HITRIplus

Heavy Ion Therapy Research Integration

Accelerator Physics - Introduction

Mariusz Sapinski (SEEIIST)

Heavy Ion Therapy School, May 2021

SEEIIST

South East European International Institute for Sustainable Technologies

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101008548

Contents

- History and motivation for accelerators
- Beam properties – transverse emittance
- RF acceleration, longitudinal dynamics, phase stability
- Cyclotrons and synchrotrons
- Strong focusing, transverse dynamics beam transport

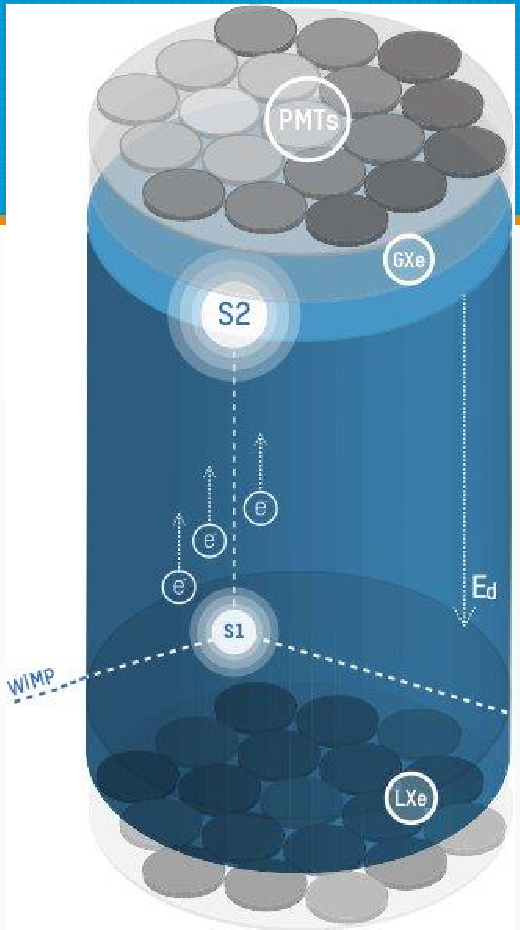
- *Beam instrumentation* – Wednesday (20 min)
- *Sarajevo Linac project* – Friday (20 min)

Other presentations this week:

- Introduction to accelerators and medical machines (Maurizio, 1.5 h)
- Linear accelerators (Giovanni, 45 min)
- Injection to synchrotrons (Elena, 20 min)
- Beam extraction (Rebecca, 30 min)
- Ion sources (Nadia, 30 min)
- Gantries and Beam Delivery (Elena, 45 min)
- Low energy accelerators (Milko, 45 min)
- Sarajevo Linac project:
  - Ion Beam Analysis (Fehima, 15 min)
  - Low energy beam transport simulations (Benjamin, 10 min)

Methods of science

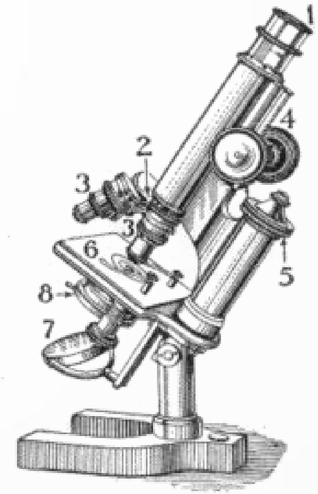
- Observation of nature:
  - Astronomy – purely observational science
  - Physics – e.g. Dark Matter search (XENON)
- Controlled experiments:
  - Many types of experiment, rich methodology
  - Various tools, including accelerators

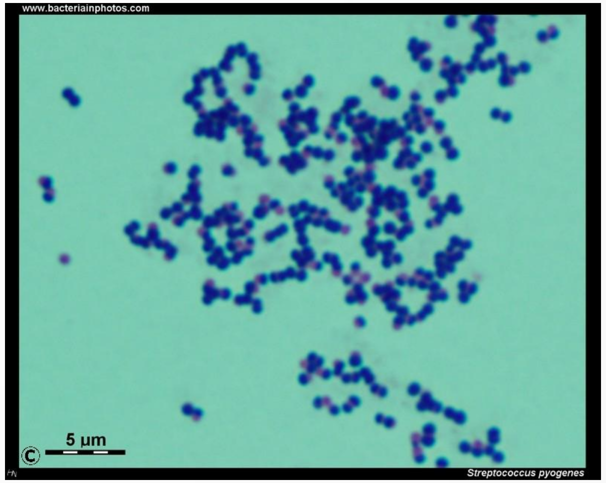


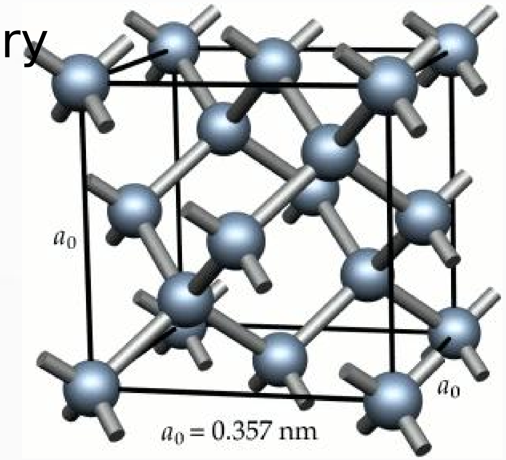
e.g. lasers are very important tools of modern physics: quantum mechanics, atomic physics, ultra-fast chemistry, etc

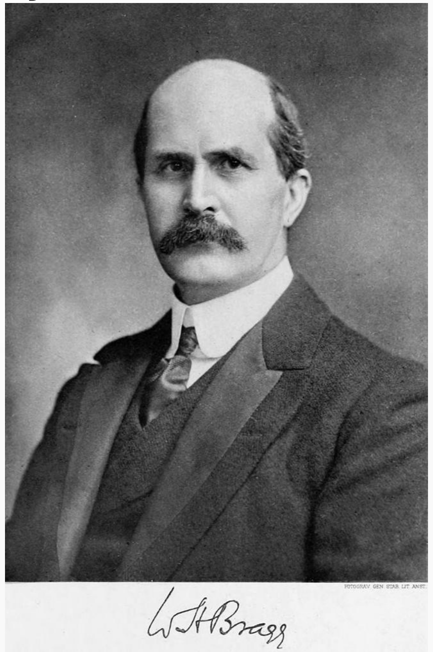
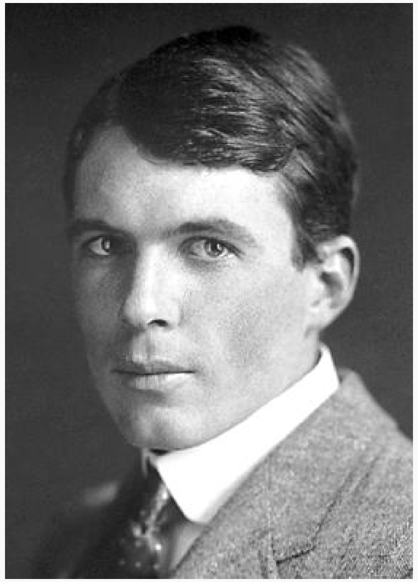
Microscopes

- Optical microscopes invented in 17th century
- Resolution 200 nm determined by wavelight length (diffraction limit λ/2, optical λ=400-800 nm)
- Bacteria size ~ 0.5-2 μm – seen by light microscopes, but SARS-Cov-2 virus size ~100 nm
- Crystalline structures need sub-nm resolution
- One could use shorter wavelengths: X-ray Crystallography (not microscopy – difficult optics)
- Bragg's law 1912, by William Henry - father and William Lorentz – son
- X-ray λ=0.1 nm
- BTW Bragg father discovered Bragg peak in 1903



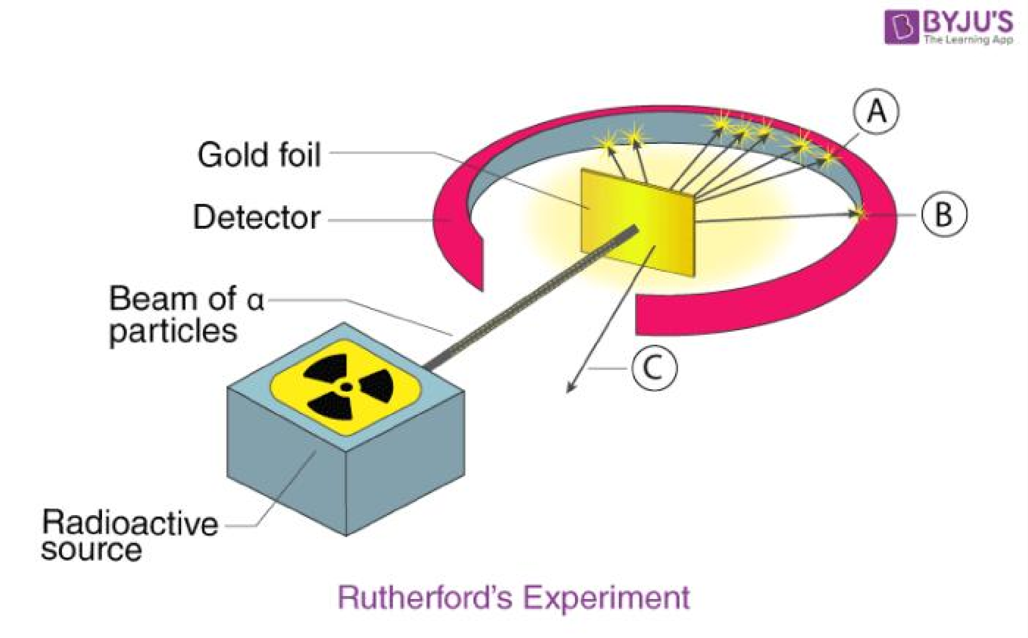




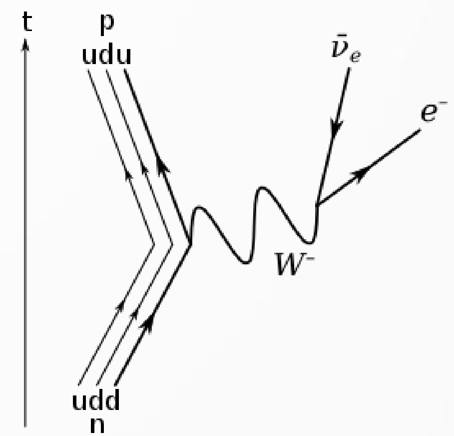
Rutheford experiment

- Electrons are also waves (de Broglie, 1924), use them instead of light (E. Ruska, 1931)
- Electron microscopes can reach: 0.1 nm resolution – atom size; they work on normal objects (not only crystals)
- What if we want to **look inside atoms**? Photons and electrons are interacting with other electrons in the atom.
- Rutheford experiment (1908) – use alpha particles – they are heavy and penetrate through electrons
- Rutheford (+Geiger+Marsden) used Radon-222, which decays emitting alpha with E_k=5.5 MeV
- Experiment lead to discovery of nucleus and further to discovery of protons (1919) and neutrons (1932)
- Note: Beam of particles can be generated from radioactive source, but we have little control on it
- Positrons, muons – discovered in cosmic radiation



Why do we like $E=mc^2$?

- Since Rutheford – enormous progress in nuclear physics thanks to accelerators
- E.g. discovery of new isotopes, often short-living and non existing in nature, or new particles
- *New* matter is produced from energy
- Why creation of something what does not exist in nature is important ?
  - Because those particles really existed during Big Bang shaping our World
  - Because they exist now, in form of virtual particles
- Virtual particles born from vacuum for a short moment
  - Disappear after Δt≤h/2E
  - Unless they interact/decay as e.g. in β-decay (^{60}Co)



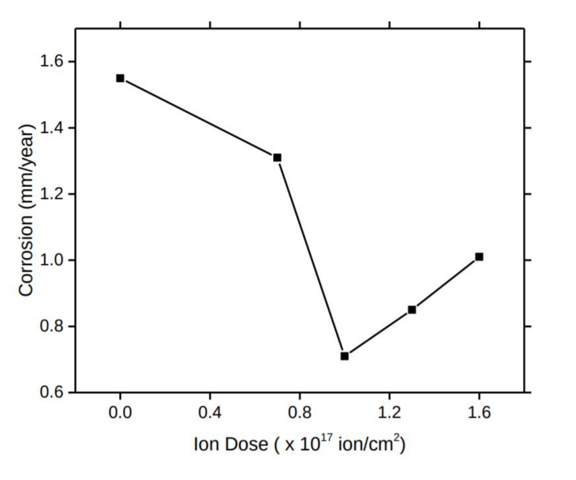
m 4.7 MeV 80 GeV 0.5 MeV

$E=mc^2$

We have to do accelerator experiments
if we want to understand the world around us

Accelerators in industry and medicine

- Ion implantation
- Ion beam analysis
- Electron beam material processing
- Radioisotope production (also medical)
- Neutron generation
- Radiotherapy, radiosurgery
- Noninvasive diagnostics

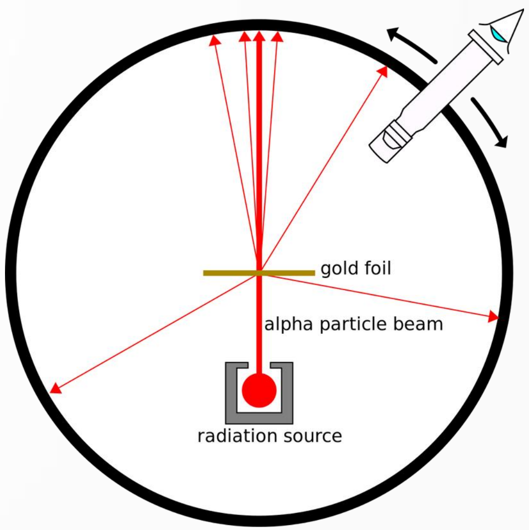


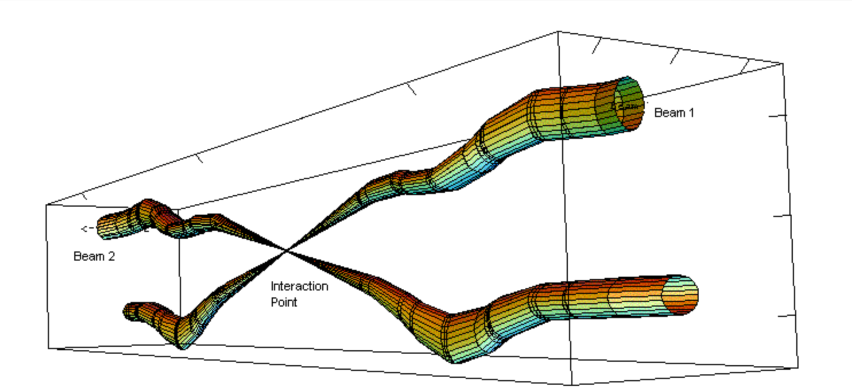
Corrosion for implanted 304 SS
(A. Nikmah et al 2019 IOP Conf. Ser.: Mater. Sci. Eng. 515 012018)

Öztürk, O. (2014). “Structural and Magnetic Characterization of Nitrogen Ion Implanted Stainless Steel and CoCrMo Alloys.”

What is a beam?

