UK Research and Innovation

Introduction to lattice QCD

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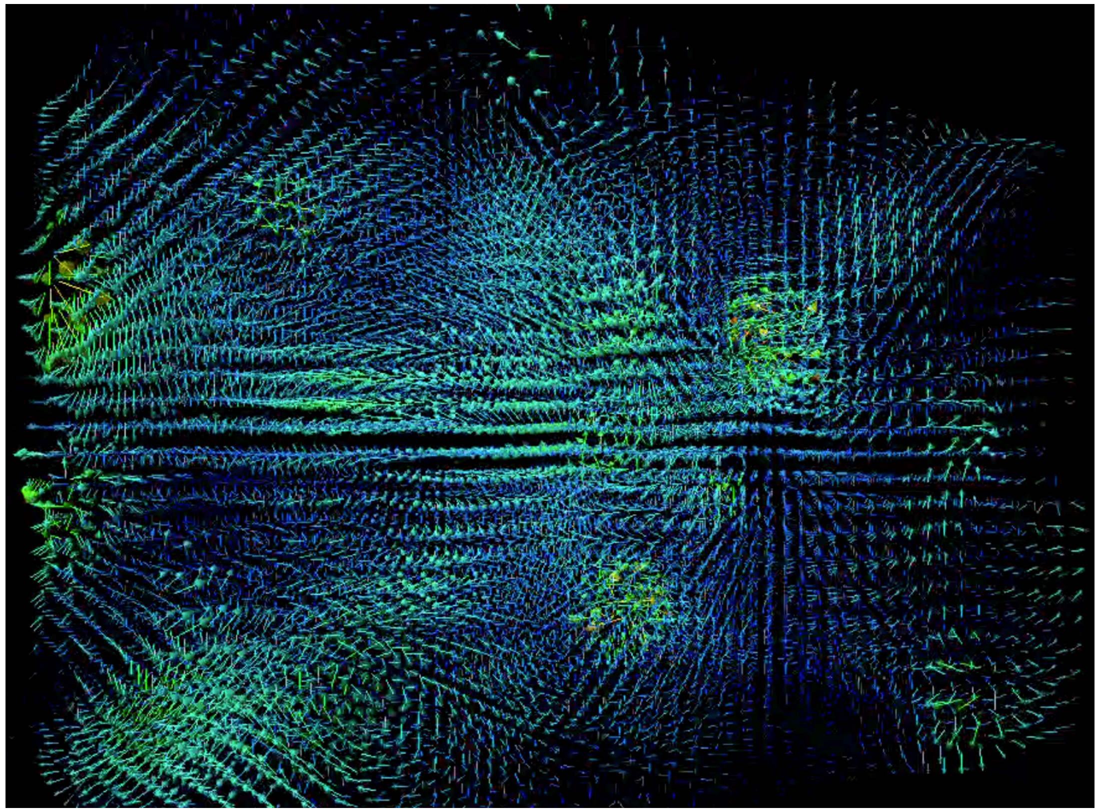
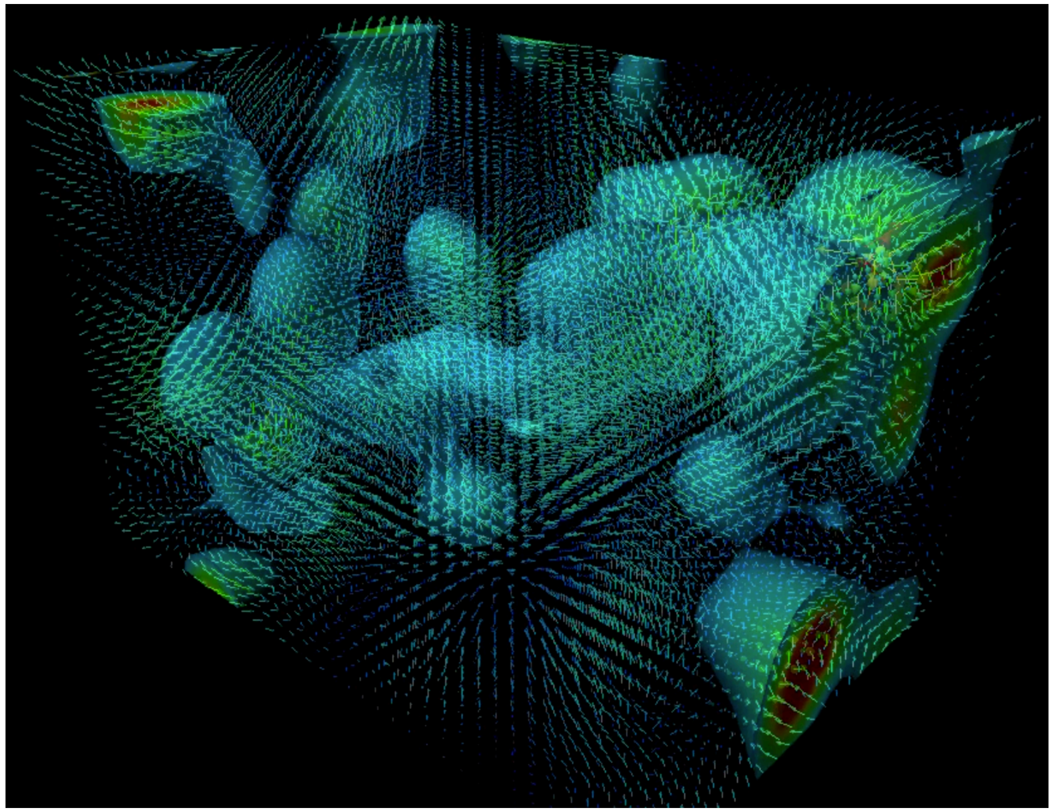
THE UNIVERSITY of EDINBURGH

Invitation: *What do we latticists think about the field/method?*

- ❑ Non-perturbative regulator of quantum field theory (QFT)
- ❑ Systematically improvable numerical method for extracting QFT's properties
- ❑ Exciting, vibrant, highly active research community
- ❑ Technical field that challenges all of us to be great communicators



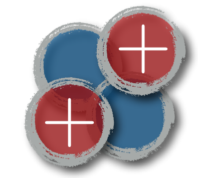


University of Adelaide, CSSM

Lattice QCD

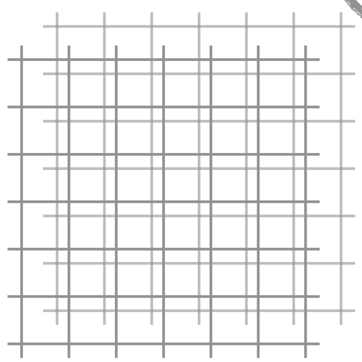
1. Lagrangian defining QCD +
2. Formal / numerical machinery (lattice QCD) +
3. A few experimental inputs (e.g. M_π, M_K, M_Ω) =

$$\mathcal{L}_{\rm QCD} = \sum_f \overline{\Psi}_f (i\not{D} - m_f)\Psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



Wide range of precision pre-/post-dictions

$m_q^{\overline{\rm MS}}(\mu)$ $\alpha_{\overline{\rm MS}}(\mu)$ m_p m_n f_π g_A f_K $(g-2)_\mu^{\rm HVP}$ f_D f_B



FLAG 2019 Flavour Lattice Averaging Group

$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1182(8)$ lattice average

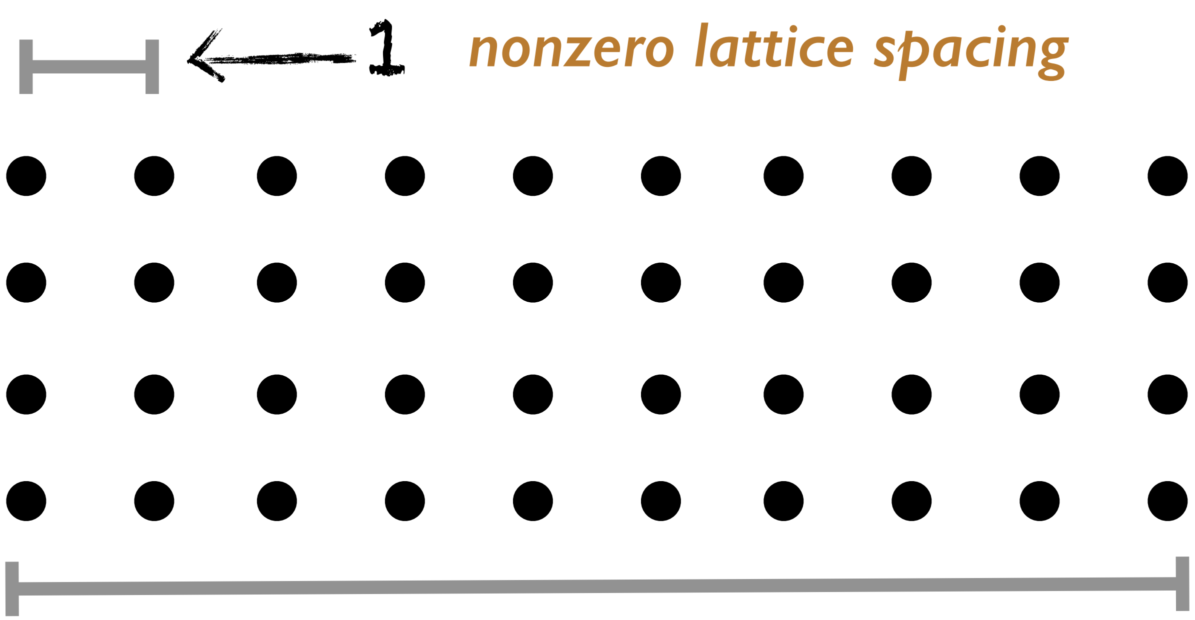
$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.1174(16)$ PDG 18 (non-lattice)

Overwhelming evidence for QCD ✓ → *Tool for new physics searches*

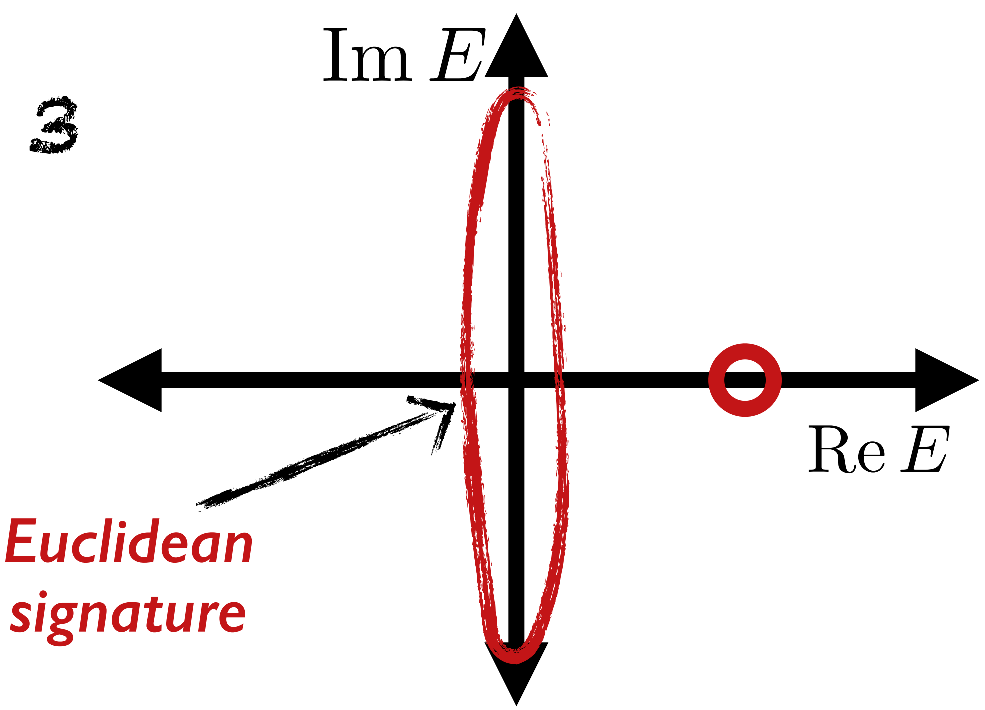
Three essential modifications

$$\text{observable?} = \int d^N\phi \, e^{-S} \left[\begin{array}{c} \textbf{interpolator} \\ \textbf{for observable} \end{array} \right]$$

To proceed we have to make *three modifications*



2 *finite volume,* $L^3 \times T$



Also… $M_{\pi,\text{lattice}} > M_{\pi,\text{our universe}}$

(but physical masses → increasingly common)

Outline

- The LQCD landscape
  - Lattice basics
  - Nielson Ninomiya
  - Many actions
- Flavor physics
  - Single-hadron matrix elements
  - Light-flavor decay constants
  - Heavy-flavor decay constants
  - Mixing
  - Form factors
- QED + QCD
  - Theoretical challenge
  - Different formulations
- $(g-2)_\mu$
  - Light-by-light
  - HVP
- Multi-hadron processes
  - Finite-volume as a tool
  - Resonances
  - $2 \to 2$ scattering
  - $1 + \mathscr{J} \to 2$ transitions
- So much more!

Discretization

- Warm up, $\lambda\phi^4$ theory

Continuum, Euclidean theory: $S[\phi] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}M^2\phi^2 + \frac{\lambda}{4!}\phi^4 \right]$

Lattice, " " : $S[\phi] = a^4 \sum_x \left[-\frac{1}{2}\phi_x \hat{\partial}^2\phi_x + \frac{1}{2}M^2\phi_x^2 + \lambda/4!\,\phi_x^4 \right]$

$\left\{ \hat{\partial}^2\phi_x \equiv 1/a^2 \sum_\mu \left(\phi_{x+ae_\mu} + \phi_{x-ae_\mu} - 2\phi_x \right) \right\}$

$$= \sum_x \left[-\sum_\mu \phi_x \phi_{x+e_\mu} + (2 + M^2/2)\phi_x^2 + \frac{\lambda}{4!}\phi_4 \right]$$

$$= \sum_x \left[-\beta \sum_\mu \phi_{x+e_\mu}\phi_x + \phi_x^2 + g(\phi_x^2 - 1)^2 \right]$$

- Calculate

$$G_n(\beta, g | x_1, \ldots, x_n) = 1/Z \prod_x \int d\phi_x\, e^{-S[\phi|\beta,g]} \phi_{x_1} \cdots \phi_{x_n}$$

$\Longrightarrow G(\tau) \equiv \int d^3x\, G_2(\beta, g | (\tau, \vec{x}), 0) = \sum_n e^{-M_n\tau} c_n \rightsquigarrow M_{n=0}(\beta, g) = a M_{phys}$

$\Longrightarrow G_4(\ldots)$ used to give $2\to 2$ scattering w/ $\vec{p} = \vec{0}$ $= \mathcal{M}_{thresh}$

Scale setting / parameter tuning

$M_{phys} = 100$ MeV

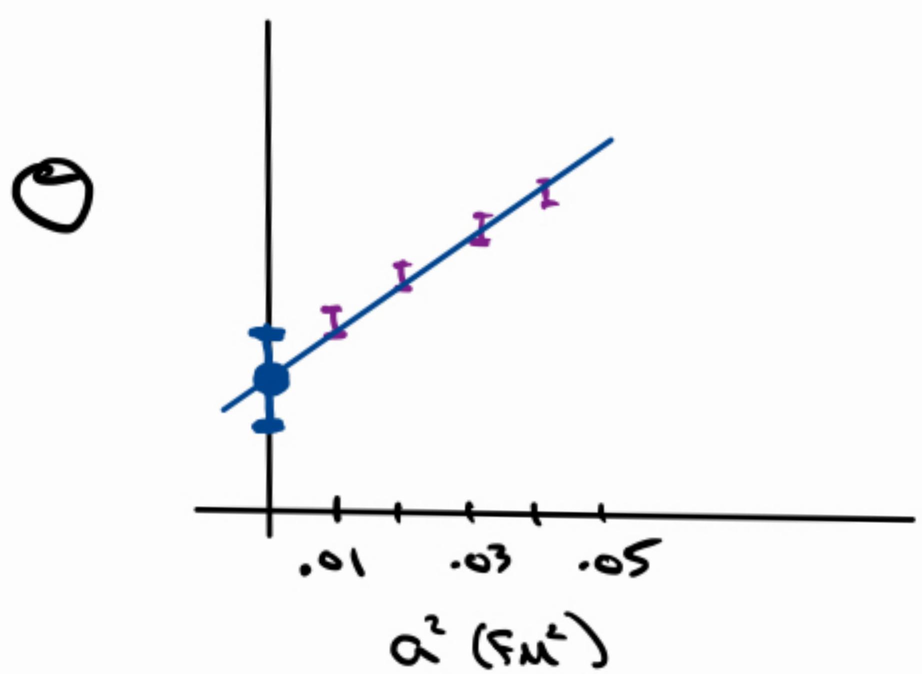
$M_{thresh} = 1.5$

- Say we want to describe physical system w/

- Tune set of $\beta, g \ldots$

β	$\#_1$	$\#_3$	...	
g	$\#_2$	$\#_4$	...	
aM_{phys}	1.0	0.5	0.4	0.3 ...
a	2fm	1fm	0.8fm	0.6fm

- For any other observable one can extract



Monte Carlo importance sampling

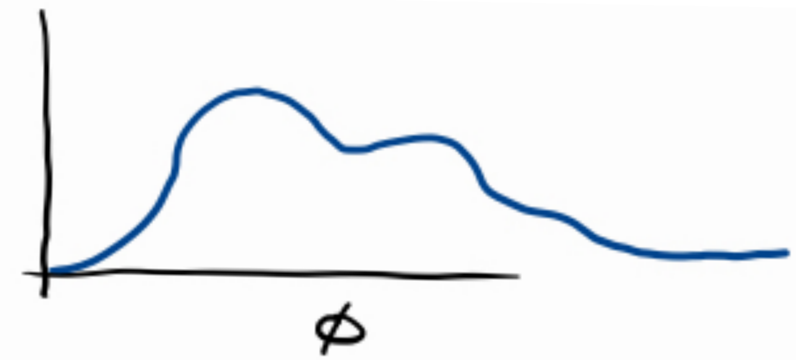
- Aim to build ensemble of configurations s.t.

$$\sum_{i=1}^{N} \mathcal{O}[\phi_i] \propto \int D\phi \, e^{-S[\phi]} \mathcal{O}[\phi] + \mathcal{O}(1/\sqrt{N})$$

Monte Carlo importance sampling

- Metropolis-Hastings algorithm
  - Probability distrib. $P(\phi)$ and $F(\phi) \propto P(\phi)$

    $\{ F(\phi) = e^{-S[\phi]} \}$



  - Choose random, arbit ϕ, define $Q(\phi' | \phi)$ as a distrib. to suggest next field ϕ'
  - For each iteration t
    - ↳ Generate ϕ' from $Q(\phi' | \phi_t)$
    - ↳ Calc. $\alpha = \frac{F(\phi')}{F(\phi_t)} = \frac{P(\phi')}{P(\phi_t)}$
    - ↳ Accept/Reject: If $\alpha > 1$ accept

      If $\alpha < 1$ generate random $u \in [0,1]$

      ↳ If $\alpha > u$ accept

      else reject

Fermion doubling

- Continuum, non-interacting, Dirac spinor

$$S = \int d^dx\, \bar{\psi}(x)\left[\partial\!\!\!/ + M\right]\psi(x)$$

$\Longrightarrow$ $\Delta(p)^{-1} = M + i p\!\!\!/$ for $p_\mu \in (-\infty, \infty)$

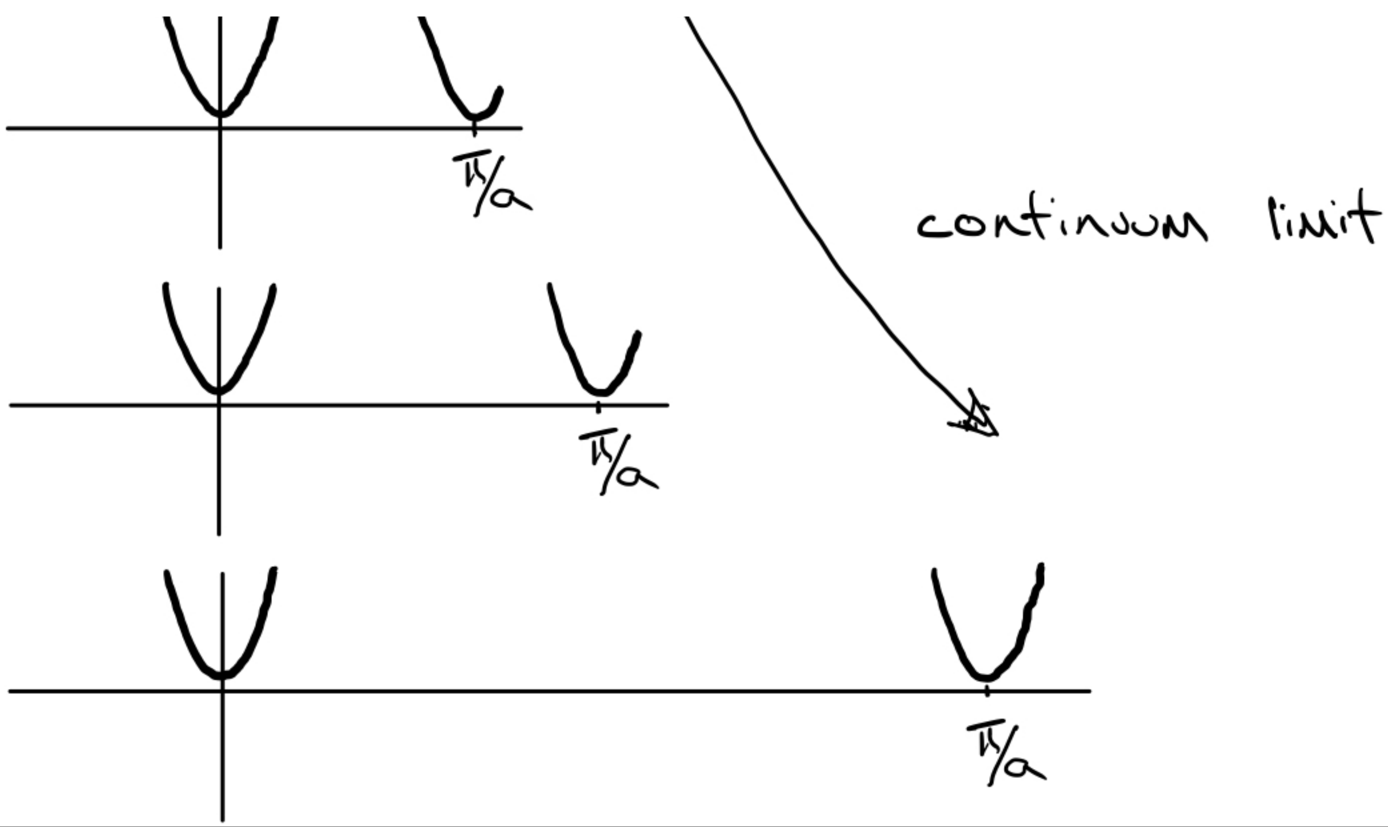
$\Longrightarrow$ $\Delta(p)$ has a single pole @ $(p^0)^2 = \vec{p}^{\,2} + M^2$

- Lattice naive, non-interacting Dirac spinor

$$S = a^d \sum_{x,\mu} \frac{1}{2a}\left[\bar{\psi}_x \gamma_\mu \psi_{x+\hat{\mu}} - \bar{\psi}_{x+\hat{\mu}} \gamma_\mu \psi_x\right] + a^d \sum_x M \bar{\psi}_x \psi_x$$

$\Longrightarrow$ $\Delta(p)^{-1} = M + i/a \sum_\mu \gamma_\mu \sin(p_\mu a)$ for $p_\mu \in \left(\frac{-\pi}{a}, \frac{\pi}{a}\right]$

$\Longrightarrow$ $\Delta(p)$ has poles @ $M^2 + \sum_\mu \frac{\sin^2(p_\mu a)}{a^2} = 0$



$\Longrightarrow$ actually a theory of 2^d Dirac fermions

Relation to the anomaly

- Usual continuum story

$$Z[\eta,\bar{\eta},j] = \int DA\, D\psi\, D\bar{\psi}\; e^{-S[\bar{\psi},\psi,A] + (\bar{\psi},\eta) + (\bar{\eta},\psi) + (j,A)}$$

↳ can't find regulator s.t. $D\psi D\bar{\psi}$ invariant under $\psi \to e^{i\alpha\gamma_5}\psi$, $\bar{\psi} \to \bar{\psi} e^{i\alpha\gamma_5}$

- On the lattice

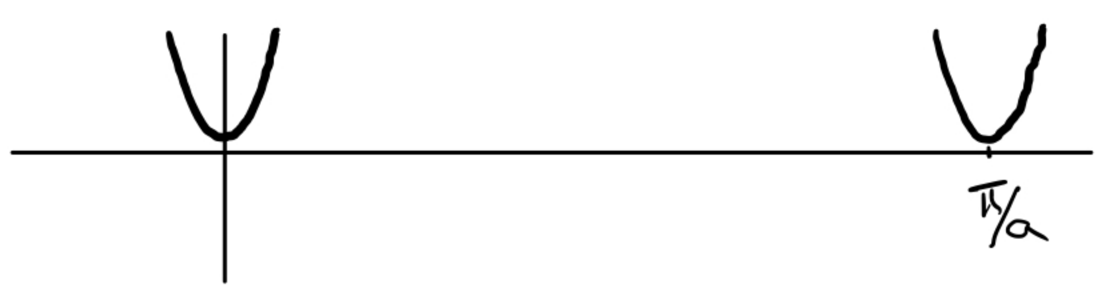
$$Z[\eta,\bar{\eta},j] = \int \left(\prod_x dA_x\right)\left(\prod_x d\psi_x\, d\bar{\psi}_x\right) e^{-S[\cdot]+\cdots}$$

↑ Manifestly invariant

$$\Longrightarrow \quad \lim_{m\to 0} \partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) = 0$$

- Resolution = doublers (cancel axial charge)

$$\lim_{m\to 0} \partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) = -\sum_Q \frac{Qg^2}{16\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} = 0$$



Nielsen-Ninomiya "no go" theorem

Define a generic fermion action: $S = \int_{-\pi/a}^{\pi/a} \frac{d^d p}{(2\pi)^d} \bar{\psi}(-p) \tilde{D}(p) \psi(p)$

Cannot simultaneously have:

(1) d (spacetime dim.) $\in 2\mathbb{Z}$ (even) ← our world

(2) $\tilde{D}(p)$ = periodic + analytic ← locality in x

(3) $\tilde{D}(p) \to \not{p}$ for $a|p_\mu| \ll 1$ ← conventional Dirac

(4) $\tilde{D}(p)$ invertible (besides $p_\mu = 0$) ← only one

(5) $\{\gamma_5, \tilde{D}(p)\} = 0$ ← chiral symmetry

Proliferation of discretization

Cannot simultaneously have: | Violated by:

(1) d (spacetime dim.) $\in 2\mathbb{Z}$ (even) | Domain wall (RBC/UKQCD)

(2) $\tilde{D}(p)$ = periodic + analytic | SLAC fermions

(3) $\tilde{D}(p) \to \not{p}$ for $a|p_\mu| \ll 1$

(4) $\tilde{D}(p)$ invertible (besides $p_\mu = 0$) | Naive fermions, Staggered fermions

(5) $\{\gamma_5, \tilde{D}(p)\} = 0$ | Wilson/Clover (CLS), Twisted mass (ETMC)

Useful to instead require (Ginsparg-Wilson): $\{\gamma_5, D\} = aD\gamma_5 D$ ← soft breaking of $\{\gamma_5, D\} = 0$

Domain wall, Overlap

Many lattice actions = many collaborations

BMW (Budapest Marseille Wuppertal)	Wilson (Clover) / Staggered
CalLatt (California Lattice)	Overlap / Staggered (mixed action)
CLS (Coordinated Lattice Effort)	Wilson (Clover)
ETMC (~~European~~ Extended Twisted Mass Collaboration)	Twisted Mass
Fermilab/MILC (MIMD Lattice Collaboration)	Staggered (HISQ)
NPLQCD (Nuclear Physics for lattice QCD)	Wilson (Clover)
RBC/UKQCD (Riken Brookhaven Columbia/United Kingdom)	Domain Wall

