Studying the QCD Phase Structure through Higher Moments at RHIC-BES at STAR

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For the STAR collaboration

ICNFP2021@Kolympari, Greece (Zoom)
Outline

- Introduction
- $C_4/C_2$ for critical point search
- $C_5/C_1$ and $C_6/C_2$ for crossover search
- BES-II and fixed-target programs
- Experimental challenges
- Summary
QCD phase diagram

✓ QCD phase structure in wide ($\mu_B,T$) region.

- Crossover at $\mu_B = 0$ MeV

- 1st-order phase transition at large $\mu_B$?

- Critical point?

Need to investigate the QCD phase structure in wide ($\mu_B, T$) region.

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>No. of events (million)</th>
<th>$T_{ch}$ (MeV)</th>
<th>$\mu_B$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>238</td>
<td>164.3</td>
<td>28</td>
</tr>
<tr>
<td>62.4</td>
<td>47</td>
<td>160.3</td>
<td>70</td>
</tr>
<tr>
<td>54.4</td>
<td>550</td>
<td>160.0</td>
<td>83</td>
</tr>
<tr>
<td>39</td>
<td>86</td>
<td>156.4</td>
<td>160</td>
</tr>
<tr>
<td>27</td>
<td>30</td>
<td>155.0</td>
<td>144</td>
</tr>
<tr>
<td>19.6</td>
<td>15</td>
<td>153.9</td>
<td>188</td>
</tr>
<tr>
<td>14.5</td>
<td>20</td>
<td>151.6</td>
<td>264</td>
</tr>
<tr>
<td>11.5</td>
<td>6.6</td>
<td>149.4</td>
<td>287</td>
</tr>
<tr>
<td>7.7</td>
<td>3</td>
<td>144.3</td>
<td>398</td>
</tr>
</tbody>
</table>

- Crossover at $\mu_B = 0$ MeV
  

- 1st-order phase transition at large $\mu_B$?

- Critical point?
Need to investigate the QCD phase structure in wide $(\mu_B,T)$ region.

- Crossover at $\mu_B = 0$ MeV
  

- 1st-order phase transition at large $\mu_B$?

- Critical point?

- Large & uniform acceptance (full azimuth, $|\eta|<1$)

- Excellent particle identification
Higher-order fluctuations

Moments and cumulants are mathematical measures of “shape” of a distribution which probe the fluctuation of observables.

- Moments: mean ($\langle M \rangle$), standard deviation ($\sigma$), skewness ($S$) and kurtosis ($\kappa$).
- $S$ and $\kappa$ are sensitive to non-gaussian fluctuations.

Cumulant $\Leftrightarrow$ Central Moment

\[
\begin{align*}
\langle \delta N \rangle &= N - \langle N \rangle \\
C_1 &= M = \langle N \rangle \\
C_2 &= \sigma^2 = \langle (\delta N)^2 \rangle \\
C_3 &= S\sigma^3 = \langle (\delta N)^3 \rangle \\
C_4 &= \kappa\sigma^4 = \langle (\delta N)^4 \rangle > -3 < (\delta N)^2 >^2
\end{align*}
\]

Cumulant : additivity

\[C_n(X + Y) = C_n(X) + C_n(Y)\]

Proportional to volume

From Wikipedia

- Skewness $\rightarrow$ asymmetry
- Kurtosis $\rightarrow$ sharpness

$k > 0$

$k < 0$
Fluctuations of conserved quantities

- Net baryon, net charge and net strangeness
  “Net” : positive - negative

\[ \Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S \]

No. of positively charged particles in one collision

No. of negatively charged particles in one collision

(1) Sensitive to correlation length

\[
C_2 = \langle (\delta N)^2 \rangle_c \approx \xi^2 \quad C_5 = \langle (\delta N)^5 \rangle_c \approx \xi^{9.5}
\]

\[
C_3 = \langle (\delta N)^3 \rangle_c \approx \xi^{4.5} \quad C_6 = \langle (\delta N)^6 \rangle_c \approx \xi^{12}
\]

\[
C_4 = \langle (\delta N)^4 \rangle_c \approx \xi^7
\]

M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009)

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)


(2) Direct comparison with susceptibilities.