- Accelerators are producing beams, so what is a beam?
- An ensemble of particles moving in the same direction
- Characterized by:
  - Particle type (usually monoparticle)
  - Intensity
  - Particle energy and energy spread
  - Transverse size and divergence (emittance)

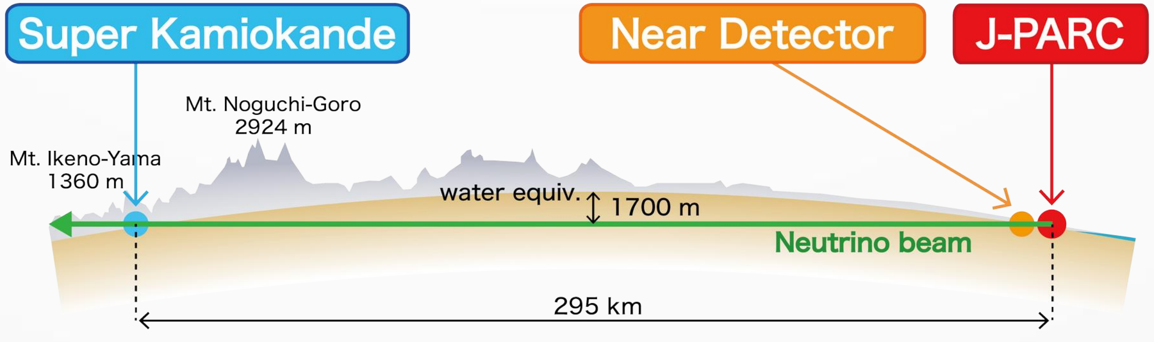




Relative beam sizes around IP1 (Atlas) in collision

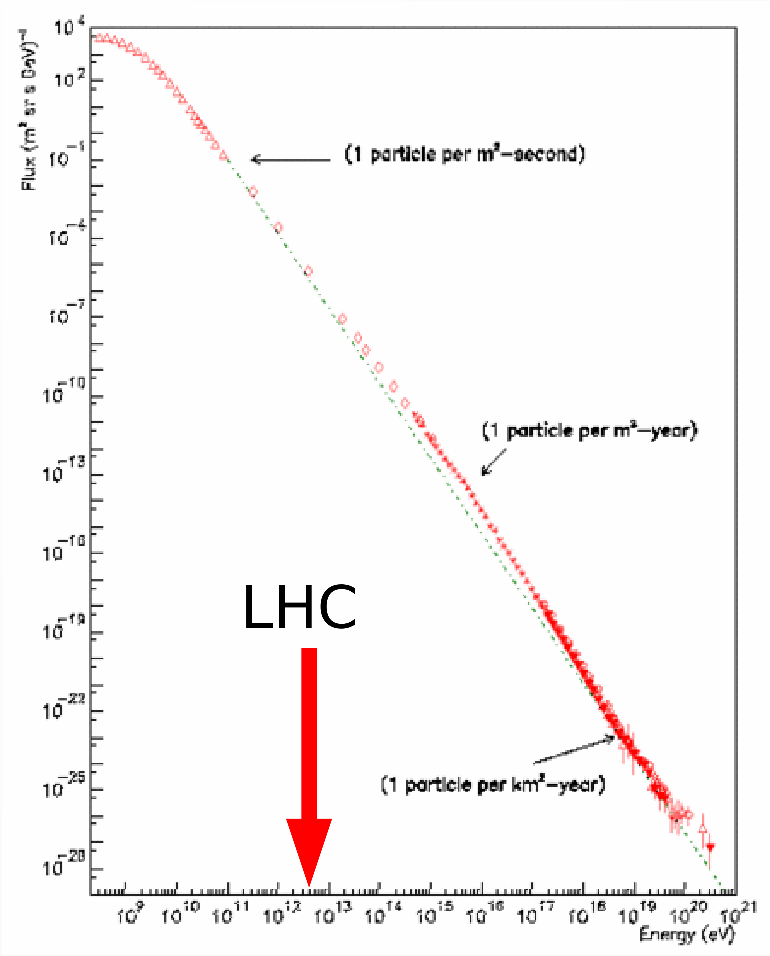
Particle types

- Electrons (the easiest, e.g. X-ray tube), positrons
- Protons, antiprotons
- Ions, e.g. $^{4}He^{2+}$, $^{12}C^{6+}$, all isotopes and charge states,
- also exotic and radioactive beams eg. $^{6}He^{2+}$ ($\tau_{½}$=0.8s) and negative ions (eg. H^{-})
- Compound particles eg. CH_3^+
- Neutral particles (eg. neutrons, neutrinos or photons) are produced as secondary beams



Particle energies

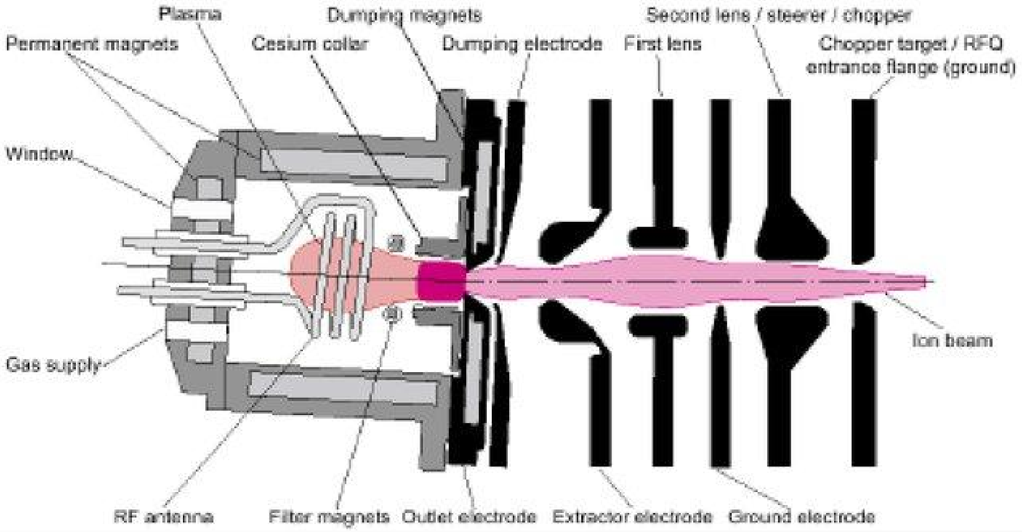
- Energy is conveniently expressed in electron-volt (eV) and for ions in eV/u (per nucleon)
- For some studies particles are deccelerated down to meV energies and trapped (e.g. anitmatter)
- The highest beam energy (per particle) is at LHC: 6.5 TeV proton beams
- Total energy stored in beams: 362 MJ (equivalent of 77,4 kg TNT!)
- Interestingly cosmic rays reach much higher energies: so called cosmic accelerators are probably driven by expanding magnetic field of exploding stars (Fermi acceleration)
- Beams are not monoenergetic; typically we talk about momentum spread $\Delta p/p \sim 10^{-3}$



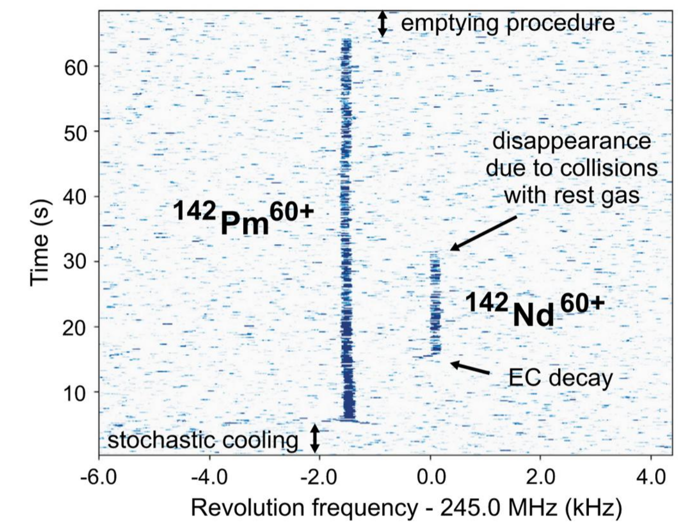
Oh-My-God particle, 50 J

Beam intensities and time structure

- Beam is typically produced as continous from the ion source and **bunched** in the accelerating structures
- Therefore ion source intensity is given mA of DC current; in the linac it is peak current and pulse duration; in synchrotron it is easier to talk about number of circulating particles
- Ion source can reach 65 mA currents, numbers of circulating particles can be in range 1-10^{14}
- Bunch length: from DC to 1 ps



M. Steck and Y.A. Litvinov / Progress in Particle and Nuclear Physics 115 (2020) 103811

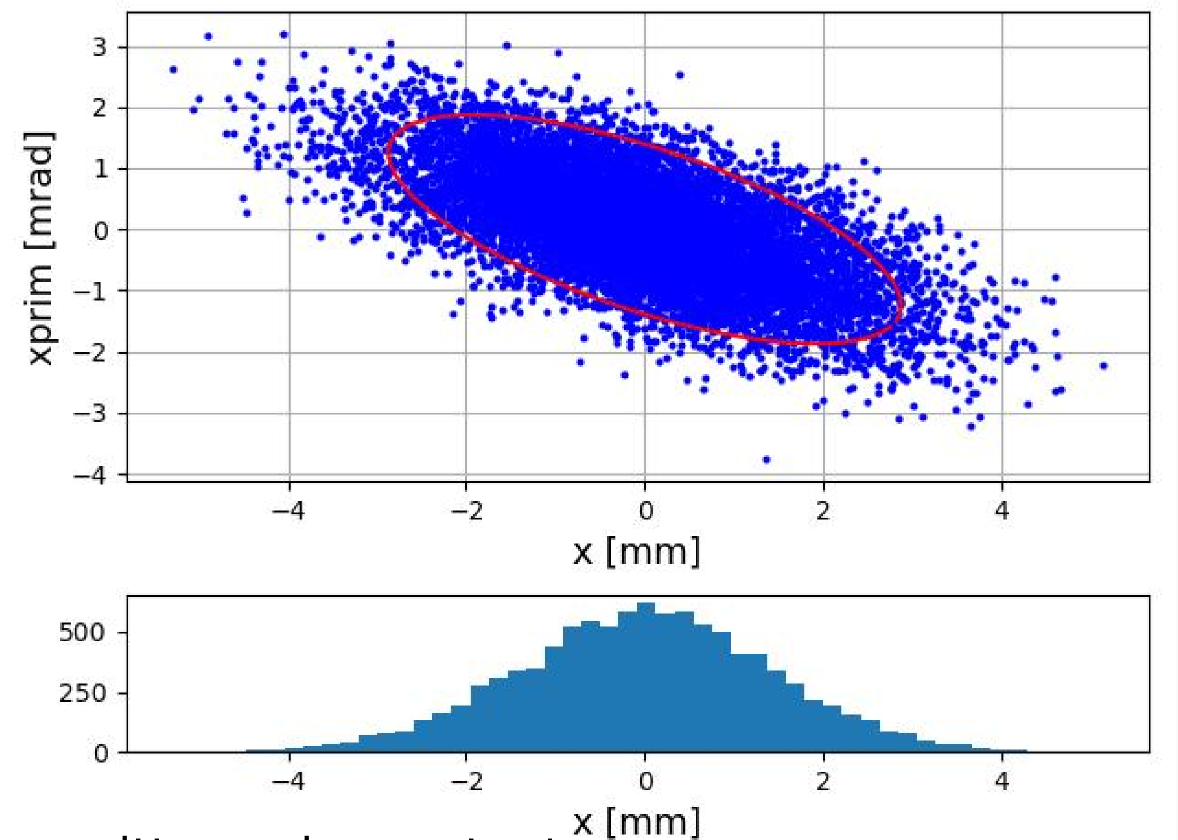


Transverse size

- Transverse sizes of beams can vary:
  - nanometers (10^{-9} m)- electron beam litography
  - micrometers (10^{-6} m) – synchrotron light sources
  - millimeters (10^{-3} m) – eg. LHC
  - centimeters (10^{-2} m) – hadron therapy synchrotrons
  - meters – neutron, neutrino beams
- Beam size changes when traveling through accelerator – for instance it is usually focused on target
- It is better to use about beam emittance

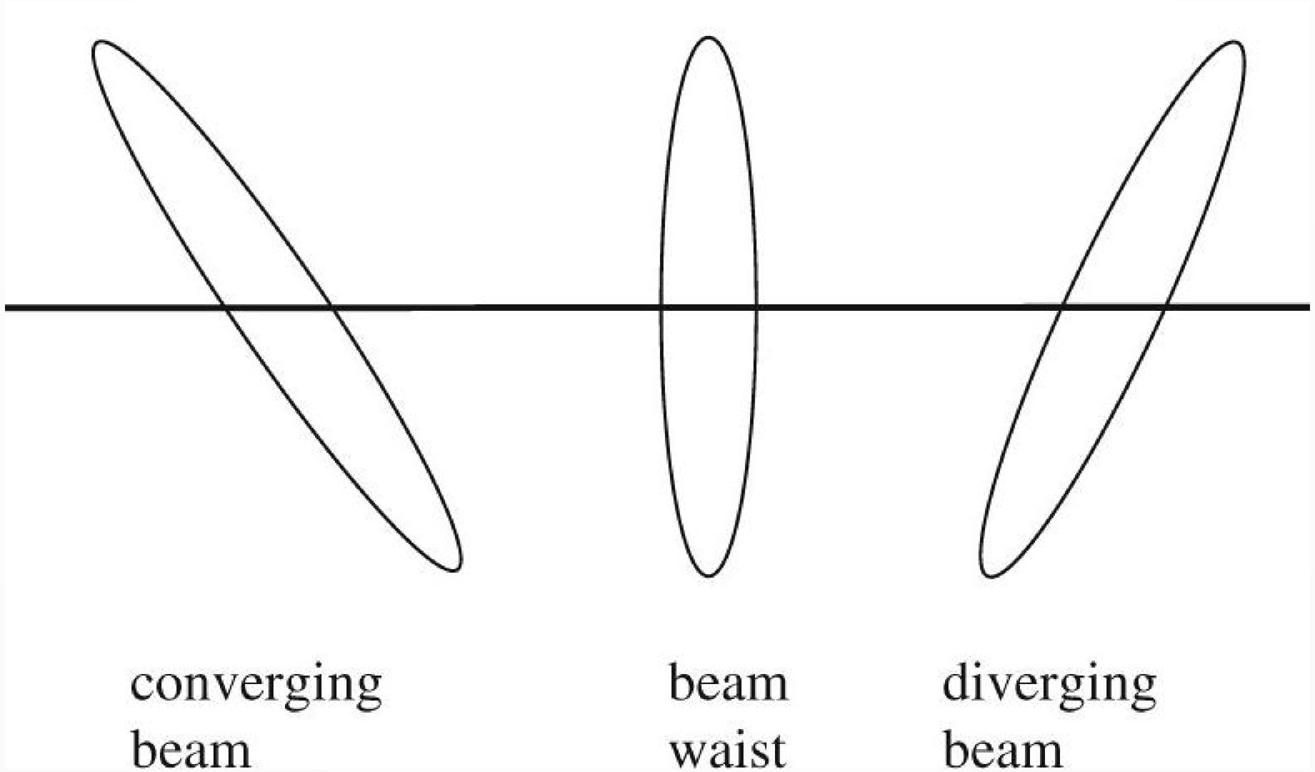
Phase space and emittance

- **Beam phase space** is defined by it transverse position (x) and divergence (x')
- Both distributions have usually approximately gaussian shape
- The surface of the ellipse containing 95% of the beam particles is called **emittance** ($\varepsilon_{95\%}$)
- People also use RMS-emittance (ε_{RMS}), surface of ellipse containing 1 Root Mean Square (RMS) of the particles (40% for 2D gaussian distribution)



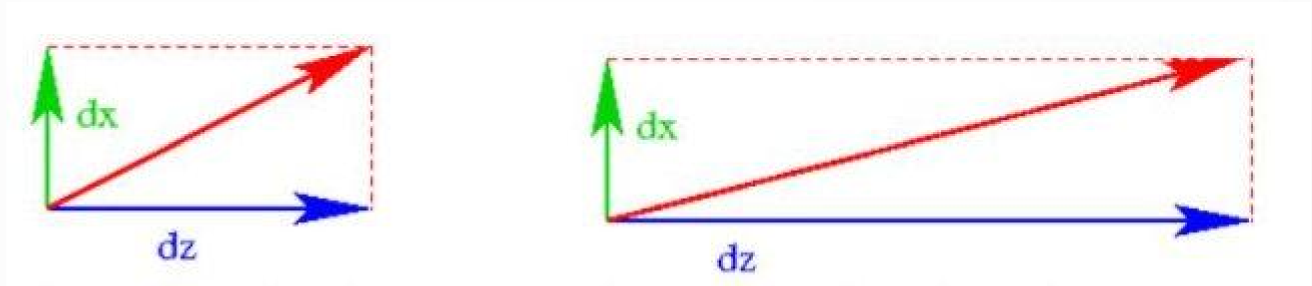
- In lack of acceleration and dissipative processes emittance is constant (Liouville's theorem) → ion source must produce good emittance as it cannot be (easily) decreased

Understanding the beam phase space



Emittance and acceleration

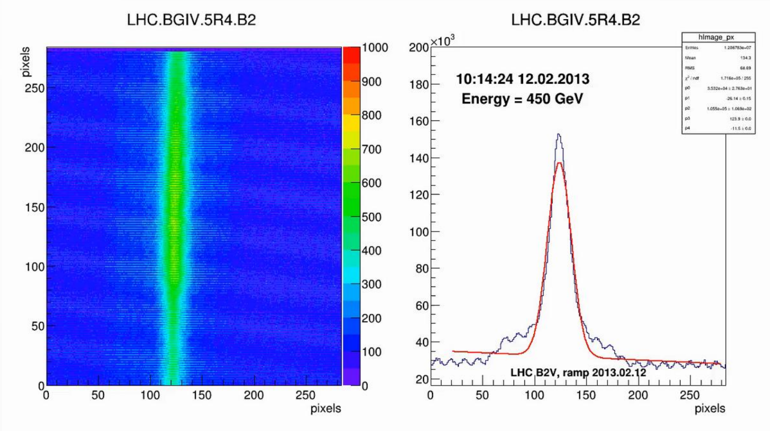
- During acceleration (energy ramp) the longitudinal momentum increases while transverse remains the same



- Therefore the divergence of particles decreases, so the emittance shrinks!
- **Normalized emittace is conserved during acceleration**:

$$\varepsilon_n = \beta\gamma\varepsilon_{RMS}$$

- Units: [mm*mrad], [π*mm*mrad]
- Typical values for medical ion beams: 0.5-1.0 [mm*mrad]
- Synchrotron light sources emittance reach ~1 nm*mrad



I hope at this point you understand the role of accelerators in our civilization and variety of beams produced.

