New Physics Beyond the Standard Model

Known knowns (= SM)
Known unknowns (e.g., DM)
Unknown unknowns

Lepton flavour violation in $B$ decays?

$g_\mu - 2$ ?
$g_\mu - 2$: from Dirac and Schwinger to Fermilab and Beyond

A story of 94 years, 8 experiments and many theorists
It began with Dirac ...

- Two fundamental papers:

  - "The quantum theory of the electron" (1928)

    The Dirac equation: \((i\gamma_{\mu} \partial^{\mu} - m)\psi = 0\) predicts that the electron’s magnetic moment \(g = 2\)

    "That was really an unexpected bonus for me, completely unexpected"

  - "The Quantum Theory of the Emission and Absorption of Radiation" (1927)

    The basis for QED (and all of quantum field theory) enables the calculations of the anomaly: \(g \neq 2\)
The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charged electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement with experiment for hydrogen-like spectra to the first order of accuracy.

The question remains as to why Nature should have chosen this particular model for the electron instead of being satisfied with the point-charge. One would like to find some incompleteness in the previous methods of applying quantum mechanics to the point-charge electron such that, when removed, the whole of the duplexity phenomena follow without arbitrary assumptions. In the present paper it is shown that this is the case, the incompleteness of the previous theories lying in their disagreement with relativity, or, alternatively, with the general transformation theory of quantum mechanics. It appears that the simplest Hamiltonian for a point-charge electron satisfying the requirements of both relativity and the general transformation theory leads to an explanation of all duplexity phenomena without further assumption. All the same there is a great deal of truth in the spinning electron model, at least as a first approximation. The most important failure of the model seems to be that the magnitude of the resultant electron moving in an orbit in a central field

\[ \mathbf{p_0} + \frac{e}{c} \mathbf{A}_0 + \rho_1 \left( \mathbf{\sigma} \cdot \mathbf{p} + \frac{e}{c} \mathbf{A} \right) + \rho_3 mc \right] \psi = 0 \]


Emission and Absorption of Radiation.

By P. A. M. Dirac, St. John's College, Cambridge, and Institute for Theoretical Physics, Copenhagen.

(Communicated by N. Bohr, For. Mem. R.S.—Received February 2, 1927.)

§ 1. Introduction and Summary.

The new quantum theory, based on the assumption that the dynamical variables do not obey the commutative law of multiplication, has by now been developed sufficiently to form a fairly complete theory of dynamics. One can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly anything has been done up to the present on quantum electromodynamics. The theory of a system in which the forces are propagated instantaneously, of the production of a magnetic field of the electron, and of the reaction of this field on the electron, and in addition, there is a serious difficulty in making the theory satisfy all the requirements of the restricted
... and then Schwinger

- First calculation of leading-order contribution to $g - 2$ in QED: $\frac{\alpha}{2\pi}$ (1947)
- Inscribed on his tombstone
$\mathcal{O}(\frac{\alpha}{\pi})^2$: André Petermann

- Co-inventor (with Stueckelberg) of the renormalisation group

- First to submit a paper proposing quarks (a few days before Gell-Mann and Zweig), not widely known because written in French, and publication delayed > year!

- Pioneer of $g_\mu - 2$ calculations: $a_\mu \equiv \frac{(g_\mu - 2)}{2} = 1 + \frac{\alpha}{2\pi} + 0.75(\frac{\alpha}{\pi})^2$ (1957)

- First correct calculation of $\mathcal{O}(\alpha^2)$ contributions to $g_e - 2, g_\mu - 2$

(Also Sommerfeld; Suura & Wichmann, previous work by Karplus & Kroll)
Fourth order magnetic moment of the electron
by A. Petermann.

CERN. Theoretical Study Division. Institute for theoretical Physics. Copenhagen.
(17. VIII. 1957.)

