Chiral Separation Effect and Kondo effect in finite-density SU(2) gauge theory

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Why chiral plasmas?

Collective motion of chiral fermions

- **High-energy physics:**
  - Quark-gluon plasma
  - Neutrinos/leptons in Early Universe
  - Neutrinos in supernovae cores ($l_{\text{free}} \sim 1\text{cm}$)

- **Condensed matter physics:**
  - Liquid He$_3$ [G. Volovik]
  - Weyl semimetals
  - Topological insulators
Chiral anomaly [Adler-Bell-Jackiw 1969]

Classical action

\[ S = \bar{\psi} \gamma_\mu (\partial_\mu - iA_\mu) \psi \]

invariant under chiral rotations

\[ \psi \rightarrow e^{i\gamma_5 \theta} \psi \]

\[ \gamma_5 \gamma_\mu + \gamma_\mu \gamma_5 = 0 \]

Corresponding conserved current: axial current

\[ j_\mu^A = \bar{\psi} \gamma_5 \gamma_\mu \psi \]

Upon quantization, one finds

\[ \partial_\mu j_\mu^A = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B} \]
Anomalous transport

Axial anomaly
\[ \partial_\mu j^A_\mu = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B} \]

Chiral Magnetic Effect
\[ \vec{j} = \frac{\mu_A}{2\pi^2} \vec{B} \]

Chiral Separation Effect
\[ \vec{j}_A = \frac{\mu \vec{B}}{2\pi^2} \]
Anomalous transport and heavy ions

Ideal hydro

Viscous hydro

Anomalous hydro

Elliptic flow

Parity-odd fluctuations

Isobar run RHIC 2018 – results announced right now!

https://indico.bnl.gov/event/12758/
Anomalous transport coefficients

• Input for hydrodynamic simulations of HICs
• Get unknown corrections in real QCD
• Due to broken chiral symmetry [PB’1312.1843]
• Perturbatively [Miransky 1304.4606] [Gursoy 1407.3282]
• Due to influence of heavy quark flavors [Suenaga 2012.15173]
Anomalous transport coefficients

Lattice studies so far:

• [Yamamoto’1105.0385]: $\sim20\%$ of Chiral Magnetic Effect
• [Braguta et al’ 1401.8095]: $\sim5\%$ of Chiral Vortical Effect
• So far hydro simulations with free-fermion transport coefficients only
• Lattice conclusions can question the hydro interpretation of RHIC results

BUT: Wilson-Dirac/Quenched overlap/non-conserved currents/energy-momentum
$T > T_c$ [PB, M. Puhr, ArXiv: 1611.07263]
Pure SU(3) gauge theory

$T > T_c$

[PB, M. Puhr, ArXiv: 1611.07263]
What can be the order of magnitude of corrections?

Sign problem in full QCD use \( SU(2) \) gauge theory, no sign problem

Features confinement-deconfinement crossover and \( \chi_{SB}, \) QCD-like dynamics at small \( \mu < m_\pi/2 \).

Diquark condensation at \( \mu > m_\pi/2 \), absent in real QCD
Phase diagram of SU(2) gauge theory

QCD-like low-temperature phase, broken chiral symmetry, pion excitations

Qualitatively similar to QCD !!!
Lattice setup: sea quarks & gauge action

- $N_f=2$ light flavours with $m_u=m_d = 0.005$, pion mass $m_\pi = 0.158$
- Rooted staggered sea quarks
- Tadpole-improved gauge action
- Spatial lattice sizes $L_s=24$ and $L_s=30$
- Single gauge coupling = single lattice spacing
- Temporal lattice sizes $L_t=4 \ldots 26$
- Standard Hybrid Monte Carlo
- Acceleration using GPUs

- Small diquark source term added for low temperatures to facilitate diquark condensation
Lattice setup: valence quarks

- Wilson-Dirac and Domain-Wall valence quarks
- HYP-smeared gauge links in the Dirac operator:
  reduces additive mass renormalization and lattice artifacts
- Better quality of signal than for staggered quarks
- Bare mass for Wilson-Dirac/Domain-Wall quarks tuned to match the pion mass calculated with sea quarks
- GMOR relation works with good precision
Measuring the CSE

