Search for QCD critical point by NA61/SHINE at CERN SPS

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  - QCD critical point
  - Exploring the phase diagram with heavy-ion collisions
- Experimental measures to search QCD critical point
  - Fluctuations in large momentum bins
    - Extensive quantities
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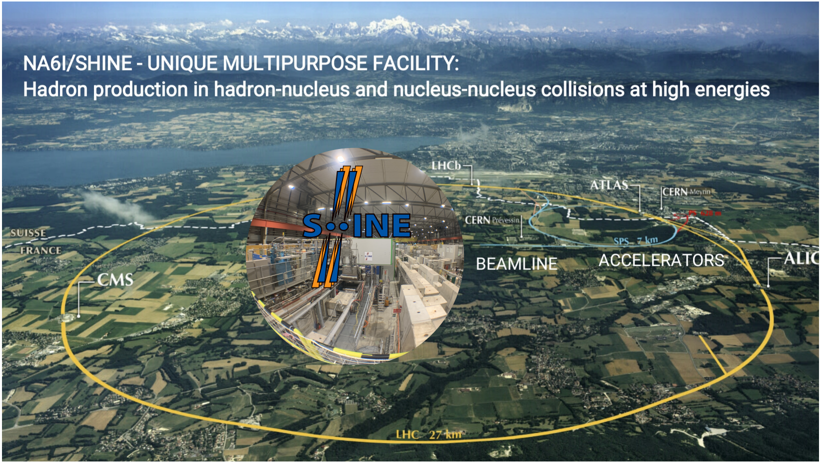
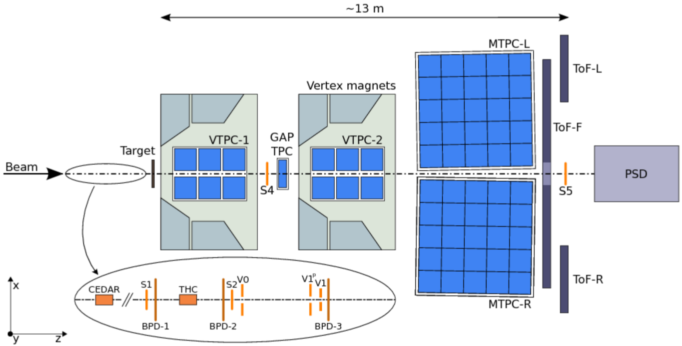
  - Fluctuation as a function of momentum bin size
    (Pb+Pb at 30A GeV/c($\sqrt{s_{NN}} \approx$ 7.5 GeV), Ar+Sc at 150A GeV/c($\sqrt{s_{NN}} \approx$ 17 GeV))
    - Proton intermittency anlysis
    - Pion intermittency analysis
- Summary

NA61/SHINE at CERN SPS



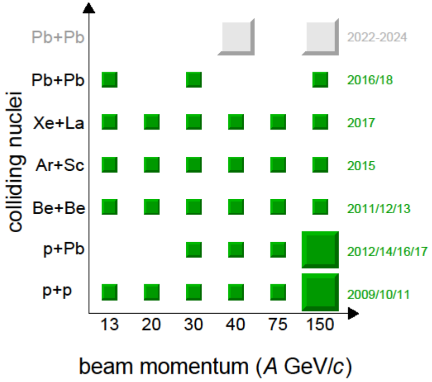
NA61/SHINE (**S**PS **H**eavy **I**on and **N**eutrino **E**xperiment) is a particle physics fixed-target experiment at CERN SPS

NA61/SHINE detector



- VTPC-1, VTPC-2 and GTPC are placed in the magnetic field
- TPC system: track reconstruction and particle identification based on specific energy loss
- Projectile Spectator Detector (PSD): hadronic calorimeter, measures projectile spectators energy

