

How the physics of quantum impurities became a central question in the relic neutrino detection

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February 11, 2022

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condensed matter



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condensed matter



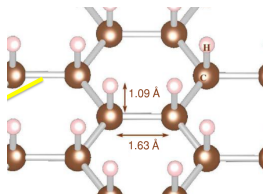
Alexey Boyarsky, Leiden University
particle physics

This talk will be about

Quantum impurity as a central question of the
astrophysical experiment

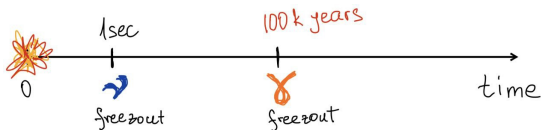
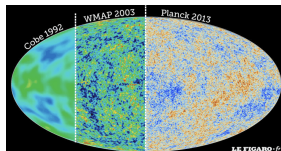
Quantum measurement devices with quantum impurities for particle physics and physics of the early universe

- ▶ New era in cosmology and particle physics through the experiments on quantum devices
- ▶ New pillar of the early universe picture comparable to CMB
- ▶ Devices with quantum impurities for the relic neutrino detection
- ▶ Zoo of condensed matter effects on small energy scales



$C\nu B$ and CMB

- ▶ Similarly to how relic photons form **CMB**, relic neutrinos form **$C\nu B$**
- ▶ **As the Universe expands** at some moments it becomes *transparent* for ν/γ .
- ▶ “Frozen picture” of the early Universe.
 - ▶ The freezout of the neutrinos is much earlier than photons.



- ▶ Right now, in your room, there are **411 relic photons** and **339 relic neutrinos** in every cm^3 .
- ▶ Most of them are **relic** neutrinos.

Observation of the cosmological neutrinos would then provide a window into the 1st second of creation

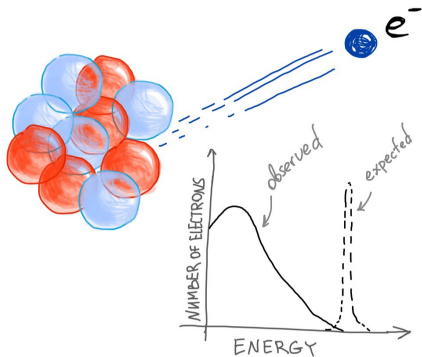
Why have we not discovered
 $C \cup B$ yet?

Riminder: what is neutrino?



- ▶ Energy and angular momentum are not conserved in β decay processes?
- ▶ W. Pauli *predicted* in 1930 a new particle to "save" the conservation laws
- ▶ Pauli originally called his new particle "neutron" (neutral one)

- ▶ Chadwick discovered a massive nuclear particle in 1932, however it was *not* Pauli's particle.
- ▶ Fermi renamed Pauli's particle to *neutrino* ("little neutral one")



How was the neutrino detected?

"I have done a terrible thing. I have postulated a particle that cannot be detected." (W. Pauli)

- ▶ Bethe and Peierls in 1934 estimated

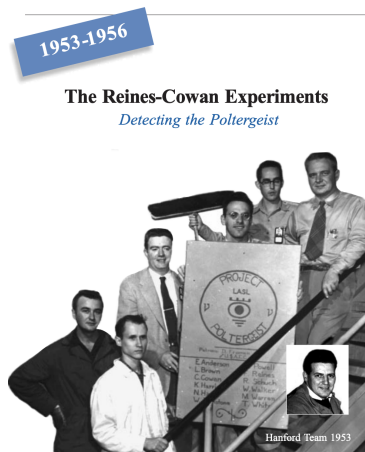
$$\sigma \sim 10^{-44} \text{ cm}^2$$

$$\sigma_{\text{Thomson}} \sim 10^{-25} \text{ cm}^2$$

$$\sigma_{\text{nuclear}} \sim 10^{-26} \text{ cm}^2$$

- ▶ Neutrino was first detected in 1956 by a group led by Clyde Cowan and Frederick Reines
- ▶ They used the enormous flux of antineutrinos from a nuclear reactor.

