Confinement/deconfinement transition in rotating gluodynamics within lattice simulation

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The Outline

- Introduction
- Rotation on the lattice, boundary conditions
- Results for gluodynamics
- Preliminary results for full QCD
- Conclusions

See the details in:

In non-central heavy ion collisions creation of QGP with angular momentum is expected.
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The recent results show that the rotation occurs with relativistic velocities.

\[ \omega \sim 0.1 - 0.2 \, \text{fm}^{-1} \sim 20 - 40 \, \text{MeV} \]

\[ \omega \sim 6 \, \text{MeV} \]

\[ \text{Au} + \text{Au}, \quad b = 7 \, \text{fm} \]


Introduction

- In non-central heavy ion collisions creation of QGP with angular momentum is expected.

- The recent results show that the rotation occurs with relativistic velocities.

\[ \omega \sim 0.1 - 0.2 \text{ fm}^{-1} \sim 20 - 40 \text{ MeV} \]

\[ \omega \sim 6 \text{ MeV} \]

- How does the rotation affect to phase transitions in QCD?
Related papers

Rotation on the lattice (phase transitions were not considered):


Properties of rotating QCD matter (mostly within NJL, focused on fermions):


Holography:

Compact QED in 2+1-D:

HRG model:

⇒ Critical temperature decreases due to the rotation.

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Related papers

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\( \Rightarrow \) Critical temperature decreases due to the rotation.
SU(3)-gluodynamics (at thermal equilibrium) is investigated in the reference frame which rotates with the system with angular velocity \( \Omega \).

In this reference frame there appears an external gravitational field

\[
g_{\mu\nu} = \begin{pmatrix}
1 - r^2 \Omega^2 & \Omega y & -\Omega x & 0 \\
\Omega y & -1 & 0 & 0 \\
-\Omega x & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}.
\]

The partition function is\(^1\)

\[
Z = \text{Tr} \exp \left[ -\beta \hat{H} \right] \quad \Rightarrow \quad Z = \int DA \exp (-S_G),
\]

where the Euclidean action can be written as

\[
S_G = \frac{1}{2g^2} \int d^4 x \sqrt{g_E} g^\mu_\nu g^\alpha_\beta \hat{F}_{\mu\alpha} \hat{F}^\alpha_{\nu\beta}.
\]


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• **Tolman-Ehrenfest effect:** In gravitational field the temperature isn’t a constant in space at thermal equilibrium:

\[ T(r)\sqrt{g_{00}} = \text{const}, \]

For the rotation one has

\[ T(r)\sqrt{1 - r^2\Omega^2} = \text{const} \equiv T, \]

One could expect, that the rotation effectively warm up the periphery of the modeling volume

\[ T(r) > T(r=0), \]

and as a result, from kinematics, the critical temperature should decreases.
Rotating reference frame: temperature

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\[ S_G = \frac{1}{4g^2} \int d^4 x \sqrt{|g_E|} g_E^{\mu \nu} g_E^{\alpha \beta} F^a_{\mu \alpha} F^a_{\nu \beta}. \]  

\[ (2) \]

Substituting the \((g_E)^{\mu \nu}\) to formula (2) one gets

\[ S_G = \frac{1}{2g^2} \int d^4 x \left[ (1 - r^2 \Omega^2) F^a_{xy} F^a_{xy} + (1 - y^2 \Omega^2) F^a_{xz} F^a_{xz} + (1 - x^2 \Omega^2) F^a_{yz} F^a_{yz} + 
+ F^a_{x \tau} F^a_{x \tau} + F^a_{y \tau} F^a_{y \tau} + F^a_{z \tau} F^a_{z \tau} - 
- 2iy \Omega (F^a_{xy} F^a_{y \tau} + F^a_{xz} F^a_{z \tau}) + 2ix \Omega (F^a_{yx} F^a_{x \tau} + F^a_{yz} F^a_{z \tau}) - 2xy \Omega^2 F^a_{xz} F^a_{zy} \right]. \]
Rotating reference frame

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+ F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} - 
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Sign problem

