PERTURBATIVE QCD

AND

JET PHYSICS

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FACTORIZATION
SCALE SEPARATION

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PREREQUISITES: QCD AS A GAUGE THEORY

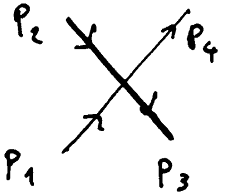
- LAGRANGIAN $\mathcal{L} = -\frac{1}{4} G^a_{\mu\mu} G^a_{\mu\nu} + \sum_i^{n_f} \bar{\psi}_i (i \not{D} - m_i) \psi_i$
  - $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} g_s A^b_\mu A^c_\nu$;
  - $D_\mu = \partial_\mu - i g_s \lambda^a A^a_\mu$;
  - λ^a GENERATORS OF SU(3); $[\lambda^a, \lambda^b] = i f^{abc} \lambda^c$ ACTING ON ψ
- $n_f = 6$; $m_u = m_d = m_s = 0$ IN THE PERTURBATIVE REGIME
- PERTURBATIVE EXPANSION PARAMETER $\alpha_s \equiv \frac{g_s^2}{4\pi}$
- DO NOT CONFUSE N_c GAUGE GROUP & n_f NUMBER OF FERMIONS (QUARKS)
- RUNNING COUPLING: $\alpha(s) = \frac{\alpha(\mu^2)}{1 + \beta_0 \frac{\alpha(\mu^2)}{2\pi} \ln \frac{s}{\mu^2}} \Rightarrow$ DECREASES AT HIGH ENERGY:

  ASYMPTOTIC FREEDOM

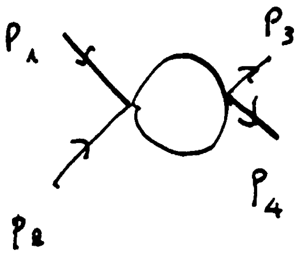
PREREQUISITES: RENORMALIZATION

THE BASIC IDEA: ϕ^4 THEORY

$\mathcal{L}_I = -\frac{g}{24}\phi^4$; $\phi\phi \to \phi\phi$ ELASTIC SCATTERING OF MASSIVE SCALAR FIELDS



$$\frac{d\sigma}{d\cos\theta} = \frac{g^2}{128\pi}\frac{1}{s}; \qquad s = (p_1 + p_2)^2$$



+ 2 more

$$\frac{d\sigma}{d\cos\theta} = \frac{g^2}{128\pi}\frac{1}{s}F(s,t)\text{: DIVERGES!;}$$

$$t = (p_1 - p_3)^2,\ u = (p_1 - p_4)^2$$

$$F(s,t) = \lim_{\Lambda\to\infty} 1 + \frac{g}{32\pi}\left(3 + \int_0^1 dx \ln\frac{M^2(s)}{\Lambda^2} + s \to t + s \to u\right); \quad M^2(s) = m^2 - x(1-x)s$$

RENORMALIZATION: EXPRESS A PHYSICAL OBSERVABLE IN TERMS OF OTHER PHYSICAL OBSERVABLES:

WHAT IS THE CHARGE g? DEFINE $g_{\rm phys}$ FROM $\left.\frac{d\sigma}{d\cos\theta}\right|_{s=t=u=\mu_0^2} = \frac{g^2_{\rm phys}}{128\pi}\frac{1}{s}$:

$$\frac{d\sigma}{d\cos\theta} = \frac{g^2_{\rm phys}}{128\pi}\frac{1}{s}F(s,t); \quad F(s,t) = 1 + \frac{g_{\rm phys}}{32\pi}\left(\int_0^1 \ln\frac{M^2(s)}{\mu_R^2} + s \to t + s \to u\right); \quad \mu_R^2 = M^2(\mu_0^2)$$

UV SINGULARITY IS UNIVERSAL $\Rightarrow$ REABSORBED IN DEF. OF THE COUPLING

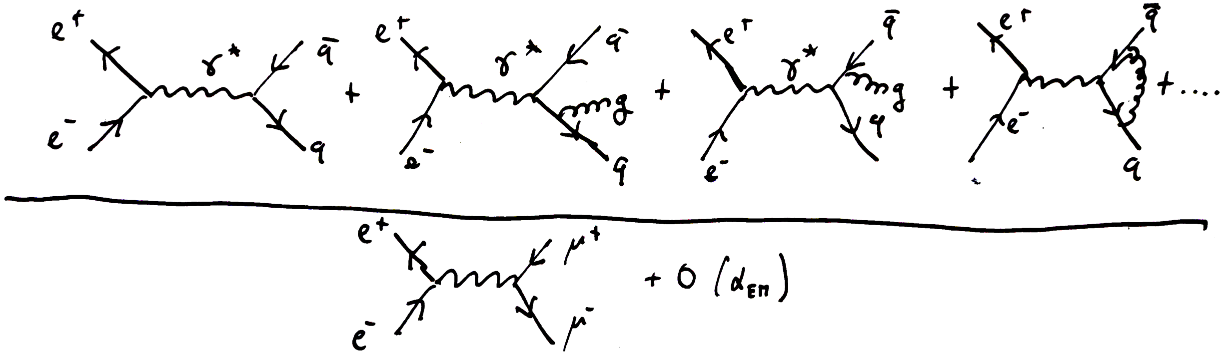
NOTE DEF. OF THE COUPLING DEPENDS ON THE RENORMALIZATION SCALE μ_R

RG: THE BASIC IDEA

PHYSICAL RESULTS CANNOT DEPEND ON RENORMALIZATION SCALE μ_R!