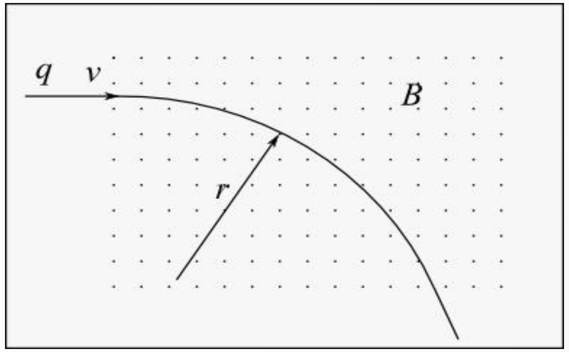
You got familiar with a concept of beam emittance.

Let’s go to some details.

Acceleration techniques

- Which field to use for the acceleration?

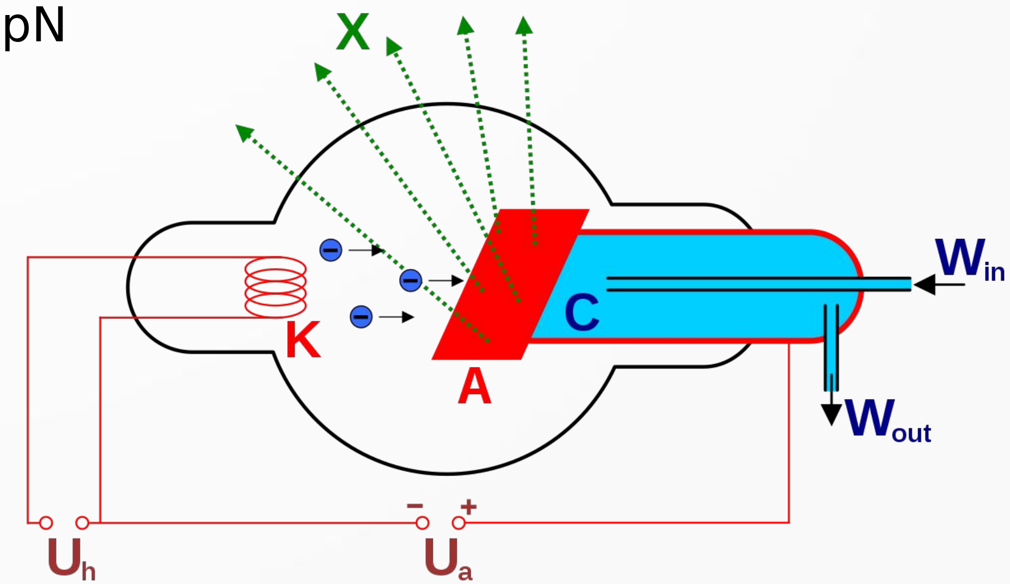
  electric force: $\underline{F}=q\underline{E}$ - acts along the field lines
  magnetic force: $\underline{F} = q(\underline{v} \times \underline{B})$ - acts perpendicular to field lines and to particle velocity **– no acceleration**



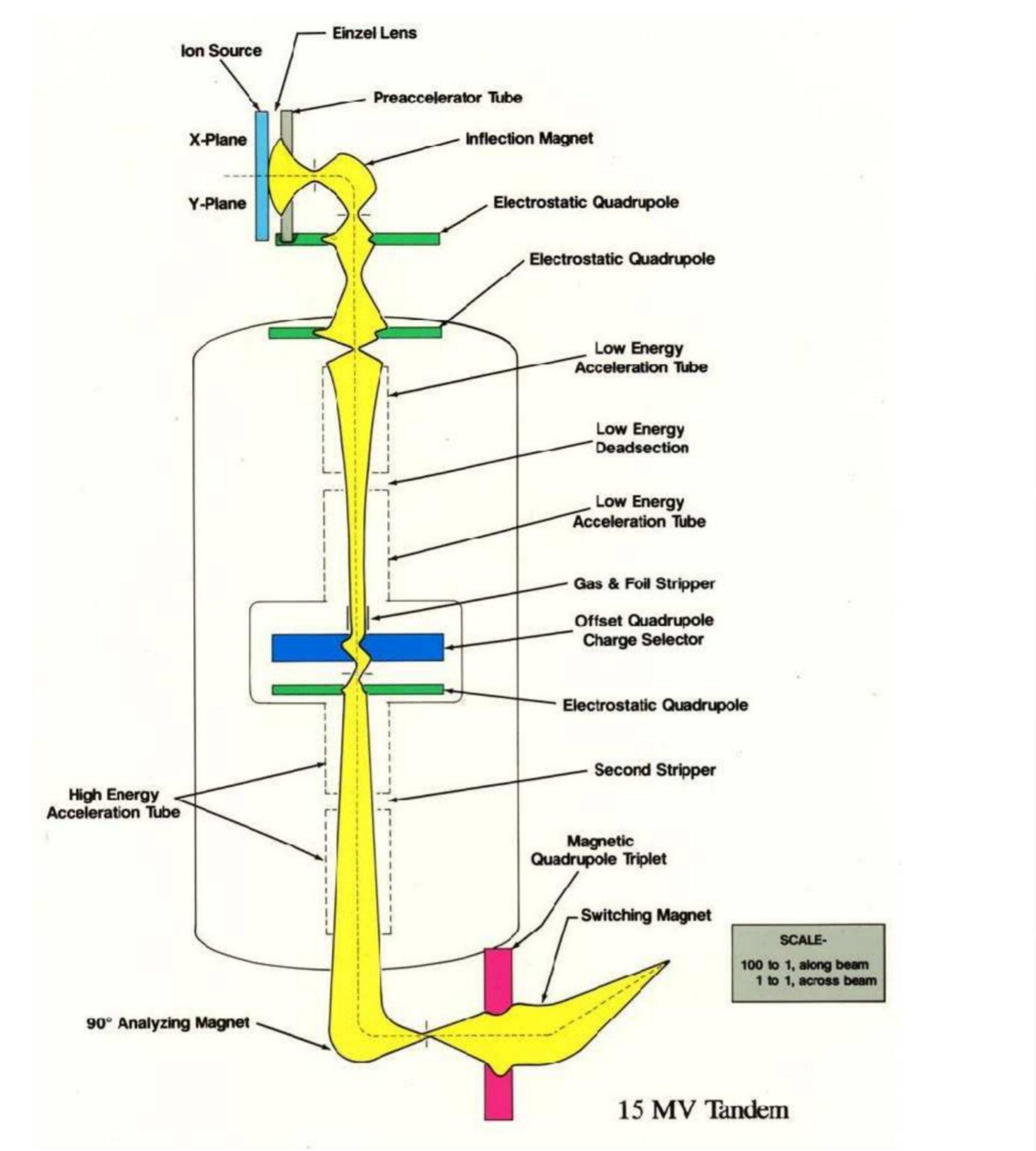
- Force magnitudes:
  - electric: 20 MV/m(*), F=3.2 pN
  - magnetic: 1.5 T(*), v(p@20 keV)=0.007c, F=0.5 pN

  (* typical values) but at v→c: F=70 pN (!)

- Electrostatic acceleration:
  - Continuous beam, small energy spread
  - Easy to tune energy



Electrostatic acceleration

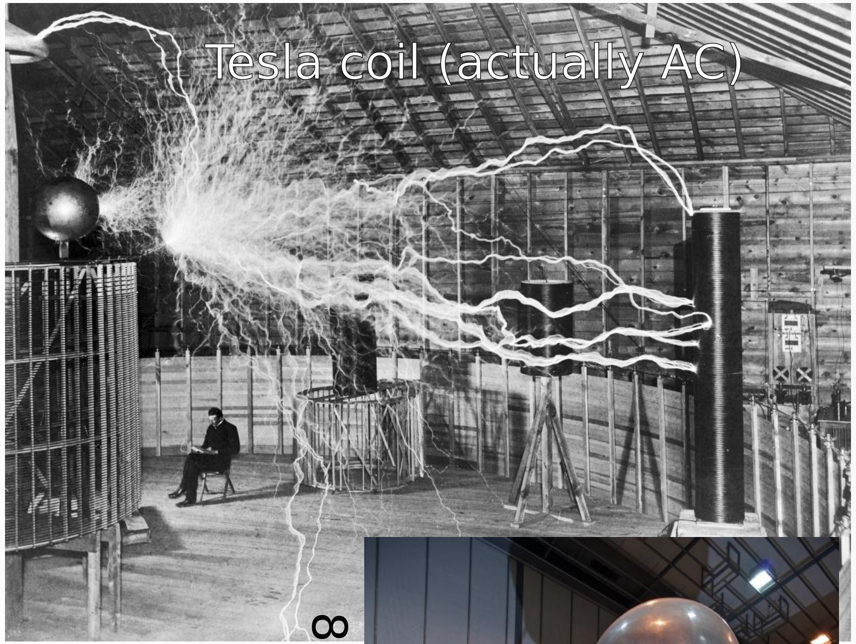


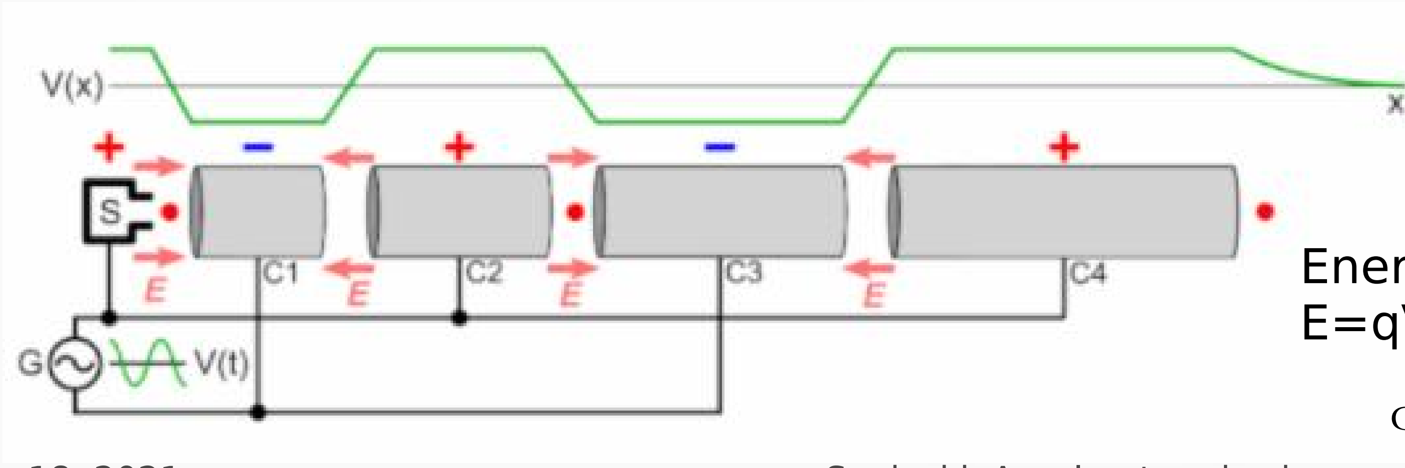
Beam envelopes in a 15 MV Tandem accelerator from NEC (courtesy of G. A. Norton, NEC)

- Tandem accelerator, doubling the energy
- Energies in range 1-40 MeV
- Energy spread 10^{-4}
- Electrostatic lenses keep the beam focused (first mention of transverse focusing)
- Still used for instance:
  - Ion Beam Analysis
  - as pre-accelerators for larger facilities
  - ion implantation
- See Giovanni's, Aris'es and Fehima's presentations

Acceleration techniques: RF

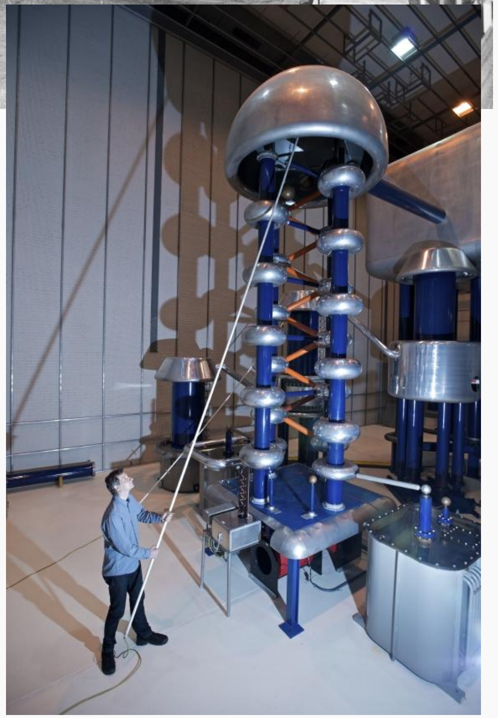
- MV electrostatic generators are huge
- Safe handling these voltages is difficult
- Idea: use oscillating electric field - Gustav Ising (1924)
- First device: Rolf Widreoe (1928)
- Note: beam is bunched and energy tuning is not as easy as for electrostatic machines
- Common name: Drift-Tube Linac (DTL)





Energy gain:
$E=qV_{RF}\sin(\phi)$,

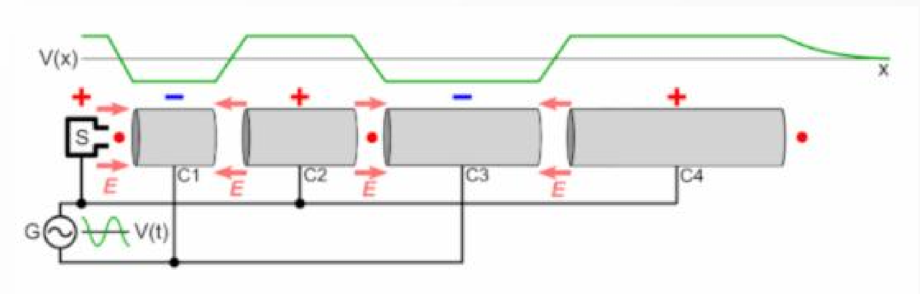
ϕ - phase

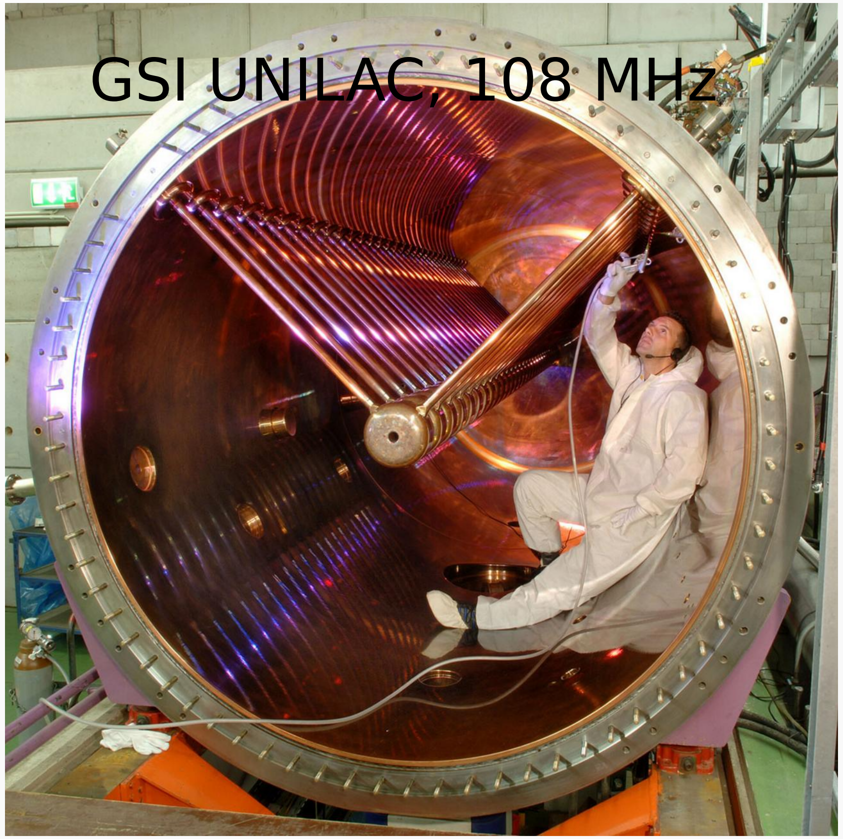


810 kV generator (PSI)

Acceleration techniques: RF

- With increase of energy, the drift tubes gets longer, higher frequency allows them to be shorter (careful, the first drift tubes can be too short)
- Typically MeV ion beams require frequencies 36-750 MHz, and elements of the system work like antennas emitting most of the energy
- Therefore the accelerator is enclosed in resonant tank and fed by RF source (no need to make electrical connections to the drift tubes)

