

Chiral Separation Effect and Kondo effect in finite- density $SU(2)$ gauge theory

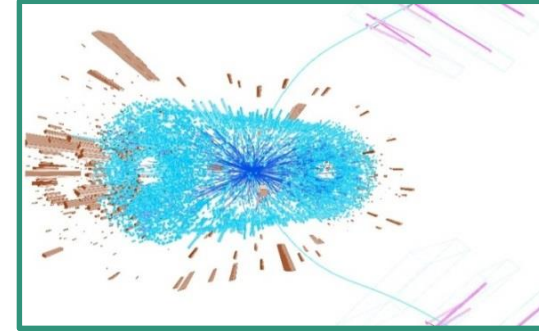
PAVEL BUIVIDOVICH (UNIVERSITY OF LIVERPOOL), LORENZ VON
SMEKAL (GIESSEN UNIVERSITY), DOMINIK SMITH (GSI DARMSTADT)

Why chiral plasmas?

Collective motion of chiral fermions

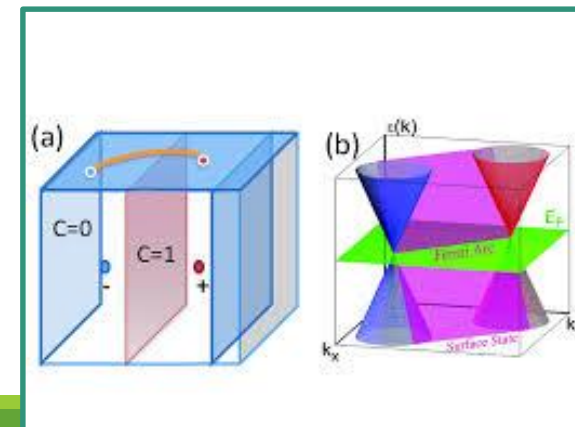
- High-energy physics:

- ✓ Quark-gluon plasma
- ✓ Neutrinos/leptons in Early Universe
- ✓ Neutrinos in supernovae cores ($l_{free} \sim 1\text{cm}$)



- Condensed matter physics:

- ✓ Liquid He₃ [G. Volovik]
- ✓ Weyl semimetals
- ✓ Topological insulators



Chiral anomaly [Adler-Bell-Jackiw 1969]

Classical action

$$S = \bar{\psi} \gamma_{\mu} (\partial_{\mu} - iA_{\mu}) \psi$$

invariant under chiral rotations

$$\psi \rightarrow e^{i\gamma_5 \theta} \psi$$

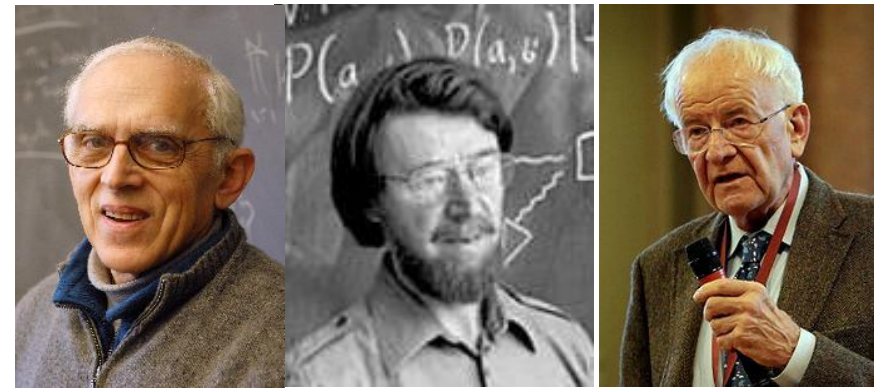
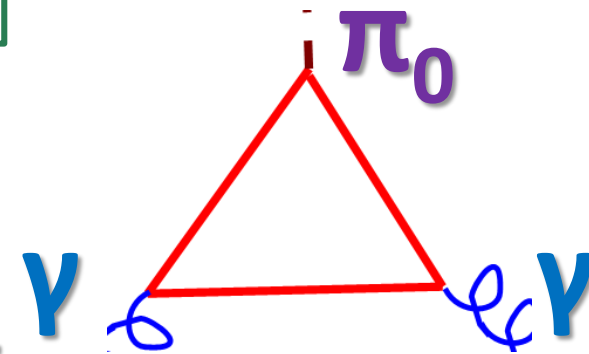
$$\gamma_5 \gamma_{\mu} + \gamma_{\mu} \gamma_5 = 0$$

Corresponding conserved current: axial current

$$j_{\mu}^A = \bar{\psi} \gamma_5 \gamma_{\mu} \psi$$

Upon quantization, one finds

$$\partial_{\mu} j_{\mu}^A = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B}$$



Anomalous transport

Axial anomaly

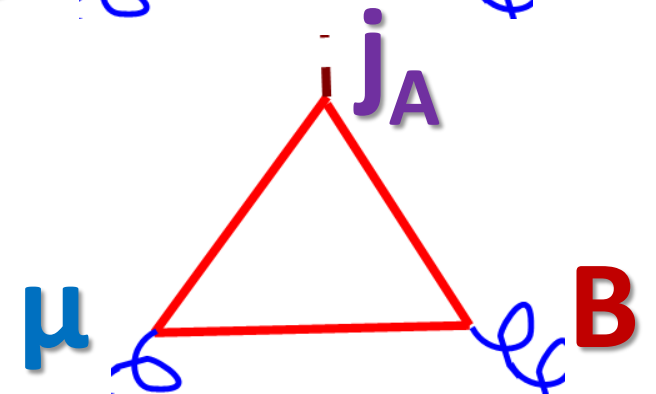
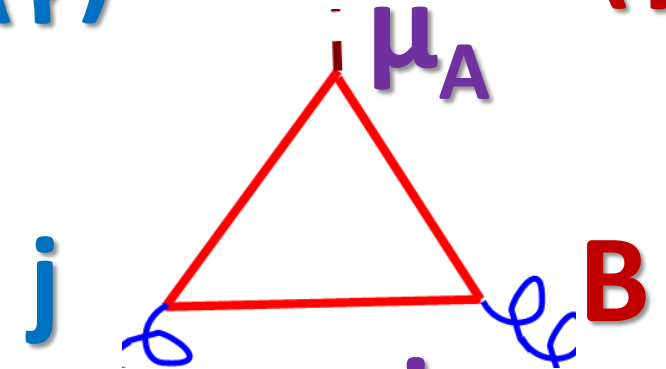
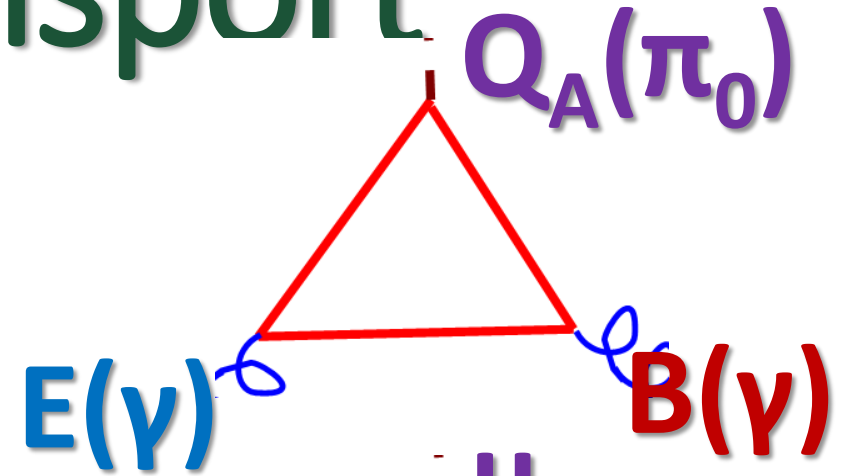
$$\partial_{\mu} j_{\mu}^A = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B}$$