1995 - Nobel Prize



Fermi theory

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Fermi: β decay is a decay of neutron inside the nucleus

$$V_F(x_1, x_2, x_3, x_4) = G_F \delta(x_1 - x_2) \delta(x_2 - x_3) \delta(x_3 - x_4)$$

One can calculate the number of **transitions per unit time** into some range of final states $d\nu_f$ using **Fermi Golden Rule**

$$dw_{if} = 2\pi \left| \langle \psi_f^0 | \hat{V}_F | \psi_i^0 \rangle \right|^2 \delta(E_i - E_f) d\nu_f$$

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} \sqrt{E_e^2 - m_e^2} E_e (Q - E_e)^2$$

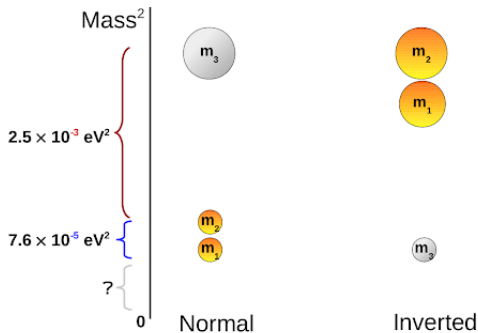
$$Q = m_N({}_Z^A X) - m_N({}_{Z+1}^A X') - m_e - m_{\bar{\nu}_e}$$



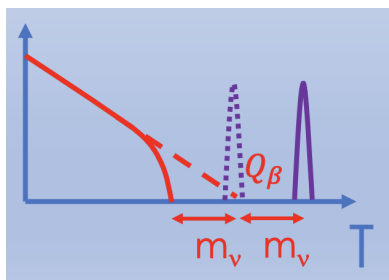
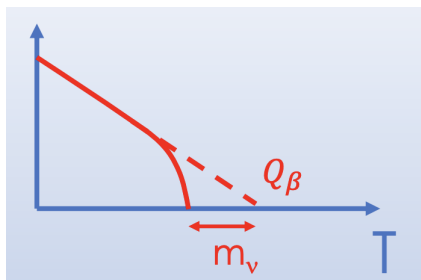
- ▶ Looks like neutrinos are **massless**
- ▶ Along with the β decay, Fermi theory predicts **neutrino capture**.

Neutrinos are massive

- ▶ In the last few decades neutrino flavor oscillations were convincingly observed, meaning that neutrinos are **massive**
- ▶ Neutrino oscillations can only measure Δm and hierarchy.



β -decay and neutrino capture



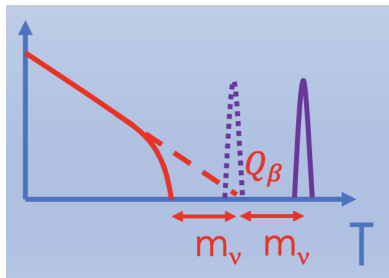
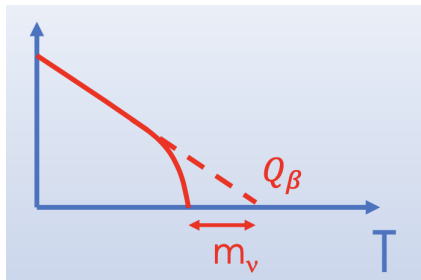
- ▶ Neutrino capture is **threshold-less** – soft relic neutrino detection [Weinberg, 1962].
- ▶ The 2 parts of the spectrum are separated by $2m_\nu$ ¹
- ▶ Before the relic neutrino detection one would be able to measure the neutrino mass m_ν

¹Schematic picture that assumes only one neutrino flavour.

**Relic neutrinos leave a
signature in the spectrum of a
radioactive atom**

**Is this goal technically
achievable?**

Challenges



- ▶ High energy precision (order of $m_\nu \sim \text{meV}$)
- ▶ Sufficient activity rate (several events per year)

Requirements to the experiment

High enough activity

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- ▶ Low emitter Q -value (Cocco et. al., 2007)

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$$(\sigma v)_\nu \propto \frac{1}{\tau Q^3}$$

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$$\lambda = (\sigma n)^{-1} = \left(R_{\text{atom}}^2 \frac{N}{L^3} \right)^{-1} > L$$

$$L > R_{\text{atom}} \sqrt{N} \sim 1 \text{ km}$$

Very naive estimate! In reality much bigger

Requirements to the experiment

High enough activity

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High enough precision

- ▶ Low emitter Q -value
- ▶ Low emitter densities - *electron free path bigger than the system size*
- ▶ Low volume

$$\Delta E \sim \frac{V_{\text{source}}}{V_{\text{detector}}}$$

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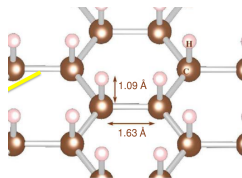
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High enough activity

