National Nuclear Security Administration

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Coulomb field correction due to virtual e+e- production in heavy ion collisions

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IN NUCLEAR TRAINING AND UNIVERSITY-BASED RESEARCH

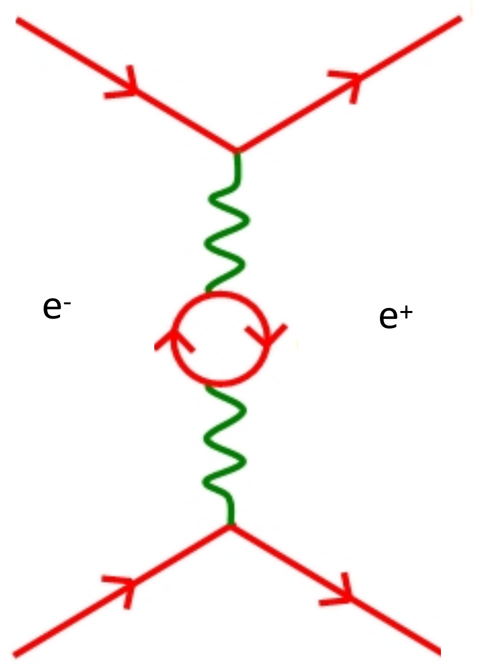
Outline

- Background/motivation
- Vacuum polarization
- Mass formula
- Sub-barrier fusion
- Fission
- Application to Constrained Molecular Dynamics (CoMD)
- Future work

Background and motivation

- production of real e+e- pairs can occur during dynamics in the presence of strong fields, when the available energy exceeds twice the electron mass.
- In the 1980s, experimentalists at GSI found some anomalous production of e^+e^-pairs in heavy ion collisions.
- Various explanations were proposed, including production of a hypothesized new light particle and experimental error. To our knowledge, there is no consensus.
- For now, we only discuss the perturbative effect on the energy. We will introduce the correction into the microscopic model Constrained Molecular Dynamics (CoMD) and in following research we will discuss actual production.

Vacuum polarization



Virtual e+e- pairs create correction to Coulomb potential.
Uehling calculated the correction in 1935 using Dirac hole theory:

$$V_{e^+e^-}(r) = -\frac{\alpha Z_1 Z_2 e^2}{\pi r}\int_0^1 (1-u^2)\, li\left(\exp\frac{-2r}{\lambda_0\sqrt{1-u^2}}\right)du$$