NA61/SHINE Physisc program

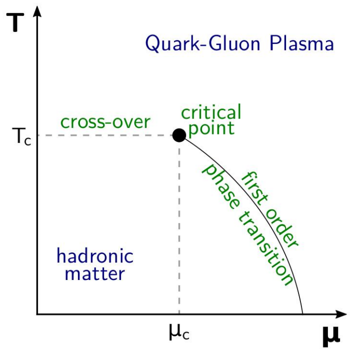


NA61/SHINE performs scan in beam momenta (13*A* - 150*A* GeV/*c*) and mass of colliding nuclei (p+p, p+Pb, Be+Be, Ar+Sc, Xe+La, Pb+Pb)

- **One of the main goals of NA61/SHINE is to search for the QCD critical point**
- Study of the properties of the onset of deconfinement
- Heavy quarks: direct measurement of open charm at SPS energies
- Measurements for the J-PARC and Fermilab neutrino programs
- Measurements of nuclear fragmentation cross sections for cosmic rays physics

QCD critical point

QCD critical point search strategies



Critical Point (CP): a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition.

Second order phase transition → scale invariance → power-law form of correlation function

These expectations are for fluctuations and correlations in the configuration space.

They are expected to be *projected* to the momentum space via quantum statistics and/or collective flow.

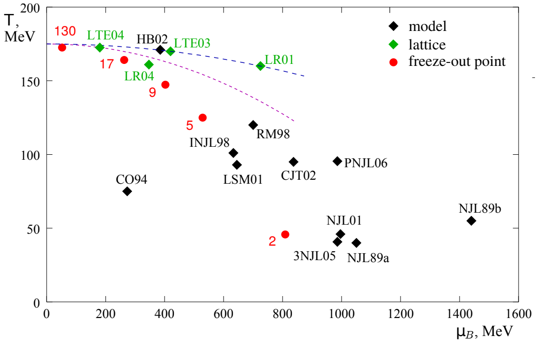
Asakawa, Yazaki NPA 504 (1989) 668

Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

QCD critical point

critical point search strategies

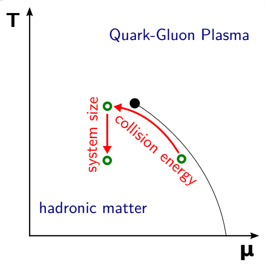
The main signal of the CP is anomaly in fluctuations in a narrow domain of the phase diagram.



However predictions on the CP existence, its location and what and how should fluctuate are model-dependent.

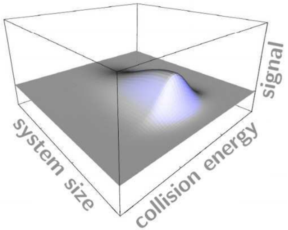
Exploring the phase diagram with heavy-ion collisions

critical point search strategies



Search for the critical point in heavy-ion collisions is performed by scan in the parameters controlled in laboratory (collision energy and nuclear mass number).

By changing them, we change freeze-out parameters (T,μ_B)



Sketch of the **critical hill** expected in the search for the critical point in the two dimensional plane of system size and collision energy.

Extensive quantities

Fluctuations in large momentum bins

A quantity proportional to W (WNM) or V in (IB-GCE) is called an extensive quantity. The most popular are particle number (multiplicity) distribution P(N) cumulants:

- $\kappa_1 = \langle N \rangle$
- $\kappa_2 = \left\langle (\delta N)^2 \right\rangle = \sigma^2$
- $\kappa_3 = \left\langle (\delta N)^3 \right\rangle = S\sigma^3$
- $\kappa_4 = \left\langle (\delta N)^4 \right\rangle - 3\left\langle (\delta N)^2 \right\rangle^2 = \kappa\sigma^4$

These multiplicity cumulants characterize the shape of multiplicity distribution and quantify fluctuations.

