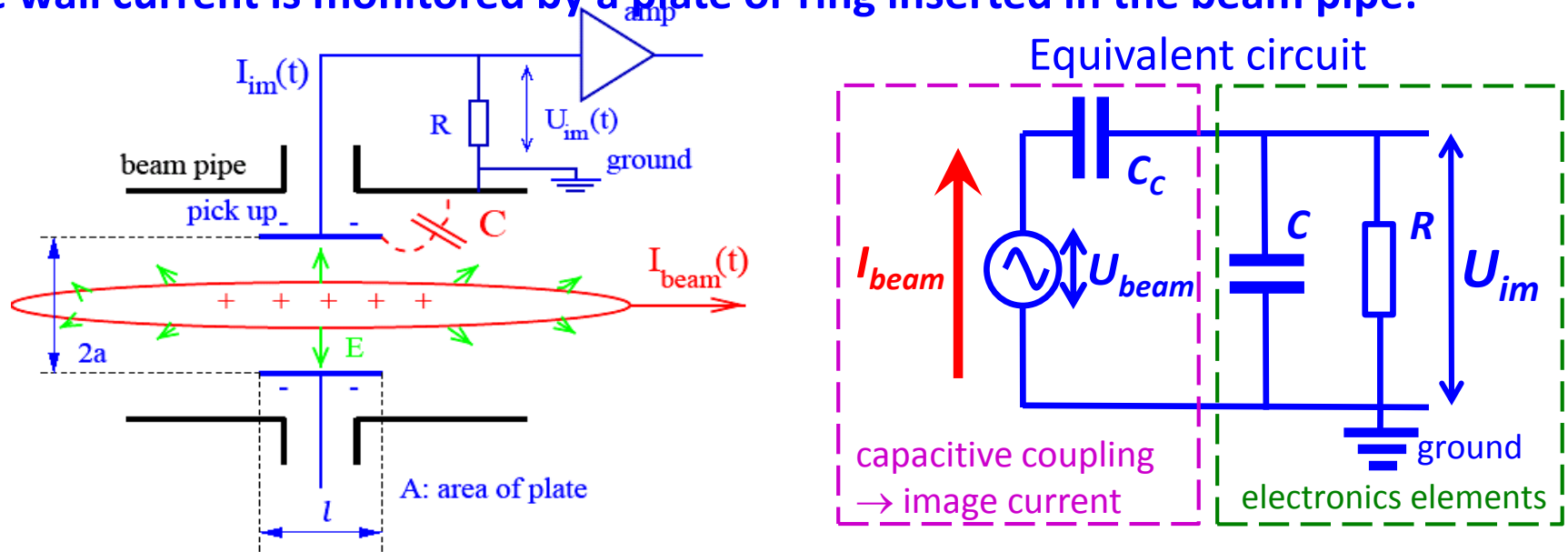
The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

Model for Signal Treatment of capacitive BPMs

The wall current is monitored by a plate or ring inserted in the beam pipe:



At a resistor R the voltage U_{im} from the image current is measured.

Goal: Connection from beam current to signal strength by transfer impedance $Z_t(\omega)$

in frequency domain: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega) \cdot I_{beam}(\omega)$

Result:
$$Z_t(\omega) = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1+i\omega RC} \in \mathbb{C} \text{ i.e. complex function}$$

The equation is annotated with boxes and arrows:

- geometry:** Points to $\frac{A}{2\pi a}$.
- stray capacitance:** Points to $\frac{1}{C}$.
- frequency response:** Points to the $\frac{i\omega RC}{1+i\omega RC}$ term.

Example of Transfer Impedance for Proton Synchrotron

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega) = Z_t(\omega) \cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega / \omega_{cut}}{\sqrt{1 + \omega^2 / \omega_{cut}^2}}$$

$$\varphi = \arctan(\omega_{cut} / \omega)$$

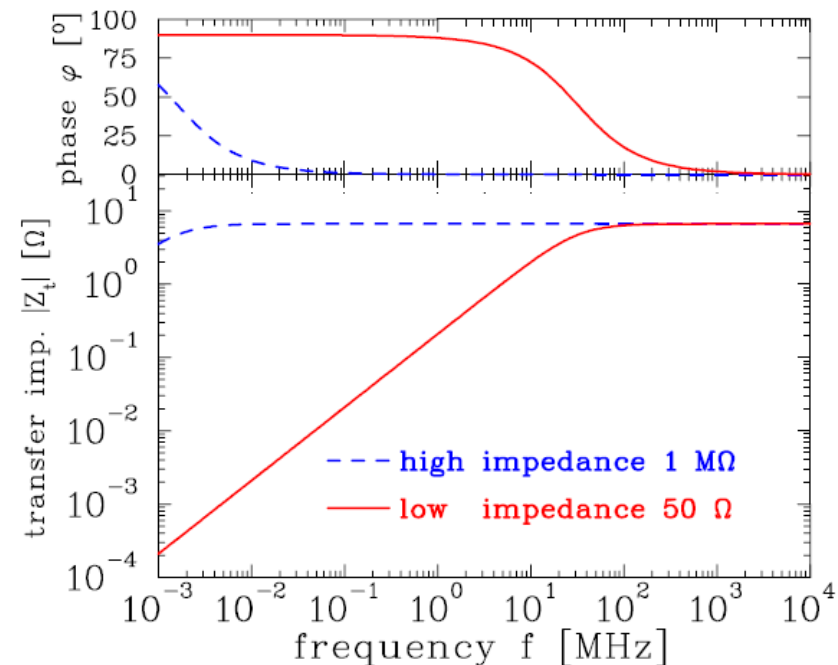
Parameter linear-cut BPM at proton synchr.:

$C = 100\text{pF}$, $l = 10\text{cm}$, $\beta = 50\%$

$$f_{cut} = \omega / 2\pi = (2\pi RC)^{-1}$$

for $R = 50 \Omega \Rightarrow f_{cut} = 32 \text{ MHz}$

for $R = 1 \text{ M}\Omega \Rightarrow f_{cut} = 1.6 \text{ kHz}$



Large signal strength for long bunches → **high impedance**

Smooth signal transmission important for short bunches → **50 Ω**

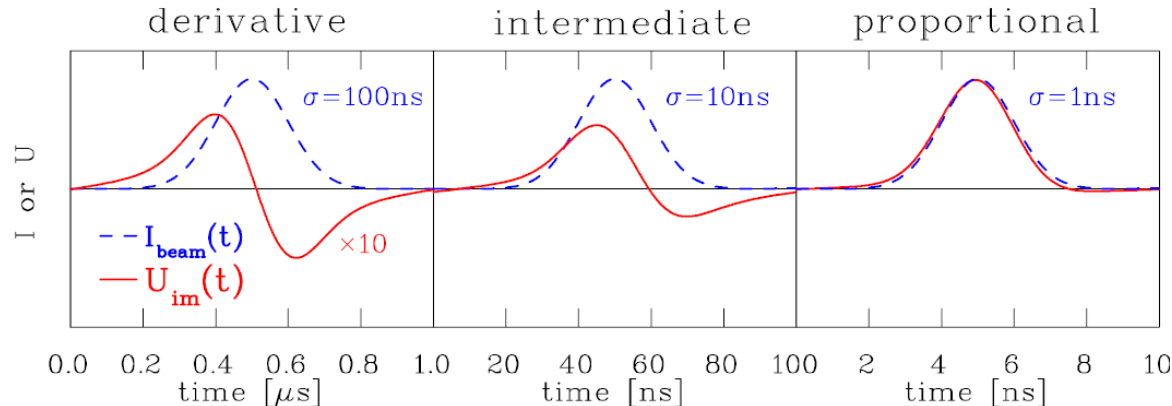
Remark: For $\omega \rightarrow 0$ it is $Z_t \rightarrow 0$ i.e. **no** signal is transferred from dc-beams e.g.

- de-bunched beam inside a synchrotron
- for slow extraction through a transfer line

Calculation of Signal Shape (here single Bunch)

The transfer impedance is used in frequency domain! The following is performed:

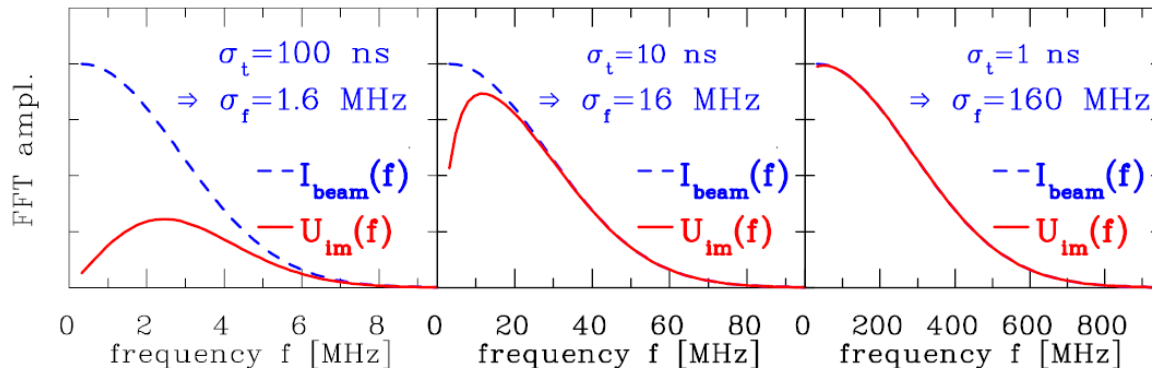
1. **Start:** Time domain Gaussian function $I_{beam}(t)$ having a width of σ_t



Fourier
trans.

inverse
Fourier
trans.

2. FFT of $I_{beam}(t)$ leads to the frequency domain Gaussian $I_{beam}(f)$ with $\sigma_f = (2\pi\sigma_t)^{-1}$



3. Multiplication with $Z_t(f)$ with $f_{cut} = 32\text{ MHz}$ leads to $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$

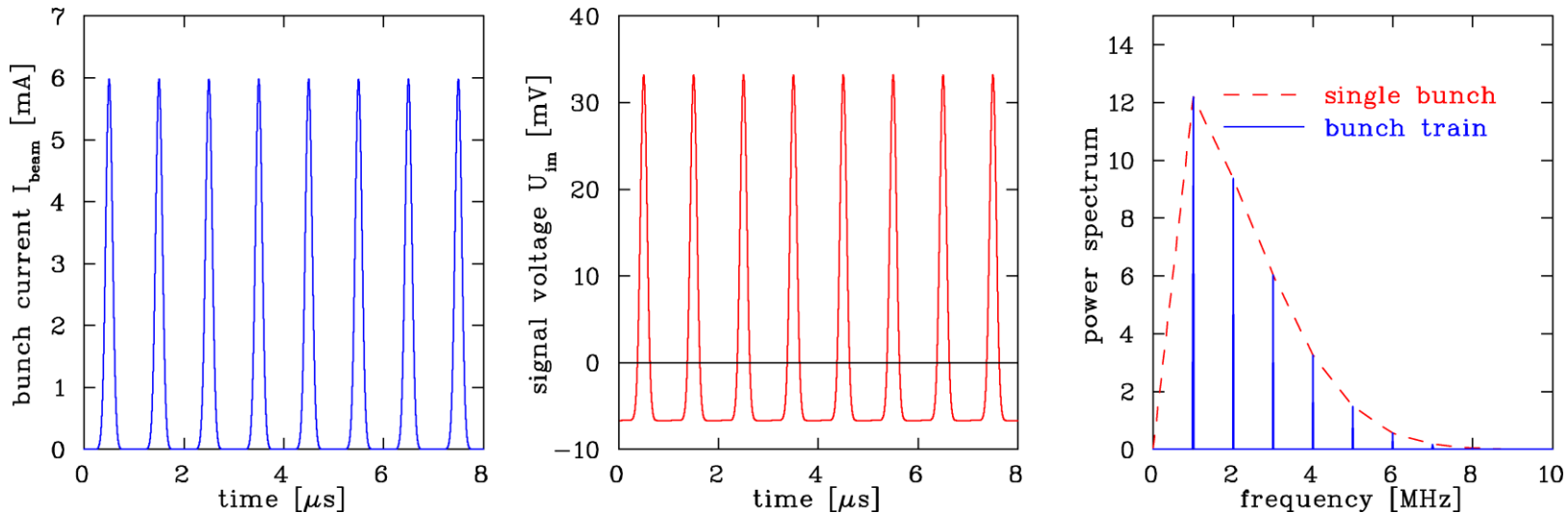
4. Inverse FFT leads to $U_{im}(t)$

Remark: Time domain processing via convolution or filters (FIR and IIR) are possible

Calculation of Signal Shape: repetitive Bunch in a Synchrotron

Synchrotron filled with 8 bunches accelerated with $f_{acc}=1$ MHz

BPM terminated with $R=1\text{ M}\Omega \Rightarrow f_{acc} \gg f_{cut}$:



Parameter: $R = 1\text{ M}\Omega \Rightarrow f_{cut} = 2\text{ kHz}$, $Z_t = 5\text{ }\Omega$, all buckets filled

$C=100\text{ pF}$, $l=10\text{ cm}$, $\beta=50\%$, $\sigma_t=100\text{ ns} \Rightarrow \sigma_f=15\text{ m}$

- Fourier spectrum is composed of lines separated by acceleration f_{rf}
- Envelope given by single bunch Fourier transformation
- Baseline shift due to ac-coupling

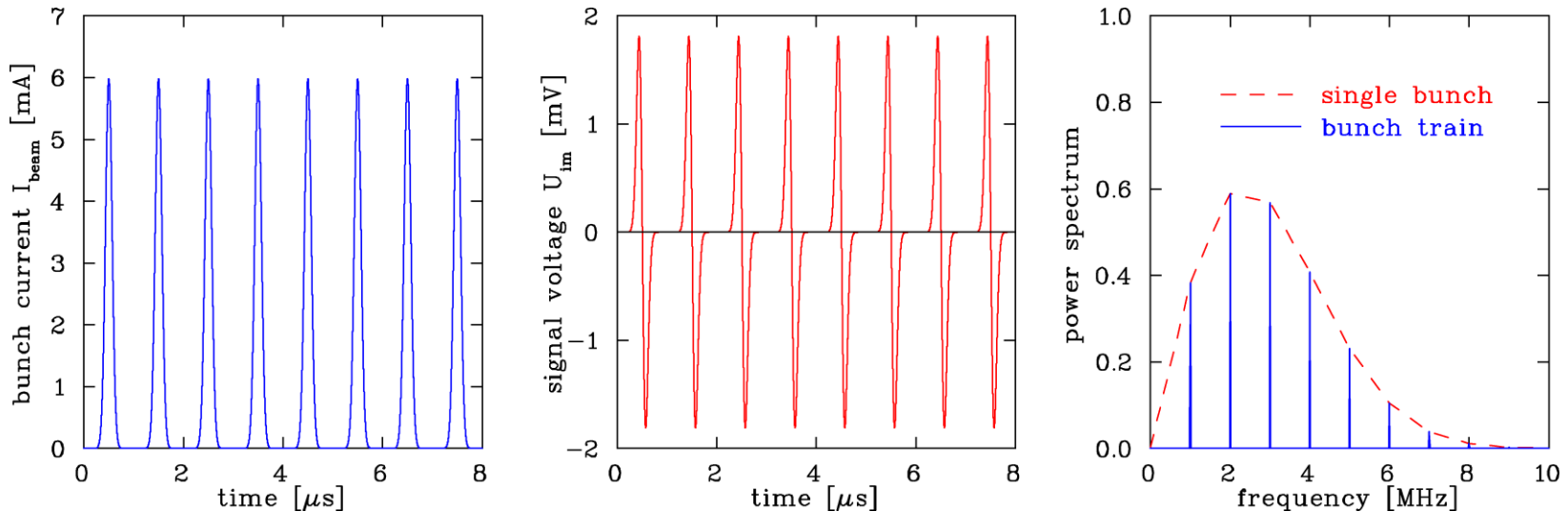
Remark: $1\text{ MHz} < f_{rf} < 10\text{ MHz} \Rightarrow \text{Bandwidth} \approx 100\text{ MHz} = 10 * f_{rf}$ for broadband observation

See lecture 'Time and Frequency Domain Signals' by Hermann Schmickler

Calculation of Signal Shape: repetitive Bunch in a Synchrotron

Synchrotron filled with 8 bunches accelerated with $f_{acc} = 1$ MHz

BPM terminated with $R=50 \Omega \Rightarrow f_{acc} \ll f_{cut}$:



