RESUMMATION II AND JETS

FINAL STATE: KINEMATICS & DYNAMICS

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

INFN
Istituto Nazionale di Fisica Nucleare

CERN-Fermilab HCP school

24 August 2021

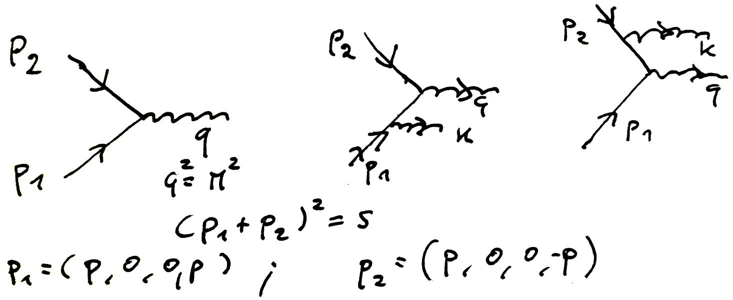
This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 740006

SUMMARY

LECTURE III: RESUMMATION II AND JETS

- THE KINEMATIC STRUCTURE OF SOFT LOGS
- THRESHOLD RESUMMATION VS. TRANSVERSE MOMENTUM RESUMMATION
- THE STRUCTURE OF RESUMMED RESULTS: TRANSVERSE MOMENTUM DEPENDENCE
- HADRONS IN THE FINAL STATE: JETS
- STERMAN-WEINBERG JETS
- JET DEFINITIONS AND IRC SAFETY
- THE k_t AND ANTI-k_t ALGORITHMS
- THE SINGLE-INCLUSIVE JET CROSS-SECTION

THE KINEMATICS OF SOFT-COLLINEAR LOGS



SUDAKOV PARAMETRIZATION: $k = (1-x)\frac{p_1+p_2}{2} + y\frac{p_1-p_2}{2} + k_T;$

$s = (p_1+p_2)^2;\ k = ((1-x)\frac{\sqrt{s}}{2}, \vec{k}_T, y\frac{\sqrt{s}}{2}),\ y = \pm\sqrt{(1-x)^2 - \frac{4|k_T|^2}{s}}$ (COLLINEAR-ANTICOLLINEAR)

PHASE SPACE $d\Phi_k = \frac{|k_T| d|k_T| d\phi dk_z}{2E(2\pi)^3} = \frac{d|k_T|^2 dE}{4|k_z|(4\pi^2)} = \frac{1}{4(4\pi^2)} \frac{dx\, d|k_T|^2}{\sqrt{(1-x)^2 - \frac{4|k_T|^2}{s}}}$

$$\bar{u}(p) \to \bar{u}(p) = \bar{u}(p)\frac{p^\mu}{p\cdot k}$$

EIKONAL AMPLITUDE $M = M_0 g\left(\frac{2p_1^\mu}{(p_1+k)^2} - \frac{2p_2^\mu}{(p_2+k)^2}\right)$

$$|M|^2 = -|M_0|^2 \pi\alpha_s \frac{2p_1\cdot p_2}{(p_1+k)^2(p_2+k)^2} = -2|M_0|^2\alpha_s 4\pi\frac{16}{s[(1-x)^2-y^2]} = -8\pi|M_0|^2 e^2\frac{4}{|k_T|^2}$$

PHASE SPACE IN THE SOFT LIMIT

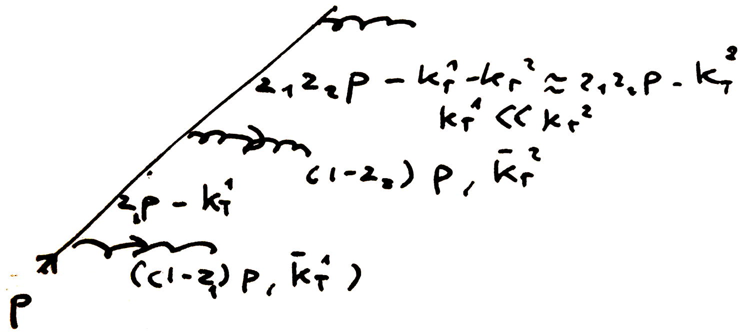
$$d\Phi_k = \frac{1}{16\pi^2}\frac{dx\, d|k_T|^2}{\sqrt{(1-x)^2 - \frac{4|k_T|^2}{s}}} = \frac{dx\, d|k_T|^2}{16\pi^2}\left[\frac{1}{(1-x)_+} - \frac{1}{2}\delta(1-x)\ln\frac{4|k_T|^2}{s}\right] + O(|k_T|^2)$$