In connection with the upper and lower bounds analysis done by the author\(^4\), which indicated a clear discrepancy with the Karplus and Kroll's result for the 4th order magnetic moment\(^3\), we have performed an analytic evaluation of the five independent diagrams contributing to this moment in fourth order\(^4\). The results are the following:

\[ \mu_1 = \frac{\alpha^2}{\pi^2} \left( \frac{1}{6} + \frac{13}{36} \pi^2 + \frac{5}{4} \zeta(3) - \frac{5}{6} \pi^2 \log 2 \right) = -0.467 \frac{\alpha^2}{\pi^2}. \]  
\[ \mu_{11a} = \frac{\alpha^2}{\pi^2} \left( \frac{11}{48} + \frac{\pi^2}{18} \right) = 0.778 \frac{\alpha^2}{\pi^2}. \]  
\[ \mu_{11b} = \frac{\alpha^2}{\pi^2} \left( -\frac{1}{24} + \frac{\pi^2}{18} \right) \left( \frac{1}{2} \zeta(3) + \frac{1}{3} \pi^2 \log 2 - \frac{1}{2} \pi^2 \frac{\zeta(3)}{24} \right) = -0.564 \frac{\alpha^2}{\pi^2} + \frac{\pi^2}{2} \log \frac{\pi^2}{24}. \]  
\[ \mu_{11c} = \frac{\alpha^2}{\pi^2} \left( \frac{119}{30} - \frac{\pi^2}{18} \right) = 0.016 \frac{\alpha^2}{\pi^2}. \]  
\[ \mu_{11d} = \frac{\alpha^2}{\pi^2} \left( \frac{197}{144} + \frac{\pi^2}{12} \right) \left( \frac{3}{4} \zeta(3) - \frac{1}{2} \pi^2 \log 2 \right) = -0.328 \frac{\alpha^2}{\pi^2}. \]

Compared with the values given in their original paper by Karplus and Kroll, one can see that two terms were in error: \( \mu_1 \) differs by

\[ \frac{\alpha^2}{\pi^2} \left( \frac{2}{3} \right) = 0.091 \frac{\alpha^2}{\pi^2}; \]

\[ \mu_{11a} \text{ by } \frac{\alpha^2}{\pi^2} \left( \frac{32}{13} \right) = 0.84 \frac{\alpha^2}{\pi^2} \]

\[ \mu_{11c} \text{ by } \frac{\alpha^2}{\pi^2} \left( \frac{32}{13} \right) = 0.614 \frac{\alpha^2}{\pi^2}. \]

The three other terms check. The error in \( \mu_1 \) remained of course undetected in the upper and lower bound analysis owing to its small-
Magnetic Moment of the Free Muon*†

T. COFFIN, R. L. GARVIN,‡ S. PENMAN, L. M. LEDERMAN, AND A. M. SACHS
Columbia University,§ New York, New York
(Received October 1, 1957)

The magnetic moment of the positive μ meson has been measured in several target materials by a magnetic resonance technique. Muons were brought to rest with their spins parallel to a magnetic field. A radio-frequency pulse was applied to effect a spin reorientation which was detected by counting the decay electrons emerging after the pulse in a fixed direction. Results are expressed in terms of a $g$ factor which for a spin $\frac{1}{2}$ particle is the ratio of the actual moment to $\hbar/2m_e$. The most accurate result obtained in a CHBr$_3$ target, is that $g=2(1.0026\pm0.0009)$ compared to the theoretical prediction of $g=2(1.0012)$. Less accurate measurements yielded $g=2.005\pm0.005$ in a copper target and $g=2.00\pm0.01$ in a lead target.

I. INTRODUCTION

The μ meson has often been described as one of the more baffling of elementary particles. It alone, among the unstable particles, has no strong interaction. Aside from its usefulness as a tool in the study of nuclear structure and the details of parity violation in weak interactions it appears to play no essential role in any organization of fundamental particles. A precise measurement of the magnetic moment of the muon offers some promise for clarification of this situation.

The Dirac equation predicts precisely 2 for the $g$ value of a spin $\frac{1}{2}$ particle. Including corrections due to the interaction of the particle with its radiation field, one obtains

$$g = \left(1 + \frac{\alpha}{2\pi} + \frac{0.75}{\pi^2} + \cdots \right)$$

(1)

and an energy $\lambda$ would alter the $g$ value as follows

$$g = 2\left[1 + \frac{1}{3}(\frac{m}{\lambda})^2 \frac{\alpha}{2\pi} + \cdots \right].$$

(2)

It might be remarked that the model used in reference 3 implies a modification in the scattering of one Dirac particle by another. Such a modification can be described by a mean square radius, the appropriate relation being $\langle r^2 \rangle = 6(h/\lambda c)^2$. Qualitatively, at least, the measured proton radius should constitute an upper limit for such an “electrodynamic radius.” Hence the fractional alteration of the muon moment from such a presumed breakdown of quantum electrodynamics should not exceed $\sim 0.02(\alpha/2\pi)$.