WNM – Wounded Nucleon Model ($\langle N_{A+B} \rangle = \langle W_{A+B} \rangle /2 \cdot \langle N_{N+N} \rangle$)

IB-GCE – Ideal Boltzmann Grand Canonical Ensamble

Bialas, Bleszynski Czyz, NPB 111 461 (1976)

Intensive quantities

Fluctuations in large momentum bins

Ratio of any two extensive quantities is independent of W (WNM) or V (IB-GCE). It is an intensive quantity.
For example:

$$\langle A\rangle / \langle B\rangle = W \cdot \langle a\rangle / W \cdot \langle b\rangle = \langle a\rangle / \langle b\rangle$$

where A and B are any extensive event quantities, i.e. $\langle \mathrm{A}\rangle \sim \mathrm{W}$, $\langle \mathrm{B}\rangle \sim \mathrm{W}$ and $\langle \mathrm{a}\rangle = \langle \mathrm{A}\rangle$ and $\langle \mathrm{b}\rangle = \langle \mathrm{B}\rangle$ for $\mathrm{W} = 1$.

Popular examples:

- $\frac{\kappa_2}{\kappa_1} = \omega[\mathrm{N}] = \frac{\sigma^2[\mathrm{N}]}{\langle \mathrm{N}\rangle} = \frac{\mathrm{W} \cdot \sigma^2[\mathrm{n}]}{\mathrm{W} \cdot \langle \mathrm{n}\rangle} = \omega[\mathrm{n}]$ (scaled variance)
- $\frac{\kappa_3}{\kappa_2} = \mathrm{S}\sigma$
- $\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$

Strongly intensive quantities

Fluctuations in large momentum bins

For an event sample with varying W, cumulants are not extensive quantities anymore. For example:

$$\kappa_2 = \sigma^2[N] = \sigma^2[n]\langle W\rangle + \langle n\rangle^2 \sigma^2[W]$$

But having two extensive event quantities, one can construct quantities that are independent of P(W)!

Popular example:

$$\Sigma[N, P_T] = \frac{1}{C}(\omega[N]\langle P_T\rangle + \omega[B]\langle N\rangle - 2(\langle NP_T\rangle - \langle P_T\rangle\langle N\rangle))$$

Where $P_T = \sum_{i=1}^{N} p_{T,i}$ and C is any extensive quantity (e.g. $\langle N\rangle$)

Gazdzicki, Gorenstein, PRC 84(2011) 014904

Gazdzicki, Gorenstein, Mackowiak-Pawlowska PRC 88(2013) 024907

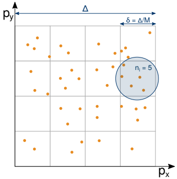
Scaled factorial moments of order r

Fluctuation as a function of momentum bin size

In NA61/SHINE at CERN SPS, intermittency analysis is performed at mid-rapidity and particle fluctuations are studied in transverse momentum plane.

At the second order phase transition (critical point), the system becomes scale invariant. This phenomenon leads to enhanced multiplicity fluctuations with special properties, that can be revealed by scaled factorial moments:

$$F_r(\delta) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i-1)\ldots(n_i-r+1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^r}$$



δ: size of each of the M sub-division intervals of Δ

n_i : number of particles in i-th bin

Scaled factorial moments of order r

Fluctuation as a function of momentum bin size

When the system is a simple fractal and $F_r(\delta)$ follows a power law dependences:

$$F_r(\delta) = F_r(\Delta) \cdot (\frac{\Delta}{\delta})^{D \cdot \phi_r}$$

D is the embedding dimension, in this case $D = 2$ (transverse plane)

Additionally, the exponent (intermittency index) ϕ_r obeys the relation:

$$D \cdot \phi_r = (r - 1) \cdot d_r$$

Where the anomalous fractal dimension d_r is independent of r.

Wosiek, APPB 19 (1988) 863
Bialas, Hwa, PLB 253 (1991) 436
Bialas,Peschanski, NPB 273(1986) 703
Antoniou, Diakonos, Kapoyannis, Kousouris, PRL 97 (2006) 032002

Scaled factorial moments in wounded nucleon model

In WNM: $N = \sum_{i=1}^{W} n_i$; W: constant number of wounded nucleon

$$\langle N \rangle = W \cdot \langle n \rangle \text{ and } \omega[N] = \omega[n]$$

Second scaled factorial moments in WNM:

$$F_2[N] = \frac{1}{W} F_2[n] + 1 - \frac{1}{W}$$

$F_2[N]$ is neither extensive ($\approx$ W), nor intensive (const(W))