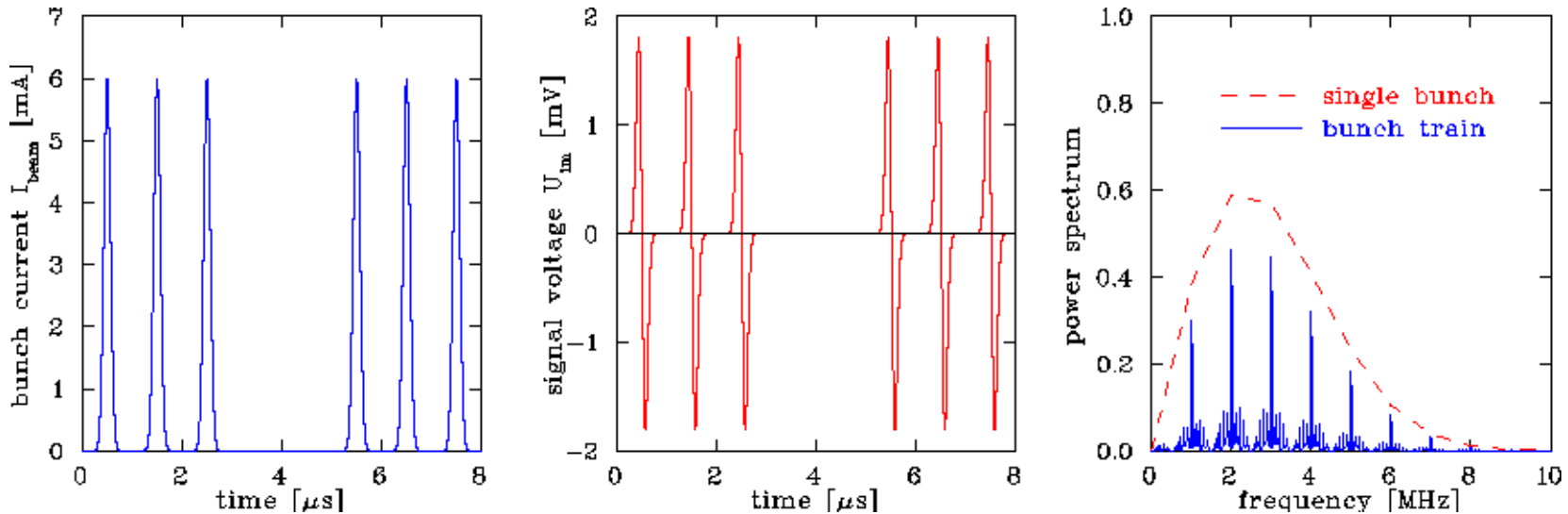
Parameter: $R=50 \Omega \Rightarrow f_{cut}=32$ MHz, all buckets filled

$C=100$ pF, $l=10$ cm, $\beta=50\%$, $\sigma_t=100$ ns $\Rightarrow \sigma_f=15$ m

- Fourier spectrum is concentrated at acceleration harmonics with single bunch spectrum as an envelope.
- Bandwidth up to typically $10 \cdot f_{acc}$

Calculation of Signal Shape: Bunch Train with empty Buckets

Synchrotron during filling: Empty buckets, $R=50 \Omega$:



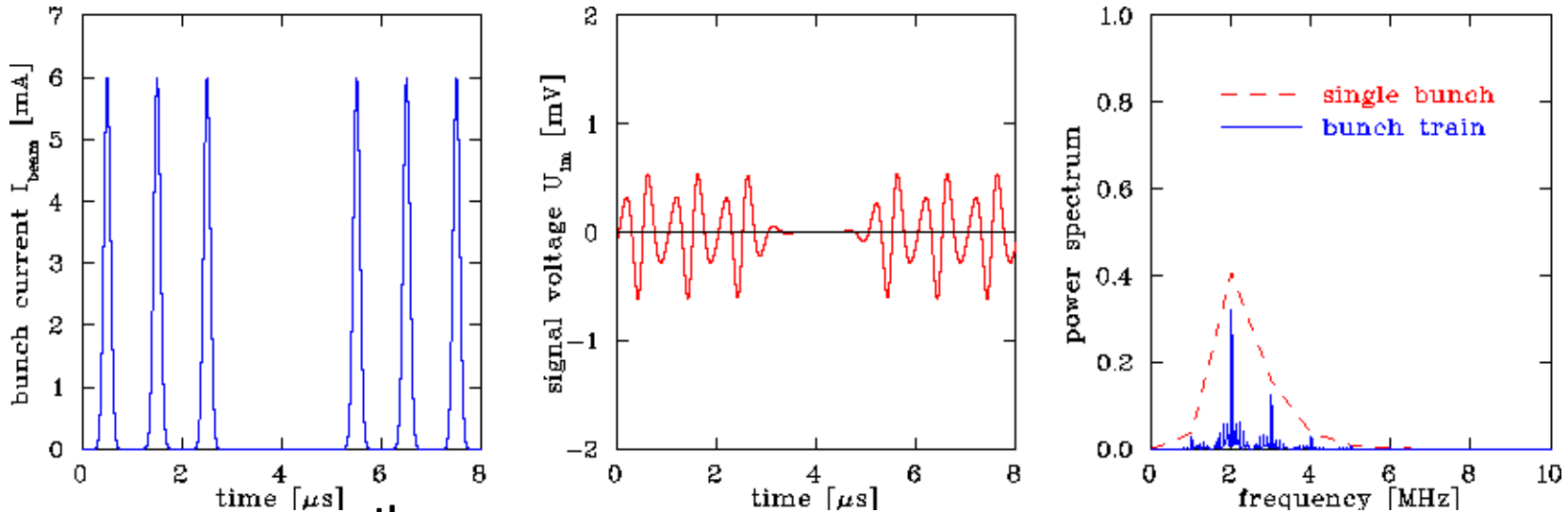
Parameter: $R=50 \Omega \Rightarrow f_{\text{cut}}=32 \text{ MHz}$, 2 empty buckets

$C=100\text{pF}$, $l=10\text{cm}$, $\beta=50\%$, $\sigma_t=100 \text{ ns} \Rightarrow \sigma_f=15\text{m}$

- Fourier spectrum is more complex, harmonics are broader due to sidebands

Calculation of Signal Shape: Filtering of Harmonics

Effect of filters, here bandpass:



Parameter: $R=50 \Omega$, 4th order Butterworth filter at $f_{cut}=2$ MHz

$C=100\text{pF}$, $l=10\text{cm}$, $\beta=50\%$, $\sigma=100$ ns

- Ringing due to sharp cutoff
- Other filter types more appropriate

*n^{th} order Butterworth filter, math. simple, but **not** well suited:*

$$|H_{low}| = \frac{1}{\sqrt{1 + (\omega / \omega_{cut})^{2n}}} \quad \text{and} \quad |H_{high}| = \frac{(\omega / \omega_{cut})^n}{\sqrt{1 + (\omega / \omega_{cut})^{2n}}}$$

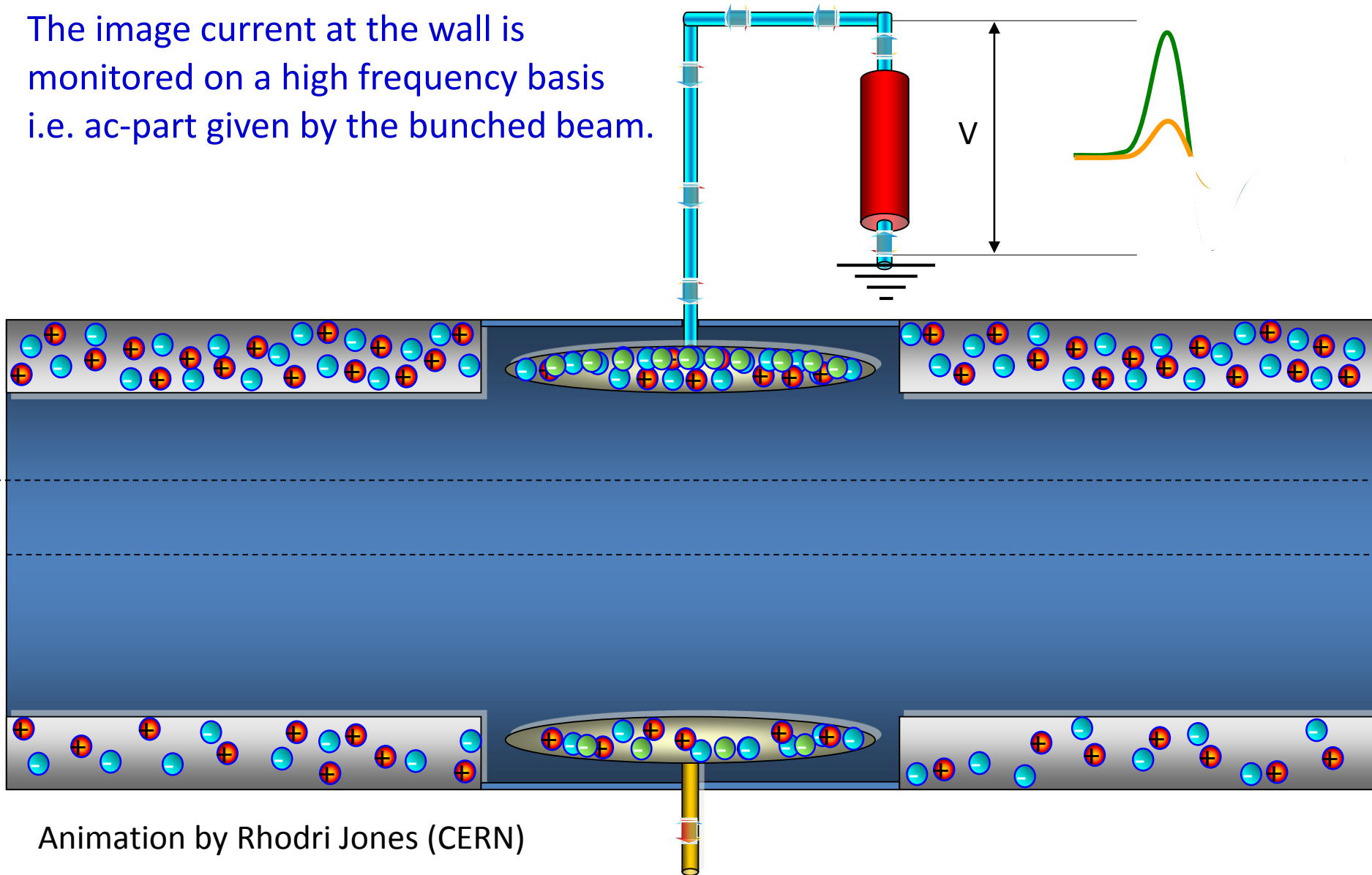
$$H_{filter} = H_{high} \cdot H_{low}$$

Generally: $Z_{tot}(\omega) = H_{cable}(\omega) \cdot H_{filter}(\omega) \cdot H_{amp}(\omega) \cdot \dots \cdot Z_t(\omega)$

Remark: For numerical calculations, time domain filters (FIR and IIR) are more appropriate

Principle of Signal Generation of a BPMs: off-center Beam

The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

Principle of Position Determination by a BPM

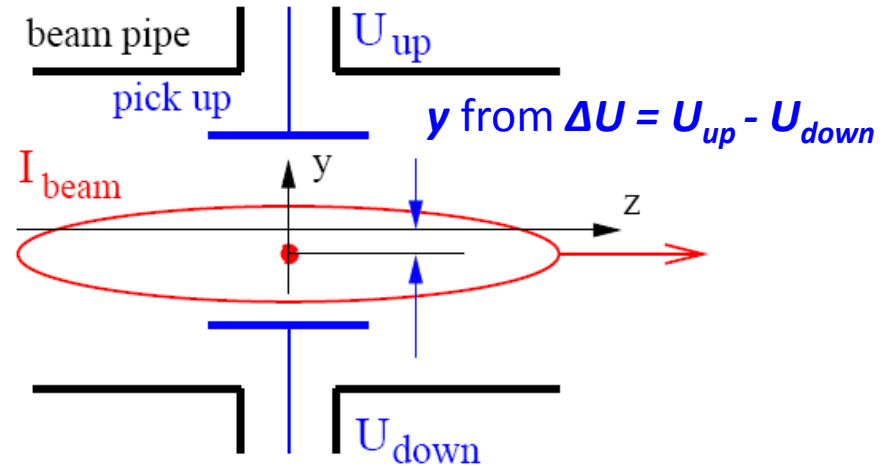
The difference voltage between plates gives the beam's center-of-mass

→ **most frequent application**

$$y = \frac{1}{S_y(\omega)} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} + \delta_y(\omega)$$

$$\equiv \frac{1}{S_y} \cdot \frac{\Delta U_y}{\Sigma U_y} + \delta_y$$

$$x = \frac{1}{S_x(\omega)} \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} + \delta_x(\omega)$$



$S(\omega, x)$ is called **position sensitivity**, sometimes the inverse is used $k(\omega, x) = 1/S(\omega, x)$

S is a geometry dependent, non-linear function,

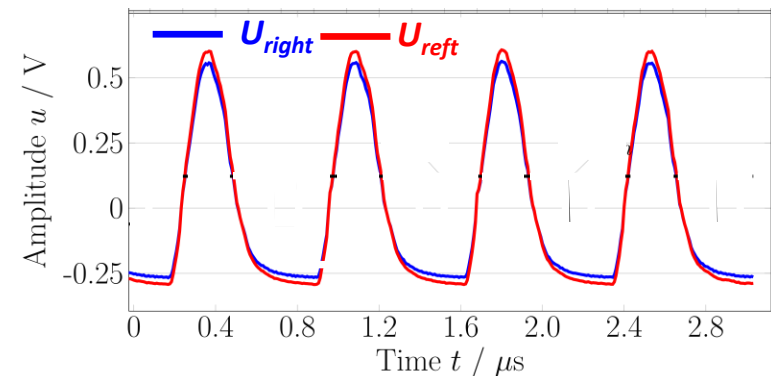
Units: $S = [\%/mm]$, sometimes $S = [dB/mm]$ or $k = [mm]$.

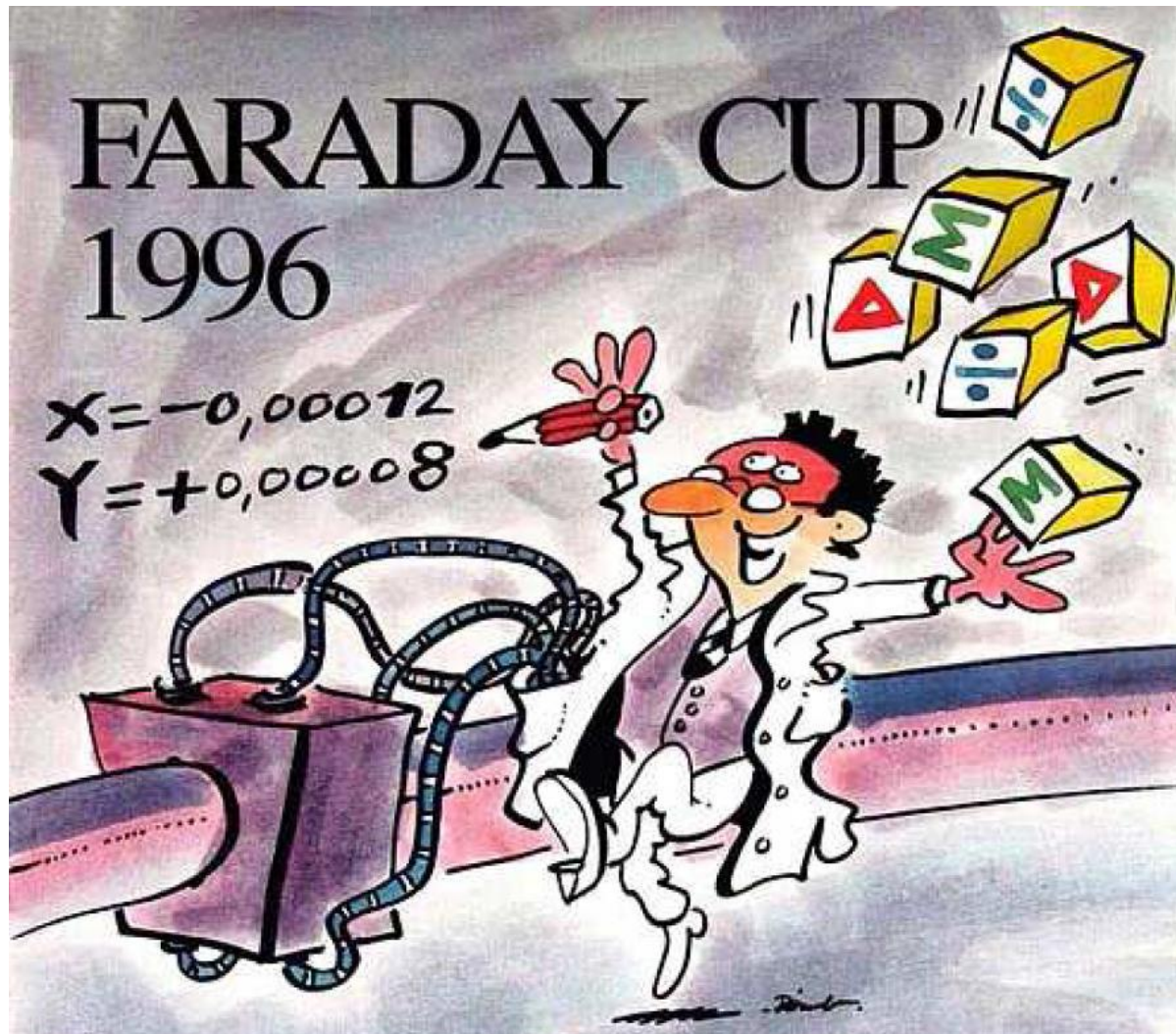
Typical desired position resolution:

$\Delta x \approx 0.1 \dots 0.3 \cdot \sigma_x$ of beam width

It is at least: $\Delta U \ll \frac{1}{10} \Sigma U$

Example: One turn = 4 bunches @ 35 MeV/u





Outline:

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- BPMs for measurement of closed orbit, tune and further lattice functions
- Summary

