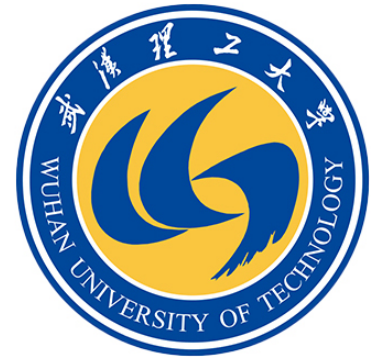


# Vorticity Effect in Heavy-ion Collisions

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*Department of Science, Wuhan University of Technology &  
Institute of Particle Physics, Central China Normal University*

**with D. J. Wang, D. M. Zhou, B. H. Sa and L. P. Csernai**



# OUTLINE

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- 1. Introduction**
- 2. Motivation**
- 3. PACIAE model**
- 4. Results**
- 5. Summary**

# Introduction

## Vortex in the nature



**Vorticity:**  $\boldsymbol{\omega} = \text{curl } \boldsymbol{v} = \nabla \times \boldsymbol{v}$   
reflects the local angular velocity of fluid

# Introduction

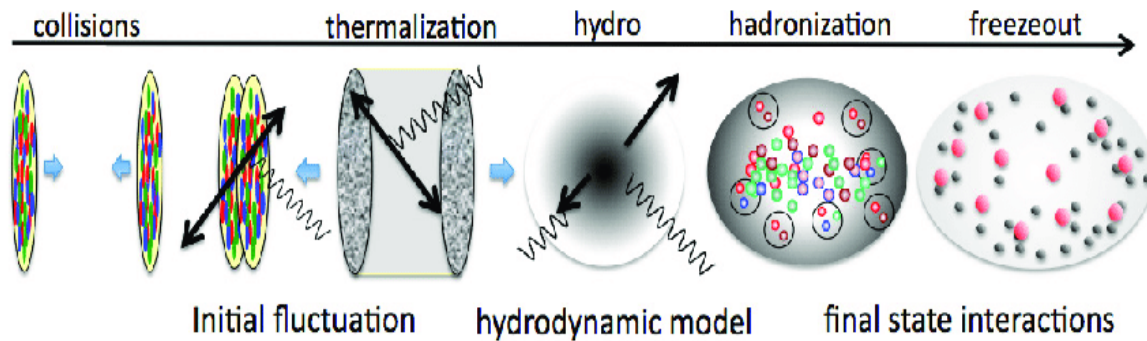


Fig. 1 The evolution of the Heavy-ion Collisions  
 [ S. Suharyo, J. Phys. : Conf. Ser. 856 012002 (2017) ]

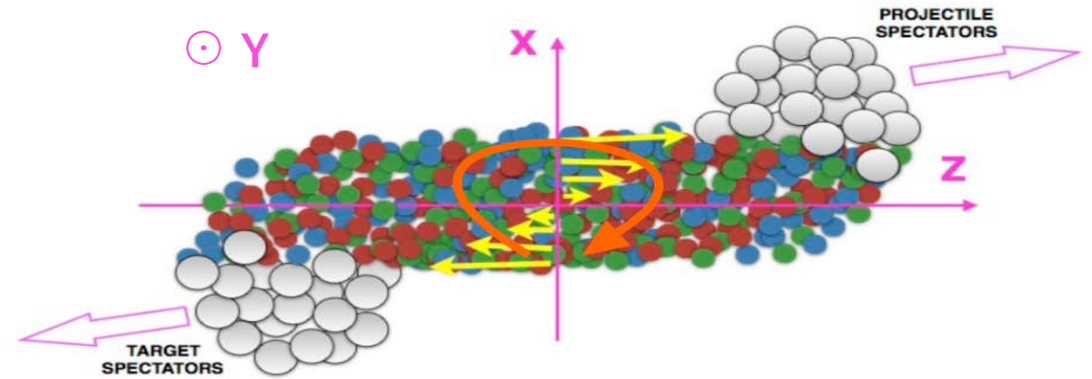


Fig. 2 Longitudinal velocity field and system rotation  
 [ F. Becattini, Phys. Rev. C 95, 054902 (2017) ]

non-central collisions  $\longrightarrow$

huge initial orbital angular momentum (OAM)

$$J_0 \sim 10^5 \hbar$$

- rotation / vorticity (-Y)

# Introduction

The polarization vector for spin-1/2 particles:

[ F. Becattini et al. (2008~2013) ]

$$\Pi^\mu(x, p) = -\frac{1}{8m} \epsilon^{\mu\nu\sigma\tau} (1 - n_{F(x,p)}) p_\tau \varpi_{\rho\sigma}(x)$$

$$\Pi^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\nu\sigma\tau} p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F (1 - n_F) \varpi_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n_F}$$

$\epsilon^{\mu\nu\sigma\tau}$  - Levi-Civita symbol (+1)

$p$  ---- four-momentum

$\Sigma$  ---- hypersurface of freeze-out

$n_F$  ---- Fermi-Dirac distribution

$\varpi$  ---- **thermal vorticity**

Recovered:

[ R. H. Fang et al. (2016) ]

$$S^\mu(x, p) = -\frac{1}{8m} (1 - n_F) \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x)$$

$$S^\mu(x, p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x)$$

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

$$\beta^\mu = u^\mu / T$$

$\beta$  ---- inverse temperature 4-vector field

$u$  ---- four velocity of fluid

# Introduction

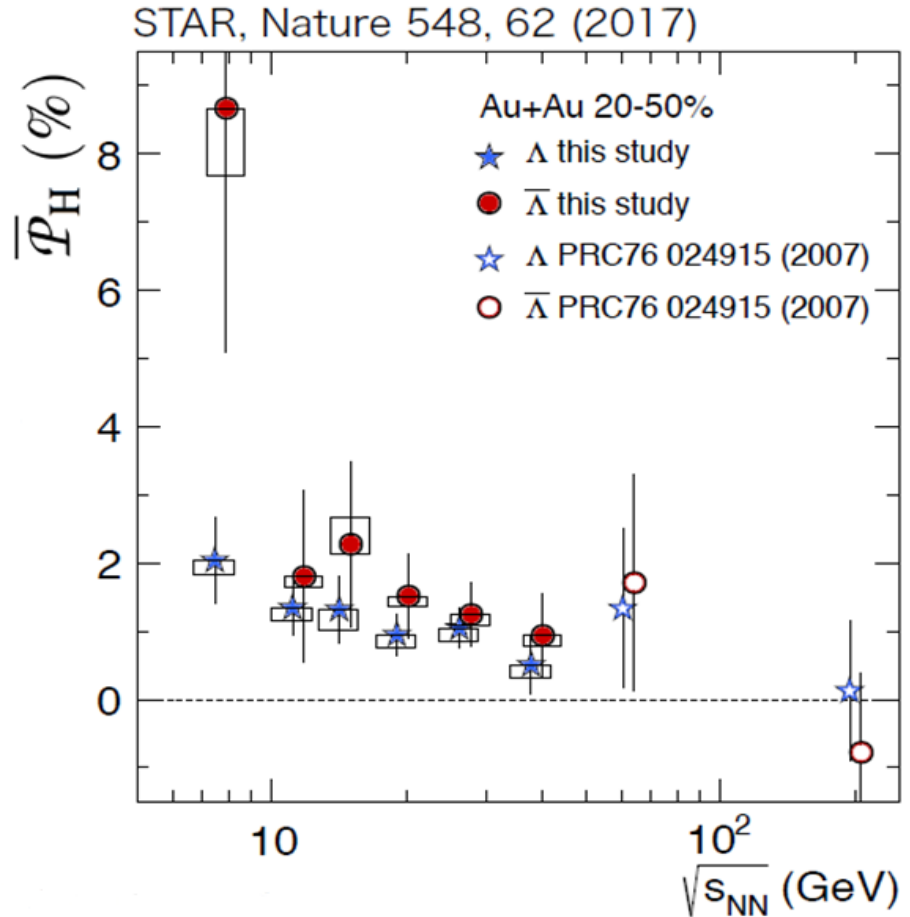


Fig. 3 The global  $\Lambda$  polarization  
 [ STAR, Nature 548,62 (2017) ]

The huge initial OAM will be transferred to the hadron polarization via *spin-vorticity coupling*.



## The Fastest Fluid

by Sylvia Morrow

Superhot material spins at an incredible rate.

$$\omega \sim 10^{-22} \text{ s}^{-1}$$

The most vortical fluid observed in the nature!

# Motivation

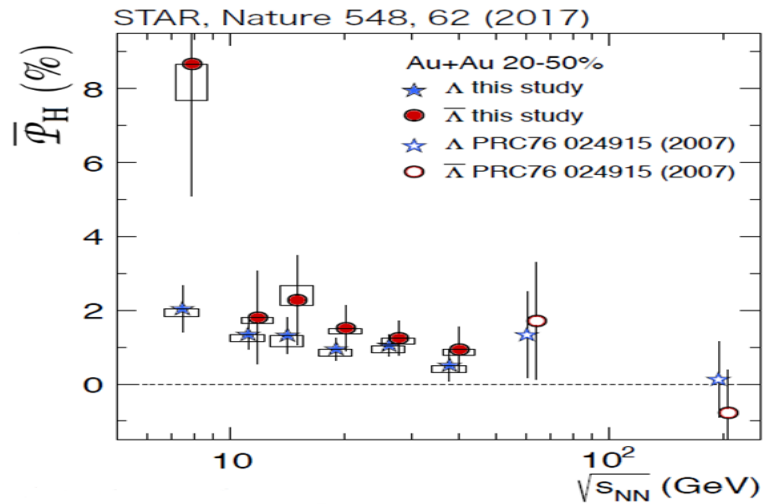


Fig. 4 STAR 2017  
[STAR, Nature 548,62(2017)]

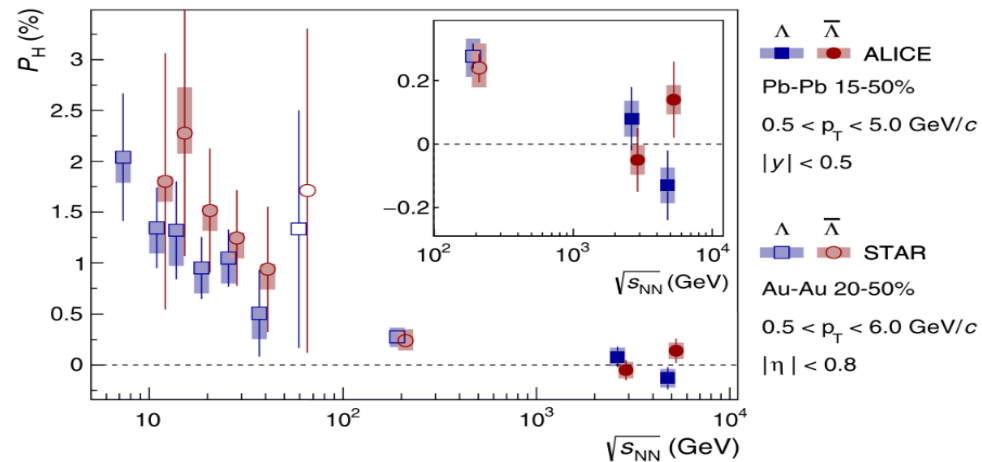


Fig. 5 ALICE 2020  
[ALICE, PRC 101, 044611 (2020)]

The energy dependence of  $\Lambda$  polarization

$$\sqrt{s_{NN}} \uparrow \text{ --- } P_H \downarrow$$

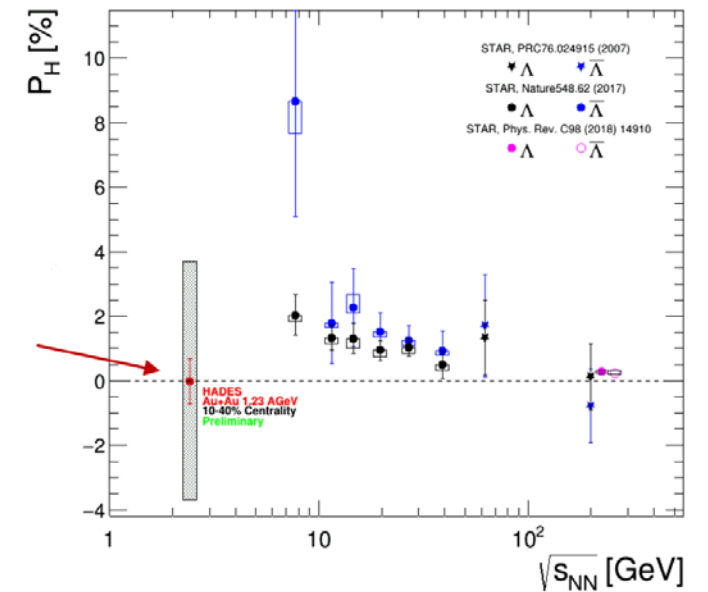


Fig. 6 HADES 2019  
[HADES, Frédéric Kornas` talk in Strange Quark Matter 2019 ]

# Motivation

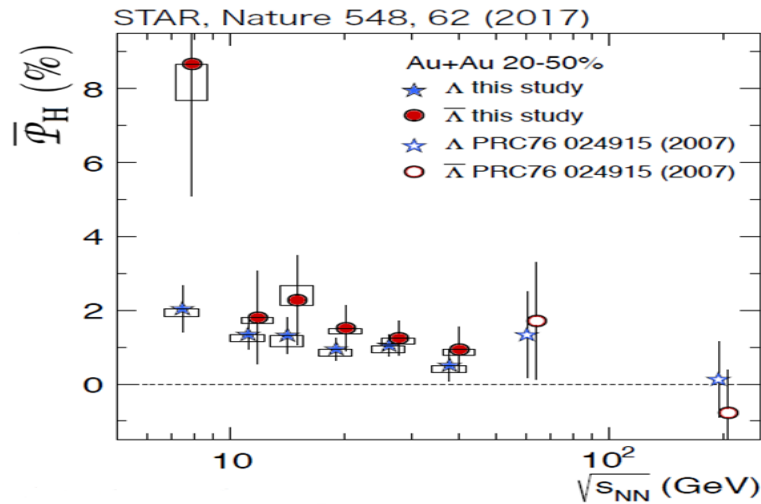


Fig. 4 STAR 2017  
[STAR, Nature 548,62(2017)]

The energy dependence of  $\Lambda$  polarization

$$\sqrt{s_{NN}} \uparrow \text{ --- } P_H \downarrow$$

$P_H \uparrow$ , then  $\downarrow$ , non-monotonic trend ?!