Chiral Magnetic Effect

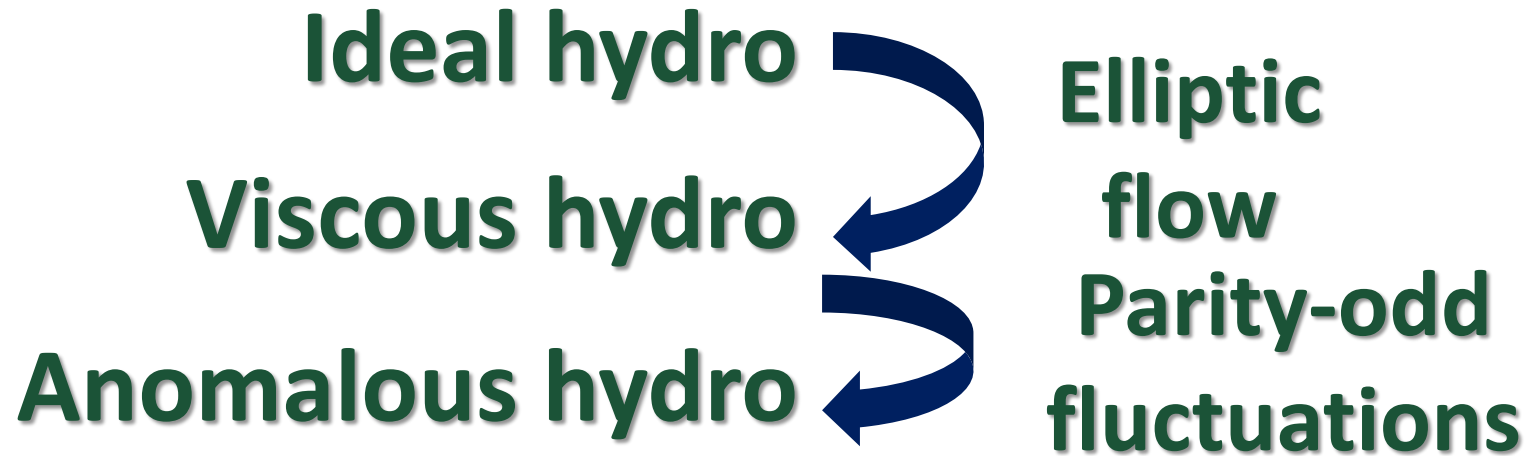
$$\vec{j} = \frac{\mu_A}{2\pi^2} \vec{B}$$

Chiral Separation Effect

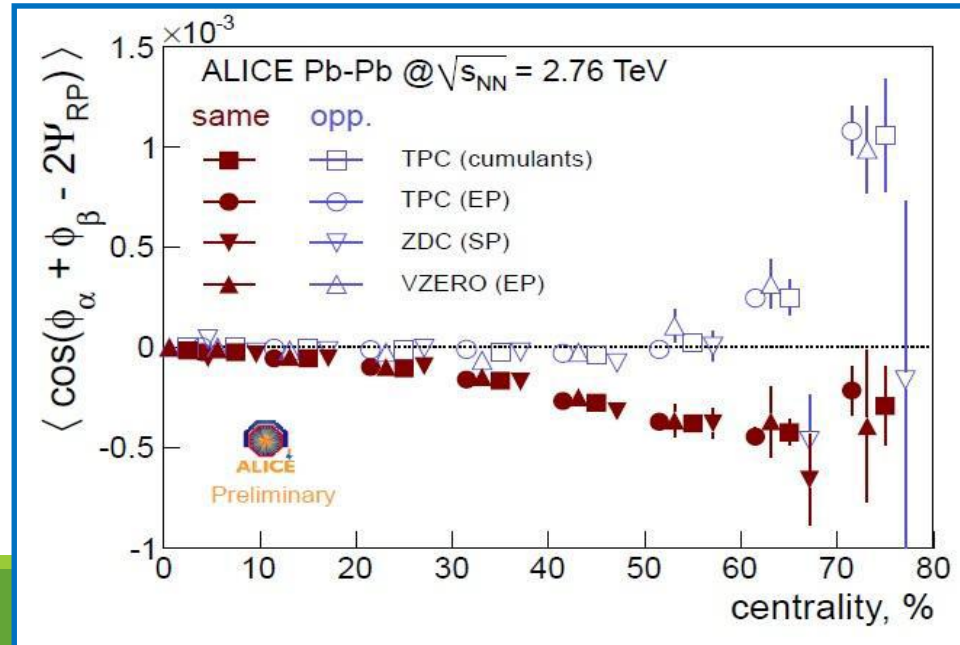
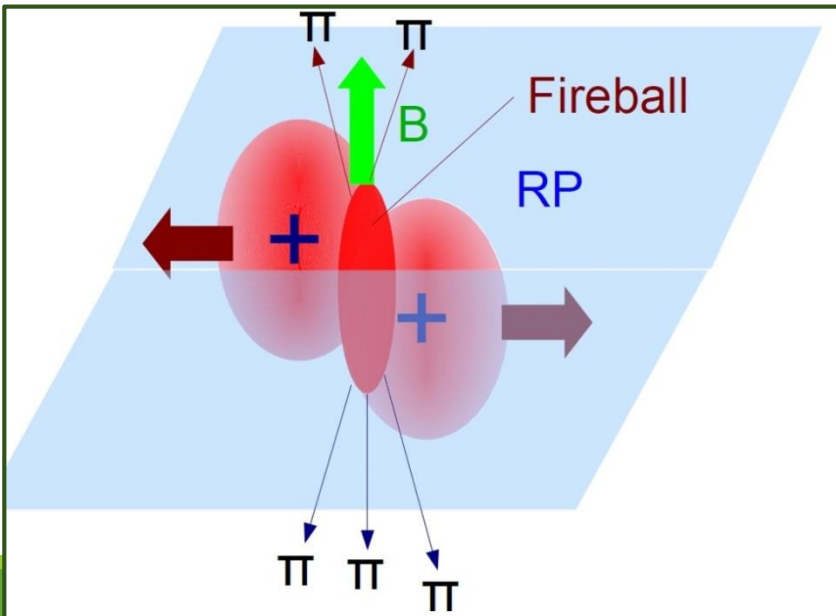
$$\vec{j}_A = \frac{\mu \vec{B}}{2\pi^2}$$



Anomalous transport and heavy ions



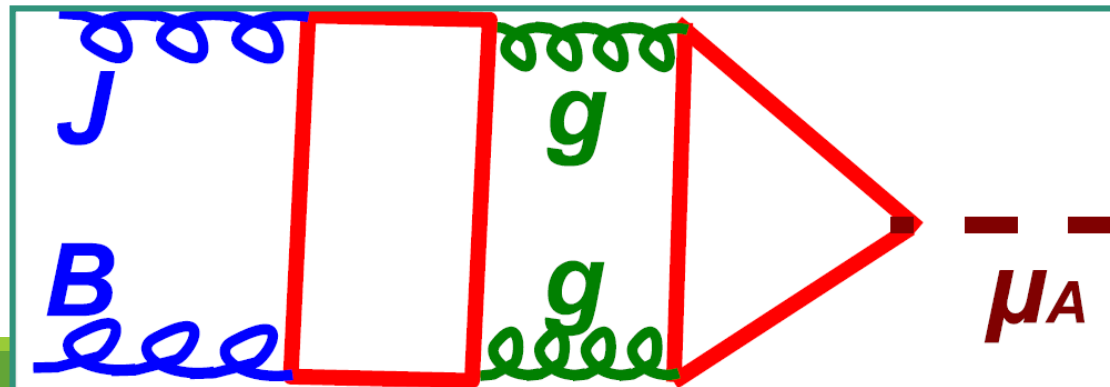
Isobar run RHIC 2018 – results announced right now!



<https://indico.bnl.gov/event/12758/>

Anomalous transport coefficients

- Input for hydrodynamic simulations of HICs
- Get unknown corrections in real QCD
- Due to broken chiral symmetry [PB'1312.1843]
- Perturbatively [Miransky 1304.4606] [Gursoy 1407.3282]
- Due to influence of heavy quark flavors [Suenaga 2012.15173]