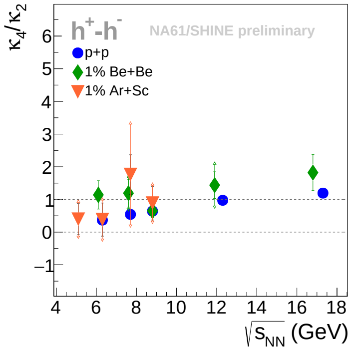
For $F_2[n]$ » 1, W » 1 limit,

$$F_2[N] = \frac{1}{W} F_2[n]$$

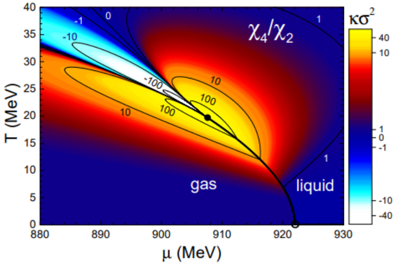
Scaled factorial moments is inversely extensive quantities

Multiplicity fluctuations

Experimental results on Fluctuations in large momentum bins



Critical fluctuations in models with van der Waals interactions



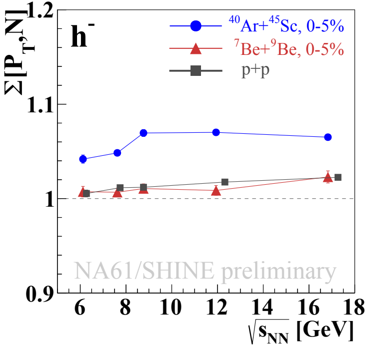
No prominent structures that could be related to the critical point are observed so far...

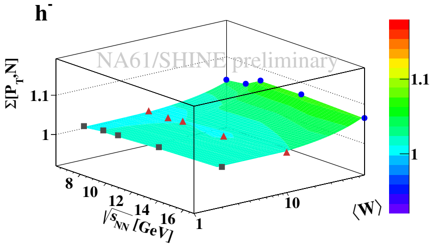
NA61/SHINE: PoS CPOD2017 (2018) 012

V. Vovchenko, D. V. Anchishkin, and M. I. Gorenstein, J. Phys. A48,305001 (2015)

Multiplicity-transverse momentum fluctuations

Experimental results on Fluctuations in large momentum bins





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NA61/SHINE: Acta Phys.Polon.Supp. 10(2017) 449

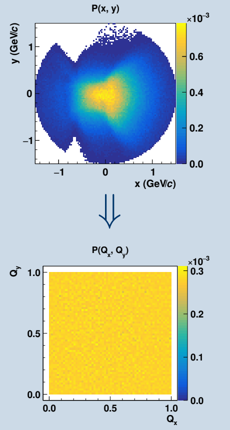
Cumulative variables

Instead of using p_x and p_y, one can use cumulative quantities:

$$Q_x = \int_{x_{min}}^{x} \rho(x)dx / \int_{x_{min}}^{x_{max}} \rho(x)dx$$

$$Q_y = \int_{y_{min}}^{y} P(x,y)dy / P(x)$$

- transform any distribution into uniform distribution (0,1)
- remove the dependence of F_2 on the shape of the single-particle distribution
- intermittency index of an ideal power-law correlation function system described in two dimensions in momentum space was proven to remain approximately invariant after the transformation

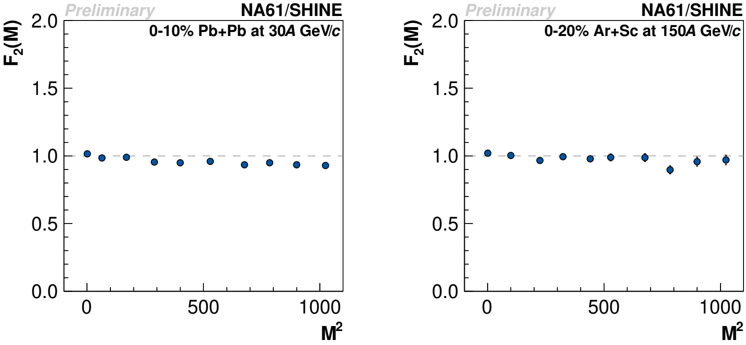


(example for 0-5% Ar+Sc at 150A GeV/c)

Bialas, Gazdzicki, PLB 252 (1990) 483
Antoniou, Diakonos, https://indico.cern.ch/event/818624/

Proton intermittency analysis result

Experimental result on fluctuations as a function of momentum bin size



statistical uncertainties only

CPOD 2021: https://indico.cern.ch/event/985460

No prominent structures that could be related to the critical point are observed so far...