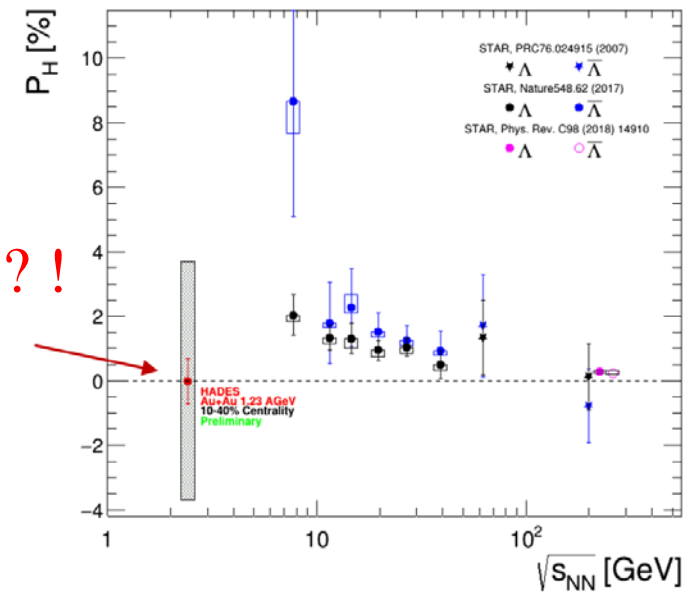


Fig. 6 HADES 2019  
[HADES, Frédéric Kornas` talk in Strange Quark Matter 2019]

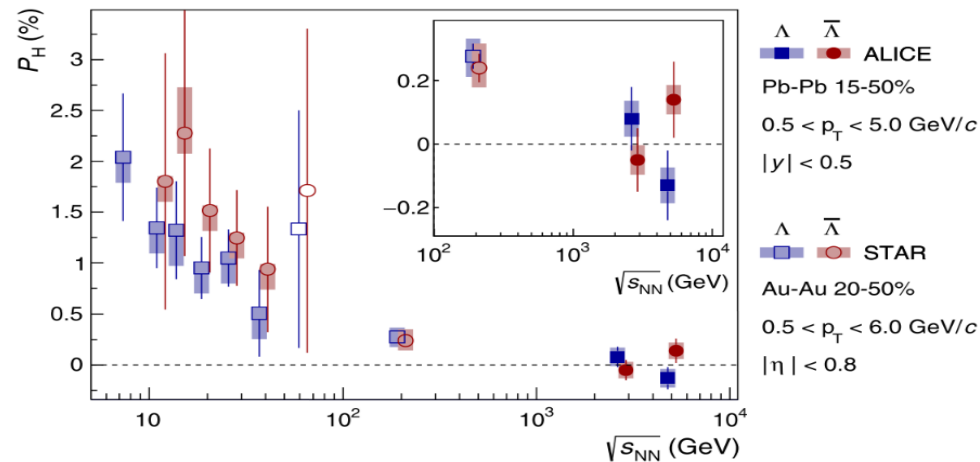


Fig. 5 ALICE 2020  
[ALICE, PRC 101, 044611 (2020)]



# Motivation

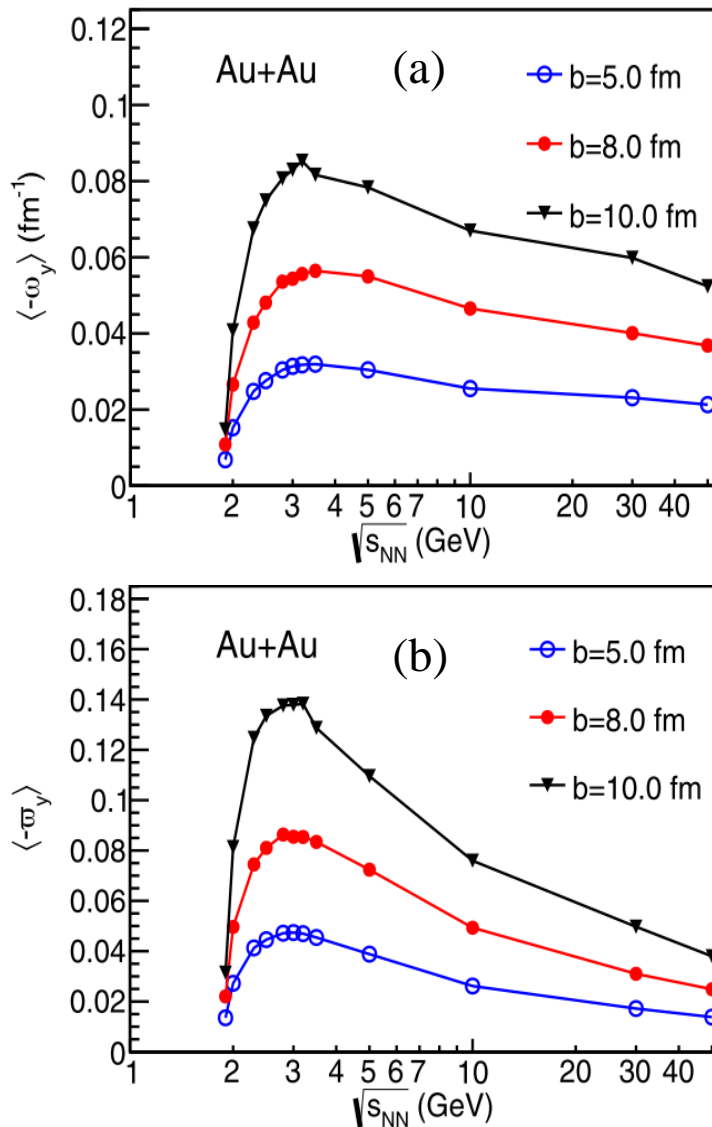


Fig. 7 The initial vorticities in **UrQMD**  
 [ X. G. Deng et al. PRC 101, 064908 (2020) ]

The energy dependence of  $\Lambda$  polarization

$$\sqrt{s_{NN}} \uparrow \text{ --- } P_H \downarrow$$

The **initial vorticities** show similar non-monotonic trend.

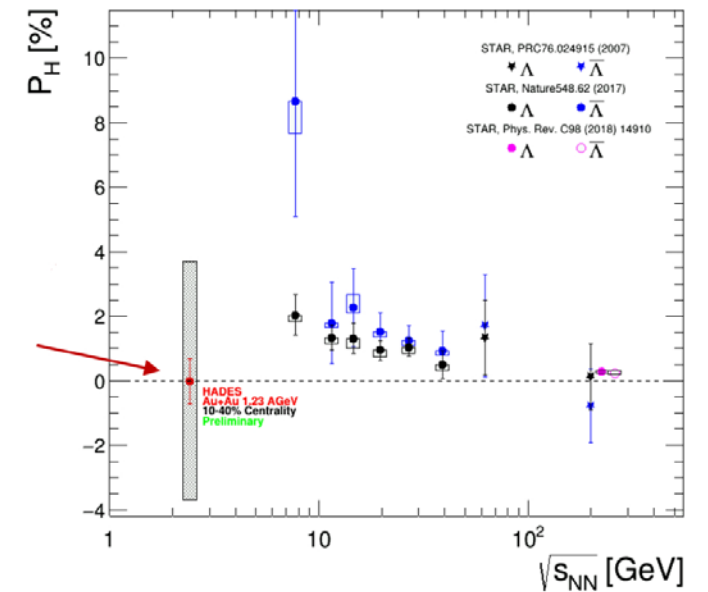


Fig. 6 HADES 2019  
 [ HADES, Frédéric Kornas` talk in Strange Quark Matter 2019 ]

# Motivation

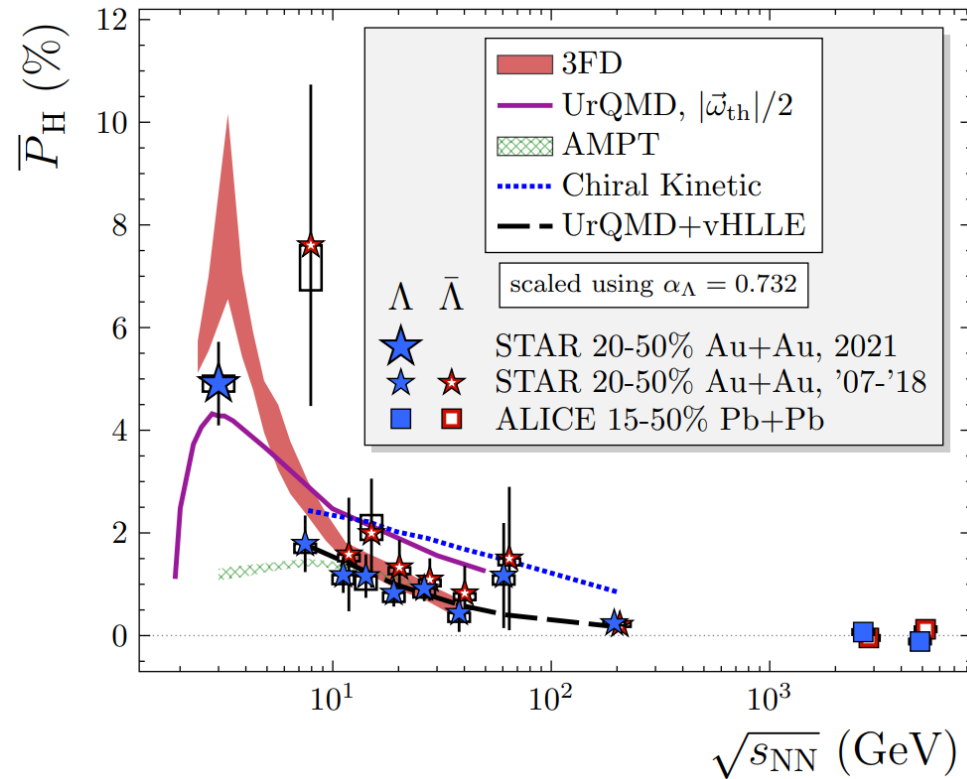


Fig. 8 STAR 2021, fixed target @ 3 GeV  
 [ arXiv:2108.00044 ]

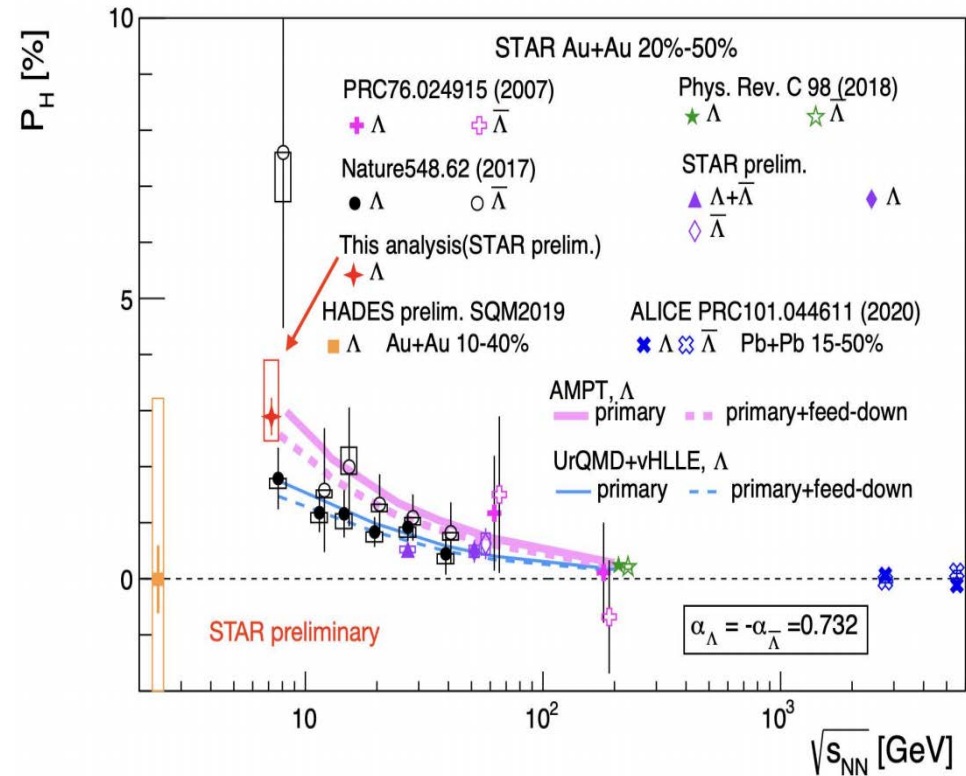


Fig. 9 STAR 2021, fixed target @ 7.2 GeV  
 [ arXiv:2108.10012 ]

Very new results from STAR

# Motivation

The “*spin puzzle*” of the longitudinal  $\Lambda$  polarization

On the transverse momentum plane ( azimuthal angle ) :

Experiment: (+, -, +, +)

Theoretical predictions: (-, +, -, +)