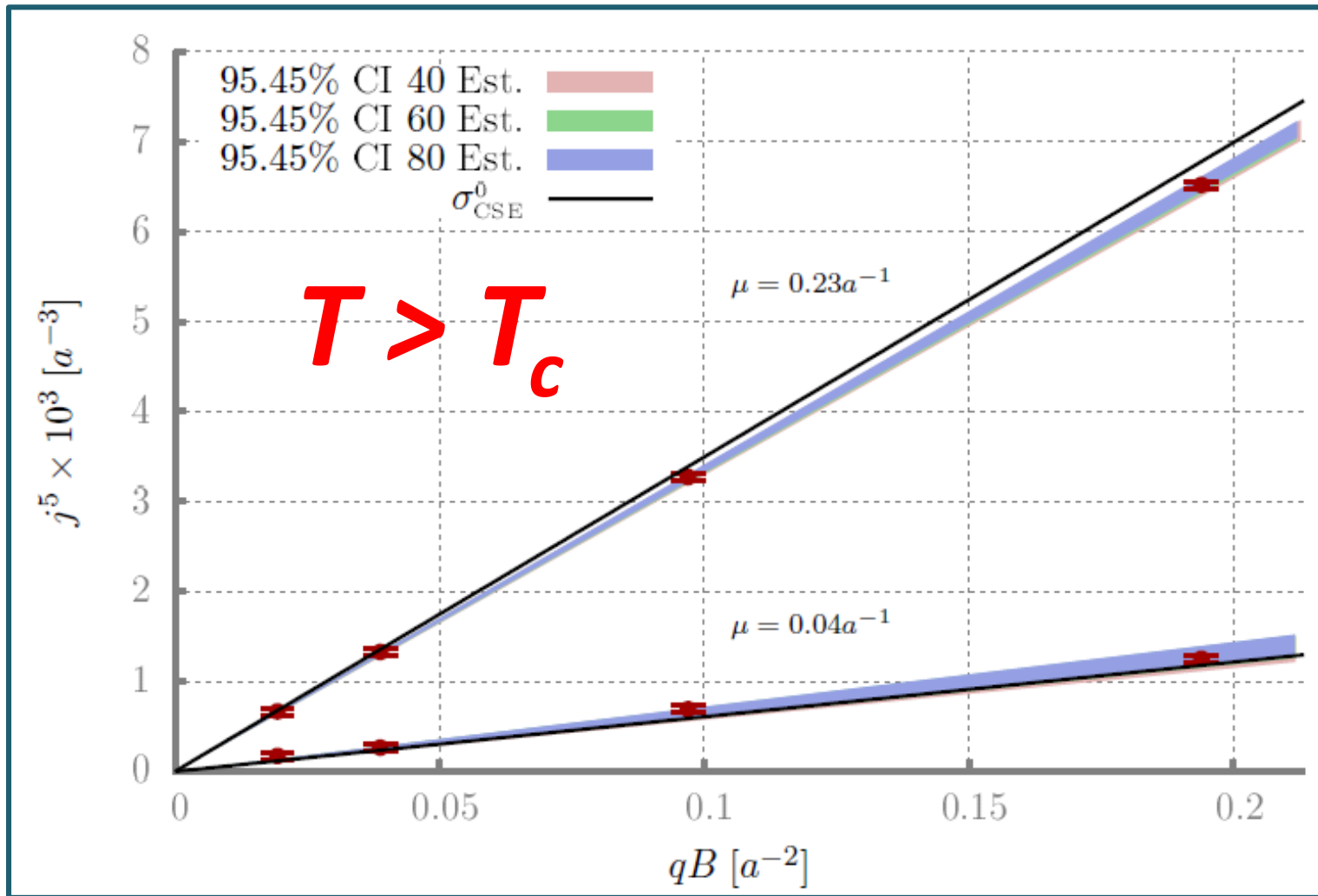
Anomalous transport coefficients

Lattice studies so far:

- [Yamamoto'1105.0385]: $\sim 20\%$ of Chiral Magnetic Effect
- [Braguta et al' 1401.8095]: $\sim 5\%$ of Chiral Vortical Effect
- So far hydro simulations with **free-fermion** transport coefficients only
- Lattice conclusions can question the **hydro interpretation** of **RHIC results**

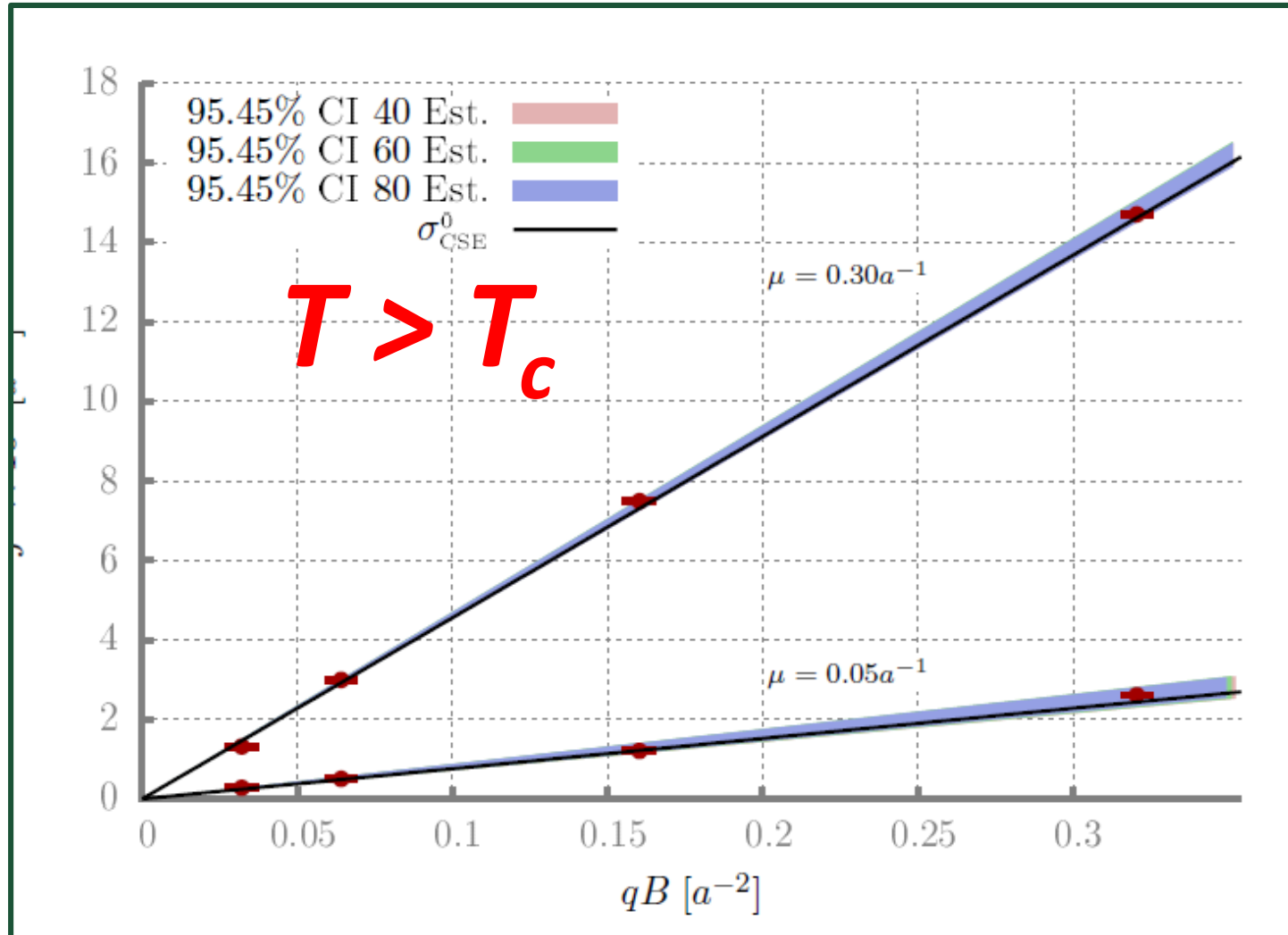
BUT: Wilson-Dirac/Quenched overlap/non-conserved currents/energy-momentum

Pure SU(3) gauge theory




[PB, M. Pühr,
ArXiv:
1611.07263]

Pure SU(3) gauge theory



[PB, M. Pühr,
ArXiv:
1611.07263]