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  - [ ] Single-hadron matrix elements
  - [ ] Light-flavor decay constants
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- [ ] QED + QCD
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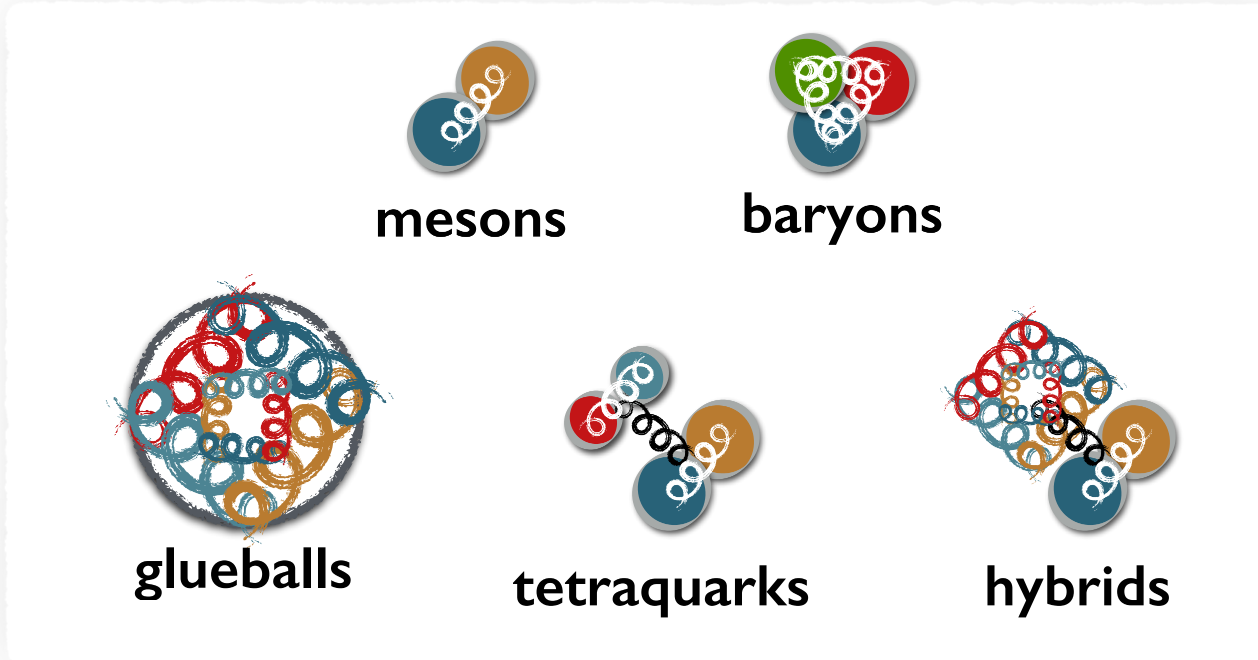
Flavor anomalies

- *Flavor anomalies* = opportunity for BSM
- QCD = crucial for ***confirming significance*** and interpreting

experiment = **SM** x *perturbative QCD* x (non-perturbative QCD)
+ **BSM** x *perturbative QCD* x (non-perturbative QCD)

- **QCD** is complicated
- Difficult to extract non-perturbative predictions



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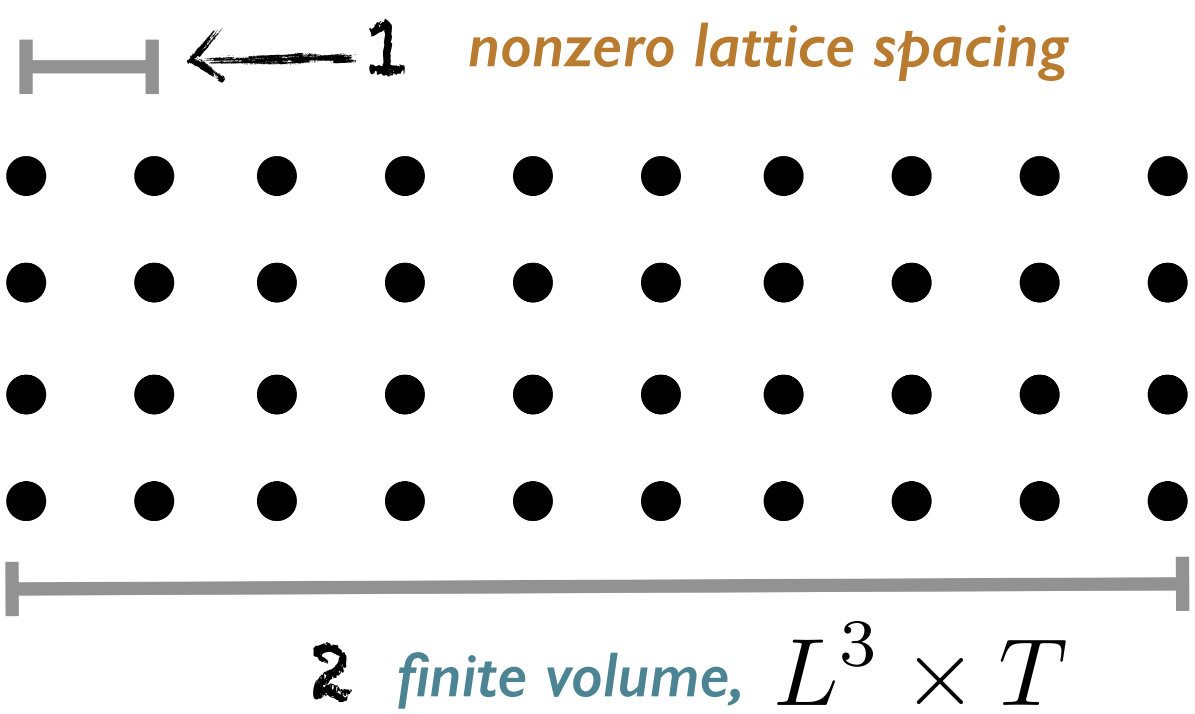
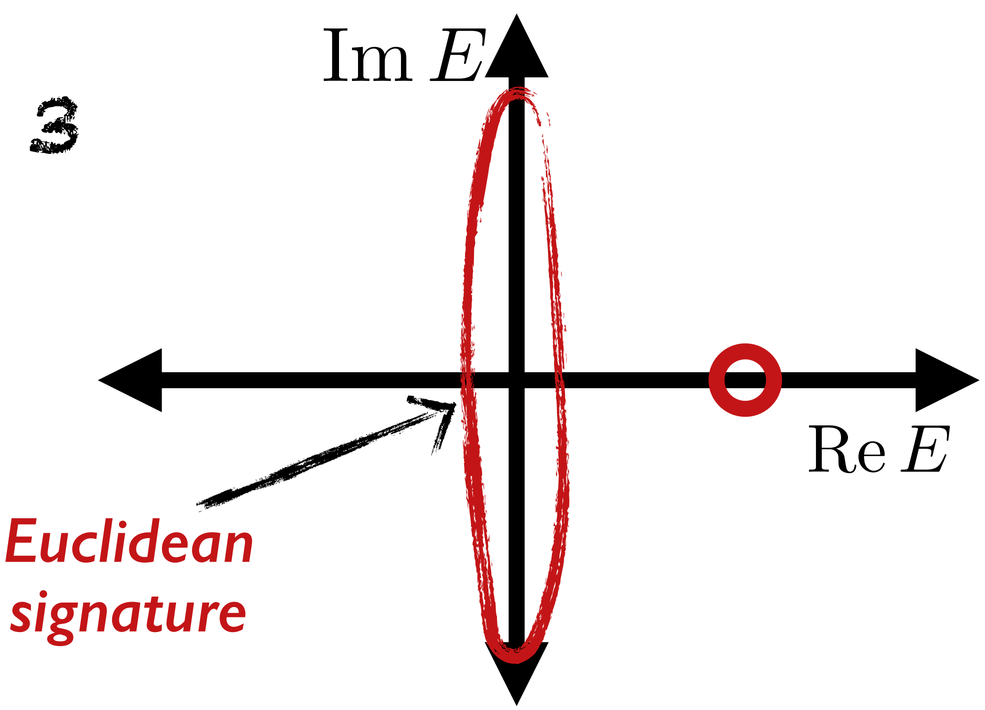




Also… $M_{\pi,\text{lattice}} > M_{\pi,\text{our universe}}$

(but physical masses → increasingly common)

Single-hadron states

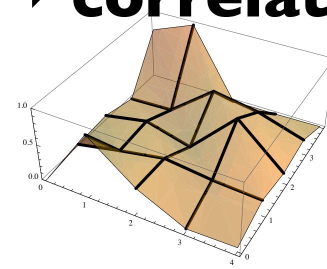
- **Three categories:**
  - **Decay constants**
  - **Form factors**
  - **Mixing parameters**

$$\langle 0|\mathcal{J}|\mathbf{1}\rangle \qquad \langle \mathbf{1}|\mathcal{J}|\mathbf{1}'\rangle \qquad \langle \overline{\mathbf{1}}|\mathcal{H}^{\Delta F=2}|\mathbf{1}\rangle$$

$$f_\pi, f_K, f_B \qquad f_+^{K^0\pi^-}(q^2),\ f_{B\to\pi}(q^2) \qquad B_{B_d}^{(n)},\ B_{B_s}^{(n)}$$

- **Summary of the approach…**
  - **Importance sampling QCD gauge fields → correlators**

$$\langle A_\mu^{\mathrm{bare}}(0)\ \pi_{\boldsymbol{p}}(-\tau)\rangle_{T,L,m_q,a} =$$



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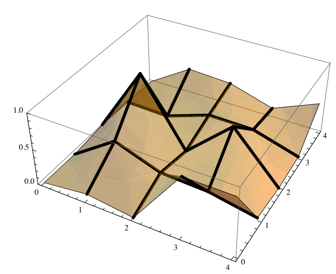

$+ \cdots$

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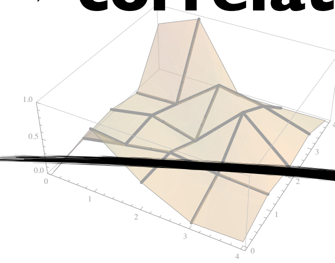
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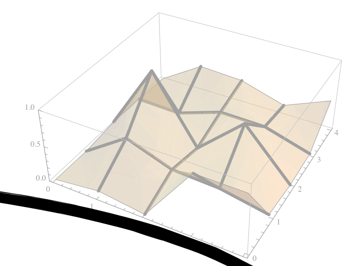
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- **temporal length**
- **volume**
- **quark masses**
- **lattice spacing**
- **Renormalization** of currents required (typically non-perturbative)

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- **Summary of the approach…**
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$$Z^{\text{renorm}} \langle A_\mu^{\text{bare}}(0)\ \pi_{\boldsymbol{p}}(-\tau)\rangle_{T,L,m_q,a} \underset{\tau \gg \delta E_\pi}{\longrightarrow} Z_\pi e^{-E_\pi \tau}\ ip_\mu f_\pi(T,L,m_q,a)$$

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  - volume
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  - **Renormalization** of currents required (typically non-perturbative)
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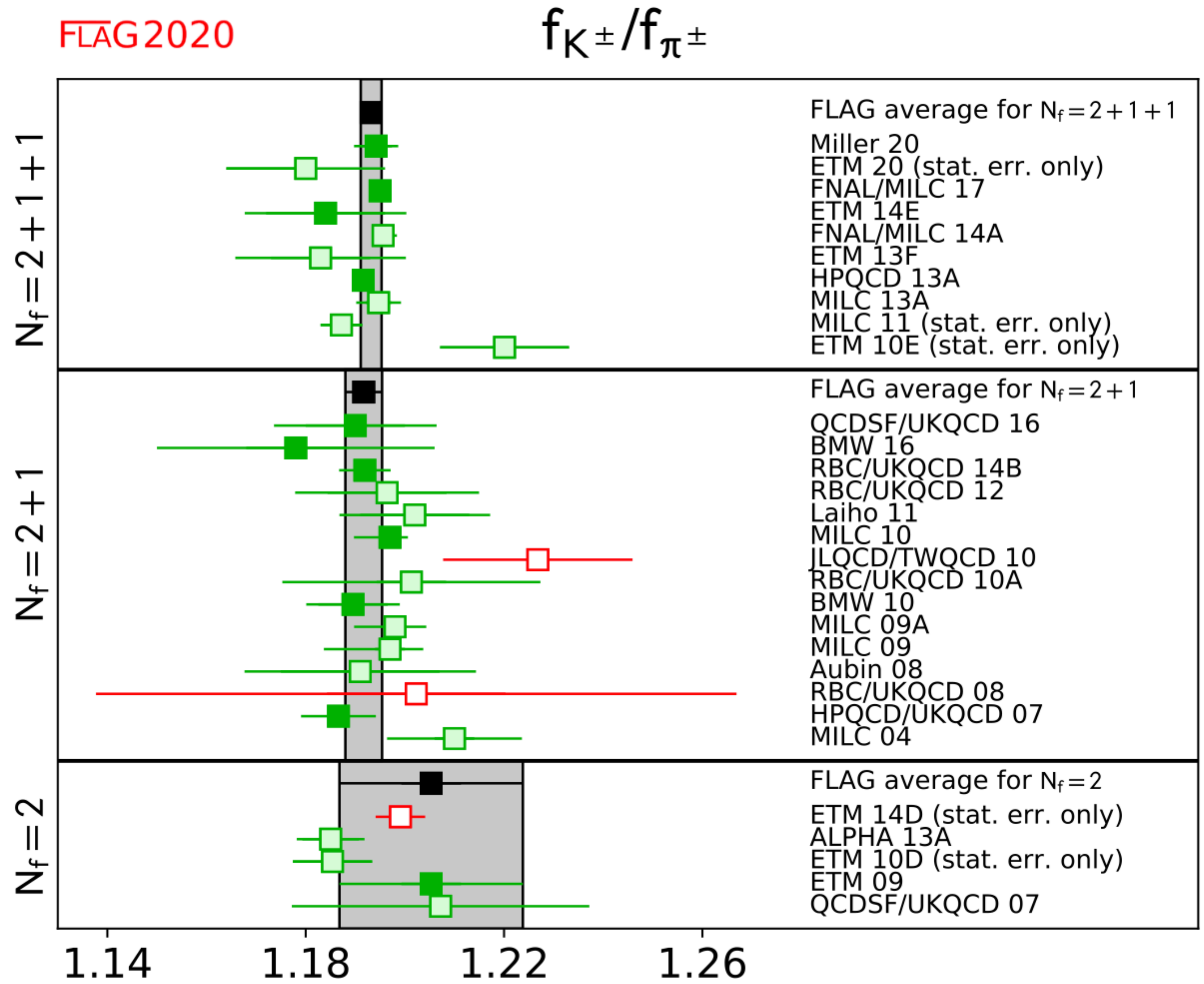
  - temporal length
  - volume
  - quark masses
  - lattice spacing
  - **Renormalization** of currents required (typically non-perturbative)
  - **Large time** separation filters excited states
  - Extrapolation/interpolation to physical point

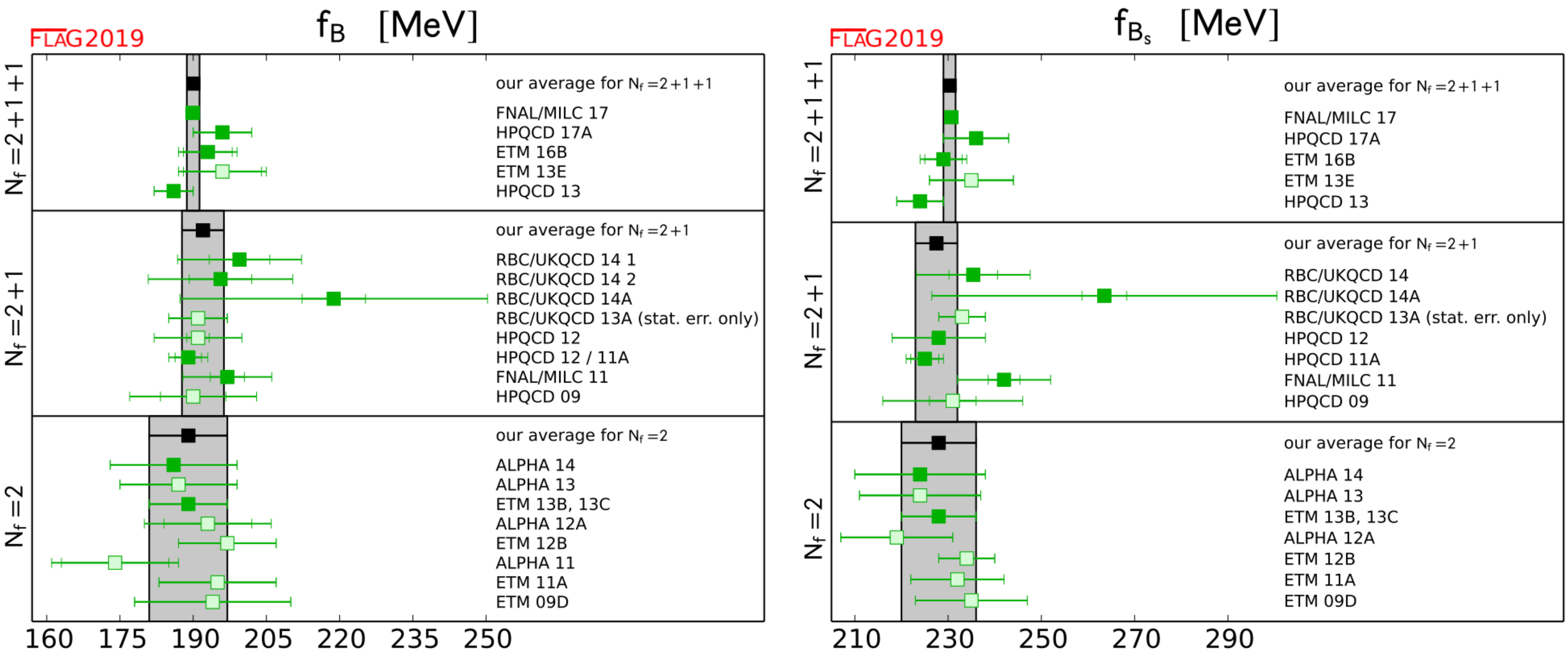
$$\lim_{T,L\to\infty}\lim_{a\to 0} f_\pi(T,L,m_q^{\text{phys}},a) = f_\pi^{\text{phys}}$$

Decay constants $\langle 0|\mathcal{J}|\mathbf{1}\rangle$



- Includes isospin breaking (but QED)
- Constrain CKM matrix elements
- Important to ask "What quantities are being sacrificed to set the calculation?"

Decay constants $\langle 0|\mathcal{J}|\mathbf{1}\rangle$

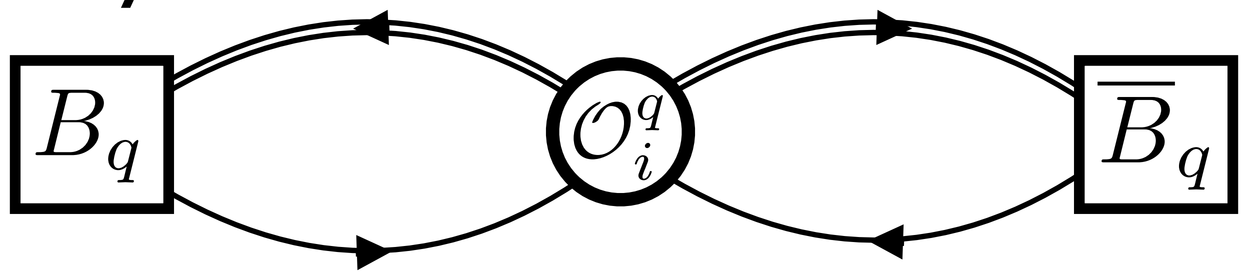


- **Current precision sufficient** for BES III, BELLE II
- Fermilab/MILC includes **QED** uncertainty (not yet rigorous)
- MILC quoting higher precision than any other 2+1(+1) calculation

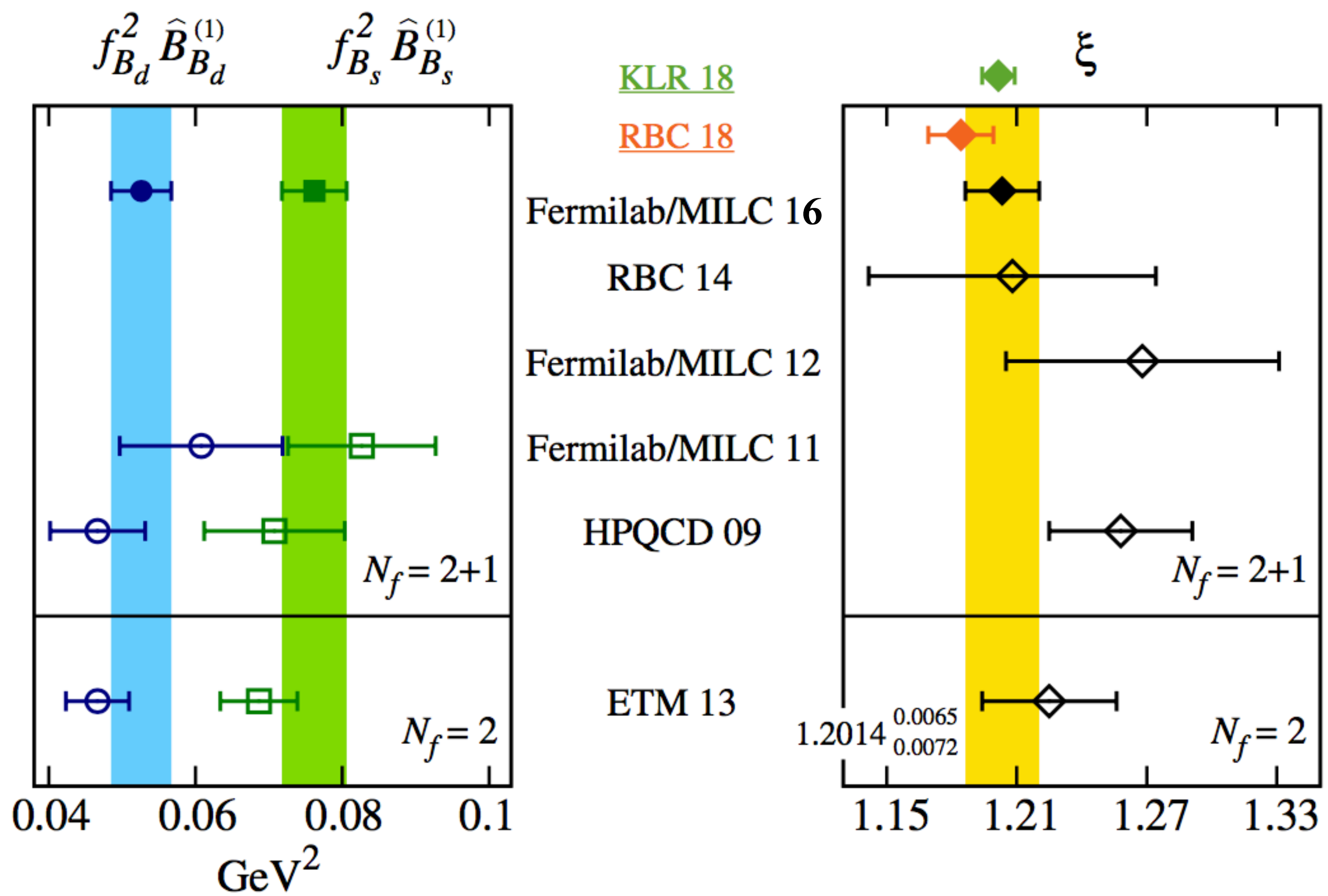
Need comparable precision from other calculations to **cross-check**

Neutral meson mixing $\langle \overline{\mathbf{1}} | \mathcal{H}^{\Delta F=2} | \mathbf{1} \rangle$

- **B-mixing dominated by local matrix element**



- **Summary (from Bazavov et al. [Fermilab/MILC] 2016)**



$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$$

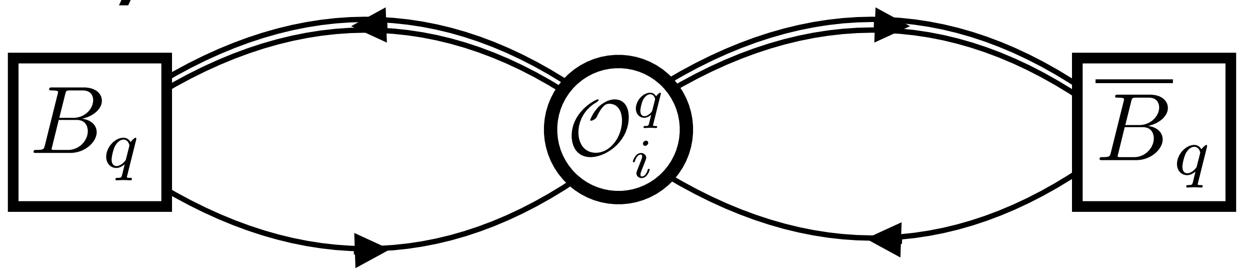
KLR 18 =
King, Lenz, Rauh *(2018)*
(QCD sum rules)

plot from Kronfeld
(Durham workshop 2019)

- **Lattice precision (~3-4%) is well behind even older experiments (~0.06 - 0.2%)**
- **Challenging to find optimal '*discretization*' (lattice definition of quarks)**

Neutral meson mixing $\langle \overline{\mathbf{1}} | \mathcal{H}^{\Delta F=2} | \mathbf{1} \rangle$

- **B-mixing dominated by local matrix element**



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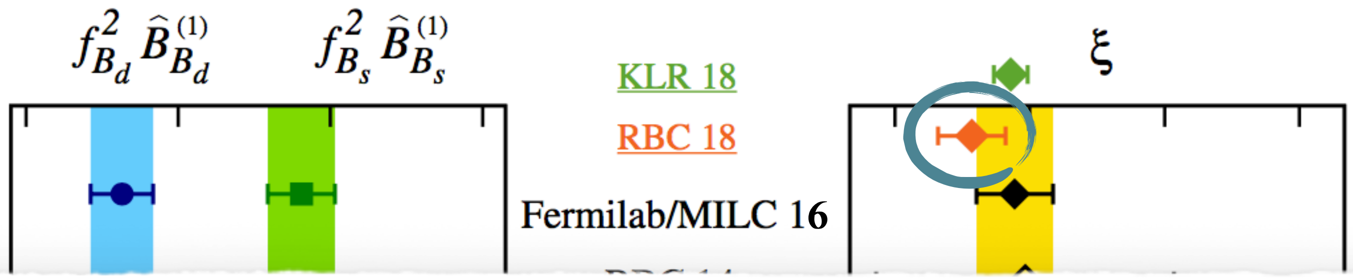
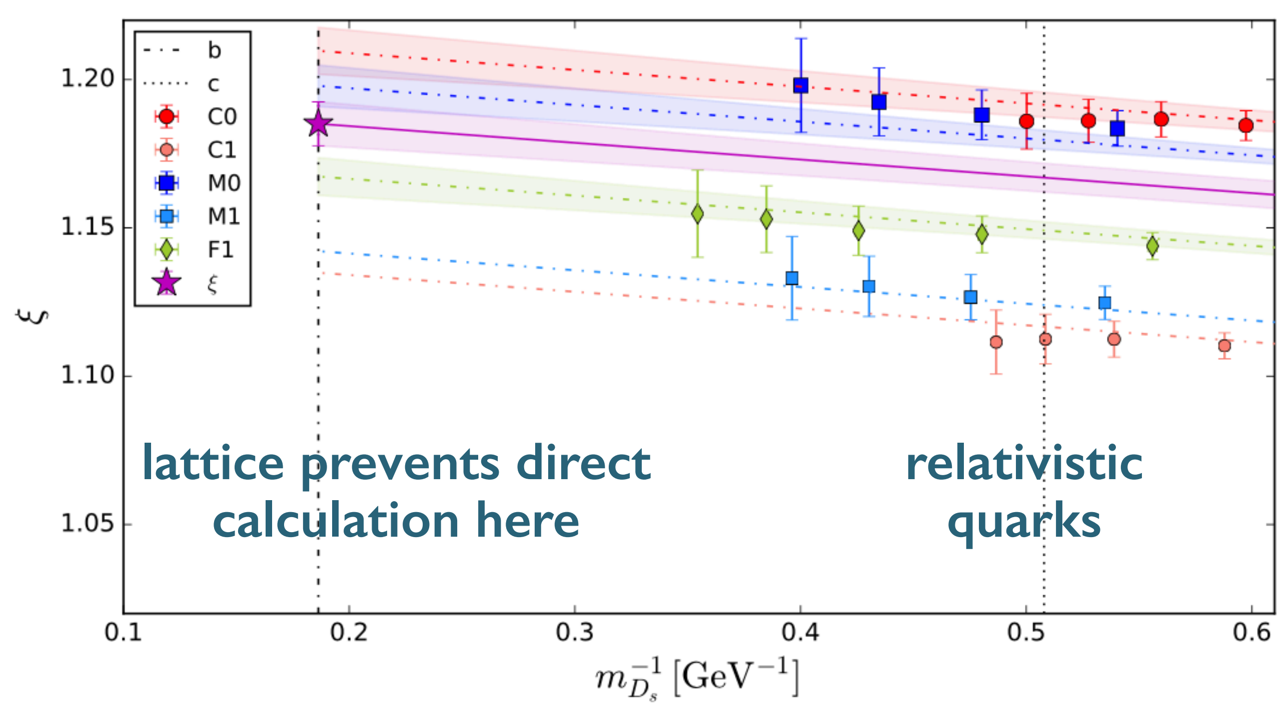