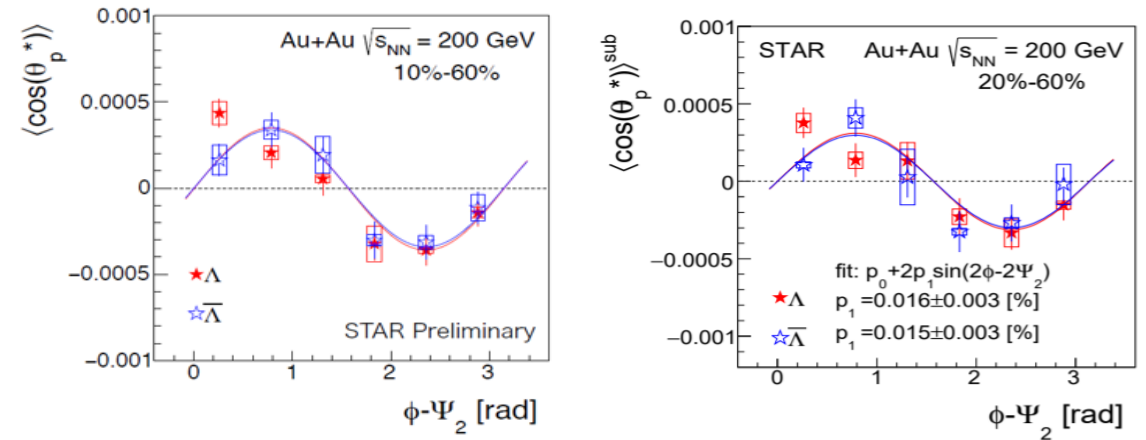


Fig. 10 STAR 2018, 2019 the longitudinal  $\Lambda$  polarization [ STAR, NPA 2019, PRL 2019 ]

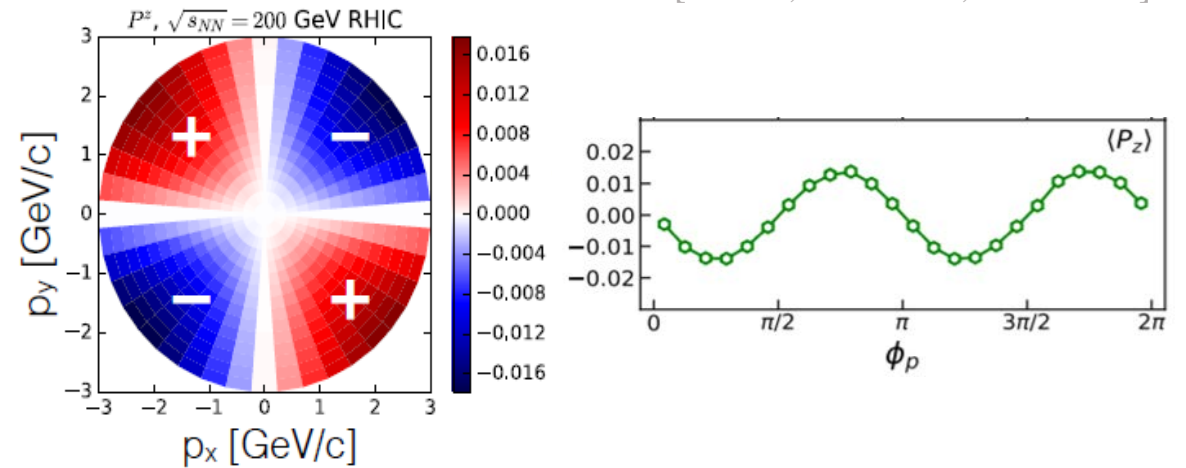


Fig. 11 Left: prediction from ECHO-QGP  
Right: prediction from AMPT

[ F. Becattini et al. PRL 2018 ] [ X. Liang et al. PRC 2018 ]

# Motivation

- Non-relativistic vorticity

$$\omega_{ij}^{NR} = -\frac{1}{2}(\partial_i v_j - \partial_j v_i)$$

- Kinematic vorticity:

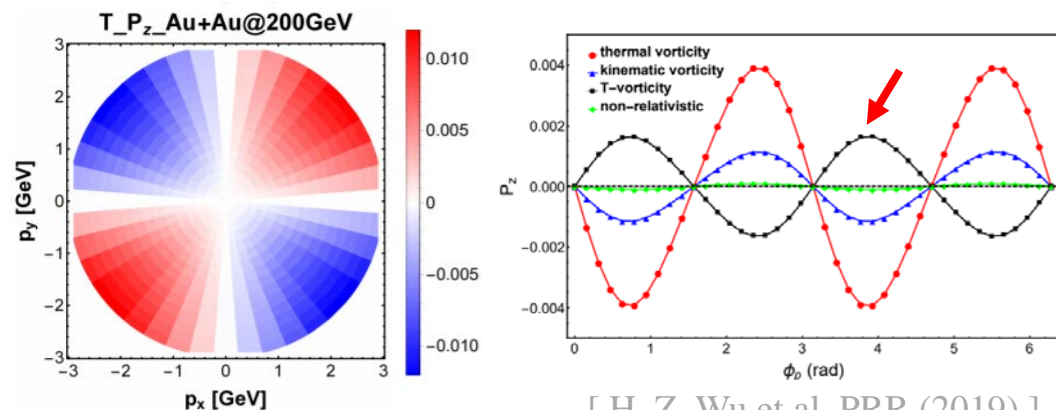
$$\omega_{\mu\nu}^K = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

- Thermal vorticity:

$$\omega_{\mu\nu}^{th} = -\frac{1}{2}[\partial_\mu (u_\nu/T) - \partial_\nu (u_\mu/T)]$$

- **Temperature vorticity:**

$$\omega_{\mu\nu}^T = -\frac{1}{2}[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)]$$



[ H. Z. Wu et al, PRR (2019) ]

Fig. 12 Longitudinal polarization based on T-vorticity and other three types of vorticities

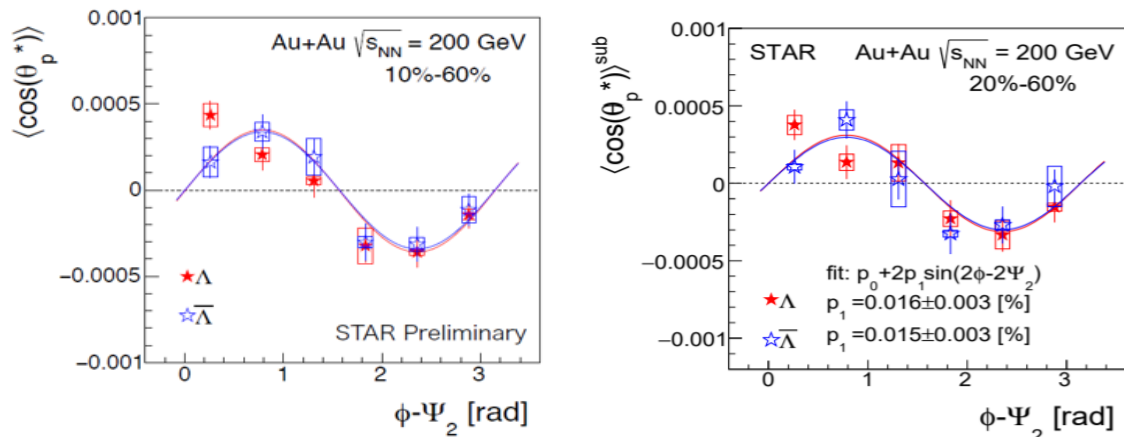


Fig. 10 STAR 2018, 2019 the longitudinal  $\Lambda$  polarization  
[ STAR, NPA 2019, PRL 2019 ]

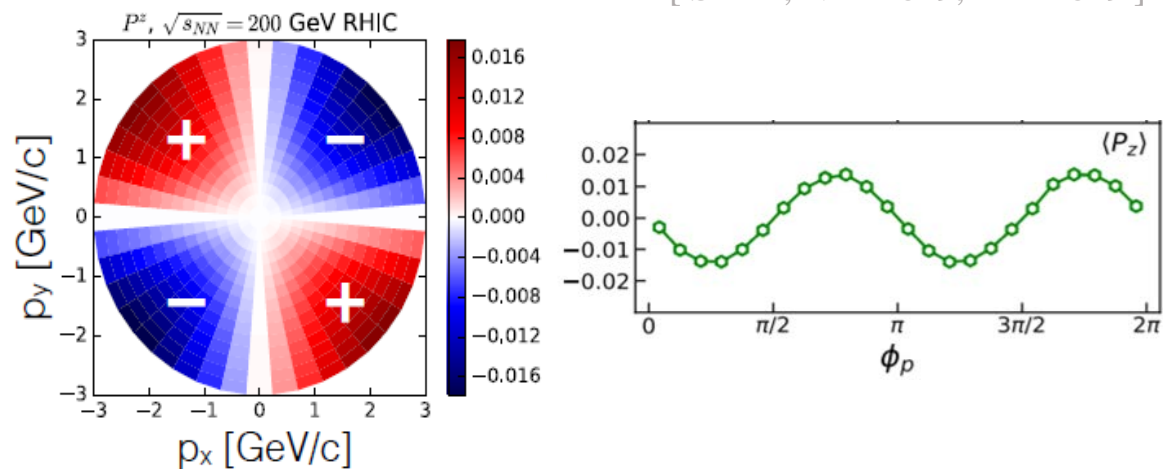


Fig. 11 Left: prediction from ECHO-QGP  
Right: prediction from AMPT

[ F. Becattini et al. PRL 2018 ] [ X. Liang et al. PRC 2018 ]

# Motivation

- Non-relativistic vorticity

$$\omega_{ij}^{NR} = -\frac{1}{2}(\partial_i v_j - \partial_j v_i)$$

- Kinematic vorticity:

$$\omega_{\mu\nu}^K = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

- Thermal vorticity:

$$\omega_{\mu\nu}^{th} = -\frac{1}{2}[\partial_\mu (u_\nu/T) - \partial_\nu (u_\mu/T)]$$

- Temperature vorticity:

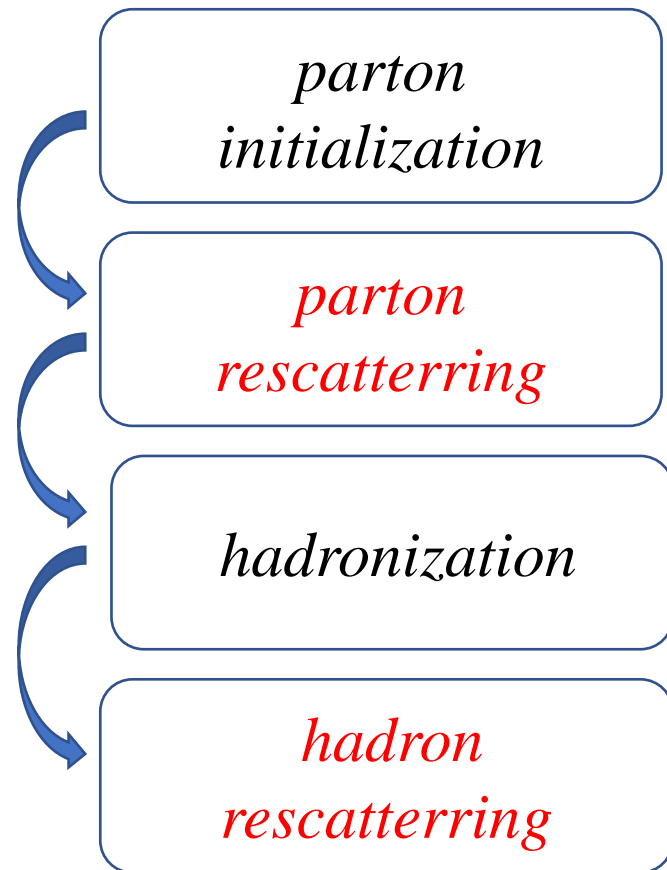
$$\omega_{\mu\nu}^T = -\frac{1}{2}[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)]$$

Behaviors of four types of vorticities?

Non-monotonic trend in other models?