THE R RATIO

$$R = \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu-}$$

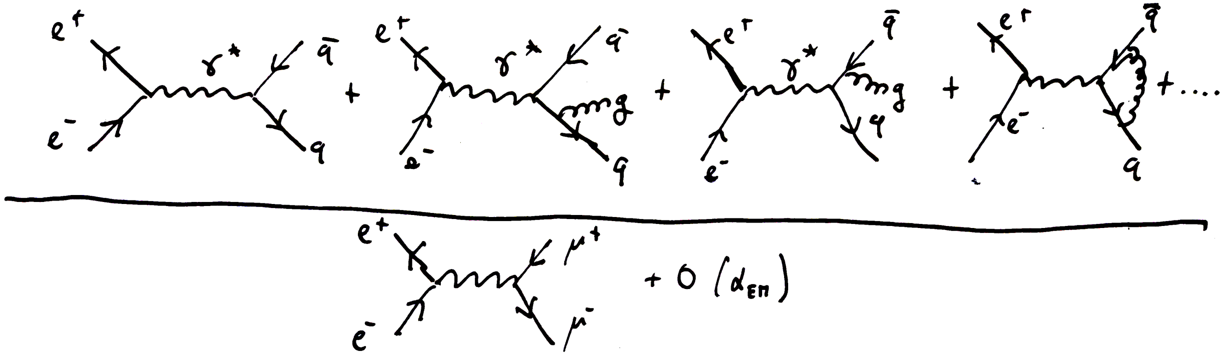


- IN THE FINAL STATE THERE ARE HADRONS, NOT QUARKS AND GLUONS!
- INFINITE ORDERS NEEDED???

THE RG ARGUMENT

- DIMENSIONAL ANALYSIS: $R = R\left(\alpha_s(\mu_R), \frac{s}{\mu_R^2}\right)$
- RG INVARIANCE: $\mu_R \frac{d}{d\mu_R} R = 0$
- SOLUTION: $R = R\left(\alpha_s(s)\right)$
- RUNNING COUPLING $\alpha_s(s)$
- SCALING PERTURBATIVELY COMPUTABLE: $\mu_R^2 \frac{d}{d\mu_R^2} \alpha_s(\mu_R) = \beta(\alpha_s(\mu_R))$

THE RUNNING COUPLING



- QCD BETA FUNCTION: $\beta(\alpha_s(\mu)) = \mu^2 \frac{d}{d\mu^2}\alpha_s(\mu)$;
  $\beta(\alpha_s) = -\beta_0 \frac{1}{2\pi}\alpha_2^2 + O(\alpha^3)$; $\beta_0 = \frac{1}{6}\left(11C_A - 2n_f\right)$;
  for $SU(N_c)$, $C_A = N_c$

- SOLUTION RESUMS ALL ORDERS IN $\alpha(\mu^2)$:
  $\alpha_s(s) = \frac{\alpha_s(\mu^2)}{1+\beta_0\alpha_s(\mu^2)\ln\frac{s}{\mu^2}} = \alpha_{\mu^2}\left(1 - \beta_0\alpha_s(\mu^2)\ln\frac{s}{\mu^2} + \ldots\right)$

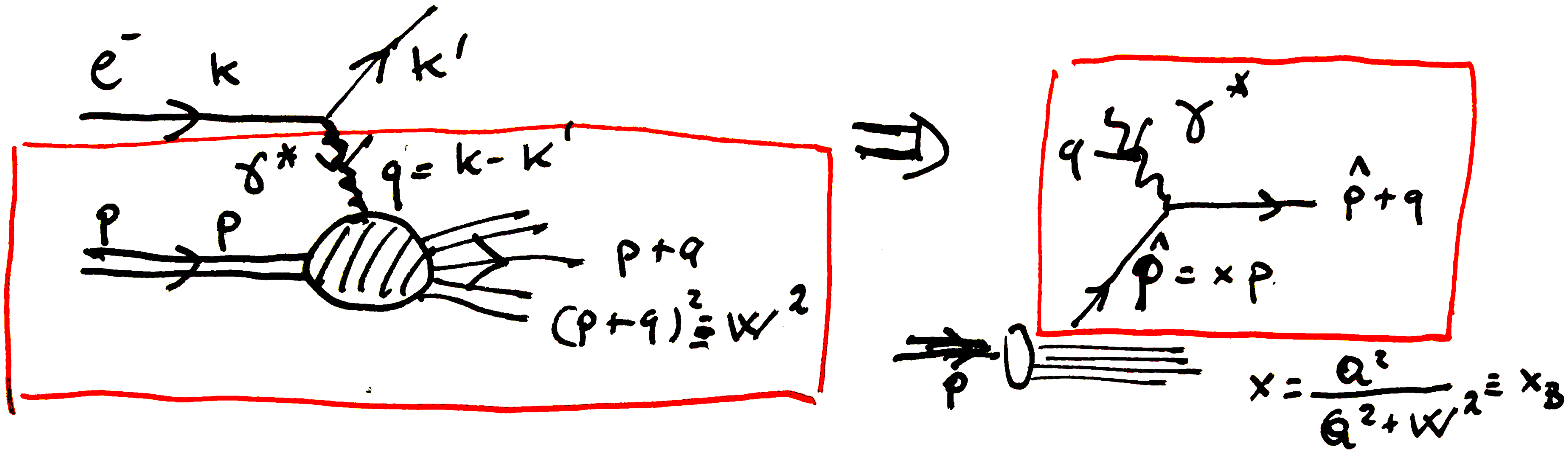
- $\alpha_s \sim \frac{1}{\ln s/\mu^2} \Leftrightarrow \alpha_s \ln\frac{s}{\mu^2} \sim 1$