THE SOFT-COLLINEAR LOGS

$$\int d\Phi_k |M|^2 = -|M_0|^2\frac{e^2}{4(4\pi^2)}\int dx\, d|k_T|^2\left[\frac{1}{(1-x)_+} - \frac{1}{2}\delta(1-x)\ln\frac{4|k_T|^2}{s}\right]\frac{8}{|k_T|^2} = -|M_0|^2\frac{\alpha}{2\pi}\ln^2\frac{s}{\mu^2}$$

μ ir cutoff

THE KINEMATICS OF RESUMMATION



- CANCEL IR SINGULARITIES:
  - DR $\xi = 4|k_T|^2/s$;

    p_t PLUS: $\frac{\ln\xi}{\xi} \to \frac{\ln\xi}{\xi^{1+\epsilon}} = -\frac{1}{\epsilon^2} + \left[\frac{\ln\xi}{\xi}\right]_+$;

    $\int_0^{\xi_{\max}} d\xi \left[\frac{1}{\xi}\right]_+ f(\xi) = \int_0^{\xi_{\max}} d\xi \left[\frac{1}{\xi}\right] [f(\xi) - \Theta(1-\xi) f(0)]$
  - VIRTUAL CONTRIBUTION REMOVES DOUBLE POLE
- $\xi_{\max} = \frac{(1-\tau)^2}{4}$ $\tau \equiv \frac{s}{M^2}$ INTEGRATION OVER $k_T \Rightarrow \frac{\ln(1-\tau)^2}{(1-\tau)_+}$ + COLLINEAR POLE $\Rightarrow \ln^2 N$
- COLLINEAR POLE FACTORIZED IN PDF
- IN ORDERED REGION $k_t^1 < k_t^2 < \ldots$ k EMISSIONS $\Rightarrow \frac{\ln^{2k-1}(1-\tau)}{(1-\tau)_+} \Rightarrow \ln^{2k} N$ (Mellin)
- IF LAST k_t NOT INTEGRATED, k EMISSIONS $\Rightarrow \frac{\ln^{2k-1}\xi}{\xi_+}$
- PHASE SPACE FACTORIZATION:
  - LONGITUDINAL $\delta(1 - x_1 x_2 \ldots x_n) \Rightarrow$ MELLIN
  - TRANSVERSE $\delta(\vec{k}_T^1 + \vec{k}_T^2 + \cdots + \vec{k}_T^n + \vec{p}_T) \Rightarrow$ FOURIER

SOFT RESUMMATION $\ln N \Leftrightarrow p_T$ RESUMMATION $\ln b$

TRANSVERSE MOMENTUM RESUMMATION

THE p_t DISTRIBUTION

$$P(p_1) + P(p_2) \to H(p) + X$$

FACTORIZATION

$$Q^2 = \left(\sqrt{M^2 + p_t^2} + p_t\right)^2, \ \tau' = \frac{Q^2}{s}$$

$$\frac{d\sigma}{dp_t^2}\left(\tau', p_t, M^2\right) = \tau' \sum_{ij} \int_{\tau'}^{1} \frac{dx}{x} \, \mathcal{L}_{ij}\left(\frac{\tau'}{x}, \mu_f^2\right) \frac{1}{x} \frac{d\hat{\sigma}_{ij}}{dp_t^2}\left(x, p_t, \alpha_s, \mu_f^2\right)$$