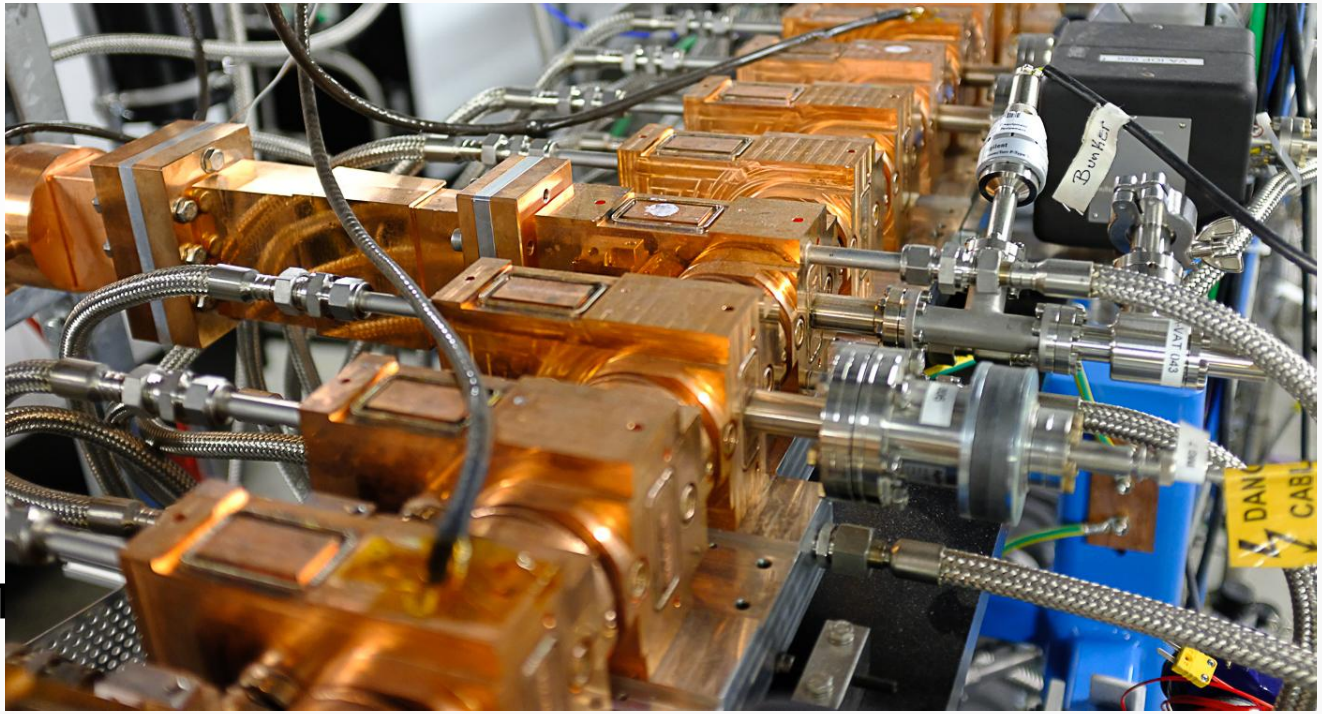




Very high frequencies

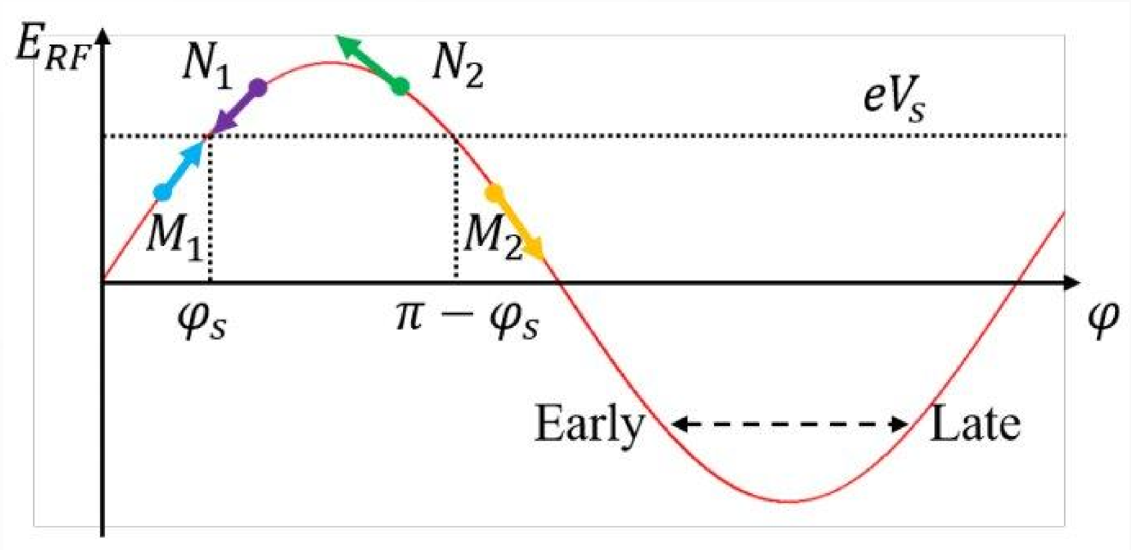
- Higher frequencies allow for smaller linac and higher acceleration gradients
- Maximum frequency depend on particle velocity and accuracy of machining of structures
- For electrons, which are fast relativistic (1 MeV – 95% c) the “golden standard” frequency is 3 GHz (SLAC)
- CLIC developments at CERN (e.g. new industrial CNC machines developed for this project) pushed it to 12 GHz
- For protons new developments:
  - 750 MHz (CERN, low energy)
  - 3 GHz (AVO-ADAM, proton therapy, 70-230 MeV)
  - Accelerating gradient >30 MV/m

AVO-ADAM
SCDTL



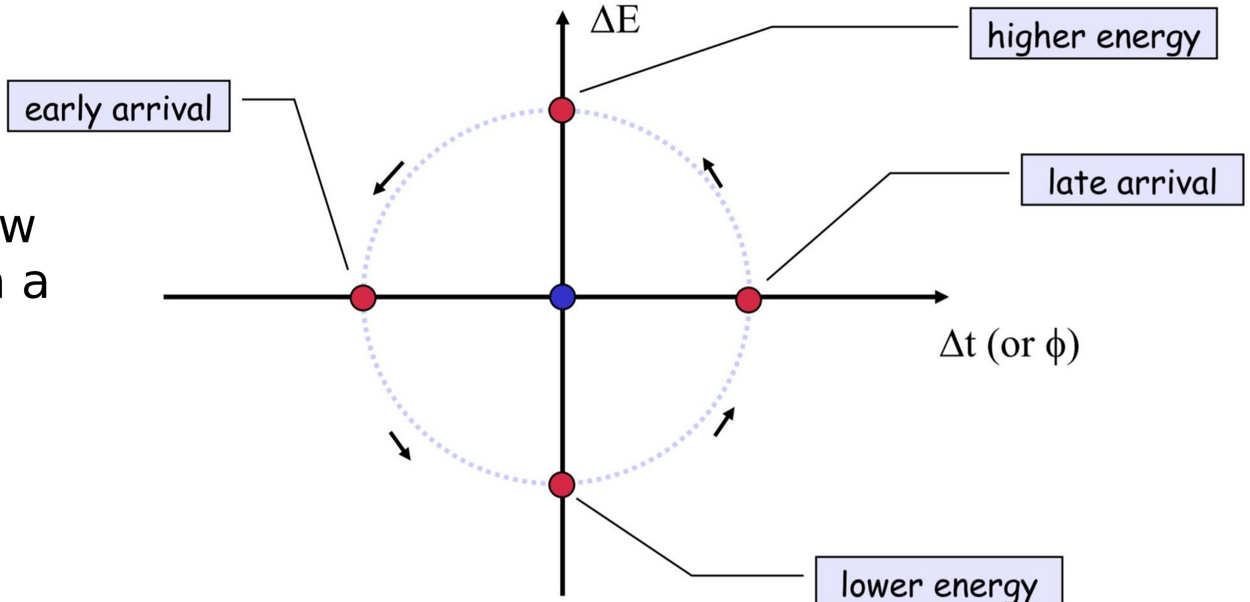
Tuning the RF cavity

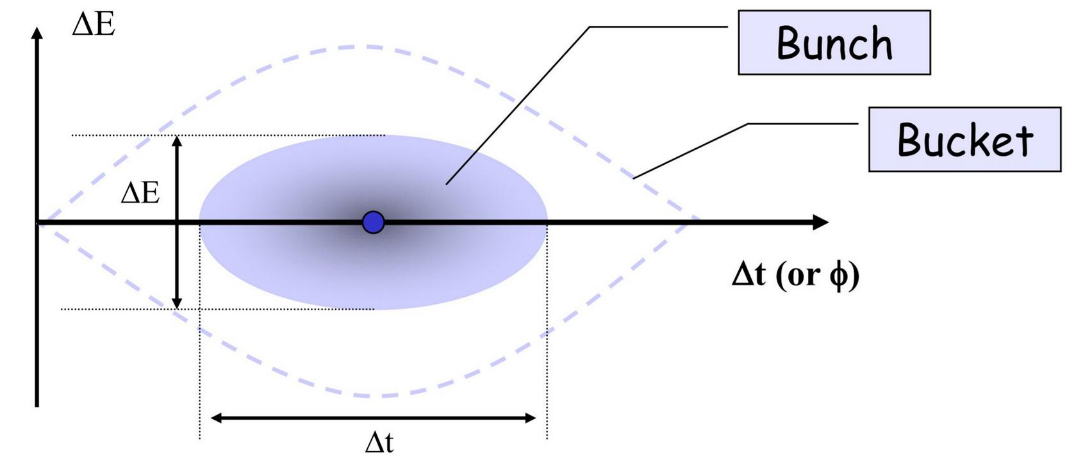
- Phase stability = longitudinal focusing
- Particles arriving too early (higher energy) to the accelerating gap experience smaller accelerating field
- Particles arriving too late (lower energy) experience higher accelerating field
- For synchrotrons it is a bit more complex because the different energy means different orbit and depends on beam energy and machine lattice (transition) – M_2 becomes stable
- The stable particle position (RF set-point, optimal phase) is usually found by optimizing the beam transmission through the linac



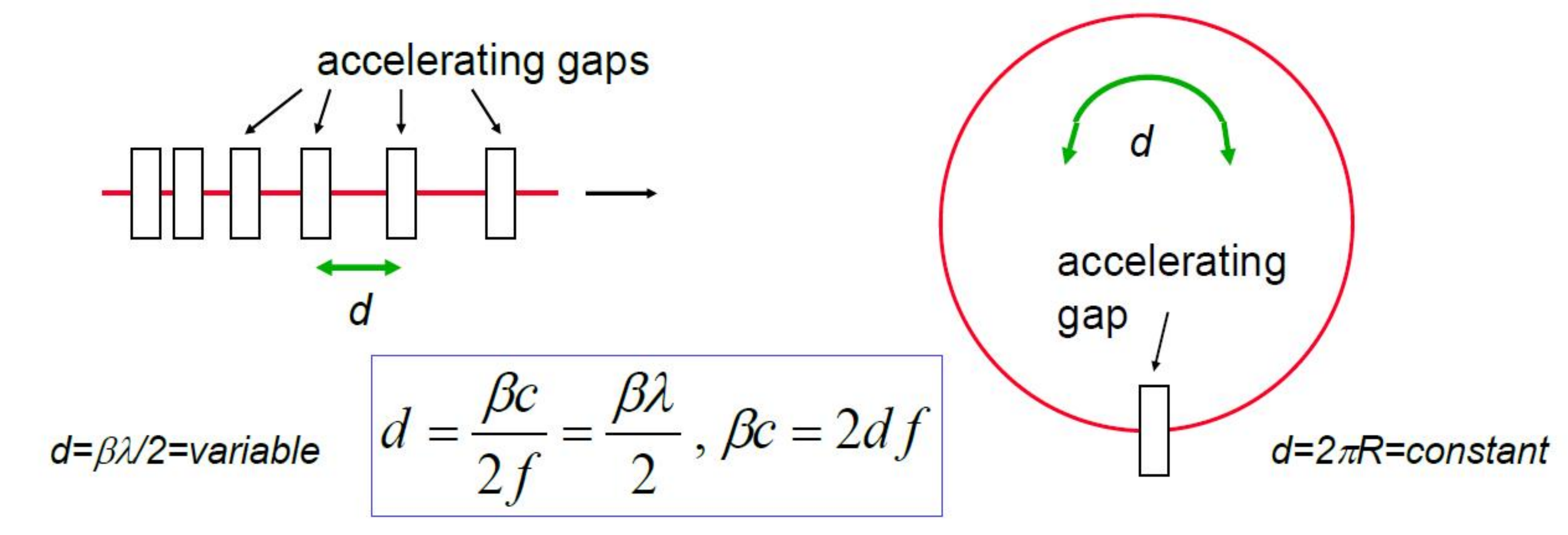
Longitudinal phase space

- Particles oscillate around the synchronous particle position
- This oscillation is called **synchrotron oscillation**, mainly because it is rather slow oscillation so particle must be circulating in a synchrotron in order to observe it
- Particles stay within separatrix
- Longitudinal phase space: energy-phase
- Longitudinal emittance is the phase space area including all particles $4 \cdot \pi \cdot \sigma_{\Delta E} \cdot \sigma_{\Delta t}$
  - Unit [eV·s]





Linear versus circular machines

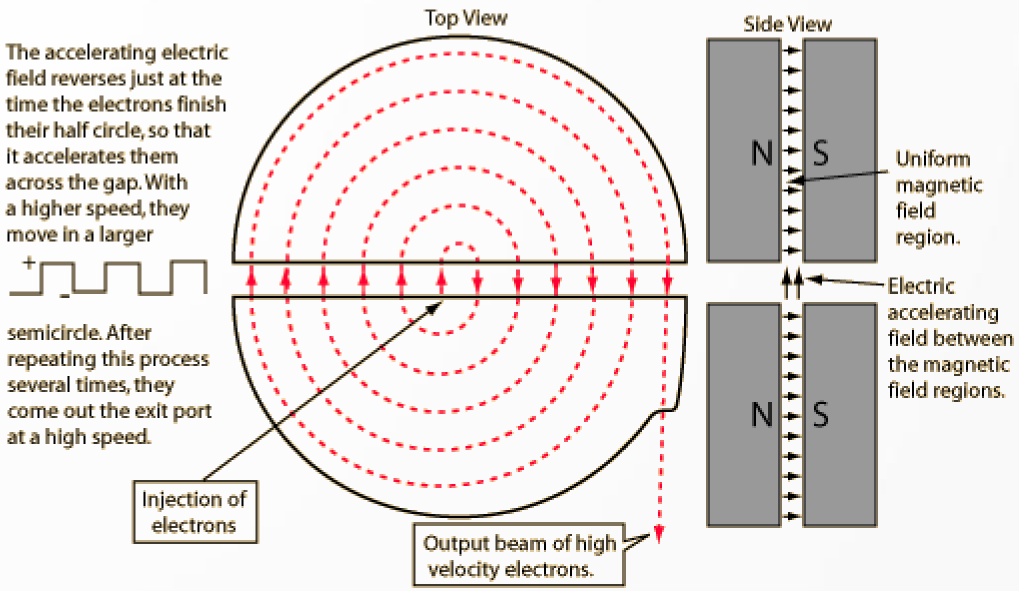


$$d = \frac{\beta c}{2f} = \frac{\beta\lambda}{2}\,,\ \beta c = 2df$$

- In linear machine each accelerating gap is used once, high accelerating gradient is important; in circular machine beam comes back to the same cavity multiple times, gradient is not so crucial
- Linear machine: distance between gaps increases; circular: frequency of the cavity increases
  (in non relativistic regime)

Circular machines: cyclotrons

- Proposed by E.O. Lawrence (1929) and build by Livingstone (1931)
- Vertical magnetic field bends the particle trajectory
- Gap between the dees is used for acceleration
- Radius of the particle increases with its energy
- Lorentz and centrifugal forces balance:
  - $qvB=mv^2/r$
  - $\omega = v/r = qB/m$
    (Larmor frequency)
- Modern cyclotrons: multiple cavities, RF frequency ~100 MHz