- Columbia Nevis and Carnegie Institute of Technology cyclotrons
- Agreement between theory and experiment
First $g_\mu - 2$ Experiment at CERN

(1958 - 1962)

$\delta a \equiv \delta \left( \frac{g_\mu - 2}{2} \right) = \pm 0.4\%$

Georges Charpak  Francis Farley
Hans Sens    Theo Muller  Nino Zichichi

(Suggested by Leon Lederman)

(Also experiment at Berkeley Cyclotron)
Principle of Storage Ring Experiments

**LIFE OF A MUON:**

**THE g-2 EXPERIMENT**

- Protons
- Hit Target.
- Pions, weighing 1/6 proton, are created.
- Pions decay to muons.
- Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.
- Muons are tiny magnets spinning on axis like tops.
- After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

Detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.
First Storage Ring Experiment at CERN
(1962 - 1968)

Design

Under construction

\[ \delta a = \pm 270 \text{ ppm} \]

Agreement with theory after inclusion of light-by-light scattering
(Aldins, Kinoshita, Brodsky, Dufner)
$O\left(\frac{\alpha}{\pi}\right)^3$ Calculations

(Kinoshita, 1967) + De Rafael

\[
A_2^{(6)}(m_\mu/m_e) = \frac{2}{9} \log^2 x - \left( \zeta(3) - \frac{2}{3} \pi^2 \log 2 + \frac{7\pi^2}{9} + \frac{31}{27} \right) \log x + \frac{97\pi^4}{360}
\]

\[
- \frac{2}{9} \pi^2 \log^2 2 - \frac{8}{3} a_4 - \frac{\log 2}{9} - 6\zeta(3) + \frac{5}{3} \pi^2 \log 2 - \frac{85\pi^2}{18} + \frac{1219}{216}
\]

\[
+ x \left( -\frac{4}{3} \pi^2 \log x - \frac{604}{9} \pi^2 \log 2 + \frac{54079\pi^2}{1080} - \frac{13\pi^3}{18} \right)
\]

\[
+ x^2 \left[ \frac{2}{3} \log^3 x + \left( \frac{\pi^2}{9} - \frac{10}{3} \right) \log^2 x + \left( \frac{16\pi^4}{135} + 4\zeta(3) - \frac{32\pi^2}{9} + \frac{194}{9} \right) \log x \right.
\]

\[
+ \frac{4}{3} \zeta(3)\pi^2 - \frac{61\pi^4}{270} + \zeta(3) + \frac{197\pi^2}{36} - \frac{2809}{108} - \frac{14}{3} \pi^2 \log 2 \right] + O(x^3)
\]

\[
= 22.86837998(20),
\]
$\mathcal{O}\left(\frac{\alpha}{\pi}\right)^4$ Calculations

\begin{align*}
A_2^{(8)}(m_\mu/m_e) &= 123.785 \, 51(44) + 8.8997 \, 59(59) = 132.6852(60)
\end{align*}
Second Storage Ring Experiment at CERN
(1969 - 1976)

Precession frequency
\[ \tilde{\omega}_a \equiv \tilde{\omega}_s - \tilde{\omega}_c = -\frac{q}{m_\mu} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\beta \cdot \vec{B}) \beta \right] \]

Precession frequency \( \propto a_\mu \)
\[ \delta a_\mu = 8 \text{ ppm} \]

Magic energy
\[ E = 3.094 \text{ GeV} \]
\[ \gamma = 29.3 \]

(Emilio Picasso → LEP)
$g_\mu - 2$ in Supersymmetry

- One-loop contribution from smuon/neutralino loop

\[ \Delta(g - 2)_\mu = -ab\left(\cos \alpha \sin \alpha / 4\pi^2\right) \left(m_\mu / m_\widetilde{\chi}\right) \]
\[ \times \left\{ 1/(1 - \eta_1) + 2\eta_1/(1 - \eta_1)^2 + [2\eta_1/(1 - \eta_1)^3] \log \eta_1 - (\eta_1 \leftrightarrow \eta_2) \right\}, \]

- where \( \eta_i \equiv (m_{\widetilde{s}_i}^2 / m_{\widetilde{\chi}}^2) \)