$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$$

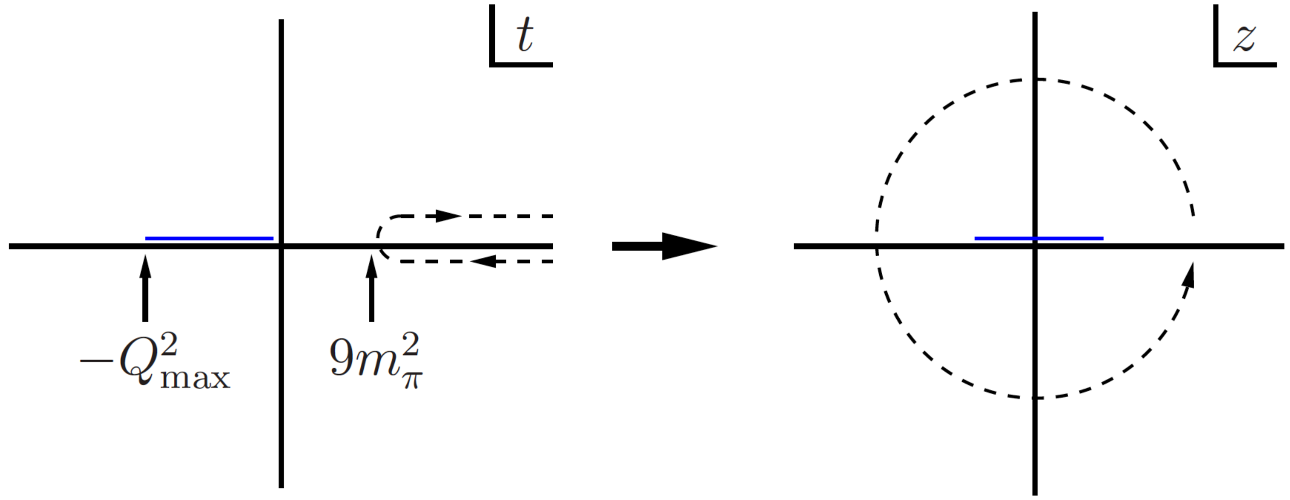
$\xi(a, m_\pi, m_H)$



- **RBC/UKQCD 2018**
- **No effective action for *b* quark**
- **Extrapolate to heavy mass**

Form factors $\langle \mathbf{1} | \mathcal{J} | \mathbf{1}' \rangle$

- Significantly more information (functions vs numbers)
- Conformal mapping → z-expansion → wider kinematic range



Bhattacharya, Hill, Paz *(2011)*

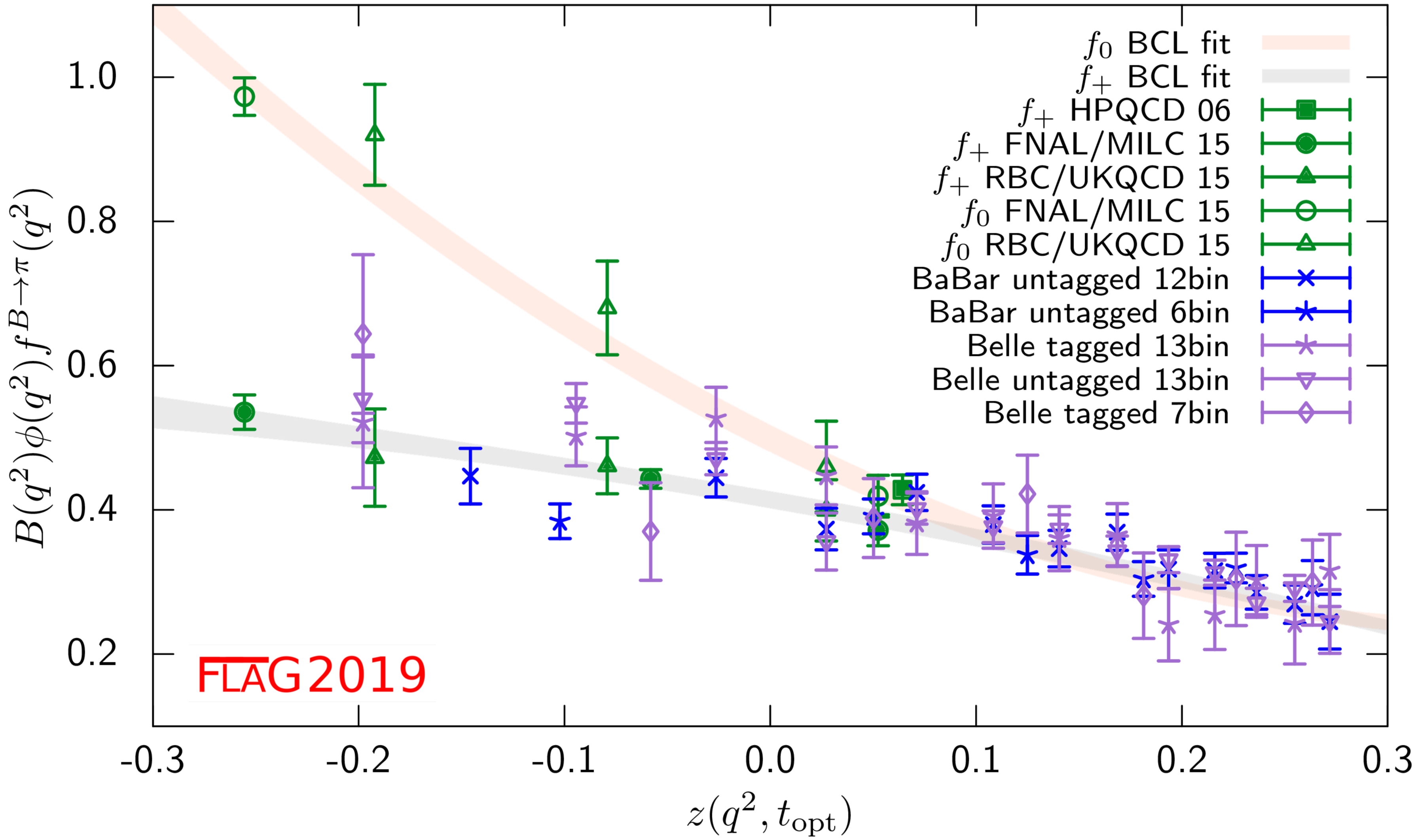
- Report z coefficients + correlations
- Joint fit to LQCD and experiment → CKM
- Better precision needed for BES III, LHCb and BELLE II

$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{ub}\rvert$
$\pi^+\to l^+\nu$	$K^+\to l^+\nu$	$B^+\to\tau^+\nu$
$\pi^+\to\pi^0 e^+\nu$	$K\to\pi l^+\nu$	$B\to\pi l^+\nu$
$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	$\lvert V_{cb}\rvert$
$D^+\to l^+\nu$	$D_s^+\to l^+\nu$	$B_c^+\to\tau^+\nu$
$D\to\pi l^+\nu$	$D\to K l^+\nu$	$B\to\pi l^+\nu$
$\lvert V_{td}\rvert$	$\lvert V_{ts}\rvert$	$\lvert V_{tb}\rvert$
$B^0\to\pi^0 l^+ l^-$	$B^0\to K^0 l^+ l^-$	
$B^0\leftrightarrow\bar{B}^0$	$B_s^0\leftrightarrow\bar{B}_s^0$	

Kronfeld (Durham workshop) *(2019)*

Form factors $\langle \mathbf{1} | \mathcal{J} | \mathbf{1}' \rangle$

- **Example:** $f^{B\to\pi}(q^2)$



- **See new FLAG report/website for details**
- **Please cite original work** ***(each figure has a .bib)***

FLAG 2019 Flavour Lattice Averaging Group

Outline

- ☑ The LQCD landscape
  - ☐ Lattice basics
  - ☐ Nielson Ninomiya
  - ☐ Many actions
- ☑ Flavor physics
  - ☐ Single-hadron matrix elements
  - ☐ Light-flavor decay constants
  - ☐ Heavy-flavor decay constants
  - ☐ Mixing
  - ☐ Form factors
- ☐ QED + QCD
  - ☐ Theoretical challenge
  - ☐ Different formulations
- ☐ $(g-2)_\mu$
  - ☐ Light-by-light
  - ☐ HVP
- ☐ Multi-hadron processes
  - ☐ Finite-volume as a tool
  - ☐ Resonances
  - ☐ $2 \to 2$ scattering
  - ☐ $1 + \mathscr{J} \to 2$ transitions
- ☐ So much more!

Why QED + QCD?

- In our Universe, up and down quarks have different masses and electric charges
- Thus far we have been ignoring these effects

$$\langle \mathcal{O} \rangle_{[1+1+1,\ \alpha_{\text{QED}}=1/137]} \approx \langle \mathcal{O} \rangle_{[2+1,\ \alpha_{\text{QED}}=0]}$$

- This is expected to induce a percent-level systematic uncertainty

$$\alpha_{\text{QED}} \approx 0.7\,\% \qquad \frac{M_n - M_p}{M_n} \approx 0.1\%$$

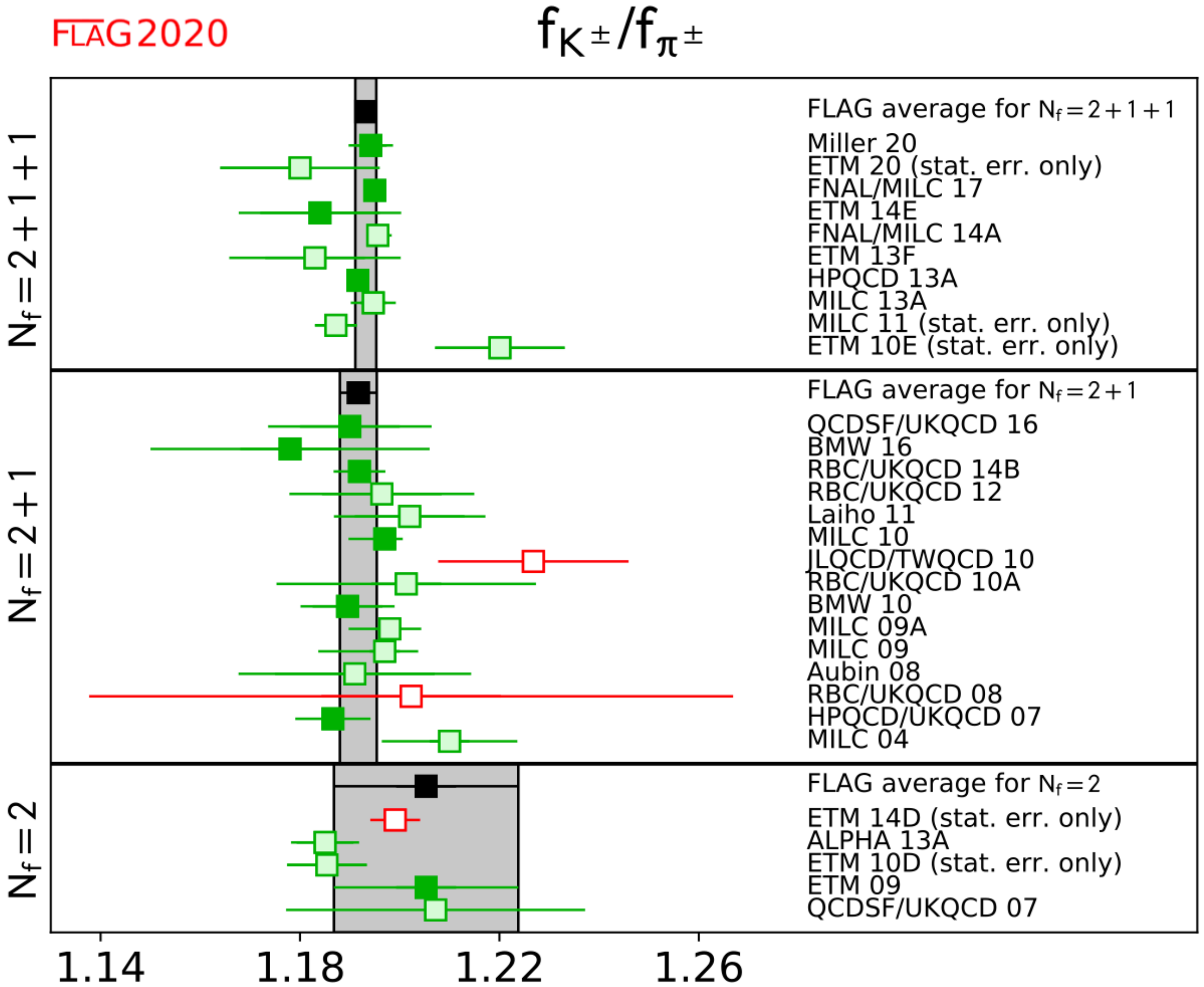
- But many LQCD observables have reached percent level determinations!

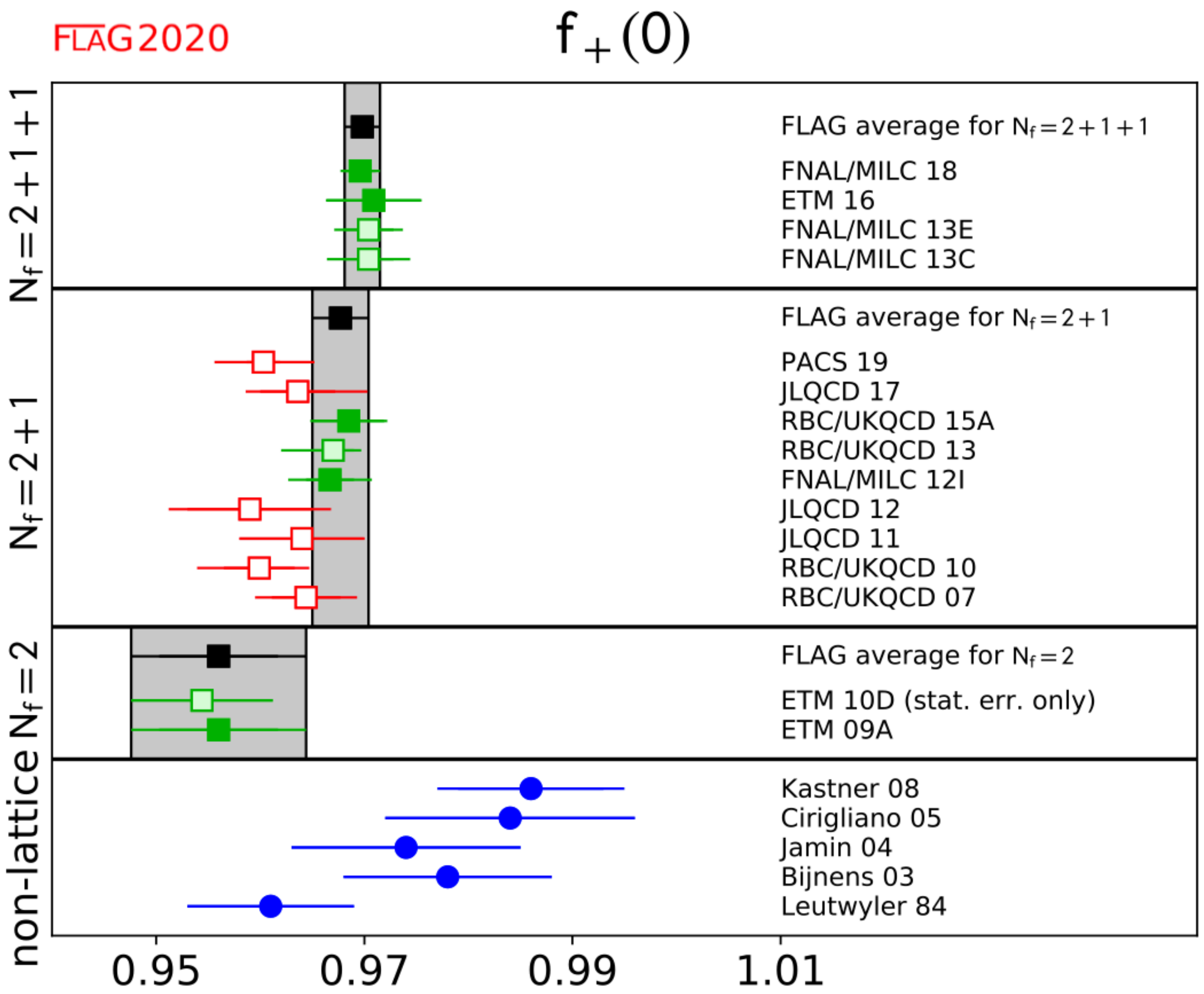
Precision era LQCD

$f_\pi = 130.2(8)$ MeV | uncertainty = 0.6% | $\delta_{\chi PT,\,\mathrm{QED}}(\pi^- \to \ell^-\bar{\nu}) = 1.8\,\%$

$f_K = 155.7(0.7)$ MeV | uncertainty = 0.5% | $\delta_{\chi PT,\,\mathrm{QED}}(K^- \to \ell^-\bar{\nu}) = 1.1\,\%$

$f_+(0) = 0.9698(17)$ | uncertainty = 0.2% | $\delta_{\chi PT,\,\mathrm{QED}}(K \to \pi\ell\bar{\nu}) = 0.5$ to $3\,\%$



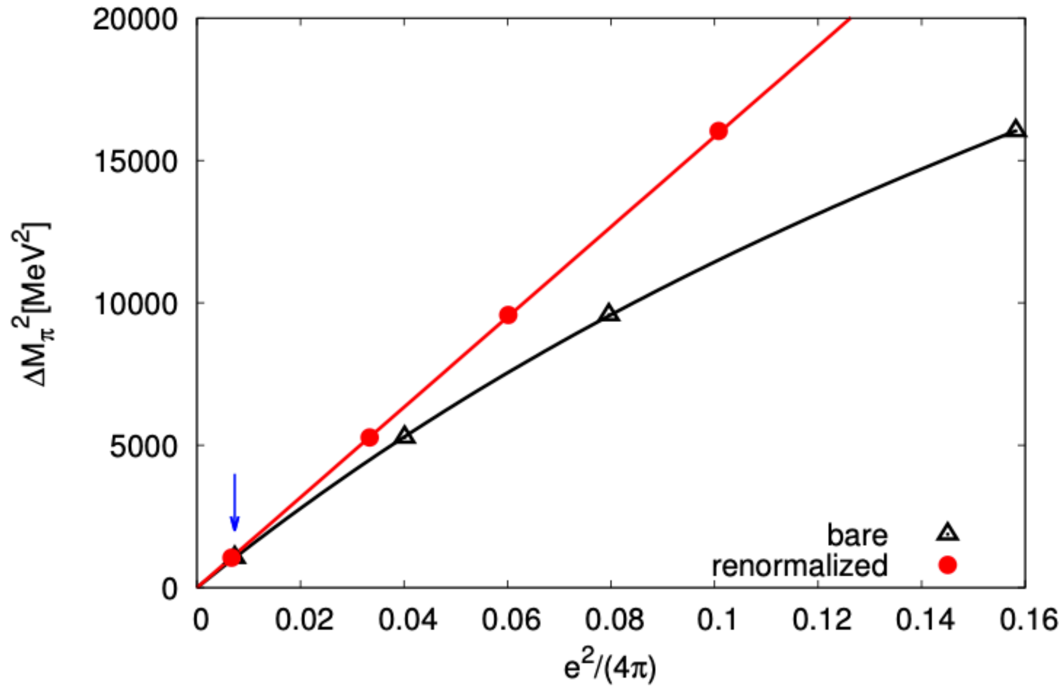


Two basic strategies for QED+QCD

- Simulate with α_{QED}, $m_u - m_d$

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S_{\text{QCD}} - S_{\text{QED+IB}}}$$

simplifies observables, but signal may be suppressed



Borsanyi et al., Science 347 (2015) 1452-1455

- Expand in α_{QED}, $m_u - m_d$

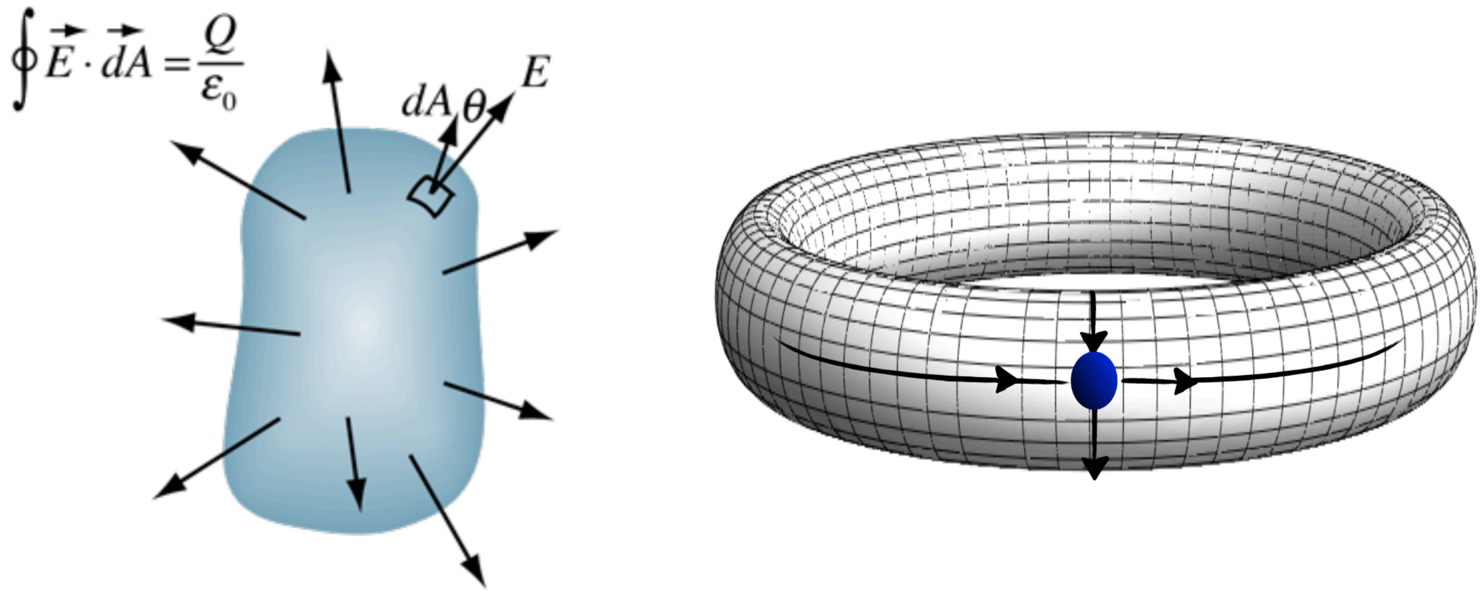
$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S_{\text{QCD}}} - \int \mathcal{D}\Phi \left[S_{\text{QED+IB}} \mathcal{O} \right] e^{-S_{\text{QCD}}} + \mathcal{O}(e^4)$$

signal is not suppressed, but observables are more complicated

QED in a box

Gauss's law prohibits a naive implementation of charged objects in a periodic volume

$$Q = \int d^3\boldsymbol{x}\, j_0(t,\boldsymbol{x}) = \int d^3\boldsymbol{x}\, \nabla \cdot \boldsymbol{E}(t,\boldsymbol{x}) = \int d\boldsymbol{S} \cdot \boldsymbol{E}(t,\boldsymbol{x}) = 0$$



Note: if expanding one can take infinite-volume QED

but putting QED and QCD in different spaces causes its own subtleties

• figures from A. Nicholson, GHP 2021 •

Many proposed methods

- Remove the global zero-mode of the gauge field ($\mathrm{QED_{TL}}$)
- Restrict the global zero-mode of the gauge field
- Remove the spatial zero-mode of the gauge field in each timeslice ($\mathrm{QED_L}$)
- Massive photon ($\mathrm{QED_M}$)
- $C^\star$ boundary conditions ($\mathrm{QED_C}$)

All equivalent if $L \to \infty$ before anything else
(before $a \to 0$, before $T \to \infty$, before $m_\gamma \to 0$, maybe before fitting)

Many proposed methods

- ❑ Remove the global zero-mode of the gauge field (QED_{TL})
- ❑ Restrict the global zero-mode of the gauge field
- ❑ Remove the spatial zero-mode of the gauge field in each timeslice (QED_L)
- ❑ Massive photon (QED_M)
- ❑ $C^\star$ boundary conditions (QED_C)

All equivalent if $L \to \infty$ before anything else
(before $a \to 0,$ before $T \to \infty,$ before $m_\gamma \to 0,$ maybe before fitting)

Don't mess with the global zero mode!

- Remove the global zero-mode of the gauge field (QED_{TL})

$$a_\mu = eL_\mu \int d^4x A_\mu(x) \overset{!}{=} 0$$

- Restrict the global zero-mode of the gauge field

$$-\pi \;<\; a_\mu \;<\; \pi$$

Both of these disrupt the transfer matrix! (i.e. the hamiltonian)

$$\int d^3\boldsymbol{x} \langle \psi(t,\boldsymbol{x})\overline{\psi}(0)\rangle \neq \sum_{n,m} C_{nm}(L) e^{-t(E_n - E_m)} e^{-TE_m}$$

Must send $L \to \infty$ before making use of spectral decomposition

Many proposed methods

- Remove the global zero-mode of the gauge field (QED_{TL})
- Restrict the global zero-mode of the gauge field
- Remove the spatial zero-mode of the gauge field in each timeslice (QED_L)
- Massive photon (QED_M)
- $C^\star$ boundary conditions (QED_C)

All equivalent if $L \to \infty$ before anything else
(before $a \to 0,$ before $T \to \infty,$ before $m_\gamma \to 0,$ maybe before fitting)

$\mathrm{QED_L}$

$$\int d^3\boldsymbol{x}\, A_\mu(x) \stackrel{!}{=} 0$$

$\mathrm{QED_L}$ has a transfer matrix. It is a nonlocal prescription. Locality is a core property of QFT, it is a fundamental assumption behind

- Renormalizability by power counting
- Volume-independence of renormalization constants
- Operator product expansion
- Effective-theory description of long-distance physics
- Symanzik improvement program
- ...

Infinite-volume limit should be taken before the continuum limit.