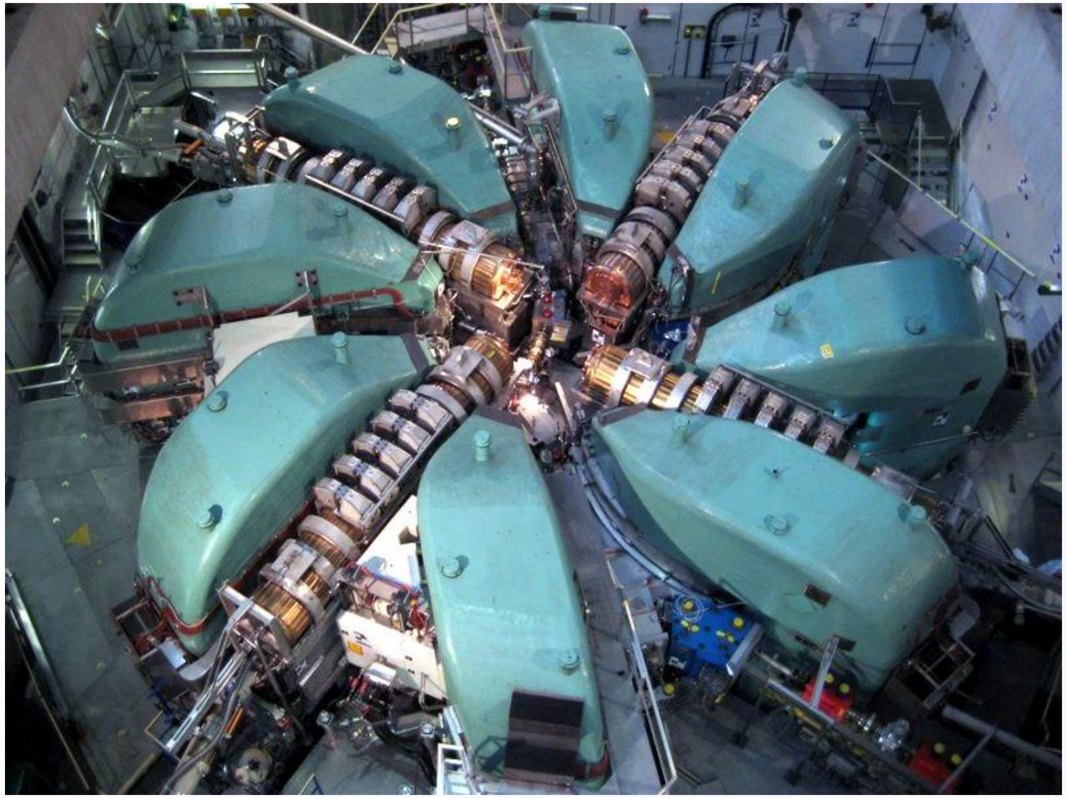
Livingstone cyclotrone, diam. 10 cm. 80 keV protons

Limitations of cyclotrons

- For relativistic particles mass increases:

$$\omega = v/r = qB/\mathbf{m(E)}$$

- Need to increase magnetic field (isochronous) or frequency (synchrocyclotrons)
- At high energies large vacuum chamber becomes difficult (large disc with vacuum)
- Most of proton therapy machines are based on cyclotrons (Varian, IBA)
- The extraction energy is constant (e.g. 230 MeV), must be degraded if needed (e.g. for shallow tumors)

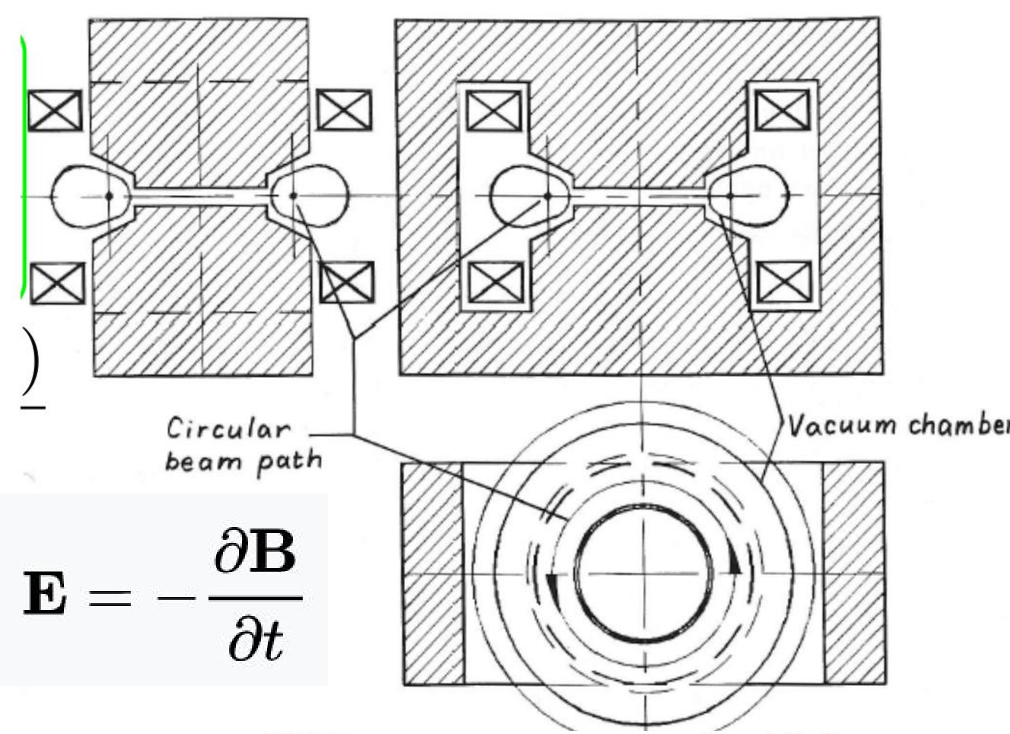


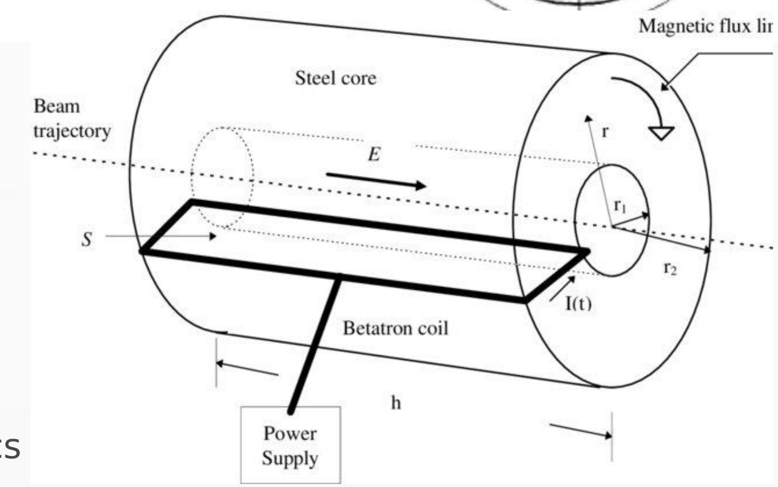
PSI cyclotron, isochronous, sectored, AVF (azimuthally varied field)
protons at 590 MeV, P_{beam} = 1.3 MW
Diameter 15 m
4 RF cavities, 8 magnets

Betatron

- Electrons are relativistic at 500 keV – classical cyclotron not useful
- Magnetic field increases with energy, orbit is constant
- Energy is transmitted through transformer effect: increasing magnetic field generates vortex electric field which accelerates electrons
- Acceleration takes place over ¼ of the RF cycle
- Betatrons were used to produce electron beams up to 300 MeV
- Similar idea of final smooth acceleration is used for slow extraction in CNAO and MedAustron (betatron core)

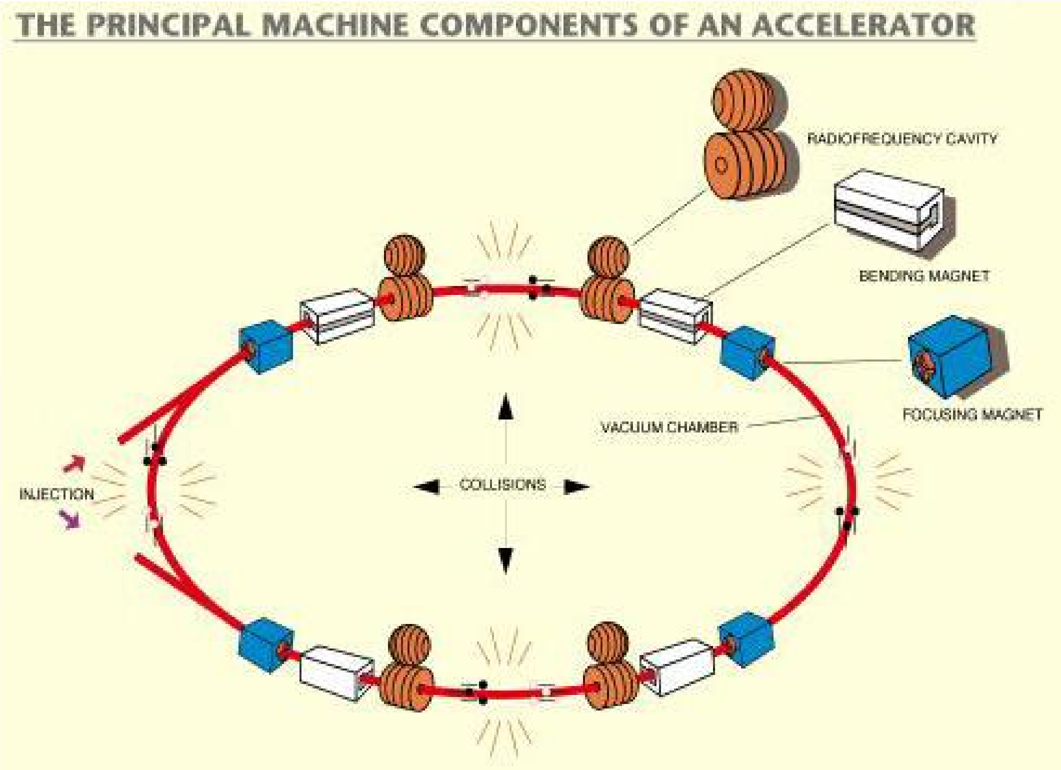
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$





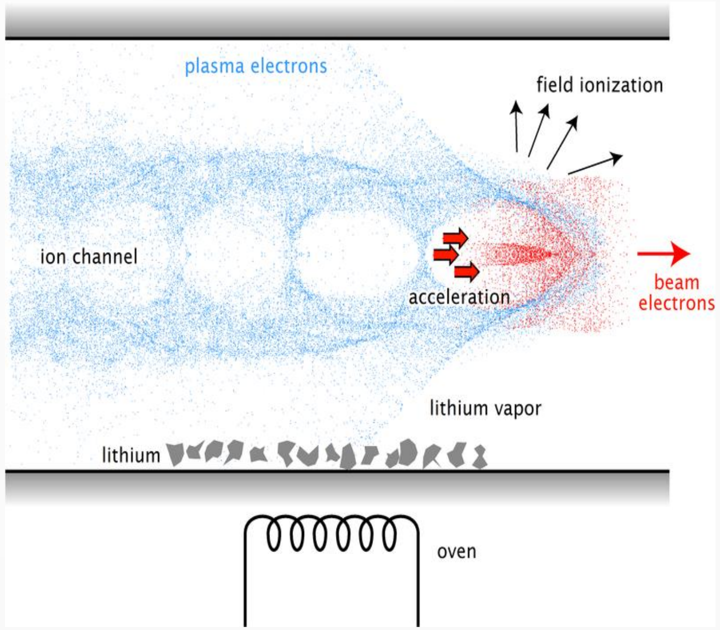
Synchrotron

- Betatron acceleration method is limited by magnet size and iron saturation
- For larger energies need to split the magnets
- And use much more efficient RF-acceleration
- Synchrotron idea: Vladimir Veksler (1944)
- Origin of the name: synchronous change of RF frequency with magnets’ current
- First electron synchrotron: Edwin McMillan (1945, independently from Veksler)
- First proton synchrotron: Marcus Oliphant (1952)



Future of acceleration techniques

- The best cavities reach 50 MV/m (less in regular operation e.g. 30 MV/m European XFEL, DESY, Germany)
- Vacuum breakdown limits possible fields
- Idea: use plasma – it is already broken down
- Separate electrons from ions using strong laser pulse, generate locally fields of 100 GV/m (factor 5000!)
- Currently plasma acceleration, dielectric acceleration, laser ion sources are very active fields of research



Accelerators started with electrostatic machines.
The crucial development was RF resonant acceleration.
Phase stability keeps beam bunched.
Cyclotrons and Synchrotrons.

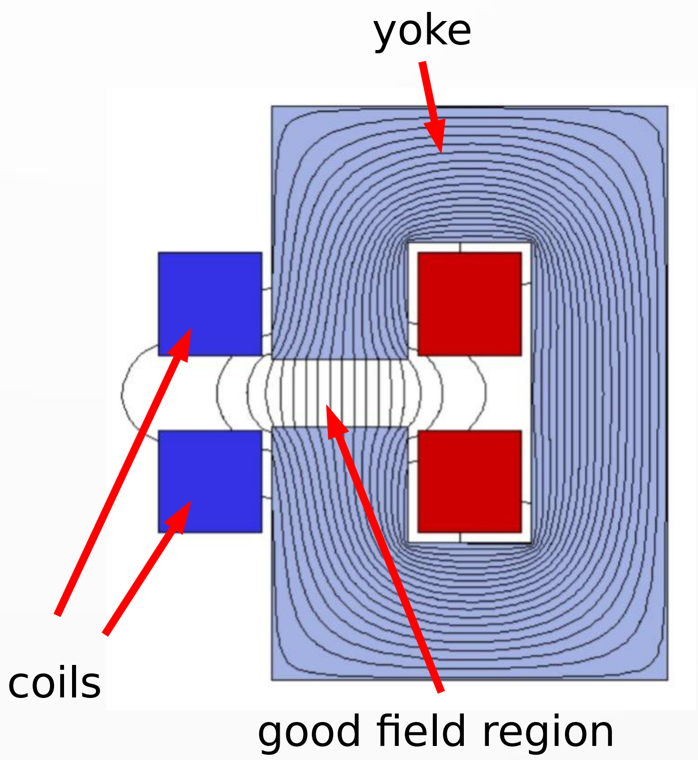
The future may be in plasma accelerators.

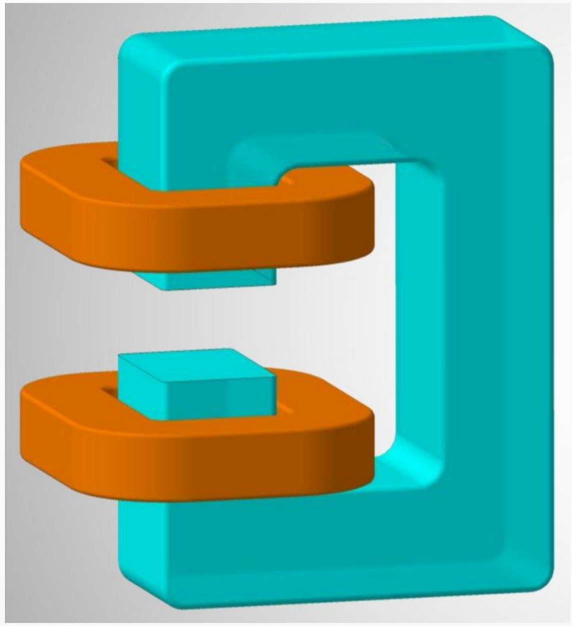
Beam trajectory

- Beam can be steered using electric or magnetic fields
- Magnetic field is more effective for high velocities, because:

  $\underline{F} = q(\underline{v} \times \underline{B})$

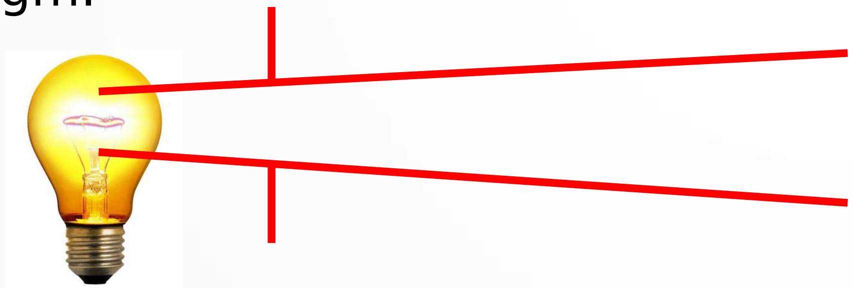
- Dipole magnets steer the beam
- Particles of the same **magnetic rigidity** have the same trajectory:
  - $p/q = B\rho$, ρ-bending radius
  - $^{12}C^{6+}$ and $^{4}He^{2+}$ can circulate in the same machine having the same kinetic energy per unit mass eg. 430 MeV/u