# PACIAE model

**PACIAE**: a microscopic parton and hadron transport model ( based on PYTHIA)



Sketch for  $pp$  dynamic simulation  
(PYTHIA & PACIAE)

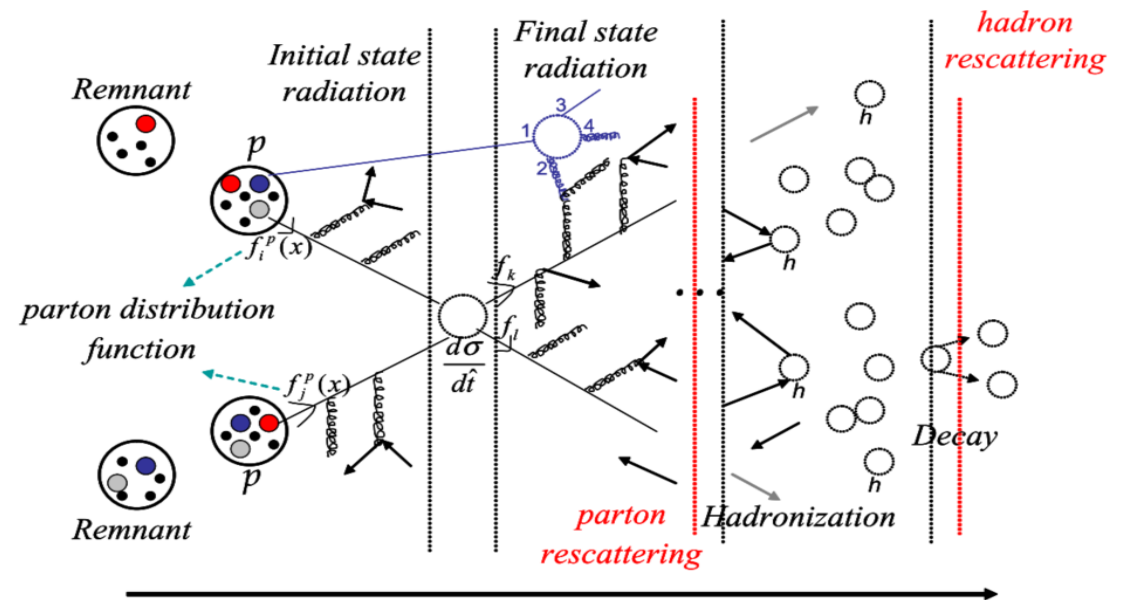


Fig. 13 Sketch of PYTHIA and PACIAE

$\pi, K, p, n, \rho(\omega), \Delta, \Lambda, \Sigma, \Xi, \Omega, J/\Psi$

# PACIAE model: fluidization

- Coarse-graining: the partons are divided into cell

$$\vec{P}_{cell} = \frac{1}{\Delta x \Delta y \Delta z} \langle \sum_i \vec{P}_i \rangle$$

$$\epsilon_{cell} = \frac{1}{\Delta x \Delta y \Delta z} \langle \sum_i E_i \rangle$$

$$\vec{v}_{cell} = \vec{P}_{cell} / \epsilon_{cell}$$

[ Y. Jiang, et al., Phys. Rev. C 94, 044901 (2016) ]

- Temperature : B-E & F-D distribution for partons

$$\epsilon_{cell} = \pi^2 (16 + 10.5 N_f) T_{cell}^4 / 30$$

$N_f$  – the number of quark flavors (3;  $u, d, s$ )

[ Z. W. Lin, Physical Review C, 2014, 90: 014904 ]

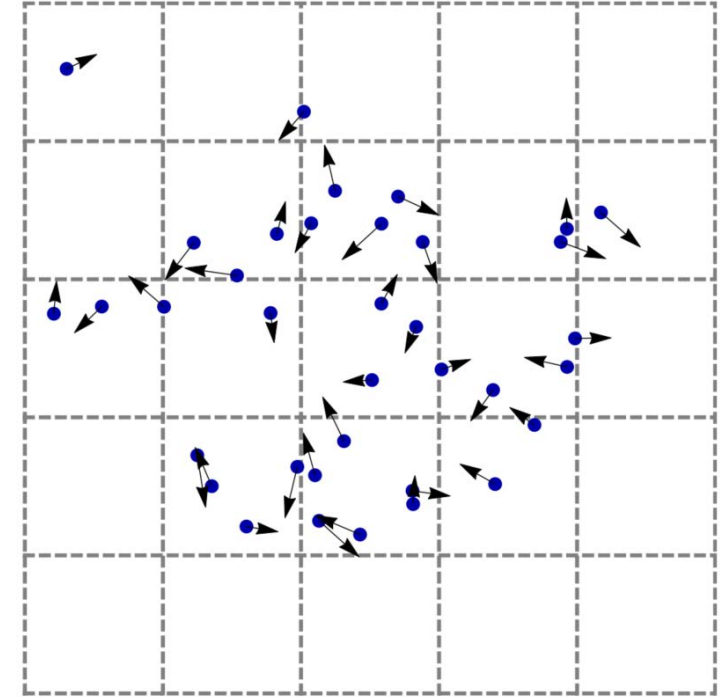


Fig. 14 The cell division in the transport model  
[ Y. Jiang, et al., Phys. Rev. C 94, 044901 (2016) ]

- Another method: smearing function  
[ Phys. Rev. C 93, 064907 (2016) ]

# PACIAE model: fluidization

- Generalized coarse-graining:  
add  $\epsilon_{cell}$  and  $\vec{P}_{cell}$  of each nearest side and corner cells into the central one, then do average

$$\bar{P}_{cell} = \frac{1}{27} \sum_{icell} \vec{P}_{icell}$$

$$\bar{\epsilon}_{cell} = \frac{1}{27} \sum_{icell} \epsilon_{icell}$$

$$\vec{v}_{cell} = \bar{P}_{cell} / \bar{\epsilon}_{cell}$$

- Temperature : B-E & F-D distribution for partons

$$\bar{\epsilon}_{cell} = \pi^2 (16 + 10.5 N_f) T_{cell}^4 / 30$$

$N_f$  – the number of quark flavors (3;  $u, d, s$ )

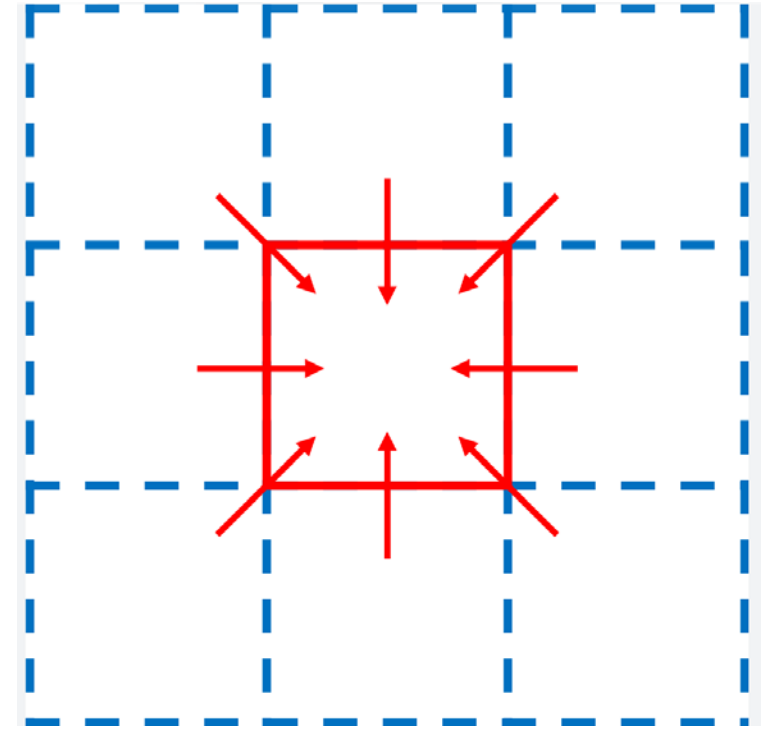


Fig. 15 The generalized coarse-graining

- Another method: smearing function  
[ Phys. Rev. C 93, 064907 (2016) ]



# PACIAE model: fluidization

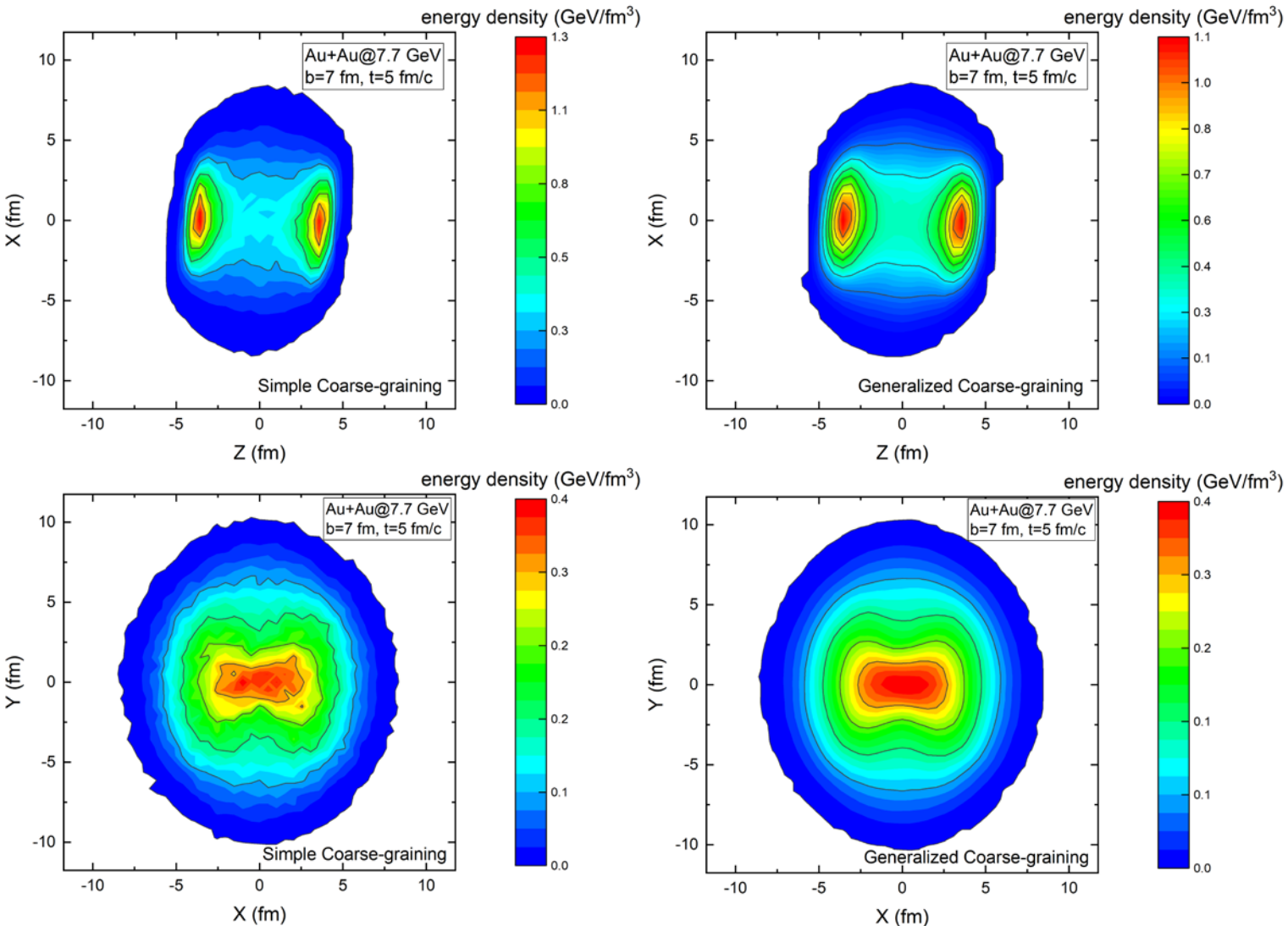


Fig. 16 The energy density distribution in PACIAE

Aug, 2021, ICNFP 2021

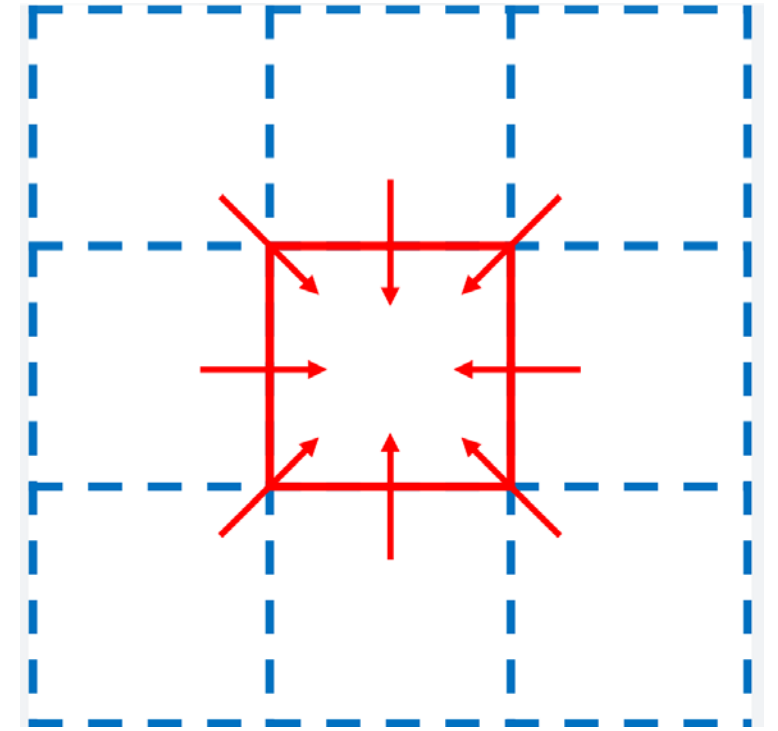


Fig. 15 The generalized coarse-graining

- Another method: smearing function  
[ Phys. Rev. C 93, 064907 (2016) ]

# Results: time and energy dependence

The energy-density-weighted average vorticity:

$$\langle -\omega_{zx} \rangle \equiv \langle -\omega_Y \rangle = \frac{\sum_i \bar{\epsilon}_i \omega_i}{\sum_i \bar{\epsilon}_i}$$

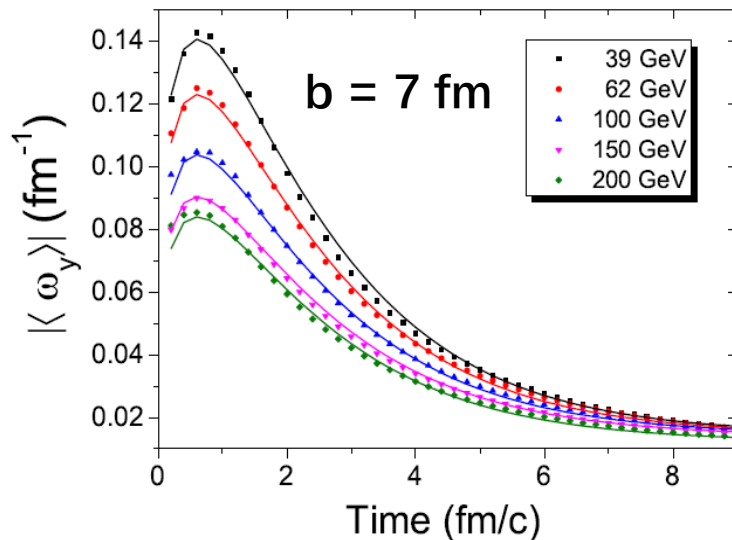


Fig. 17 Results of K-vorticity from **AMPT**  
 [ Y. Jiang, et al., Phys.Rev. C 94, 044901 (2016) ]