• Sign problem even in SU(2) gauge theory at finite $\mu$ and magnetic field
• We use linear response approximation w.r.t. magnetic field

\[ \vec{j}_A = \sigma_{CSE} (\mu, T) \vec{B} \]

\[ \langle j^A_1 (k_3) j^V_2 (-k_3) \rangle = \sigma_{CSE} k_3 \]
Numerical results

$L_t = 12$, $a\mu = 0.05$

High T, Small $\mu$

Disconnected contribution

$\sigma_{CSE}(k)/\mu$ vs $k$ (Lattice momentum)
Numerical results

$L_t = 16, a\mu = 0.20$

Critical $T$, Large $\mu$

Disconnected contribution
Numerical results

$L_t = 20, a\mu = 0.05$

Low $T$, Small $\mu$

Disconnected contribution
Numerical results

$L_t = 16, a\mu = 0.50$

Critical $T$, Large $\mu$

Disconnected contribution
$\sigma_{\text{CSE}}$ vs temperature, low $\mu$

Significant suppression towards low temperatures!

Rescaled charge density
$\sigma_{\text{CSE}}$ vs temperature, medium $\mu$

Data moving closer to the free fermion results

Rescaled charge density (same coefficient)
$\sigma_{\text{CSE}}$ vs temperature, large $\mu$

Data quite close to the free fermion results
Describing CSE suppression

• ChPT result for flavor-non-singlet axial current [Avdoshkin,Sadofyev,Zakharov’ 1712.01256]:

\[ \vec{j}_A = \frac{N_c \text{Tr} (Q)}{(2\pi f_\pi)^2} \rho_V^a \vec{B} \]

• We work with flavor-singlet axial current, has different status in ChPT
• Singlet and non-singlet currents become similar at large $N_c$
• Phenomenological formula works well in the low-$T$, low-$\mu$ regime even for singlet axial current in SU(2) gauge theory

Disconnected contribution appears to be small!

\[ \sigma_{\text{CSE}} (\mu, T) = \alpha \rho_V (\mu, T) \]
Kondo effect in non-Abelian gauge theory

• Suppression of an interesting effect feels somewhat unfortunate...

• Is there something that can enhance the CSE?

• Yes, QCD Kondo Effect [Suenaga et al., 2012.15173]
Kondo effect in non-Abelian gauge theory

- **Kondo effect**: scattering of light fermion near a Fermi surface off a heavy fermion of mass $M$ enhanced as $\log(M)$
- Mean-field approach for QCD [Yasui, Suzuki, Itakura, 1604.07208]: spontaneous emergence of Kondo condensate $\langle \bar{Q} q \rangle$
- Suppresses low-$T$, finite-$\mu$ conductivity [Yasui, Ozaki, 1710.03434]
- But... **CSE** is enhanced [Suenaga, Araki, Suzuki, Yasui, 2012.15173]
- We only consider CSE of light quarks
Numerical results for CSE in $N_f=2+1$ SU(2) LGT

- $\mu = 0.2$
- $\mu = 0.5$
- $\mu = 1.0$
- $\mu = +\infty$

- $L_t=20$, low-temperature regime
- CSE enhanced by more than 30% for $\mu_Q=0.2 \ a$
Numerical results for CSE in \( N_f=2+1 \) SU(2) LGT

- \( m_Q = 0.2 \)
- \( m_Q = 0.5 \)
- \( m_Q = 1.0 \)
- \( m_Q = +\infty \)

- \( L_t=18 \), a bit higher temperature
- Enhancement not so large
- Not a conventional Kondo, Fermi surface not well-defined
Conclusions

• CSE close to free-quark result at high temperatures and/or high densities
• Significant suppression at low temperatures and low densities
• $\sigma_{\text{CSE}}$ approximately proportional to charge density rather than chemical potential
• Similar to ChPT calculation of [Avdoshkin, Sadofyev, Zakharov’ 1712.01256] for axial non-flavor-singlet current, although non-singlet and singlet axial currents are physically quite different
• CSE can be enhanced in the presence of additional fermion flavors – signature of Kondo effect
• Next step: conductivity at finite density with heavy quarks