- ▶ Low emitter Q -value
 - ▶ High number of emitters (order of 10^{25})
 - ▶ Lifetime of emitter: small enough to have a high decay rate, but large enough not to decay instantly
-
- Radioactive material in gaseous form does not suit (0.93 eV resolution)
 - Need in the solid-state based experiment

High enough precision

- ▶ Low emitter Q -value
- ▶ Low emitter densities - *electron free path bigger than the system size*
- ▶ Low volume



PTOLEMY² - state of the art

$C\nu B$ detection experiment challenge:

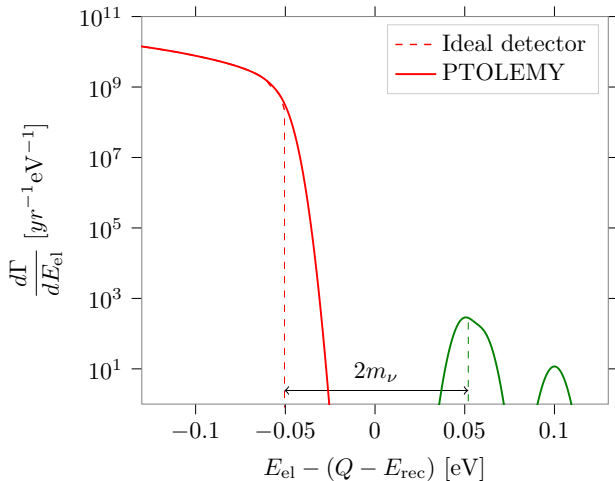
- ▶ High energy resolution combined with sufficient number of events.

- ▶ **Tritium** as a β -decay emitter.
- ▶ Tritium is **deposited on graphene sheets** (vdW forces).
- ▶ ≈ 4 $C\nu B$ events per year.
- ▶ Outstanding **energy resolution** of the apparatus ≈ 10 meV.
- ▶ Strong collaboration funded by Simon's foundation
- ▶ Collaboration between Princeton (PI Chris Tully), Amsterdam, Milan, Rome



P on-
T ecorvo
O bservatory for
L ight,
E arly-universe,
M assive-neutrino
Y ield

β -decay in Tritium ³



³Mind the **log** scale

So that is it?

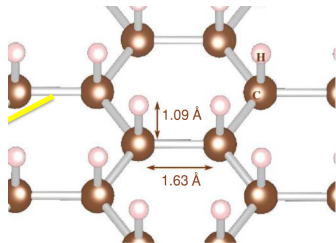
So that is it?

No

One needs to account for the intrinsic energy resolution

The **width of the peak** that serves as a signature of $C\sqrt{B}$ is defined by

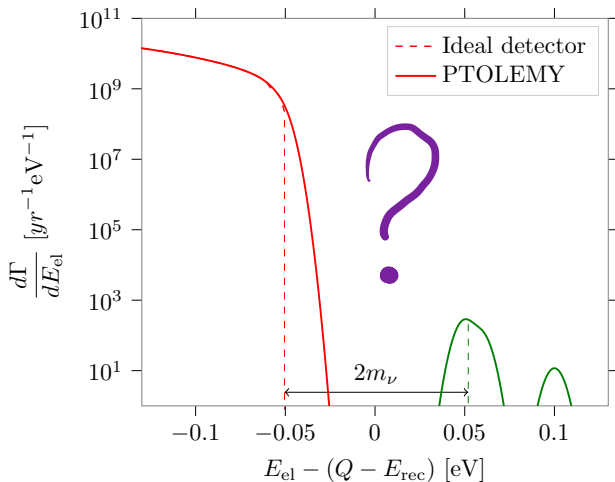
- ▶ *energy resolution* of the measurement
- ▶ physical *smearing* of the energies of individual electrons



- ▶ The presence of the substrate changes the **intrinsic** (before measurement) **energy spectrum of the emitted electron**.
- ▶ Introducing additional **broadening of the electron spectrum**.
- ▶ Which leads to **intrinsic irreducible limitations** on the energy resolution.

Real spectrum

Along with the finite energy resolution of the measurement device one has to account for the **intrinsic physical smearing** of the energies of individual electrons.