- DIMENSIONAL TRANSMUTATION: DEFINE Λ SUCH THAT
  $\alpha_s(\Lambda) = \infty \Leftrightarrow \alpha_s(s) = \frac{1}{\beta_0 \ln s/\Lambda^2}$

- CAN COMPUTE R FROM PERTURBATIVE DIAGRAMS $\Rightarrow$ RESUMMED PERT. EXPANSION

PERTURBATION AT LEADING ORDER:
THE PARTON MODEL
EXAMPLE: $e - p$ SCATTERING

- LEPTON-PROTON REDUCED TO PHOTON (W, Z)-QUARK; $k' - k = q$
- CROSS-SECTION DEPENDS ON MOMENTA (p, q)
  $\Rightarrow$ SCALE $Q^2 = -q^2$ & DIMENSIONLESS RATIO $x_B = \frac{Q^2}{2p \cdot q}$ (Bjorken variable)
- LEADING-ORDER: $(xp + q)^2 = 0 \Rightarrow x = x_B$; CROSS-SECTION $\sigma \propto \delta(x - x_B)$

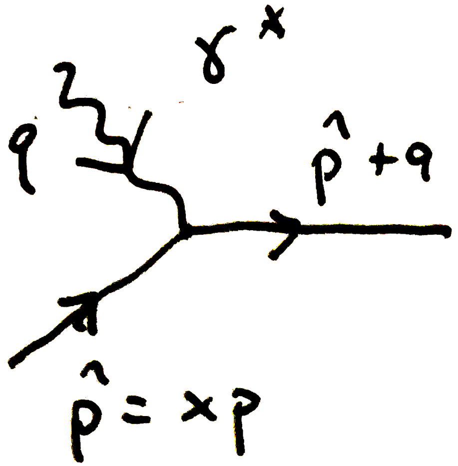


BEYOND LEADING ORDER

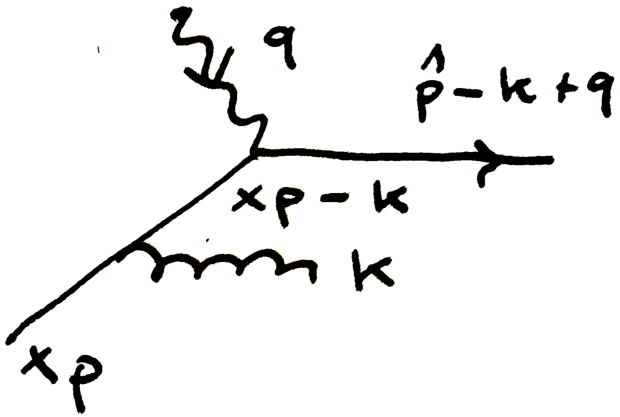
- CROSS-SECTION DEPENDS ON MOMENTA (p, q)
  $\Rightarrow$ SCALE $Q^2 = -q^2$ & DIMENSIONLESS RATIO $x_B = \frac{Q^2}{2p \cdot q}$ (Bjorken variable)
- LEADING-ORDER: $(xp + q)^2 = 0 \Rightarrow x = x_B$; CROSS-SECTION $\sigma \propto \delta(x - x_B)$
- NEXT-TO-LEADING-ORDER:
  - INTEGRATE OVER MOMENTUM OF EMITTED GLUON k: $d^3k = d^2k_t dk_z$; $k_z = \frac{\vec{k} \cdot \hat{\vec{p}}}{|\hat{\vec{p}}|}$
  - PROPAGATOR OF INTERMEDIATE QUARK $\sim \frac{1}{(\hat{p} - k)^2} = \frac{1}{k_t^2}$
  - LOGARITHMICALLY DIVERGENT $\frac{dk_t^2}{k_t^2}$

COLLINEAR SINGULARITY $\Rightarrow$ SENSITIVE TO IR PHYSICS???

LEADING-ORDER



NEXT-TO-LEADING-ORDER

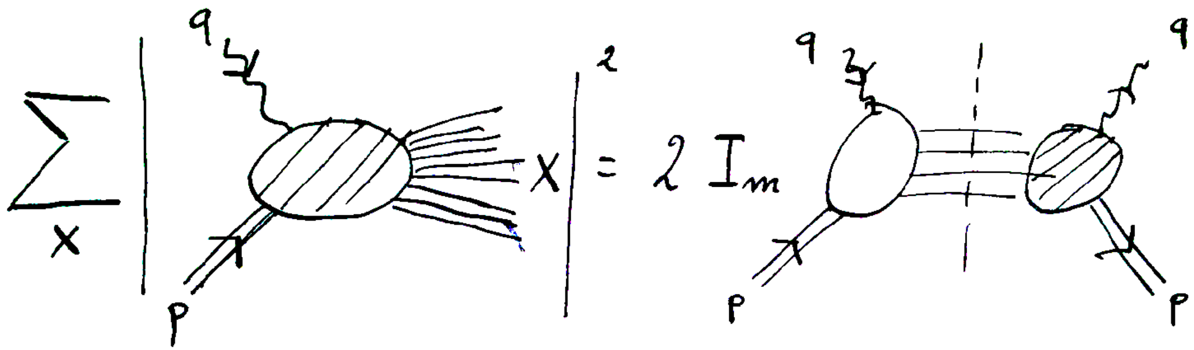


SCALE SEPARATION

THE WILSON EXPANSION

OPTICAL THEOREM FOR THE INCLUSIVE CROSS SECTION

$$\sigma = \sum_X |\langle X|p\gamma^*\rangle|^2 = \frac{2}{\pi}\mathrm{Im}\langle p\gamma^*|p\gamma^*\rangle$$