THE LEADING-ORDER PARTONIC CROSS-SECTION

$$\frac{d\sigma}{dp_t^2} = P_1 \alpha_s \left[\frac{\ln(p_T^2/M^2)}{p_t^2/M^2}\right]_+ + P_2 \alpha_s \left[\frac{1}{p_t^2/M^2}\right]_+ + Q_1(p_t^2/M^2) + D_1 \delta(p_t^2/M^2)$$

P_1, P_2, D_1 NUM. COEFFICIENTS; Q_1 FUNCTION

- TRANSVERSE MOMENTUM DEPENDENCE REQUIRES AT LEAST ONE EMISSION $\Rightarrow$ STARTS AT $O(\alpha_s)$
- RESUMMATION REQUIRED IN ORDER TO OBTAIN RELIABLE PREDICTIONS FOR SMALL p_t

REMINDER:
THE STRUCTURE OF SOFT-RESUMMED TOTAL CROSS-SECTIONS

$$C_{\rm res}(N,\alpha_s) = \hat{g}_0(\alpha_s)\exp\left[2\int_0^1 dx\,\frac{x^{N-1}-1}{1-x}\int_{M^2}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_g^{\rm th}\left(\alpha_s\left(q^2\right)\right)+D_g^{\rm th}\left(\alpha_s\left(\left(1-z\right)^2M^2\right)\right)\right]$$

$$C_{\rm res}(N,\alpha_s) = \hat{g}_0(\alpha_s)\exp -\int_1^{N^a}\frac{dn}{n}\left[\int_{n\mu^2}^{M^2}\frac{dk^2}{k^2}A(\alpha_s(k^2/n))+\tilde{B}(\alpha_s(M^2/n))\right]$$

PDFs AT SCALE M^2

$$C_{\rm res}(N,\alpha_s) = \hat{g}_0(\alpha_s)\exp\int_{M^2/N^a}^{M^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q^2))\log\frac{q^2}{M^2}+\hat{B}(\alpha_s(q^2),N)\right]$$

PDFs AT SCALE M^2/N^2

$$C_{\rm res}(N,\alpha_s) = \hat{g}_0(\alpha_s)\exp\int_{M^2/N^a}^{M^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q^2))\log\frac{q^2}{M^2}+\tilde{B}(\alpha_s(q^2))\right]$$

THE STRUCTURE OF TRANSVERSE MOMENTUM RESUMMATION

PHASE-SPACE FACTORIZATION

LONGITUDINAL ↔ MELLIN; TRANSVERSE ↔ FOURIER

$$\frac{d\hat\sigma}{dp_t^2}(\alpha_s, p_t^2) = \frac{M^2}{2\pi}\int d^2b\, e^{-i\vec{p}_t\cdot\vec{b}}\,\Sigma(\alpha_s, b^2) = \int_0^{+\infty} db\, b\, J_0(bq_T)\,\Sigma(\alpha_s, b^2)$$

RESUMMATION

PDFS AT SCALE M^2

$$\frac{d\hat\sigma_{ij}}{dp_t^2}\left(N, p_t, \alpha_s\left(M^2\right), M^2\right) = \sigma_0 \int_0^\infty db\, \frac{b}{2}\, J_0\left(bp_t\right) H_{ij}\left(N, \alpha_s\left(M^2\right)\right) S(M, N, b)$$

- ij PARTONIC SUBCHANNEL
- RESUMMATION ⇒ SUDAKOV EXPONENT

$$S(M,b) = \exp\left[-\int_{\frac{1}{b^2}}^{M^2} \frac{dq^2}{q^2}\left[A^{pt}\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2} + B^{pt}\left(\alpha_s\left(q^2\right), N\right)\right]\right]$$

- LEADING LOG → LEADING ORDER A; DEFINES A-B SEPARATION
  NOTE BEYOND NNLL A DIFFERS FROM CUSP ANOMALOUS DIMENSION

HARD FUNCTION

$$H_{ij}(\alpha_s) = [C_i(N, \mathbf{b})C_i(N, \mathbf{b}) + G_i(N, \mathbf{b})G_j(N, \mathbf{b})]$$

C, G UNIVERSAL (DEP. ON PARTON)