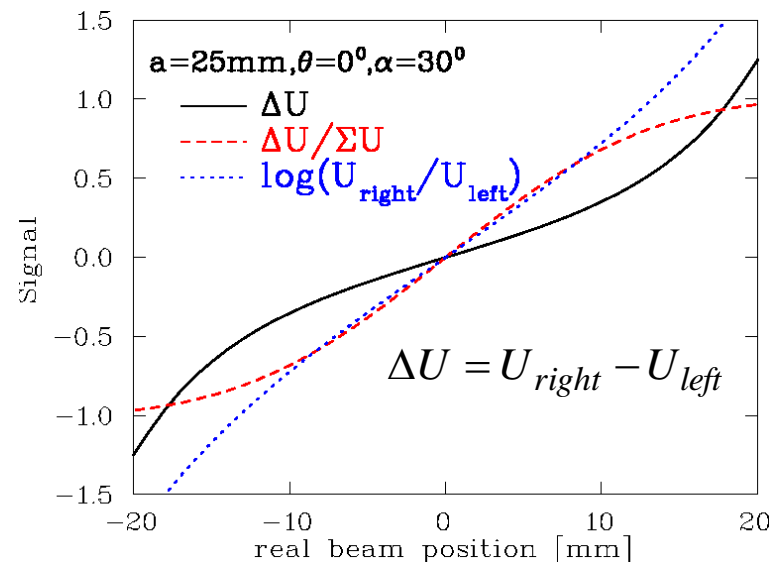
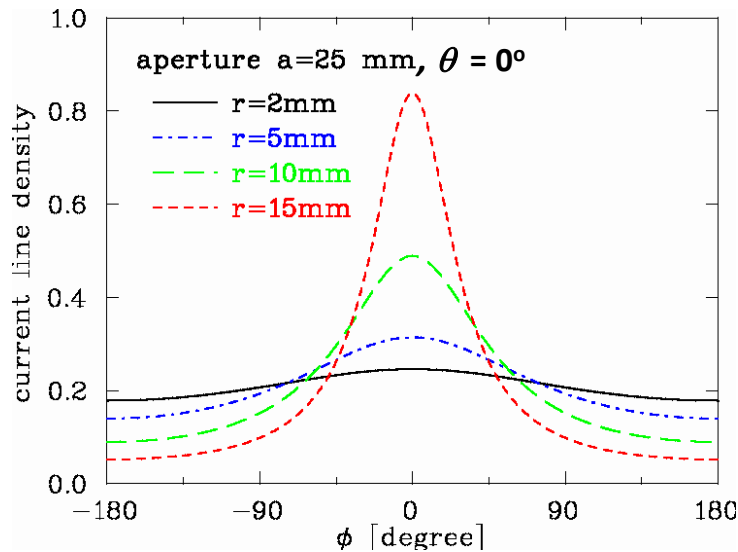
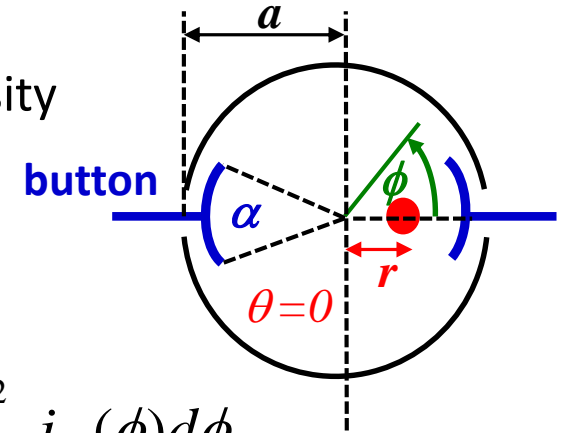
2-dim Model for a Button BPM

‘Proximity effect’: larger signal for closer plate

Ideal 2-dim model: Cylindrical pipe → image current density
via ‘image charge method’ for ‘pencil’ beam:

$$j_{im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

Image current: Integration of finite BPM size: $I_{im} = a \cdot \int_{-\alpha/2}^{\alpha/2} j_{im}(\phi) d\phi$



2-dim Model for a Button BPM

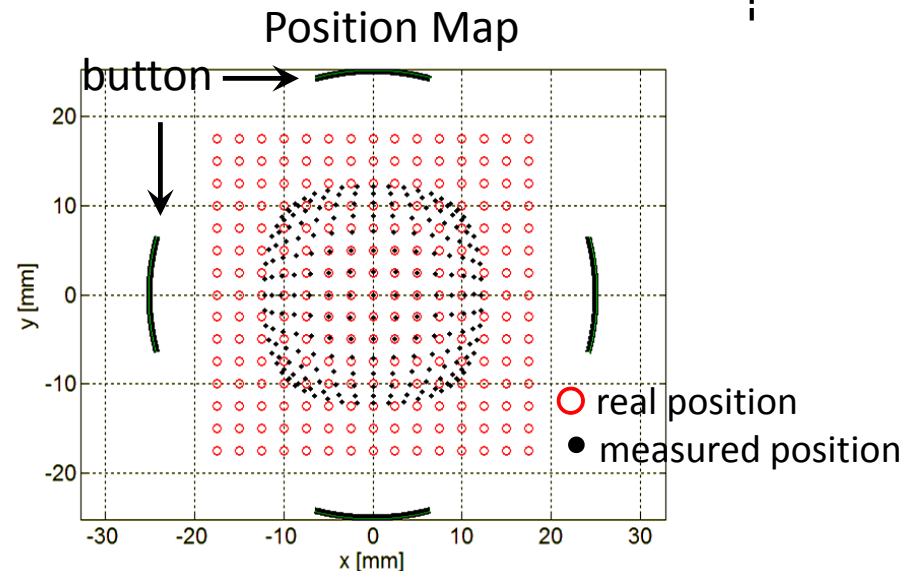
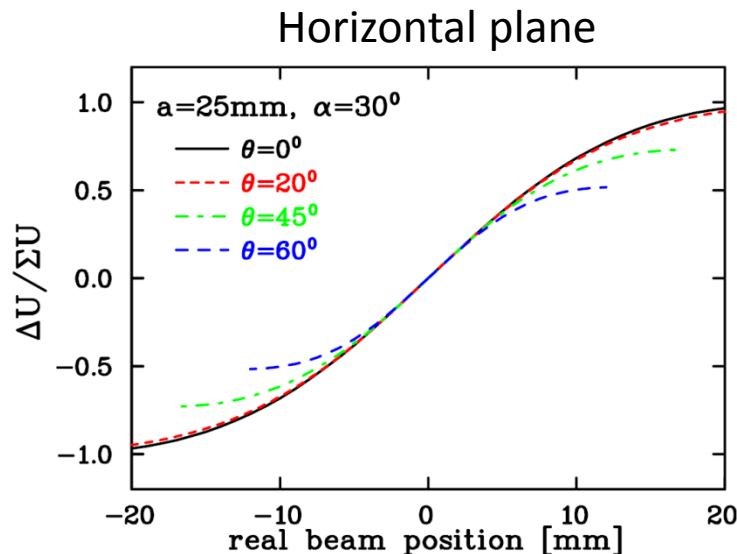
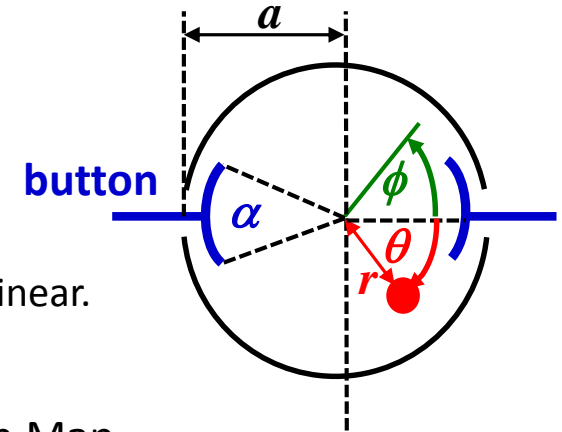
Ideal 2-dim model: Non-linear behavior and hor-vert coupling:

Sensitivity S converts signal to position $x = \frac{1}{S} \cdot \frac{\Delta U}{\Sigma U}$

with S [%/mm] or [dB/mm]

i.e. S is the derivative of the curve $S_x = \frac{\partial(\frac{\Delta U}{\Sigma U})}{\partial x}$, here $S_x = S_x(x, y)$ i.e. non-linear.

For this example: central part $S=7.4\%/mm \Leftrightarrow k=1/S=14mm$



Button BPM Realization

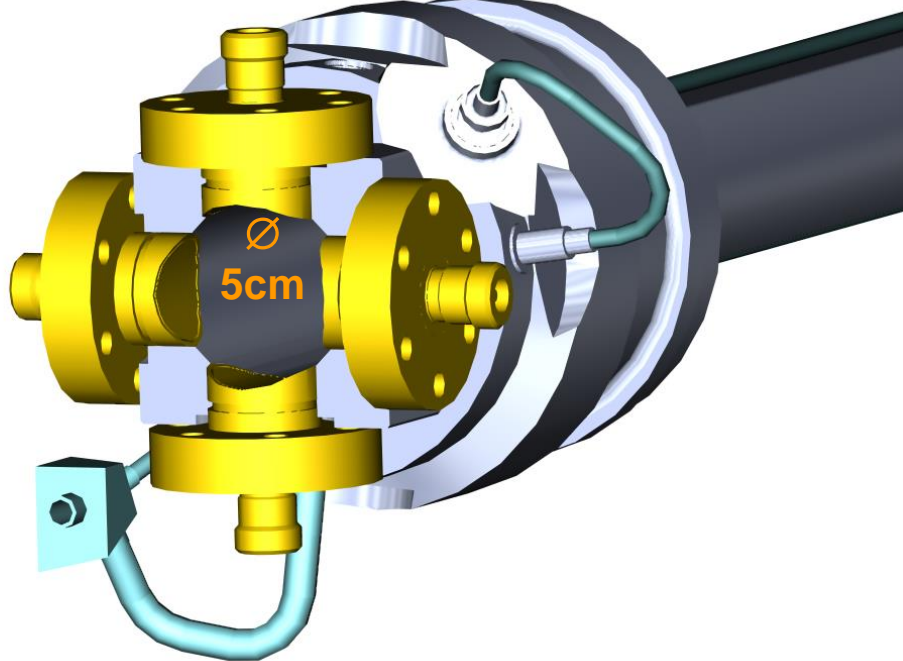
LINACs, e⁻-synchrotrons: $100 \text{ MHz} < f_{rf} < 3 \text{ GHz} \rightarrow \text{bunch length} \approx \text{BPM length}$

$\rightarrow 50 \Omega$ signal path to prevent reflections

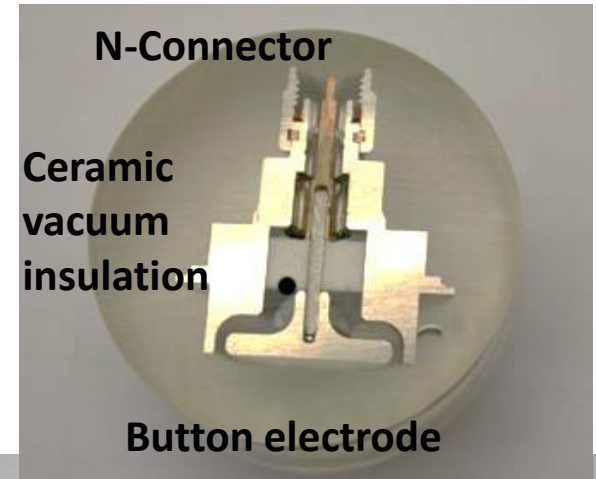
Example: LHC-type inside cryostat:

$\varnothing 24 \text{ mm}$, half aperture $a = 25 \text{ mm}$, $C = 8 \text{ pF}$

$\Rightarrow f_{cut} = 400 \text{ MHz}$, $Z_t = 1.3 \Omega$ above f_{cut}



Courtesy C. Boccard (CERN)

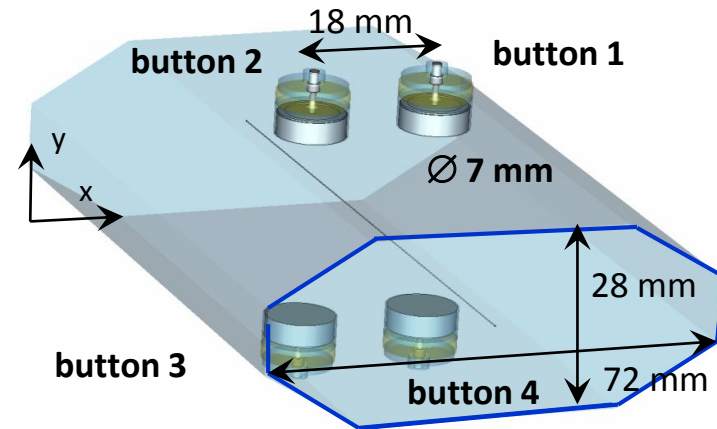
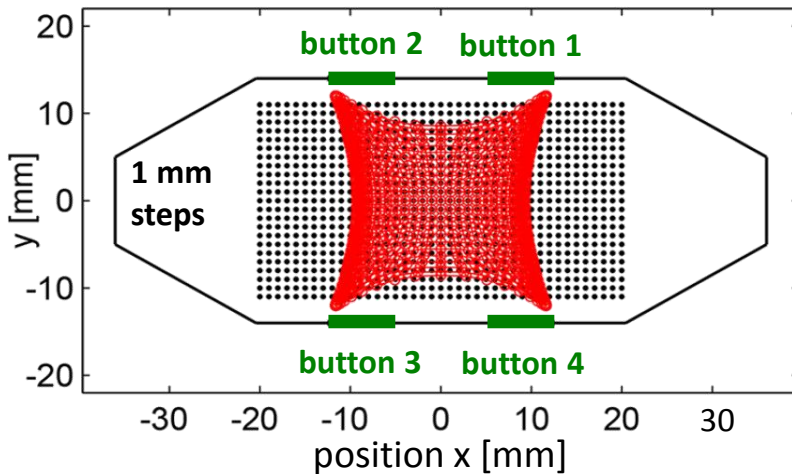
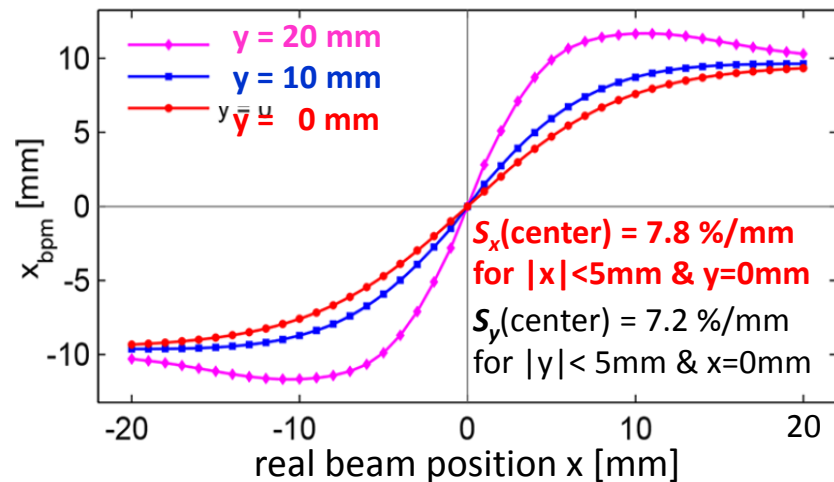
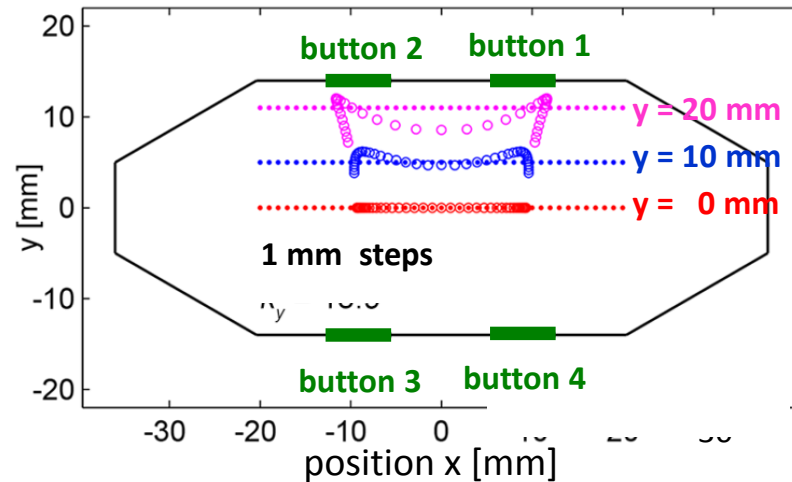


Simulations for Button BPM at Synchrotron Light Sources

Example: Simulation for ALBA light source for 72 x 28 mm² chamber

Optimization: horizontal distance and size of buttons

from A.A. Nosych et al., IBIC'14



Result: non-linearity and **xy**-coupling occur in dependence of button size and position

Outline:

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- Summary

Linear-cut BPM for Proton Synchrotrons

Frequency range: $1 \text{ MHz} < f_{rf} < 100 \text{ MHz} \Rightarrow \text{bunch-length} \gg \text{BPM length}$.