- and \( \mathcal{L} = a\sqrt{2} \tilde{s}_\mu \tilde{\mu}_L \tilde{\chi} + b\sqrt{2} t_\mu \tilde{\mu}_R \tilde{\chi} \)

During the present resurgence of interest in supersymmetry broken at low energies [1] new significance is attached to the classical phenomenological playgrounds of gauge theories such as the anomaly magnetic moments of the electron and muon [2], flavour-changing neutral interactions [3-5] parity [6] and CP violation [7,8] in the strong interactions. The three latter phenomena make life rather difficult [3,7] for the most general form of soft supersymmetry breaking, whereas simple models [9-11] of spontaneously broken supersymmetry naturally [3,4,7] respect the \( \Delta F \neq 0, P \) and CP violation constraints. As for the anomalous magnetic moments of the leptons, it has long been known that they vanish in an exactly supersymmetric theory [12], and Fayet [2] showed that in his model of supersymmetry breaking \( g - 2, \mu \) would be compatible with experiment if the spin-zero muon (smuon) masses were heavier than 15 GeV. Direct experimental searches [13] now exclude the existence of lighter smuons. Fayet’s analysis [2] was in the context of a model with a very light photino \( \tilde{\gamma} \) (see fig. 1a), and Grifols and Méndez [14] have recently made the interesting observation that his analysis is significantly altered for massive gauginos (see figs. 1b, 1c). They show that there are potentially nontrivial constraints on the smuon masses in models of broken supersymmetry.

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The anomalous magnetic moment of the muon \( g - 2, \mu \) imposes constraints on the masses and mixing of spin-zero leptons (sleptons). We develop the predictions of models of spontaneous supersymmetry breaking for the slepton mass matrix, and show that they are comfortably consistent with the \( g - 2, \mu \) constraints.

![Fig. 1. One-loop diagrams contributing to \( g - 2, \mu \): (a) essentially massless photino \( \tilde{\gamma} \) exchange, (b) \( \tilde{W} \) and slepton (\( \tilde{\nu} \)) exchange, and (c) \( \tilde{B} \) or \( \tilde{Z} \) exchange.](image-url)
Possible Discrepancy with Theory?

$\delta a = \pm 0.47$ ppm

BNL E821 experiment, 2001 - 2006
$g_\mu - 2$ in Supersymmetry v2: the CMSSM

Abstract

We combine the constraint suggested by the recent BNL E821 measurement of the anomalous magnetic moment of the muon on the parameter space of the constrained MSSM (CMSSM) with those provided previously by LEP, the measured rate of $b \to s\ell^+\ell^-$ decay and the cosmological relic density $\Omega_\chi h^2$. Our treatment of $\Omega_\chi h^2$ includes carefully the direct-channel Higgs poles in annihilation of pairs of neutralinos $\chi$ and a complete analysis of $\chi - \ell$ coannihilation. We find excellent consistency between all the constraints for $\tan \beta \gtrsim 10$ and $\mu > 0$, for restricted ranges of the CMSSM parameters $m_0$ and $m_{1/2}$. All the preferred CMSSM parameter space is within reach of the LHC, but may not be accessible to the Tevatron collider, or to a first-generation $e^+e^-$ linear collider with centre-of-mass energy below 1.2 TeV. © 2001 Published by Elsevier Science B.V.
Calculations

Complete Tenth-Order QED Contribution to the Muon $g - 2$

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2Nishina Center, RIKEN, Wako, Japan 351-0198
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(Dated: August 21, 2012)

We report the result of our calculation of the complete tenth-order QED terms of the muon $g - 2$. Our result is $a^{(10)} = 753.29$ (1.04) in units of $(\alpha/\pi)^5$, which is about 4.5 s.d. larger than the leading-logarithmic estimate 663 (20). We also improved the precision of the eighth-order QED term of $a^{(8)}$, obtaining $a^{(8)} = 130.8794$ (63) in units of $(\alpha/\pi)^4$. The new QED contribution is $a^{(QED)} = 116\ 584\ 718\ 951\ (80) \times 10^{-14}$, which does not resolve the existing discrepancy between the standard-model prediction and measurement of $a$.