Simple power-law model

A simple model that generates momentum of particles for a given number of events with a given multiplicity distribution.

It has two main parameters:

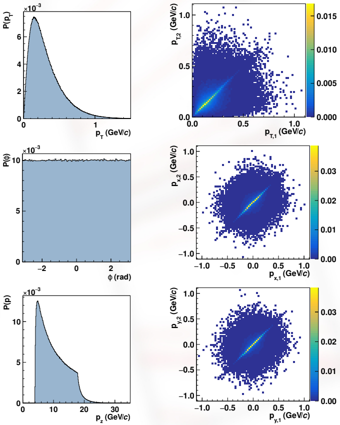
- ratio of correlated to uncorrelated particles
- power-law exponent

Uncorrelated particles (background)

$$\rho_B(p_T) = p_T \cdot e^{-6p_T}$$

Correlated pairs (signal)

$$\rho_S(p_{T,1}, p_{T,2}) = \rho_B(p_{T,1}) \cdot \rho_B(p_{T,2}) \cdot \left[|\Delta p_x|^{\phi} + \epsilon\right]^{-1} \cdot \left[|\Delta p_y|^{\phi} + \epsilon\right]^{-1}$$



Example for:
$\phi = 0.80$
$\epsilon = 10^{-5}$
$N_B = \text{Poisson}(30)$
$N_S = 2$

Simple power-law model

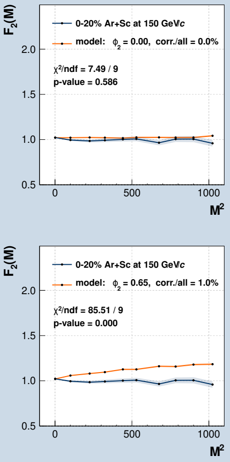
Lots of model data sets are generated:

- correlated-to-all ratio: vary from 0.0 to 4.0% (with 0.2 steps)
- power-law-exponent: vary from 0.0 to 1.0 (with 0.05 steps)

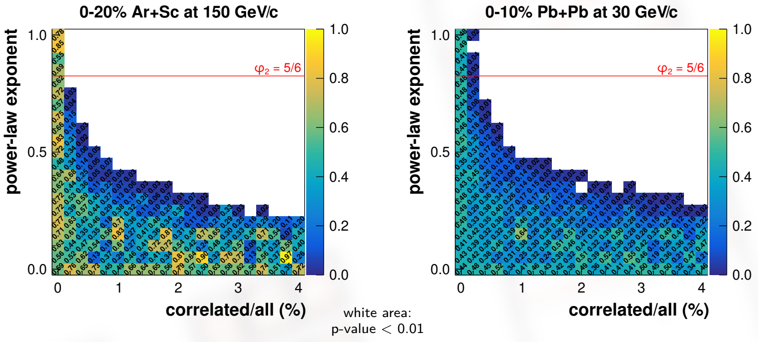
and compared with the experimental data

For the construction of exclusion plots, statistical uncertainties were calculated using model with statistics corresponding to the data.

M = 1 ... 32



Exclusion plot

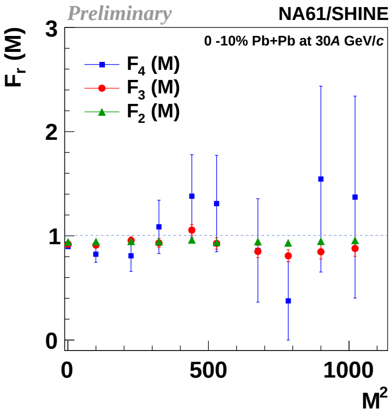


exclusion plots for parameters of simple power-law model

The intermittency index φ_2 for a system freezing out at the QCD critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that the latter belongs to the 3-D Ising universality class.