THE STRUCTURE OF TRANSVERSE MOMENTUM RESUMMATION

PHASE-SPACE FACTORIZATION

LONGITUDINAL $\leftrightarrow$ MELLIN; TRANSVERSE $\leftrightarrow$ FOURIER

$$\frac{d\hat{\sigma}}{dp_t^2}(\alpha_s, p_t^2) = \frac{M^2}{2\pi}\int d^2b\, e^{-i\vec{p}_t\cdot\vec{b}}\,\Sigma(\alpha_s, b^2) = \int_0^{+\infty} db\, b\, J_0(bq_T)\,\Sigma(\alpha_s, b^2)$$

RESUMMATION

PDFS AT SCALE $1/b^2$

$$\frac{d\hat{\sigma}_{ij}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(M^2\right), M^2\right) = \sigma_0 \int_0^\infty db\, \frac{b}{2}\, J_0\left(bp_t\right) H_{ij}\left(N, \alpha_s\left(M^2\right)\right)\bar{S}(M, b)$$

- RESUMMATION $\Rightarrow$ SUDAKOV EXPONENT

$$\bar{S}(M, b) = \exp\left[-\int_{\frac{b_0^2}{b^2}}^{M^2} \frac{dq^2}{q^2}\left[\bar{A}^{p_t}\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2} + \bar{B}^{p_t}\left(\alpha_s\left(q^2\right)\right)\right]\right]$$

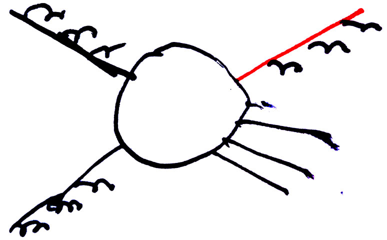
- B NOW N-INDEPENDENT
- $\bar{A}$ AND $\bar{B}$ DETERMINED FROM A, B, β FUNCTION & ANOMALOUS DIMENSIONS

HARD FUNCTION

$$H_{ij}(\alpha_s) = [C_i(N, \mathbf{b})C_i(N, \mathbf{b}) + G_i(N, \mathbf{b})G_j(N, \mathbf{b})]$$

C, G UNIVERSAL (DEP. ON PARTON)

STRONGLY INTERACTING FINAL STATES



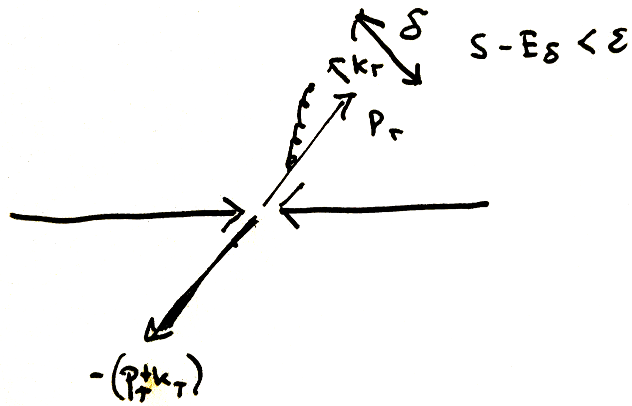
THE p_T DISTRIBUTION OF, SAY, A FINAL STATE QUARK?
QUESTIONS

- THERE IS NO SUCH THING AS A FINAL STATE QUARK!
- A BUNCH OF HADRONS (JET)?
  - FOR EACH REAL EMISSION, MUST INCLUDE THE VIRTUAL CORRECTION THAT CANCELS IR SINGULARITIES
  - WHAT ABOUT THE COLLINEAR SINGULARITY?
- NEED AN IRC SAFE JET DEFINITION
  - IR $O_n(k_1, \ldots, k_i \ldots, k_n) \underset{k_i \to 0}{\to} O_{n-1}(k_1, \ldots, k_{i-1}, k_{i+1} \ldots, k_n)$
  - C $O_n(k_1, \ldots, k_i, k_j, \ldots, k_n) \underset{k_i \| k_j}{\to} O_{n-1}(k_1, \ldots, k_i + k_j, \ldots, k_n)$

STERMAN-WEINBERG JETS

e^+e^- ANNIHILATION

TOTAL CROSS SECTION IS FINITE: IR CANCEL, COLLINEAR LOG IS $\ln|\vec{k}_T - \vec{p}_t|$, INTEGRAL OVER p_t FINITE!