PACS numbers: 13.40.Em,14.60.Ef,12.20.Ds
Electroweak Contributions

• Leading one-loop order

$$a_{\mu}^{\text{EW}(1)} = \frac{G_F m_{\mu}^2}{\sqrt{2} 8\pi^2} \left[ \frac{5}{3} + \frac{1}{3} (1 - 4s_W^2)^2 \right] = 194.79(1) \times 10^{-11}$$

• Two-loop contributions

$$a_{\mu}^{\text{EW}(2), \text{logs}} = -4 \frac{\alpha}{\pi} \log \frac{M_Z}{m_{\mu}} a_{\mu}^{\text{EW}(1)}$$

$$+ \frac{G_F m_{\mu}^2}{8\pi^2 \sqrt{2}} \frac{\alpha}{\pi} \log \frac{M_Z}{m_{\mu}} \left[ -\frac{47}{9} - \frac{11}{9} (1 - 4s_W^2)^2 \right]$$

$$+ \frac{G_F m_{\mu}^2}{8\pi^2 \sqrt{2}} \frac{\alpha}{\pi} \sum_f \log \frac{M_Z}{\max(m_f, m_{\mu})} \left[ -6g_A^f g_A^f N_f Q_f^2 + \frac{4}{9} g_V^f g_V^f N_f Q_f \right]$$

$$= -41.2(1.0) \times 10^{-11}$$

Combined result

Hadronic Contribution to Light-by-Light Scattering

\[ k = p' - p \]

\[ \pi^0, \eta, \eta' \]

\[ \pi^+ \]

Exchanges of other resonances \((f_0, a_1, f_2, \ldots)\)

\[ q \]

\[ + \ldots + \]

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<th>PdRV(09) [475]</th>
<th>N/JN(09) [476, 596]</th>
<th>J(17) [27]</th>
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<td>105(26)</td>
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<td>100.4(28.2)</td>
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The anomalous magnetic moment of the muon in the Standard Model


Hadronic Vacuum Polarization

- Most important contribution is from low energies \( \lesssim 1 \text{ GeV} \), dominated by \( \rho \) and \( \omega \) peaks, taking account of interference effects.

- Uncertainties dominated by \( \rho \) and \( \omega \) region, and by region between 1 and 2 GeV (\( \phi \), etc.)

- High energies under good control from perturbative QCD

\[
\alpha_{\mu}^{\text{HVP,LO}} = 693.1(2.8)_{\exp}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} = 693.1(4.0) \times 10^{-10}. 
\]

τ Decays?

- Relation between τ decays and $I = 1$ portion of hadronic vacuum polarization:
  \[
  \sigma_{X^0}^{I=1}(s) = \frac{4\pi\alpha^2}{s} \nu_{1,X^-}(s)
  \]
  \[
  \nu_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2} \frac{B_{X^-}}{B_e} \frac{1}{N_X} \frac{dN_X}{ds} \times \left[ \left( 1 - \frac{s}{m_\tau^2} \right)^2 \left( 1 + \frac{2s}{m_\tau^2} \right) \right]^{-1} \frac{R_{IB}(s)}{S_{EW}}
  \]

- BUT what about $I = 0$ portion?

- AND what about isospin breaking?

- AND uncertainties in τ decay data?

- NOT INCLUDED by Theory Initiative

Comparison of Calculations of Hadronic Vacuum Polarization

\[ \alpha_{\mu}^{\text{HVP}} + \left[ \alpha_{\mu}^{\text{QED}} + \alpha_{\mu}^{\text{Weak}} + \alpha_{\mu}^{\text{HLebL}} \right] = \alpha_{\mu}^{\text{SM}} \]

HVP from:
- LM20
- BMWV20
- ETM18/19
- Mainz/CLS19
- FHM19
- PACS19
- RBC/UKQCD18
- BMW17
- RBC/UKQCD data/lattice
- BDJ19
- J17
- DHMZ19
- KNT19
- WP20

Fermilab uncertainty goal

(not used in WP20)

RBC/UKQCD Hybrid Method

Replace lattice data at very short and long distances by experimental e+e- scattering data

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- Intermediate window:
  expect reduced FV effects and discretization errors

Blum et al, arXiv:1801.07224

BMW Lattice Calculation

High statistics, accurate continuum extrapolation

\[ a_\mu^{\text{EXP}} - a_\mu^{\text{BMW}} = 107(70) \times 10^{-11} \]
How to Accommodate BMW?