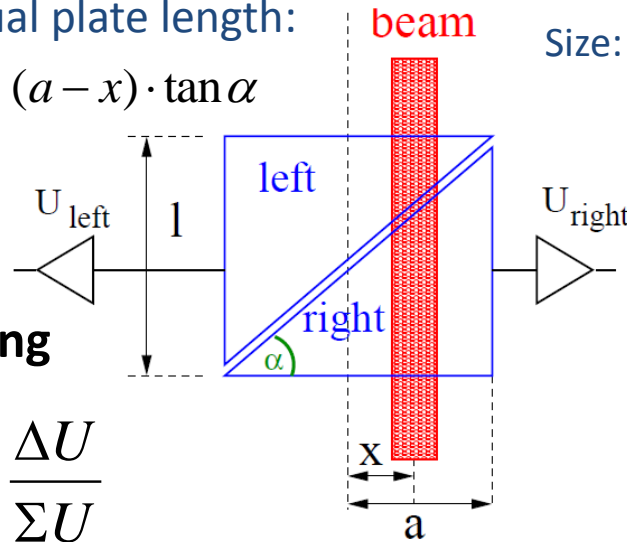
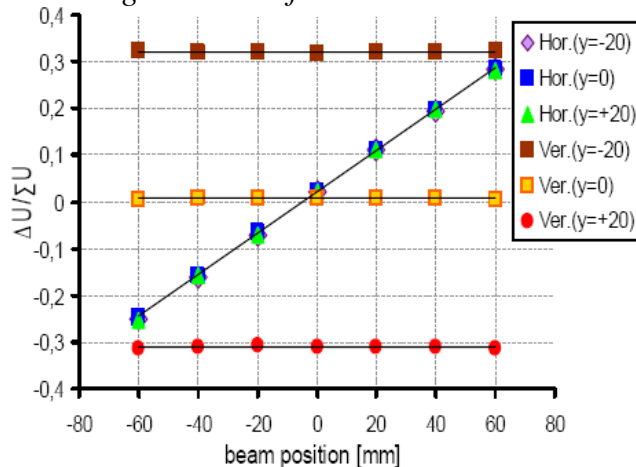
Signal is proportional to actual plate length:

$$l_{\text{right}} = (a + x) \cdot \tan \alpha, \quad l_{\text{left}} = (a - x) \cdot \tan \alpha$$

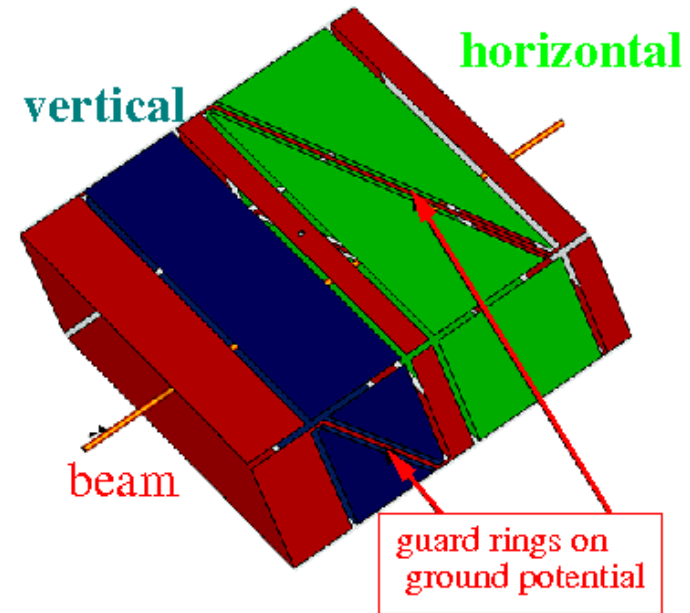
$$\Rightarrow x = a \cdot \frac{l_{\text{right}} - l_{\text{left}}}{l_{\text{right}} + l_{\text{left}}}$$

In ideal case: linear reading

$$x = a \cdot \frac{U_{\text{right}} - U_{\text{left}}}{U_{\text{right}} + U_{\text{left}}} \equiv a \cdot \frac{\Delta U}{\Sigma U}$$



Size: 200x70 mm²



Linear-cut BPM:

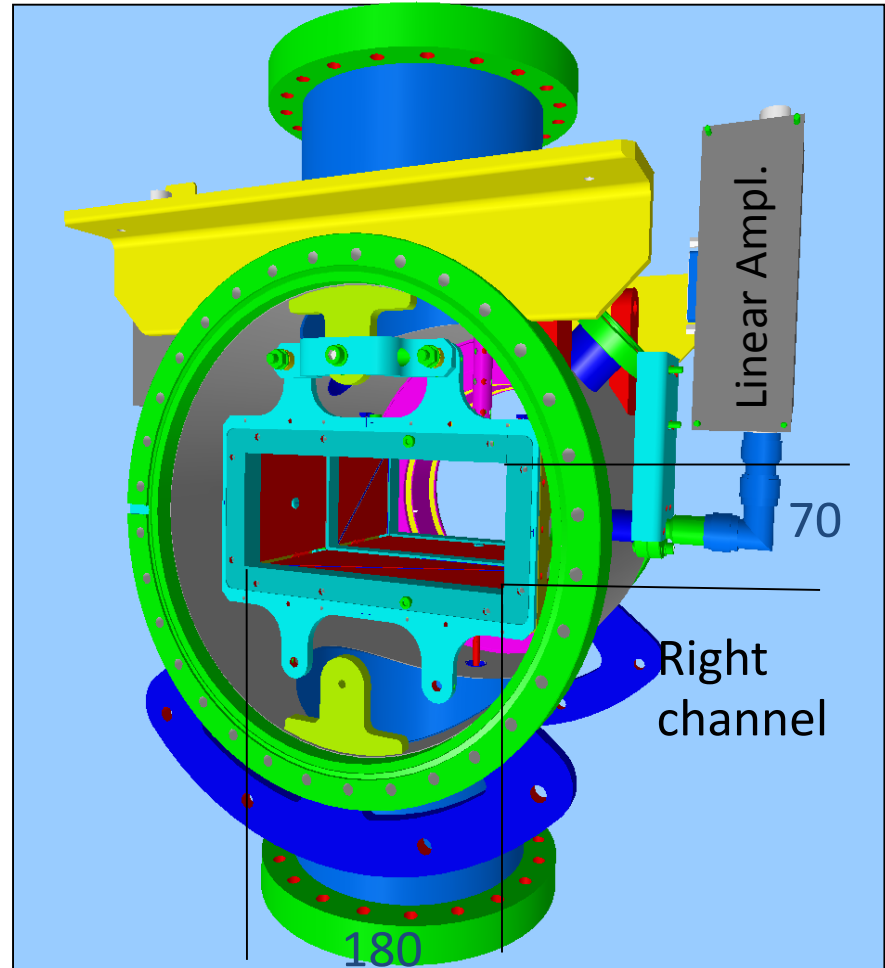
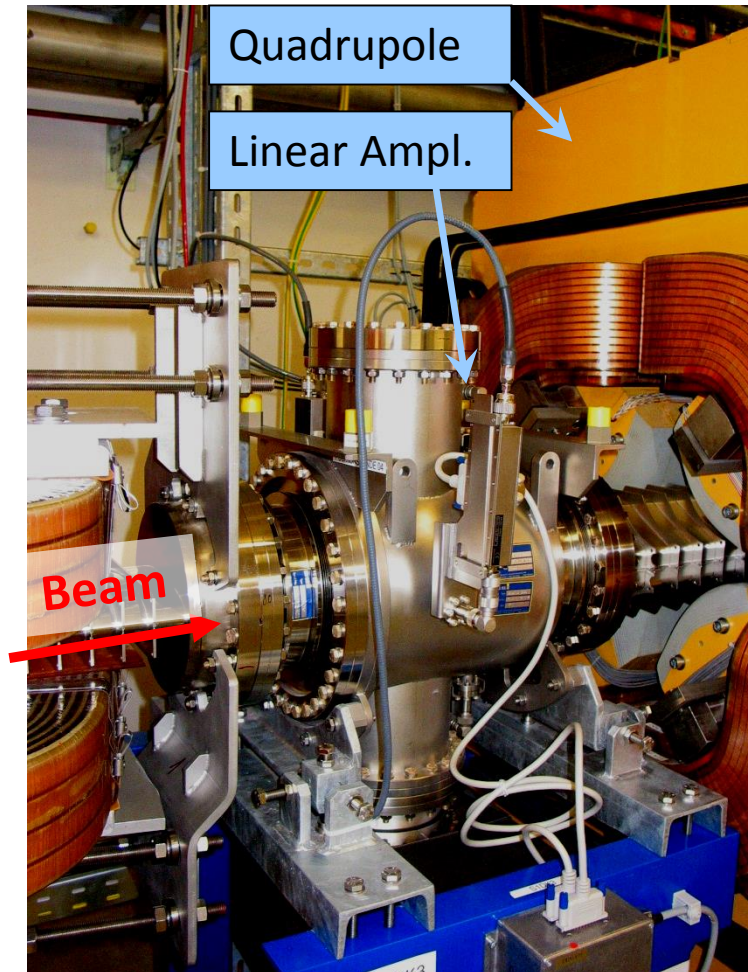
Advantage: Linear, i.e. constant position sensitivity S

\Leftrightarrow no beam size dependence

Disadvantage: Large size, complex mechanics
high capacitance

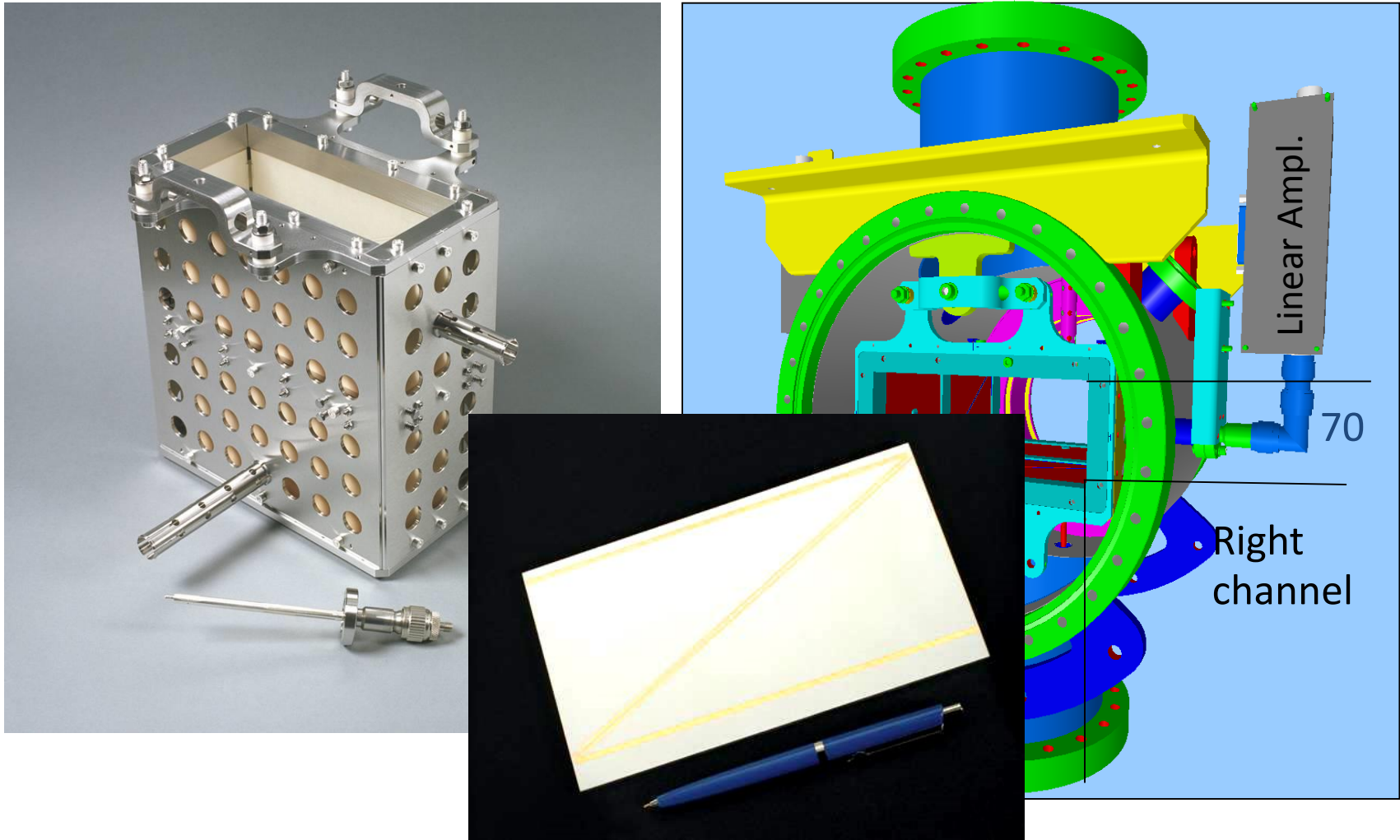
Technical Realization of a linear-cut BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u \rightarrow 440 MeV/u
 BPM clearance: 180x70 mm², standard beam pipe diameter: 200 mm.



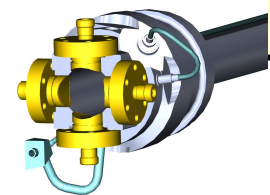
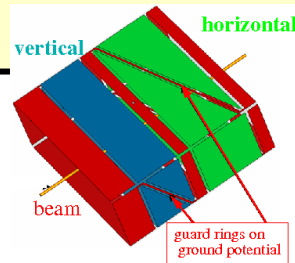
Technical Realization of a linear-cut BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u \rightarrow 440 MeV/u
 BPM clearance: 180x70 mm², standard beam pipe diameter: 200 mm.



Comparison linear-cut and Button BPM

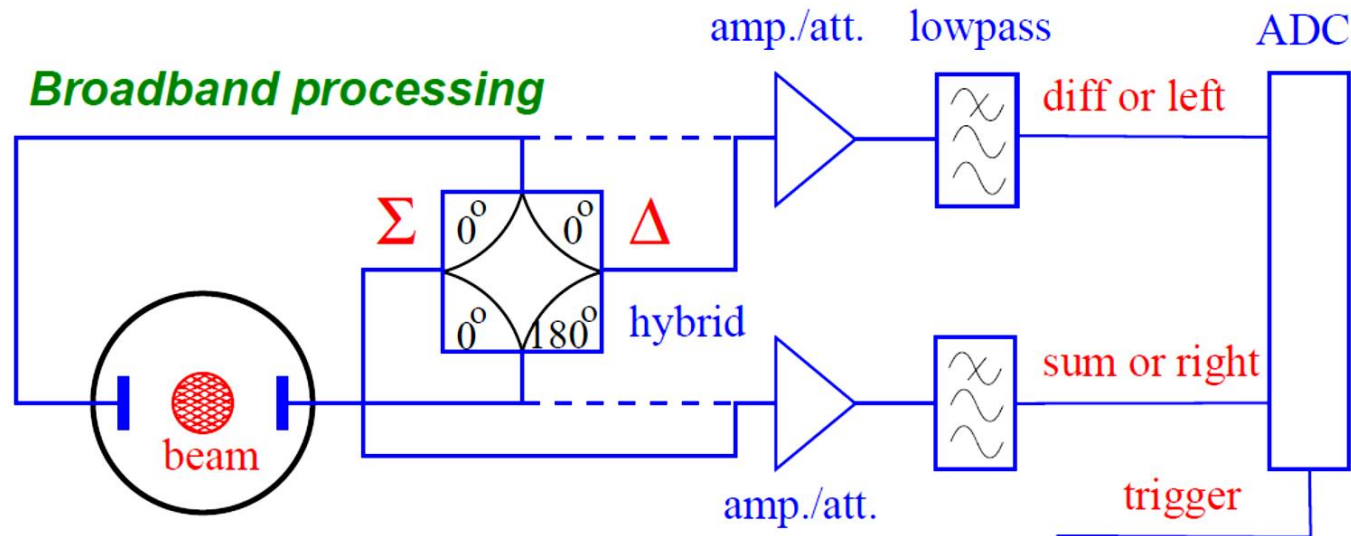
	Linear-cut BPM	Button BPM
Precaution	Bunches longer than BPM	Bunch length comparable to BPM
BPM length (typical)	10 to 20 cm length per plane	Ø1 to 5 cm per button
Shape	Rectangular or cut cylinder	Orthogonal or planar orientation
Bandwidth (typical)	0.1 to 100 MHz	100 MHz to 5 GHz
Coupling	1 MΩ or ≈ 1 kΩ (transformer)	50 Ω
Cutoff frequency (typical)	0.01... 10 MHz ($C=30\ldots 100$ pF)	0.3... 1 GHz ($C=2\ldots 10$ pF)
Linearity	Very good, no x-y coupling	Non-linear, x-y coupling
Sensitivity	Good, care: plate cross talk	Good, care: signal matching
Usage	At proton synchrotrons, $f_{rf} < 10$ MHz	All electron acc., proton Linacs, $f_{rf} > 100$ MHz



Remark: Other types are also some time used: e.g. wall current monitors, inductive antenna, BPMs with external resonator, cavity BPM, slotted wave-guides for stochastic cooling etc.

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analog signal conditioning to achieve small signal processing
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- Summary



- Hybrid or transformer close to beam pipe for analog ΔU & ΣU generation or U_{left} & U_{right}
- Attenuator/amplifier
- Filter to get the wanted harmonics and to suppress stray signals
- ADC: digitalization \rightarrow followed by calculation of $\Delta U / \Sigma U$

Advantage: Bunch-by-bunch observation possible, versatile post-processing possible

Disadvantage: Resolution down to $\approx 100 \mu\text{m}$ for shoe box type , i.e. $\approx 0.1\%$ of aperture, resolution is worse than narrowband processing, see below

Challenge: Precise analog electronics with very low drift of amplification etc.