- The Euclidean action is complex-valued function!
- The Monte–Carlo simulations are conducted with imaginary angular velocity \(\Omega_I = -i\Omega\).
- The results are analytically continued to the region of the real angular velocity.
The lattice action can be written as

\[ S_G = \beta \sum_x \left( (1 + r^2 \Omega_I^2)(1 - \frac{1}{N_c} \text{Re Tr} \, \bar{U}_{xy}) + (1 + y^2 \Omega_I^2)(1 - \frac{1}{N_c} \text{Re Tr} \, \bar{U}_{xz}) + \\ + (1 + x^2 \Omega_I^2)(1 - \frac{1}{N_c} \text{Re Tr} \, \bar{U}_{yz}) + 3 - \frac{1}{N_c} \text{Re Tr} \, \left( \bar{U}_{x\tau} + \bar{U}_{y\tau} + \bar{U}_{z\tau} \right) - \\ - \frac{1}{N_c} \text{Re Tr} \left( y\Omega_I (\bar{V}_{xy\tau} + \bar{V}_{xz\tau}) - x\Omega_I (\bar{V}_{yx\tau} + \bar{V}_{yz\tau}) + xy\Omega_I^2 \bar{V}_{xzy} \right) \right), \]

where \( \beta = 2N_c/g^2 \),

\( \bar{U}_{\mu\nu} \) denotes clover-type average of four plaquettes,

\( \bar{V}_{\mu\nu\rho} \) is asymmetric chair-type average of 8 chair.
Lattice setup

- Simulation is performed on the lattice $N_t \times N_z \times N_s^2$ ($N_s = N_x = N_y$), which rotates around $z$-axis.
- The system should be limited in the directions, which are orthogonal to the rotation axis: $\Omega_I(N_s - 1)a/\sqrt{2} < 1$
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The following types of BC were systematically checked:

- Open b.c. – OBC
  - All $U_{\mu\nu}$, $V_{\mu\nu\rho}$, which contain links sticking out of the lattice, excluded.
  - Does not break any symmetries.
  - $U_P = 1$ for all $P \in \text{out}$; or $F_{\mu\nu} = 0$ $\Rightarrow$ "low" temperature on the boundary.
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- Periodic b.c. – PBC
  - The velocity distribution is not periodic.
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- **Periodic b.c. – PBC**
  - The velocity distribution is not periodic.

- **Dirichlet b.c. – DBC**
  - $U_\mu(x) = \hat{1}$ for all $x, x + \mu \in \text{boundary}$
  - Violate $\mathbb{Z}_3$ center symmetry.
  - $L(x, y) = 3$ on the boundary $\Rightarrow$ „high“ temperature on the boundary.
The Polyakov loop is an order parameter. The lattice version is defined as usual:

\[
L(\vec{x}) = \text{Tr} \left[ \prod_{\tau=0}^{N_t-1} U_4(\vec{x}, \tau) \right], \quad L = \frac{1}{N_s^2 N_z} \sum_{\vec{x}} L(\vec{x}).
\] (3)

In confinement \(\langle L \rangle = 0\); in deconfinement \(\langle L \rangle \neq 0\) (\(\mathbb{Z}_3\) center symmetry is broken).

The critical temperature \(T_c\) is determined using the Polyakov loop susceptibility

\[
\chi = N_s^2 N_z \left( \langle |L|^2 \rangle - \langle |L| \rangle^2 \right),
\] (4)

by means of the Gaussian fit.
Polyakov loop

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\[ \chi = N_s^2 N_z (\langle |L|^2 \rangle - \langle |L| \rangle^2), \tag{4} \]

by means of the Gaussian fit.

- Non-periodic b.c. changes the critical temperature \( T_c(0) \)
  - \( T_c(0)^{OBC} > T_c(0)^{PBC} \)
  - \( T_c(0)^{DBC} < T_c(0)^{PBC} \)
- With \( N_s/N_t \to \infty \) their influence wanes, and \( T_c(0) \to T_c(0)^{(PBC)} \)
Open boundary conditions

![Graphs showing Polyakov loop and susceptibility as functions of temperature with different angular velocities Ω_I.](image)

**Figure:** The Polyakov loop (a) and Polyakov loop susceptibility (b) as a function of temperature for different values of imaginary angular velocity Ω_I. The results are obtained on the lattice 8 × 24 × 49².

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$T_c$ depends on $\Omega_I^2$ and is well described by

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2 \Omega_I^2$$

- The coefficient $C_2$ depends on the transverse lattice size ($N_s/N_t$) and almost independent of both the lattice spacing and the lattice size along the rotation axis ($N_z/N_t$).