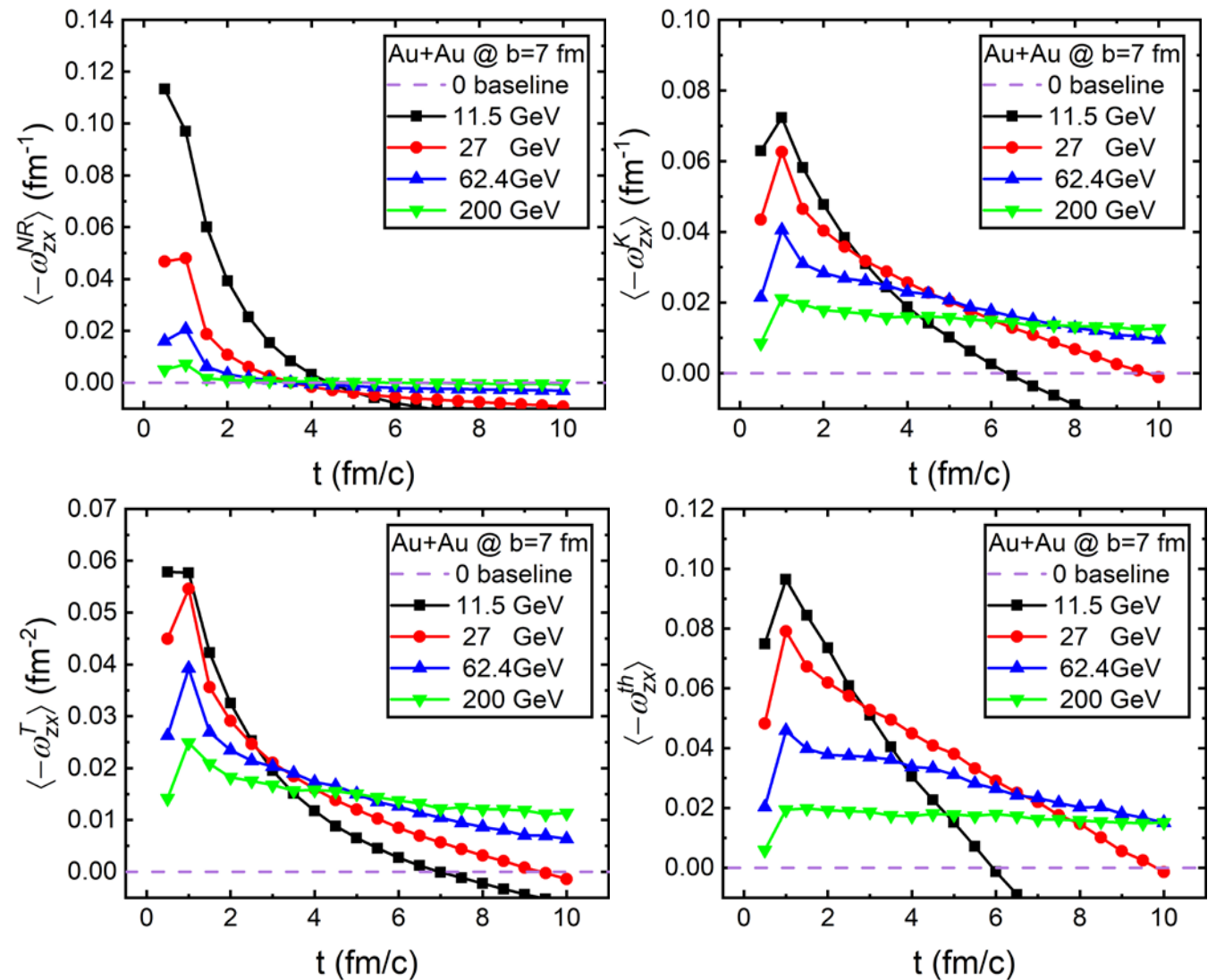


Fig. 18 Results from **PACIAE**

# Results: centrality dependence

The energy-density-weighted average vorticity:

$$\langle -\omega_{zx} \rangle \equiv \langle -\omega_Y \rangle = \frac{\sum_i \bar{\epsilon}_i \omega_i}{\sum_i \bar{\epsilon}_i}$$

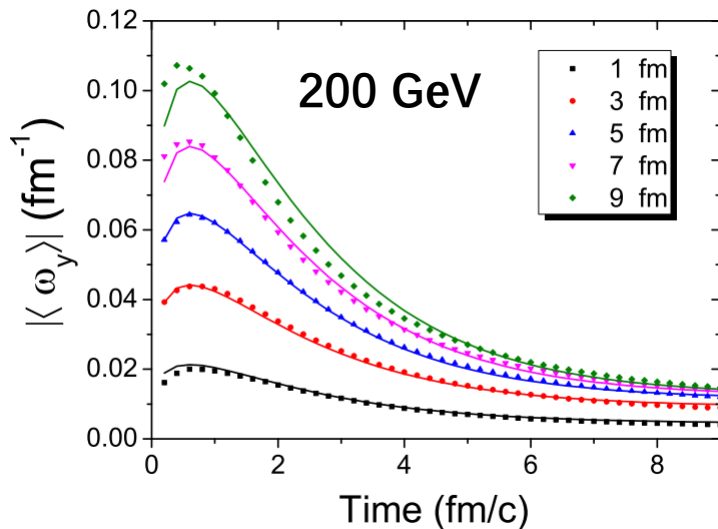


Fig. 19 Results of K-vorticity from **AMPT**  
 [ Y. Jiang, et al., Phys.Rev. C 94, 044901 (2016) ]

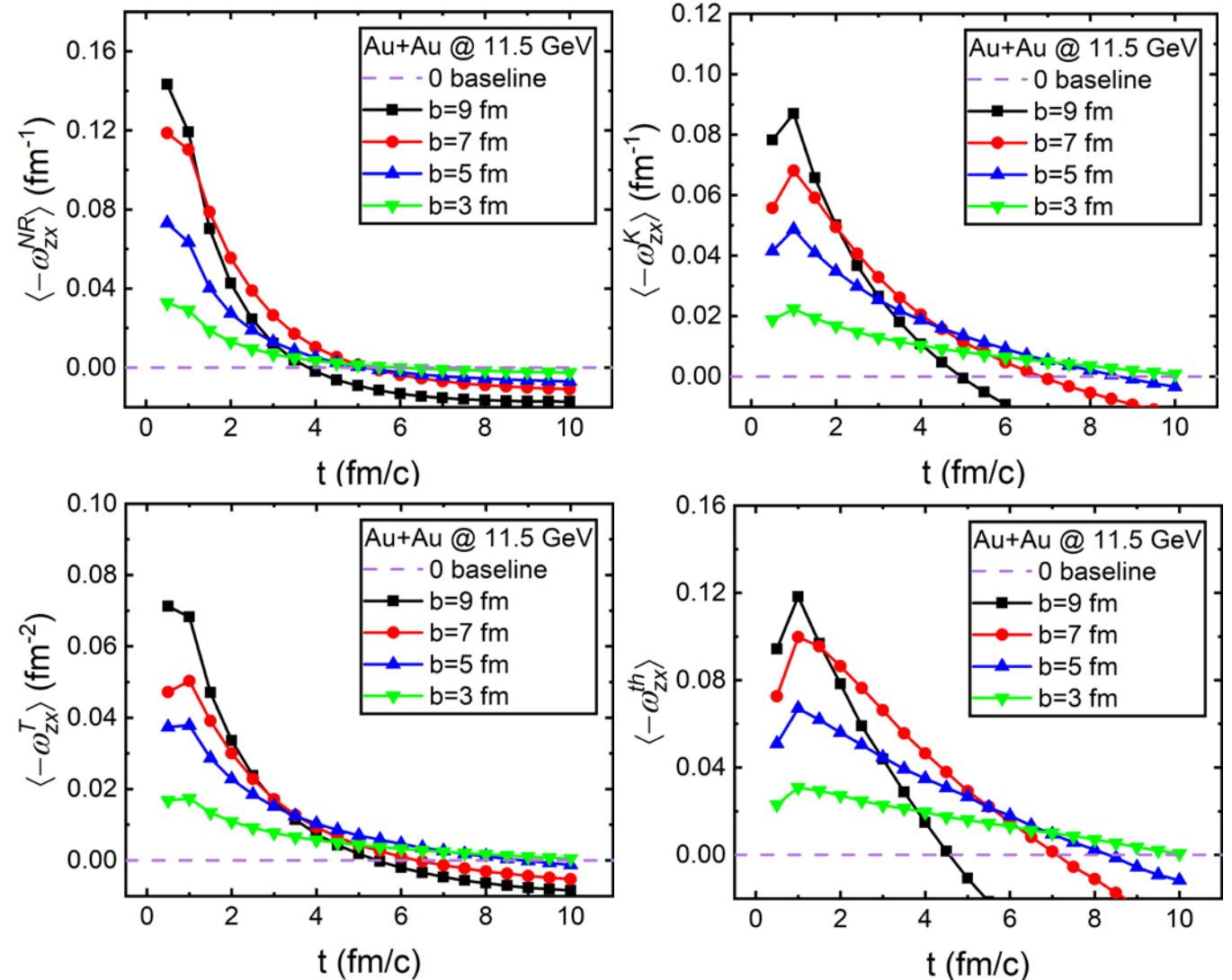


Fig. 20 Results from **PACIAE**

# Results: spatial distribution

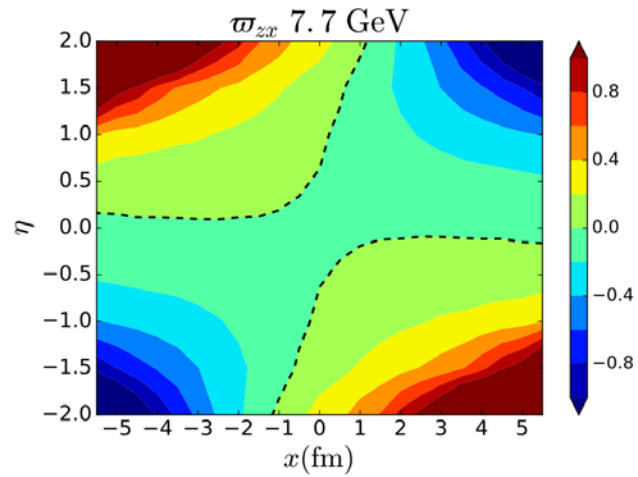


Fig. 21 Thermal vorticity from **AMPT**

[ H. Li, et al. , Phys. Rev. C96, 054908 (2017) ]

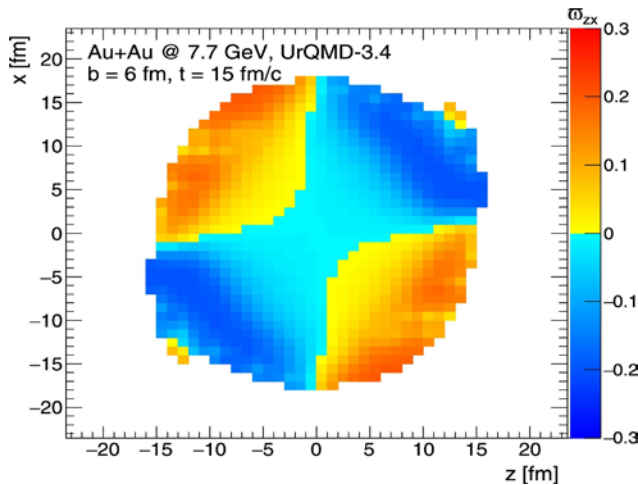


Fig. 22 Thermal vorticity from **UrQMD**

[ O. Vitiuk, et al. , Phys. Lett.B 803, 135298 (2020) ]