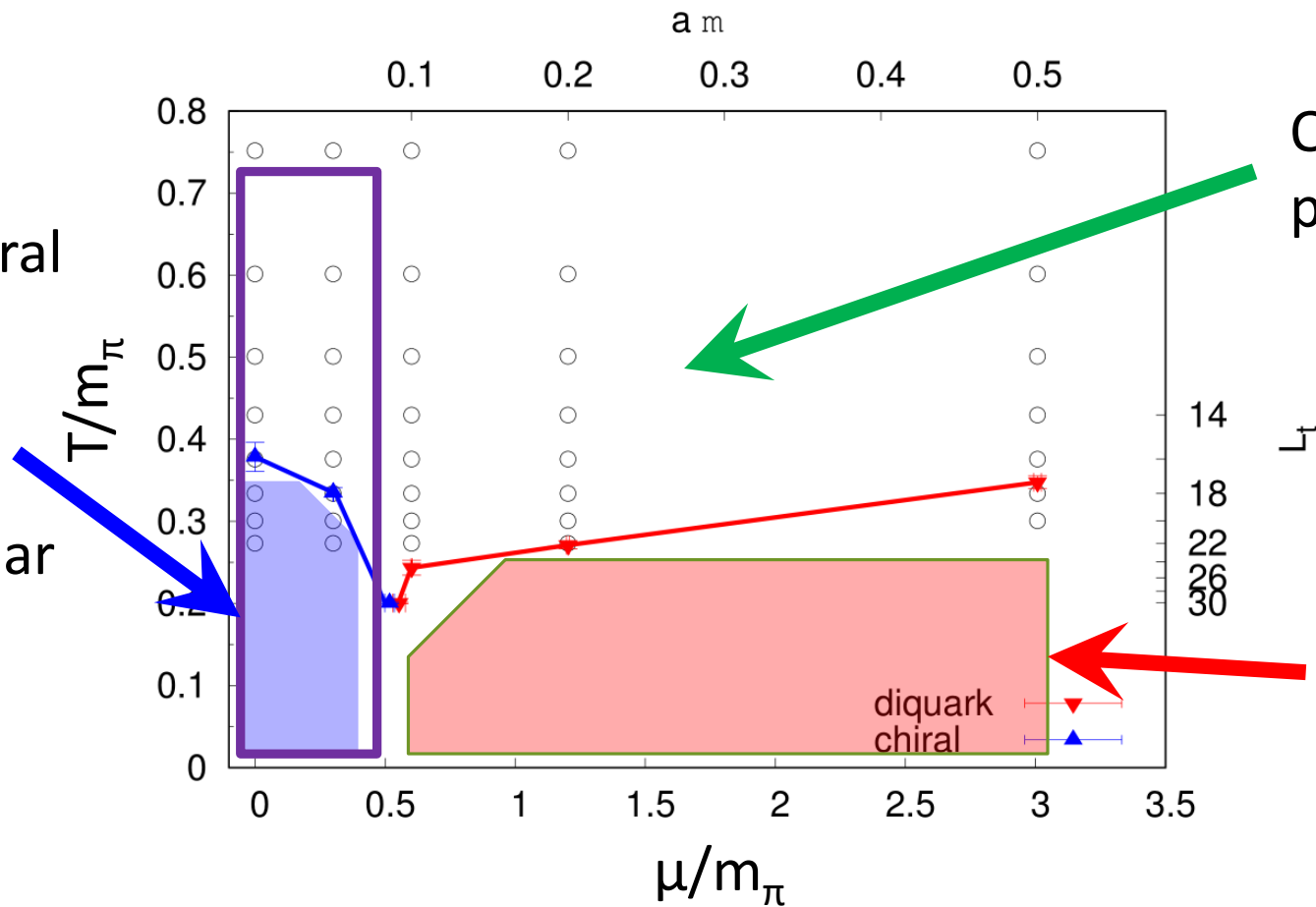
CSE with dynamical fermions

- What can be the order of magnitude of **corrections**?
- **Sign problem** in full **QCD**  use **SU(2)** gauge theory, no sign problem
- Features **confinement-deconfinement crossover** and χ SB, **QCD**-like dynamics at small $\mu < m_\pi/2$.
- **Diquark** condensation at $\mu > m_\pi/2$, absent in real **QCD**

Phase diagram of $SU(2)$ gauge theory

QCD-like
low-temperature
phase, broken chiral
symmetry,
pion excitations

Qualitatively similar
to QCD !!!



Quark-gluon plasma
phase

Diquark condensation
phase, absent in QCD

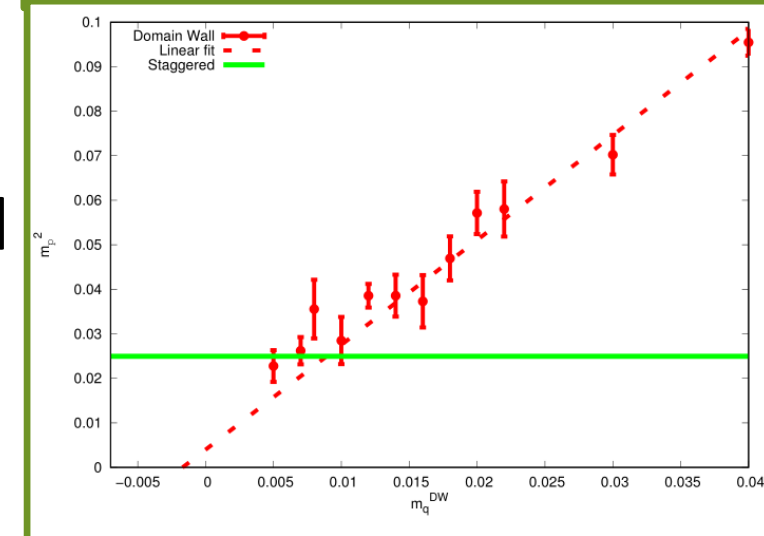
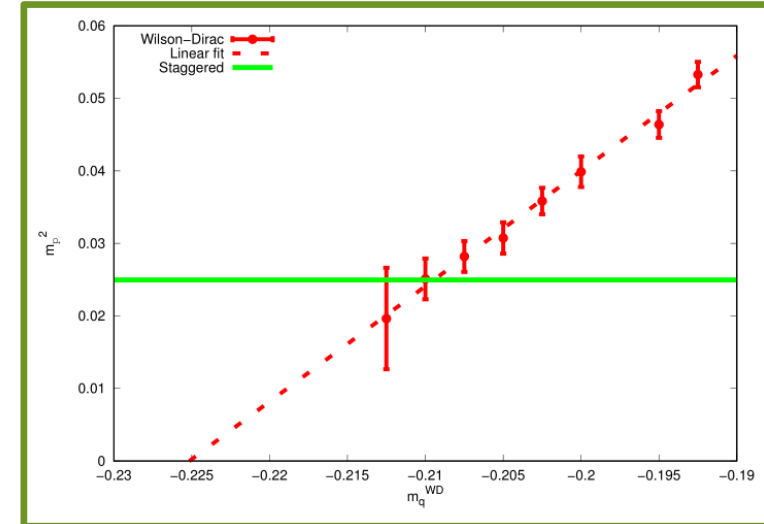
Lattice setup: sea quarks & gauge action

- $N_f=2$ light flavours with $m_u=m_d = 0.005$, pion mass $m_\pi = 0.158$
- Rooted staggered sea quarks
- Tadpole-improved gauge action
- Spatial lattice sizes $L_s=24$ and $L_s=30$
- Single gauge coupling = single lattice spacing
- Temporal lattice sizes $L_t=4 \dots 26$
- Standard Hybrid Monte Carlo
- Acceleration using **GPUs**
- Small **diquark source term** added for low temperatures to facilitate **diquark condensation**



Lattice setup: valence quarks

- **Wilson-Dirac** and **Domain-Wall** valence quarks
- **HYP-smearred gauge links** in the Dirac operator:
reduces additive mass renormalization and lattice artifacts
- Better quality of signal than for staggered quarks
- Bare mass for Wilson-Dirac/Domain-Wall quarks tuned to match the pion mass calculated with sea quarks
- **GMOR relation** works with good precision



Measuring the CSE

- Sign problem even in $SU(2)$

gauge theory at finite μ and

magnetic field

- We use linear response

approximation w.r.t.