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Open boundary conditions: critical temperature

$T_c$ depends on $\Omega_I^2$ and is well described by

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2 \Omega_I^2$$

$$\Omega_I^2 = -\Omega^2$$

$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2 \Omega^2$$

The critical temperature increases with the angular velocity ($C_2 > 0$)

- The coefficient $C_2$ depends on the transverse lattice size ($N_s/N_t$) and almost independent of both the lattice spacing and the lattice size along the rotation axis ($N_z/N_t$).
The linear velocity on the boundary $v_I = \Omega_I (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 \frac{v_I^2}{c^2} \implies \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The coefficient $B_2$ slightly depends on the transverse lattice size ($N_s/N_t$).
- For lattices with sufficiently large $N_s$ and OBC the coefficient is $B_2 \sim 0.7$. 
Open boundary conditions: Polyakov loop distribution

The local Polyakov loop in $x, y$-plane

$$L(x, y) = \frac{1}{N_z} \sum_z L(x, y, z)$$
Open boundary conditions: Polyakov loop distribution

Figure: The local Polyakov loop $|\langle L(x, y) \rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x, y) \rangle|$ is zero for all spatial points in the confinement phase, both with and without rotation $\Rightarrow$ Polyakov loop still acts as the order parameter.
- In deconfinement phase the boundary is screened.
Periodic boundary conditions: critical temperature

\[ T_c(v_I)/T_c(0) = 1 - B_2 \frac{v_I^2}{c^2} \implies \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2} \]

- The results for the finest lattices with \( N_t = 10, 12 \) are close to each others, and for PBC the coefficient is \( B_2 \sim 1.3 \).
Periodic boundary conditions: Polyakov loop distribution

**Figure:** The local Polyakov loop $|\langle L(x, y) \rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x, y) \rangle|$ is zero for all spatial points in the confinement phase, both without rotation and with nonzero angular velocity.
- The local Polyakov loop demonstrates weak dependence on the coordinate in the deconfinement phase.
Dirichlet boundary conditions: critical temperature

The linear velocity on the boundary $v_I = \Omega_I (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 \frac{v_I^2}{c^2} \implies \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- For lattices with sufficiently large $N_s$ and DBC the coefficient goes to plateau $B_2 \sim 0.5$. 
### Dirichlet boundary conditions: Polyakov loop distribution

**Figure:** The local Polyakov loop \(|\langle L(x, y) \rangle|\) as a function of coordinate for OBC and \(\Omega_I = 0\) MeV (left), \(\Omega_I = 24\) MeV (right). Points with \(x \neq 0, y = 0\) from the lattice \(8 \times 24 \times 49^2\) are shown.

- The local Polyakov loop \(|\langle L(x, y) \rangle|\) is equal three on the boundary in both phases.
- The boundary is screened.
Rotation and susceptibility scaling

**Figure:** The height of the susceptibility peak for various lattices $8 \times N_z \times 41^2$ and zero/nonzero angular velocities.

Rotation does not change the order of the phase transition (in studied region of $\Omega$):

- **OBC:** $\chi^{(max)} \sim V$
- **PBC:** $\chi^{(max)} \sim V$
- **DBC:** $\chi^{(max)} \sim const$
Including fermions (preliminary results)

The rotation affects both gluon and fermionic degrees of freedom.

\[ Z = \int D\psi D\bar{\psi} DA \exp \left( - S_G[A, \Omega] - S_F[\bar{\psi}, \psi, A, \Omega] \right). \]  (5)
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There is the sign problem for the lattice quark action. After the same substitution \((\Omega = -i\Omega_I)\) it has the following form

\[ S_F = \sum_{x_1, x_2} \bar{\psi}(x_1) \left\{ \delta_{x_1, x_2} - \kappa \left[ (1 - \gamma^x)T_{x+} + (1 + \gamma^x)T_{x-} \right. \right. \]
\[ + (1 - \gamma^y)T_{y+} + (1 + \gamma^y)T_{y-} + (1 - \gamma^z)T_{z+} + (1 + \gamma^z)T_{z-} \]
\[ \left. \left. + (1 - \gamma^\tau) \exp \left( ia\Omega_I \frac{\sigma^{12}}{2} \right) T_{\tau+} + (1 + \gamma^\tau) \exp \left( - ia\Omega_I \frac{\sigma^{12}}{2} \right) T_{\tau-} \right] \right\} \psi(x_2), \] (6)

where \(T_{\mu+} = U_\mu(x_1)\delta_{x_1+\mu, x_2}, \, T_{\mu-} = U_\mu(x_1)\delta_{x_1-\mu, x_2}\) and

\[ \gamma^x = \gamma^1 - y\Omega_I \gamma^4, \quad \gamma^y = \gamma^2 + x\Omega_I \gamma^4, \quad \gamma^z = \gamma^3, \quad \gamma^\tau = \gamma^4. \]