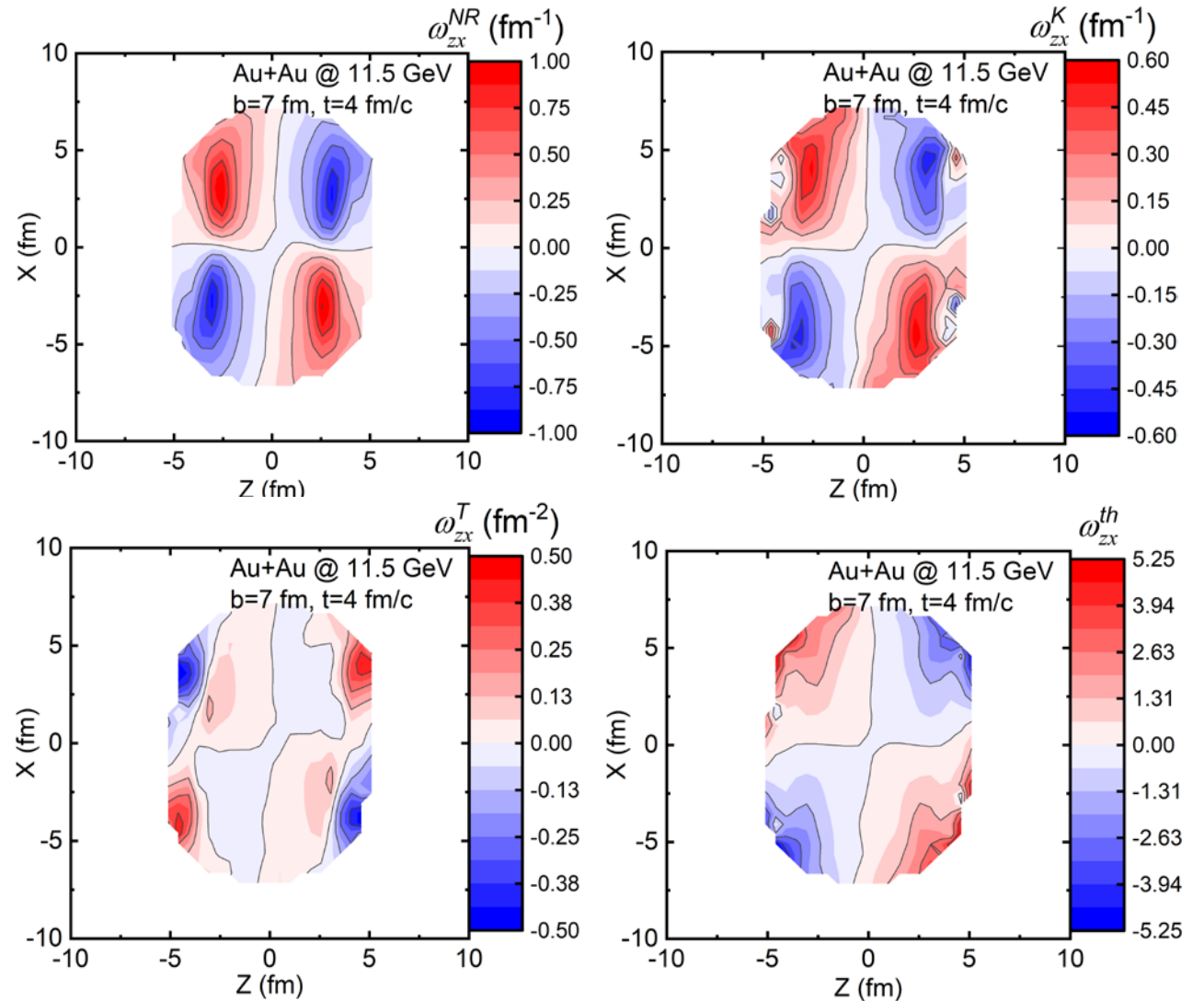


Fig. 23 Results from **PACIAE**

# Results: initial vorticity

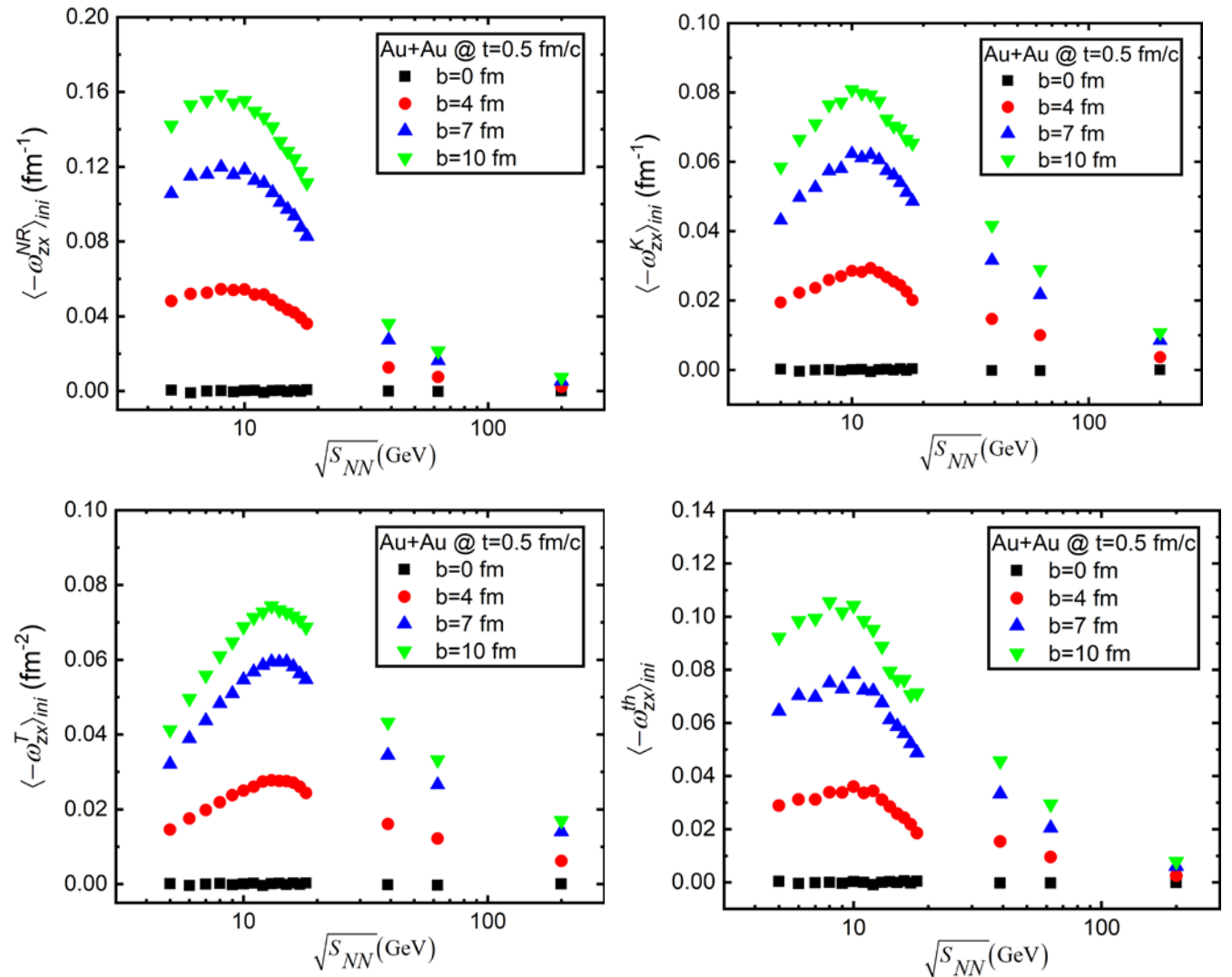
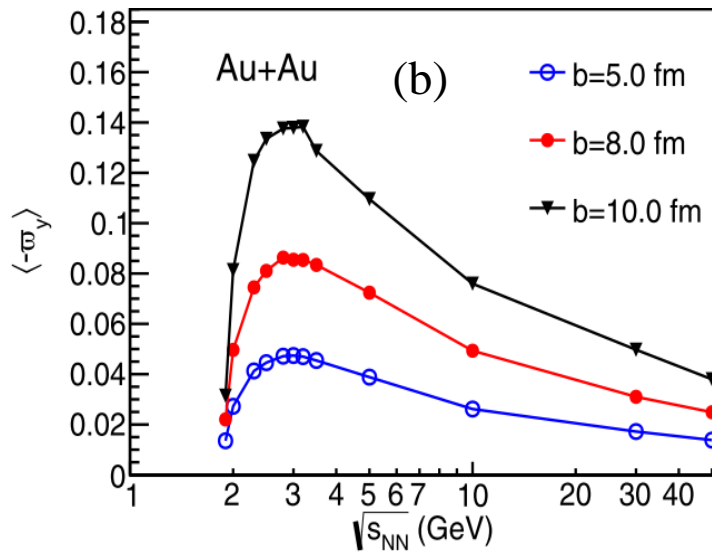
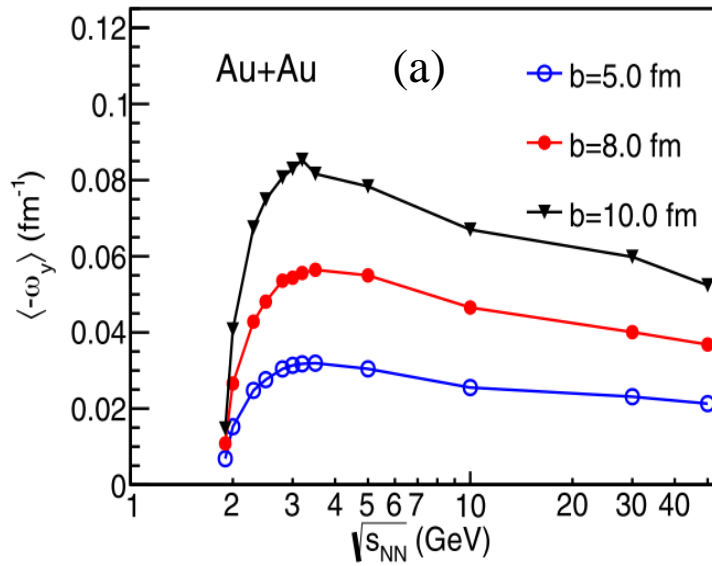


Fig. 7 The initial vorticities in **UrQMD**  
[ X. G. Deng et al. PRC 101, 064908 (2020) ]

Aug, 2021, ICNFP 2021

Fig. 24 Results from **PACIAE**

# Results: initial vorticity

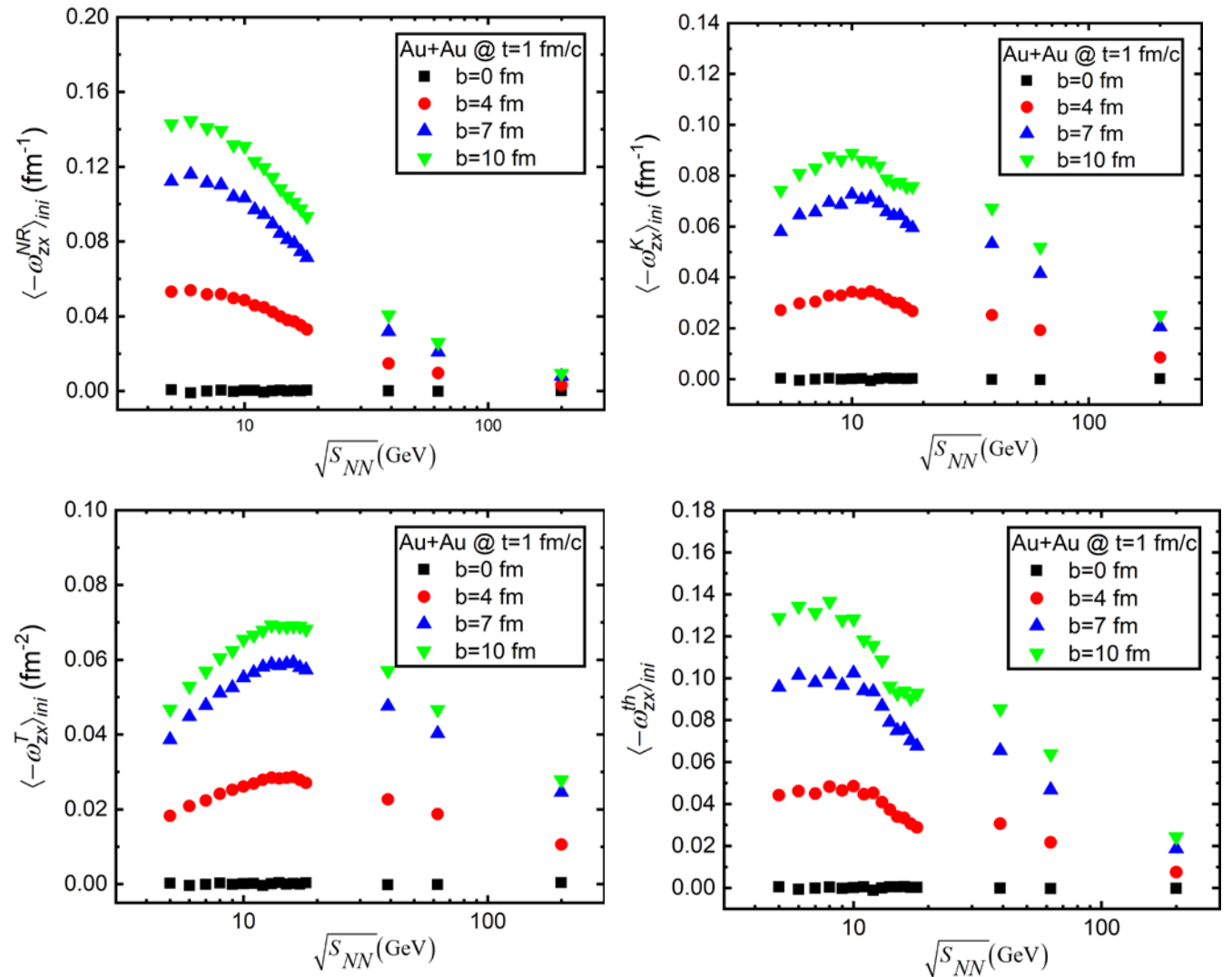
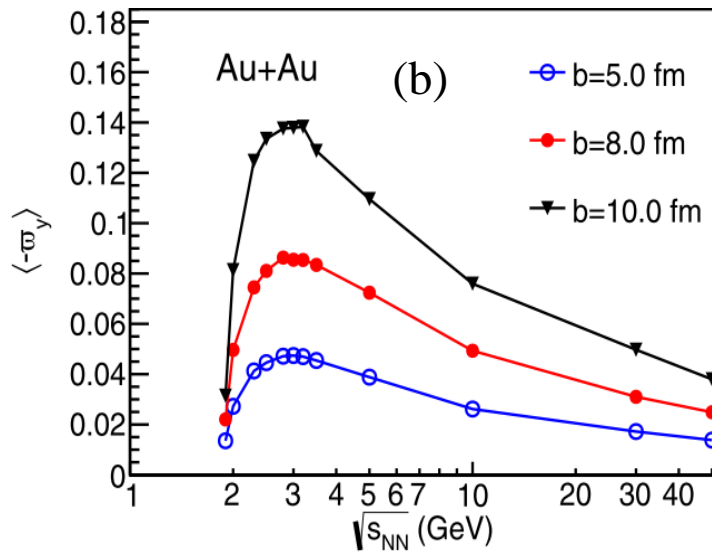
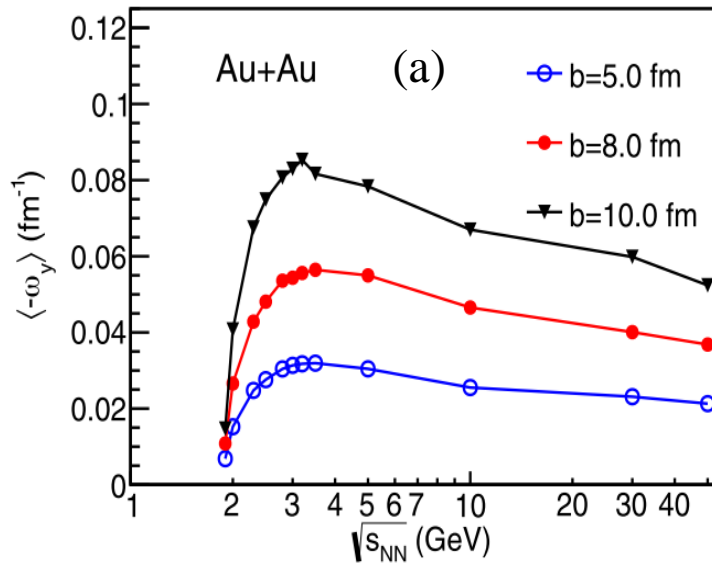