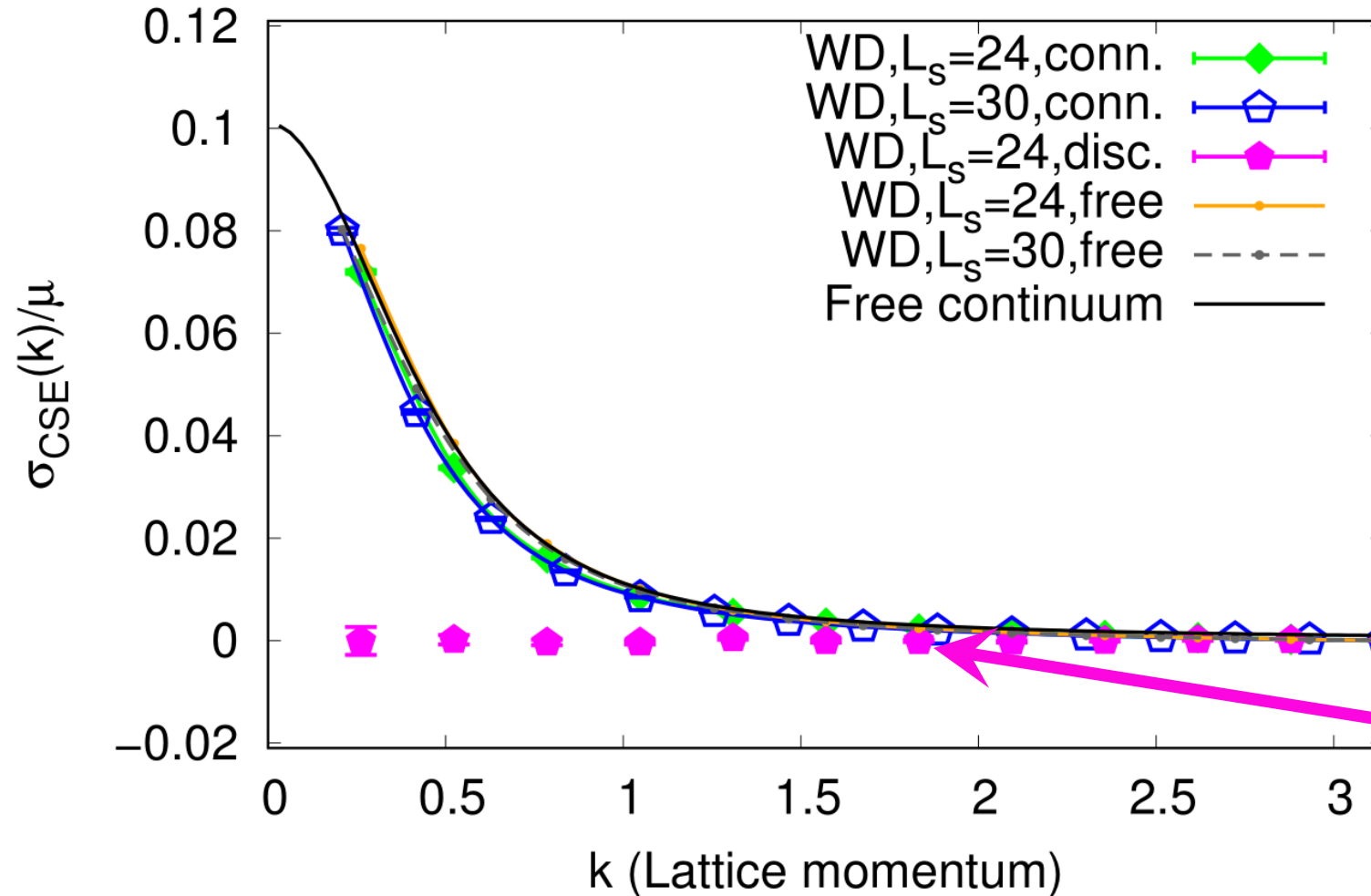
magnetic field

$$\vec{j}_A = \sigma_{CSE}(\mu, T) \vec{B}$$

$$\langle j_1^A(k_3) j_2^V(-k_3) \rangle = \sigma_{CSE} k_3$$

Numerical results

$L_t = 12, a\mu = 0.05$

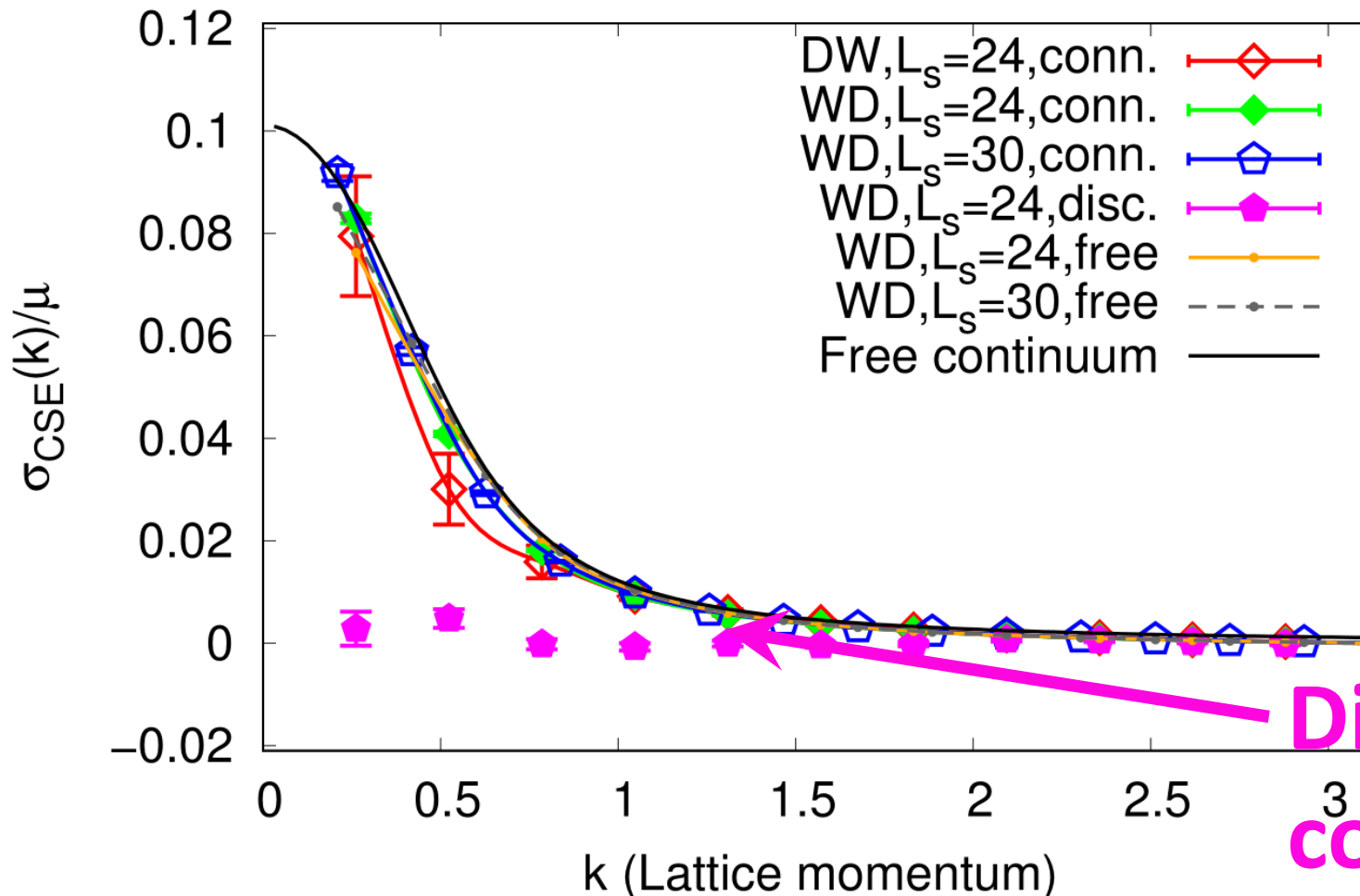


**High T,
Small μ**

**Disconnected
contribution**

Numerical results

$L_t = 16, a\mu = 0.20$

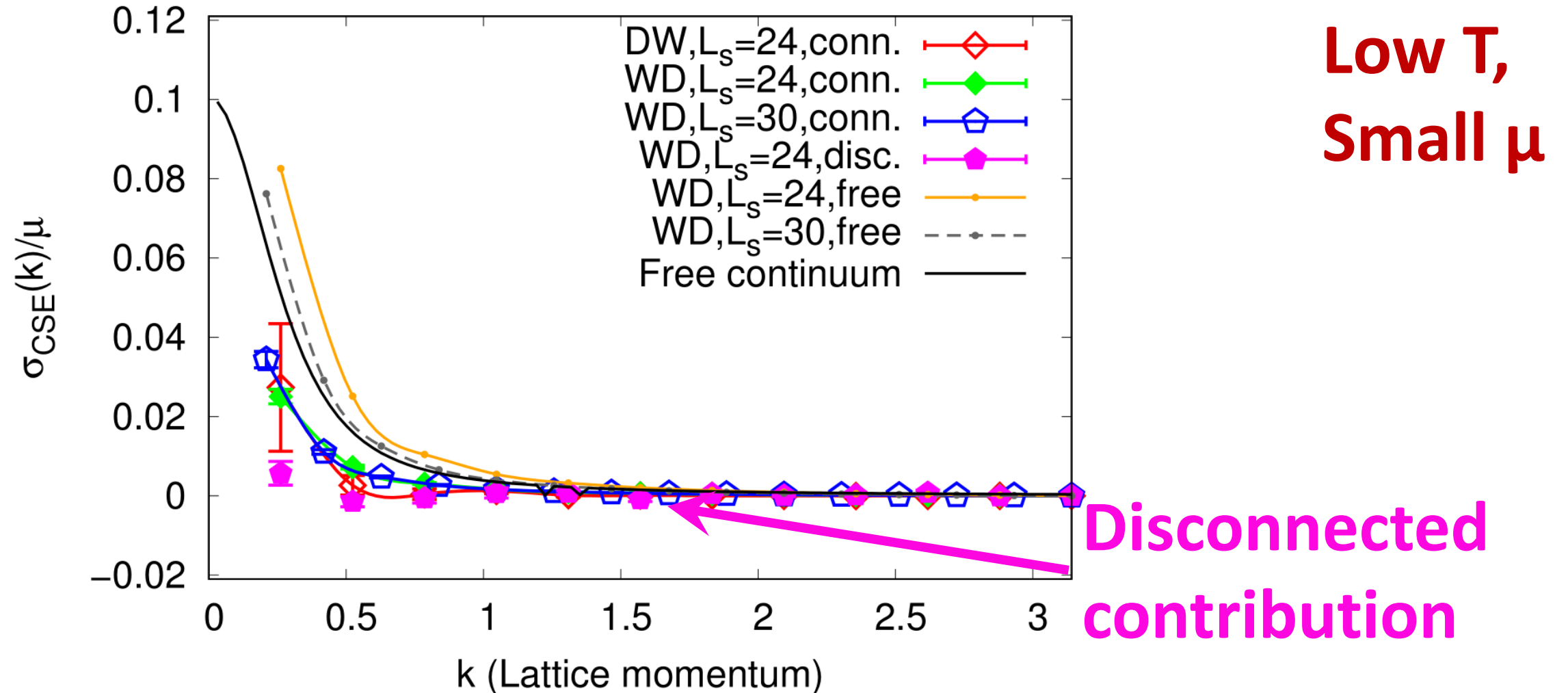


**Critical T,
Large μ**

**Disconnected
contribution**

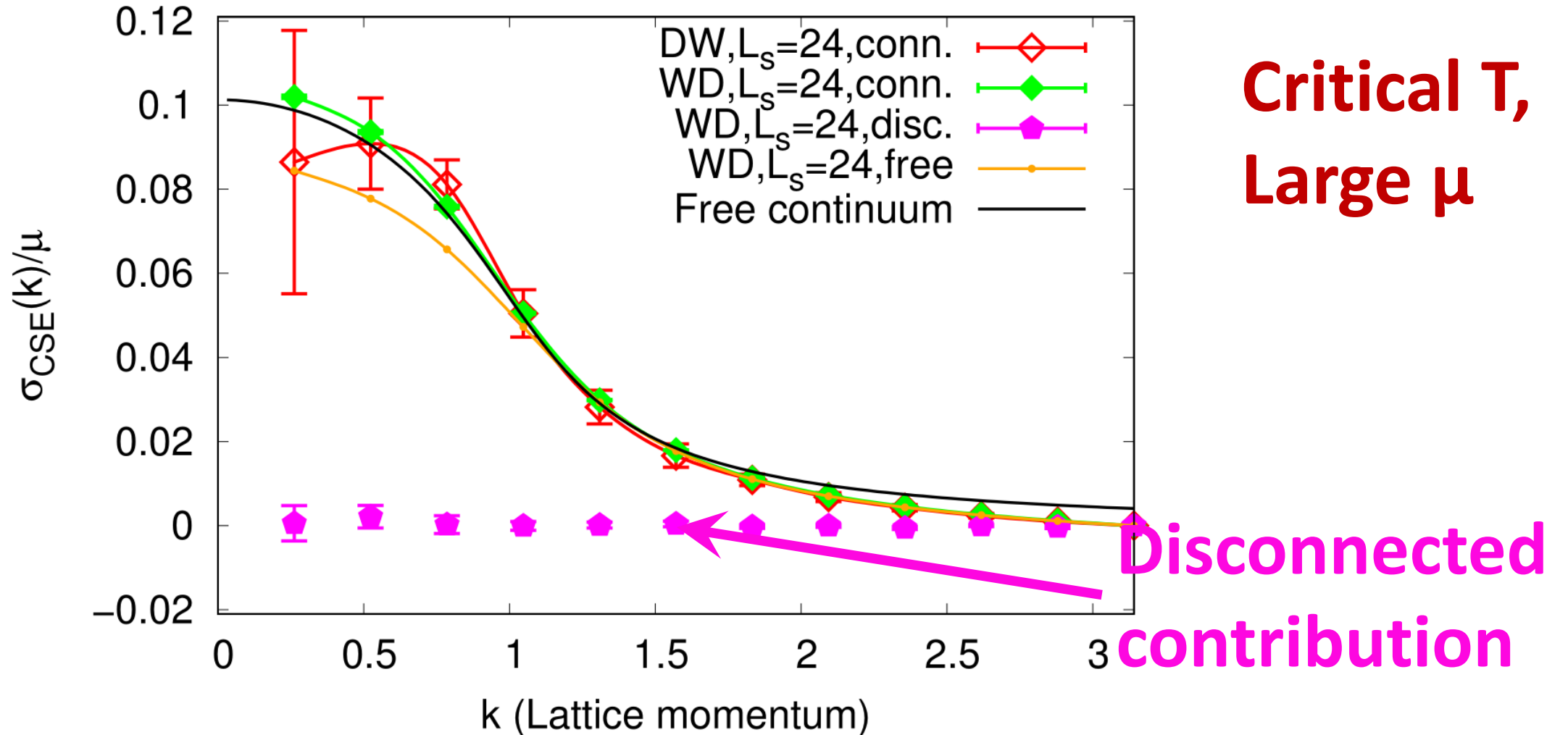
Numerical results

$L_t = 20, a\mu = 0.05$