\[
S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \quad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}
\]

\[
\chi_n^q = \frac{1}{VT^3} \times C_n^q \frac{\partial n_p/T^4}{\partial \mu_n^q}, \quad q = B, Q, S
\]

\[ \rightarrow \text{neutrons cannot be measured} \]


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Particle identification

✓ dE/dx measured with TPC is used for proton identification at low $p_T$ region.
✓ The combined PID with $m^2$ from TOF is used at high $p_T$ region.
Raw net-proton distribution

✓ Avoid auto-correlation effects: New centrality definition
✓ Suppress initial volume fluctuation: Centrality bin width correction
✓ Detector efficiency correction: Binomial model

M. Kitazawa: PRC.86.024904 (2012)
A. Bzdak and V. Koch: PRC.86.044904 (2012), X. Luo: PRC.91.034907 (2016)
T. Nonaka, M. Kitazawa, S. Esumi: PRC.95.064912 (2017), NIMA906 10-17 (2018),
NIMA984 (2020) 164632

STAR Collaboration,
PRL.126.092301 (2021)
PRC.104.024902 (2021)
Efficiency correction

Detector efficiencies are studied in embedding simulations.

Efficiencies are assumed to follow binomial distributions.

Multiplicity and $p_T$ dependence are taken into account.

M. Kitazawa : PRC.86.024904(2012)
A. Bzdak and V. Koch : PRC.86.044904(2012)
X. Luo : PRC.91.034907(2016)
T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912(2017)
Centrality bin width correction

STAR Collaboration, PRC.104.024902(2021)

- Final state multiplicity and initial geometry are not one-to-one corresponding → volume fluctuation
- Data driven approach (CBWC) is applied.
$C_4/C_2$ for critical point search

✓ Net-proton $\kappa\sigma^2 (C_4/C_2)$ shows a non-monotonic behaviour. The trend is consistent with the expectation from theoretical calculations having a critical point.

✓ Enhancement at low beam energies cannot be explained by baryon number conservation.

STAR Collaboration, PRL.126.092301(2021)

STAR Data
- 0 - 5%
- 70 - 80%
- Stat. uncertainty
- Syst. uncertainty
- Projected BES-II
- Stat. uncertainty
- UrQMD 0-5%

Au+Au Collisions at RHIC
Net-proton $|y| < 0.5, ~ 0.4 < p_T < 2.0$ (GeV/c)

Collision Energy $\sqrt{s_{NN}}$ (GeV)

$T$, GeV

$\kappa\sigma^2$

M.A. Stephanov, PRL107, 052301 (2011)
Non-monotonicity

- Polynomial fits are done varying the data point within uncertainties.
- Check the probability that at least one point of derivatives at 8 energies has different sign from others → **3.1σ significance of non-monotonicity for κσ²**

![Graph](image)

**STAR Collaboration, PRL.126.092301(2021)**
Net-charge and net-kaon

\[ \text{error}(\kappa \sigma^2) \propto \frac{\sigma^2}{\epsilon^2} \frac{1}{\sqrt{N_{\text{evts}}}} \]

✓ Large statistical uncertainties, need more data.
**C₆/C₂ for crossover search**

✓ There isn’t yet any direct experimental evidence for the smooth crossover at $\mu_B \sim 0$.

✓ $C_6/C_2 < 0$ is predicted as a signature of crossover transition.

✓ High-statistics data sets at $\sqrt{s_{NN}} = 27$, 54.4, and 200 GeV are analyzed to look for the experimental signature of crossover transition.

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**Predicted scenario for this measurement**

A. Bazavov et al, *PhysRevD.95.054504* : LQCD


<table>
<thead>
<tr>
<th>Freeze-out conditions</th>
<th>$\chi_B^4 / \chi_2^B$</th>
<th>$\chi_6^B / \chi_2^B$</th>
<th>$\chi_4^Q / \chi_2^Q$</th>
<th>$\chi_6^Q / \chi_2^Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRG</td>
<td>1</td>
<td>1</td>
<td>$\sim 2$</td>
<td>$\sim 10$</td>
</tr>
<tr>
<td>QCD: $T_{\text{freeze}} / T_{pc} \lesssim 0.9$</td>
<td>$\gtrsim 1$</td>
<td>$\gtrsim 1$</td>
<td>$\sim 2$</td>
<td>$\sim 10$</td>
</tr>
<tr>
<td>QCD: $T_{\text{freeze}} / T_{pc} \simeq 1$</td>
<td>$\sim 0.5$</td>
<td>$&lt; 0$</td>
<td>$\sim 1$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

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**STAR**

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Centrality dependence

✓ $C_6/C_2$ values are progressively negative from peripheral to central collisions at 200 GeV, which is consistent with LQCD calculations.