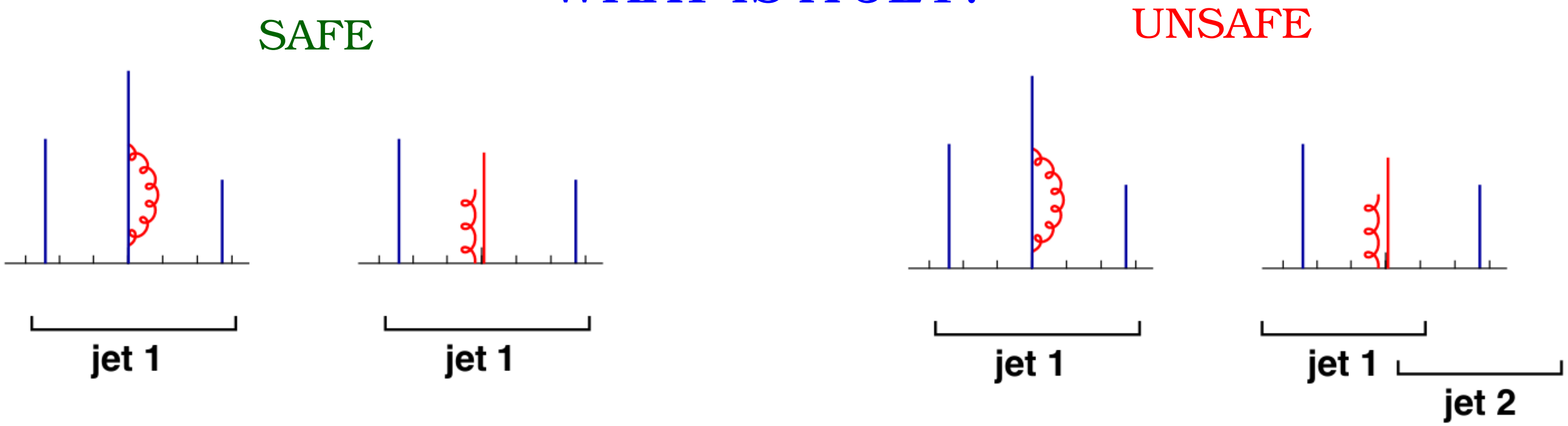


- IDEA
  - START WITH LO $q\bar{q}$ FINAL STATE
  - COMPUTE NLO $q\bar{q}g$ FINAL STATE
  - DRAW CONES OF APERTURE δ ABOUT Q AND $\bar{q}$
  - DEFINE NLO QG CROSS-SECTION WITH CONDITION
    ENERGY OUTSIDE CONE $E_{\text{out}} < \epsilon s$
    $\Leftrightarrow$ ENERGY INSIDE CONE $E_{\text{in}} \geq (1-\epsilon)s$
- THREE CONTRIBUTIONS:
  - qg OR $\bar{q}g$ IN JET
  - $q\bar{q}$ JETS & SOFT GLUON
  - $q\bar{q}$ ONE LOOP
- IRC SINGULARITIES CANCEL

$$\sigma^{\text{NLO}} \sim \sigma_0 \left[1 + \alpha_s \left(K \ln\delta + K' \ln\delta \ln\epsilon + \text{non log}\right)\right]$$

- FINITE, MAY NEED RESUMMATION
- $\ln\delta\ln\epsilon$ SOFT-COLLINEAR; $\ln\delta$ PURE COLLINEAR

WHAT IS A JET?