General: Noise Consideration

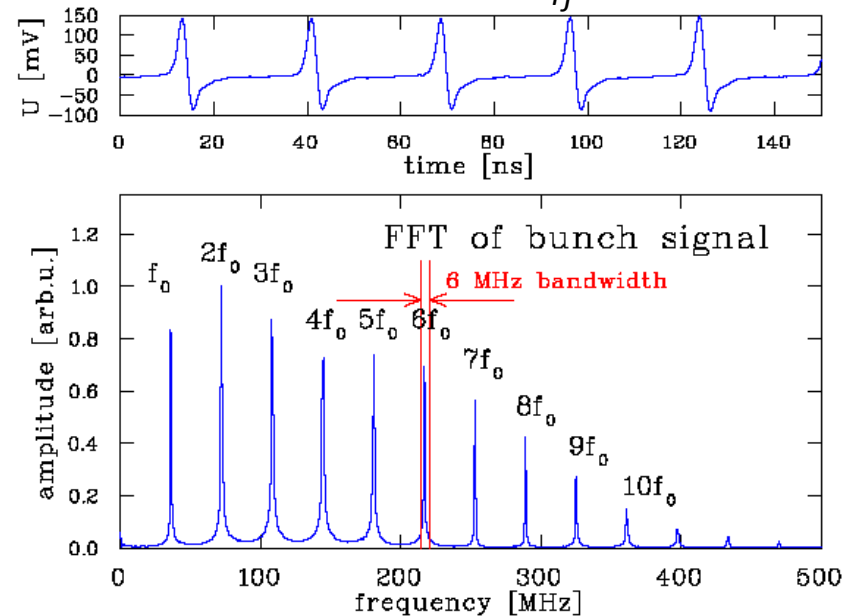
1. Signal voltage given by: $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$
2. Position information from voltage difference: $x = 1/S \cdot \Delta U / \Sigma U$
3. Thermal noise voltage given by: $U_{noise}(R, \Delta f) = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$

Signal-to-noise $\Delta U_{im}/U_{noise}$ is influenced by:

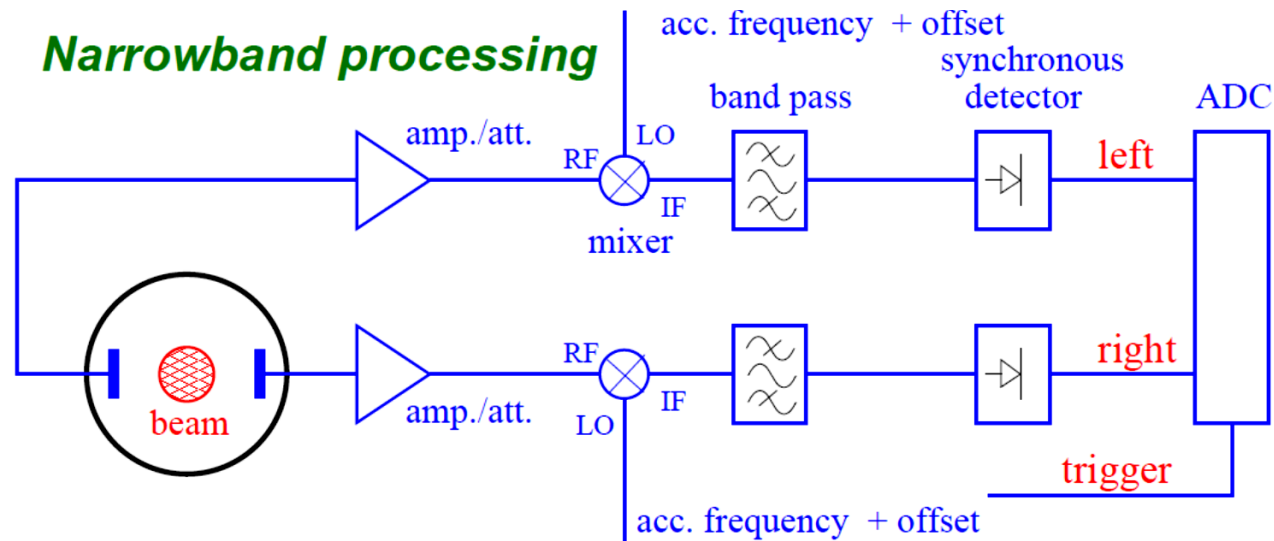
- Input signal amplitude
- Thermal noise from amplifiers etc.
- Bandwidth Δf

⇒ Restriction of frequency width
as the power is
concentrated at harm. nf_{rf}

Example: GSI-LINAC with $f_{rf}=36$ MHz



Narrowband Processing for improved Signal-to-Noise



Narrowband processing equals heterodyne receiver (e.g. AM-radio or spectrum analyzer)

- Attenuator/amplifier
- Mixing with accelerating frequency $f_{rf} \Rightarrow$ signal with difference frequency
- Bandpass filter of the mixed signal (e.g at 10.7 MHz)
- Rectifier: synchronous detector
- ADC: digitalization \rightarrow followed calculation of $\Delta U / \Sigma U$

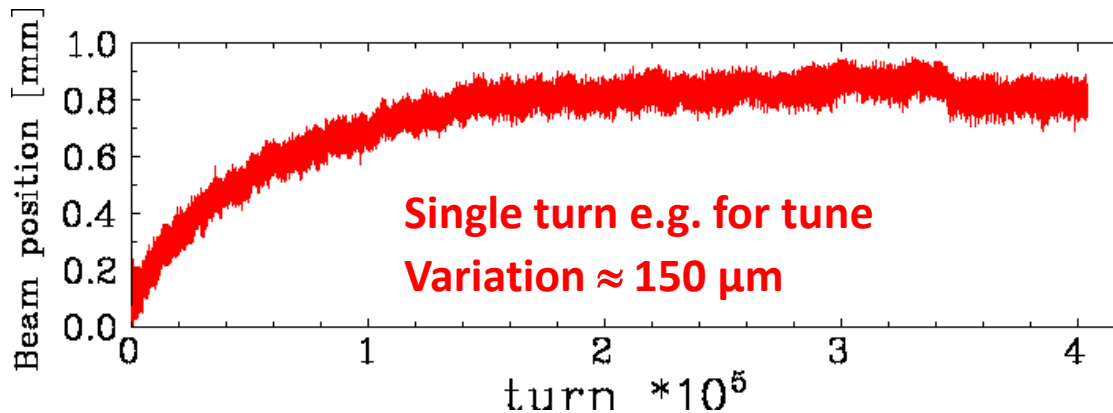
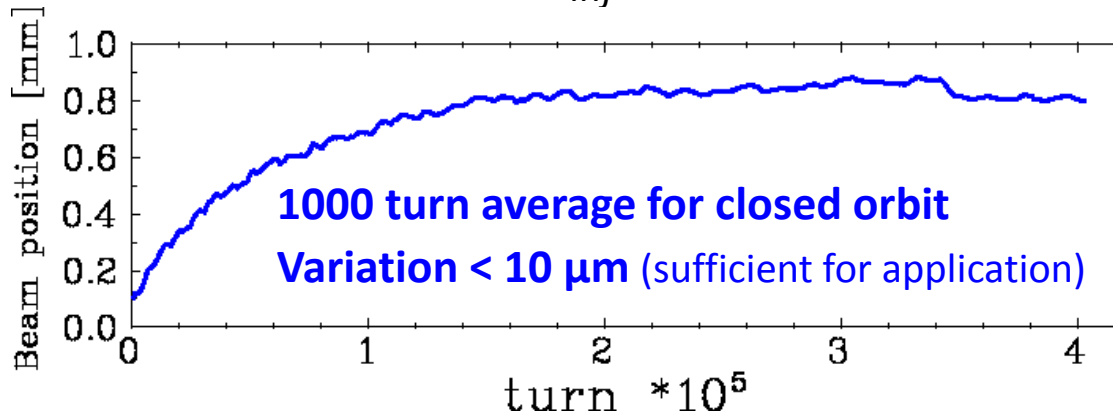
} Digital
correspondence:
I/Q demodulation

Advantage: Spatial resolution about 100 time better than broadband processing

Disadvantage: No turn-by-turn diagnosis, due to mixing = 'long averaging time'

Comparison: Filtered Signal \leftrightarrow Single Turn

Example: GSI Synchr.: U^{73+} , $E_{inj} = 11.5 \text{ MeV/u} \rightarrow E_{out} = 250 \text{ MeV/u}$ within 0.5 s, 10^9 ions



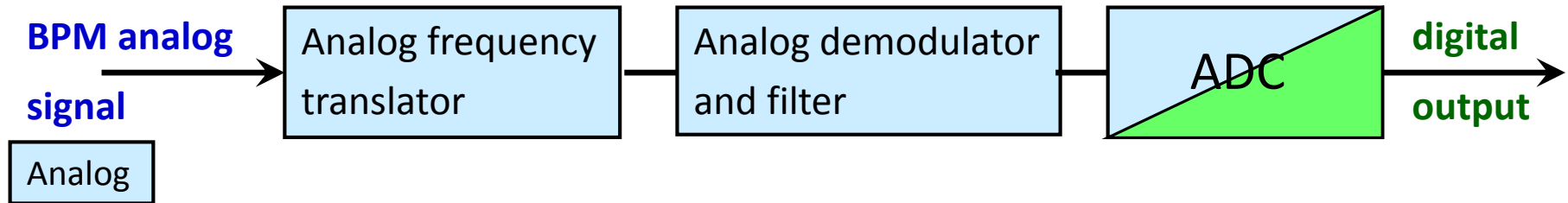
- Position resolution < 30 μm (BPM diameter $d=180 \text{ mm}$)
- average over 1000 turns corresponding to $\approx 1 \text{ ms}$ or $\approx 1 \text{ kHz}$ bandwidth
- Turn-by-turn data have much larger variation

However: Not only noise contributes but additionally **beam movement** by betatron oscillation \Rightarrow broadband processing i.e. turn-by-turn readout for tune determination.

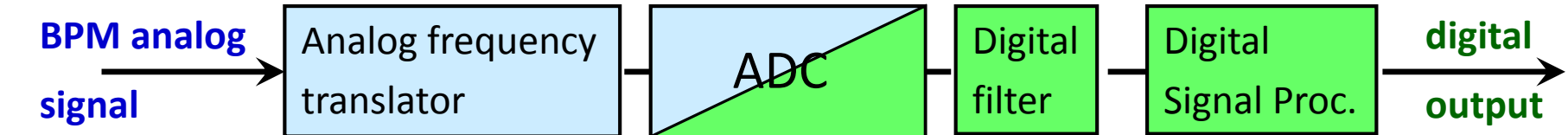
Analog versus Digital Signal Processing

Modern instrumentation uses **digital** techniques with extended functionality.

Traditional analog processing



Modern digital processing



Digital receiver as modern successor of super heterodyne receiver

- Basic functionality is preserved but implementation is very different
- Digital transition just after the amplifier & filter or mixing unit
- Signal conditioning (filter, decimation, averaging) on FPGA

Advantage of DSP: Versatile operation, flexible adoption without hardware modification

Disadvantage of DSP: non, good engineering skill requires for development, expensive

Comparison of BPM Readout Electronics (simplified)

Type	Usage	Precaution	Advantage	Disadvantage
Broadband	p-sychr.	Long bunches	Bunch structure signal Post-processing possible Required for transfer lines with few bunches	Resolution limited by noise
Narrowband	all synchr.	Stable beams >100 rf-periods	High resolution	No turn-by-turn Complex electronics
Digital Signal Processing	all	ADC sample typ. 250 MS/s	Very flexible & versatile High resolution Trendsetting technology for future demands	Basically non! Limited time resolution by ADC → under-sampling Man-power intensive

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- Electronics for position evaluation
analog signal conditioning to achieve small signal processing
- **BPMs for measurement of closed orbit, tune and further lattice functions**
frequent application of BPMs
- **Summary**

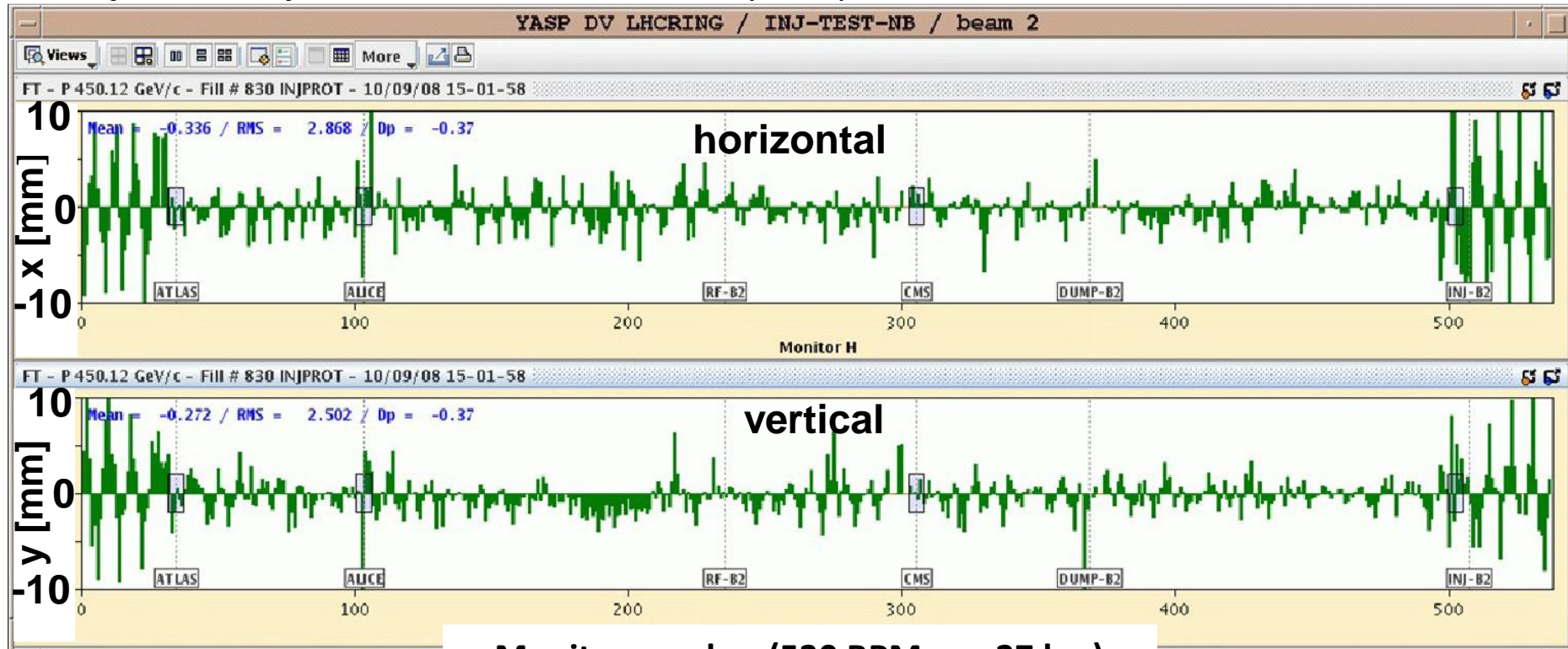
Trajectory Measurement with BPMs

Trajectory:

The position delivered by an **individual bunch** within a transfer line or a synchrotron.

Main task: Control of matching (center and angle), first-turn diagnostics

Example: LHC injection 10/09/08 i.e. first day of operation !



Monitor number (530 BPMs on 27 km)

Courtesy R. Jones (CERN)

Tune values at LHC: $Q_h = 64.3$, $Q_v = 59.3$

Closed Orbit Feedback: Typical Noise Sources

Beam movement:

Short term (min to 10 ms):

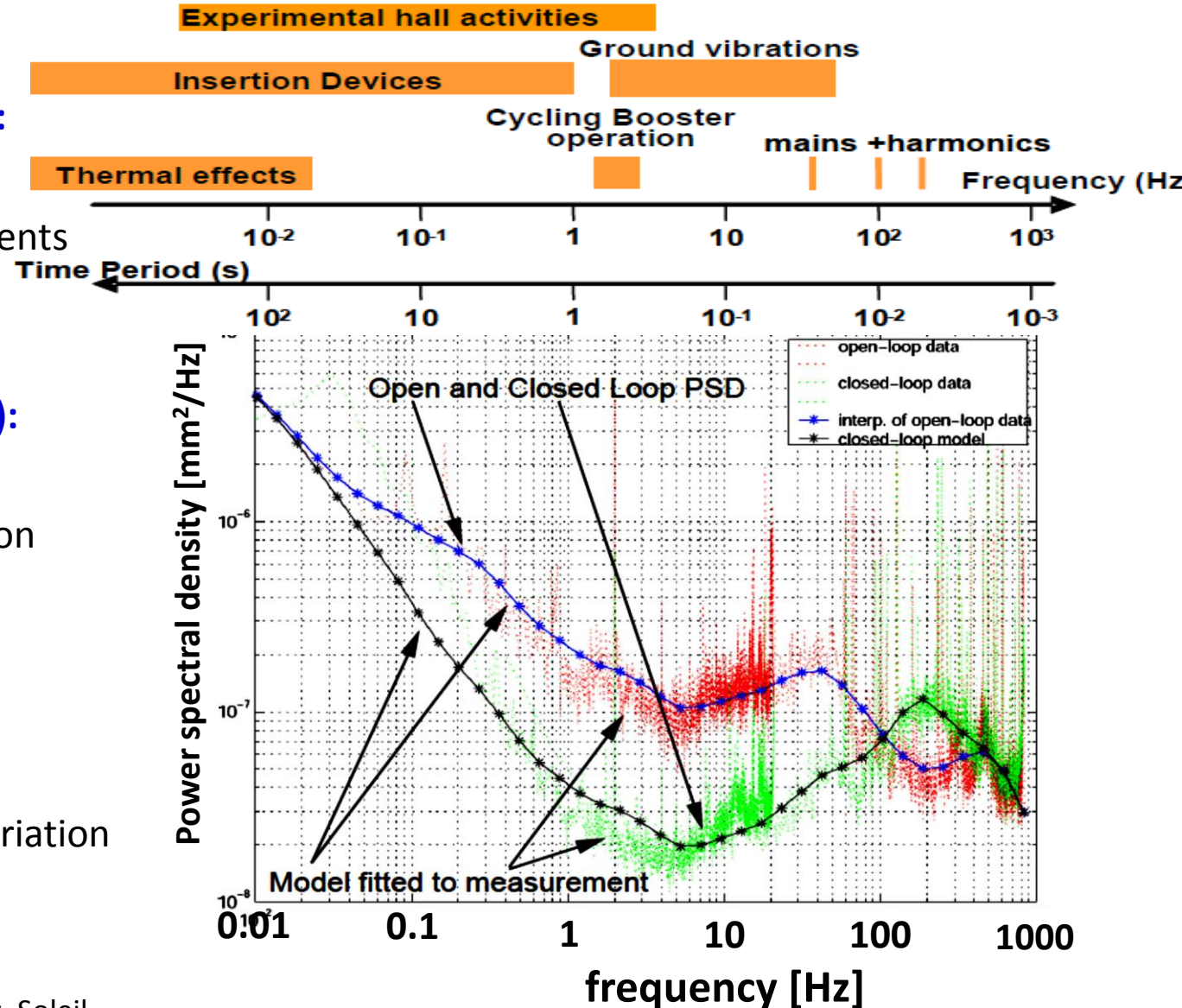
- Traffic
- Machine (crane) movements
- Water & vacuum pumps
- 50 Hz main power net