• slide from A. Patella, Lattice 2016 plenary •

QED$_L$

$$\int d^3\boldsymbol{x}\, A_\mu(x) \stackrel{!}{=} 0$$

- In matrix elements of high-dimension operators one expects terms scaling as

$$\frac{1}{aL^n}$$

Vanishes if $L \to \infty$ at fixed a,
diverges if $a \to 0$ at fixed L

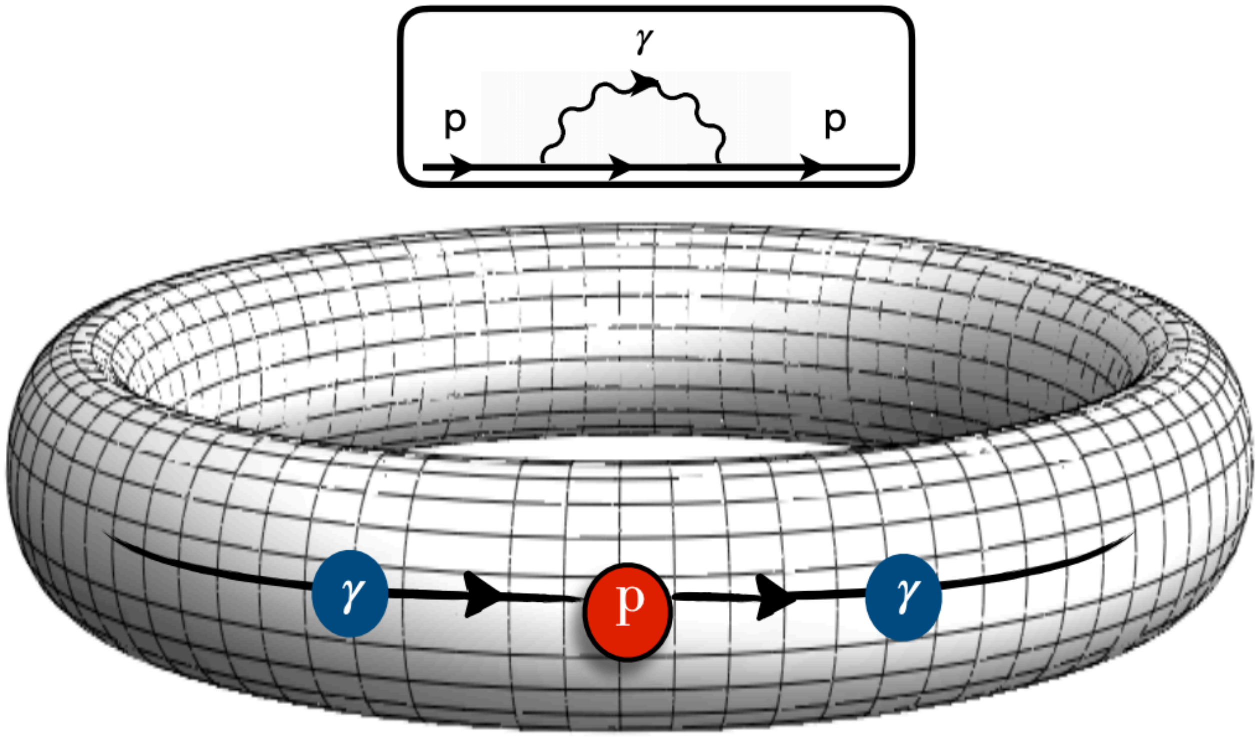
- No specific evidence of a problem for quantities currently being calculated
- Failure of NREFT = important lesson
- Would be very interesting to improve understanding of these issues

Practical consequence of QED$_L$ = modified Feynman rules in calculating volume effects

$$\frac{1}{L^3}\sum_{\boldsymbol{k}} \longrightarrow \frac{1}{L^3}\sum_{\boldsymbol{k}\neq\boldsymbol{0}}$$

QED$_M$

- Combine Landau gauge + mass term for the photon
- Gauge invariance is broken, but in a controlled way



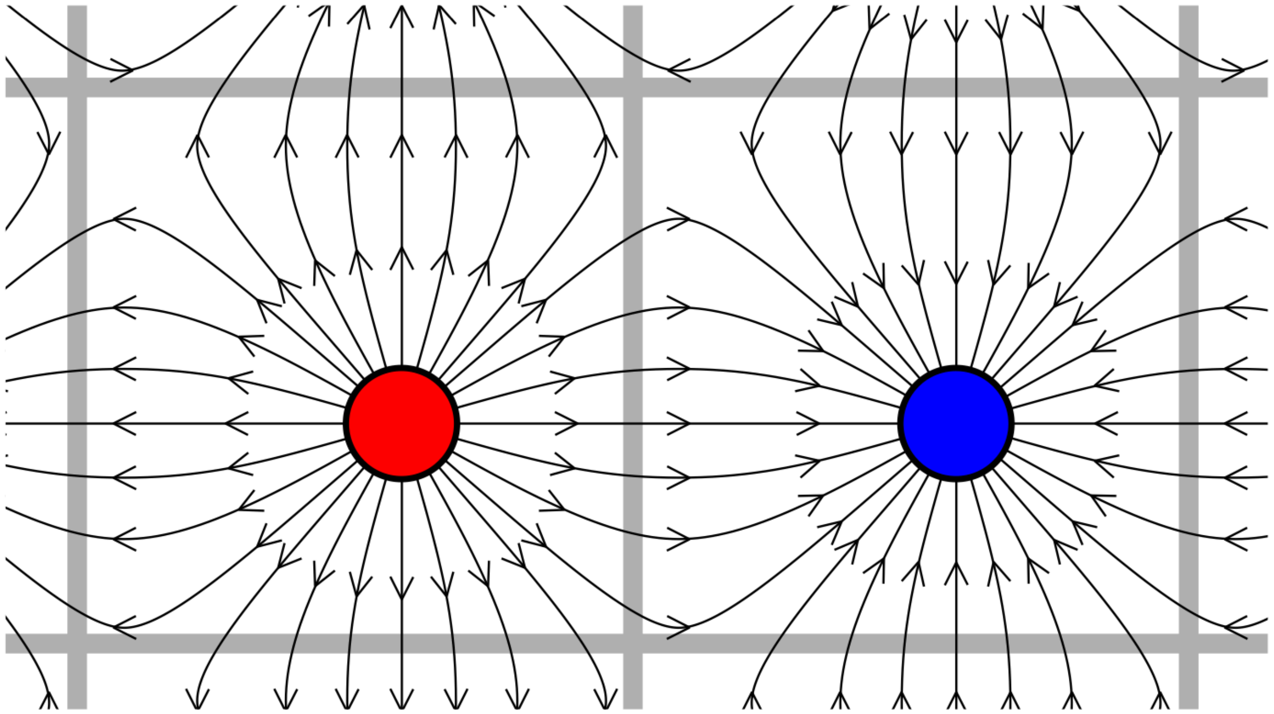
$$\delta_{\mathrm{FV}} \sim e^{-m_\gamma L}$$

• Endres et al., arXiv:1507.08916, (2015) • *figure from A. Nicholson* •

QED$_C$

- Use charge-conjugation-like boundary conditions

$$A_\mu(x + L\hat{\boldsymbol{e}}_i) = -A^*_\mu(x)$$
$$\psi(x + L\hat{\boldsymbol{e}}_i) = C^{-1}\overline{\psi}^T(x)$$



- Still power-like volume effects, but leading non-universal term is removed

- Exponentially suppressed flavor mixing

• Wiese (1992) • Polley (1993) • Lucini *et al.* (2016) • *figure from A. Patella* •

QCD+QED observables

- Masses and mass splittings
- Meaning of decay constants
  - Pure QCD $$\Gamma(K^- \to \ell^- \overline{\nu}_\ell) = \frac{G_F^2 |V_{us}|^2 f_K^2}{8\pi} m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2$$
  - QCD + QED (GRS scheme) $$\Gamma(K^- \to \mu^- \overline{\nu}_\mu [\gamma]) = (1.0032 \pm 0.0011)\Gamma^{(0)}(K^- \to \mu^- \overline{\nu}_\mu)$$

C. Sachrajda *(Durham flavour workshop)* • Di Carlo et al. (2020)

- Different soft scales for different particles
  - Well-understood for pions and kaons
  - *B* and *D* = different soft scale → **requires theory developments**

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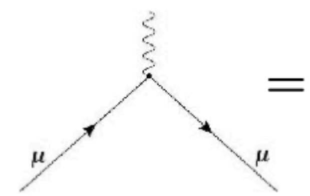
$(g-2)_\mu$ general

Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

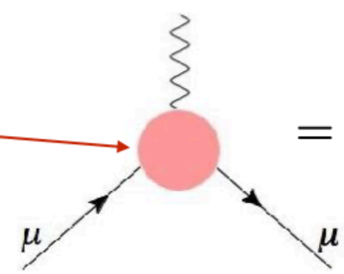
Dirac (leading order): $g = 2$



$= (-ie)\,\bar{u}(p')\gamma^\mu u(p)$

Quantum effects (loops):

All SM particles contribute



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

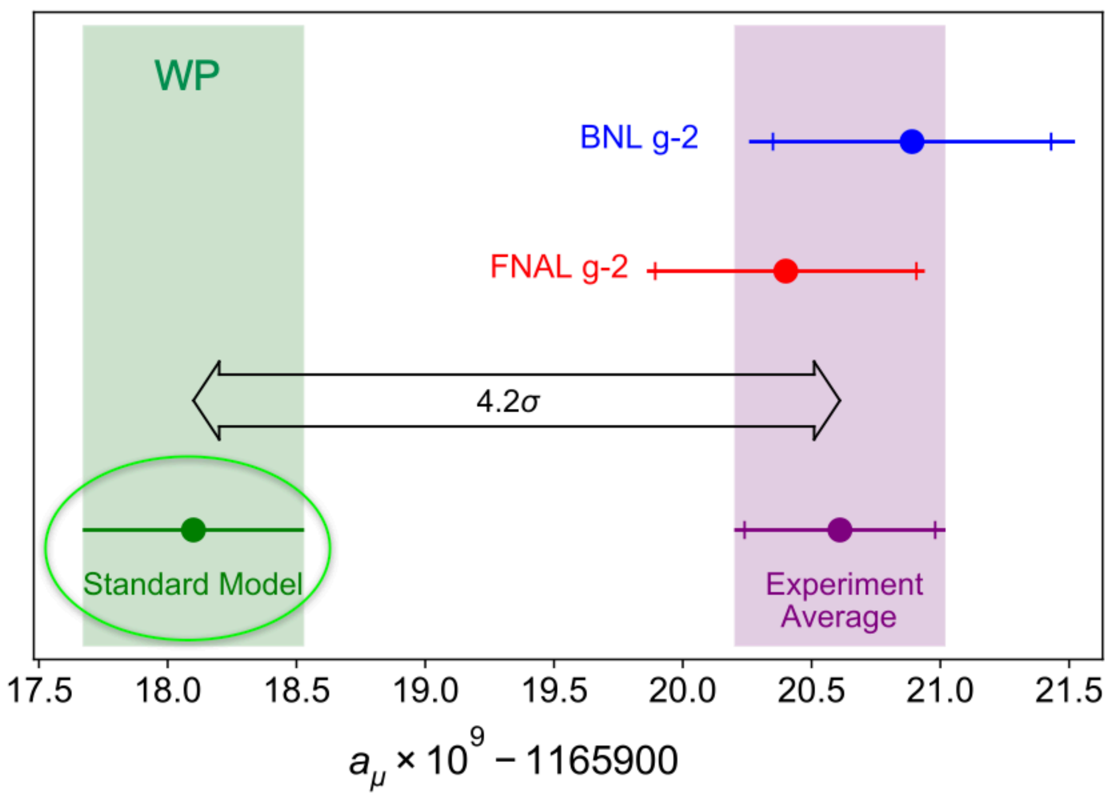
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$

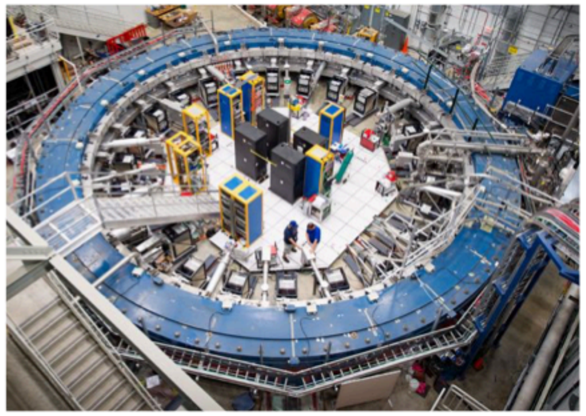
Slides from Aida El-Khadra, Lattice 2021

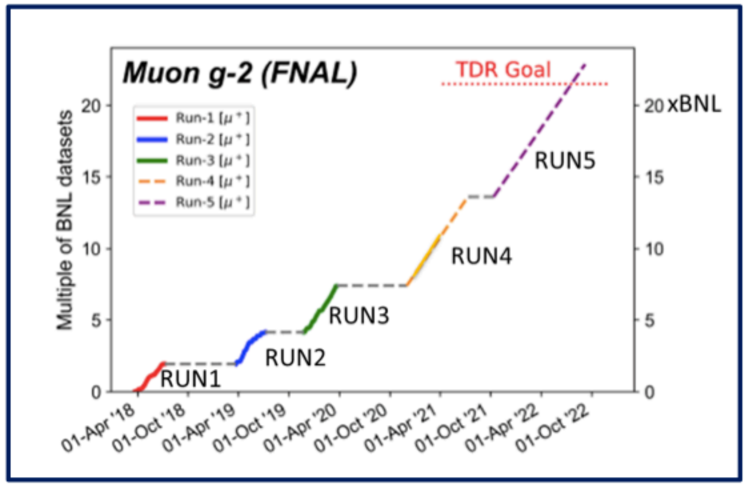
$(g-2)_\mu$ general

Muon g-2: experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021. [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Analysis of runs 2 and 3 is now underway.







Slides from Aida El-Khadra, Lattice 2021

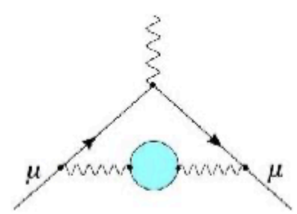
$(g-2)_\mu$ general

Muon g-2: SM contributions

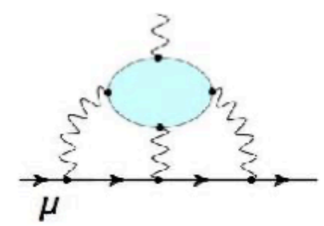
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

leading hadronic

α^2



α^3



✦ The hadronic contributions are written as:

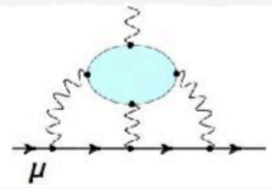
$$a_\ell(\text{hadronic}) = a_\ell^{\text{HVP, LO}} + a_\ell^{\text{HVP, NLO}} + a_\ell^{\text{HVP, NNLO}} + \ldots$$
$$+ a_\ell^{\text{HLbL}} + a_\ell^{\text{HLbL, NLO}} + \ldots$$

α^2 α^3 α^4

$\sim \quad 10^{-7}$

Slides from Aida El-Khadra, Lattice 2021

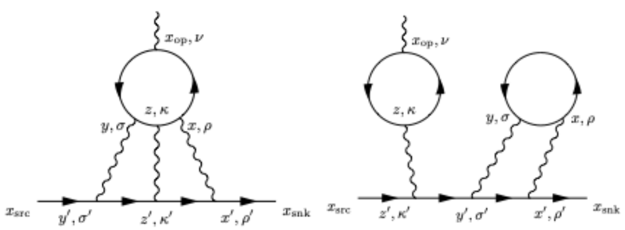
$(g-2)_\mu$ light-by-light

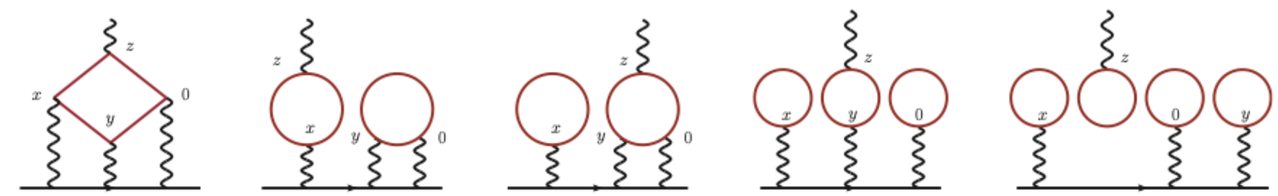


HLbL: lattice

Hadronic light-by-light: Target: ≲ 10% total error

Two independent and complete direct lattice calculations of a_μ^{HLbL}





- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + $\mathrm{QED_L}$ (finite volume)
  ➠ $1/L^2$ FV effects
  stochastic evaluation of position space sums
  Feynman gauge photon propagators
  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm},\ L \sim 4.5 - 9.3\,\mathrm{fm}$

- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum)
  ➠ $e^{-m_\pi L}$ FV effects
  semi-analytic QED kernel function

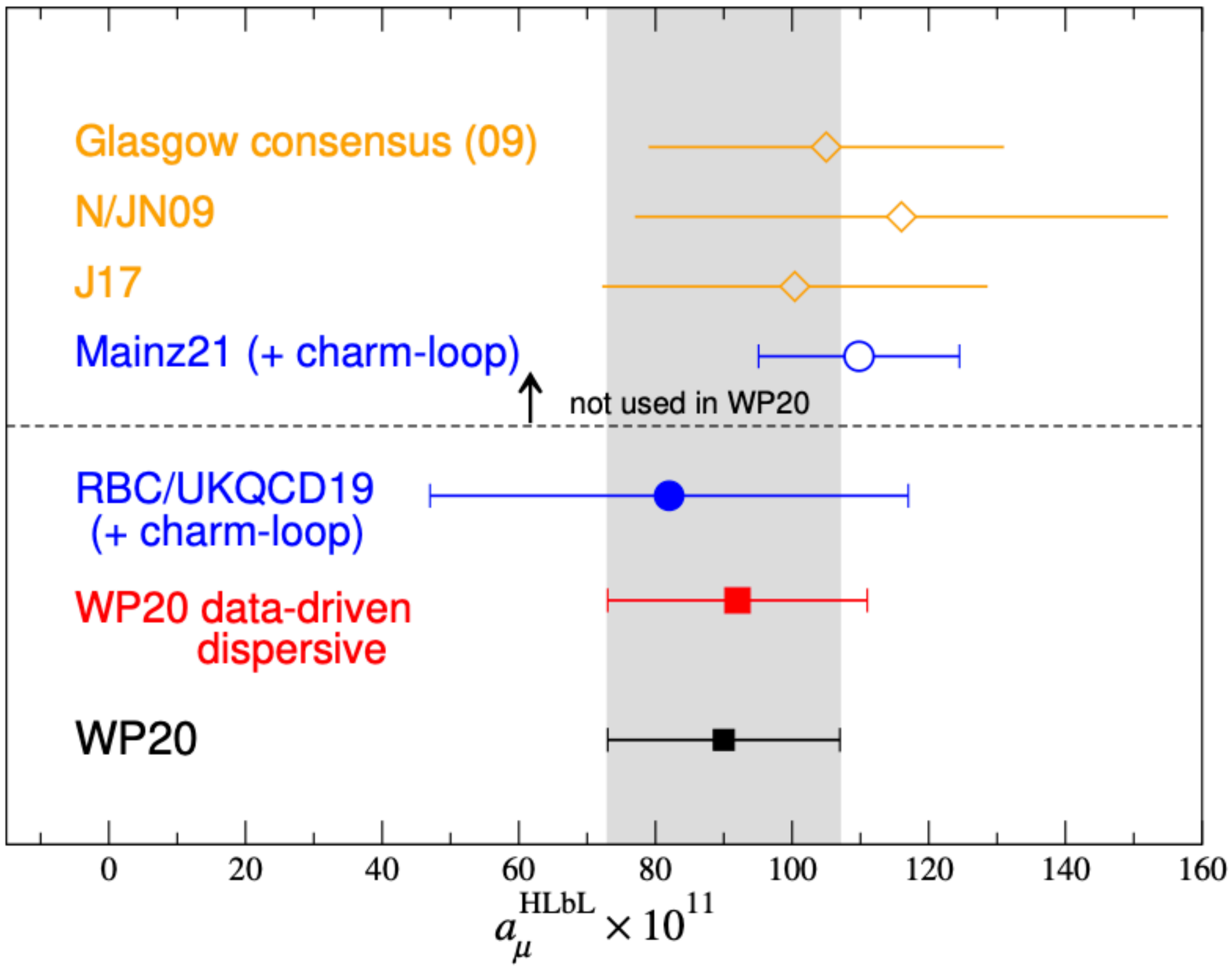
  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\ \mathrm{MeV},\ a \approx 0.05 - 0.1\,\mathrm{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass

Slides from Aida El-Khadra, Lattice 2021

$(g-2)_\mu$ light-by-light

a_μ^{HLbL}



models of QCD +EFT, large N_c

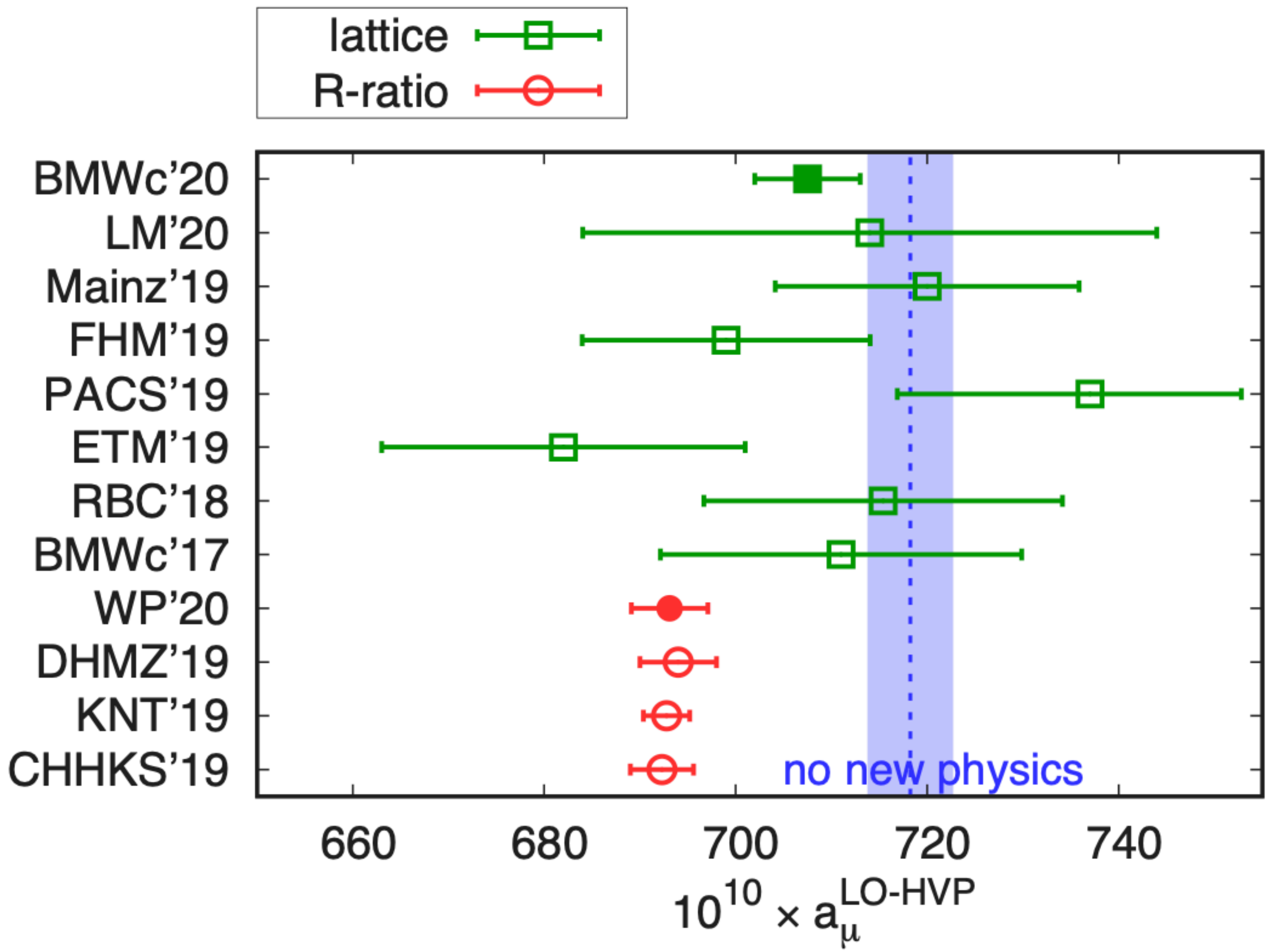
Lattice QCD + QED

data + dispersive approach

Now well-determined in two independent approaches, systematically improvable

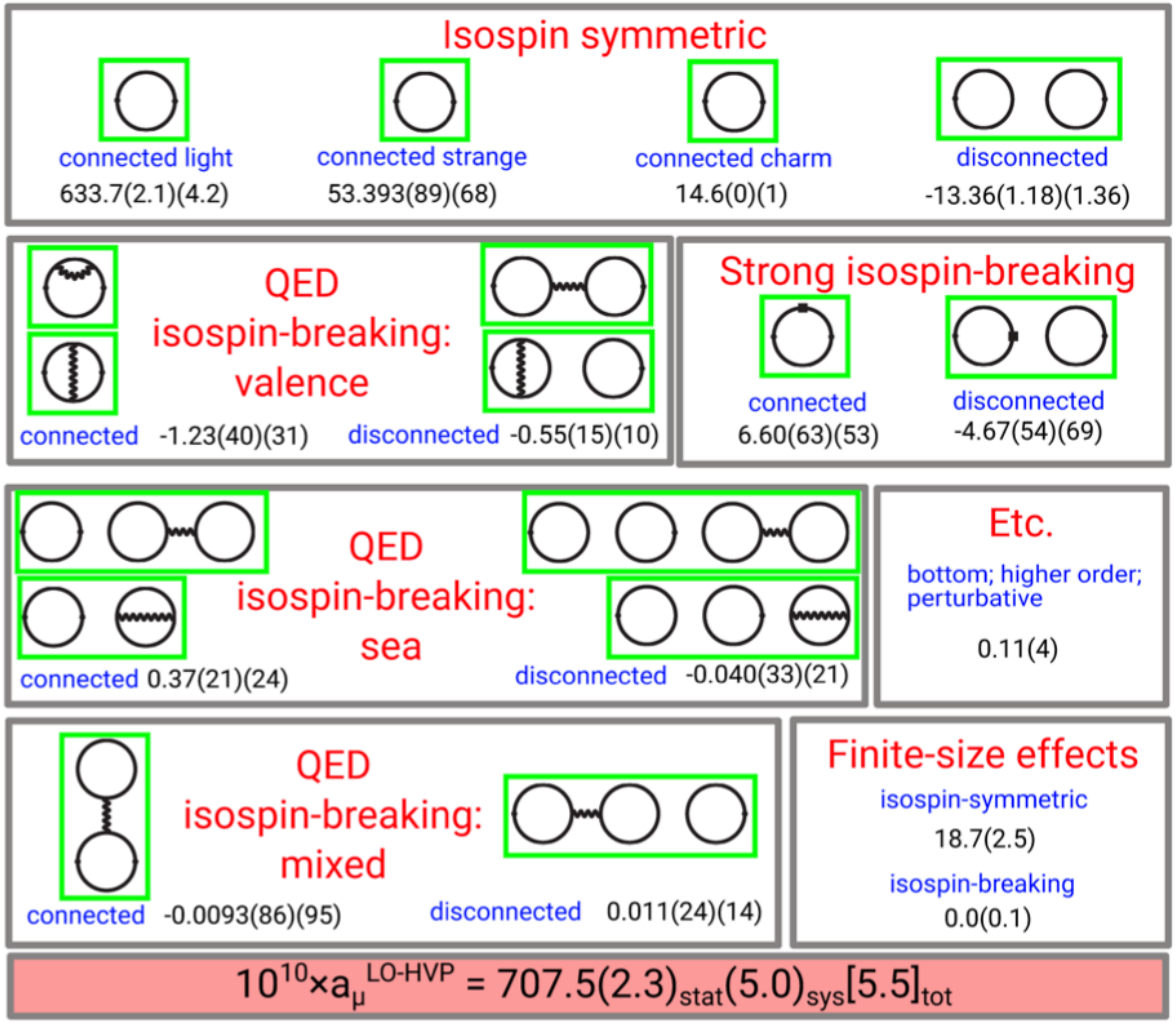
Slides from Aida El-Khadra, Lattice 2021

$(g-2)_\mu$ HVP



Slides from B.C. Toth, Lattice 2021

$(g-2)_\mu$ HVP



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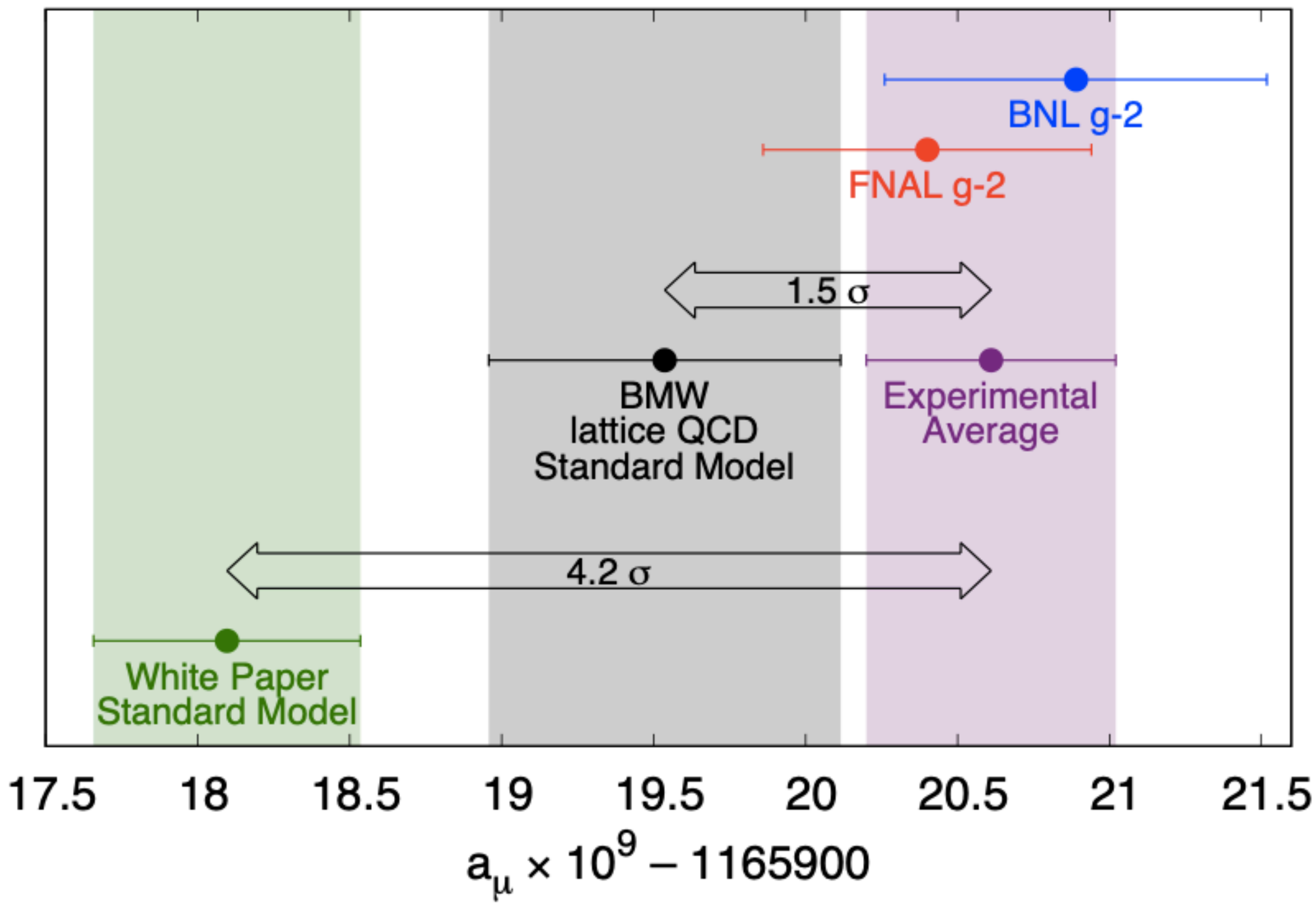
$(g-2)_\mu$ HVP