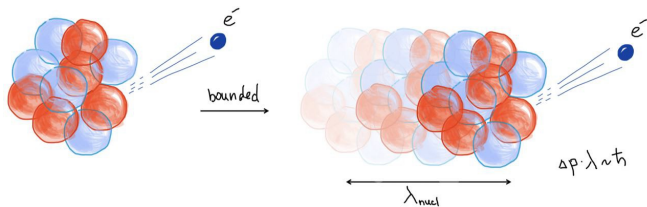


Mechanisms of the intrinsic energy broadening

- ▶ **Chemical bonding of the atom to the substrate.**
- ▶ **Impurity screening by charges in the substrate.**
- ▶ **X-ray edge singularity.**
- ▶ Lattice vibrations
- ▶ Emission of plasmons and surface polaritons
- ▶ Creation of shock wave emission due to the motion of the emitted electron at grazing angles at speeds exceeding the Fermi velocity
- ▶ Inhomogeneous broadening
- ▶ ...

Chemical bonding of the atom to the substrate

General mechanism of the broadening



- ▶ For a bonded system, **recoil energy** of the nucleus **is not fixed** by the kinematics but has some distribution.
- ▶ **Uncertainty⁴** in the velocity of the centre of mass of the nucleus

$$\Delta u \approx \frac{\hbar}{m_{\text{nucl}} \lambda_{\text{nucl}}}.$$

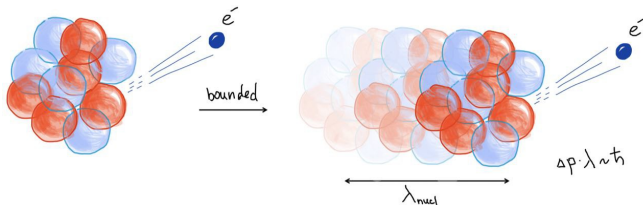
- ▶ **The energy of the electron** is measured in the laboratory frame of reference, where it **acquires an uncertainty⁵**

$$\Delta E \approx m_e v_e \Delta u.$$

⁴from the Heisenberg uncertainty principle.

⁵ ΔE has the same distribution as Δu .

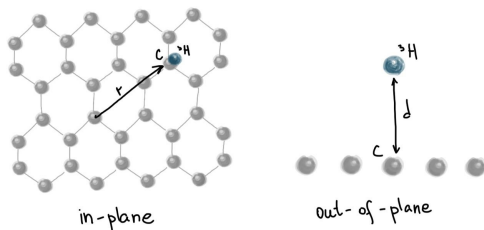
General mechanism of the broadening



$$\Delta E \approx \hbar \frac{m_e v_e}{m_{\text{nucl}} \lambda_{\text{nucl}}},$$

- ▶ λ_{nucl} is the **spread of the ground state** of the nucleus that is defined by the bonding potential.

Bonding potential



For the **heavy atom** one can expand the potential near its minimum

$$U = \frac{1}{2} \kappa_{i,j} r_i r_j + U_0$$

The energy uncertainty very **weakly** depends on the binding potential

$$\Delta E \propto \lambda_{\text{nucl}}^{-1} \propto \kappa^{1/4}$$

Energy broadening for the β -decay of the Tritium on graphene

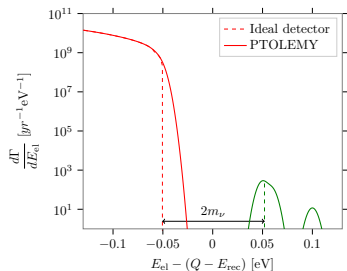
$$\frac{\Delta E}{\sqrt{\hbar m_e}} \approx \underbrace{\kappa^{1/4}}_{\text{potential}} \underbrace{\sqrt{\frac{Q}{m_{\text{nucl}}^{3/2}}}}_{\text{nucleus}}$$

The uncertainty in the electron energy ΔE :

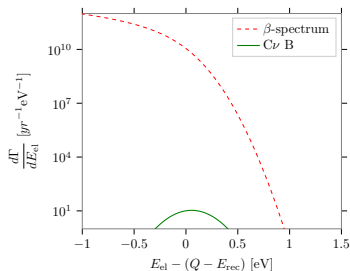
- ▶ Is of the order of **0.5 eV**.
- ▶ Is 2 orders of magnitude greater than the resolution needed to see the $C\nu B$ signal.
- ▶ Weakly depends on the potential stiffness.
- ▶ For molecular tritium the estimate is of the same order.
- ▶ Strongly depends on the radioactive nucleus.
- ▶ Agrees with the the fully quantum calculation⁶

⁶Fermi Golden Rule.