Medium term (day to min):

- Movement of chambers due to heating by radiation
- Day-night variation
- tide, moon cycle

Long term (> days):

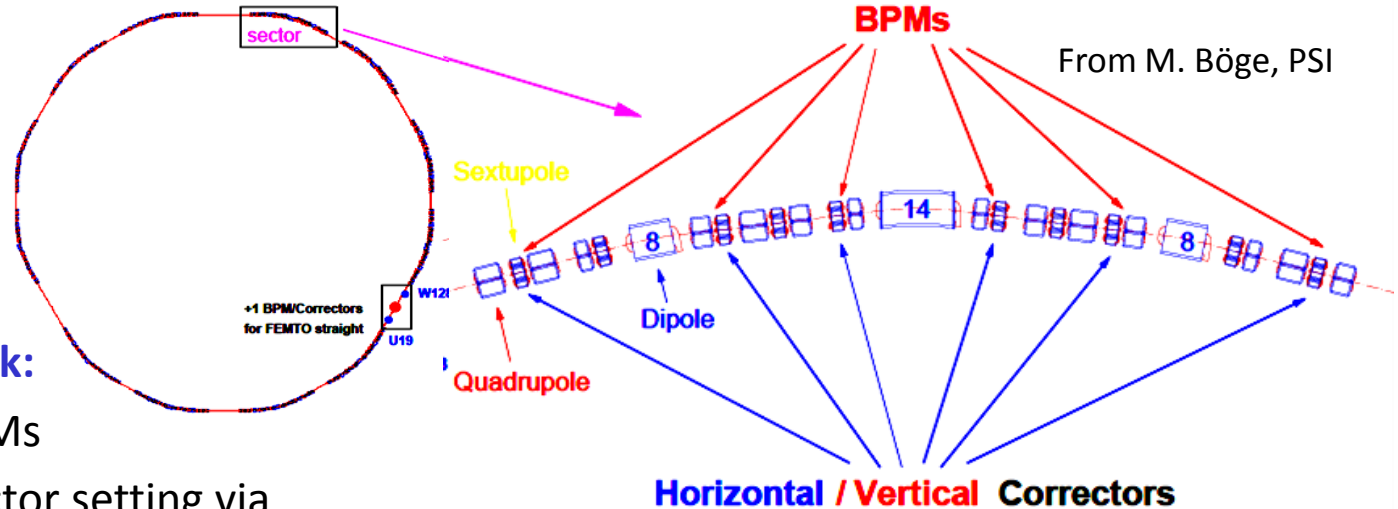
- Ground settlement
- Seasons, temperature variation



Courtesy M. Böge, PSI, N. Hubert, Soleil

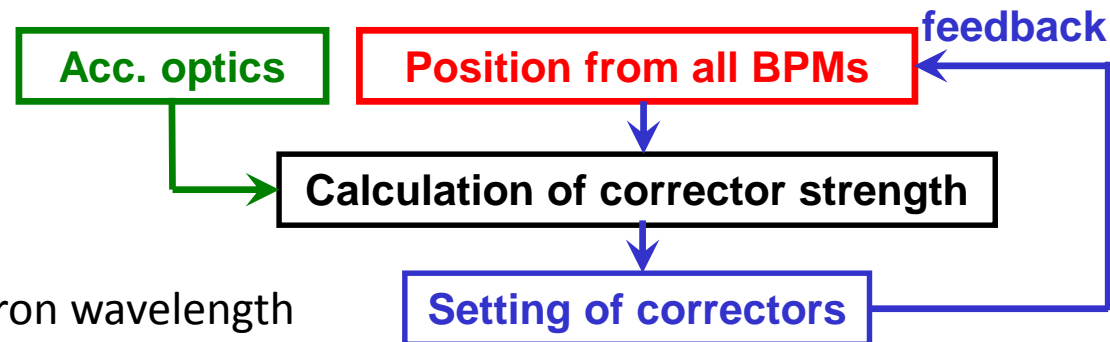
Orbit feedback: Synchrotron light source → spatial stability of light beam

Example: SLS-Synchrotron at Villigen, Switzerland



Procedure of a feedback:

1. Position from all BPMs
 2. Calculation of corrector setting via Orbit Response Matrix
 3. Change of magnet setting
 - 1.' New position measurement
- ⇒ regulation time down to 10 ms
- ⇒ Role of thumb: ≈ 4 BPMs per betatron wavelength



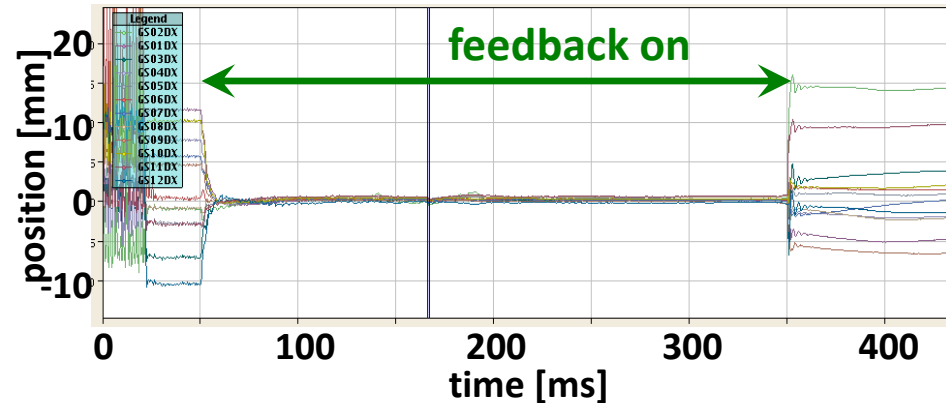
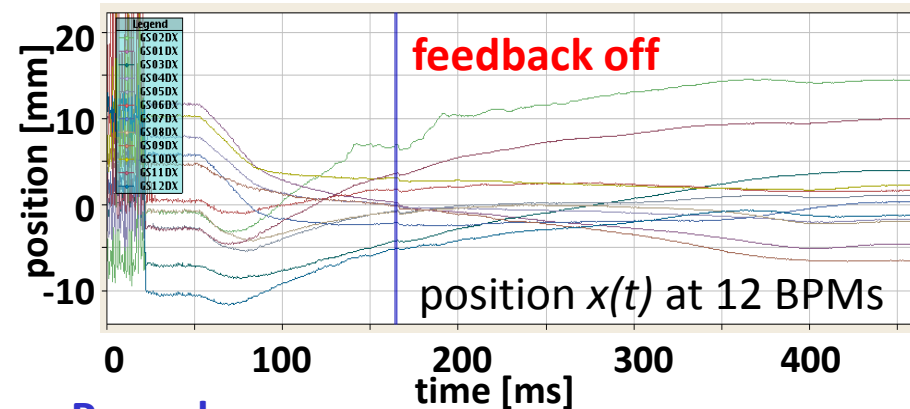
Uncorrected orbit: typ. $\langle x \rangle_{rms} \approx 1 \text{ mm}$

Corrected orbit: typ. $\langle x \rangle_{rms} \approx 1 \mu\text{m}$ up to $\approx 100 \text{ Hz}$ bandwidth!

Close Orbit Feedback: Results

Orbit feedback:

Example: 12 beam positions at GSI-SIS during ramping from 8.6 to 500 MeV/u for Ar^{18+}



Procedure:

1. Position from all 12 BPMs
 2. Calculation of corrector setting on fast (FPGA-based) electronics
 3. Submission to corrector magnets
 4. New position measurement
- ⇒ regulation time down to 10 ms

Role of thumb:

Movement related to tune i.e. 'natural oscillations by periodic focusing'

To determine the 'sine-like' oscillation 4 BPMs per oscillation are required

⇒ 4 BPMs per tune value (but detailed investigation required to determine the # of BPMs)

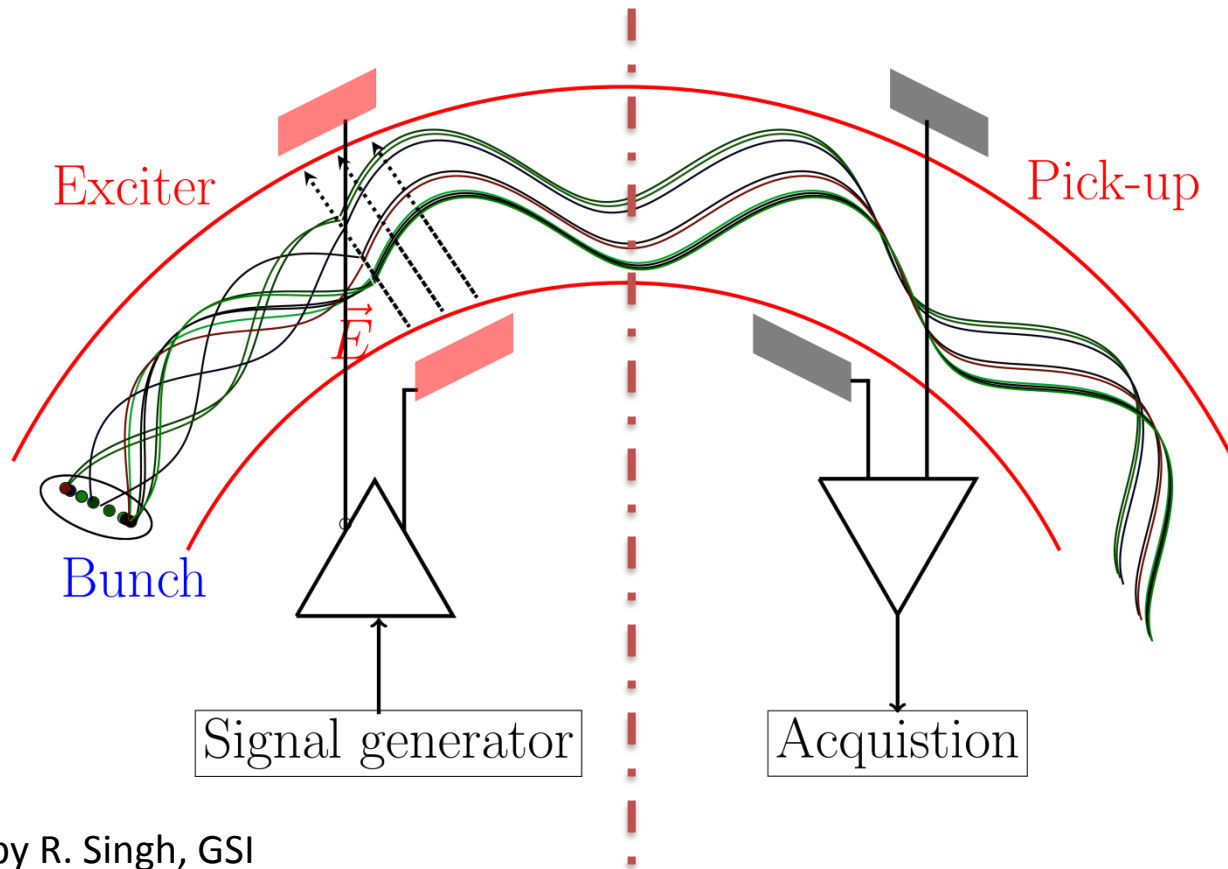
Tune Measurement: General Considerations

Coherent excitations are required for the detection by a BPM

Beam particle's *in-coherent* motion \Rightarrow center-of-mass stays constant

Excitation of **all** particles by rf \Rightarrow *coherent* motion

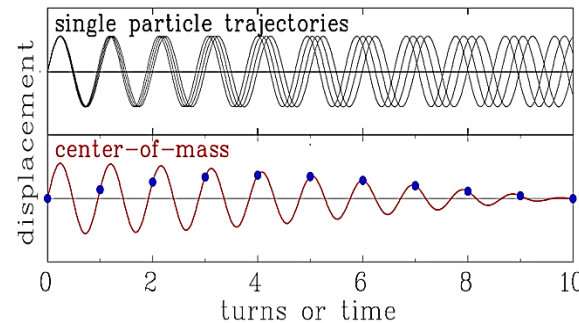
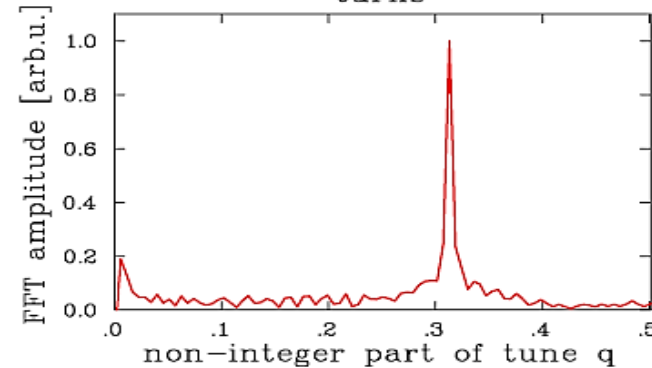
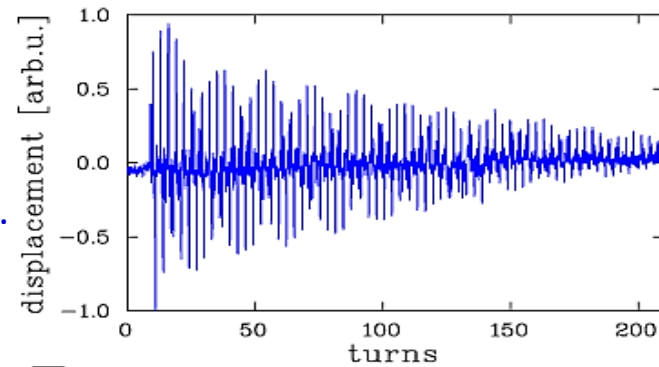
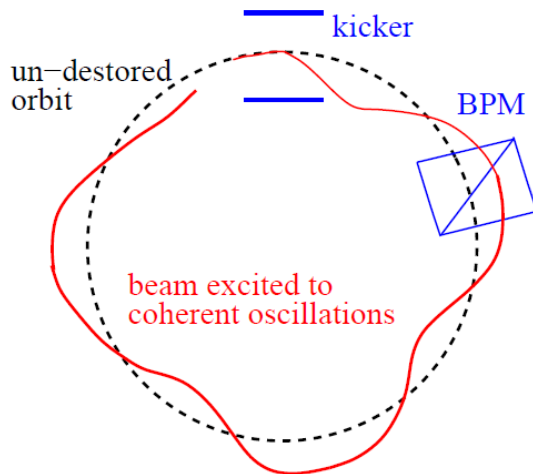
\Rightarrow center-of-mass variation turn-by-turn i.e. center acts as **one** macro-particle



Graphics by R. Singh, GSI

Tune Measurement: The Kick-Method in Time Domain

The beam is excited to
coherent betatron oscillation
 → Beam position measured
 each revolution ('turn-by-turn')
 → Fourier Trans. gives the non-integer tune q .
 Short kick compared to revolution.



Decay is caused by
 de-phasing,
not by decreasing
 single particle
 amplitude.

The de-coherence time limits the **resolution**:

N non-zero samples
 ⇒ General limit of discrete FFT: $\Delta q > \frac{1}{2N}$

Here: $N = 200$ turn $\Rightarrow \Delta q > 0.003$
 (tune spreads can be $\Delta q \approx 0.001$!)