- CONE ALGORITHMS
  - BASIC IDEA LIKE STERMAN-WEINBERG
  - SPLIT-MERGE: IS IT STABLE?
  - ADDITION OF SOFT OR COLLINEAR GLUON MAY CHANGE THE NUMBER OF JETS $\to$ UNSAFE
- SEQUENTIAL RECOMBINATION ALGORITHMS: BASIC IDEA
  - FOR EACH PAIR OF PARTICLES, DEFINE DISTANCE
  - SMALL DISTANCE $\Leftrightarrow$ COLLINEAR OR SOFT
  - COMBINE PARTICLES WITH SMALL DISTANCE INTO JET UNTIL THRESHOLD VALUE
  - ALWAYS SAFE

k_T SEQUENTIAL RECOMBINATION

DISTANCE: RELATIVE $d_{ij} = \min({p_t^i}^2, {p_t^j}^2)\frac{\Delta_{ij}}{R^2}$; $\quad \Delta_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

TO BEAM $d_{iB} = {p_t^i}^2$

THE ALGORITHM

1. DETERMINE d_{ij} AND d_{iB} FOR EACH PARTICLE i AND PAIR OF PARTICLES i, j
2. SELECT SMALLEST AMONG ALL d_{ij}, d_{iB}
3. - IF SMALLEST IS A d_{ij} RECOMBINE i AND j
   - IF SMALLEST IS A d_{iB} REMOVE i FROM LIST OF PARTICLES (IT IS A JET)
   - IF THERE ARE PARTICLES LEFT GO BACK TO 1, IF NOT STOP

KEEP JET i ONLY IF $|p_t^i| > p_t^{\min}$

ANTI k_T SEQUENTIAL RECOMBINATION

(Cacciari, Salam, Soyez, 2008)

DISTANCE: RELATIVE $d_{ij} = \min(\frac{1}{{p_t^i}^2}, \frac{1}{{p_t^j}^2})\frac{\Delta_{ij}}{R^2}$; $\quad \Delta_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

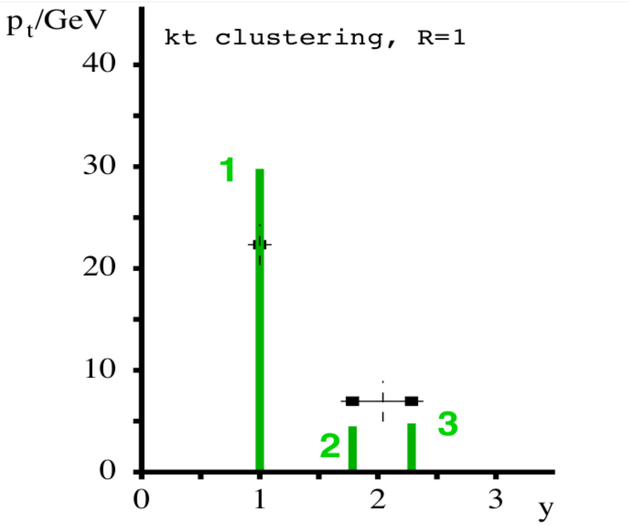
TO BEAM $d_{iB} = \frac{1}{{p_t^i}^2}$

THE ALGORITHM

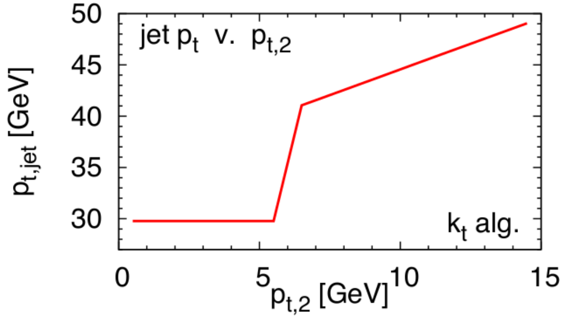
1. DETERMINE d_{ij} AND d_{iB} FOR EACH PARTICLE i AND PAIR OF PARTICLES i, j
2. SELECT SMALLEST AMONG ALL d_{ij}, d_{iB}
3.
   - IF SMALLEST IS A d_{ij} RECOMBINE i AND j
   - IF SMALLEST IS A d_{iB} REMOVE i FROM LIST OF PARTICLES (IT IS A JET)
   - IF THERE ARE PARTICLES LEFT GO BACK TO 1, IF NOT STOP