The Monte-Carlo simulations with dynamical fermions \((N_f = 2\) Wilson fermions) for an imaginary angular velocity were performed.
Figure: The Polyakov loop (a) and the chiral condensate (b) as a function of $\beta$ for different values of imaginary angular velocity $\Omega_I$. Lattice $4 \times 20 \times 21^2$, the hopping parameter $\kappa = 0.170$ ($m_\pi \simeq 690$ MeV, $T \simeq 171$ MeV for $\beta = 5.15$).

- Critical couplings $\beta_c$ for chiral transition and confinement-deconfinement transition coincide with each other (up to the error).
Figure: The Polyakov loop (a) and the chiral condensate (b) as a function of $\beta$ for different values of imaginary angular velocity $\Omega_I$. Lattice $4 \times 20 \times 21^2$, the hopping parameter $\kappa = 0.170$ ($m_\pi \simeq 690$ MeV, $T \simeq 171$ MeV for $\beta = 5.15$).

- Critical couplings $\beta_c$ for chiral transition and confinement-deconfinement transition coincide with each other (up to the error).

One can split the full action as $S_G(\Omega_G) + S_F(\Omega_F)$ and rotate each part separately!
Figure: The Polyakov loop (a) and the chiral condensate (b) as a function of $\beta$ for different values of imaginary angular velocity $\Omega_I$. Lattice $4 \times 20 \times 21^2$, the hopping parameter $\kappa = 0.170$.

- Rotation of fermions and gluons separately has the opposite influence on the critical coupling (temperature).
Rotation of fermions and gluons separately has the **opposite** influence on the critical coupling (temperature).

Figure: The critical value $\beta_c$ as function of **imaginary** linear velocity on the boundary.
Figure: The critical value $\beta_c$ as function of imaginary linear velocity on the boundary.

- Rotation of fermions and gluons separately has the opposite influence on the critical coupling (temperature).
- The results are qualitatively the same for OBC.
Conclusions

- The critical temperature of the confinement/deconfinement transition in gluodynamics increases with angular velocity

\[ \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2 \Omega^2, \]

- The result does not depend on the boundary condition used

\[ \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}, \]

where for OBC \( B_2 \sim 0.7 \), for PBC \( B_2 \sim 1.3 \) and for DBC \( B_2 \sim 0.5 \)

- Rotation does not change the order of the phase transition.

- It should be noted, that NJL (and other phenomenological models) predicts that critical temperature decreases due to the rotation.

- Preliminary results for QCD show that the separate rotation of quarks and gluons has the opposite influence on \( \beta_c \) (for \( m_\pi \sim 690 \) MeV gluons win).
Thank you for your attention!
Figure: The Polyakov loop (a) and Polyakov loop susceptibility (b) as a function of temperature for different values of imaginary angular velocity $\Omega_I$. The results are obtained on the lattice $8 \times 24 \times 49^2$.

- The height of the peak $\chi^{(max)}$ slightly grows with angular velocity for OBC.
Periodic boundary conditions

Figure: The Polyakov loop (a) and Polyakov loop susceptibility (b) as a function of temperature for different values of imaginary angular velocity $\Omega_I$. The results are obtained on the lattice $8 \times 24 \times 49^2$.

- The height of the peak $\chi^{(max)}$ falls down with angular velocity.
Dirichlet boundary conditions

Figure: The Polyakov loop (a) and Polyakov loop susceptibility (b) as a function of temperature for different values of imaginary angular velocity $\Omega_I$. The results are obtained on the lattice $8 \times 24 \times 49^2$.

- The height of the Polyakov loop susceptibility $\chi^{(max)}$ falls down with rotation.
- Polyakov loop is not zero for low temperatures. Contribution from boundary is $\delta L_{b.c.} = 12(N_s - 1)/N_s^2$, or $\delta L_{b.c.} \simeq 0.24$. 