OPERATOR-PRODUCT MATRIX ELEMENT

$$\langle p\gamma^*(q)|p\gamma^*(q)\rangle = i\int \frac{d^4x}{(2\pi)^4} e^{iqx}\langle p|J^\mu(x)J^\nu(0)|p\rangle \equiv W^{\mu\nu}$$

SCALE SEPARATION

EXPANSION OF THE OPERATOR PRODUCT

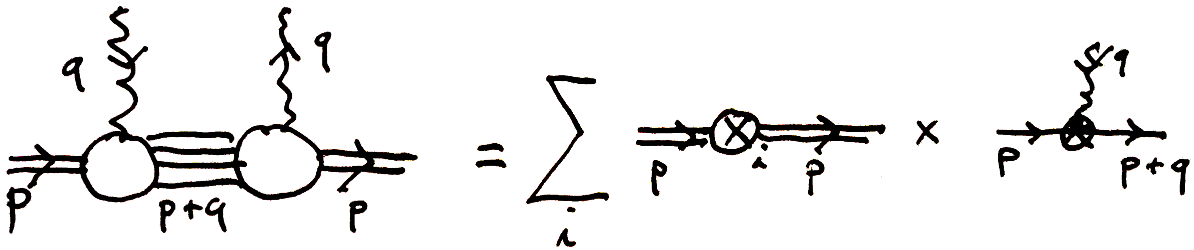
$$J^\mu(x)J^\nu(0) = \sum_i C_i(x^2) O_i^{\mu\nu}$$

AS $||x^\mu|| \to 0$ ONLY LOW-DIM OPERATORS SURVIVE

$||x^\mu|| \to 0$ SHORT DISTANCE $\Leftrightarrow$ $Q^2 \to \infty$ HIGH ENERGY

MATRIX ELEMENT OF OPE

$$\int \frac{d^4x}{(2\pi)^4} e^{iqx} \langle p|J(x)J(0)|p\rangle \quad = \quad \sum_i \quad \langle p|O_i|p\rangle \quad \times \quad C_i(Q^2)$$



- $\langle O_i \rangle$ DEPENDS ON STATE $|p\rangle$, DOES NOT DEPEND ON q
- $C_i(Q^2)$ DEPENDS ON q RECALL $q^2 = -Q^2$, DOES NOT DEPEND ON STATE $|p\rangle$

FACTORIZATION!

THE LIGHT-CONE EXPANSION

A SUBTLETY: $x^2 \to 0$ DOES NOT IMPLY $x^\mu \to 0$ $\quad Q^2 \to \infty$ DOES NOT IMPLY $q^\mu \to \infty$:

$\frac{Q^2}{2p\cdot q} \sim q^\mu$ BUT $\frac{Q^2}{2p\cdot q} = x$ FIXED!!

LEADING TWIST OPERATORS

$$J^\mu(x)J^\nu(0) = \sum_k C_k(x^2) O_k^{\mu\nu\alpha_1\ldots\alpha_{k-2}} x_{\alpha_1}\ldots x_{\alpha_k}$$

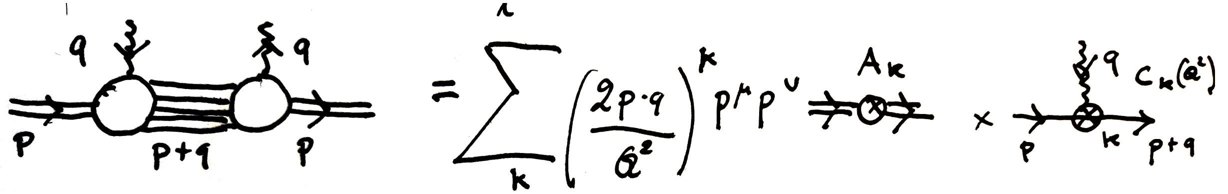
TWIST OF THE OPERATOR= dim.-spin

TWIST 2 OPERATORS

$$O_k^{(2,\,q)\,\mu\ldots\alpha_{k-2}} = \bar{\psi}\gamma^\mu\partial^\nu\partial^{\alpha_1}\ldots\partial^{\alpha_{k-2}}\psi; \quad O_k^{(2,\,g)\,\mu\ldots\alpha_{k-2}} = G_{\mu\alpha}\partial^{\alpha_1}\ldots\partial^{\alpha_{k-2}}G^\nu{}_\alpha$$

OPE MATRIX ELEMENTS

$$i\int \frac{d^4x}{(2\pi)^4} e^{iqx}\langle p|J^\mu(x)J^\nu(0)|p\rangle = \sum_k \langle p|O^{(2,\,q)\,\mu\ldots\alpha_{k-2}}|p\rangle i\int \frac{d^4x}{(2\pi)^4} e^{iqx} C_k(x^2) x_{\alpha_1}\ldots x_{\alpha_k}$$