Shape of the spectrum for the β -decay of the Tritium on graphene⁷



smearing \rightarrow



$$\tilde{\mathcal{G}}(v) = \int du \mathcal{F}(u) \mathcal{G}(v + u).$$

- ▶ \mathcal{G} - distr. of the electron velocity in the centre of mass ref. frame.
- ▶ \mathcal{F} - distr. of the velocity of the centre of mass.
- ▶ $\tilde{\mathcal{G}}$ - distr. of the electron velocity in the laboratory ref. frame.

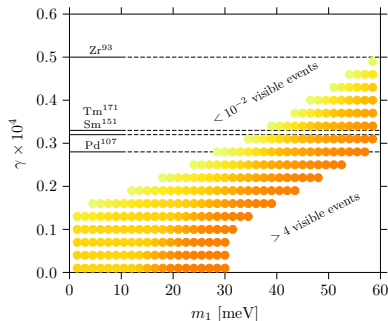
⁷ $\Delta E \approx mu\Delta u$, therefore ΔE has the same distribution as Δu .

Solution ⁸

$$\frac{\Delta E}{\sqrt{\hbar m_e}} \approx \varkappa^{1/4} \sqrt{\frac{Q}{m_{\text{nucl}}^{3/2}}} \equiv \varkappa^{1/4} \gamma$$

Change the β -emitter to minimize γ

- ▶ Define the **visibility** as the number of $C\nu B$ events that overlap with the continuous spectrum.
- ▶ ^{107}Pd , ^{151}Sm , ^{171}Tm seem to work
- ▶ ^{107}Pd has a very low activity
- ▶ ^{171}Tm has ~ 10 times less events per year than ^3H
- ▶ ^{151}Sm has $\sim 10^3$ times less events per year than ^3H



⁸Mikulenکو A., Cheipesh Y., Cheianov V., Boyarsky A., soon to appear

Is that it?

Is that it?

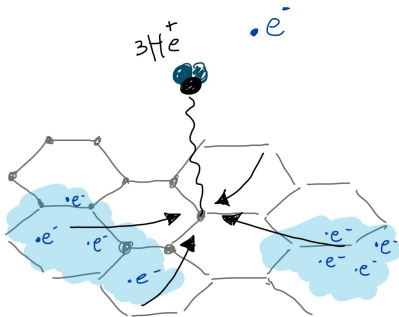
No, it is only beginning...

Mechanisms of the intrinsic energy broadening

- ▶ Chemical bonding of the atom to the substrate.
- ▶ **Impurity screening by charges in the substrate.**
- ▶ **X-ray edge singularity.**
- ▶ Lattice vibrations
- ▶ Emission of plasmons and surface polaritons
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- ▶ Inhomogeneous broadening
- ▶ ...

Charge screening

Once the electron is emitted, positive ion is formed. As a response to this, the **graphene will polarize** to screen the positive ion.



A screened weaker Coulomb potential **effectively increases the energy of the electron.**

Charge screening. Rough dimensional analysis

The only 2 **dimensionfull quantities** are

- ▶ $d \approx 3 \text{ \AA}$ - the **distance of the tritium atom**.
- ▶ $v_F \approx 10 \text{ \AA fs}^{-1}$ - **Fermi velocity** in graphene.

The rough estimate of the **relaxation time** is

$$\tau_{\text{relax}} = \frac{d}{v_f} \approx 0.3 \text{ fs} \quad (1)$$

The **velocity of the electron** near the edge of the spectrum is almost unchanged during the flight and is

$$\frac{v_0}{c} \approx 0.27 \sqrt{\frac{E}{Q}} \quad (2)$$

Charge screening. Rough dimensional analysis

Such an electron will fly away from the atom on the distance

$$\lambda = v_e \tau_{\text{relax}} \approx \sqrt{\frac{E}{Q}} 0.27c \times 0.3 \text{ fs} \approx \sqrt{\frac{E}{Q}} \times 243 \text{ \AA} \quad (3)$$

The shift in the spectrum compared to unscreened case will be

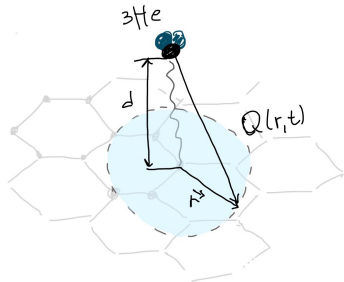
$$\Delta E(E) = k \frac{Ze^2}{\lambda} \approx \sqrt{\frac{E}{Q}} \times 59 \text{ meV} \quad (4)$$

We see that the effect is significant and, most importantly, it is **energy-dependent**.