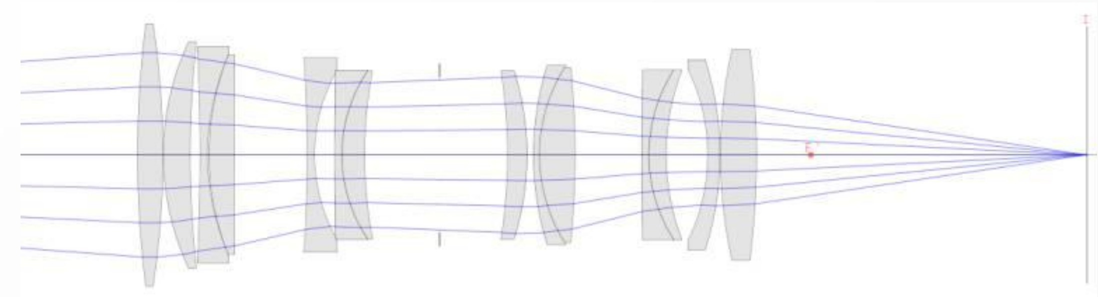




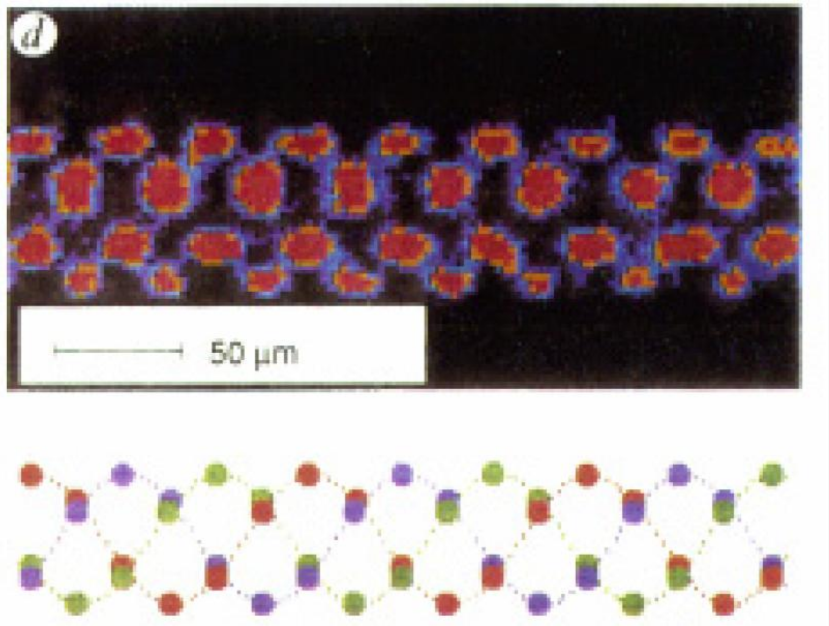
Beam stability

- Beam is naturally divergent; think about a “beam of light” (from a lamp) and a diaphragm:



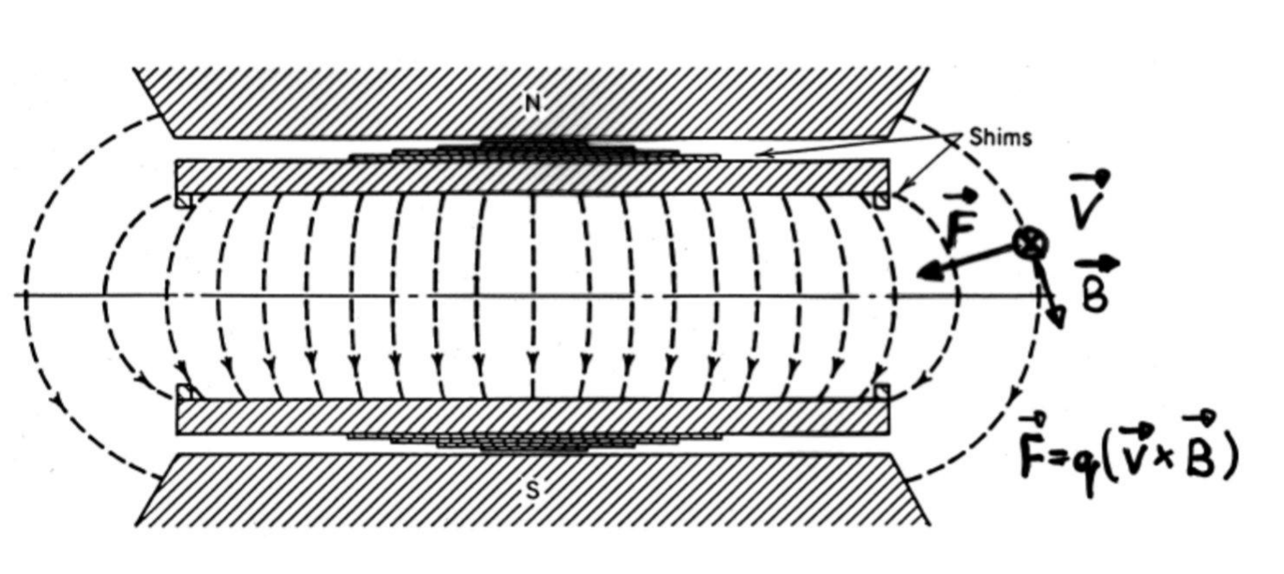


- Set of lenses focus the beam of light
- BTW laser light stays “collimated” without lenses: this is because of spatial coherence of photons in the laser beam
- Can we do similar with ion beams? Not really! Ions are fermions not bosons, they are charged (Coulomb repulsion forces);
- The crystalline ion beams reach limits of ion beam emittance, beam density

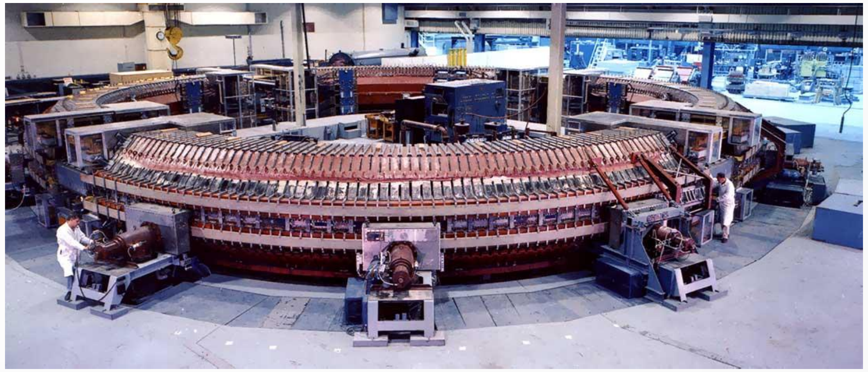


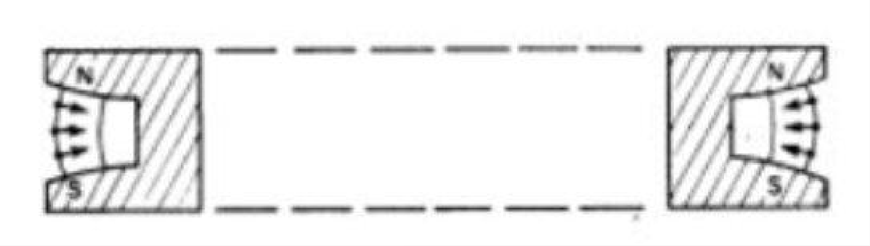
Weak focusing

- The principle – radial field gradient leads to forces which focus the beam
- This mechanism is called weak focusing
- Every dipole magnet gives vertical focusing at its edges
- In synchrotrons, which store the beam over long time, weak focusing is not enough!



Weak focusing in cyclotron

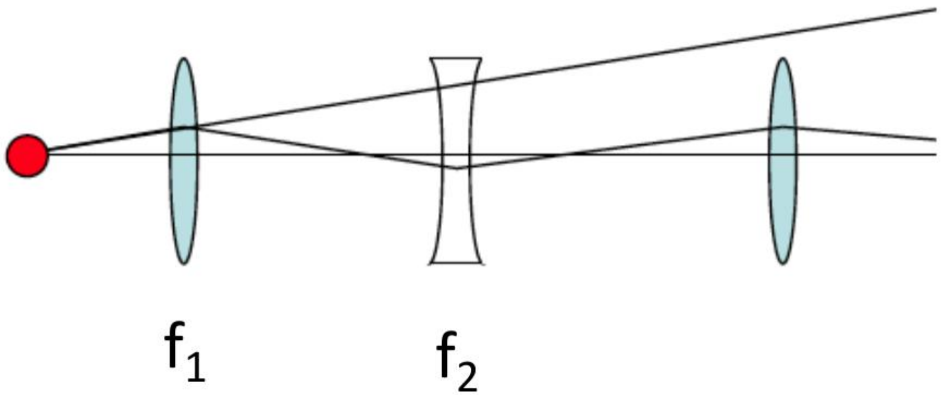




Cosmotron – 3 GeV proton synchrotron, BNL 1953

Strong focusing

- Idea: N. Christofilos, 1949 (patented but not published), rediscovered independently by E. Courant, M. Livingston, H. Snyder in 1952
- Strong focusing principle: the net effect on a particle beam of charged particles passing through alternating field gradients is to make the beam converge



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Consider $f_1 = f$, $f_2 = -f$ ➔ $F = f^2/d > 0$

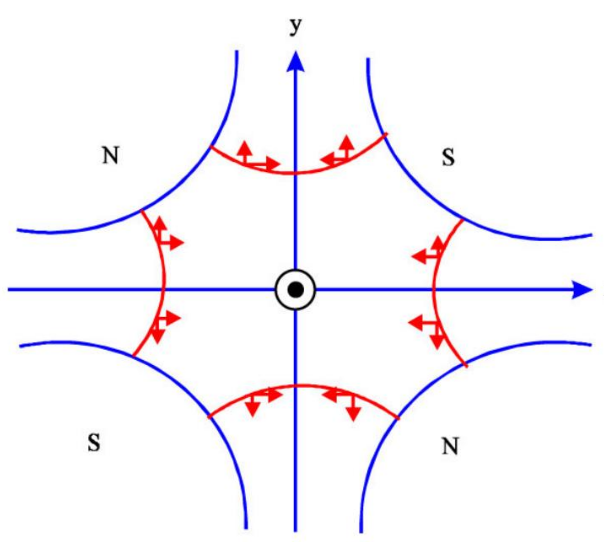
Quadrupoles

- Quadrupole magnet provides focusing in one plane and defocusing in other
- F = qvB(x) = qv(g*x)
- Magnetic field gradient:

$$g = \frac{2\mu_0 nI}{r^2}\left[\frac{T}{m}\right]$$

- Gradient normalized to rigidity:

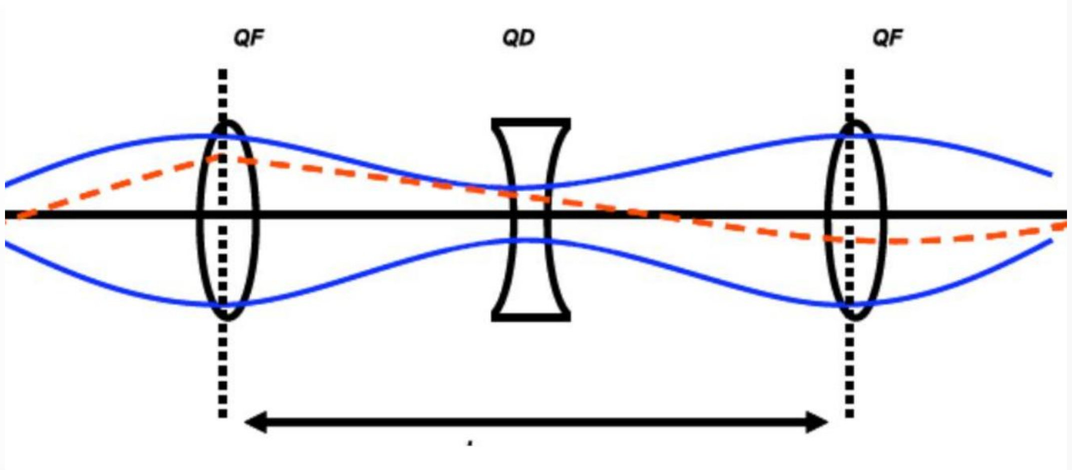
$$k = \frac{g}{p/q}\left[m^{-2}\right]$$

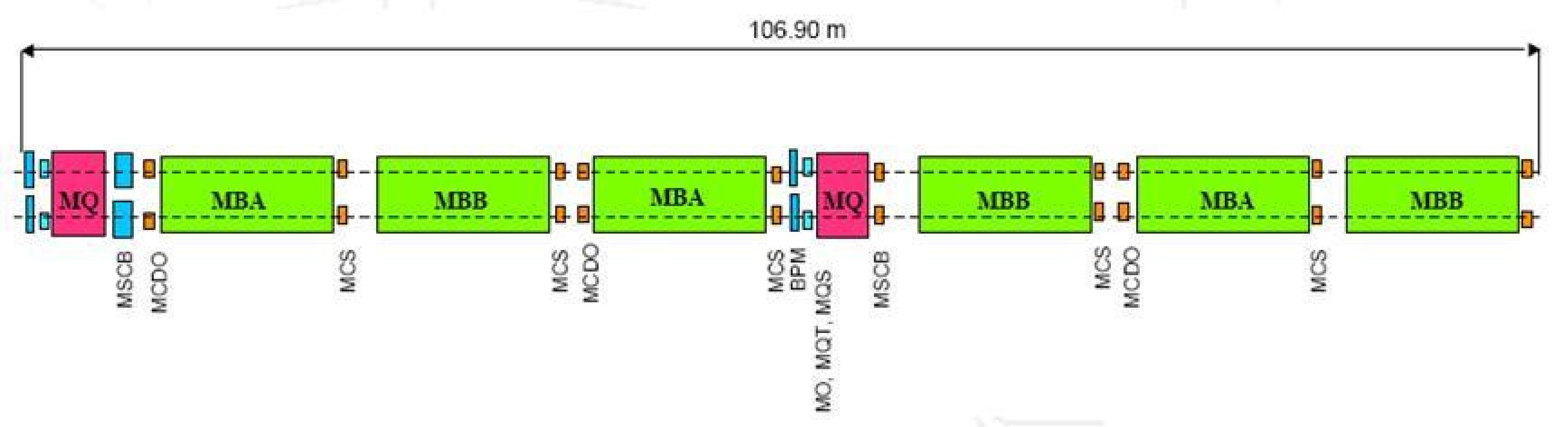


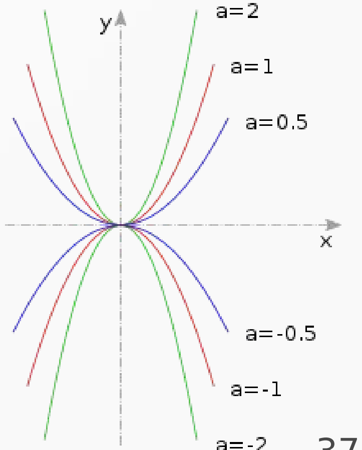
The red arrows show the direction of the force on the particle

Synchrotron cells

- As we've seen we need a system of lenses (i.e. of quadrupoles)
- Focusing-Defocusing (FODO) – the easiest elementary cell layout of a synchrotron
- Dipoles, placed between quadrupoles
  - add weak focusing
- Real example: LHC FODO
  184 FODO cells in LHC arcs
- Other cells often used (e.g. multi-bend achromat to minimize beam size in light sources)

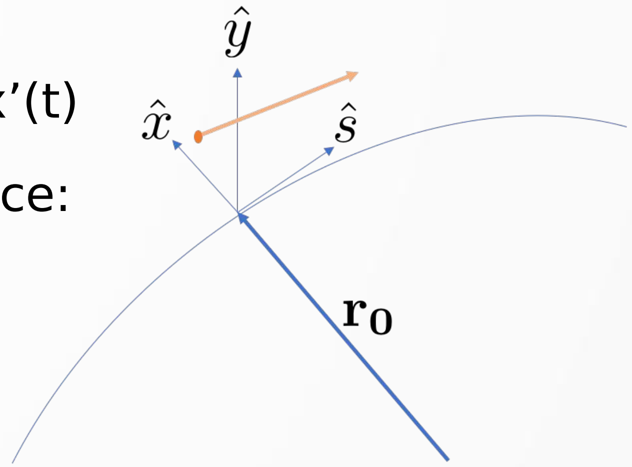






Equation of transverse motion (I)

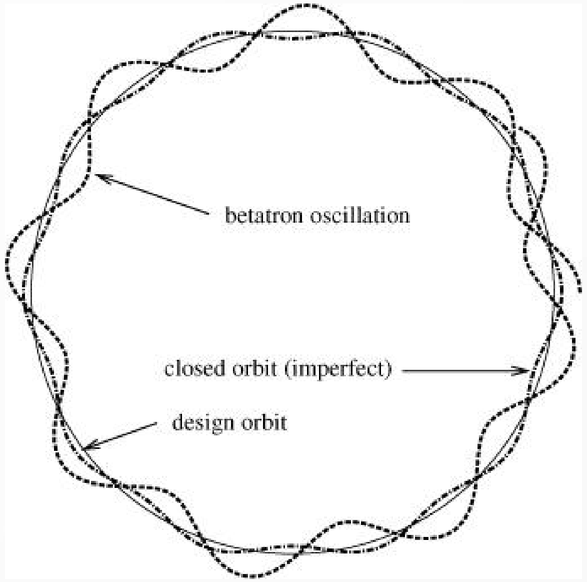
- First: reference system
- Equation of motion: position of particles in function of time x(t), x'(t)
- Particles move through lattice with constant velocity, so we replace: time (t) →position along the machine (s)
  - dx/dt = dx/ds*ds/dt = (dx/ds)*v
  - $d^2x/dt^2=d/dt(dx/ds)*ds/dt+dx/ds*d^2s/dt^2=(d^2x/ds^2)*v^2$
- Equation of motion F=ma=qvB:
  - $d^2x/ds^2=qv(g*x)/mv^2$;   k=-g/(p/q); p/q - rigidity
  - $d^2x/ds^2=-kx$ (focusing, harmonic oscillator!)
- Solution is periodic: $x(s) = x(0)*\cos(k^{½}x)+x'(0)*\sin(k^{½}x)$
  (focusing) $x'(s) = x(0)*k^{½}*\sin(k^{½}x)+x'(0)*k^{½}*\cos(k^{½}x)$



reference orbit
or trajectory

Equation of transverse motion (II)