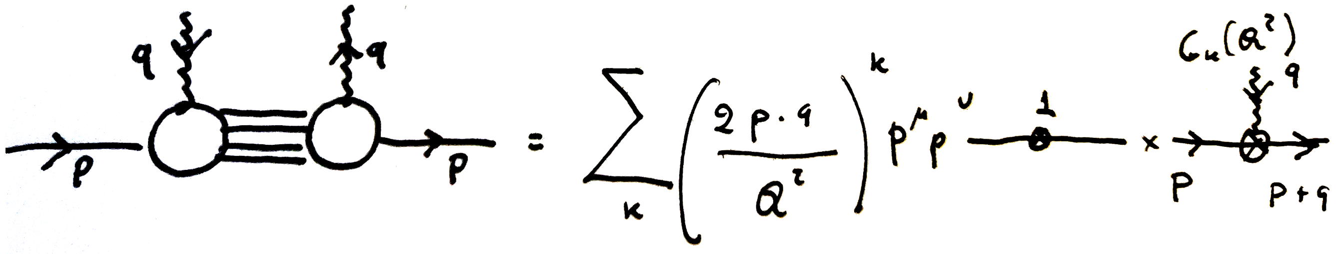
$$= \sum_k \left(\frac{2p\cdot q}{Q^2}\right)^{k-2} \frac{p^\mu p^\nu}{Q^2} \quad A_K \qquad C_k(Q^2) + \ldots$$

- $\langle p|O^{(2,\,q)\,\mu\nu\alpha_1\ldots\alpha_{k-2}}|p\rangle = A_k p^\mu p^\nu p^{\alpha_1}\ldots p^{\alpha_{k-2}}$; A_k reduced mat. el.
- $i\int \frac{d^4x}{(2\pi)^4} e^{iqx} C_k(x^2) x_\alpha = \frac{q^\alpha}{Q^2} C_k(Q^2)$ etc.

FACTORIZATION
THE UNIVERSALITY TRICK

- THE COEFFICIENTS DO NOT DEPEND ON THE STATE
- WHAT HAPPENS IF WE EVALUATE THE OPE IN A QUARK STATE?
- WHAT IS THE PROBABILITY OF FINDING A QUARK INSIDE A QUARK?

$$W^{\mu\nu}(x,Q^2) = \sum_k \left(\frac{1}{x}\right)^{k-2} \frac{p^\mu p^\nu}{Q^2} C_k(Q^2)$$



WILSON COEFFICIENTS ⇔ SCATTERING OF A FREE QUARK! (PARTON MODEL, ANYONE?)

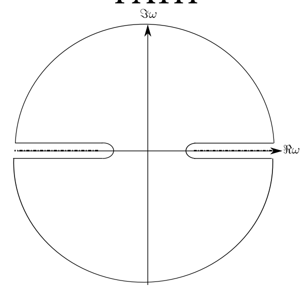
- PARAMETRIZE $W\mu\nu$ W. FORM FACTORS W_i:
  $$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(p^\mu - q^\mu \frac{p\cdot q}{q^2}\right)\left(p^\nu - q^\nu \frac{p\cdot q}{q^2}\right) \frac{2x}{Q^2} W_2$$
- LAURENT EXPANSION $W_2(x,Q^2) = \sum_k \left(\frac{1}{x}\right)^{k-1} C_k(Q^2)$

FACTORIZATION

WHAT ABOUT THE MATRIX ELEMENTS?

EXTRACTION OF THE k-TH COEFFICIENT

INTEGRATION PATH



$$C_k(Q^2) = \oint \frac{d\omega}{2\pi i}\omega^{-k} W_2(\omega, Q^2) = \int_0^1 dx x^{k-2} \mathrm{Im} W_2(x, Q^2)$$
$$C_k(Q^2) = \int_0^1 dx x^{k-2} F_2(x, Q^2)$$

QUARK (PARTON)

$$\int_0^1 dx x^{k-1} \frac{\hat{F}_2(x, Q^2)}{x} = C_k(Q^2)$$: COMPUTABLE!

$\Rightarrow$ HADRON

$$\int_0^1 dx x^{k-1} \frac{F_2(x, Q^2)}{x} = A_K C_k(Q^2)$$ WHAT ARE THE A_k?

THE MELLIN TRANSFORM

$$F(N) = \int_0^1 dx x^{N-1} f(x) \Leftrightarrow f(x) = \int_{-i\infty}^{i\infty} dN x^{-N} F(N)$$ (INVERSION)

$$F(N)G(N) = \int_0^1 dx x^{N-1} h(x) \Leftrightarrow h(x) = \int_0^x \frac{dy}{y} f(y) g\left(\frac{x}{y}\right)$$ (CONVOLUTION)

THE PARTON DISTRIBUTION

$$\int_0^1 dx x^{k-1} q(x) = A_K \Leftrightarrow F(x, Q^2) = x\int_0^x \frac{dy}{y} C\left(\frac{x}{y}, Q^2\right) q(y);\ C \equiv \frac{\hat{F}}{x}$$

PHYSICAL INTERPRETATION

THE PARTON DISTRIBUTION

$$\int_0^1 dx x^{N-1} q(x) = A_N \Leftrightarrow A_N p^\mu p^\nu p^{\alpha_1} \dots p^{\alpha_{N-2}} = \langle p|\bar\psi \gamma^\mu \partial^\nu \partial^{\alpha_1} \dots \partial^{\alpha_{k-2}} \psi|p\rangle$$

EXAMPLE: $N = 2$, THE ENERGY-MOMENTUM TENSOR:

$$\left(\int_0^1 dx x q(x)\right) A_N p^\mu p^\nu p^{\alpha_1} = \langle p|\bar\psi \gamma^{\{\mu} \partial^{\nu\}} \psi|p\rangle$$

FRACTION OF THE PROTON ENERGY-MOMENTUM CARRIED BY QUARK $\Rightarrow q(x)$ MOMENTUM FRACTION "PROBABILITY"