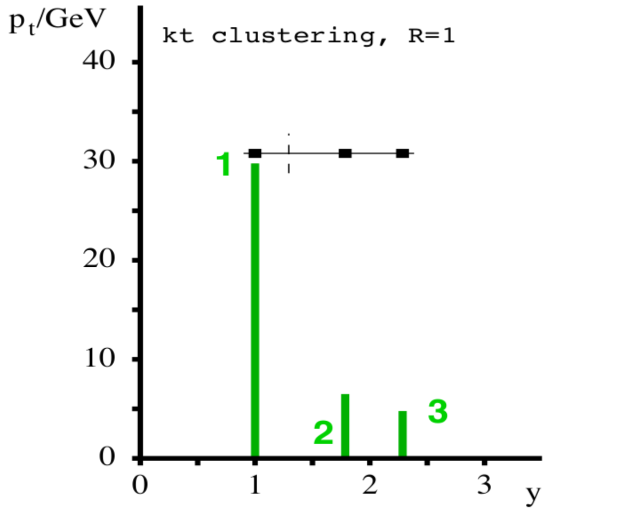
KEEP JET i ONLY IF $|p_t^i| > p_t^{\min}$

k_T VS ANTI-k_T

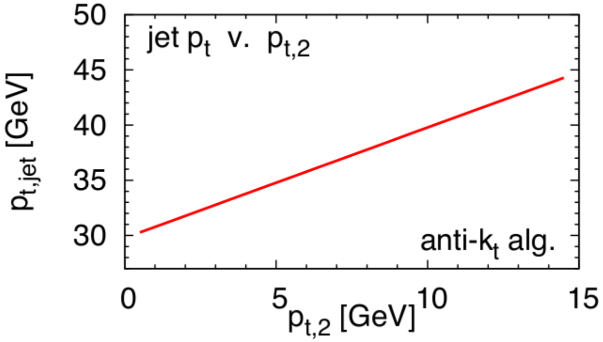


k_t: JUMPS





ANTI-k_t: LINEARITY



- k_t CLUSTERS STARTING WITH SOFTEST AND RECOMBINING UNTIL HARD ENOUGH
- ANTI-k_t CLUSTERS STARTING WITH HARDEST AND RECOMBINING UNTIL THERE IS NO SOFT STUFF LEFT

THE SIMPLEST JET OBSERVABLE

THE SINGLE-JET INCLUSIVE CROSS-SECTION

A BIZARRE DEFINITION:

$$\frac{d\sigma}{dp_t} = \sum_N \frac{d\sigma_{N\ \text{JETS}}}{dp_t}; \ \frac{d\sigma_{N\ \text{JETS}}}{dp_t} = \int dp_{t1}\dots dp_{ti}\dots dp_{tN}\ \frac{d\sigma_{N\ \text{JETS}}}{dp_{t1}\dots dp_{ti}\dots dp_{tN}} \sum_{i=1}^{N} \delta(p_{ti} - p_t)$$

- EVENT WITH N JET IS BINNED N TIMES $\Rightarrow$ NON-UNITARY!
- SHOULD WE DIVIDE BY N? INTRODUCE WEIGHTS

$$w^{(N)}(p_t; p_{t1}, \dots, p_{tN}) = \begin{cases} 1 & \text{(STANDARD)} \\ \dfrac{p_t^r}{\sum_{j=1}^{N} p_{tj}^r} & \text{(WEIGHTED)} \end{cases}$$

THE SINGLE-JET INCLUSIVE CROSS-SECTION
UNITARY vs. NONUNITARY

$$\frac{d\sigma^{(k)}_{N\text{ JETS}}}{dp_t} = \sum_{m=2}^{k+2} \int d\Phi_m \frac{d\hat{\sigma}^{(k)}_m}{d\Phi_m} G_{m\to N\text{ JETS}}(\Phi_m, p_t)$$

$$G_{2\to 2} = \Theta(p_t > p_t^{\text{CUT}}) \left\{2\, w^{(2)}(p_t; p_t, p_t)\, \delta(p_t - k_{t1})\right\}$$