Charge screening. Quasiclassics

$$m_e \ddot{z} = -\nabla \varphi_{\text{eff}}(z, t) = -\nabla (\varphi_{\text{bare}}(z, t) - \varphi_{\text{polariz}}(z, t))$$

$$\varphi_{\text{polariz}}(r, t) = \int d\vec{r}' \frac{keQ(\vec{r}', t)}{\sqrt{(z+d)^2 + r'^2}}$$



Where $Q(\vec{r}, t)$ is the **renormalized** charge which is defined by the **dielectric permittivity** of graphene. For intrinsic graphene^a

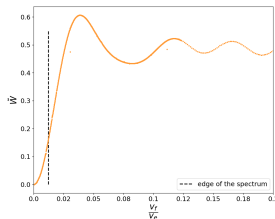
$$\epsilon(q, t) = \left(\delta(t) + \frac{\alpha\pi}{4} q e^{-qd} J_0(v_F q t) \right)$$

^aHwang, Das Sarma, PRB, 2007

Charge screening. Quasiclassics

The work performed on the electron by graphene

$$W = \underbrace{\frac{\pi\alpha^2}{16E_F d^2}}_{\approx 0.5 \text{ eV}} \tilde{W} \left(\frac{v_F}{v_e} \right)$$



- ▶ For the electron at the end of the spectrum $W \approx 75 \text{ meV}$
- ▶ Fermi velocity defines the relaxation time scale
- ▶ Near the edge of the spectrum $W(E)$ is almost linear.
- ▶ **The gap will not be "eaten" by this effect** as near the edge

$$|dW| = \frac{2\gamma}{E} \frac{dW}{d\gamma} dE \sim 10^{-6} dE$$

- ▶ Important to consider also **quantum fluctuations**

$$\text{Var}(W) = \langle \hat{W}^2 \rangle - W^2$$

X-ray edge ⁹

- ▶ Same Fermi Golden Rule but also taking into account graphene

$$|\psi\rangle_i^{(0)} = |1\rangle_H |0\rangle_{He} |1\rangle_\nu |0\rangle_e |FS\rangle_{gr}$$

$$|\psi\rangle_f^{(0)} = |0\rangle_H |1\rangle_{He} |0\rangle_\nu |1\rangle_e |\lambda\rangle_{gr}$$

$$dw_{if} = 2\pi \left| \langle \psi_f^0 | \hat{V}_F | \psi_i^0 \rangle \right|^2 \delta(E_i - E_f) d\nu_f$$

- ▶ Ion of ${}^3\text{He}^+$ is a scattering center (core level hole) in the X-ray edge singularity problem
- ▶ Energy transfer between the β emitters and the graphene system leading to the **smearing of the spectrum**
- ▶ The smeared beta decay spectrum is the convolution between the spectral density function of graphene $A(E)$ and the original beta decay spectrum $d\Gamma/dE$.

$$\left. \frac{d\Gamma}{dE} \right|_{\text{smeared}} = \left(\frac{d\Gamma}{dE} \star A \right) (E)$$

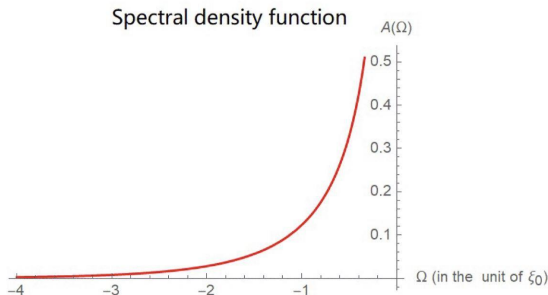
⁹Work done by T. Zhiyang and V. Cheianov