THE HADRONIC OBSERVABLE

$$\sigma(x) = \int_x^1 \frac{dy}{y} \hat\sigma\left(\frac{x}{y}, Q^2\right) q(y)$$

$\sigma \to$ HADRONIC OBSERVABLE; $\hat\sigma \to$ PARTONIC OBSERVABLE

$\frac{x}{y} = \frac{Q^2}{2yp \cdot q} \to p \to yp$ AT PARTON LEVEL

THE PARTON MODEL FROM QCD

- COMPUTE PERTURBATIVELY PROCESS WITH INCOMING QUARKS
- LET INCOMING QUARK MOMENTUM $\hat p = yp$ WHERE p INCOMING HADRON MOMENTUM
- INTEGRATE OVER ALL VALUES OF $x \le y \le 1$ WHERE x HADRONIC VALUE
- INTEGRATION WEIGHT GIVEN BY PDF

TECHNICALITIES AND SUBTLETIES

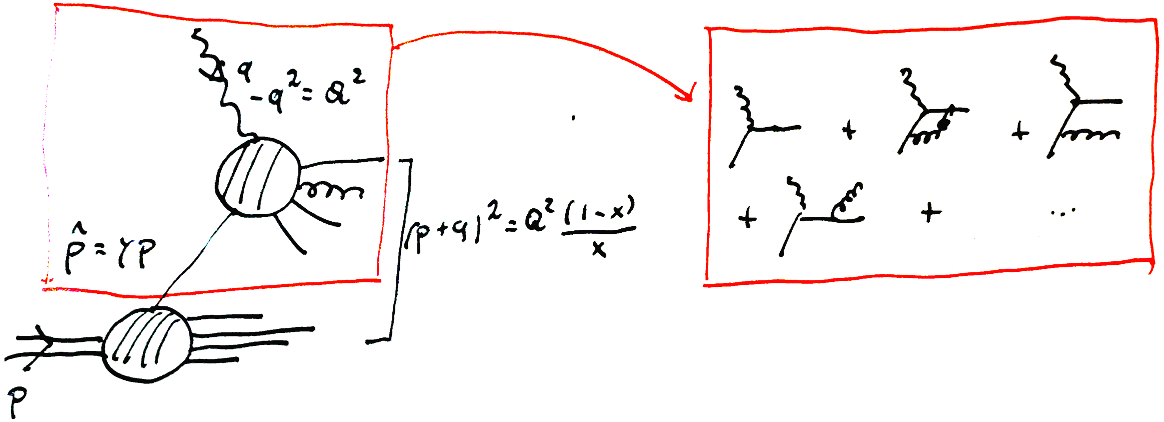
- QUARK AND GLUON OPERATORS/PDFS
- ONE FERMION OPERATOR FOR EACH QUARK FLAVOR
  - EVEN MOMENTS $\Rightarrow$ C-EVEN (QUARK+ANTIQUARK), ODD MOMENTS $\Rightarrow$ C-ODD (QUARK-ANTIQUARK)
  - ANALYTIC CONTINUATION $\Rightarrow$ ONE PDF FOR QUARK, ONE FOR ANTIQUARK
- CONSTRUCTION OF THE OPERATOR BASIS:
  - SYMMETRIZE AND SUBTRACT TRACE $\Leftrightarrow$ SPIN EIGENSTATES
  - BEYOND LEADING ORDER $\partial^\mu \to D^\mu$ $\Rightarrow$ GAUGE INVARIANT
  - COLOR INDICES SUMMED $\Rightarrow$ COLOR SINGLET

FACTORIZATION

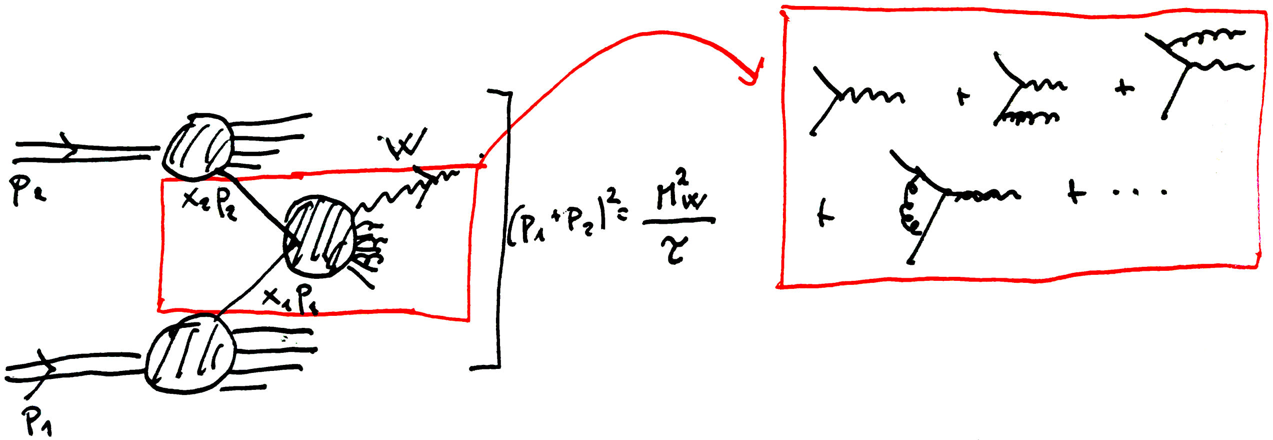
HADRONIC PROCESSES?