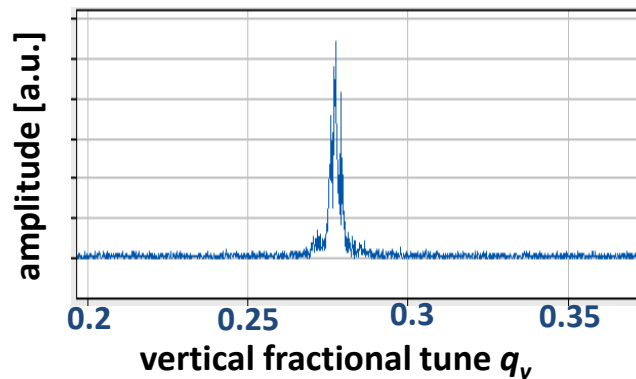
Tune Measurement: *Gentle* Excitation with Wideband Noise

Instead of a sine wave, noise with adequate bandwidth can be applied

→ beam picks out its resonance frequency:

- Broadband excitation with white noise of ≈ 10 kHz bandwidth
 - Turn-by-turn position measurement
 - Fourier transformation of the recorded data
- ⇒ Continues monitoring with low disturbance

vertical tune at fixed time ≈ 15 ms

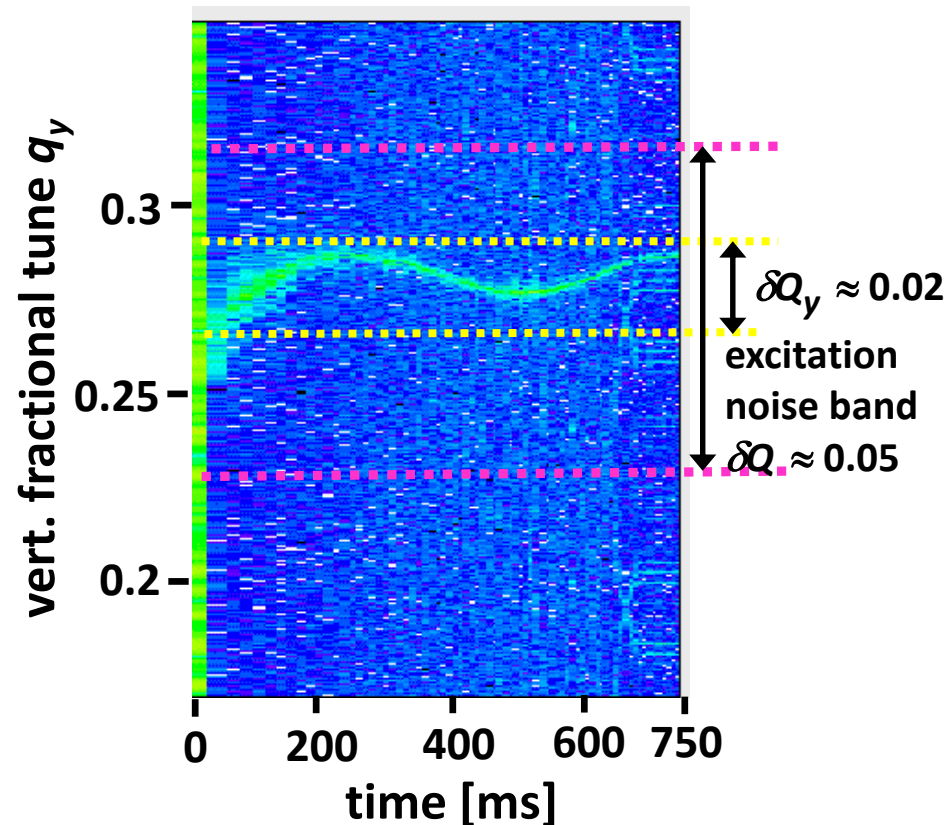


Advantage:

Fast scan with good time resolution

U. Rauch et al., DIPAC 2009

Example: Vertical tune within 4096 turn duration ≈ 15 ms
at GSI synchrotron 11 → 300 MeV/u in 0.7 s
vertical tune versus time



Chromaticity Measurement from Closed Orbit Data

Chromaticity ξ : Change of tune for off-momentum particle $\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$

Two step measurement procedure:

1. Change of momentum p by detuned rf-frequency $\frac{\Delta p}{p} = \eta^{-1} \cdot \frac{\Delta f_{acc}}{f_{acc}}$

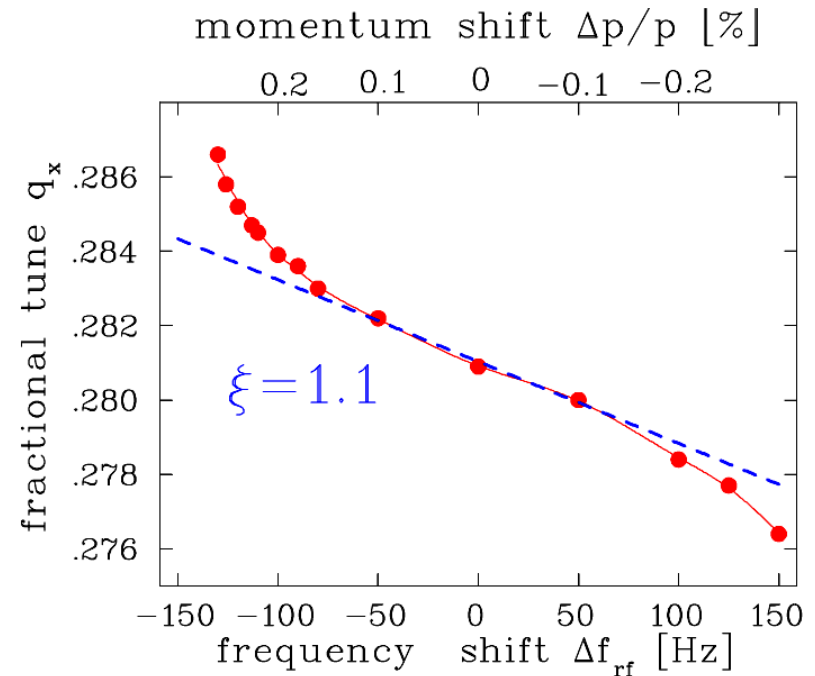
2. Excitation of coherent betatron oscillations
and tune measurement

(kick-method, BTF, noise excitation):

Plot of $\Delta Q/Q$ as a function of $\Delta p/p$

\Rightarrow slope is dispersion ξ .

Example: Measurement at LEP:



From M Minty, F. Zimmermann,
Measurement and Control of charged Particle Beam,
Springer Verlag 2003

β -Function Measurement from Bunch-by-Bunch BPM Data

Excitation of **coherent** betatron oscillations:

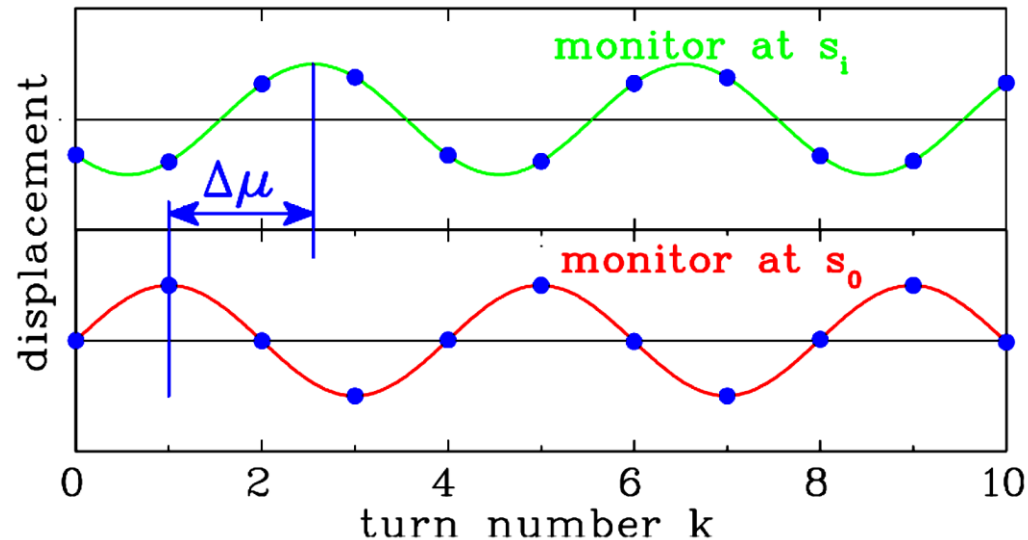
→ Time-dependent position reading results the phase advance between BPMs

The phase advance is:

$$\Delta\mu = \mu_i - \mu_0$$

β -function from

$$\Delta\mu = \int_{s_0}^{s_i} \frac{ds}{\beta(s)}$$



Remark: Determination of β -function with 3 BPMs:

$$\beta_{meas}(BPM_1) = \beta_{model}(BPM_1) \cdot \frac{\cot[\mu_{meas}(1 \rightarrow 2)] - \cot[\mu_{meas}(1 \rightarrow 3)]}{\cot[\mu_{model}(1 \rightarrow 2)] - \cot[\mu_{model}(1 \rightarrow 3)]}$$

See e.g.: R. Tomas et al., Phys. Rev. Acc. Beams **20**, 054801 (2017)

A. Wegscheider et al., Phys. Rev. Acc. Beams **20**, 111002 (2017)

'Beta-beating' from Bunch-by-Bunch BPM Data

Example: 'Beta-beating' at BPM $\Delta\beta = \beta_{meas} - \beta_{model}$ with measured β_{meas} & calculated β_{model} for each BPM at BNL for RHIC (proton-proton or ions circular collider with 3.8 km length)

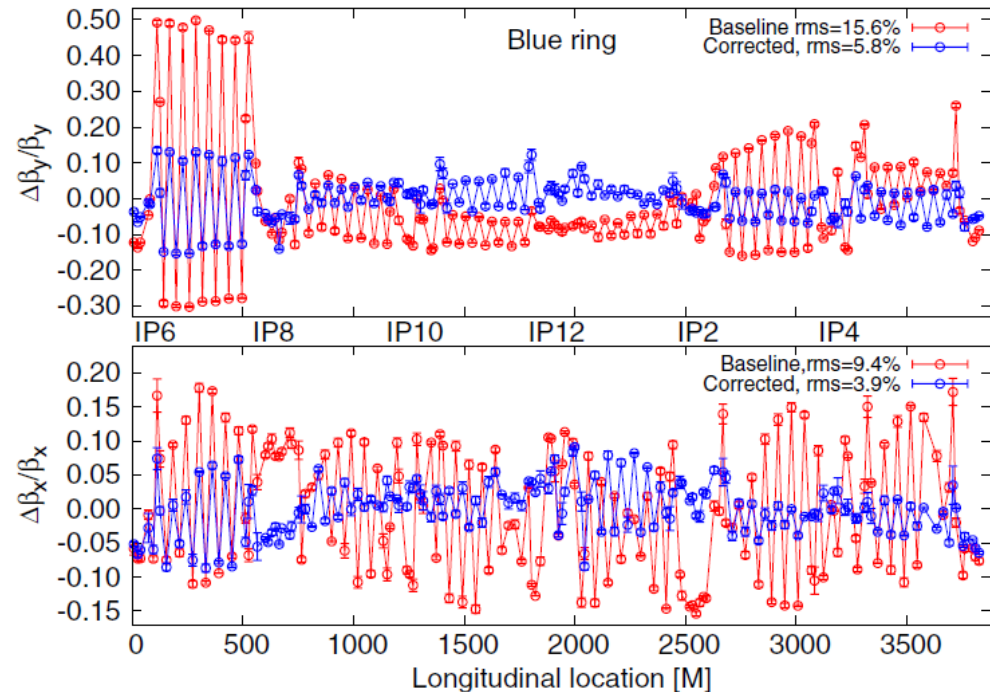
Result concerning 'beta-beating':

- Model doesn't fit reality completely
e.g. caused by misalignments
- Corrections executed
- Increase of the luminosity

Remark:

Measurement accuracy depends on

- BPM accuracy
- Numerical evaluation method



From X. Shen et al.,
Phys. Rev. Acc. Beams **16**, 111001 (2013)

See lecture 'Imperfections and Corrections' by Volker Ziemann

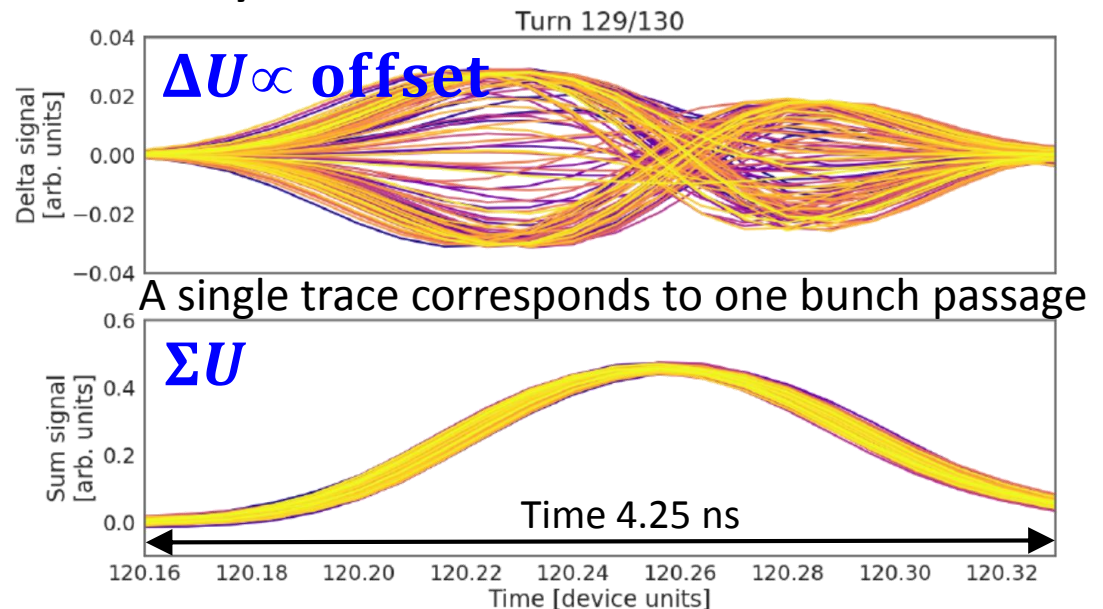
High band-width measurements delivers:

- Bunch shape given by the sum $\Sigma U(t) = U_{right}(t) + U_{left}(t)$ of two plates
- Intra-bunch movement of the **center** by $x_{center}(t) \propto \Delta U(t) = U_{right}(t) - U_{left}(t)$

Example: Single bunch observation on **turn-by-turn** basis with beam excitation at SPS

Goal: Monitoring instabilities

See lecture
'Collective Effects'
by Kevin Li



(a) Headtail mode 1 for chromaticity $\xi = 0.2$

Courtesy Kevin Li, CAS Proceedings 2021

Summary Pick-Ups for bunched Beams

The electric field is monitored for bunched beams using rf-technologies ('frequency domain'). Beside transformers they are the most often used instruments!

Differentiated or proportional signal: rf-bandwidth \leftrightarrow beam parameters

Proton synchrotron: 1 to 100 MHz, mostly 1 M Ω \rightarrow proportional shape

LINAC, e⁻-synchrotron: 0.1 to 3 GHz, 50 Ω \rightarrow differentiated shape

Important quantity: Transfer impedance $Z_t(\omega, \beta)$.

Types of capacitive pick-ups:

Linear-cut (p-synch.), button (p-LINAC, e⁻-LINAC and synch.)

Position reading: Difference signal of two or four pick-up plates (BPM):

➤ Non-intercepting reading of center-of-mass \rightarrow online measurement and control

Synchrotron: Fast reading, '**bunch-by-bunch**' \rightarrow trajectory, **slow** reading \rightarrow closed orbit

➤ **Synchrotron:** Excitation of **coherent** betatron oscillations \Rightarrow tune $q, \xi, \beta(s), D(s)$...

Remark: BPMs have high pass characteristic \Rightarrow no signal for dc-beams

Thank you for your attention!

Backup slides

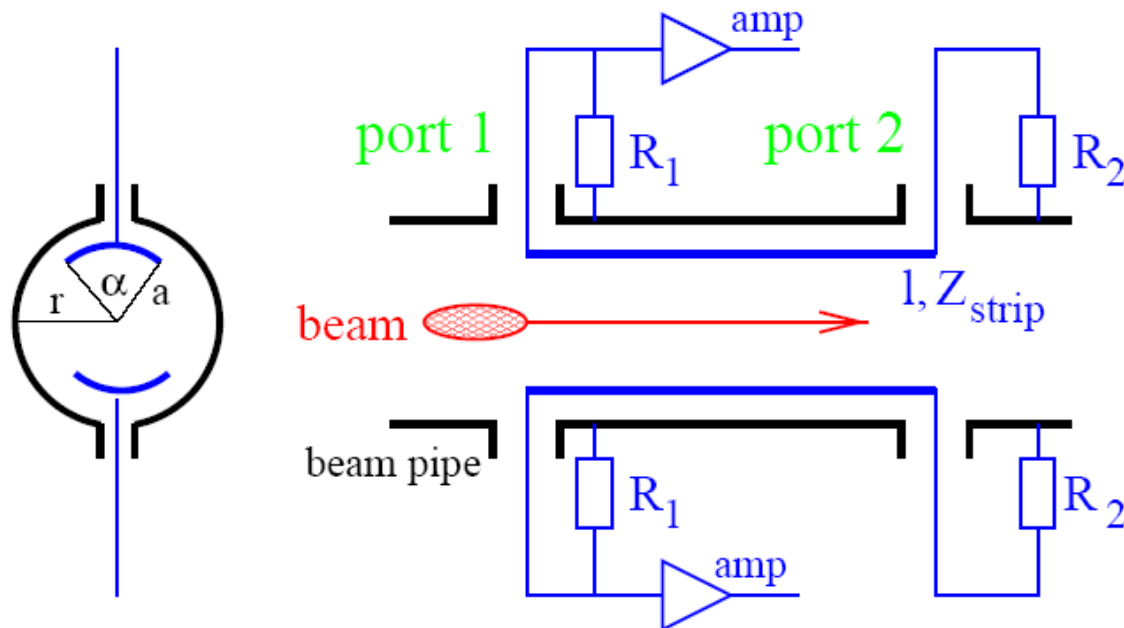
Stripline BPM: General Idea

For short bunches, the **capacitive** button deforms the signal

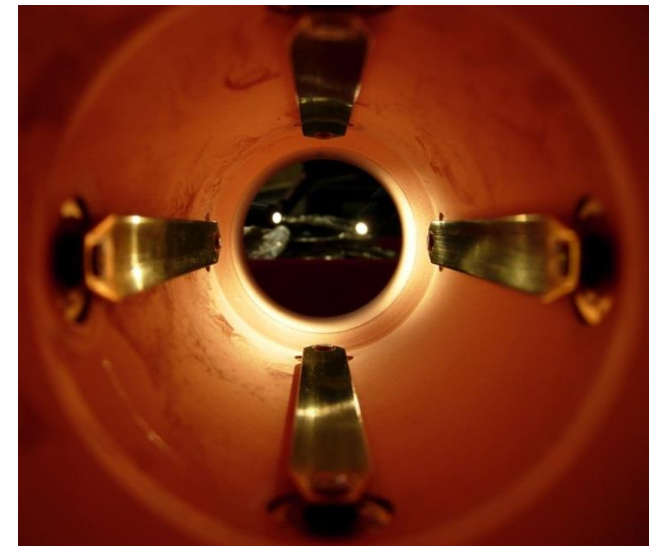
→ Relativistic beam $\beta \approx 1 \Rightarrow$ field of bunches nearly TEM wave