- For defocusing quadrupole: x(s) = x(0)*cosh(k½x)+x'(0)*sinh(k½x)
  x'(s) = x(0)*k½*sinh(k½x)+x'(0)*k½*cosh(k½x)
- General equation of motion (Hill's equation):
  x''(s) + K(s)x(s) = 0
  where K_x=1/ρ + k (includes weak focusing)
- K(L+s)=K(s) – where L is lattice period (eg. length of FODO cell)
- General solution describes quasi-harmonic movement called **betatron oscillations**:



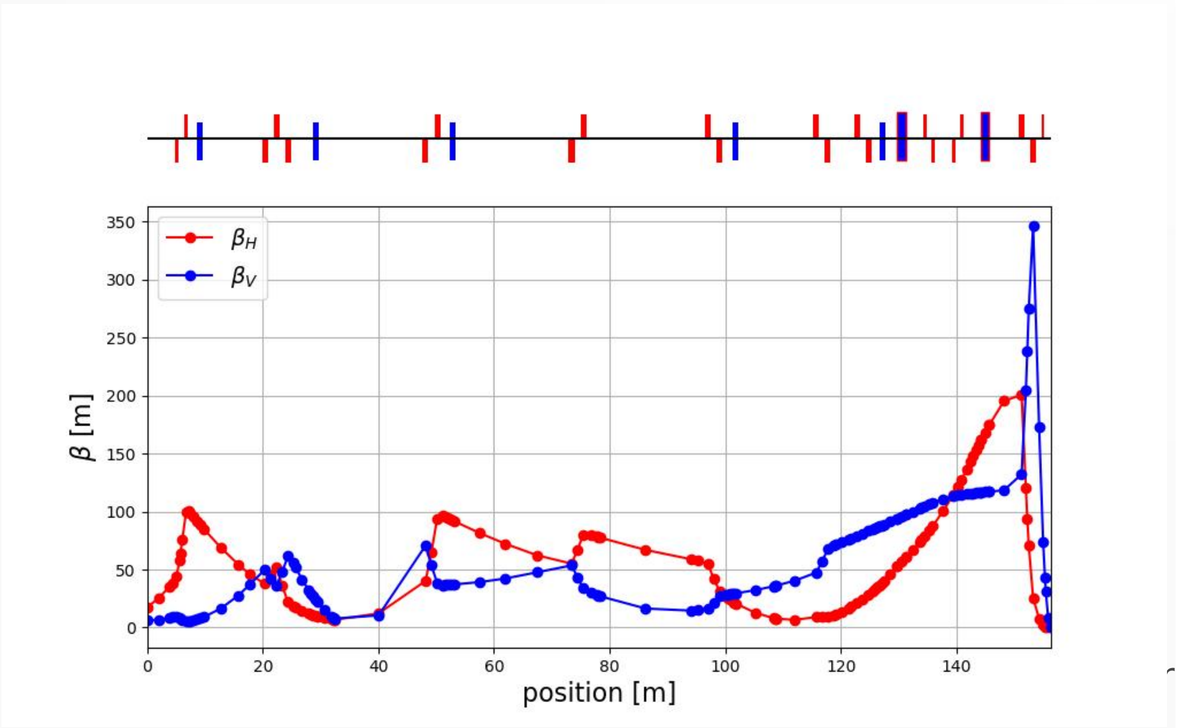
$$x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s)+\phi)$$

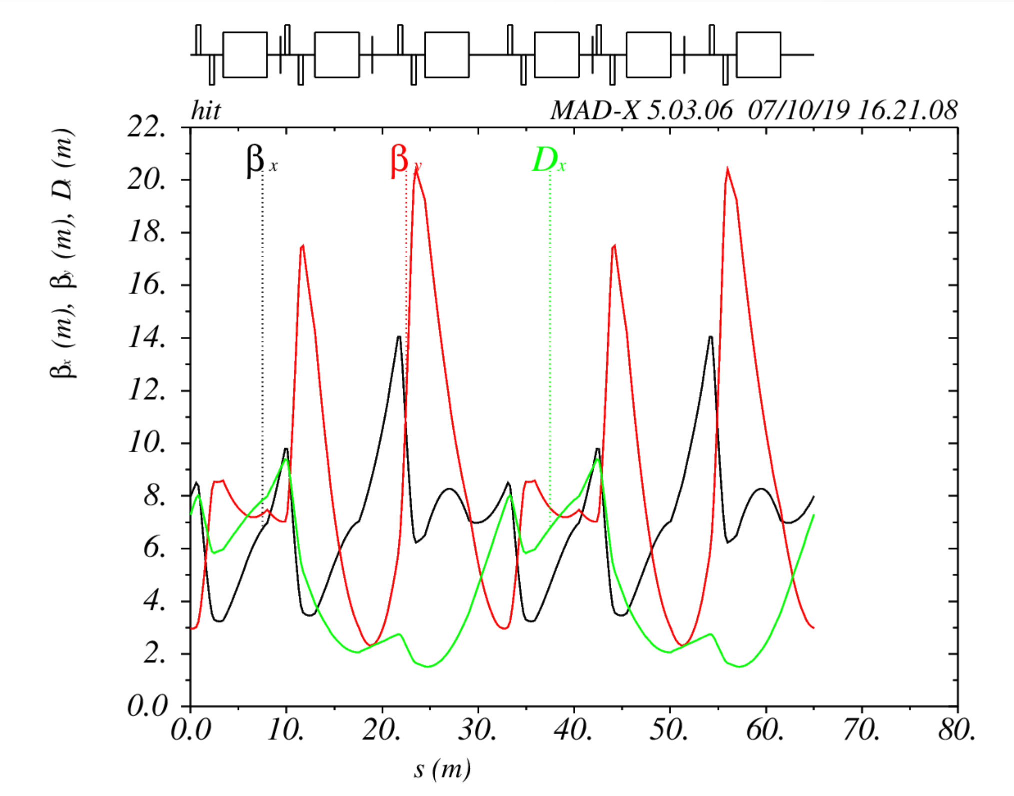
J_x and ϕ – depend on initial conditions

Beta function

- **betatron oscillations** – transverse oscillations of particle in the beam around the design orbit (reminder: synchrotron oscillations are longitudinal oscillations around the stable RF phase)
- **Beta function** β(s) describes amplitude of betatron oscillations along the accelerator or transfer line, often called *beam optics*
- Beam size (often called beam envelope):

$$\sigma = \sqrt{\beta\varepsilon}$$





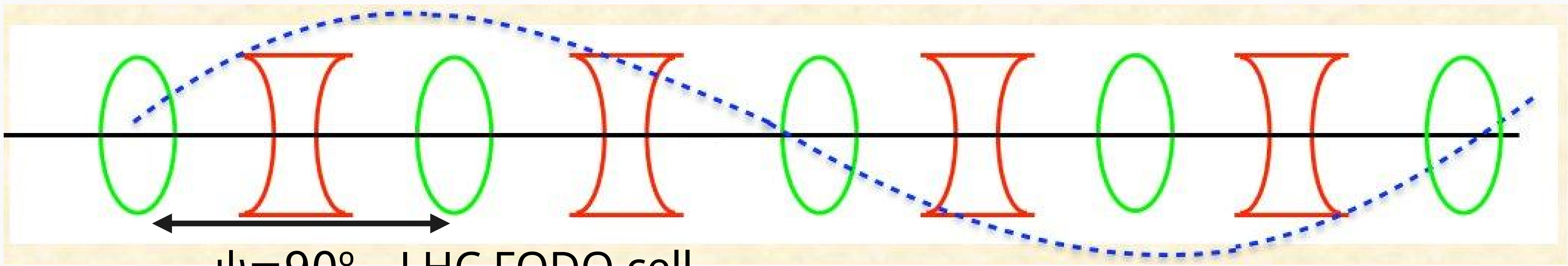
Phase advance and tune

- The difference of betatron motion phase between two points is called **phase advance**:

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

- Number of betatron oscillations per turn is called **tune**:

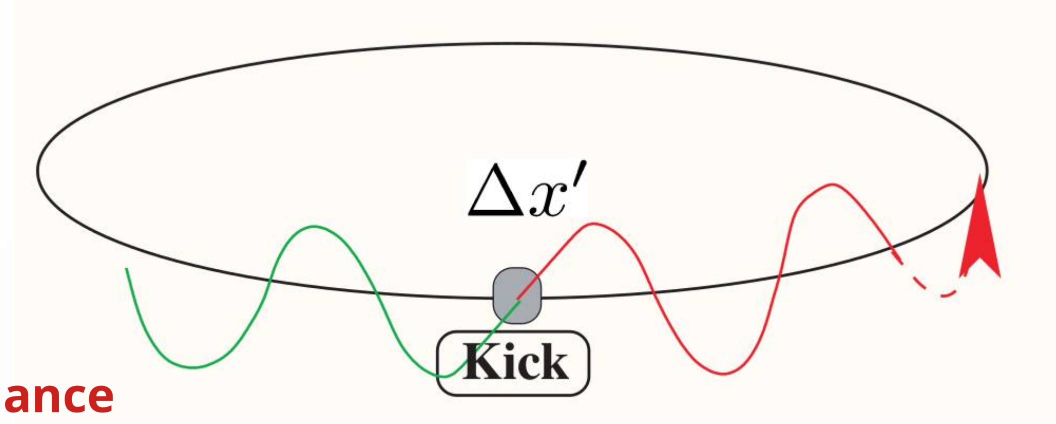
$$Q = \frac{\psi(L_{turn})}{2\pi} = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}$$



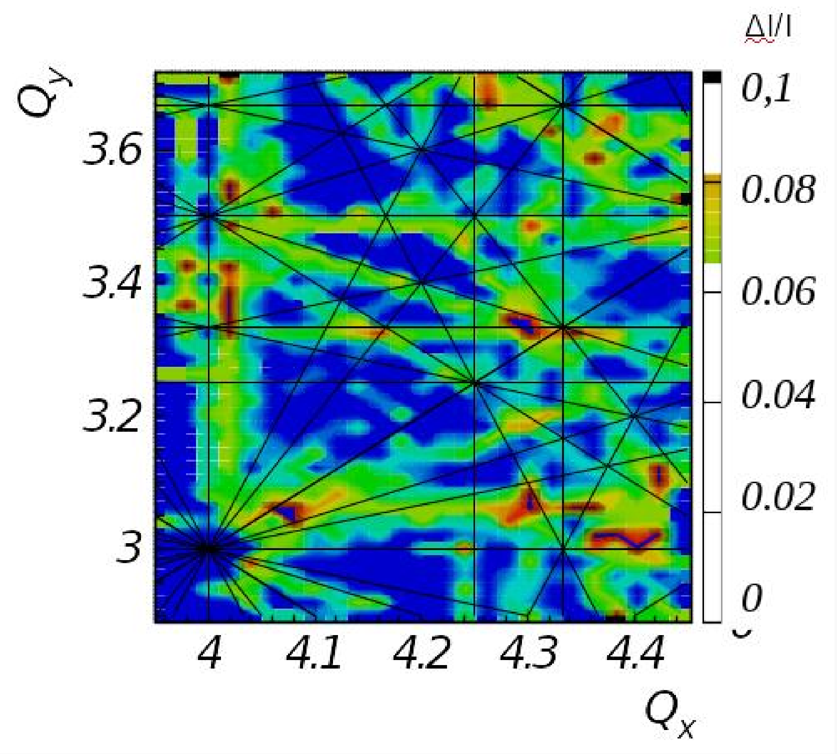
ψ=90º – LHC FODO cell

Tune and resonances

- **Tune** depends on optics (setting of quadrupoles) and can be regulated
- Each small field errors or magnet misalignments create perturbation of the beam trajectory
- If tune is integer (N) or N/2, N/3… the effect of those perturbations add up every turn, **machine is in resonance** and operation is unstable
- This is bad for storage rings but is also a basics of **resonant slow extraction** used in medical machines to extract beam to the patient
- e.g. CNAO/MedAustron working point: $Q_{x,y}$=(**1.672**,1.72)
- Q_x=1.666 is third order resonance which can be excited by sextupole magnets (see later)



GSI SIS18 tune diagram:



Dispersion

- Transverse and longitudinal motions are not independent; they are coupled via dispersion
- **Dispersion is a deviation of the particle trajectory due to momentum difference**:

$D_x(s)=dx(s)/(dp/p)$

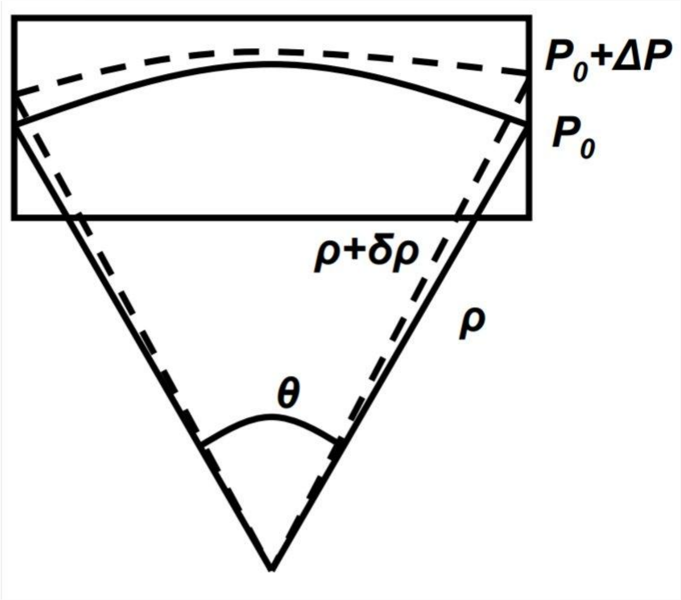
- Similar for angle:

  $D'_x=dx'/(dp/p)$

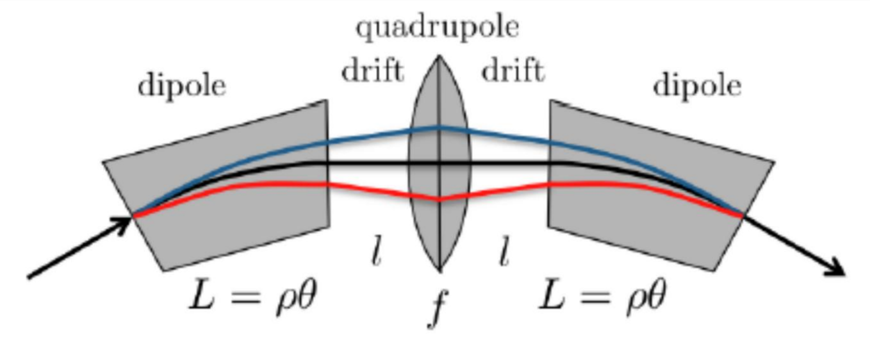
- Dispersion leads to increase of beam size:

$$\sigma = \sqrt{\beta\varepsilon + D^2(\frac{\Delta p}{p})^2}$$

- Dispersion-free regions are often needed: minimize beam size and movement on the patient, maximize luminosity, measure emittance

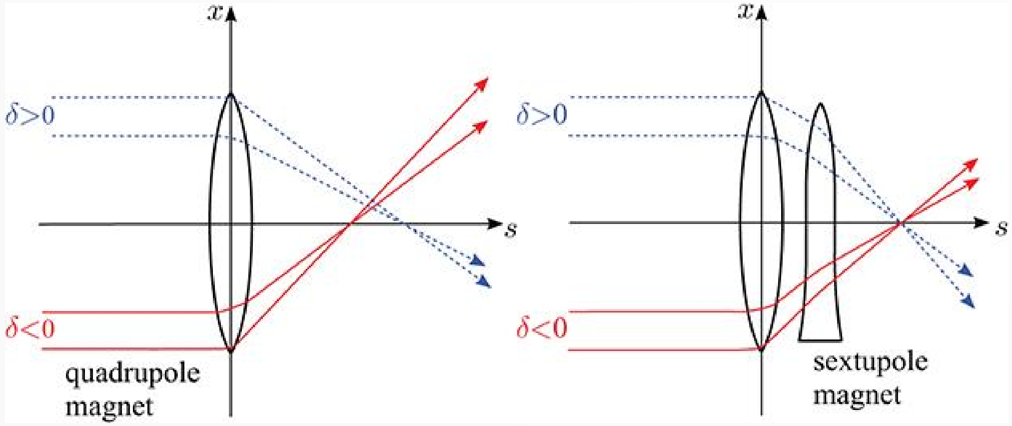


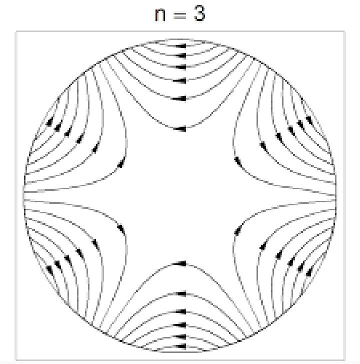
There are many ways to remove dispersion, eg. double-bend achromat (DBA):