- Analyticity and unitarity constrain increase in $\pi^+\pi^-$ cross section $< 1$ GeV
- Maximum allowed conflicts with data, does not change greatly prediction for $a_\mu$
- Increase in cross section at higher energies affects electroweak observables

Does the magnet look familiar?
Fermilab Data

Abi et al, arXiv:2104.03281
Fermilab Measurement

FNAL result: $a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$ (0.46 ppm)

Combined result: $a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11}$ (0.35 ppm)

Difference from Standard Model: $a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$

Abi et al, arXiv:2104.03281
### InterpretaLon Papers

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**Legend:**
- B: Baryon
- B, H decays: Baryon and Higgs decays
- DM: Dark Matter
- Darmé: Dark Matter
- Chiang: Chiang
- Li: Li
- Extra U(1): Extra U(1)
- Extra Higgs: Extra Higgs
- B, Cabibbo: B, Cabibbo
- Marzocca: Marzocca
- Boyarkin: Boyarkin
- Zhou: Zhou
- Anchoodoi: Anchoodoi
- Ban: Ban
- Boynkina: Boynkina
- Raff: Raff
- Buras: Buras
- Baier: Baier
- Carcino: Carcino
- Baur: Baur
- Angel: Angel
- Dacsila: Dacsila
- Coban: Coban
- Cacciapaglia: Cacciapaglia
- Boynkina: Boynkina
- Altmannshofer: Altmannshofer
- Aboubrahim: Aboubrahim
- Dev: Dev
- Ma: Ma
- Jueil: Jueil
- Fileviez: Fileviez
- Alonso: Alonso
- Arbuov: Arbuov
- Dutta: Dutta
- Zhang: Zhang
- Baum: Baum
Scalar Leptoquarks

- Consider two types of leptoquarks with couplings

\[ - \mathcal{L}_{Q_L}^Y \supset Y_2 Q_L \Phi_3 (\nu^c)_L + Y_2 \ell_L \Phi_4 (u^c)_L + Y_4 \ell_L \Phi_3^\dagger (d^c)_L + Y_4 Q_L \Phi_4^\dagger (e^c)_L + h.c. \]

- Consider constraints from \( B_s \to \mu^+ \mu^-, \mathcal{R}_K, \mathcal{R}_{K^*} \)

---

Fileviez, Margui & Plascencia, arXiv:2103.13397
Scalar Leptoquarks

- Consider 2 scenarios for mixing between leptoquarks:

\[-\mathcal{L} \supset \bar{e}^i \left( -\sin \theta_{LQ} c_L^{ij} P_L + \cos \theta_{LQ} c_R^{ij} P_R \right) d^j \phi_A^{-2/3} + \bar{e}^i \left( \cos \theta_{LQ} c_L^{ij} P_L + \sin \theta_{LQ} c_R^{ij} P_R \right) d^j \phi_B^{-2/3} + \text{h.c.}\]

- Constraints from $g_\mu - 2$, $R_K$, $B_S \rightarrow \mu^+ \mu^-$

Fileviez, Margui & Plascencia, arXiv:2103.13397
Leptophilic Z’ Gauge Boson

- LHC sets strong bounds only if Z’ boson couples to quarks
- Weaker constraints on Z’ bosons coupled to leptons only
- $\ell \rightarrow \ell'\nu\bar{\nu}, \ell \rightarrow \ell'\gamma, \ell \rightarrow 3\ell'$, mixing with Z and anomalous magnetic moments
- Can explain $g_\mu - 2$ with Z' coupled to $L_\mu - L_\tau$
- Search for lepton universality violation in $\tau \rightarrow \mu\nu\bar{\nu}/\tau \rightarrow e\nu\bar{\nu}$

$$\frac{A[\tau \rightarrow \mu\nu\bar{\nu}]}{A[\mu \rightarrow e\nu\bar{\nu}]}_{\text{EXP}} = 1.0029 \pm 0.0014$$
$$\frac{A[\tau \rightarrow \mu\nu\bar{\nu}]}{A[\tau \rightarrow e\nu\bar{\nu}]}_{\text{EXP}} = 1.0018 \pm 0.0014$$
$$\frac{A[\tau \rightarrow e\nu\bar{\nu}]}{A[\mu \rightarrow e\nu\bar{\nu}]}_{\text{EXP}} = 1.0010 \pm 0.0014$$