LEPTON-HADRON

- SCALE $Q^2 = -q^2$
- SCALING VARIABLE $x = \frac{Q^2}{2p\cdot q}$
- FACTORIZATION $\Leftrightarrow$ ONLY ONE INCOMING PARTON (OPE)



HADRON-HADRON



- SCALE M^2
- SCALING VARIABLE $\tau = \frac{M^2}{s}$
- FACTORIZATION $\Leftrightarrow$ NO INTERFERENCE BETWEEN INCOMING HADRONS (POWER COUNTING)

(Ellis, Georgi, Machecheck, Politzer, Ross, 1978; Collins, Soper, Sterman, 1982)

FACTORIZATION:

LEPTON-HADRON VS HADRON HADRON

ONE HADRON IN THE INITIAL STATE

$$\sigma(x) = \int dz \int_x^1 dy \delta(x - yz) q(y) \hat{\sigma}(z) = \int_x^1 \frac{dy}{y} q(y) \hat{\sigma}\left(\frac{x}{y}\right) = \int_x^1 \frac{dy}{y} q\left(\frac{x}{y}\right) \hat{\sigma}(y) = [\sigma \otimes q](x)$$

- $x = x_B$: SCALING VARIABLE FOR HADRONIC PROCESS (MEASURED HADRON KINEMATICS)
- z: SCALING VARIABLE FOR PARTONIC PROCESS (COMPUTED PARTONIC FEYNMAN DIAGRAM)
- y: MOMENTUM FRACTION CARRIED BY INCOMING PARTON
- Q^2 DEP. OF σ, $\hat{\sigma}$ & q OMITTED

TWO HADRONS IN THE INITIAL STATE

$$\sigma(\tau) = \int dz \int_x^1 dx_1 dx_2 \delta(\tau - x_1 x_2 z) q_1(x_1) q_2(x_2) \hat{\sigma}(z) = \int_\tau^1 \frac{dx_1}{x_1} \int_\tau^1 \frac{dx_2}{x_2} dy q_1(x_1) q_2(x_2) \hat{\sigma}\left(\frac{\tau}{x_1 x_2}\right)$$

$$= \int_\tau^1 \frac{dy}{y} \mathcal{L}(y) \hat{\sigma}\left(\frac{\tau}{z}\right) = [\mathcal{L} \otimes \hat{\sigma}](\tau)$$

$$\mathcal{L}(y) \equiv \int_y^1 \frac{dx_1}{x_1} q_1(y) q_2\left(\frac{y}{x_1}\right) = [q_1 \otimes q_2](y)$$

- τ: SCALING VARIABLE FOR HADRONIC PROCESS (MEASURED HADRON KINEMATICS)
- z: SCALING VARIABLE FOR PARTONIC PROCESS (COMPUTED PARTONIC FEYNMAN DIAGRAM)
- x_1, x_2: MOMENTUM FRACTIONS CARRIED BY INCOMING PARTONS
- $\mathcal{L}$: PARTON LUMINOSITY

PARTON KINEMATICS VS. HADRON KINEMATICS

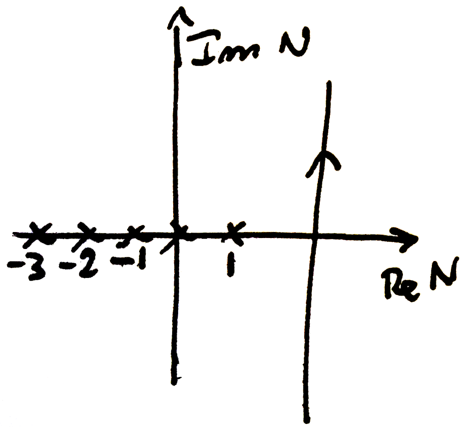
$$\sigma(\tau) = \sum_{ij} \int_\tau^1 \frac{dy}{y} \mathcal{L}_{ij}(y) \hat{\sigma}\left(\frac{\tau}{y}\right); \quad \mathcal{L}_{ij}(y) \equiv \int_y^1 \frac{dx_1}{x_1} f_i(y) f_j\left(\frac{y}{x_1}\right)$$

WHICH PARTON MOMENTUM FRACTION CONTRIBUTES TO A GIVEN HADRONIC PROCESS ?

INVERSION OF MELLIN TRANSFORMS

$$\Sigma(N) = \int_x^1 x^{N-1} \sigma(x) \Leftrightarrow \sigma(x) = \int_{-i\infty}^{+i\infty} x^{-N} \Sigma(N)$$

integrate to the right of convergence abscissa



MELLIN INVERSION DOMINATED BY SADDLE POINT

SADDLE: $\frac{d}{dN}\left[-N \ln \tau + \ln f(N)\right] = 0$

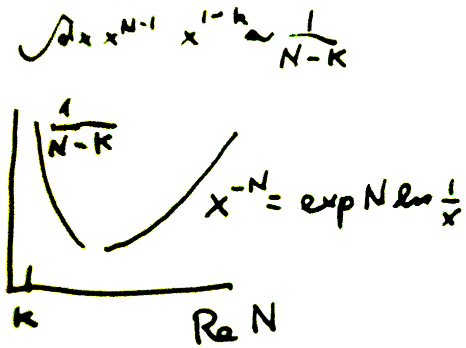
$\Sigma(N) = \hat{\sigma}(N) f_1(N) f_2(N)$; $\mathcal{L}(N) = f_1(N) f_2(N)$