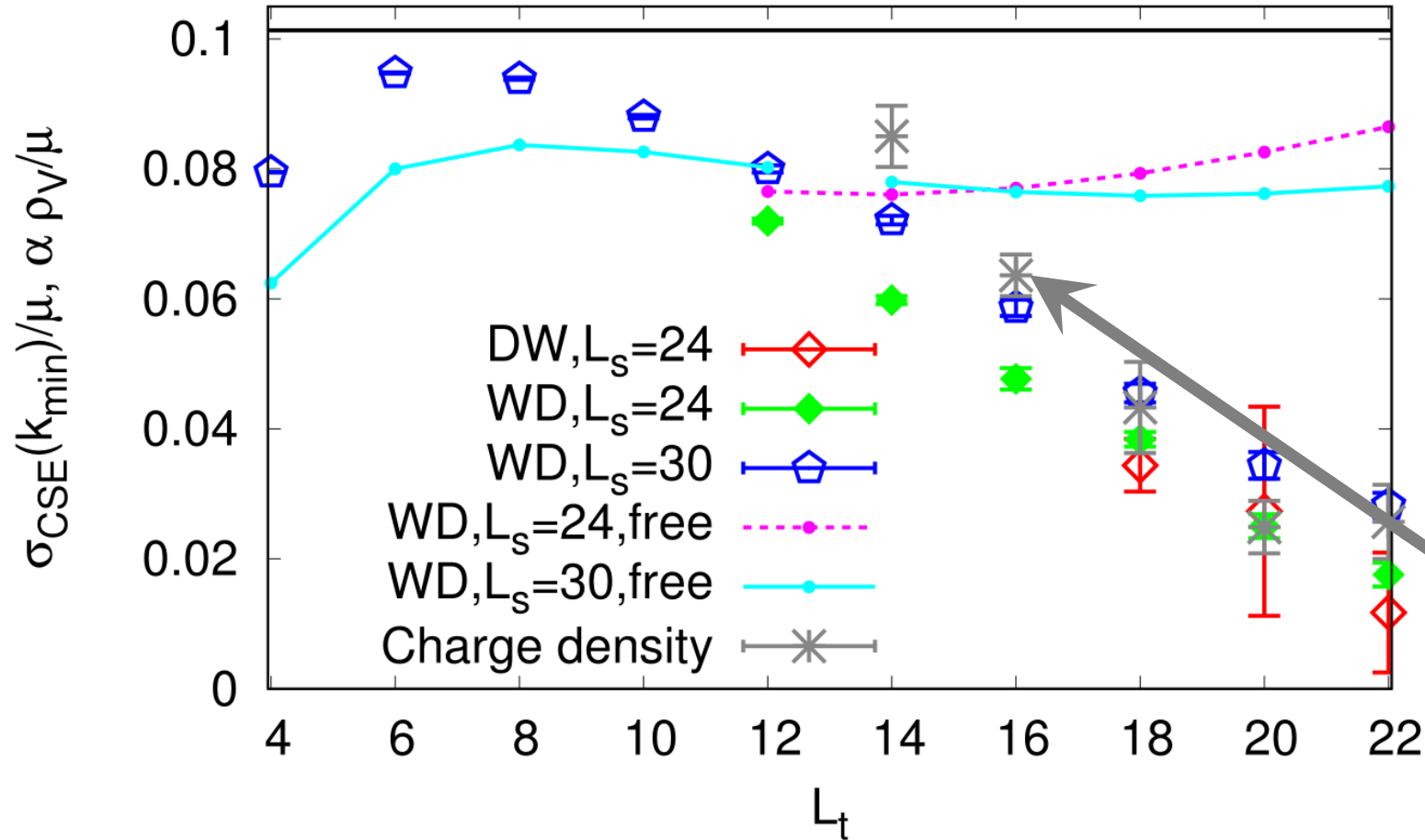
Numerical results

$L_t = 16, a\mu = 0.50$



σ_{CSE} vs temperature, low μ

$a\mu = 0.05$

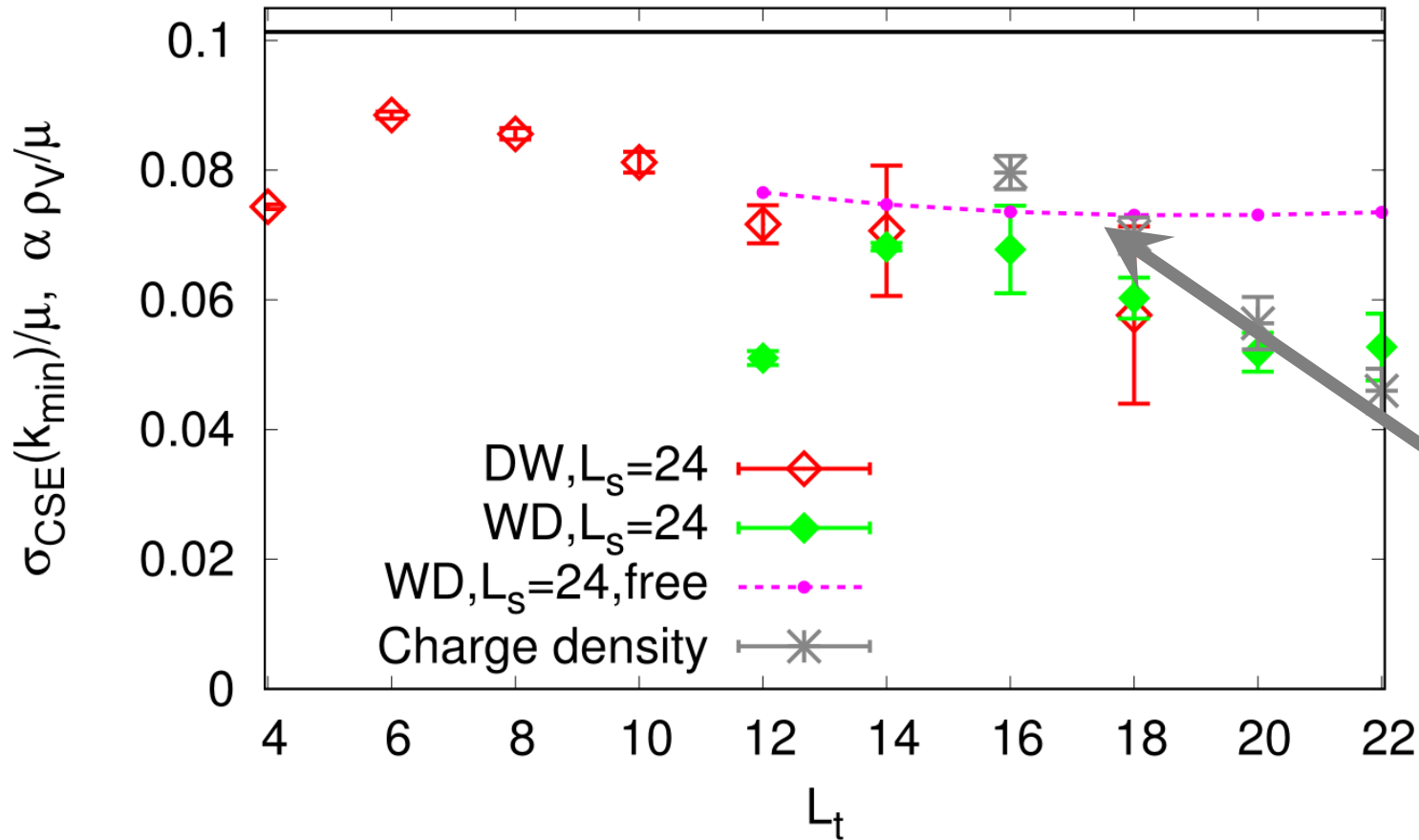


Significant suppression towards low temperatures!

Rescaled charge density

σ_{CSE} vs temperature, medium μ

$a\mu = 0.10$

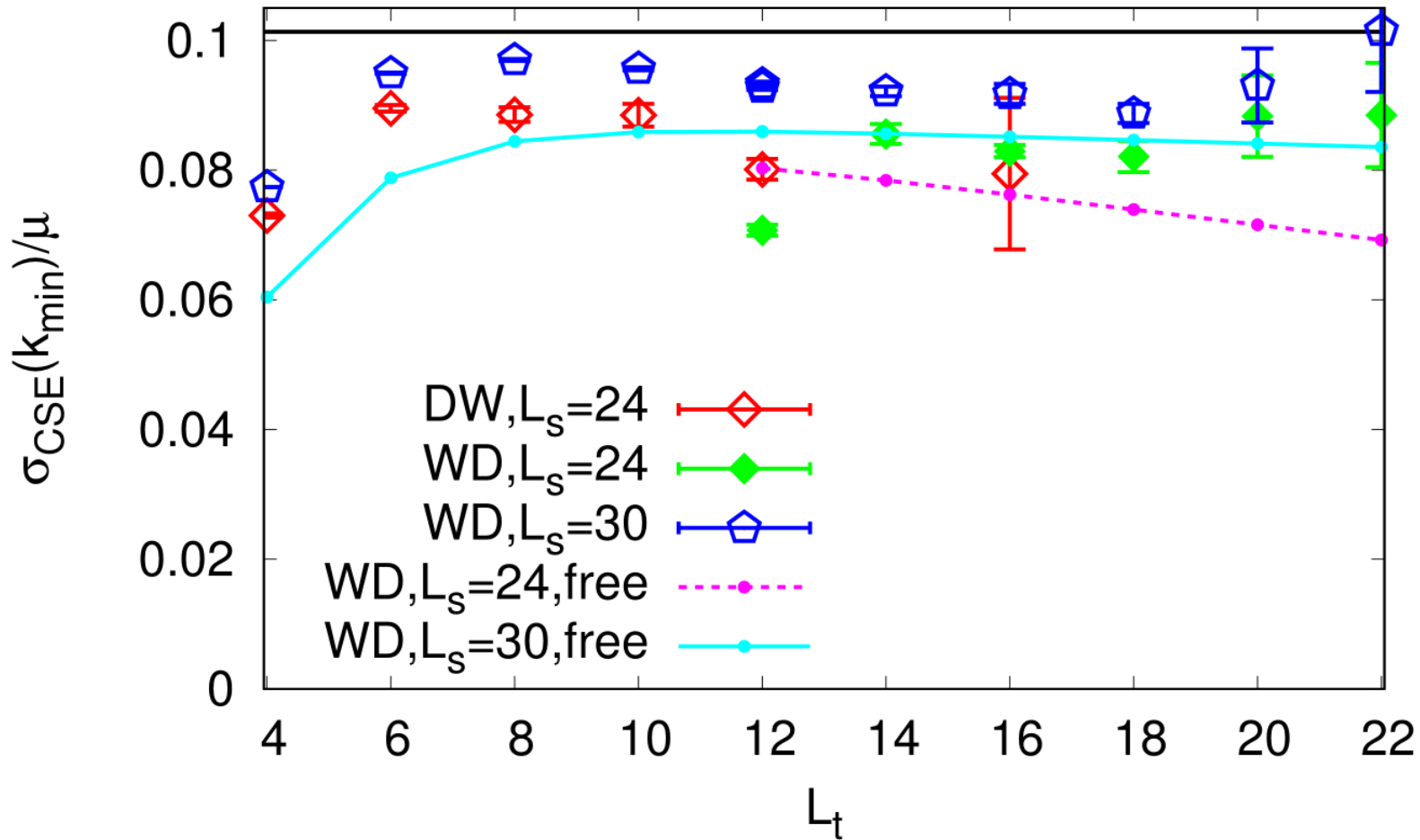


Data moving closer to the free fermion results

Rescaled charge density (same coefficient)

σ_{CSE} vs temperature, large μ

$$a\mu = 0.20$$



Data quite close to the free fermion results

Describing CSE suppression

- ChPT result for flavor-**non-singlet axial current** [Avdoshkin,Sadofyev,Zakharov' 1712.01256]:
- We work with flavor-**singlet axial current**, has different status in ChPT
- Singlet and non-singlet currents become similar at **large N_c**
- **Phenomenological formula** works well in the low- T , low- μ regime even for singlet axial current in $SU(2)$ gauge theory

$$\vec{j}_A^a = \frac{N_c \text{Tr} (Q)}{(2\pi f_\pi)^2} \rho_V^a \vec{B}$$

Disconnected contribution appears to be small!