X-ray edge ¹⁰

- ▶ Spectral density function has an X-ray edge

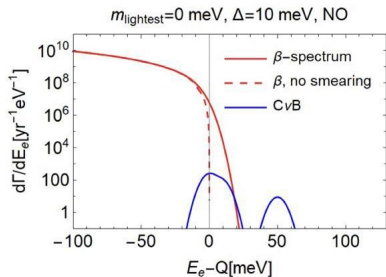
$$A(\Omega) = \Theta(-\Omega) \frac{\sin(\pi g)}{\pi} \Gamma(1-g) \frac{\exp(\Omega)}{\xi_0 (-\Omega)^{1-g}}, \quad \Omega = \frac{E + E_i}{\xi_0},$$

where $g = e^4/2\varepsilon^2 v_F^2$ and $\xi = v_F/2d$ is the cut-off energy fixed by the distance d from the impurity to the graphene sheet.

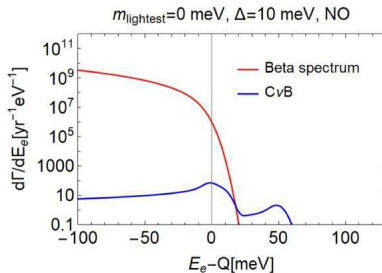


¹⁰Work done by T. Zhiyang and V. Cheianov

X-ray edge ¹¹



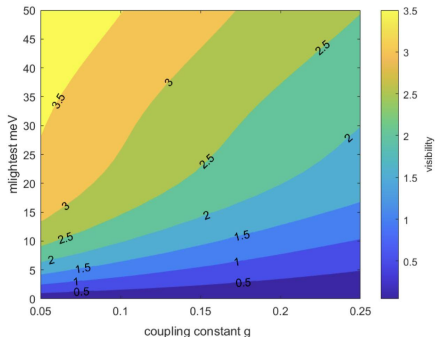
(a) original beta decay spectrum when the lightest neutrino mass equals zero.



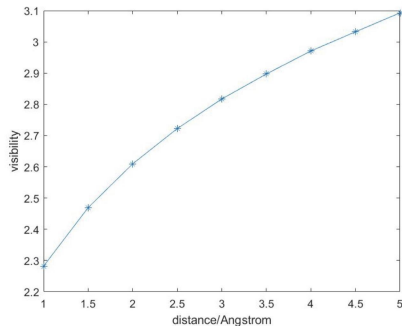
(b) smeared beta decay spectrum

¹¹Work done by T. Zhiyang and V. Cheianov

X-ray edge. Solutions ¹²



Increase dielectric permittivity ϵ of the substrate



Increase the distance between the substrate and the β emitter

Conclusions

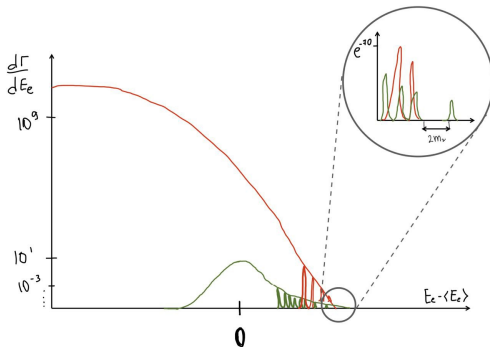
- ▶ Quantum impurity problems play a key role in the new large-scale astro-particle experiment capable to open new era in Cosmology
- ▶ Fundamental questions of particle physics and the origin of our Universe could realistically be accessed only through the understanding of the condensed matter effects
- ▶ Limitations due to intrinsic effects in the solid state part of the measurement setup are crucial for the feasibility of this large-scale experiment.
- ▶ Before scaling to to a huge number of beta emitters, a program of theoretical and experimental study of small quantum devices with relevant impurities is needed.
- ▶ Zoo of effects appear in the $E \sim 10 \text{ meV}$ which needs a much better understanding than usually in CM.
- ▶ Energy and time scales not accessible in the condensed matter experiments before
- ▶ More questions than answers ¹³

Backup slides

General shape of the spectrum

In case when the **bonding potential is harmonic**, the spectrum is

- ▶ Discrete near the edge.
- ▶ Continuous further from the edge.
- ▶ The envelope has a gaussian distribution.
- ▶ The distance between the discrete lines¹⁴ is $\varepsilon = \hbar \sqrt{\frac{\kappa}{m_{\text{nucl}}}}$.
- ▶ Biggest part of the C ν B channel overlaps with the continuum.