Fig. 7 The initial vorticities in **UrQMD**  
[ X. G. Deng et al. PRC 101, 064908 (2020) ]

Aug, 2021, ICNFP 2021

Fig. 25 Results from **PACIAE**

# Results: initial vorticity of **Hadronic Matter**

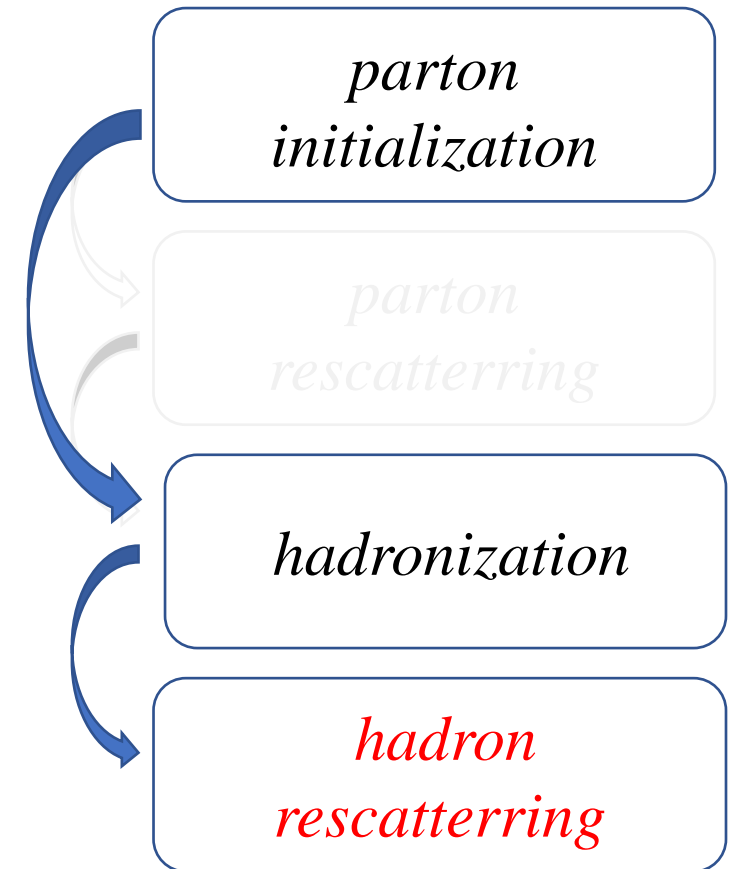
**UrQMD**: hadron transport model,  
hadron only (Mean-Field, resonances...),  
without parton

**AMPT** (version string-melting) :  
parton & hadron mixed phase, without gluon

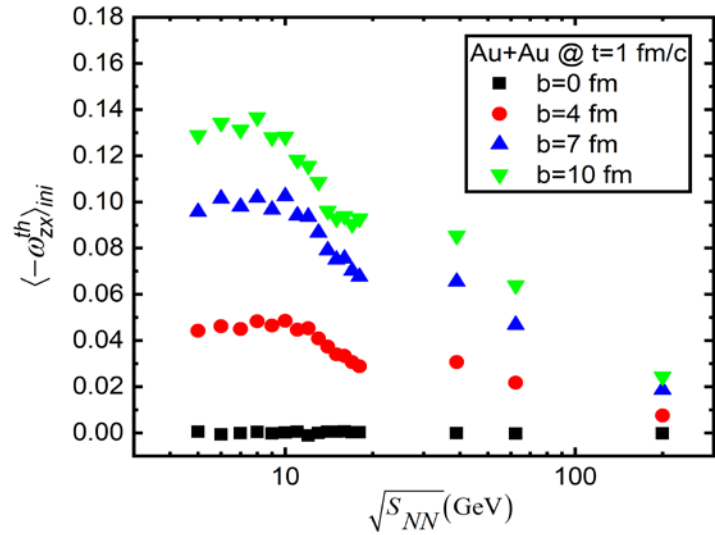
**PACIAE**: with quark & gluon, but instant freeze-out

close the partonic evolution stage at low energies

Pure Hadronic Matter



# Results: initial vorticity of **Hadronic Matter**



Thermal vorticity of **partons** in **PACIAE**

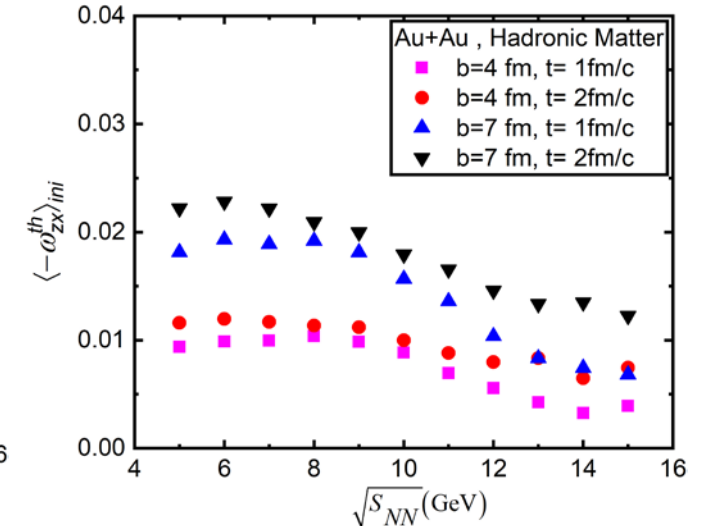
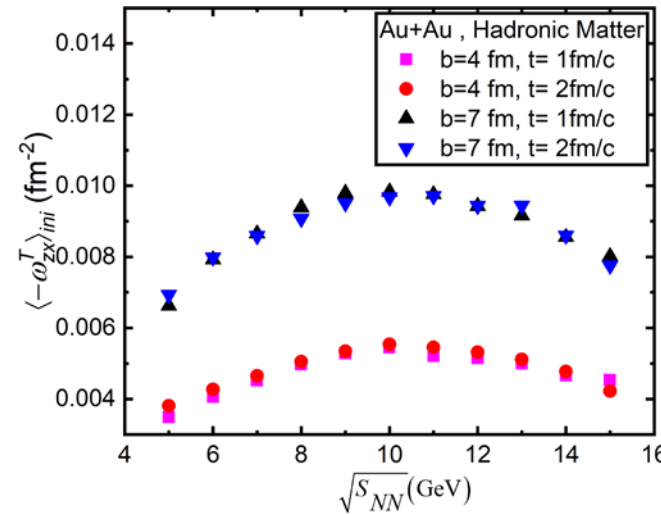
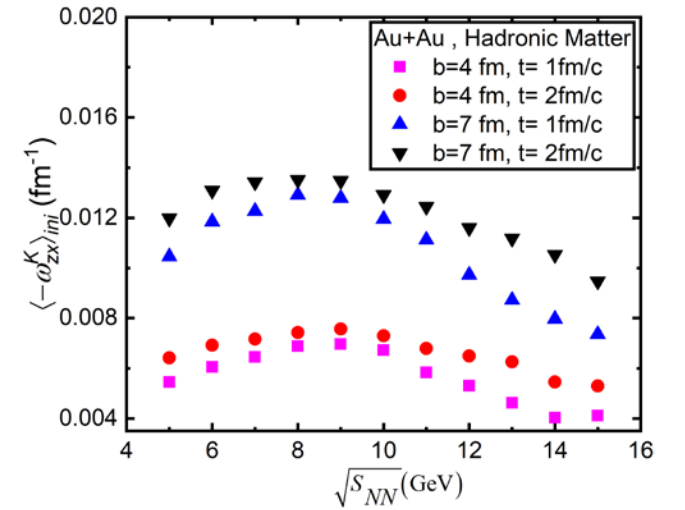
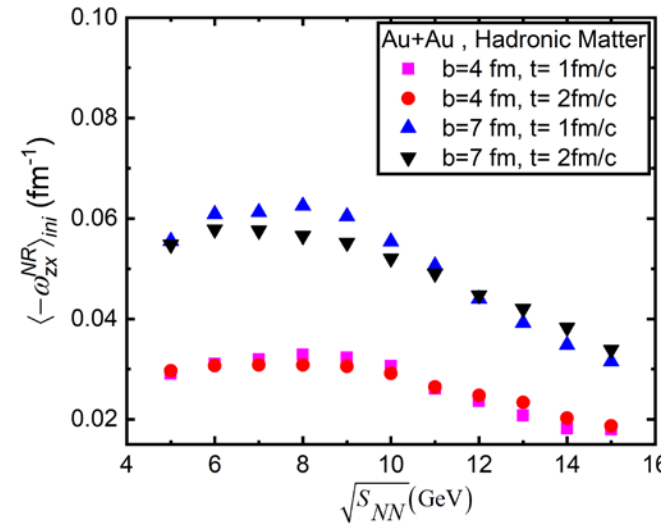
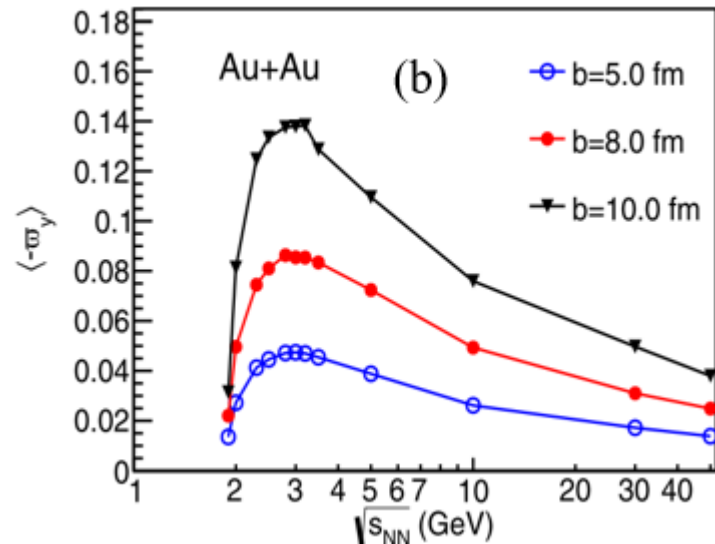


Fig. 7 The initial vorticities in **UrQMD**

Aug, 2021, ICNFP 2021

Fig. 26 Results from **PACIAE**

[ X. G. Deng et al. PRC 101, 064908 (2020) ]



# Summary

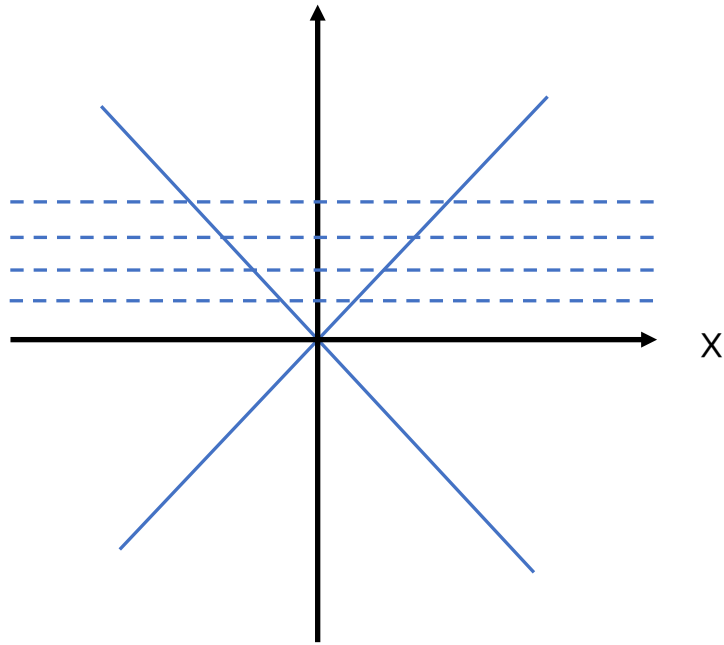
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- We gave a systematic study of four types of vorticities in PACIAE.
- The non-monotonic dependence of the initial vorticities on the collision energies was reconfirmed.  $\sim 10-15$  GeV
- The initial vorticities of pure Hadronic Matter (Hadron Gas) at 5-15 GeV were studied.