Leptophilic $Z'$ Gauge Boson

- Scenario with no $Z - Z'$ mixing, left- and right-handed couplings to $\mu, \tau$ only

$g_\mu - 2$ in Supersymmetry

- Muon $\psi_f$, 4 neutralinos $\psi_i$, 2 smuons $\phi_k$ ($\tilde{\mu}_{L,R}$)

$$- \mathcal{L}_{\text{int}} = \sum_{ik} \bar{\psi}_f (K_{ik} \frac{1 - \gamma_5}{2} + L_{ik} \frac{1 + \gamma_5}{2}) \psi_i \phi_k + H.c.$$ 

- One-loop contributions from smuon/neutralino loops:

  - Left-right mixing:  
    $$a^{11}_f = \sum_{ik} \frac{m_f^2}{8\pi^2 m_i^2} \text{Re}(K_{ik} L_{ik}^*) I_1 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right)$$

  - Unmixed:  
    $$a^{12}_f = \sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) I_2 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right)$$
$g_\mu - 2$ in Phenomenological Supersymmetry (pMSSM11)

No relation between squark & gluino masses and sleptons and neutralino
No problem accommodating BNL/FNAL result
Neutralino DM, smuon masses $\sim 300/400$ GeV
LHC vs Supersymmetry

- LHC does not exclude (relatively) light electroweakly-interacting particles, e.g., sleptons

- LHC favours squarks & gluinos $> 2$ TeV (but loopholes)
Supersymmetry

- $g_\mu$ — 2-friendly scenario with light neutralino, chargino & slepton

- Red star points include all relevant LHC and direct scattering constraints

- Prospects for the ILC?

Chakraborti, Heinemeyer & Saha, arXiv:2104.03287
\( g_\mu - 2 \) in Supersymmetric SU(5) GUT (CMSSM)

Scenario relates squark & gluino masses to sleptons and neutralino
Cannot accommodate BNL/FNAL result
Smuon masses \( \gtrsim 4 \) TeV

Very small contribution to \( g_\mu - 2 \)
Extend GUT SU(5) with additional U(1)

“Flipped” fermion assignments to representations:

\[ f_i(\bar{5}, -3) = \{U_i^c, L_i\}, \quad F_i(10, 1) = \{Q_i, D_i^c, N_i^c\}, \quad l_i(1, 5) = E_i^c, \quad i = 1, 2, 3 \]

Break GUT symmetry with 10-dimensional Higgses, electroweak symmetry with 5-dimensional Higgses:

\[ H(10, 1) = \{Q_H, D_H^c, N_H^c\}, \quad \bar{H}(10, -1) = \{\bar{Q}_H, \bar{D}_H^c, \bar{N}_H^c\} \]

\[ h(5, -2) = \{T_{H_c}, H_d\}, \quad \bar{h}(\bar{5}, 2) = \{\bar{T}_{H_c}, H_u\} \]

Superpotential:

\[ W = \lambda_{ij} F_i F_j h + \lambda_{ij}^2 F_i \bar{f}_j \bar{h} + \lambda_{ij}^3 \bar{f}_i \ell_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \]

\[ + \lambda_{6}^{ia} F_i \bar{H} \phi_a + \lambda_{7}^{a} h \bar{h} \phi_a + \lambda_{8}^{abc} \phi_a \phi_b \phi_c + \mu_{\phi} \phi_a \phi_b , \]

Scan free parameters of model:

\[ M_5, M_{X1}, m_{10}, m_5, m_1, \mu, M_A, A_0, \tan \beta \]
$g_\mu - 2$ in Flipped SU(5)

Lightest supersymmetric particle charged

LEP lower limit on slepton mass

$x =$ best-fit point

LHC constraints

x = best-fit point
$g_\mu - 2$ in CMSSM & Flipped SU(5) vs Lattice, Data-Driven Calculation

Reduce discrepancy between experiment and data-driven estimate to $< 2\sigma$
Remove residual discrepancy between experiment and BMW