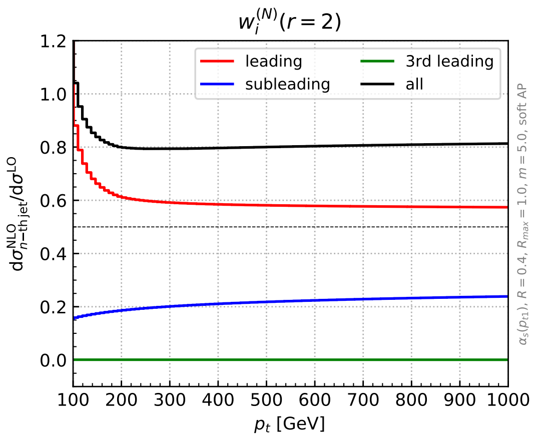
$$G_{3\to 1} = \Theta(\Delta R_{23} > R)\, \Theta(k_{t1} > p_t^{\text{CUT}} > k_{t2} > k_{t3}) \left\{w^{(1)}(p_t; p_t)\, \delta(p_t - k_{t1})\right\}$$

k_{ti} parton transverse momenta, $k_{t1} \geq k_{t2} \geq k_{t3}$

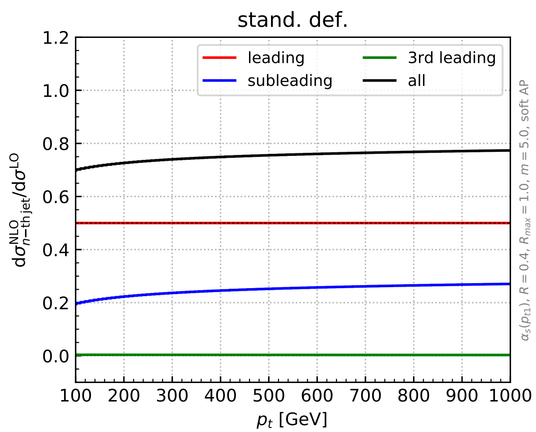
- THREE PARTON FINAL STATE $\Rightarrow$ ONE, TWO OR THREE JETS ACCORDING TO WHETHER $k_T^i > p_t^{\text{cut}}$
- UNITARY $\int dp_t G_{3\to 1} + G_{3\to 2} + G_{3\to 1} = \Theta(k_T^1 > p_t^{\text{CUT}})$ BUT $\frac{d\sigma}{dp_t} \sim \ln p_t^{\text{CUT}}$
- STANDARD $G_{3\to 1} + G_{3\to 2} + G_{3\to 1} \propto \Theta(p_t > p_t^{\text{CUT}})$, $\frac{d\sigma}{dp_t}$ INDEPENDENT OF p_t^{CUT}
- IN STANDARD DEFINITION, 2-JET CONTRIBUTION p_t^{CUT} AS UPPER LIMIT, 3-JET CONTRIBUTION AS LOWER LIMIT $\Rightarrow$ DEPENDENCE CANCELS
- IN UNITARY DEFINITION: WEIGHT SPOILS CANCELLATION $\ln \frac{p_t}{p_t^{\text{CUT}}}$ DEPENDENCE $\Rightarrow$ PERTURBATIVE INSTABILITY

NLO/LO

UNITARY



STANDARD



SUMMARY

- SOFT-COLLINEAR DOUBLE LOG $\Leftrightarrow$ PHASE SPACE VS AMPLITUDE
- NESTED EMISSIONS $\Leftrightarrow$ EXPONENTIATION
- p_T INTEGRATED: SOFT RESUMMATION $\Leftrightarrow$ UNINTEGRATED: k_T RESUMMATION
- FINAL STATE RADIATION $\Rightarrow$ JETS
- ENERGY-MOMENTUM RESOLUTION $\Leftrightarrow$ FINITE OBSERVABLE
- ALL-ORDER FINITENESS $\Leftrightarrow$ IRC SAFETY
- PERTURBATIVE STABILITY $\Leftrightarrow$ CUTOFF DEPENDENCE