¹⁴10 meV for the Tritium on graphene and 0.5 eV for the molecular tritium.

Comparison with molecular Tritium

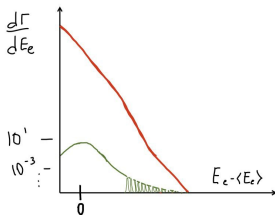
Similarities:

- ▶ Bonded by a harmonic potential ($\alpha_{\text{graphene}} \approx 0.1, \alpha_{\text{mol}} \approx 75$).
- ▶ Localized and therefore are subjects to Heisenberg's uncertainty principle $m\Delta v\Delta x \sim \hbar$.

Differences:

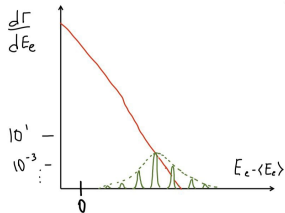
Atomic Tritium on graphene:

- ▶ All of the recoil energy goes to the harmonic modes.
- ▶ May break the bound after the recoil.

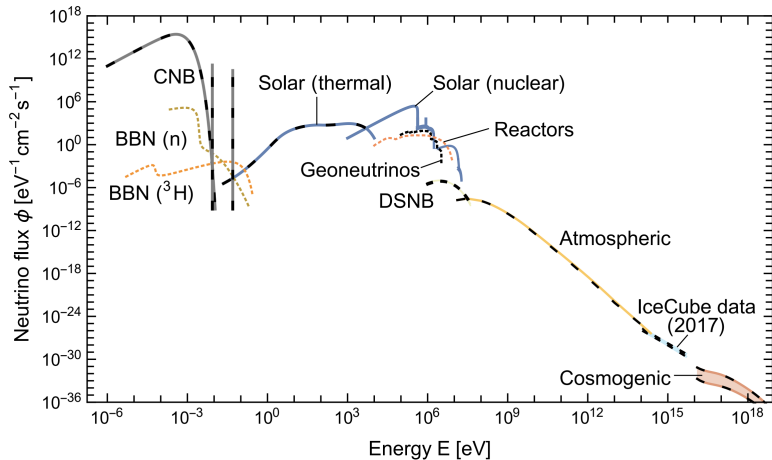


Gaseous molecular Tritium:

- ▶ Half of the recoil energy goes to the translational motion.
- ▶ Remains bound after the recoil.

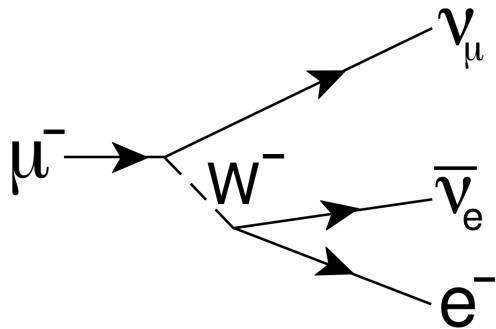


Neutrino flux¹⁵

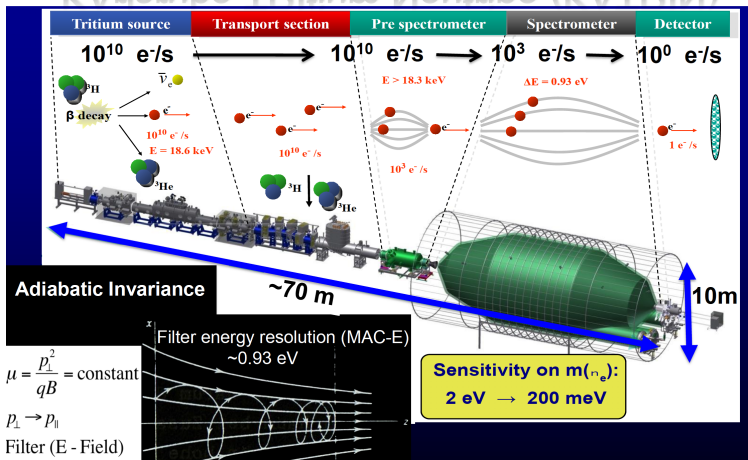


¹⁵E. Vitagliano et.al. "Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components", (2020)

Neutrino flavours



KARlsruhe TRItium Neutrino (KATRIN)



PTOLEMY