✓ Could suggest a smooth crossover transition at top RHIC energy.
Energy dependence of $C_5/C_1$ and $C_6/C_2$

- Weak collision energy dependence observed for 0-40% centrality.
- Deviations from zero at a level of $< 2\sigma$ observed for 0-40% centrality.

✓ $C_5/C_1$ and $C_6/C_2$ are positive for p+p collisions, while negative for central Au+Au collisions.
✓ Lattice calculations imply chiral phase transition in the thermalized QCD matter, which is not the case in 200 GeV p+p collisions.

STAR Preliminary

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PRC.104.024902(2021)

BES-II and Fixed-target programs

✓ 10-20 times larger statistics than BES-I have been successfully collected.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>Beam Energy (GeV/nucleon)</th>
<th>Collider or Fixed Target</th>
<th>$p_T$ (MeV)</th>
<th>Run Time (days)</th>
<th>No. Events Collected (Request)</th>
<th>Date Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>100</td>
<td>C</td>
<td>0</td>
<td>25</td>
<td>138 M (140 M)</td>
<td>Run-19</td>
</tr>
<tr>
<td>27</td>
<td>13.5</td>
<td>C</td>
<td>0</td>
<td>156</td>
<td>555 M (700 M)</td>
<td>Run-18</td>
</tr>
<tr>
<td>19.6</td>
<td>9.8</td>
<td>C</td>
<td>0</td>
<td>206</td>
<td>582 M (400 M)</td>
<td>Run-19</td>
</tr>
<tr>
<td>17.3</td>
<td>8.65</td>
<td>C</td>
<td>0</td>
<td>230</td>
<td>256 M (250 M)</td>
<td>Run-21</td>
</tr>
<tr>
<td>14.6</td>
<td>7.3</td>
<td>C</td>
<td>0</td>
<td>262</td>
<td>324 M (310 M)</td>
<td>Run-19</td>
</tr>
<tr>
<td>13.7</td>
<td>100</td>
<td>FXT</td>
<td>2.69</td>
<td>276</td>
<td>52 M (50 M)</td>
<td>Run-21</td>
</tr>
<tr>
<td>11.5</td>
<td>5.75</td>
<td>C</td>
<td>0</td>
<td>316</td>
<td>235 M (230 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>11.5</td>
<td>70</td>
<td>FXT</td>
<td>2.51</td>
<td>316</td>
<td>50 M (50 M)</td>
<td>Run-21</td>
</tr>
<tr>
<td>9.2</td>
<td>4.59</td>
<td>C</td>
<td>0</td>
<td>372</td>
<td>162 M (160 M)</td>
<td>Run-20+20b</td>
</tr>
<tr>
<td>9.2</td>
<td>4.45</td>
<td>FXT</td>
<td>2.28</td>
<td>372</td>
<td>50 M (50 M)</td>
<td>Run-21</td>
</tr>
<tr>
<td>7.7</td>
<td>3.85</td>
<td>C</td>
<td>0</td>
<td>420</td>
<td>100 M (100 M)</td>
<td>Run-21</td>
</tr>
<tr>
<td>7.7</td>
<td>31.2</td>
<td>FXT</td>
<td>2.10</td>
<td>420</td>
<td>50 M + 112 M + 100 M (100 M)</td>
<td>Run-19+20+21</td>
</tr>
<tr>
<td>7.2</td>
<td>26.5</td>
<td>FXT</td>
<td>2.02</td>
<td>443</td>
<td>155 M + 317 M</td>
<td>Run-18+20</td>
</tr>
<tr>
<td>6.2</td>
<td>19.5</td>
<td>FXT</td>
<td>1.87</td>
<td>487</td>
<td>118 M (100 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>5.2</td>
<td>13.5</td>
<td>FXT</td>
<td>1.68</td>
<td>541</td>
<td>103 M (100 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>4.5</td>
<td>9.8</td>
<td>FXT</td>
<td>1.52</td>
<td>589</td>
<td>108 M (100 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>3.9</td>
<td>7.3</td>
<td>FXT</td>
<td>1.37</td>
<td>633</td>
<td>117 M (100 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>3.5</td>
<td>5.75</td>
<td>FXT</td>
<td>1.25</td>
<td>666</td>
<td>116 M (100 M)</td>
<td>Run-20</td>
</tr>
<tr>
<td>3.2</td>
<td>4.59</td>
<td>FXT</td>
<td>1.13</td>
<td>699</td>
<td>200 M (200 M)</td>
<td>Run-19</td>
</tr>
<tr>
<td>3.0</td>
<td>3.85</td>
<td>FXT</td>
<td>1.05</td>
<td>721</td>
<td>259 M -&gt; 2B(100 M -&gt; 2B)</td>
<td>Run-18+21</td>
</tr>
</tbody>
</table>