Slides from B.C. Toth, Lattice 2021

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Multi-hadron lattice quantities

- 'On the lattice' we calculate ***finite-volume energies*** and ***matrix elements***

$$\langle \mathcal{O}_j(\tau)\mathcal{O}_i^\dagger(0)\rangle = \sum_n \langle 0|\mathcal{O}_j(\tau)|E_n\rangle\langle E_n|\mathcal{O}_i^\dagger(0)|0\rangle = \sum_n e^{-E_n(L)\tau} Z_{n,j} Z^*_{n,i}$$

- Determine ***optimized operators*** by diagonalizing correlator matrix (GEVP)

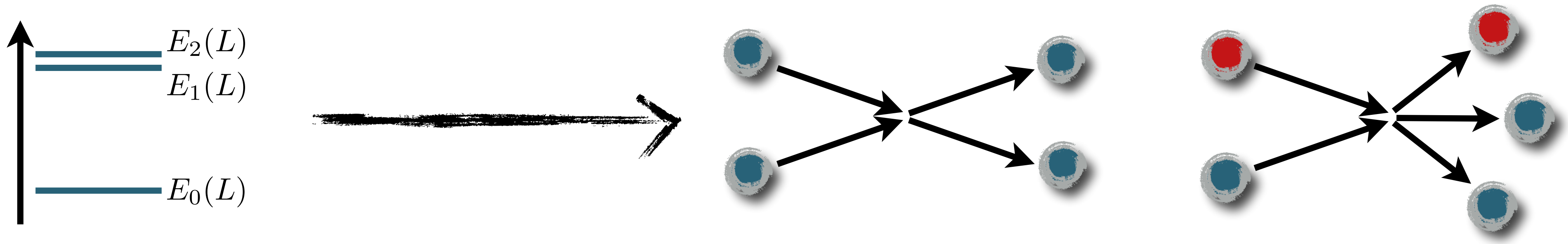
$$\big\langle \Omega_m(\tau)\Omega_m^\dagger(0)\big\rangle \sim e^{-E_m(L)\tau} + \cdots$$
$$\big\langle \Omega_{m'}(\tau)\ \mathcal{J}(0)\ \Omega_m^\dagger(-\tau)\big\rangle \sim e^{-E_{m'}\tau}\ e^{-E_m\tau}\ \langle E_{m'}|\mathcal{J}(0)|E_m\rangle + \cdots$$

- Our task is relate $E_n(L)$ and $\langle E_{m'}|\mathcal{J}(0)|E_m\rangle$ to experimental observables

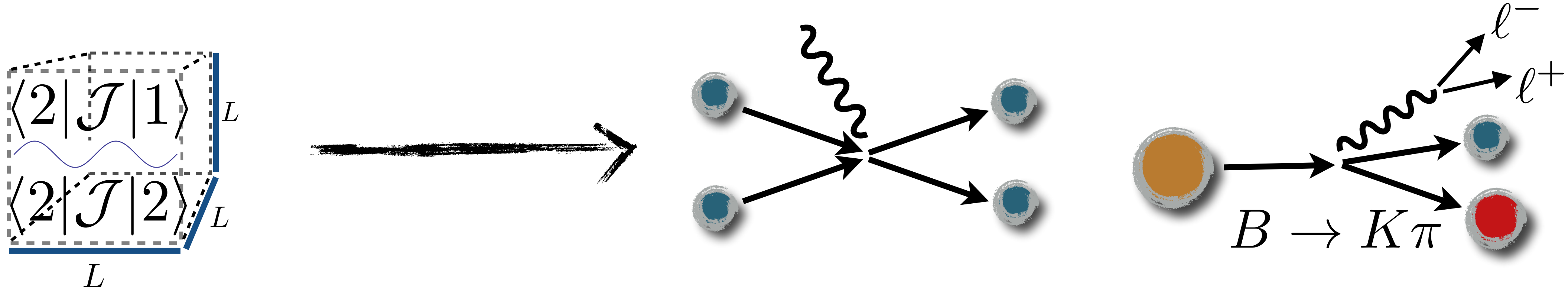
Multi-hadron processes from LQCD

Use the finite volume as a tool to extract multi-hadron observables

- Scattering (from finite-volume energies)



- Transitions (from finite-volume energies + matrix elements)



Multi-hadron processes from LQCD

Use the finite volume as a tool to extract multi-hadron observables

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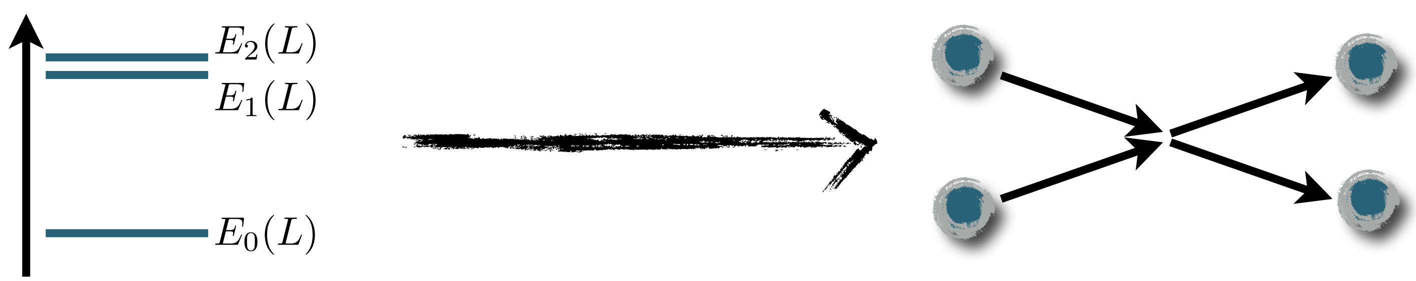

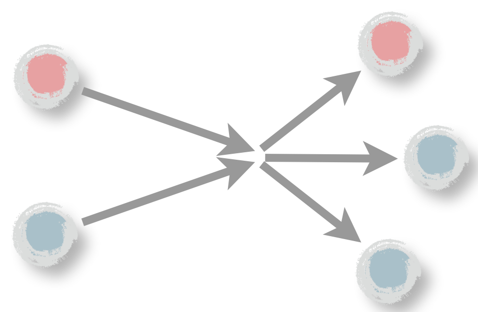

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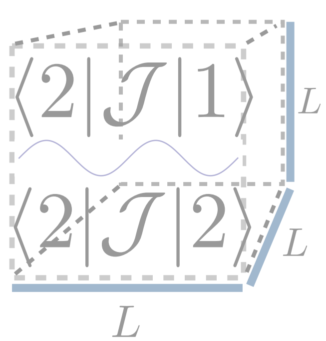
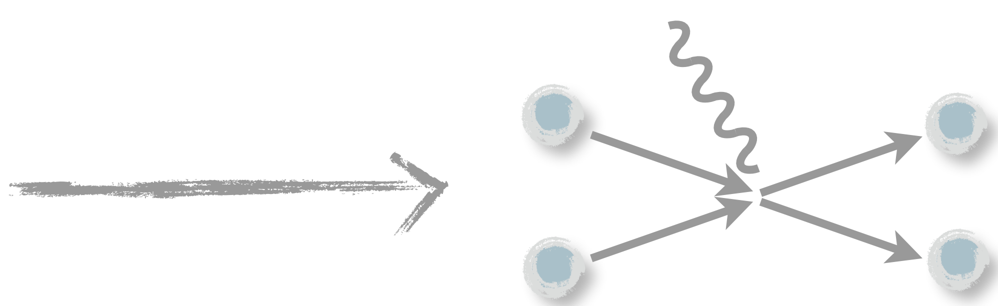
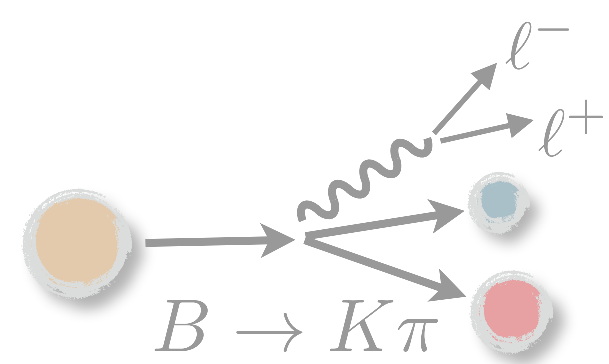
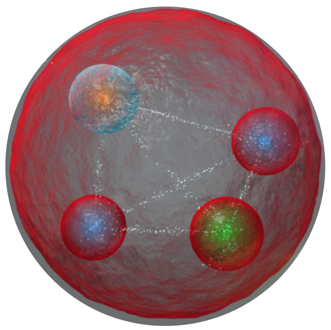






Multi-hadron observables

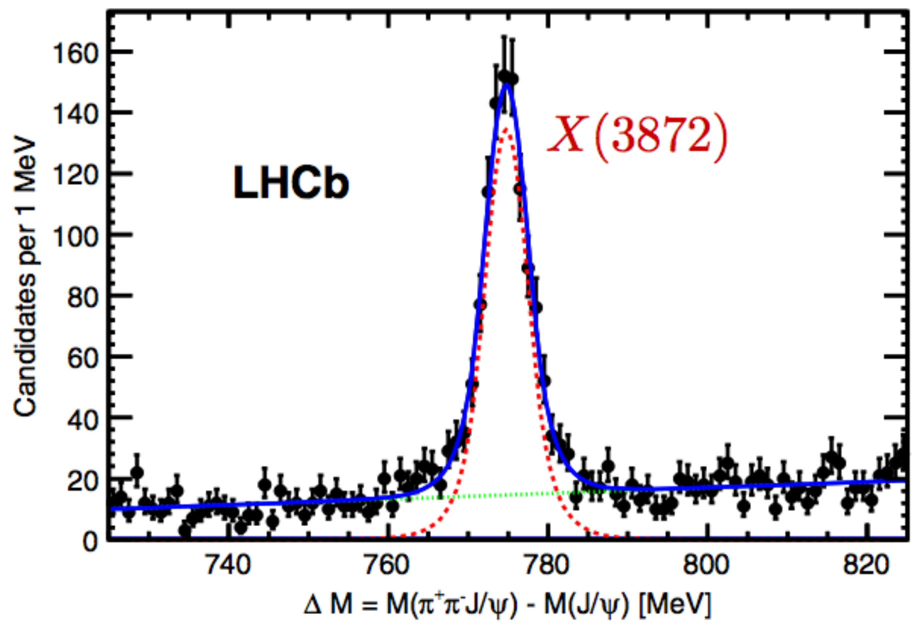
- Exotics, XYZs, tetra- and penta-quarks, *H* dibaryon



e.g. X(3872)

$$\sim |D^0\overline{D}^{*0} + \overline{D}^0 D^{*0}\rangle\ ?$$

• LHCb (PRD92, 2015) •



- Eletroweak, CP violation, resonant enhancement

CP violation in charm $D \to \pi\pi, K\overline{K}$

• LHCb (PRL, 2019) •

$f_0(1710)$ could enhance ΔA_{CP}

• Soni (2017) •

Resonant B decays $B \to K^* \ell\ell \to K\pi\, \ell\ell$

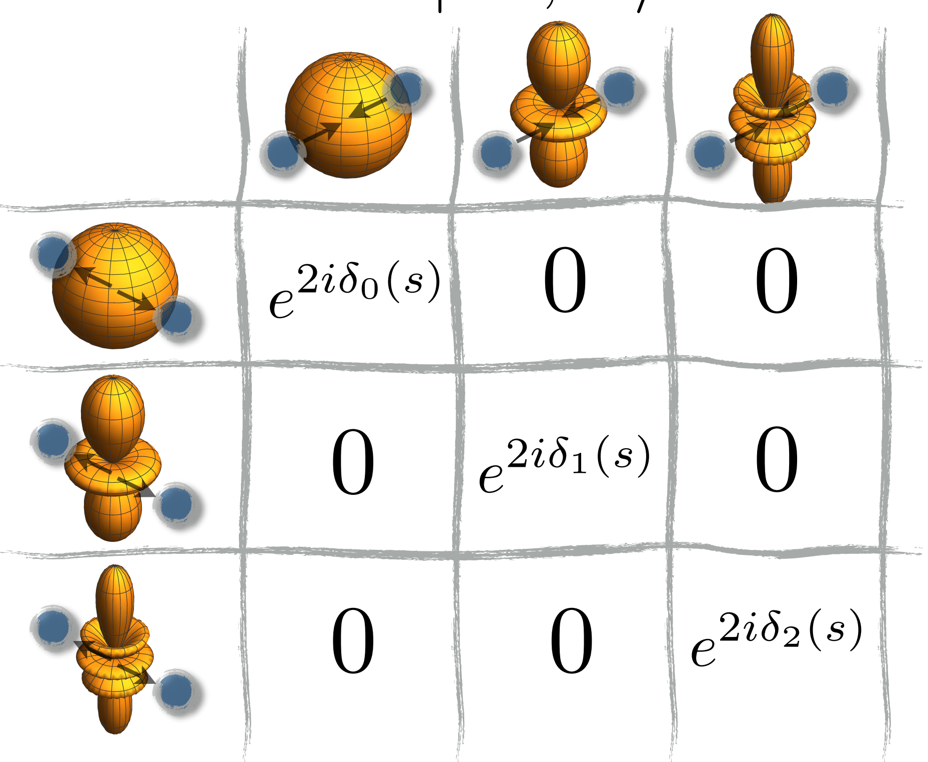
$|X\rangle, |\rho\rangle, |K^*\rangle, |f_0\rangle \notin$ **QCD Fock space**

QCD Fock space

- At low-energies QCD = hadronic degrees of freedom $\pi \sim \bar{u}d, \;\; K \sim \bar{s}u, \;\; p \sim uud$
- Overlaps of multi-hadron ***asymptotic states*** → S matrix

$|\pi\pi, \mathrm{in}\rangle$

$S(s) \equiv \langle \pi\pi, \mathrm{out}|$



depends on $s = E_{\mathrm{cm}}^2$
and angular variables

diagonal in angular momentum

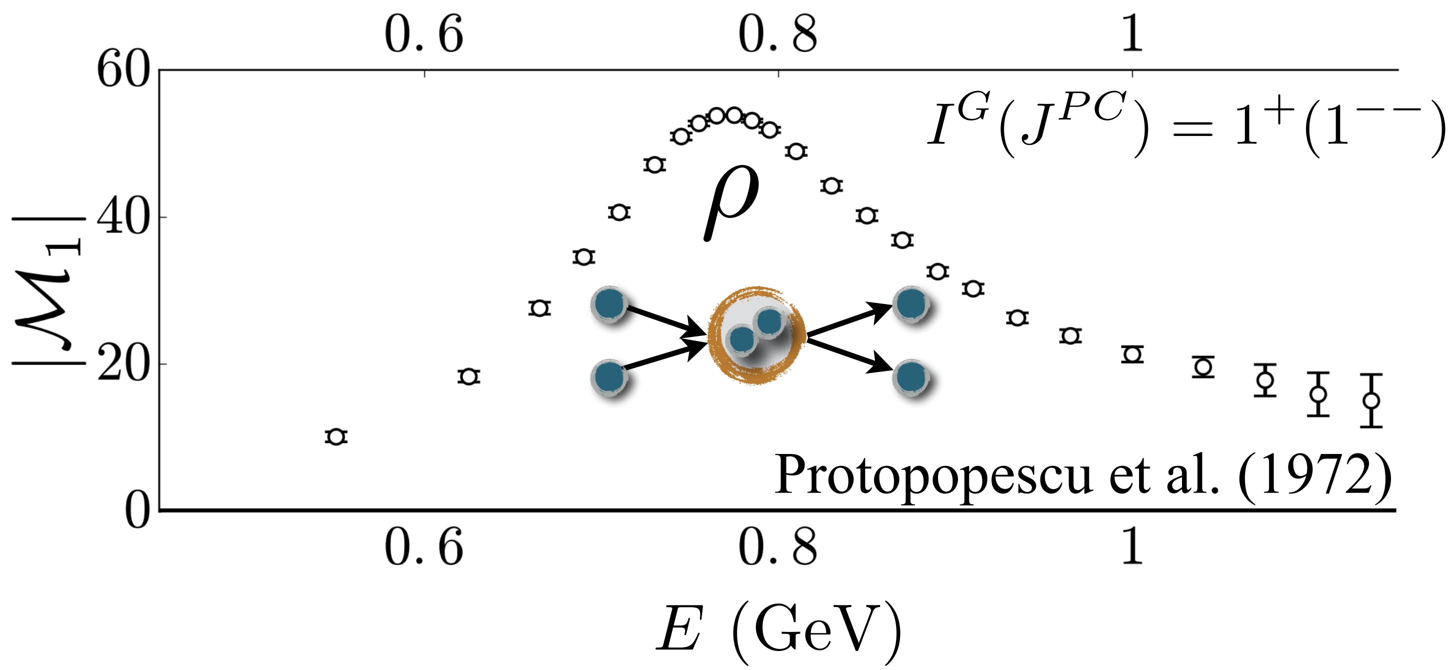
$$\mathcal{M}_\ell(s) \propto e^{2i\delta_\ell(s)} - 1$$

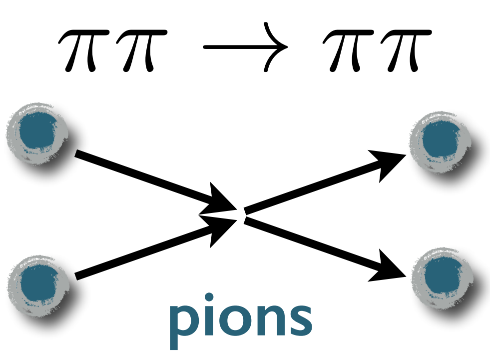
- ***An enormous space of information*** $|\pi\pi\pi\pi, \mathrm{in}\rangle \quad |K\overline{K}, \mathrm{in}\rangle \quad \cdots$

QCD resonances

- Roughly speaking, a bump in: $|\mathcal{M}_\ell(s)|^2 \propto |e^{2i\delta_\ell(s)} - 1|^2 \propto \sin^2 \delta_\ell(s)$

scattering rate

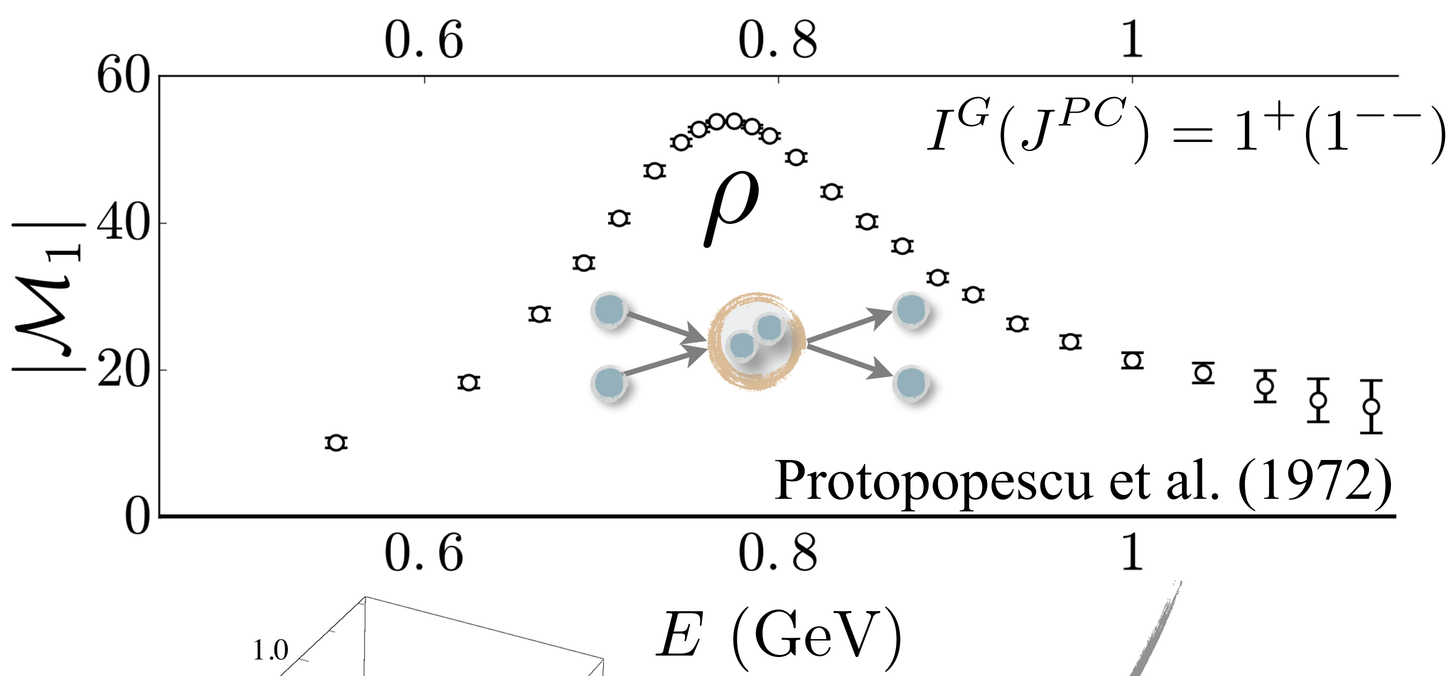


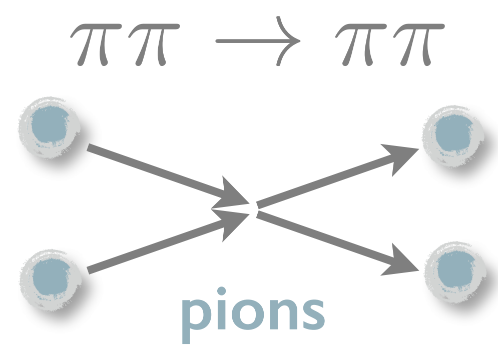


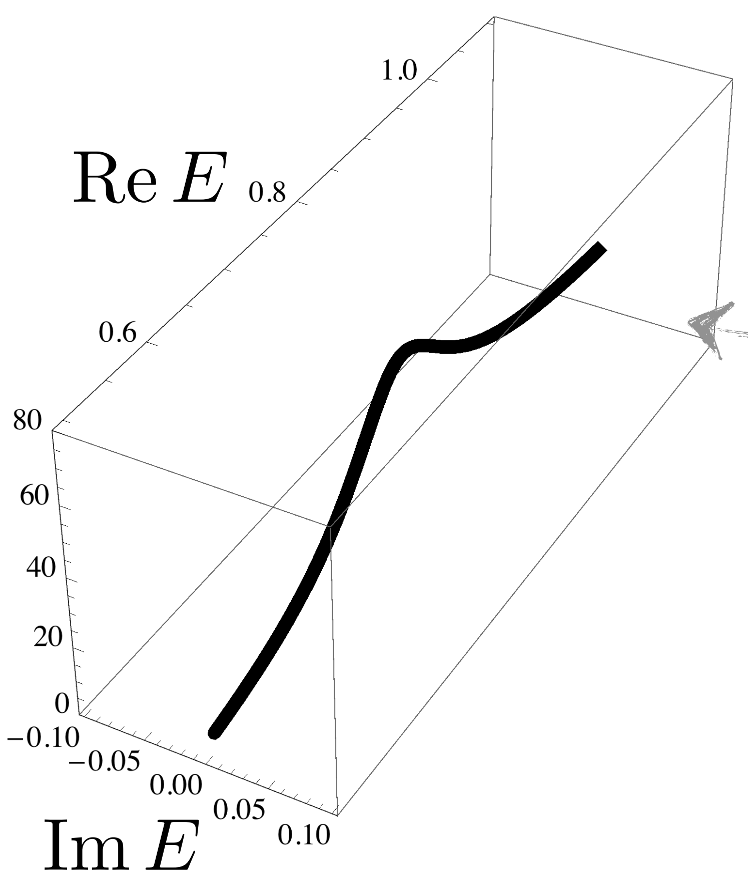
QCD resonances

- Roughly speaking, a bump in: $|\mathcal{M}_\ell(s)|^2 \propto |e^{2i\delta_\ell(s)} - 1|^2 \propto \sin^2 \delta_\ell(s)$

scattering rate





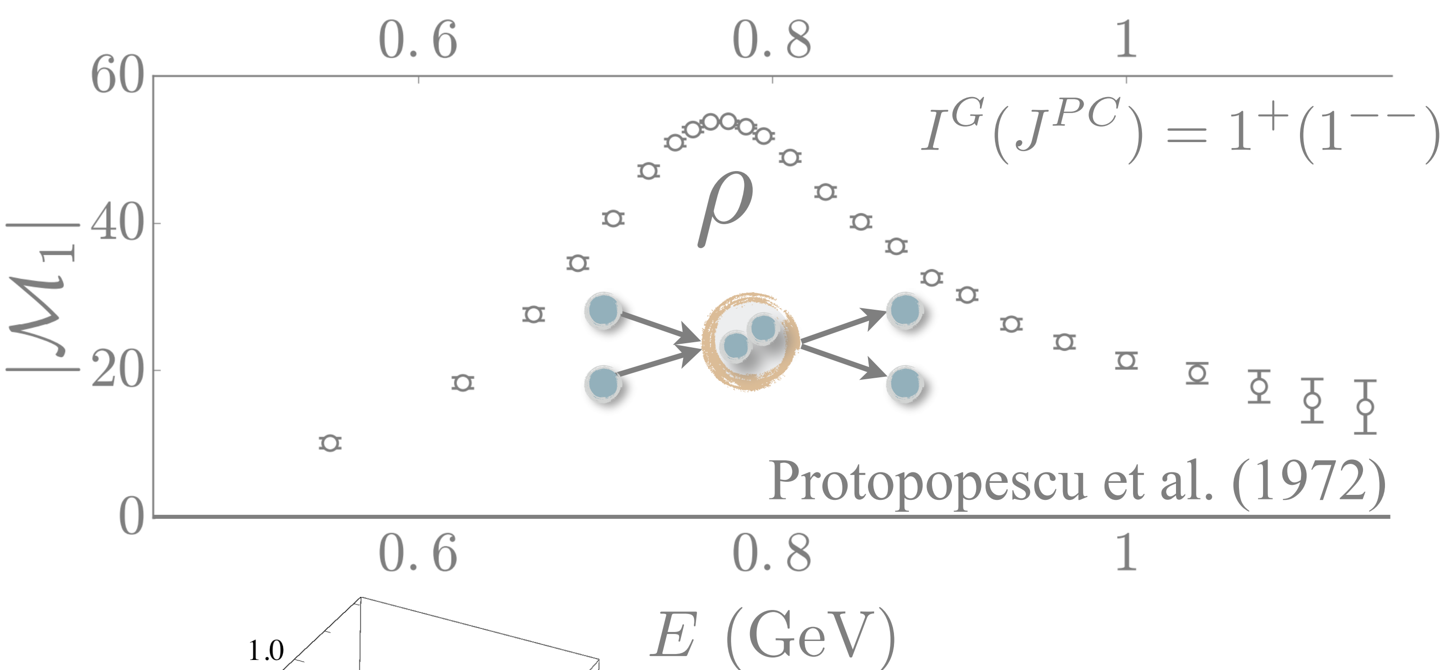


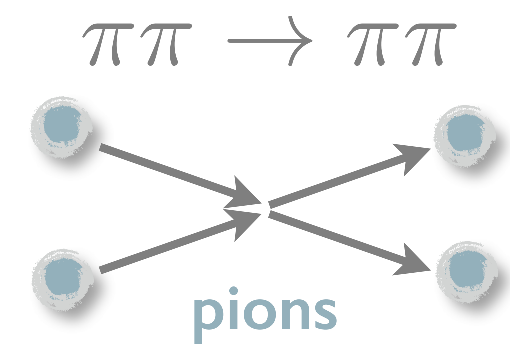
Extend to the
complex plane

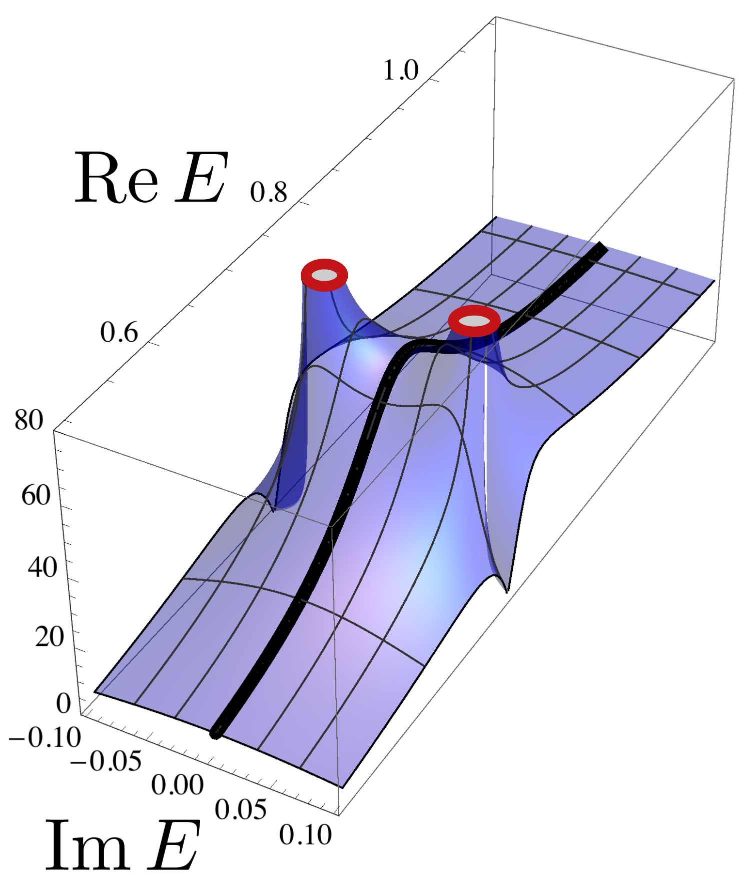
QCD resonances

- Roughly speaking, a bump in: $|\mathcal{M}_\ell(s)|^2 \propto |e^{2i\delta_\ell(s)} - 1|^2 \propto \sin^2 \delta_\ell(s)$