$\Delta a_\mu (\times 10^{11})$: GUT models vs Standard Model calculations

# $g\mu - 2$ in Flipped SU(5)

## Parameters & predictions at best-fit point

| Input GUT parameters (masses in units of $10^{16}$ GeV) |
|-----------------|-----------------|-----------------|
| $M_{GUT} = 1.00$ | $M_X = 0.79$    | $V = 1.13$      |
| $\lambda_4 = 0.1$ | $\lambda_5 = 0.3$ | $\lambda_6 = 0.001$ |
| $g_5 = 0.70$ | $g_X = 0.70$ | $m_{r_3} = 0.05$ eV |

| Input supersymmetry parameters (masses in GeV units) |
|-----------------|-----------------|-----------------|
| $M_5 = 2460$    | $M_1 = 240$     | $\mu = 4770$    |
| $m_{10} = 930$  | $m_\tau = 450$  | $m_1 = 0$       |
| $M_A = 2100$    | $A_0/M_5 = 0.67$ | $\tan \beta = 35$ |

| MSSM particle masses (in GeV units) |
|-----------------|-----------------|-----------------|
| $m_X = 84$      | $m_{t_1} = 4030$ | $m_\tilde{b} = 5090$ |
| $m_{\chi_2} = 2160$ | $m_{\chi_3} = 5080$ | $m_{\chi_4}$ |
| $m_{\tilde{\mu}_R} = 101$ | $m_{\tilde{\mu}_L} = 1600$ | $m_{\tilde{\tau}_1}$ |
| $m_{\tilde{\mu}_L} = 4470$ | $m_{\tilde{d}_R} = 4250$ | $m_{\tilde{\tau}_2}$ |
| $m_{t_2} = 4410$ | $m_{b_1} = 4170$ | $m_{b_2}$ |
| $m_{\chi^\pm} = 2160$ | $m_{H^+_A} = 2100$ | $m_{H^\pm} = 2100$ |

| Other observables |
|-----------------|-----------------|-----------------|
| $\Delta a_\mu = 150 \times 10^{-11}$ | $\Omega_\chi h^2 = 0.13$ | $m_h = 122$ GeV |
| Normal-ordered $\nu$ masses: $\tau_{p \to e^+\pi^0}\big|_{NO} = 1.1 \times 10^{36}$ yrs | $\tau_{p \to \mu^+\pi^0}\big|_{NO} = 1.1 \times 10^{37}$ yrs |
| Inverse-ordered $\nu$ masses: $\tau_{p \to e^+\pi^0}\big|_{IO} = 3.2 \times 10^{37}$ yrs | $\tau_{p \to \mu^+\pi^0}\big|_{IO} = 2.3 \times 10^{36}$ yrs |

Opportunities to search for light smuon, neutralino at LHC
Other sparticles too heavy?

MuonE: Proposed CERN Experiment to Measure HVP in Space-Like Region

Scattering of 150 GeV muons on electrons at CERN SPS

\[
\alpha^\text{HLO}_\mu = \frac{\alpha}{\pi} \int_0^1 dx \ (1 - x) \Delta \alpha_{\text{had}}[t(x)] \quad \alpha(t) = \frac{\alpha(0)}{1 - \Delta \alpha(t)} \quad t(x) = -\frac{x^2 m^2_{\mu}}{1 - x} < 0
\]

Future

Different technique: ultra-cold muon beam from muonium, accelerate to 300 MeV, inject into storage ring with radius 66cm

M. Abe et al, Progr. Theor. Exp. Phys., 2019, 053C02
Magnetic Dipole Moment of the Electron

- Discrepancies between determinations of $\alpha$ from atomic measurements and $a_e \equiv (g_e - 2)/2 + \text{QED}$

- New determination of $\alpha$ via rubidium recoil measurement allows BSM contribution to $a_e$ in range $-3.4 \times 10^{-13} < \delta a_e < 9.8 \times 10^{-13}$, comparable to $\delta a_\mu \times (m_e/m_\mu)^2$

- Experiment underway to improve precision on $a_e$ by order of magnitude

Quo Vadis $g_\mu - 2$?

- **Never forget**: the (near-) consistency between theory and experiment for $g_\mu - 2$ (and $g_e - 2$) is among the greatest successes of particle physics, particularly quantum field theory.

- **Need no reminder**: the discrepancy between theory and experiment for $g_\mu - 2$ may be a window on physics beyond the Standard Model.

- Still some **debate** about Standard Model calculation (lattice?)

- **Plenty** of theoretical interpretations proposed: many possible connections to other physics areas (B decays, dark matter, ...)

- **More experimental results** on the way: FNAL, J-PARC, $a_e$, MuonE, ...

- **We live in interesting times!**
Summary

Visible matter

$g_\mu - 2$?