$$\sigma_{CSE} (\mu, T) = \alpha \rho_V (\mu, T)$$

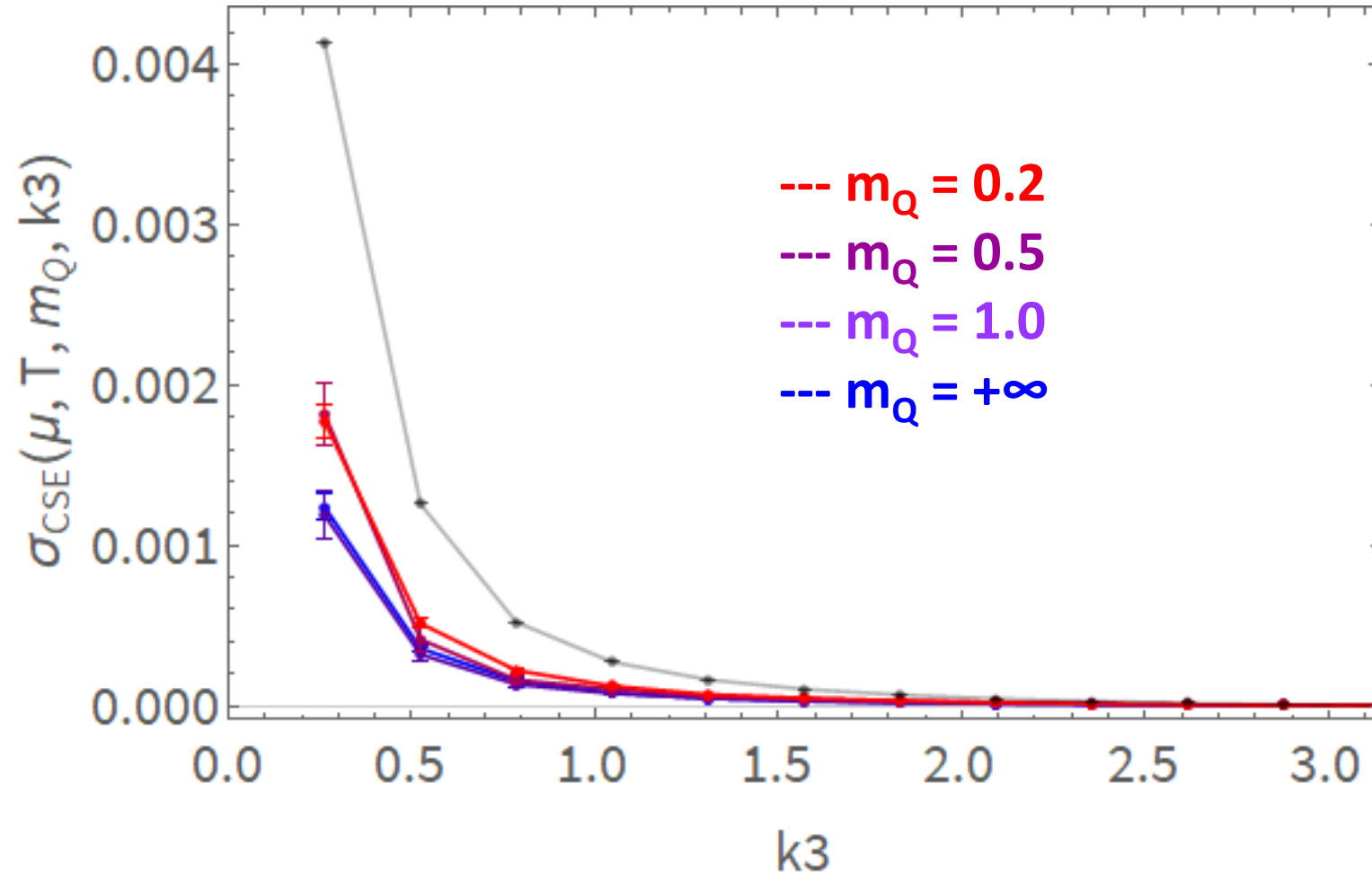
Kondo effect in non-Abelian gauge theory

- Suppression of an interesting effect feels somewhat unfortunate...
- Is there something that can **enhance the CSE?**
- Yes, **QCD** Kondo Effect [Suenaga et al., 2012.15173]

Kondo effect in non-Abelian gauge theory

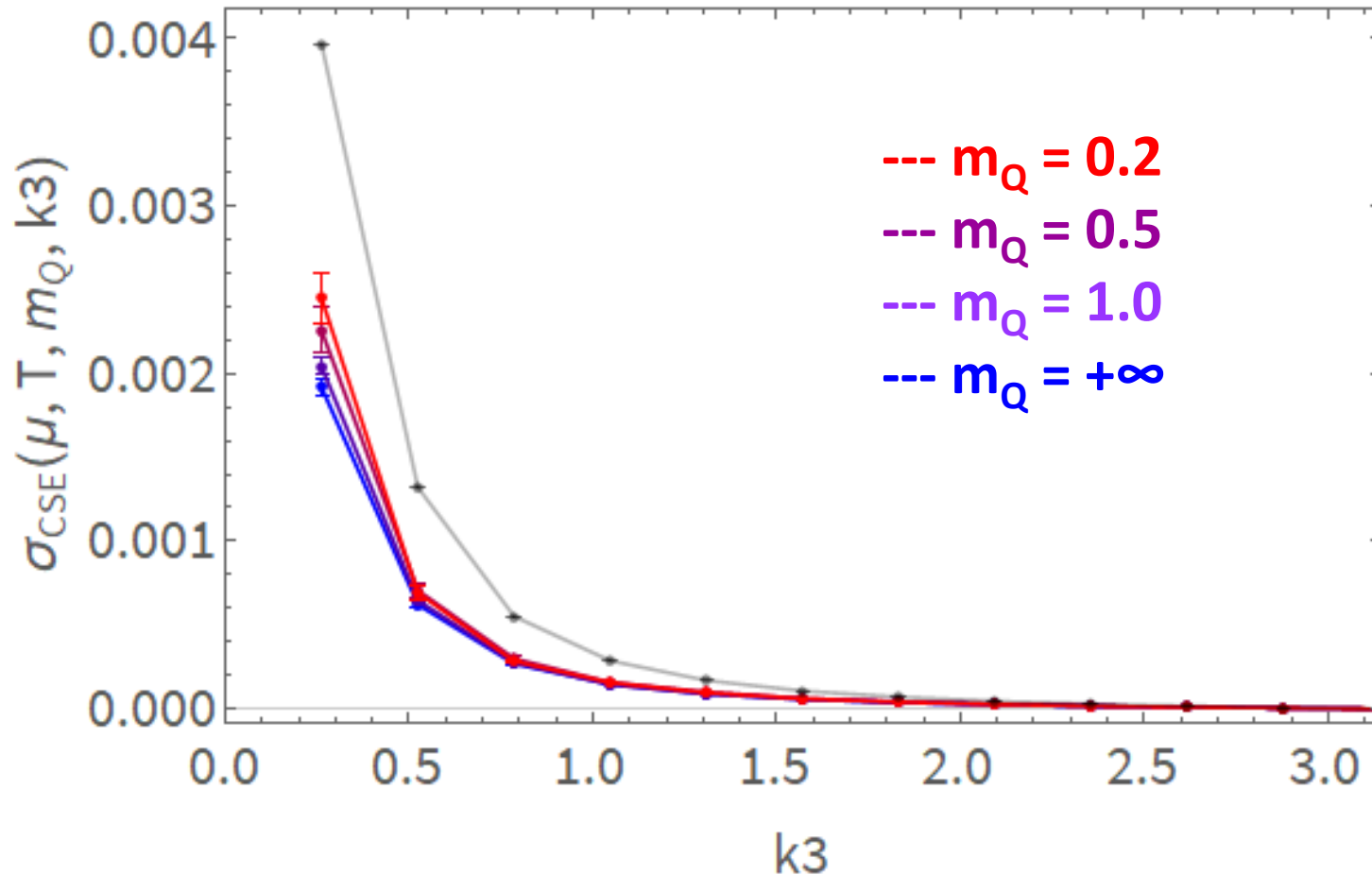
- **Kondo effect**: scattering of **light fermion** near a Fermi surface off a **heavy fermion** of mass M enhanced as $\log(M)$
- Mean-field approach for **QCD** [Yasui,Suzuki,Itakura, 1604.07208]: **spontaneous emergence of Kondo condensate** $\langle \bar{Q} q \rangle$
- **Suppresses low-T, finite- μ conductivity** [Yasui,Ozaki, 1710.03434]
- **But... CSE is enhanced** [Suenaga,Araki,Suzuki,Yasui, 2012.15173]
- **We only consider CSE of light quarks**

Numerical results for CSE in $N_f=2+1$ SU(2) LGT



- $Lt=20$, low-temperature regime
- CSE enhanced by more than **30%** for $m_Q=0.2 a$

Numerical results for CSE in $N_f=2+1$ SU(2) LGT



- $Lt=18$, a bit higher temperature
- Enhancement not so large
- Not a conventional Kondo, Fermi surface not well-defined

Conclusions

- CSE close to **free-quark result** at high temperatures and/or high densities
- Significant **suppression** at low temperatures and low densities
- σ_{CSE} approximately proportional to charge density rather than chemical potential
- Similar to **ChPT calculation** of [Avdoshkin,Sadofyev,Zakharov' 1712.01256] for axial non-flavor-singlet current, although non-singlet and singlet axial currents are physically quite different
- CSE can be **enhanced** in the presence of additional fermion flavors – signature of **Kondo effect**
- Next step: **conductivity** at finite density with **heavy quarks**