Pion intermittency analysis result with higher order moments

Experimental result on fluctuations as a function of momentum bin size



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Summary

- No prominent structures are observed related to QCD critical point in experimental result on multiplicity and multiplicity-transverse momentum fluctuation
- No significant critical signal observed via proton and pion intermittency analysis of Pb+Pb at 30A GeV/c and proton intermittency analisis of Ar+Sc at 150A GeV/c data of NA61/SHINE at CERN SPS
- We are continuing proton and pion intermittency analysis to search for critical point of strongly interacting matter for Pb+Pb at 13A GeV/c and Ar+Sc at 13A - 75A GeV/c data of NA61/SHINE

Acknowledgement

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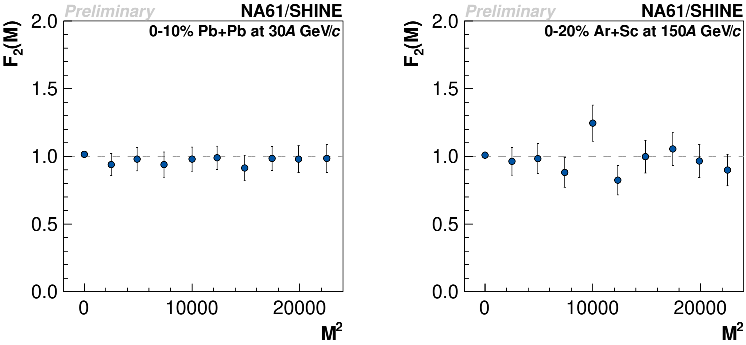
Back-ups!

Proton intermittency analysis result

Experimental result on fluctuations as a function of momentum bin size

Full M range:



CPOD 2021: https://indico.cern.ch/event/985460

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In practice ...

modified equivalent formula

Factorial moment in general for any order r is given by;

$$F_r(M) = \frac{r!(M^2)^{r-1}}{\langle N\rangle^r}\left\langle \sum_{m=1}^{M^2}\binom{n_m}{r}\right\rangle$$

$$F_2(M) = \frac{2M^2}{\langle N\rangle^2}\langle N_2(M)\rangle$$

$$F_3(M) = \frac{6M^4}{\langle N\rangle^3}\langle N_3(M)\rangle$$

$$F_4(M) = \frac{24M^6}{\langle N\rangle^4}\langle N_4(M)\rangle$$

M: number of bins in p_x and p_y

N: event multiplicity

n_m: numbers of particles in ith bin

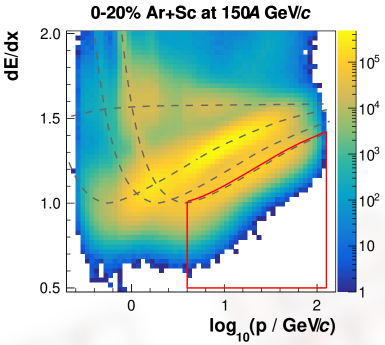
$\langle \ldots \rangle$: averaging over events

$N_2(M)$: number of pairs of particles in M bins

$N_3(M)$: number of triplets of particles in M bins

$N_4(M)$: number of quadruplets of particles in M bins

Proton candidates selection



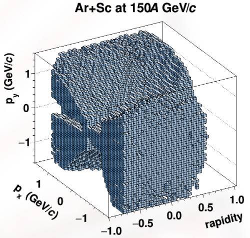
Selection of protons is based on dE/dx measurements in TPCs:

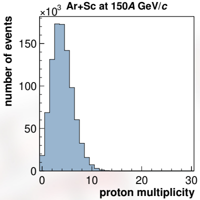
- $0.60 < \log_{10}(p \; / \; \mathrm{GeV}/c) < 2.10$
- $0.5 < dE/dx < BB_p + 0.15(BB_K - BB_p)$

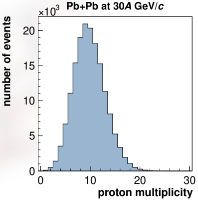
Within the selected range, the cut selects more than 50% of protons and a few percent kaons.