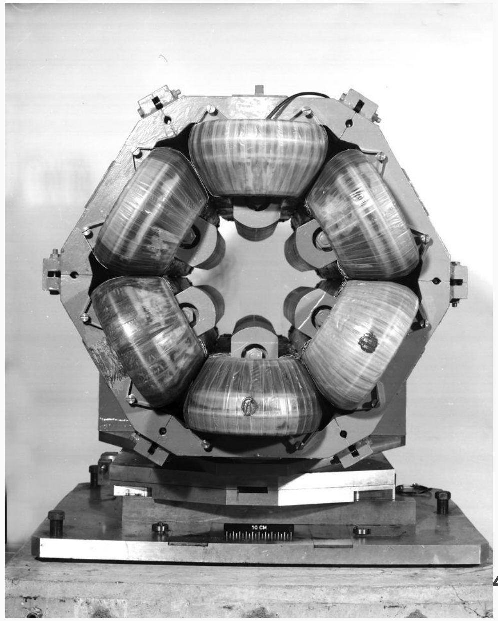


Chromaticity and sextupoles

- As dispersion is a change of trajectory with the momentum deviation, chromaticity is change of machine tune with the momentum deviation: $Q'=dQ/(dp/p)$ [dimensionless]
- Reminder: typical momentum spread in a synchrotron ($\Delta p/p \sim 10^{-3}$)
- Chromaticity is controlled by sextupole magnets installed in dispersive region
- Typically small negative chromaticity is needed to make machine stable
- Higher order effects demand octupoles, decatupoles to correct







Twiss parameters and beam ellipse

- Beam ellipse can be described in terms of emittance and Twiss parameters (called also Courant-Snyder parameters):

$$\alpha(s) \equiv -\frac{1}{2}\frac{d\beta(s)}{ds}$$

$$\gamma(s) \equiv \frac{1+\alpha^2(s)}{\beta(s)}$$

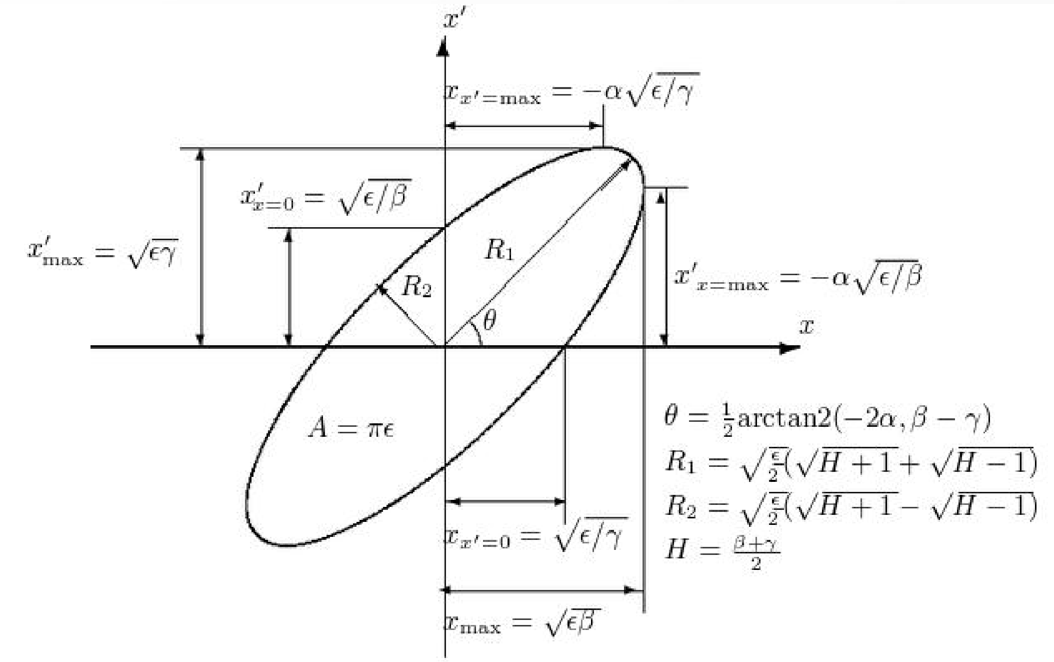


Figure 9: Emittance ellipse geometry with the most important dimensions

- Alpha (α) is *slope* of beta;
- “parallel beam”: α=0
- Gamma is dependent parameter and it is *beta for angle*

Beam transport

- Matrix formalism is used to transfer the beam from one element to another:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

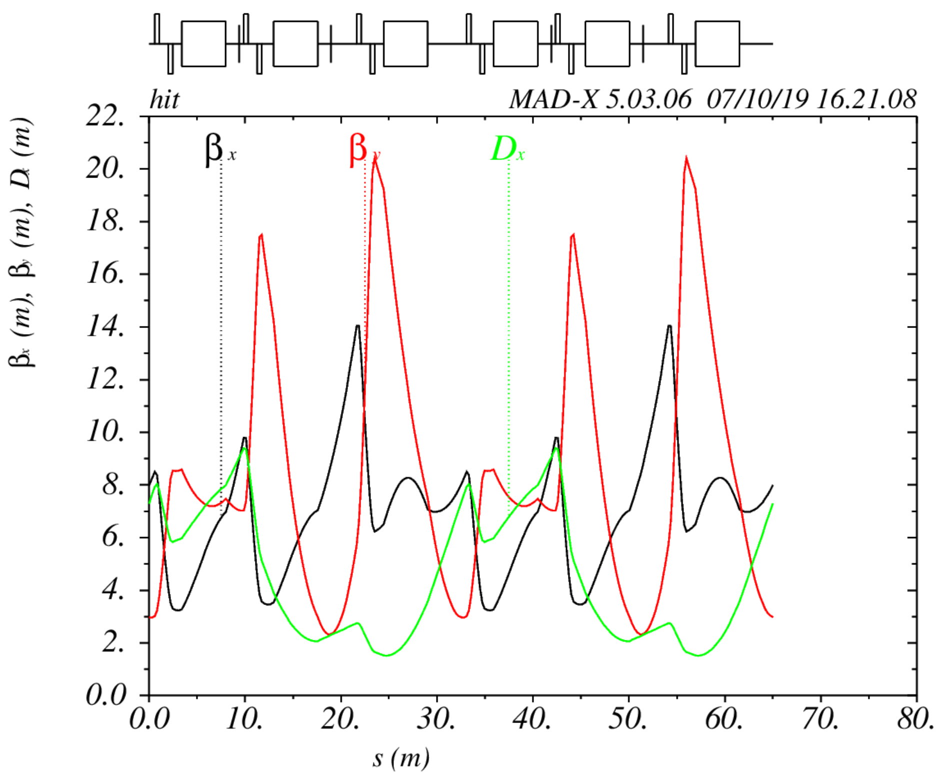
- e.g. transfer matrix for focusing quad:

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}s) \\ -\sqrt{K}\sin(\sqrt{K}s) & \cos(\sqrt{K}s) \end{pmatrix}$$

- Transport through multiple elements:

$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

- These are first steps in designing a synchrotron or a beam line



Using this formalism, or tracking of the particles in magnet fiels, programs like MAD-X, allow to compute Twiss parameters and dispersion

Beam transport - 6D

- Equation of ellipse can be also written in form of matrix Σ:
  $[x]^T\Sigma[x] = 1$
- Beam matrix Σ(s) describes the beam ellipse at a given position; determinant of the ellipse is emittance
- Beam matrix is transformed using matrix formalism:
  $\Sigma(s) = M\,\Sigma(0)\,M^T$
- Beam has 2 independent parameters per dimension, so total 6-D is needed to write full beam matrix
- Transverse-longitudinal coupling via dispersion (D) and D', here included in η

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \epsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \varepsilon_x\beta_x & -\varepsilon_x\alpha_x & 0 & 0 & 0 & \eta_x\sigma_\delta^2 \\ -\varepsilon_x\alpha_x & \varepsilon_x\gamma_x & 0 & 0 & 0 & \eta_{p_x}\sigma_\delta^2 \\ 0 & 0 & \varepsilon_y\beta_y & -\varepsilon_y\alpha_y & 0 & \eta_y\sigma_\delta^2 \\ 0 & 0 & -\varepsilon_y\alpha_y & \varepsilon_y\gamma_y & 0 & \eta_{p_y}\sigma_\delta^2 \\ 0 & 0 & 0 & 0 & \sigma_z^2 & 0 \\ \eta_x\sigma_\delta^2 & \eta_{p_x}\sigma_\delta^2 & \eta_y\sigma_\delta^2 & \eta_{p_y}\sigma_\delta^2 & 0 & \sigma_\delta^2 \end{pmatrix}$$

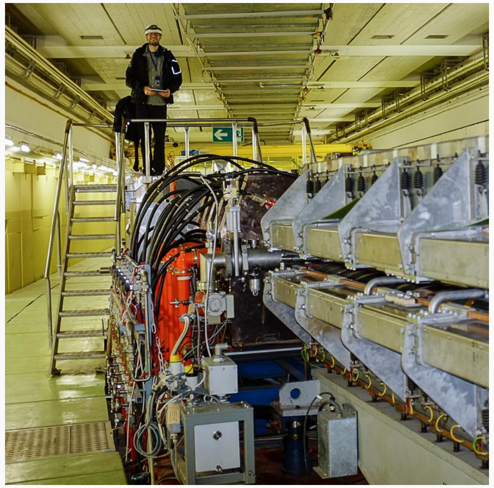
Strong focusing principle allows to construct stable storage rings and transport the beam efficiently.

Things to remember:

synchrotron cells, Twiss parameters, dispersion, tune, chromaticity, resonances and matrix formalism

Conclusions

- Accelerators are one of the most important tools in science, medicine and industry
- They produce beam of particles with a given energy and emittance
- RF acceleration allows to reach very high energies; phase stability assures longitudinal focusing
- Strong focusing made possible large machines able to produce, transport and store high intensity beams for hours (or days)
- The most important concepts: elementary cell, Twiss parameters (α,β) , beam phase space, beam ellipse, dispersion, tune, resonances, chromaticity and matrix formalism



Acknowledgments:
- Preparing these slides I used presentations of several CERN Accelerator Schools and summer student lectures

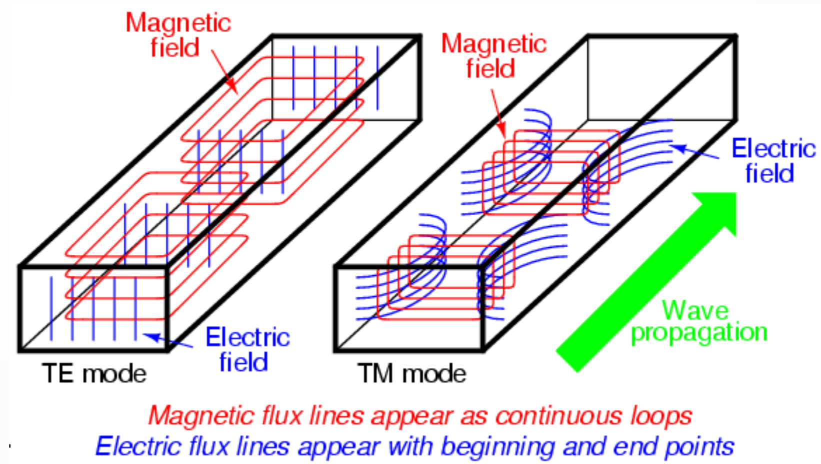
Thank you for your attention!

Please contact me if you have questions concerning this lecture: mariusz.sapinski@cern.ch

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101008548

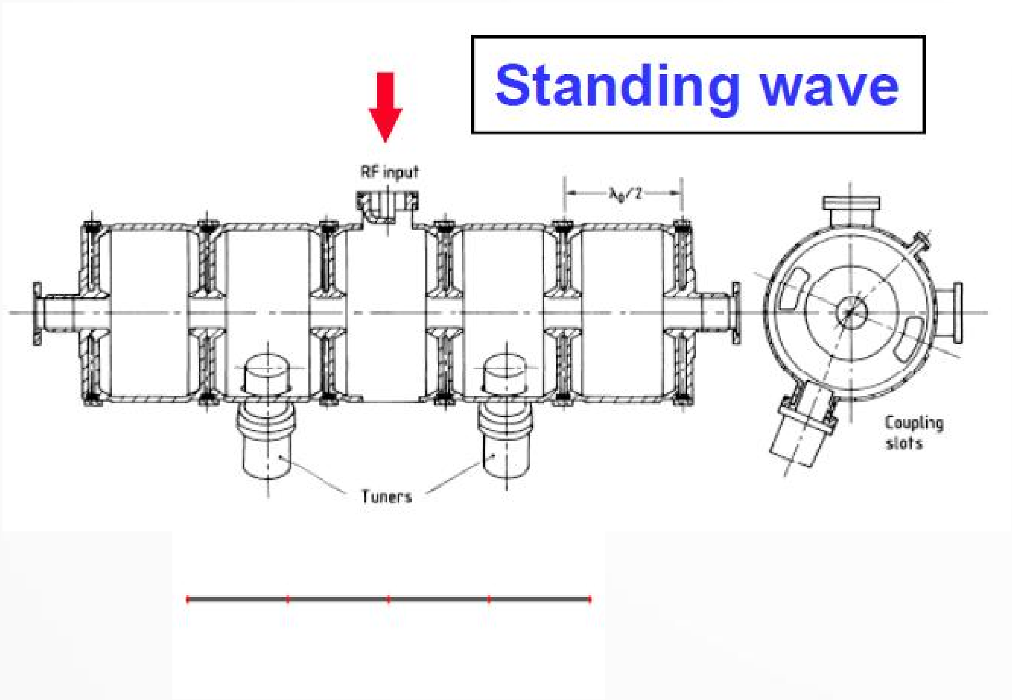
RF sources

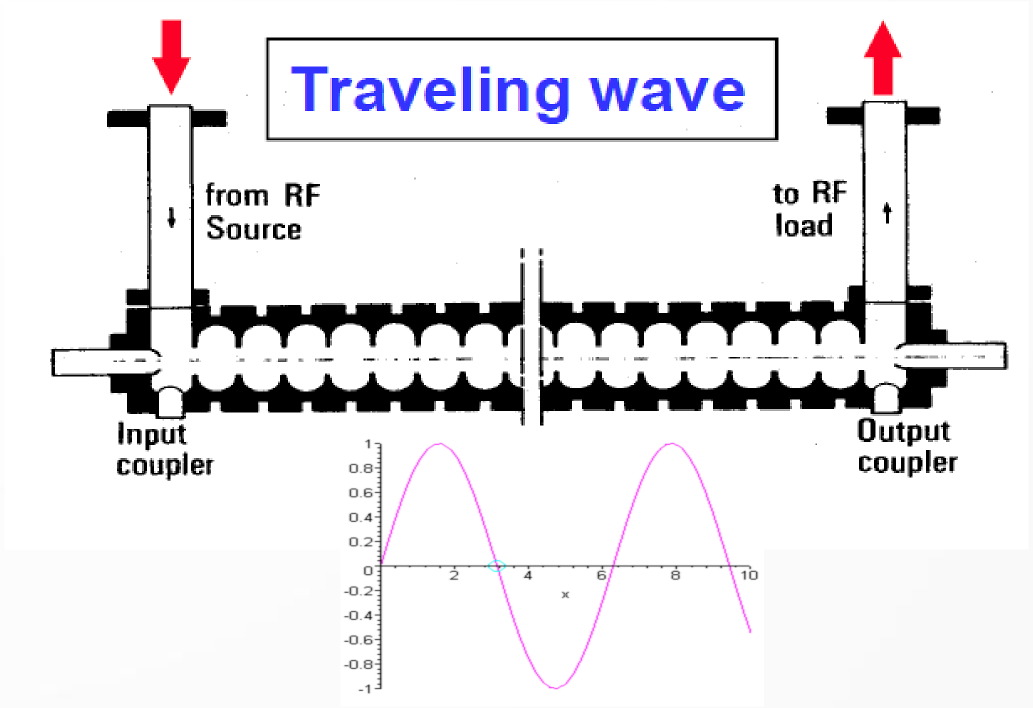
- High-frequency transmission lines are waveguides, not cables, because cables are antennas
- (But… the simplest waveguide is a concentric cable)
- **RF accelerators need powerful RF sources**



- The RF sources are closely related
- One of the first devices, still in use, was klystron, developed by Varian brothers (yes, they set up Varian company known for cyclotrons)
- Klystrons by themselves are small electron accelerators
- Trend: solid state RF generators

Standing and travelling wave





Acceleration ~ 5 MV/m

Acceleration ~ 30 MV/m

Radio-Frequency Quadrupole

- DTL can accept ion beams from energy of hundreds of keV/u (limits on frequency and size of the tank)
- Ion sources provide ion energies of ~5-50 keV/u
- Acceleration in between is difficult, space charge forces act to dirupt the beam
- Electrostatic acceleration is a valid option, but
- RFQ proves to be a very efficient and compact acceleration element
- It provides focusing and smooth bunching
- Increases the transmission from source to DTL from 50% to 90%
  - crucial for high-power machines