STAR Collaboration, PRL.126.092301(2021)

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Fixed-target program (FXT)

✓ $\mu_B$ region has been extended up to 720 MeV.
✓ Huge datasets were collected in FXT at 9 beam energies from 3 GeV to 7.7 GeV to confirm the peak structure of $\kappa\sigma^2$.
✓ Pileup is a critical issue on higher-moment analysis especially at most central collisions.

Data-driven approach of the pileup correction is available once true and pileup multiplicity distributions are determined by simulations.
Challenges: Non-binomial efficiency correction

- Conventional correction method cannot be applied if response functions of detector efficiencies deviate from binomial distribution.
- Detector response functions need to be carefully studied via simulations.

Unfolding: Esumi, Nakagawa, Nonaka, NIMA.987.164802(2021)

Moment expansion: Nonaka, Kitazawa, Esumi, NIMA.906.10-17 (2018)
Challenges: Volume fluctuation

✓ Data driven approach (CBWC) and model dependent method (VFC) are consistent with each other in BES-I data.

✓ Due to less centrality resolution in lower collision energies, results will need to be carefully checked among different ways of centrality determination.

\[
\kappa_1(\Delta N) = \left\langle N_W \right\rangle \kappa_1(\Delta n),
\]
\[
\kappa_2(\Delta N) = \left\langle N_W \right\rangle \kappa_2(\Delta n) + \left\langle \Delta n \right\rangle^2 \kappa_2(N_W),
\]
\[
\kappa_3(\Delta N) = \left\langle N_W \right\rangle \kappa_3(\Delta n) + 3 \left\langle \Delta n \right\rangle \kappa_2(\Delta n) \kappa_2(N_W) + \left\langle \Delta n \right\rangle^3 \kappa_3(N_W),
\]
\[
\kappa_4(\Delta N) = \left\langle N_W \right\rangle \kappa_4(\Delta n) + 4 \left\langle \Delta n \right\rangle \kappa_3(\Delta n) \kappa_2(N_W)
+ 3 \kappa_2(\Delta n) \kappa_2(N_W) + 6 \left\langle \Delta n \right\rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \left\langle \Delta n \right\rangle^4 \kappa_4(N_W).
\]

Additional terms appear from the event by event participant fluctuation.

P. Braun-Munzinger, A. Rustamov, J. Stachel: *NPA.2017.01.011*

\[\Delta n: \text{net-proton per } N_W\]
\[\Delta N: \text{net-proton}\]
✓ Non-monotonic beam energy dependence of $C_4/C_2$ has been observed in BES-I. Stay tuned for precise measurement from BES-II and FXT energies (3.3-19.6 GeV) to have more definitive messages.

✓ Negative value of $C_6/C_2$ at $\sqrt{s_{NN}} = 200$ GeV central collisions could suggest a smooth crossover transition at RHIC top energy. This is not the case for p+p 200 GeV high multiplicity events.

✓ FXT 3 GeV analysis is ongoing. Pileup effects can be corrected.

✓ Detector efficiencies and volume fluctuations need to be carefully studied for each experiment.
Thank you for your attention