ISOLATED POLES, CONVERGENCE ABSCISSA, SADDLE ON REAL AXIS

PDFS $f(x) \sim x^a (1-x)^b$; $\hat{\sigma} \sim \ln x + \ln(1-x) \Rightarrow$

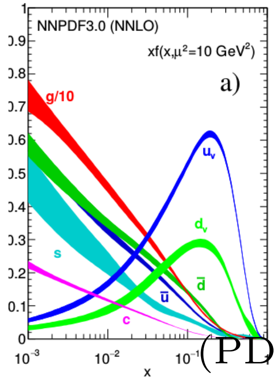
POSITION OF SADDLE CONTROLLED BY PDFS (LUMINOSITY)

FOR GIVEN HADRONIC KINEMATICS



HADRONIC VS PARTONIC
QUALITATIVE BEHAVIOUR

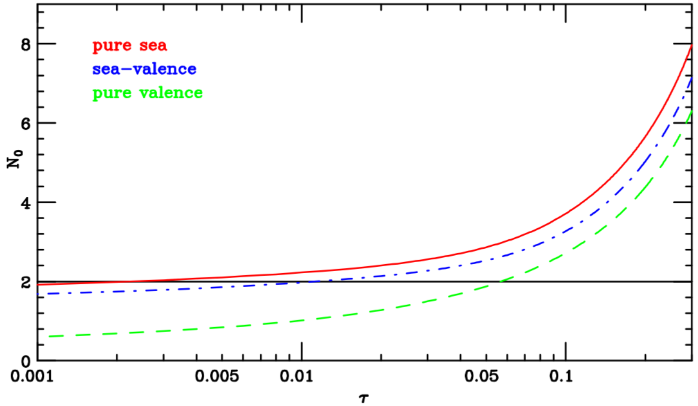
$$\sigma(\tau) = \sum_{ij} \int_\tau^1 \frac{dy}{y} \mathcal{L}_{ij}(y) \hat\sigma\left(\frac{\tau}{y}\right); \quad \mathcal{L}_{ij}(y) \equiv \int_y^1 \frac{dx_1}{x_1} f_i(y) f_j\left(\frac{y}{x_1}\right)$$



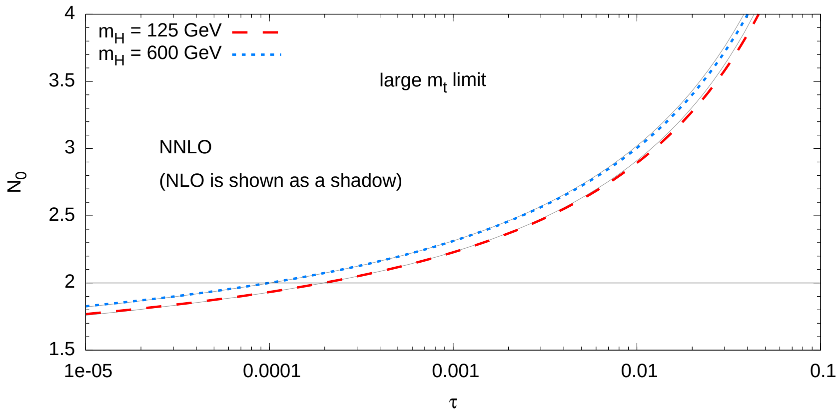
(PDG 2020)

- f_i QUARKS AND GLUONS, DEPEND ON PROCESS
- GLUON & ANTIQUARK SEA GROW A SMALL $x \Rightarrow$ SMALL N
- "VALENCE" UP AND DOWN PEAK AT LARGE $x \sim 0.3 \Rightarrow$ LARGE N

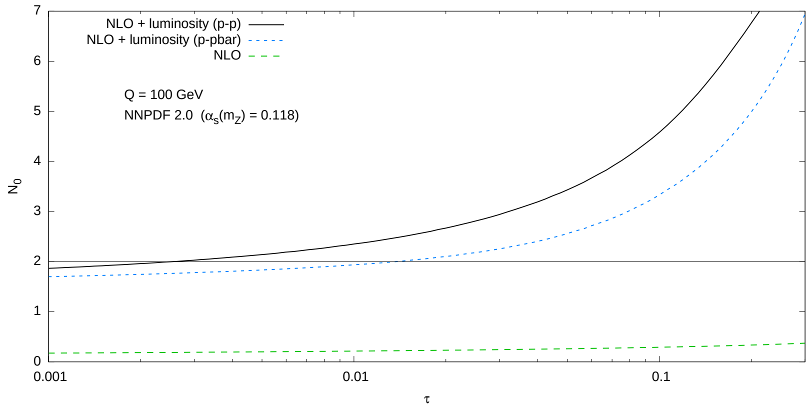
SADDLE VS $\tau = Q^2/s$



HIGGS



Z DRELL-YAN



SUMMARY

- ASYMPTOTIC FREEDOM FOLLOWS FROM RG IMPROVING THE COUPLING
- RG INVARIANCE $\Rightarrow$ PERTURBATIVE CALCULATION OF INCLUSIVE QUANTITIES WITH FINAL-STATE HADRONS (PARTON-HADRON DUALITY)
- HADRONS IN THE INTIAL STATE? NLO INSTABILITY?
- SCALE SEPARATION $\Rightarrow$ FACTORIZATION
  - SHORT DISTANCE $\Rightarrow$ PERTURBATIVE PARTONIC CROSS SECTION
  - LONG DISTANCE $\Rightarrow$ PARTON DISTRIBUTION
- MELLIN-SPACE PARTONIC KINEMATICS DETERMINED BY PHYSICAL SPACE HADRONIC KINEMATICS