→ Bunch's electro-magnetic field induces a **traveling pulse** at the strips

→ Assumption: Bunch shorter than BPM, $Z_{strip} = R_1 = R_2 = 50 \Omega$ and $v_{beam} = c_{strip}$



LHC stripline BPM, $l = 12 \text{ cm}$



From C. Boccard, CERN

Stripline BPM: General Idea

For relativistic beam with $\beta \approx 1$ and short bunches:

→ Bunch's electro-magnetic field induces a **traveling pulse** at the strip

→ **Assumption:** $l_{bunch} \ll l$, $Z_{strip} = R_1 = R_2 = 50 \Omega$ and $v_{beam} = c_{strip}$

Signal treatment at upstream port 1:

$t=0$: Beam induced charges at **port 1**:

→ half to R_1 , half toward **port 2**

$t=l/c$: Beam induced charges at **port 2**:

→ half to R_2 , **but** due to different sign, it cancels with the signal from **port 1**

→ half signal reflected

$t=2 \cdot l/c$: reflected signal reaches **port 1**

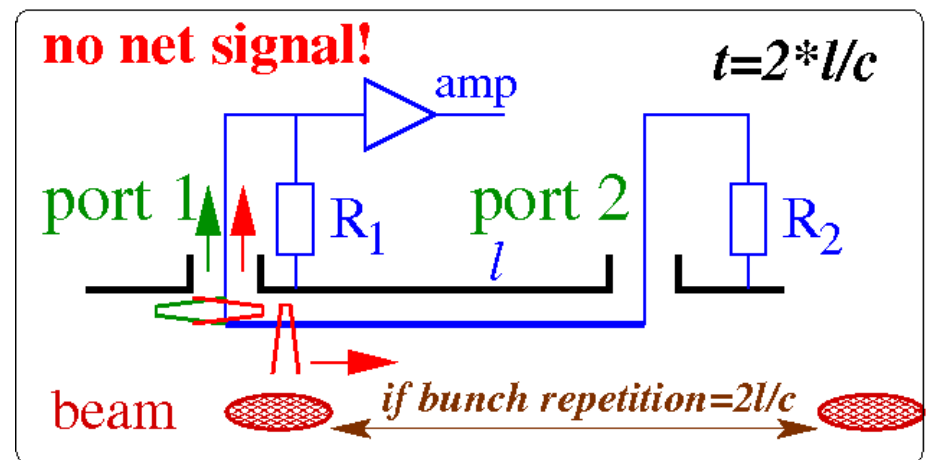
$$\Rightarrow U_1(t) = \frac{1}{2} \cdot \frac{\alpha}{2\pi} \cdot Z_{strip} (I_{beam}(t) - I_{beam}(t - 2l/c))$$

If beam repetition time equals $2 \cdot l/c$: reflected preceding port 2 signal cancels the new one:

→ no net signal at **port 1**

Signal at downstream port 2: Beam induced charges cancel with traveling charge from port 1

⇒ Signal depends on direction ⇔ **can distinguish between counter-propagation beams**

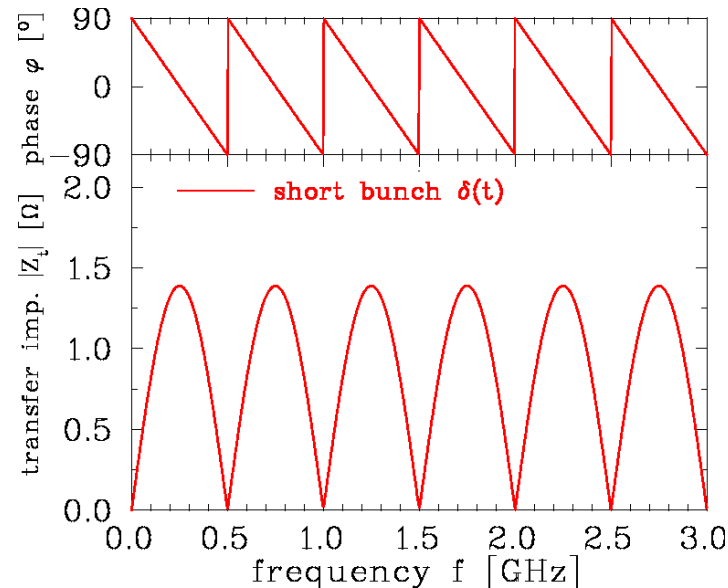
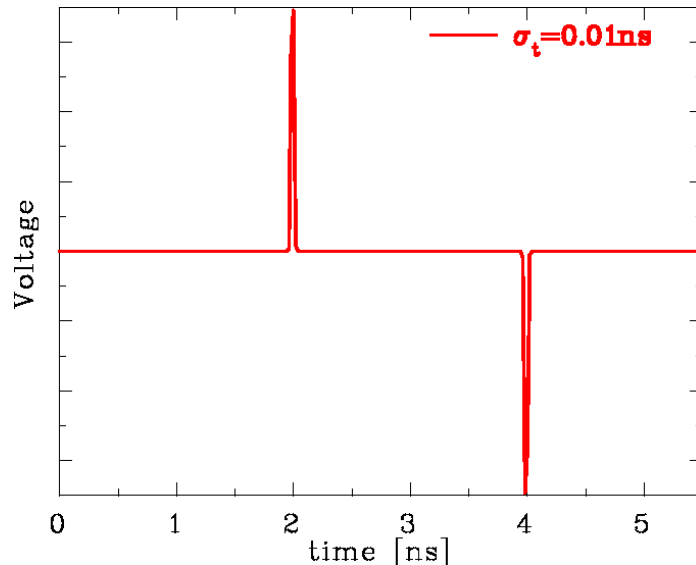


Stripline BPM: Transfer Impedance

The signal from port 1 and the reflection from port 2 can cancel \Rightarrow minima in Z_t .

For short bunches $I_{beam}(t) \rightarrow Ne \cdot \delta(t)$: $Z_t(\omega) = Z_{strip} \cdot \frac{\alpha}{2\pi} \cdot \sin(\omega l / c) \cdot e^{i(\pi/2 - \omega l / c)}$

Stripline length $l=30$ cm, $\alpha=10^0$



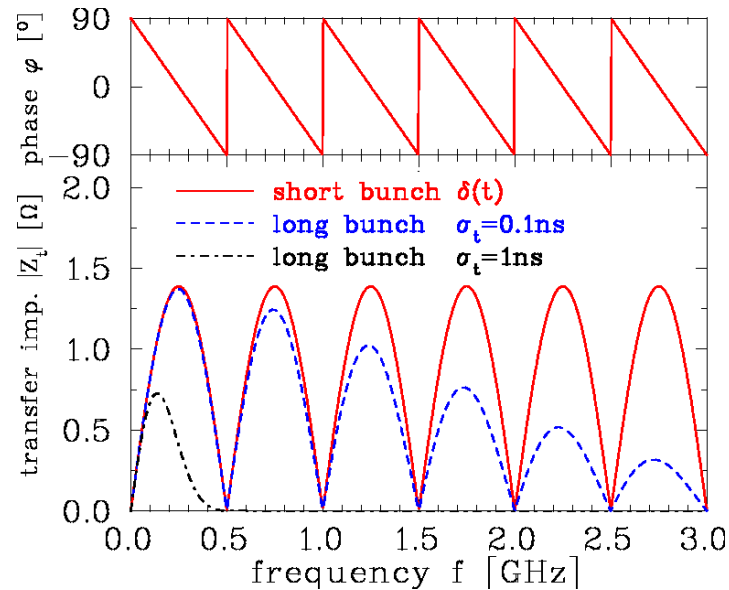
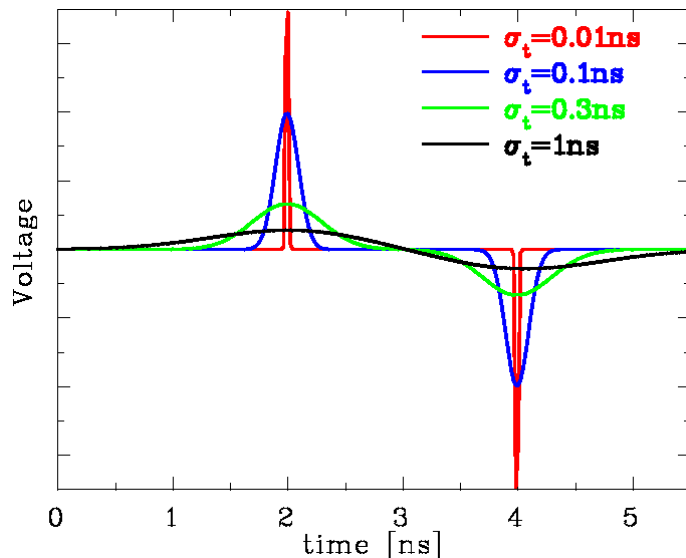
- Z_t show maximum at $l=c/4f=\lambda/4$ i.e. 'quarter wave coupler' for bunch train
 $\Rightarrow l$ has to be matched to v_{beam}
- No signal for $l=c/2f=\lambda/2$ i.e. destructive interference with **subsequent** bunch
- Around maximum of $|Z_t|$: phase shift $\varphi=0$ i.e. direct image of bunch
- $f_{center}=1/4 \cdot c/l \cdot (2n-1)$. For first lobe: $f_{low}=1/2 \cdot f_{center}$ $f_{high}=3/2 \cdot f_{center}$ i.e. bandwidth $\approx 1/2 \cdot f_{center}$
- Precise matching at feed-through required to preserve 50 Ω matching.

Stripline BPM: Transfer Impedance

The signal from port 1 and the reflection from port 2 can cancel \Rightarrow minima in Z_t .

For bunches of length σ : $\Rightarrow Z_t(\omega) = Z_{strip} \cdot \frac{\alpha}{2\pi} \cdot e^{-\omega^2 \sigma^2 / 2} \cdot \sin(\omega l / c) \cdot e^{i(\pi/2 - \omega l / c)}$

Stripline length $l=30$ cm, $\alpha=10^0$



➤ $Z_t(\omega)$ decreases for higher frequencies

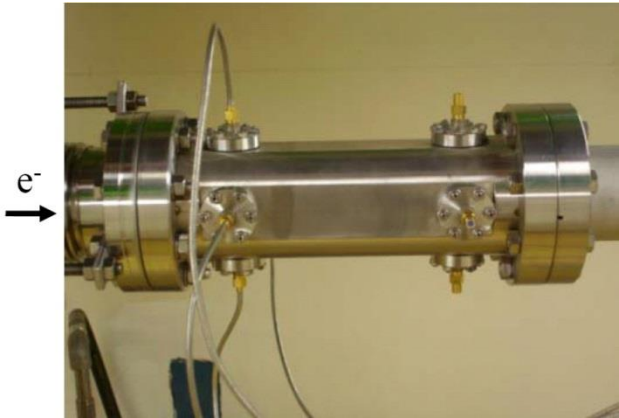
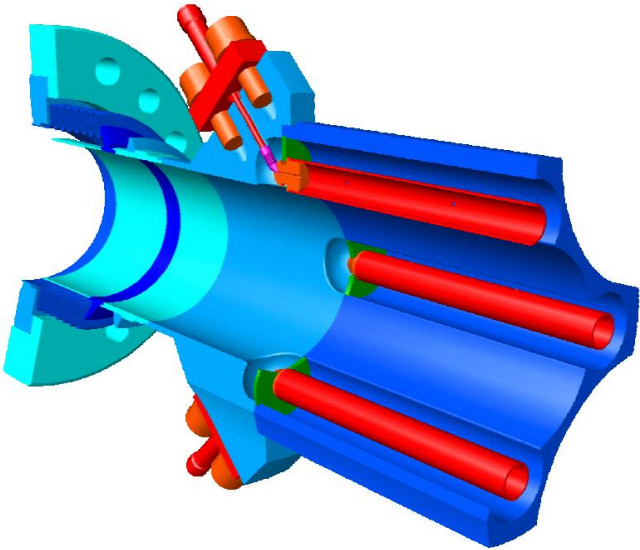
➤ If total bunch is too long $\pm 3\sigma_t > l$ destructive interference leads to signal damping

Cure: length of stripline has to be matched to bunch length

Comparison: Stripline and Button BPM (simplified)

	Stripline	Button
Idea	traveling wave	electro-static
Requirement	Careful $Z_{strip} = 50 \Omega$ matching	
Signal quality	Less deformation of bunch signal	Deformation by finite size and capacitance
Bandwidth	Broadband, but minima	Highpass, but $f_{cut} < 1 \text{ GHz}$
Signal strength	Large Large longitudinal and transverse coverage possible	Small Size $< \varnothing 3\text{cm}$, to prevent signal deformation
Mechanics	Complex	Simple
Installation	Inside quadrupole possible \Rightarrow improving accuracy	Compact insertion
Directivity	YES	No

FLASH BPM inside quadrupole



From . S. Vilkins, D. Nölle (DESY)

Estimation of finite Beam Size Effect for Button BPM

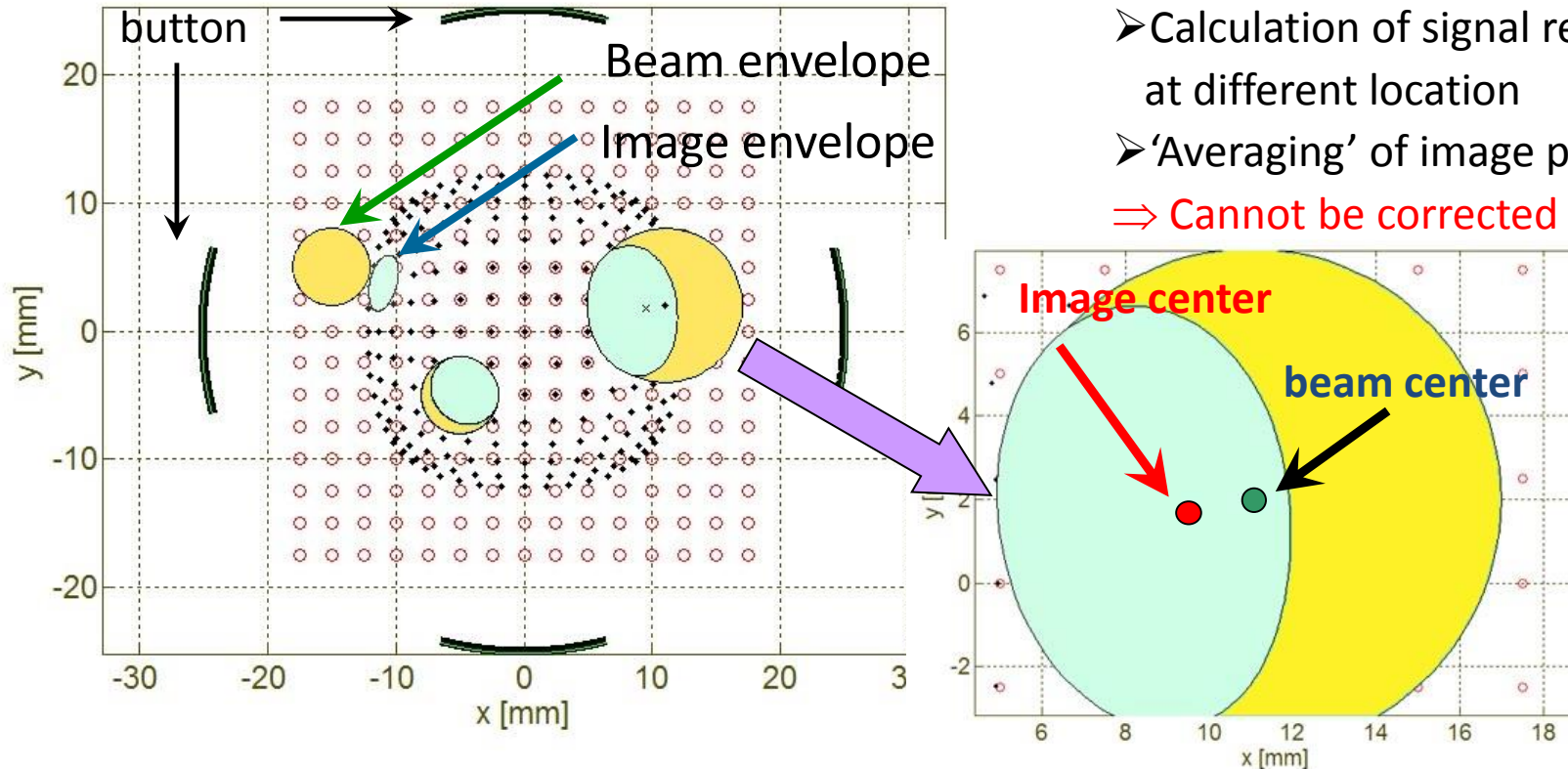
Ideal 2-dim model:

Due to the non-linearity, the beam size enters in the position reading.

Finite beam size:

- Calculation of signal response at different location
- 'Averaging' of image position

⇒ **Cannot be corrected !**



Remark: For most LINACs: Linearity is less important, because beam has to be centered
Position correction as feed-forward for next macro-pulse.