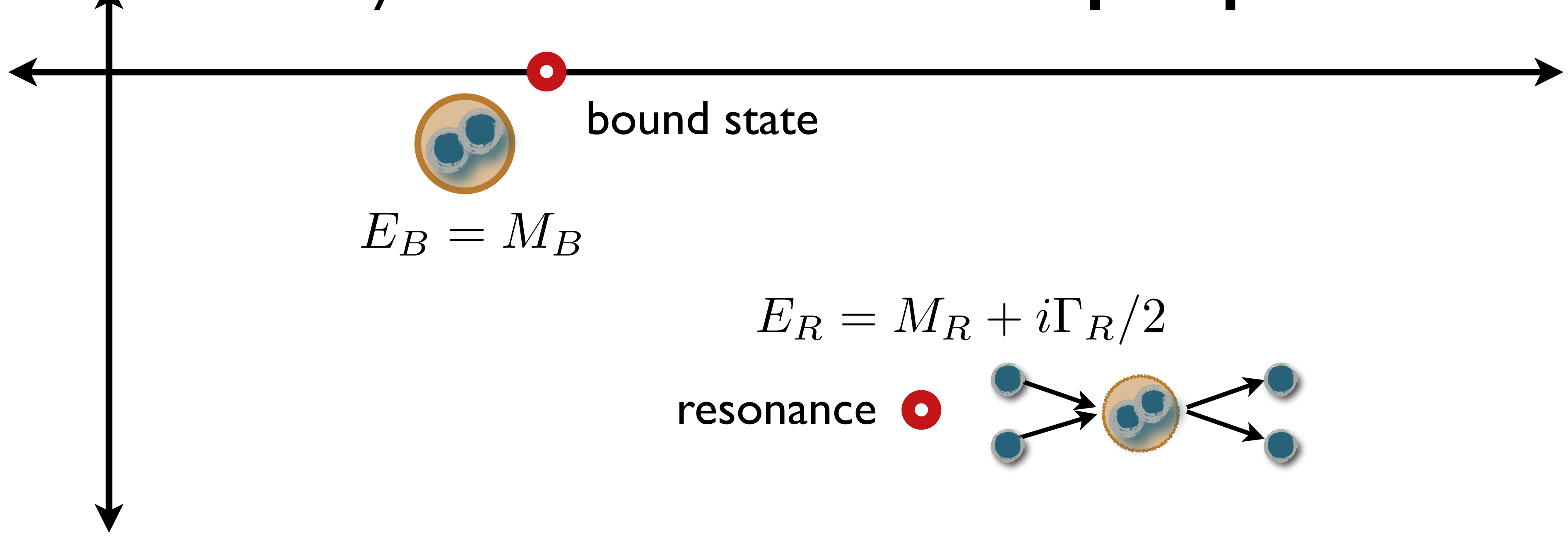
scattering rate







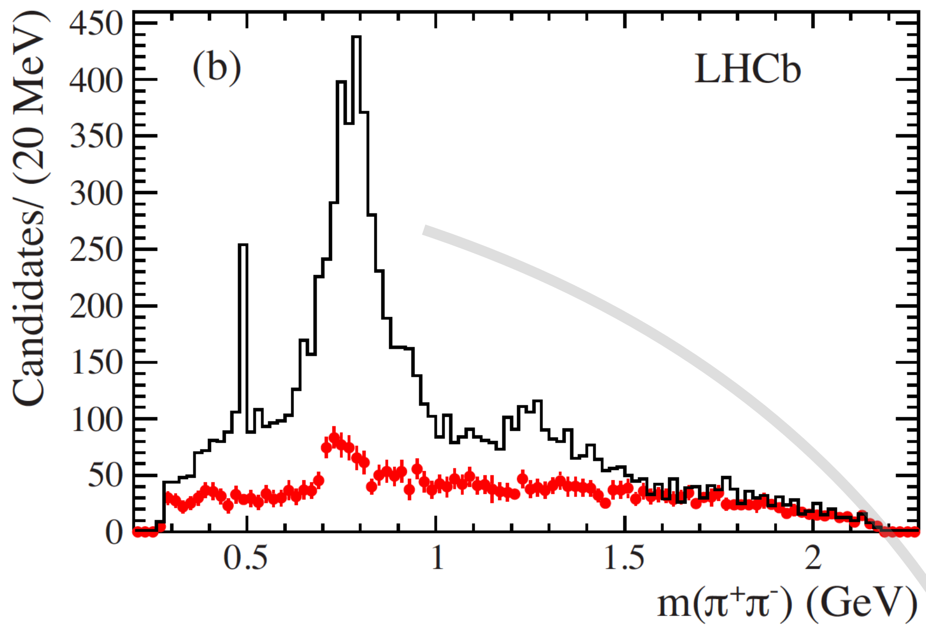
Analytic continuation reveals a **complex pole**



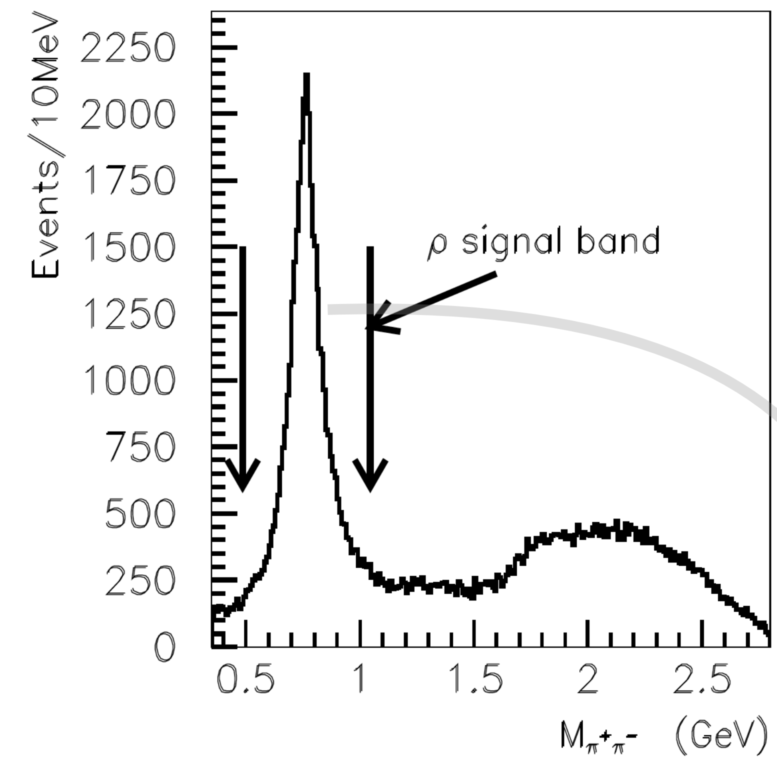
Pole is universal

- Resonances often seen in “production”

$\overline{B}^0 \to J/\psi\,\pi^+\pi^-$

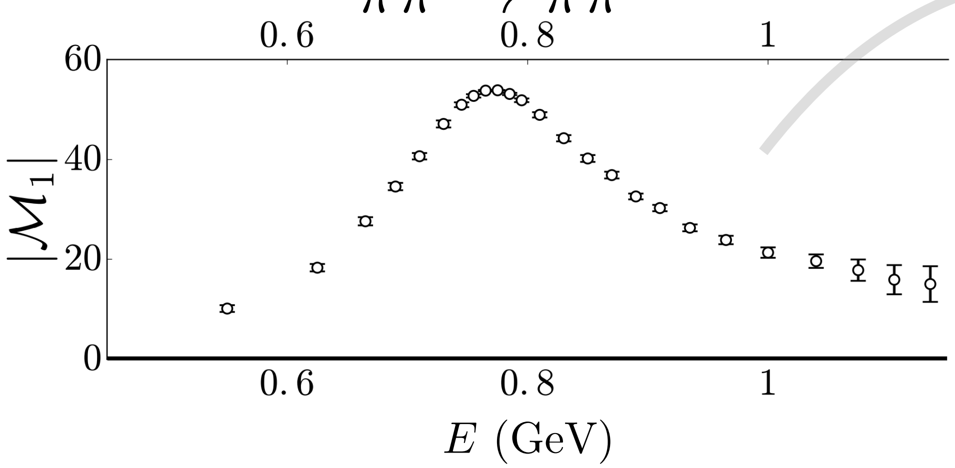


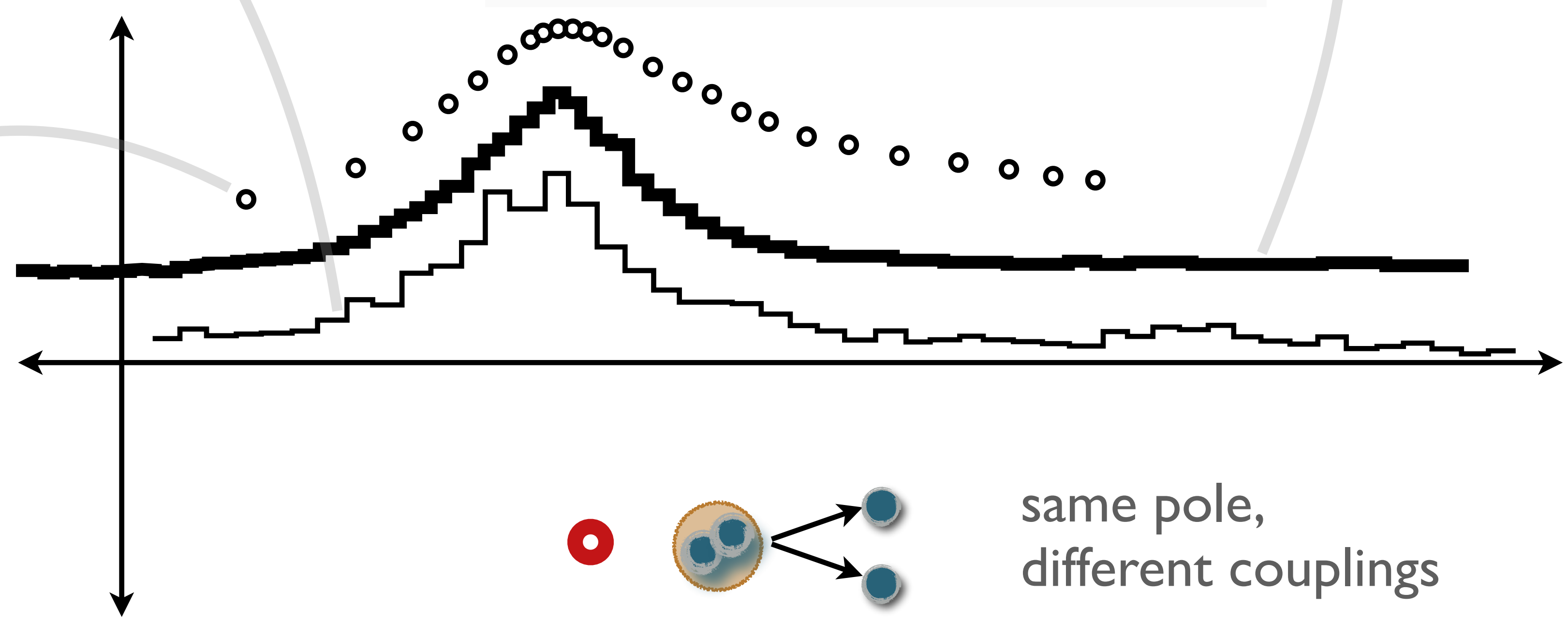
$J/\psi \to \gamma\gamma\rho$



(as opposed to scattering)

$\pi\pi \to \pi\pi$

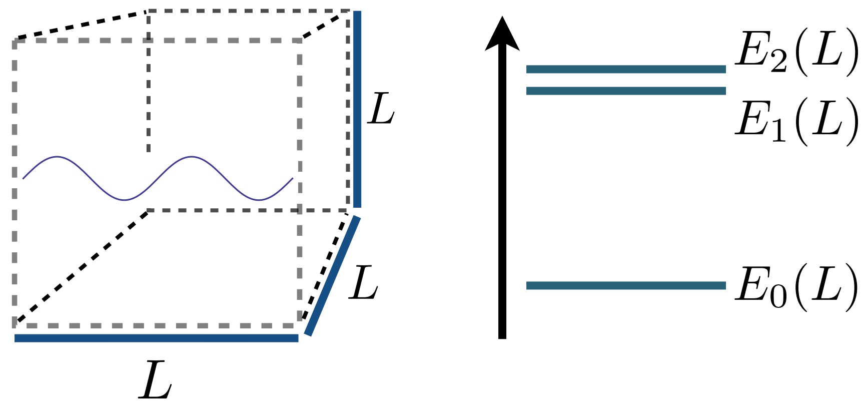




same pole,
different couplings

The finite-volume as a tool

- Finite-volume set-up

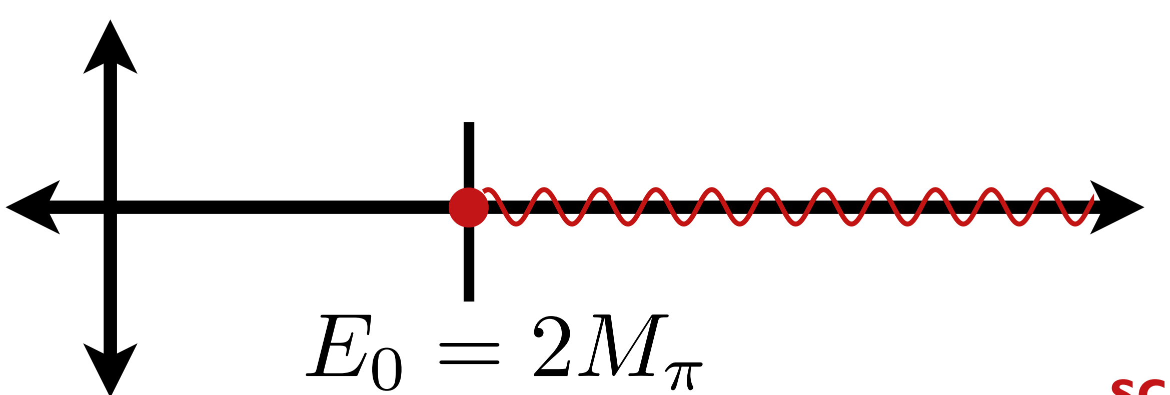


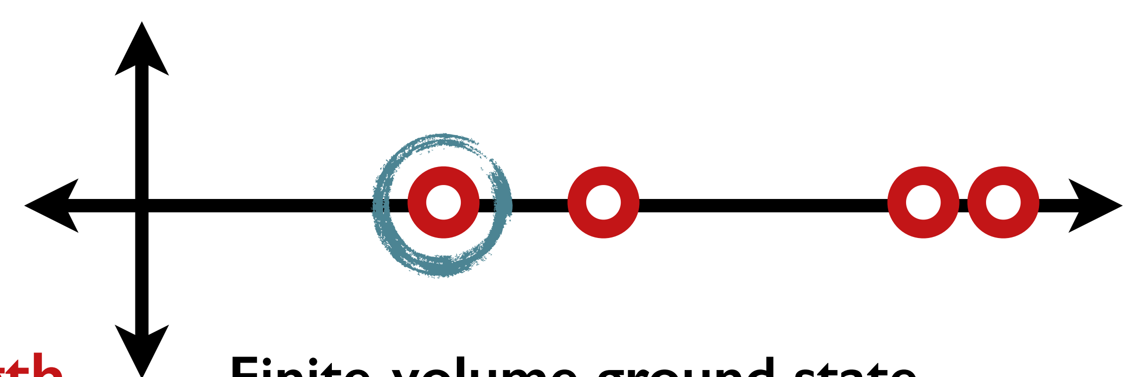
- **cubic**, spatial volume (extent *L*)
- **periodic**

$$\vec{p} = \frac{2\pi}{L}\vec{n}\,, \qquad \vec{n} \in \mathbb{Z}^3$$

- *L* is large enough to neglect $e^{-M_\pi L}$
- *T* and lattice also negligible

- Scattering leaves an *imprint* on finite-volume quantities





scattering length

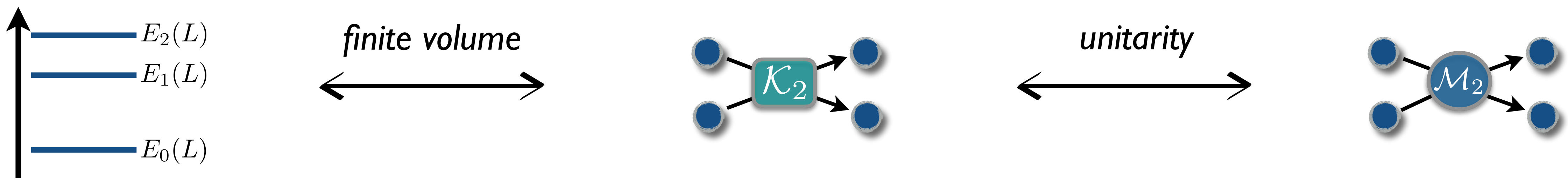
$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi a$$

$$E_0(L) = 2M_\pi + \frac{4\pi a}{M_\pi L^3} + \mathcal{O}(1/L^4)$$

• Huang, Yang (1958) •

Result

$$\det[\mathcal{K}^{-1}(s) + F(P,L)] = 0 \qquad F(P,L) \equiv$$ Matrix of known geometric functions



Holds only for two-particle energies $s < (4m)^2$ Neglects e^{-mL}

Generalized to ***non-degenerate masses, multiple channels, spinning particles***

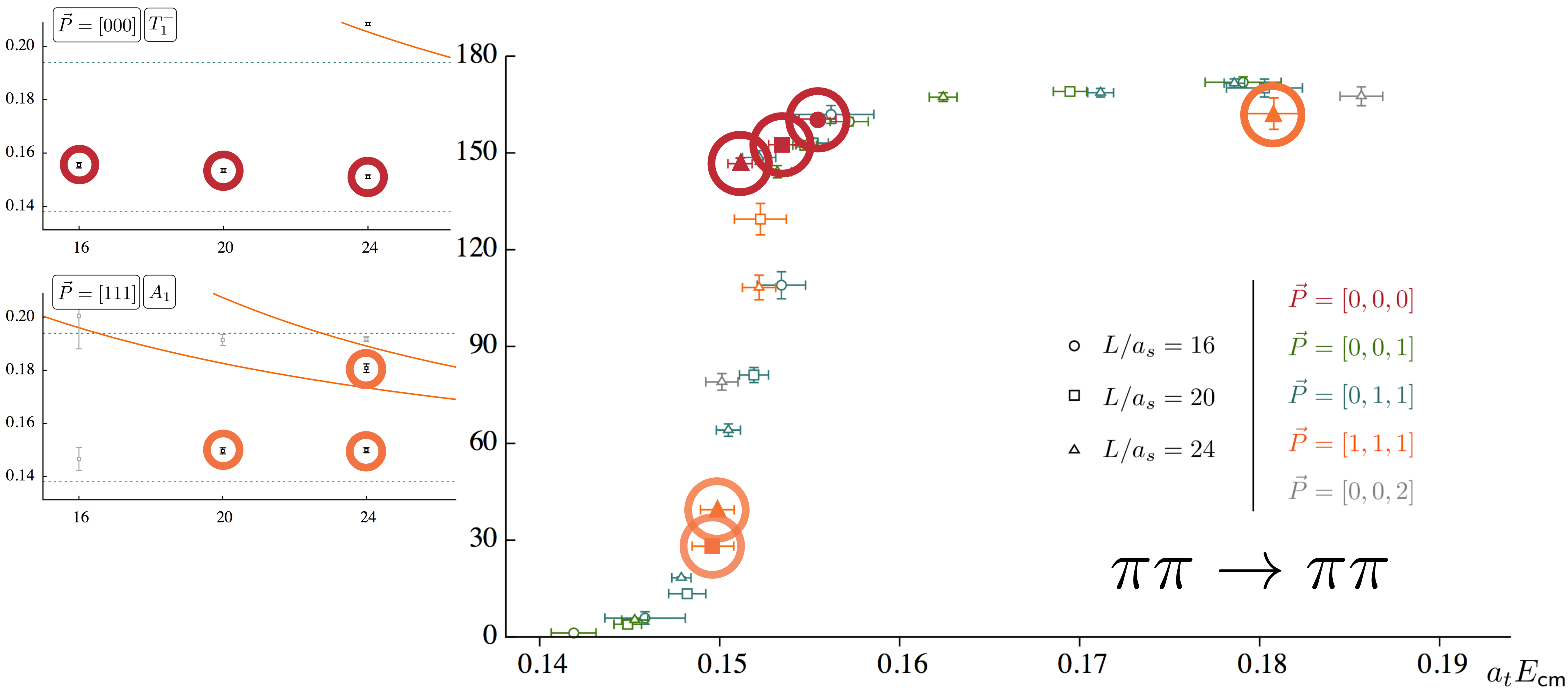
Encodes angular momentum mixing

Huang, Yang (1958) • Lüscher (1986, 1991) • Rummukainen, Gottlieb (1995)
Kim, Sachrajda, Sharpe (2005) • Christ, Kim, Yamazaki (2005) • He, Feng, Liu (2005)
Leskovec, Prelovsek (2012) • Bernard *et. al.* (2012) • MTH, Sharpe (2012) • Briceño, Davoudi (2012)
Li, Liu (2013) • Briceño (2014)

Using the result

- Single-channel case *(pions in a p-wave)*

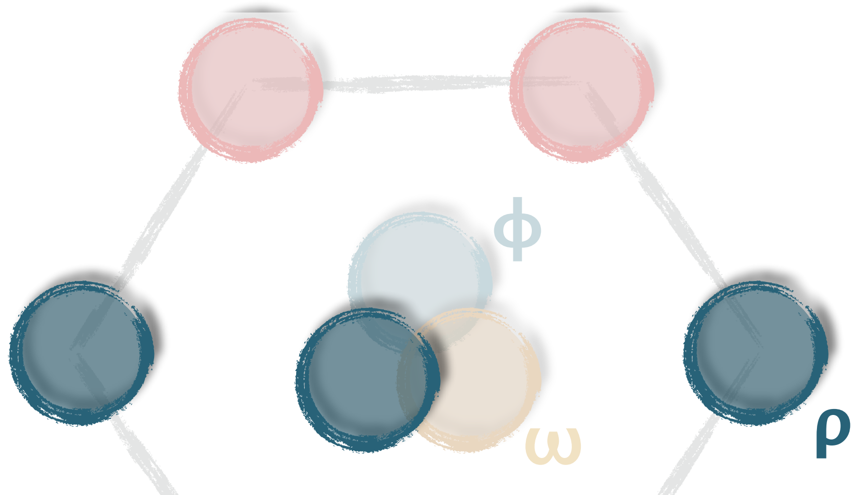
$$\mathcal{K}(s_n)^{-1} = \rho \cot \delta(s_n) = -F(E_n, \vec{P}, L)$$