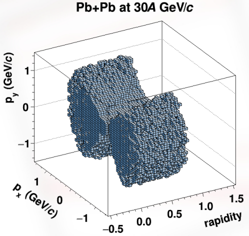
Analysis acceptance

|p_x| < 1.5 GeV/c
|p_y| < 1.5 GeV/c
Ar+Sc: -0.75 < rapidity < 0.75
Pb+Pb: 0.00 < rapidity < 0.75

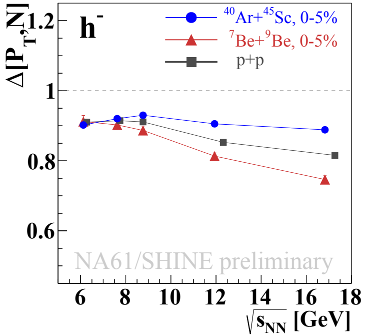








Multiplicity-transverse momentum fluctuations



No prominent structures that could be related to the critical point are observed so far...

NA61/SHINE: Acta Phys.Polon.Supp. 10(2017) 449

Strongly intensive quantities

For an event sample with varying W, cumulants are not extensive quantities any more. For example:

$$\kappa_2 = \sigma^2[N] = \sigma^2[n] \langle W \rangle + \langle n \rangle^2 \sigma^2[W]$$

But having two extensive event quantities, one can construct quantities that are independent of P(W)!

Popular examples:

- $\langle K \rangle / \langle \pi \rangle$
- $\Delta[N, P_T] = \frac{1}{C}(\omega[N] \langle P_T \rangle - \omega[P_T] \langle N \rangle)$
- $\Sigma[N, P_T] = \frac{1}{C}(\omega[N] \langle P_T \rangle + \omega[B] \langle N \rangle - 2(\langle NP_T \rangle - \langle P_T \rangle \langle N \rangle)$

where $P_T = \sum_{i=1}^{N} p_{T,i}$ and C is any extensive quantity (e.g.$\langle N \rangle$).

Gazdzicki, Gorenstein, PRC 84 (2011) 014904

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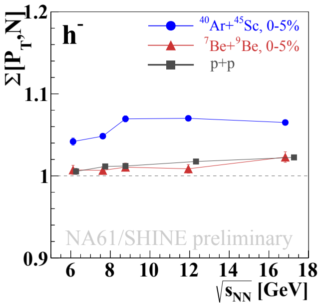

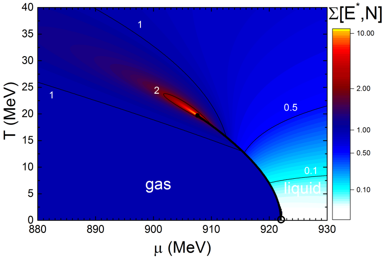

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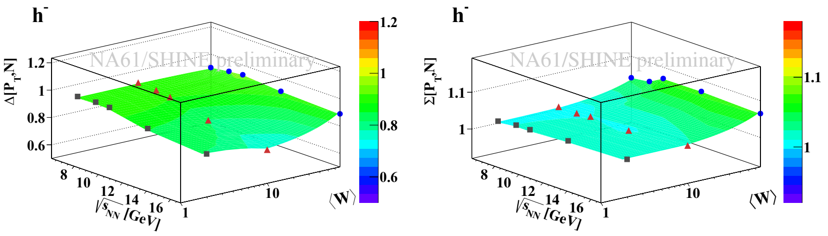
NA61/SHINE: Acta Phys.Polon.Supp. 10(2017) 449

Vovchenko, Anchinshkin, Gorenstein, Poberezhnyuk, Stoecker, Acta Phys.Polon.Supp. 10 (2017) 753

Multiplicity-transverse momentum fluctuations



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