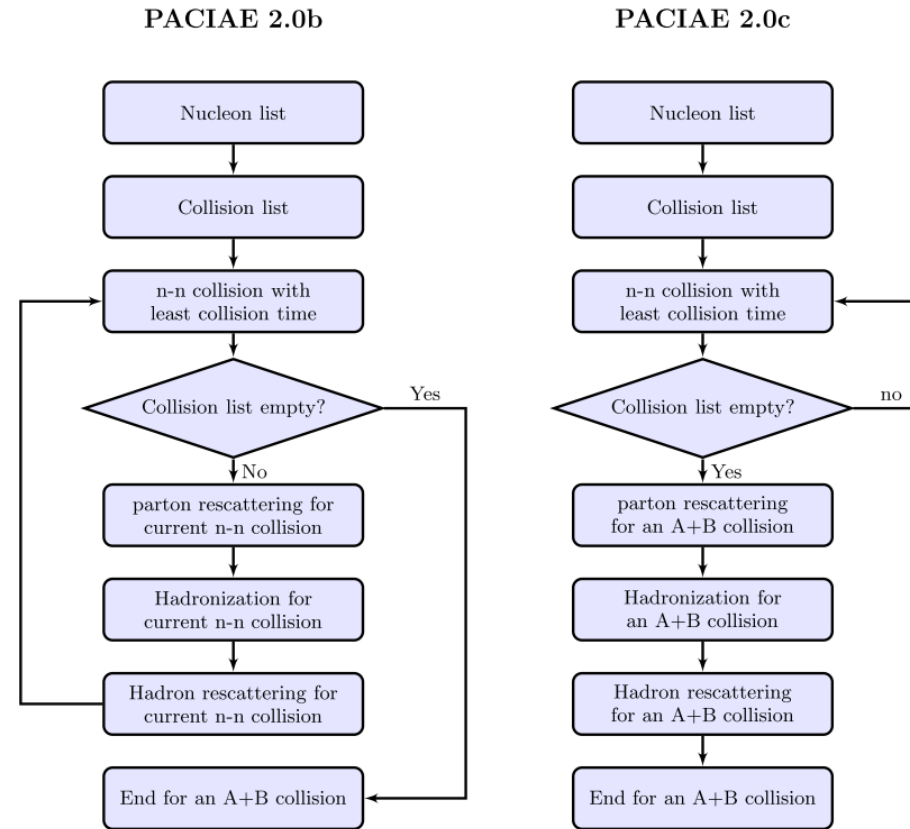
*Thanks !*

Thank Prof. Larissa and Evgeny for their hospitality during my visit in Norway.  
Thank Prof. Yilong for helpful discussion.

# Backup



Time slice division



Topological structure of PACIAE version b and c

# Backup: extracting temperature in PACIAE b

- multiplicity weighted  $p_t$  distribution fitting

$$P(p_t)_{\text{sys}} = \frac{1}{p_t} \frac{dN}{dp_t}$$

fitting

$$P_T(p_t)_{\text{sys}} = A \exp\left[-\frac{p_t}{T}\right]$$

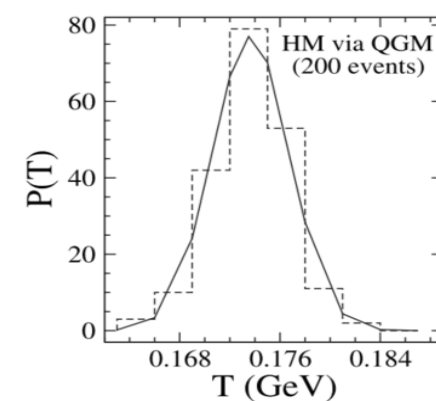
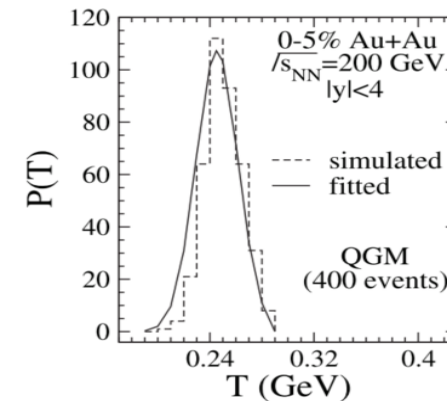
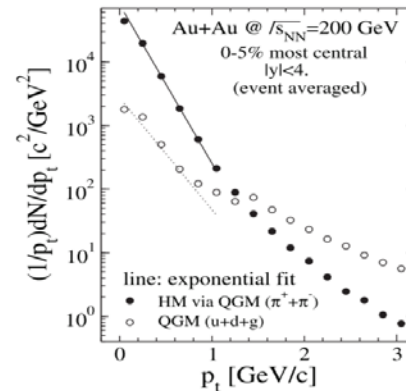
$A$  : normalization factor

$p_t$  : transverse momentum of particles

$T$  : average temperature of the system

$$P(p_t)_{\text{HM}} = \frac{M_{\pi^+}}{M_{\pi^+} + M_{\pi^-}} P(p_t)_{\pi^+} + \frac{M_{\pi^-}}{M_{\pi^+} + M_{\pi^-}} P(p_t)_{\pi^-}$$

$$P(p_t)_{\text{QGM}} = \frac{M_u}{M_u + M_d + M_g} P(p_t)_u + \frac{M_d}{M_u + M_d + M_g} P(p_t)_d + \frac{M_g}{M_u + M_d + M_g} P(p_t)_g$$



$p_t$  temperature fitting and distribution

(Sa, Li et al. 2007)

# Backup: extracting temperature in PACIAE b

$$\frac{1}{p_t} \frac{dN}{dp_t} \rightarrow A \exp\left[-\frac{p_t}{T}\right]$$

A : normalization factor

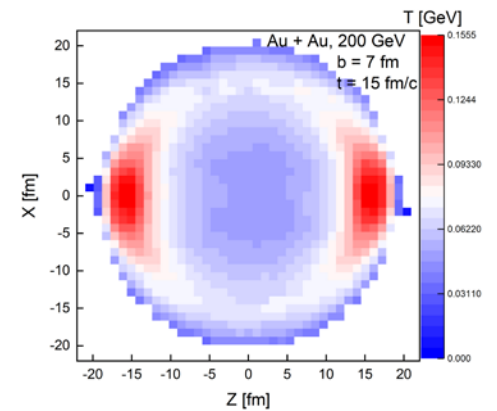
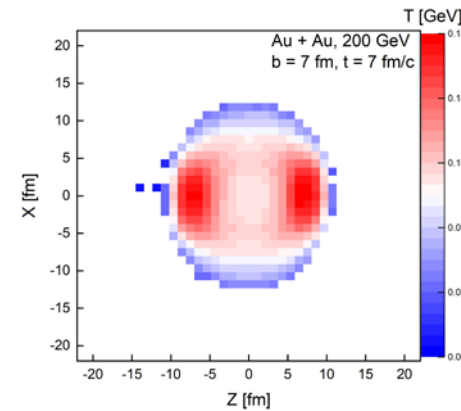
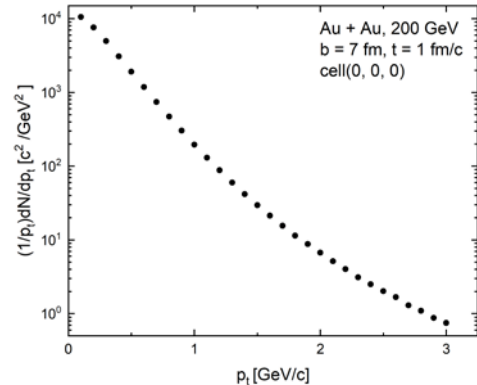
$$P(p_t)_{\text{sys}} = \frac{M_q}{M_{\text{tot}}} P(p_t)_q + \frac{M_g}{M_{\text{tot}}} P(p_t)_g + \frac{M_H}{M_{\text{tot}}} P(p_t)_H$$

$$M_{\text{tot}} = M_q + M_g + M_H \quad \text{quark + gluon + hadron}$$

In single cell,

$$T_i = \frac{-p_t}{\ln\left(\left(\frac{1}{p_t} \frac{dN}{dp_t}\right)_i / A\right)}$$

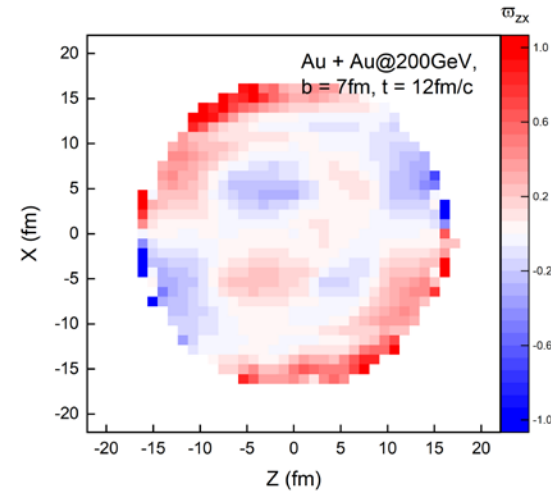
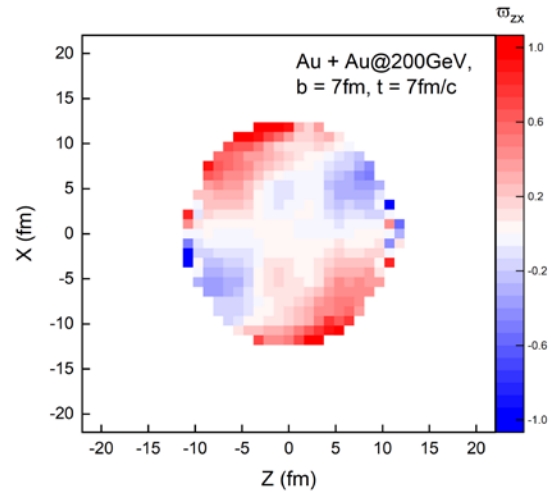
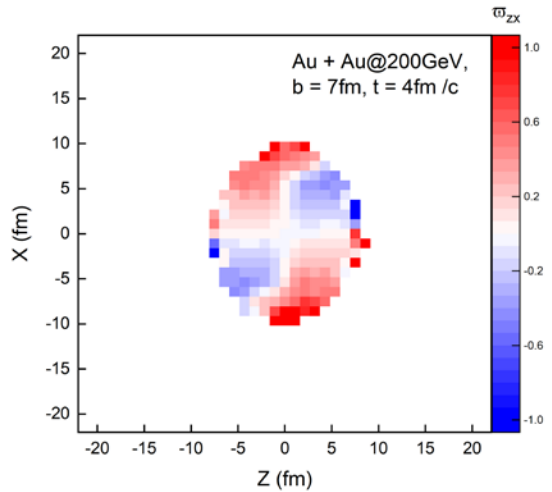
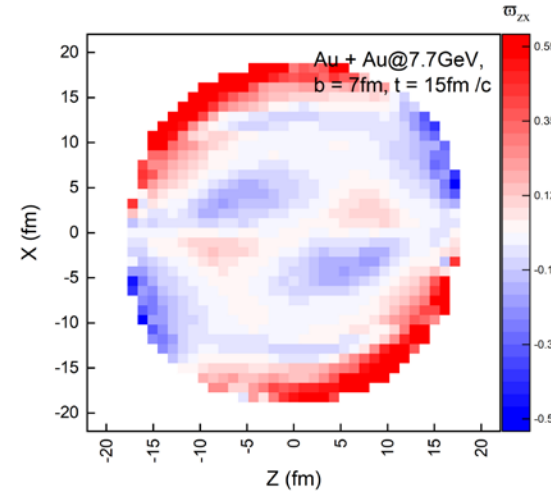
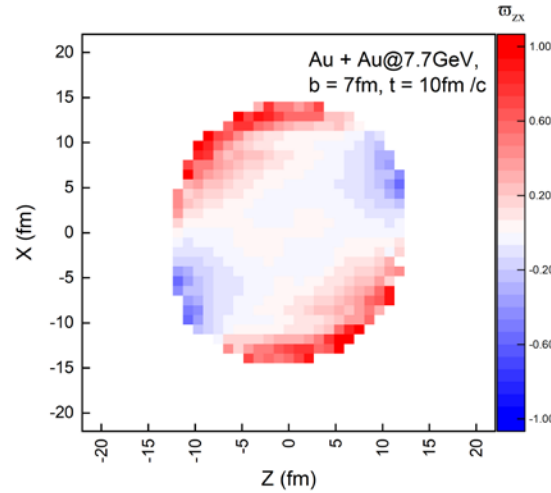
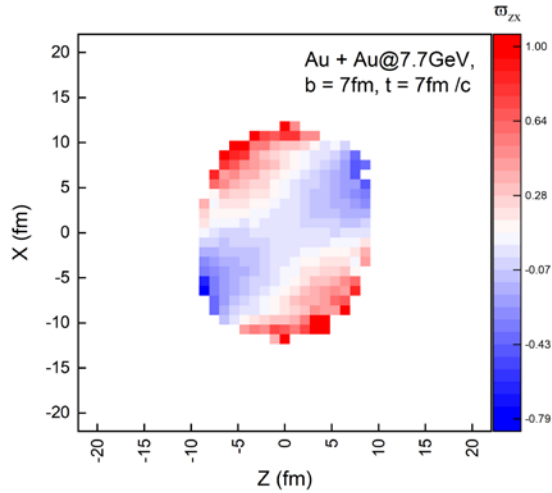
$$T_{\text{cell}} = \sum_i^N \frac{T_i}{N}$$



$p_t$  spectrum in cell (0,0,0) , temperature distribution on the reaction plane at different time

$i, N$ : the selected distribution point of  $p_t$

# Backup: vorticity in PACIAE b



$$\begin{aligned} \vec{\omega}_T &= (\omega_{0x}, \omega_{0y}, \omega_{0z}) = \frac{1}{2} \left[ \nabla \left( \frac{\gamma}{T} \right) + \partial_t \left( \frac{\gamma \mathbf{v}}{T} \right) \right] \\ \vec{\omega}_S &= (\omega_{yz}, \omega_{zx}, \omega_{xy}) = \frac{1}{2} \nabla \times \left( \frac{\gamma \mathbf{v}}{T} \right). \end{aligned}$$