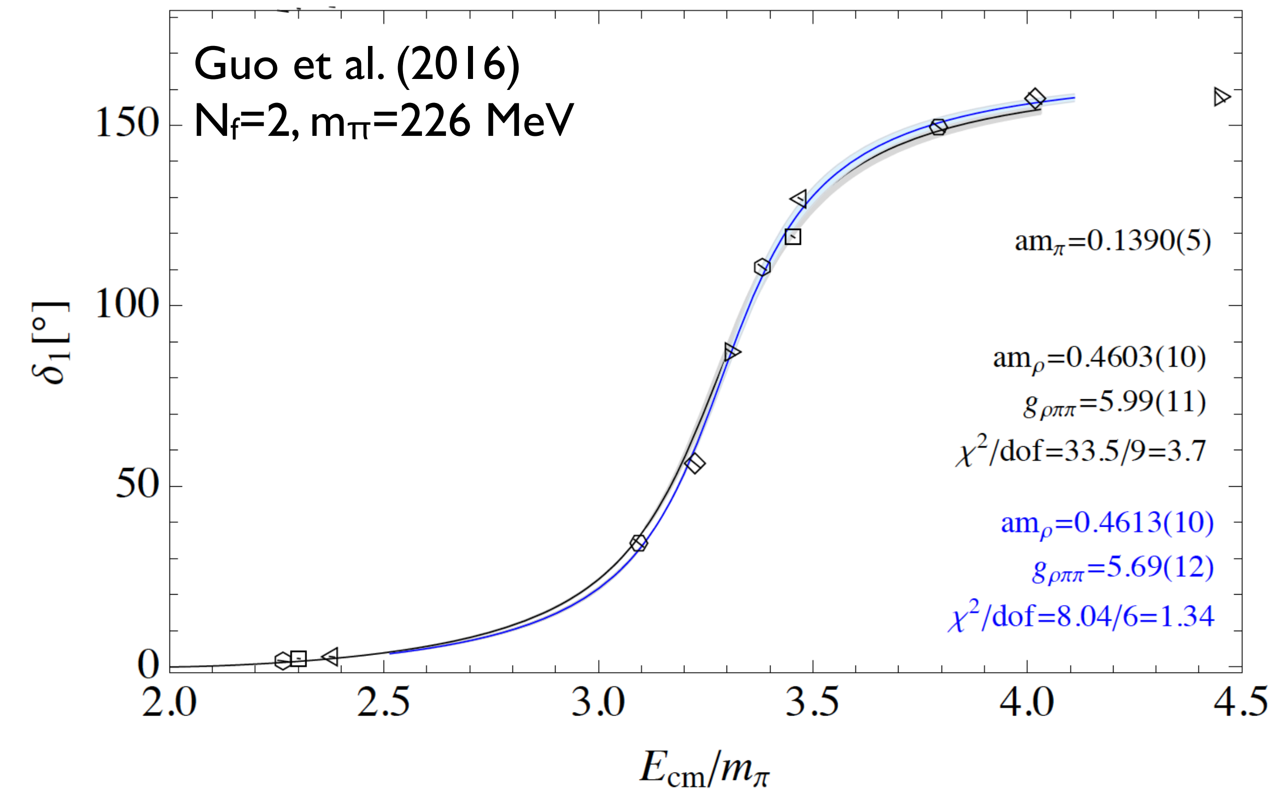


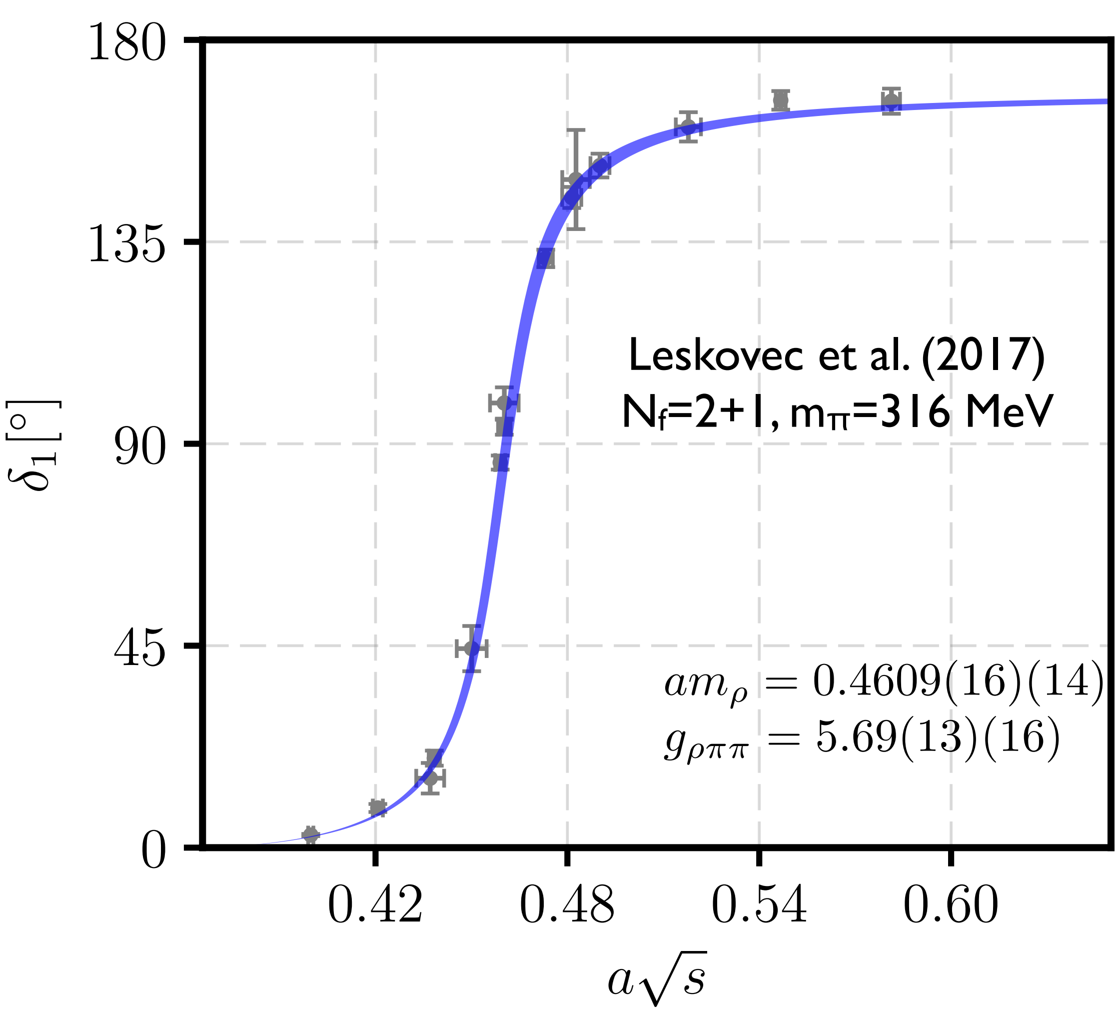
• Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505 •

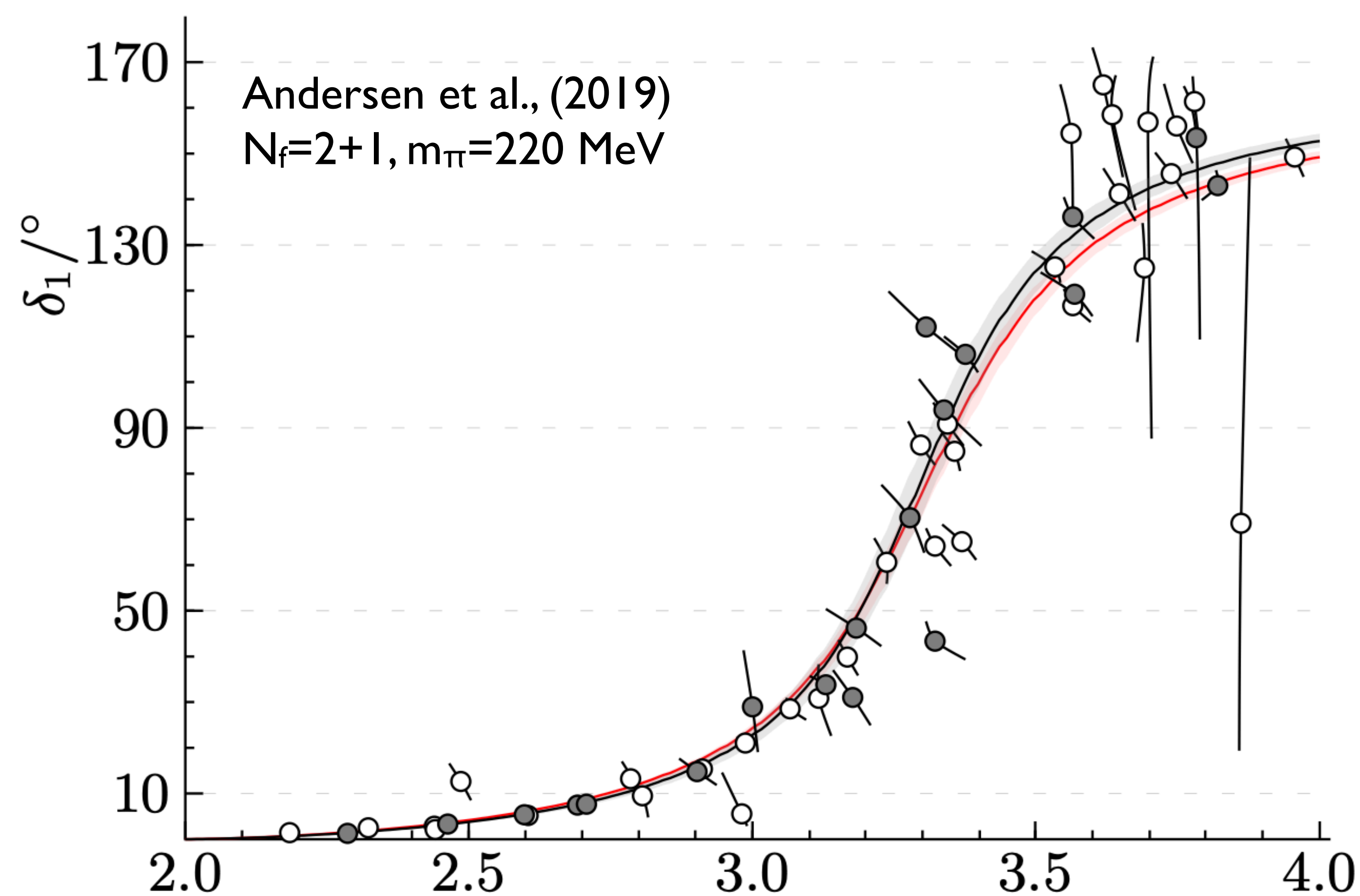
$$\rho \to \pi\pi$$

$$I^G(J^{PC}) = 1^+(1^{--})$$



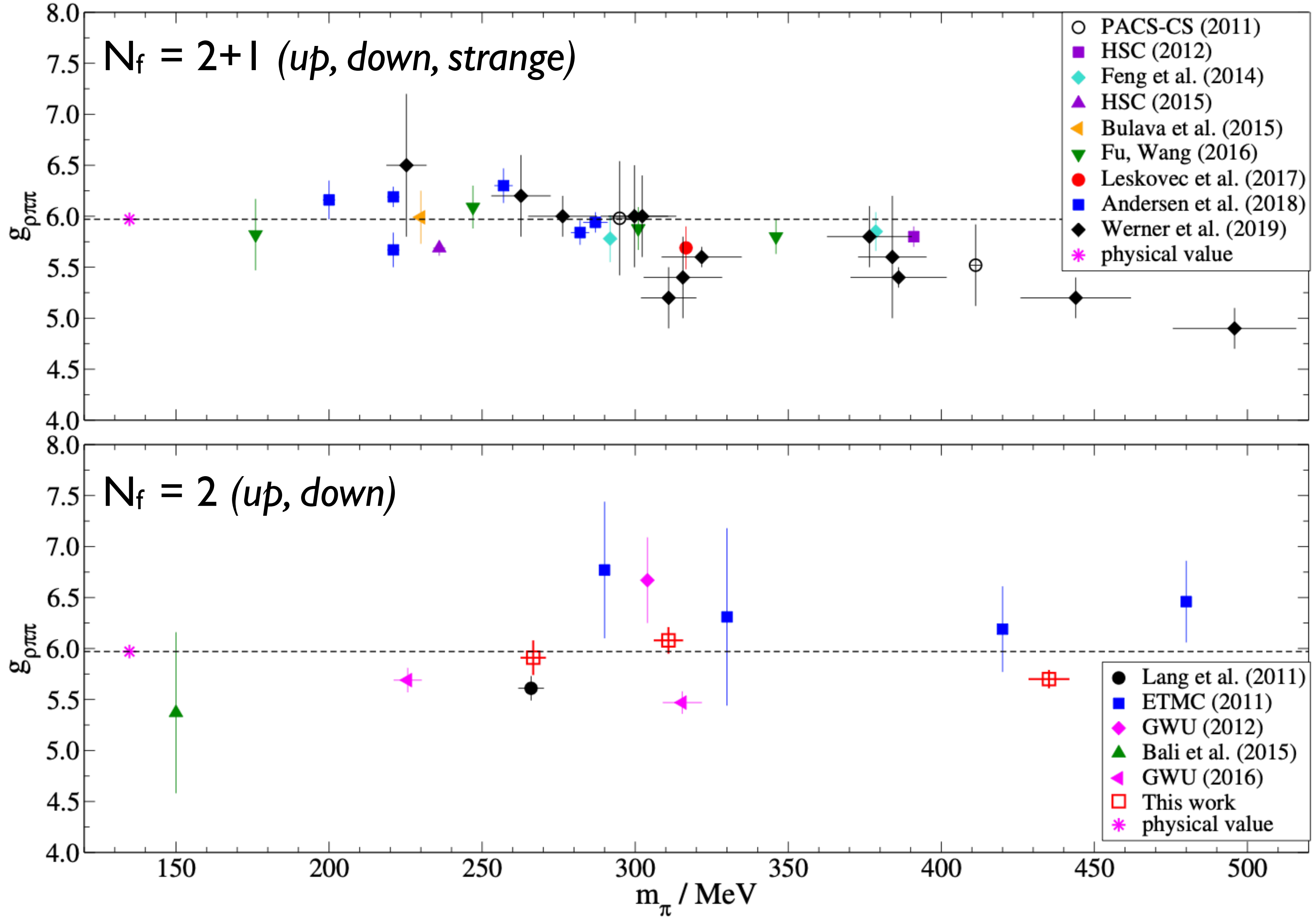






$\rho \rightarrow \pi\pi$

$$I^G(J^{PC}) = 1^+(1^{--})$$



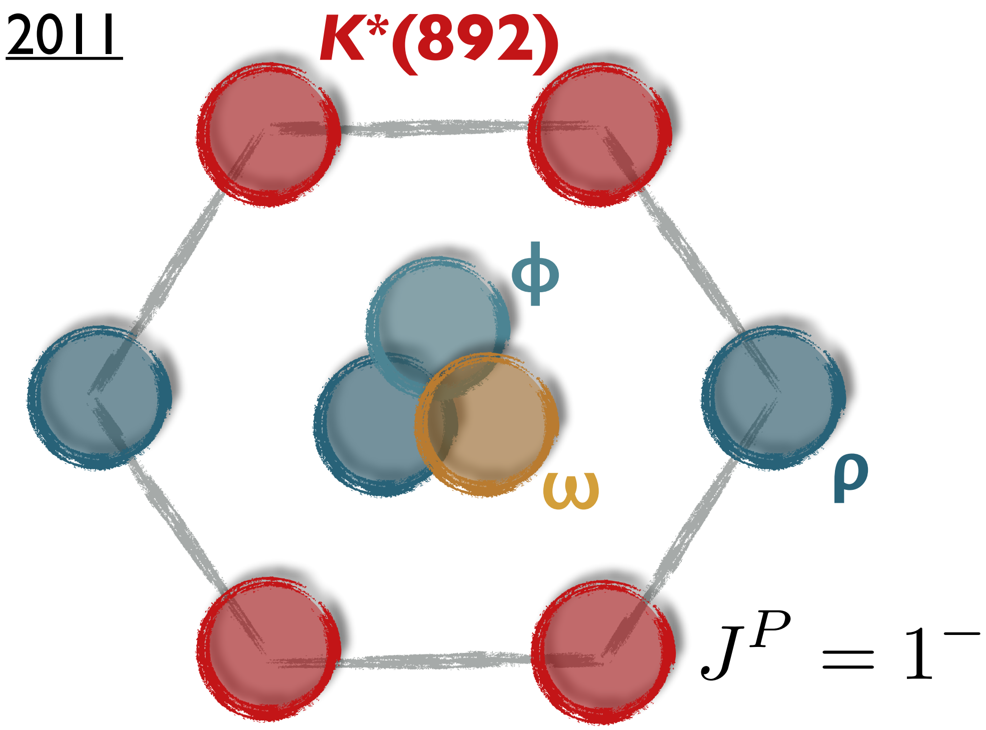
• Erben et al., Phys.Rev. **D** 101 (2020) 5, 054504 •

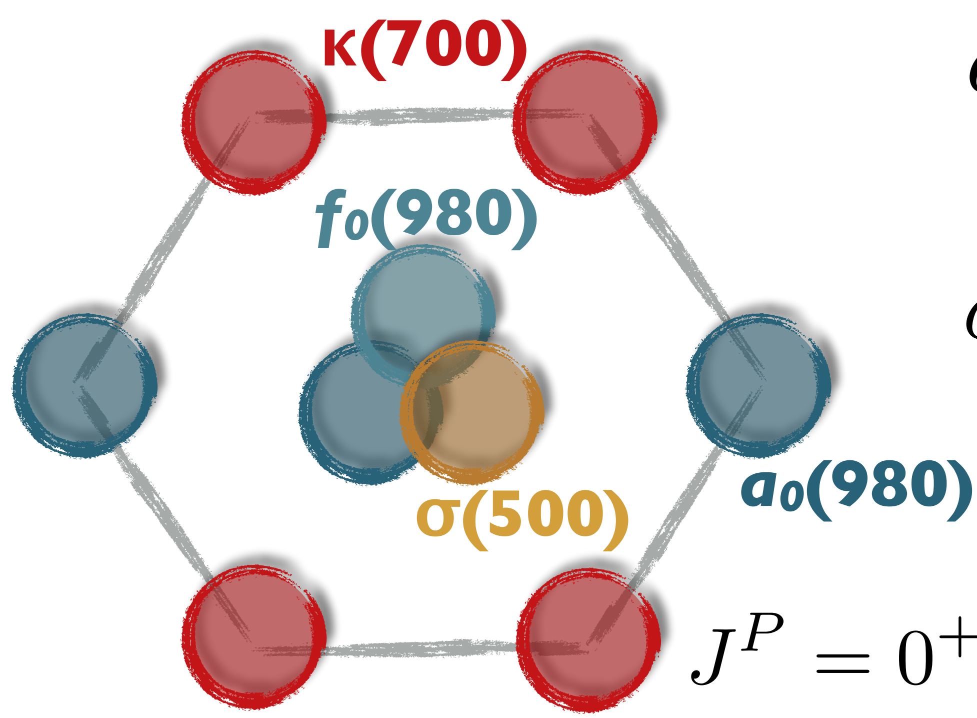
$\rho \rightarrow \pi\pi$

- CP-PACS/PACS-CS 2007, 2011
- ETMC 2010
- Lang et al. 2011
- HadSpec 2012, **2016**
- Pellisier 2012
- RQCD 2015
- Guo et al. 2016
- Fu et al. 2016
- Bulava et al. 2016
- Alexandrou et al. 2017
- Andersen et al. 2018
- Fischer et al. 2020
- Erben et al. 2020

$\sigma \rightarrow \pi\pi$

- Prelovsek et al. 2010
- Fu 2013
- Wakayama 2015
- Howarth and Giedt 2017
- Briceño et al. 2017
- Guo et al. 2018





$\kappa \rightarrow K\pi$

$K^* \rightarrow K\pi$

- Lang et al. 2012
- Prelovsek et al. 2013
- **Wilson et al. 2015**
- RQCD 2015
- **Brett et al. 2018**
- Wilson et al. 2019
- Rendon et al. 2020

$b_1 \rightarrow \pi\omega, \pi\phi$

- **Woss et al. 2019**

$a_0(980) \rightarrow \pi\eta, K\overline{K}$

- **Dudek et al. 2016**

$\sigma, f_0, f_2 \rightarrow \pi\pi, K\overline{K}, \eta\eta$

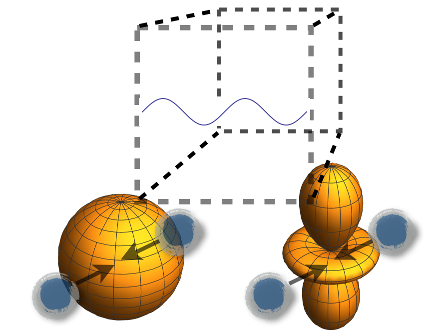
- **Briceño et al. 2017**

See the recent review by Briceño, Dudek and Young

Coupled channels

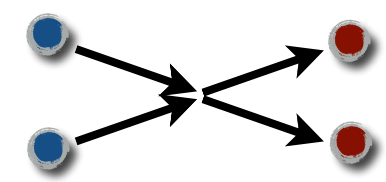
- The cubic volume mixes different partial waves…

e.g. $K\pi \to K\pi$, $\vec{P} \neq 0$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_s^{-1} & 0 \\ 0 & \mathcal{K}_p^{-1} \end{pmatrix} + \begin{pmatrix} F_{ss} & F_{sp} \\ F_{ps} & F_{pp} \end{pmatrix}\right] = 0$



…as well as different flavor channels…

e.g. $a = \pi\pi$, $b = K\overline{K}$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_{a\to a} & \mathcal{K}_{a\to b} \\ \mathcal{K}_{b\to a} & \mathcal{K}_{b\to b} \end{pmatrix}^{-1} + \begin{pmatrix} F_a & 0 \\ 0 & F_b \end{pmatrix}\right] = 0$



- Workflow…

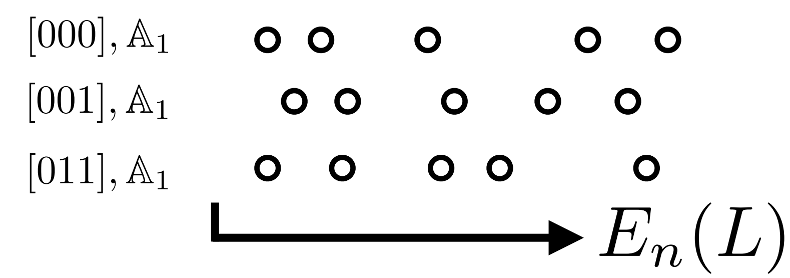
Correlators with a large operator basis
$\langle \mathcal{O}_a(\tau)\mathcal{O}_b^\dagger(0)\rangle$

→

Reliably extract finite-volume energies
$\langle \Omega_m(\tau)\Omega_m^\dagger(0)\rangle \sim e^{-E_m(L)\tau}$

→

Vary L and P to recover a dense set of energies



↓

hadspec

Identify a broad list of K-matrix parametrizations

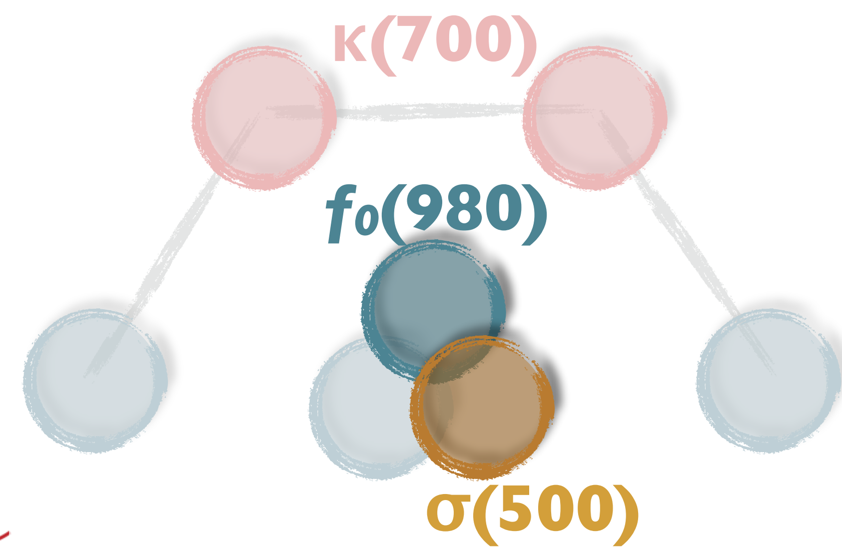
polynomials and poles | EFT based | dispersion theory based

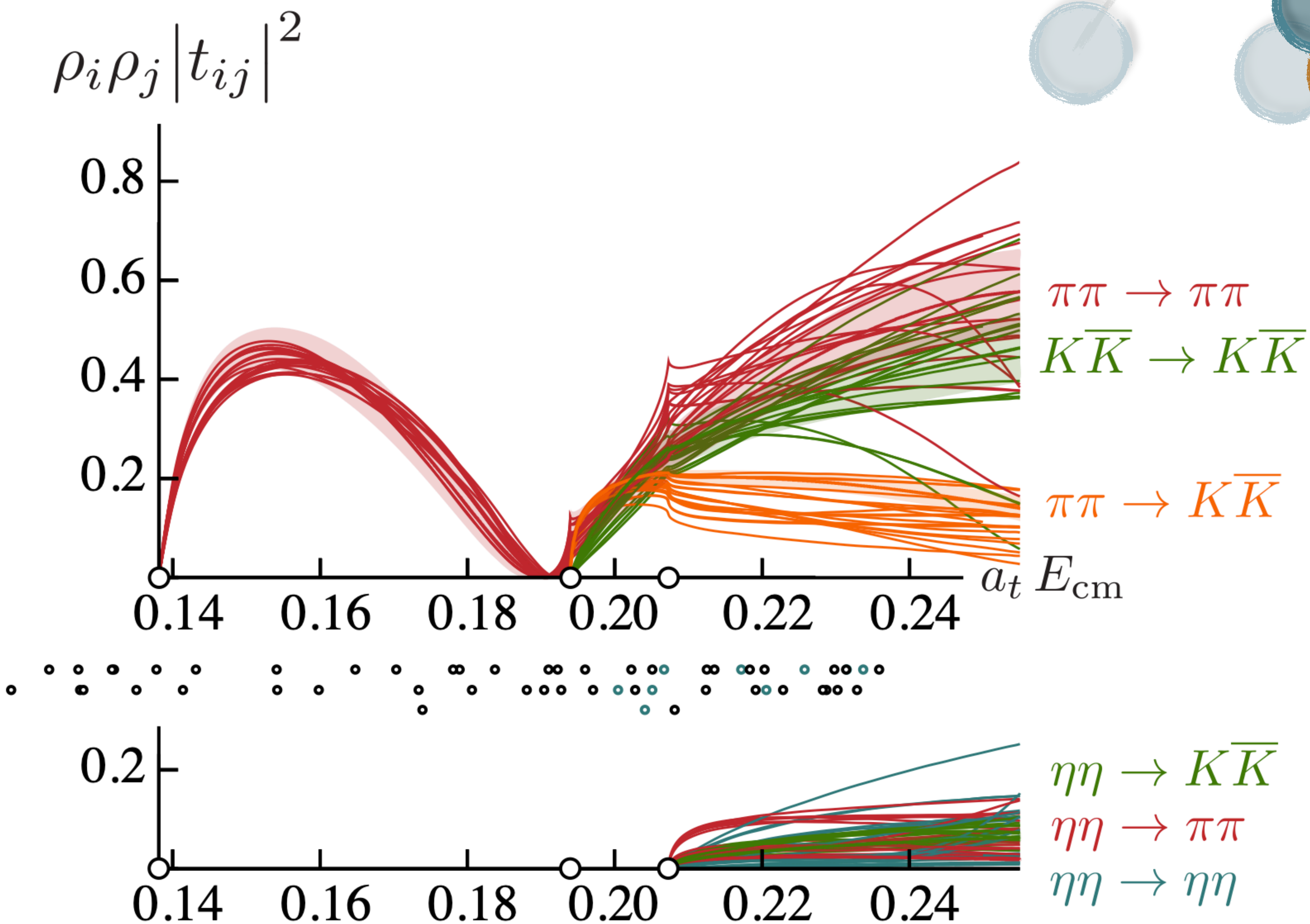
→

Perform global fits to the finite-volume spectrum

$$\sigma, f_0 \to \pi\pi, K\overline{K}, \eta\eta$$

$$I^G(J^{PC}) = 0^+(0^{++})$$

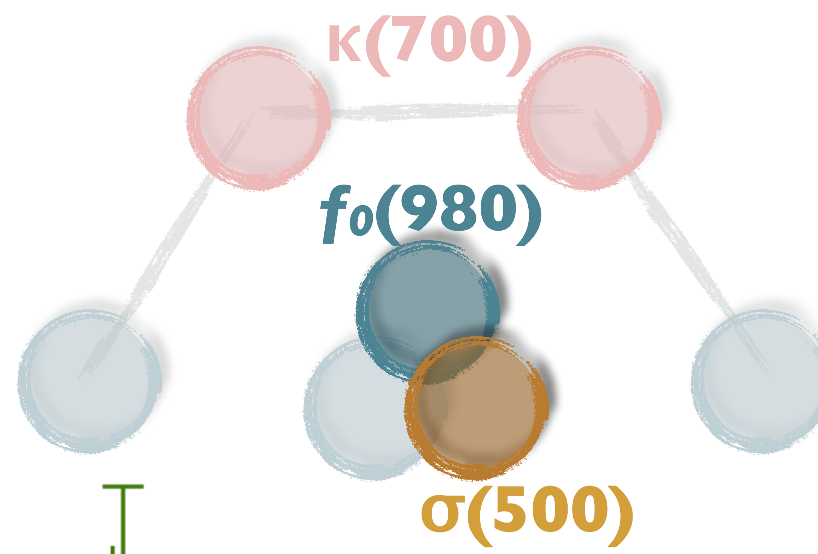


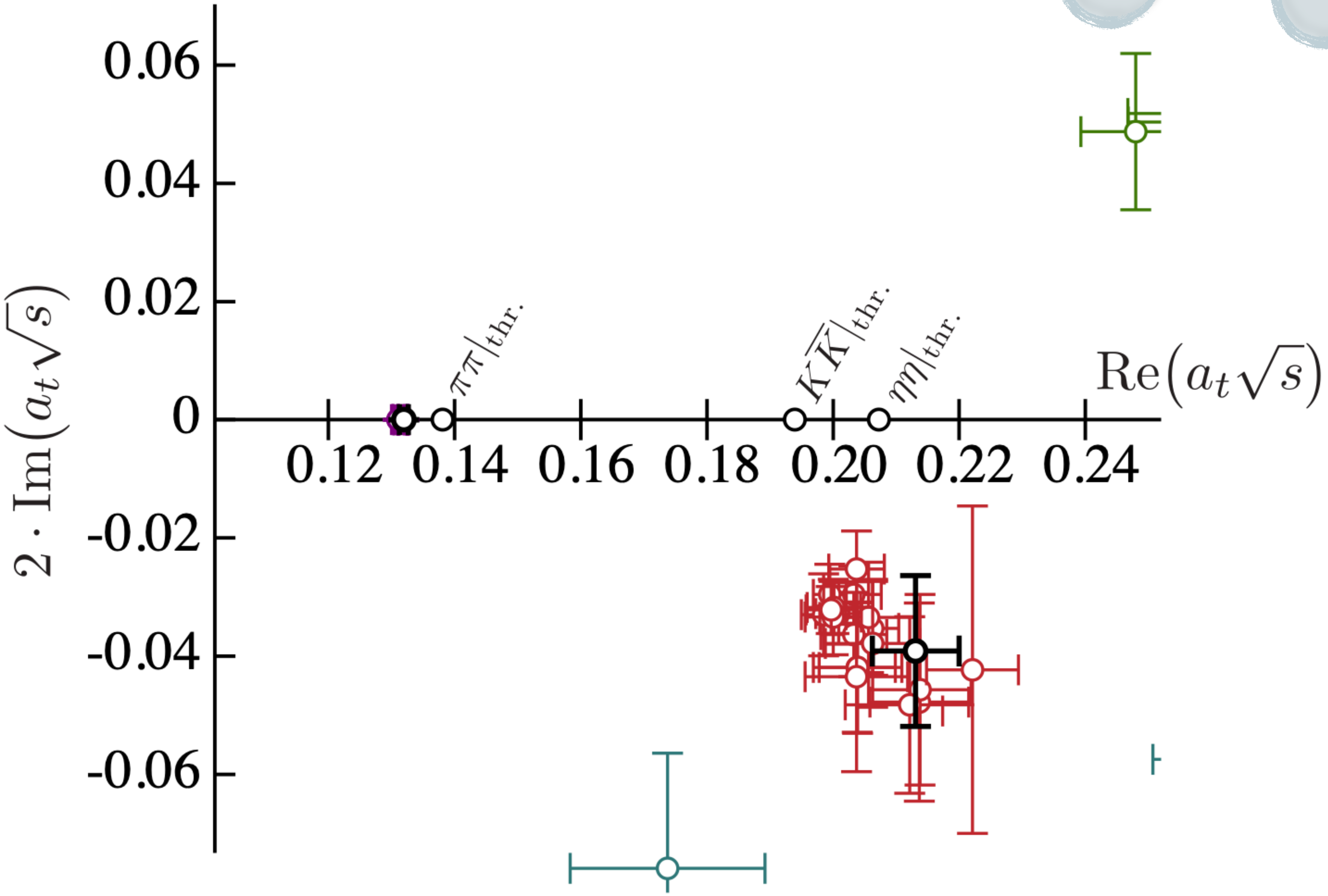


- Briceno et al., Phys.Rev. **D** 97 (2018) 5, 054513 •

$\sigma, f_0 \to \pi\pi, K\overline{K}, \eta\eta$

$$I^G(J^{PC}) = 0^+(0^{++})$$



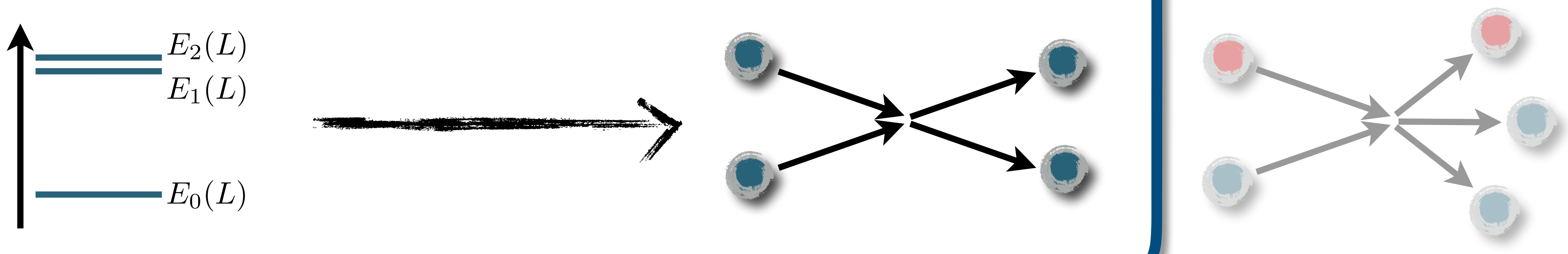


- Briceno et al., Phys.Rev. **D** 97 (2018) 5, 054513 •

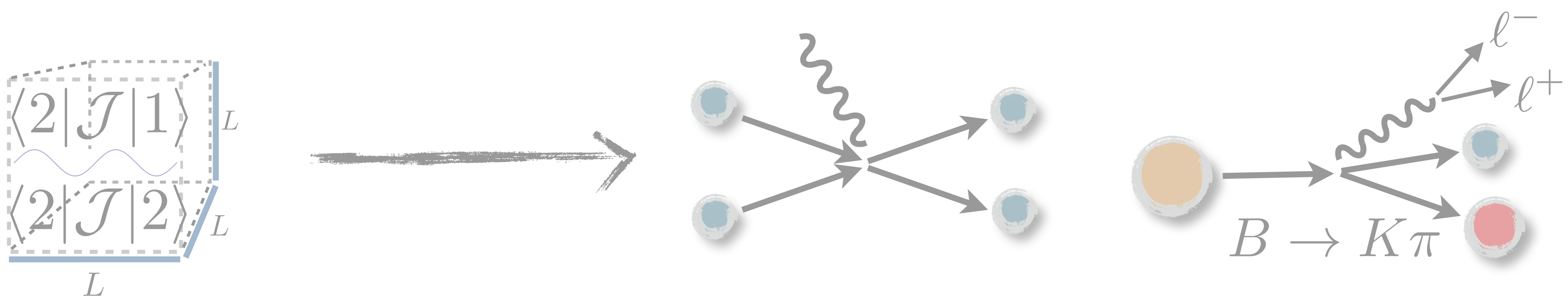
Multi-hadron processes from LQCD

Use the finite volume as a tool to extract multi-hadron observables

☑ Scattering (from finite-volume energies)



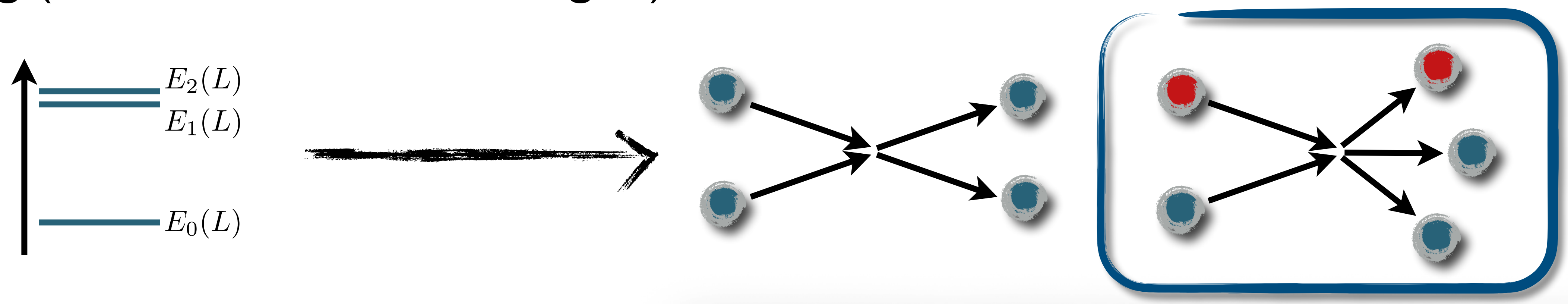
☐ Transitions (from finite-volume energies + matrix elements)



Multi-hadron processes from LQCD

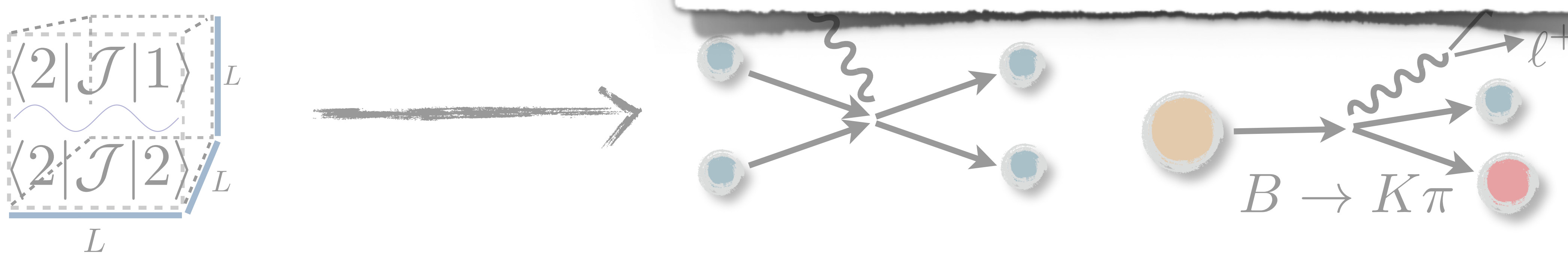
Use the finite volume as a tool to extract multi-hadron observables

☑ Scattering (from finite-volume energies)



☐ Transitions (from finite-volume energies

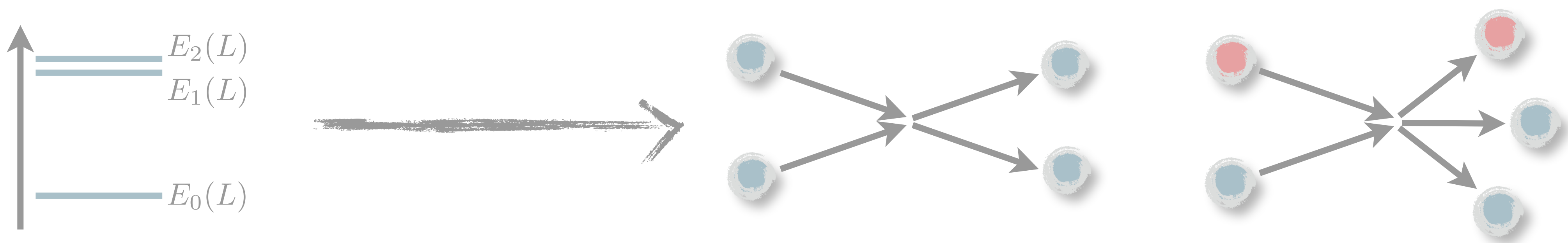
Review articles:
MTH and Sharpe (2019), Mai, Döring
Rusetsky (2020)



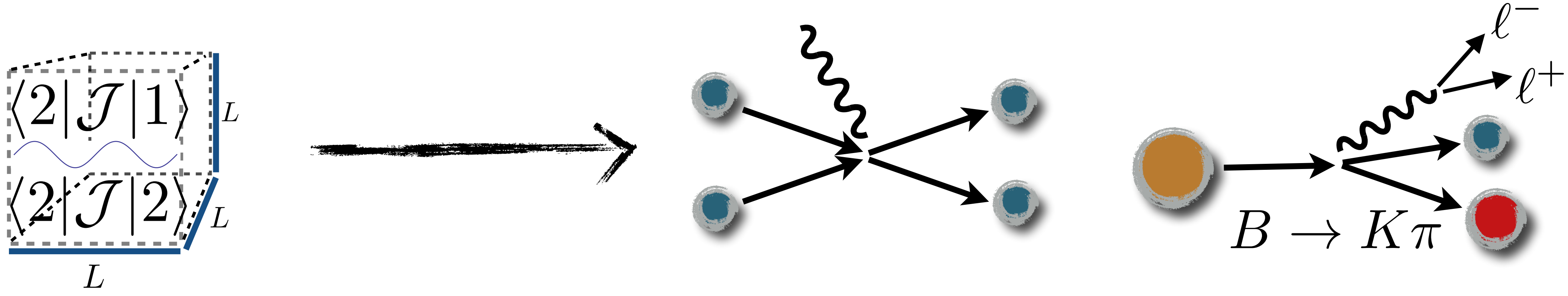
Multi-hadron processes from LQCD

Use the finite volume as a tool to extract multi-hadron observables

- [x] Scattering (from finite-volume energies)



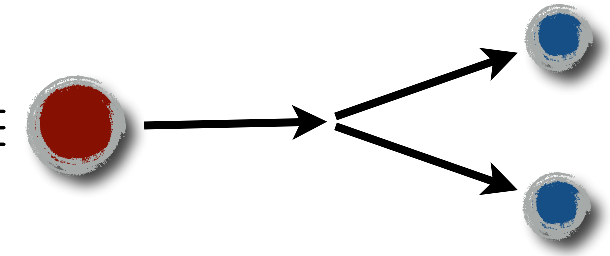
- [ ] Transitions (from finite-volume energies + matrix elements)



Multi-hadron matrix elements

Kaon decay

$$\langle \pi\pi, \mathrm{out}|\mathcal{H}|K\rangle \equiv$$

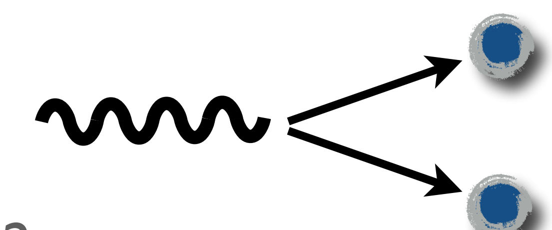


Implementation by RBC/UKQCD collaboration

Lellouch, Lüscher (2001) • Kim, Sachrajda, Sharpe (2005) • Christ, Kim, Yamazaki (2005)

Time-like form factors

$$\langle \pi\pi,\ \mathrm{out}|\mathcal{J}_\mu|0\rangle \equiv$$

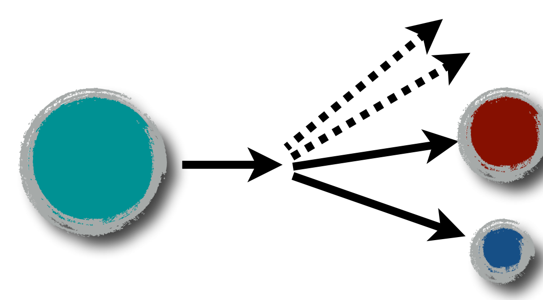


Relevant for muon HVP contribution to muon g-2

Meyer (2011)

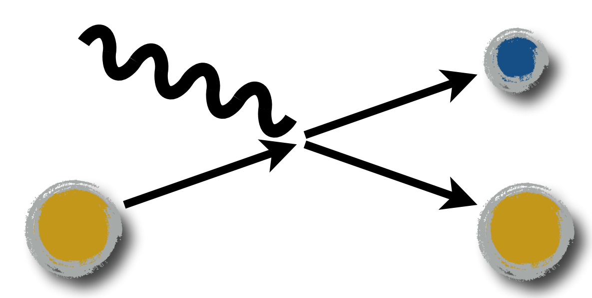
Resonance transition amplitudes

$$\langle K\pi, \mathrm{out}|\mathcal{J}_{\alpha\beta}|B\rangle \equiv$$



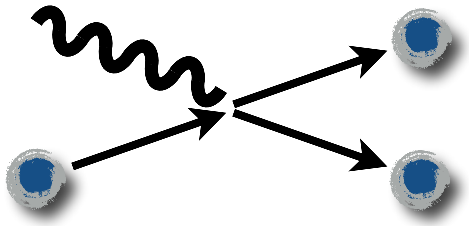
Particles with spin

$$\langle N\pi, \mathrm{out}|\mathcal{J}_\mu|N\rangle \equiv$$



Agadjanov *et al*. (2014) • Briceño, MTH, Walker-Loud (2015) • Briceño, MTH (2016)

Pion photo-production

$\langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle \equiv$ 

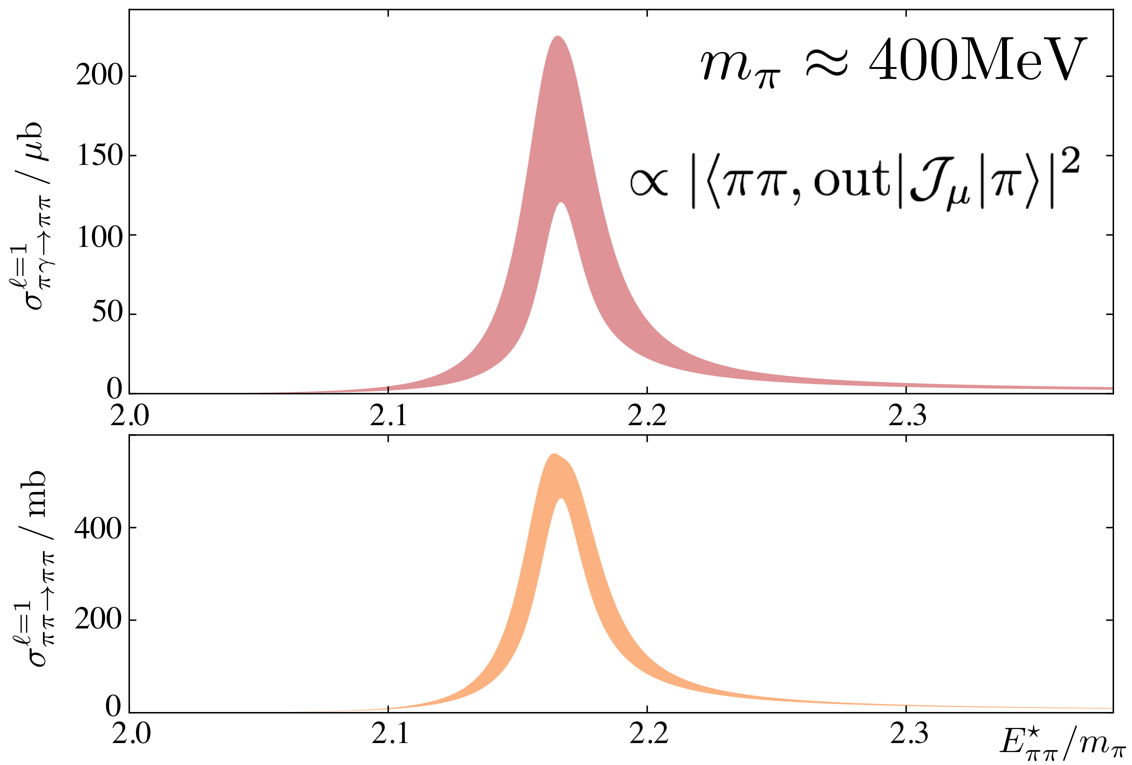
Formal relation

get this from the lattice **experimental observable**

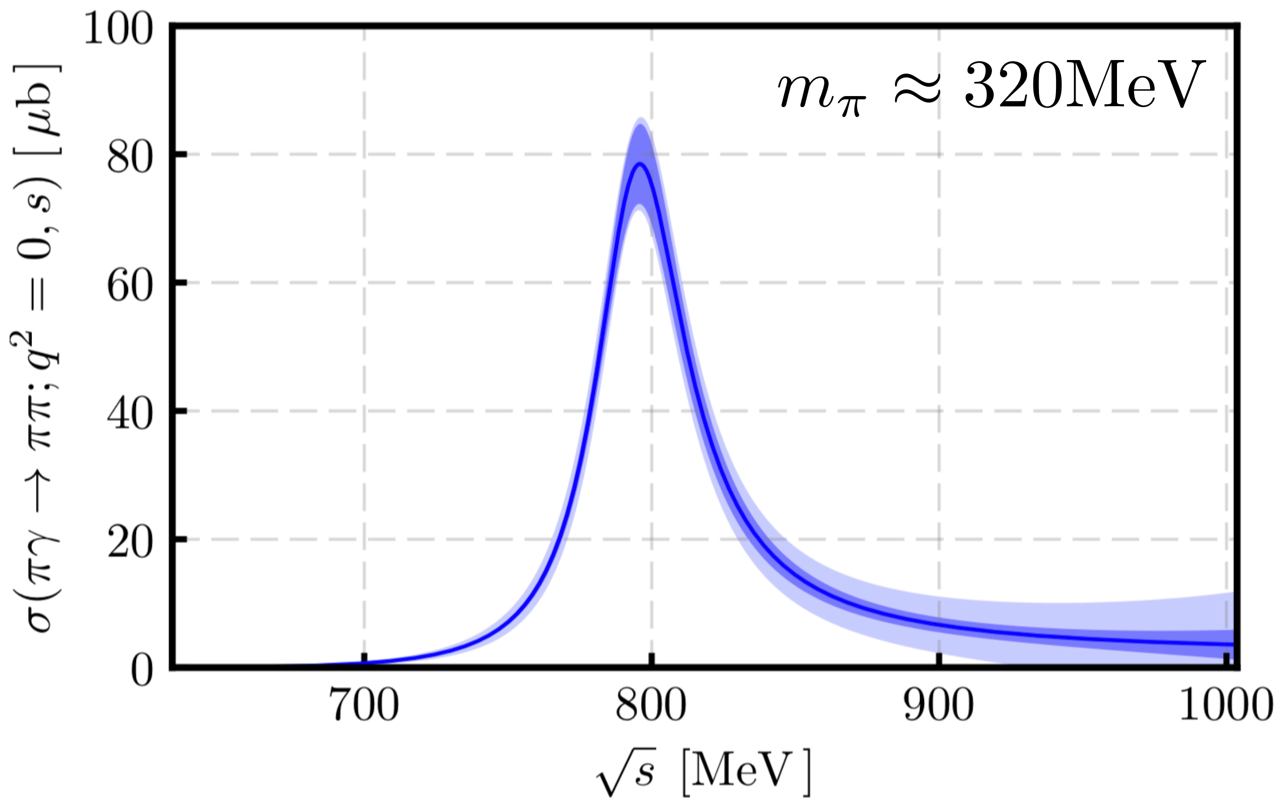
$$|\langle n, L | \mathcal{J}_\mu | \pi \rangle|^2 = \langle \pi | \mathcal{J}_\mu | \pi\pi, \mathrm{in} \rangle \mathcal{R}(E_n, L) \langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle$$

Briceño, MTH, Walker-Loud (2015)

Numerical implementation



Briceño et. al., Phys. Rev. D93, 114508 (2016)



Alexandrou et. al., Phys. Rev. D98, 074502 (2018)

Final result for ε'

- Combining our new result for $\mathrm{Im}(A_0)$ and our 2015 result for $\mathrm{Im}(A_2)$, and again using expt. for the real parts, we find

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 0.00217(26)(62)(50)$$

stat sys IB + EM

Consistent with experimental result:

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$$

Conclusions

- Completed update on our 2015 lattice determination of A_0 and ε'
  - 3.2x increase in statistics.
  - Improved systematic errors, notably use of multi-operator techniques essentially removes excited-state systematic.
- Reproduce experimental value for ΔI=1/2 rule, demonstrating that QCD sufficient to solve this decades-old puzzle.
- Result for ε' consistent with experimental value.
- Total error is ~3.6x that of experiment.
- ε' remains a promising avenue to search for new physics, but greater precision is required.
- The work goes on....

Outline

- ☑ The LQCD landscape
  - ☐ Lattice basics
  - ☐ Nielson Ninomiya
  - ☐ Many actions
- ☑ Flavor physics
  - ☐ Single-hadron matrix elements
  - ☐ Light-flavor decay constants
  - ☐ Heavy-flavor decay constants
  - ☐ Mixing
  - ☐ Form factors
- ☑ QED + QCD
  - ☐ Theoretical challenge
  - ☐ Different formulations
- ☑ $(g-2)_\mu$
  - ☐ Light-by-light
  - ☐ HVP
- ☑ Multi-hadron processes
  - ☐ Finite-volume as a tool
  - ☐ Resonances
  - ☐ $2 \to 2$ scattering
  - ☐ $1 + \mathscr{J} \to 2$ transitions
- ☐ So much more!

loads of material not covered here

especially…

nuclear physics

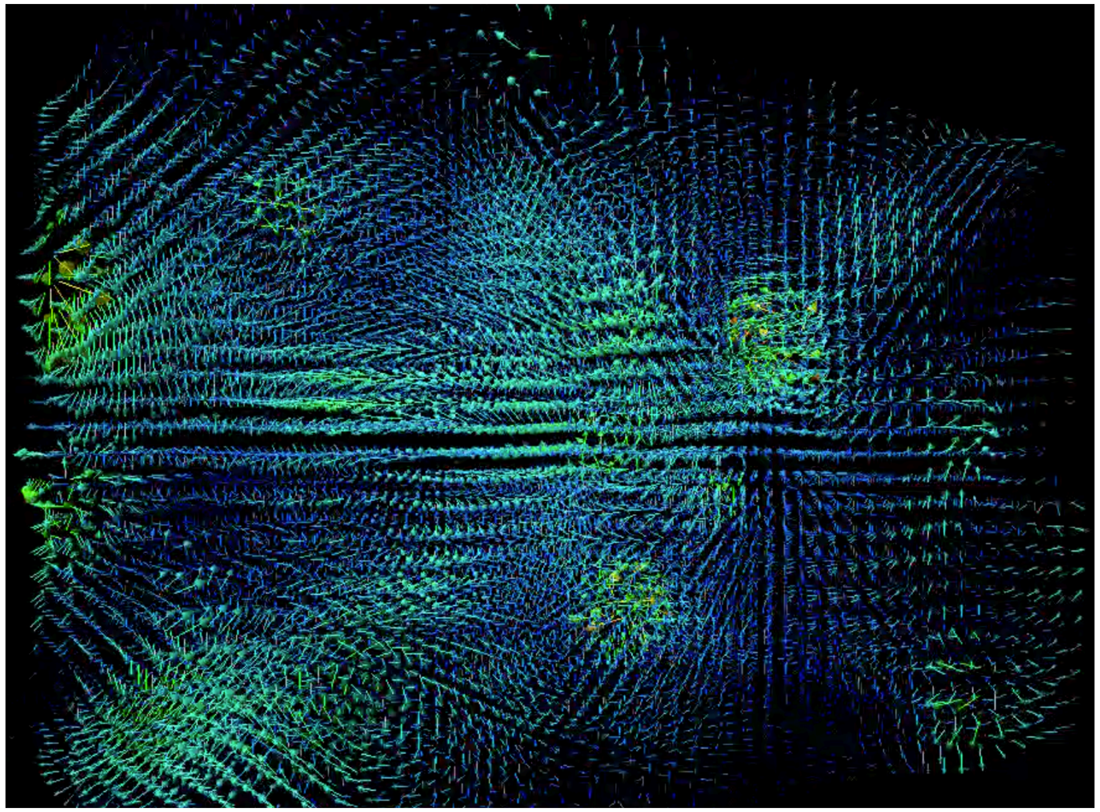
determination of alpha strong

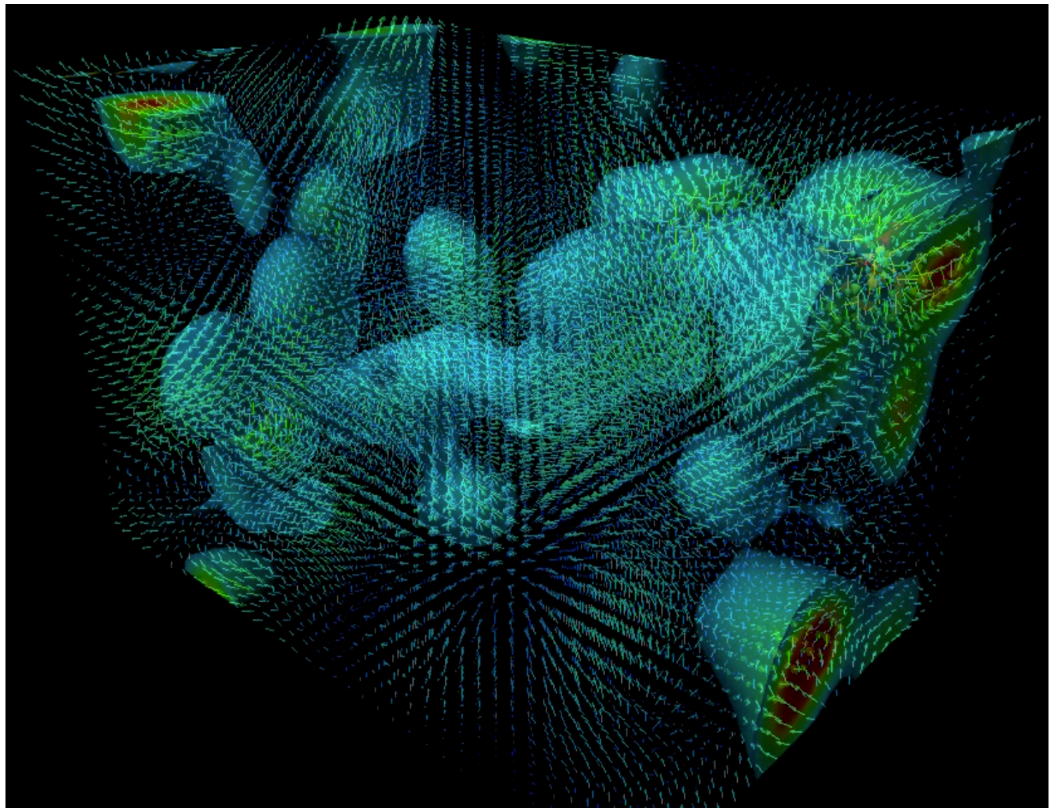
Great resources are

lattice conferences

FLAG!

Thanks for listening!





University of Adelaide, CSSM