$$\lambda_0 = \frac{\hbar}{m_e c} \approx 386\,\mathrm{fm}$$

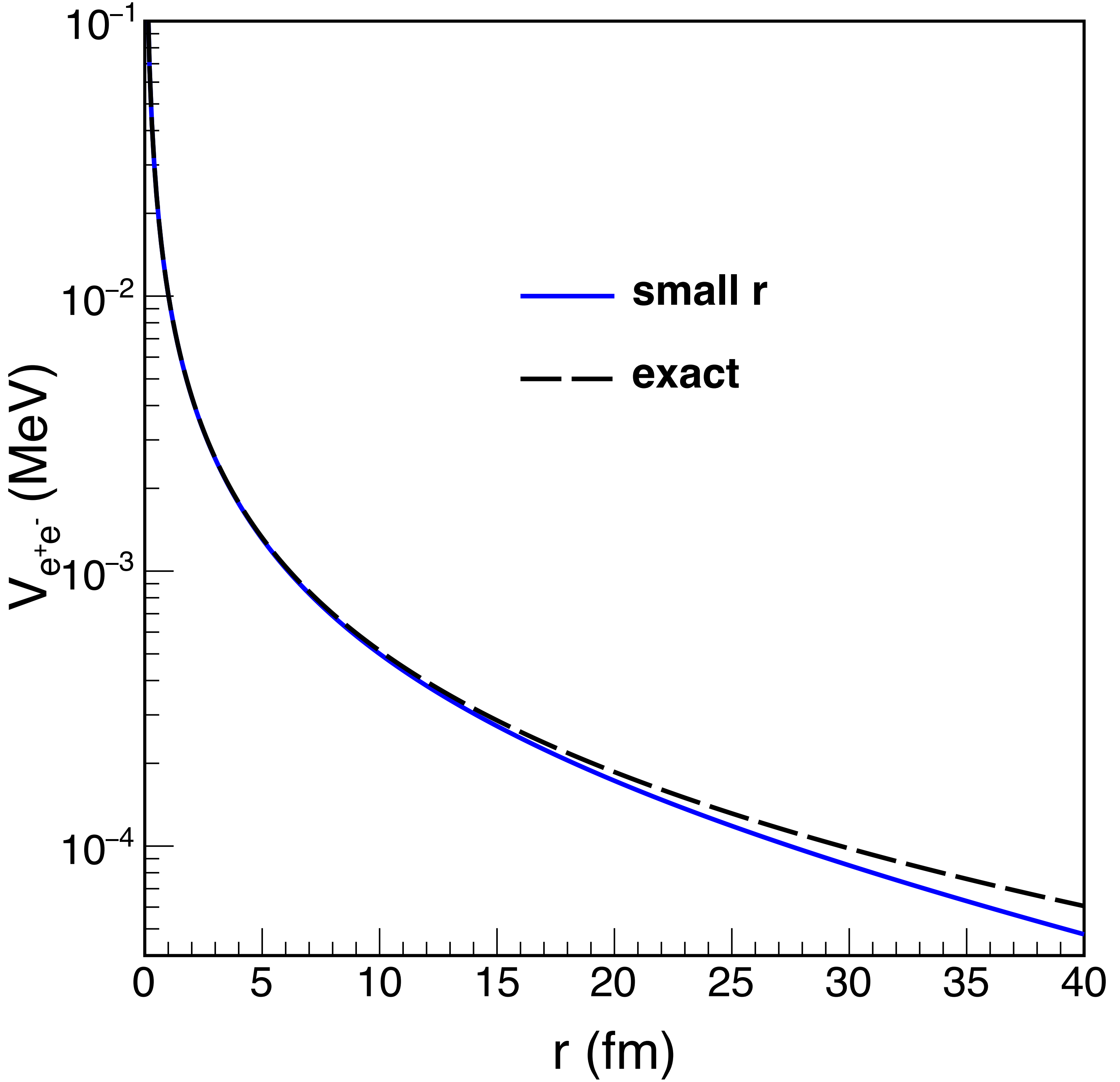
Logarithmic integral: $li(x) = \int_0^x \frac{dt}{\log t}$

For small r: $$V_{e^+e^-}(r) = -\frac{2\alpha}{3\pi}\frac{Z_1 Z_2 e^2}{r}\left(\ln\frac{r}{\lambda_0} + \gamma + \frac{5}{6}\right)$$

$$\gamma = 0.5772\ldots$$

Vacuum polarization

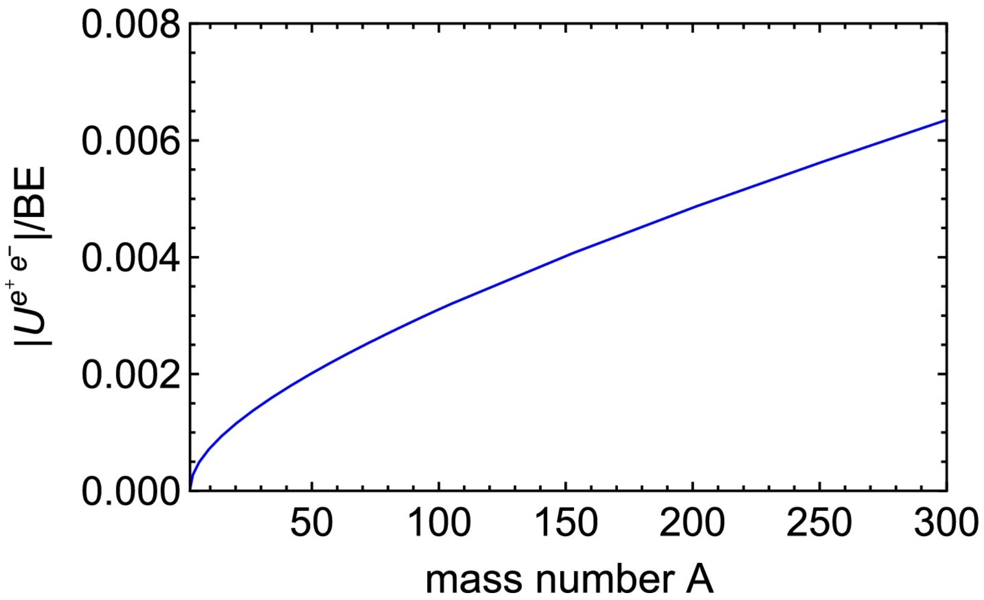
The small r approximation is fairly good in the range we consider.

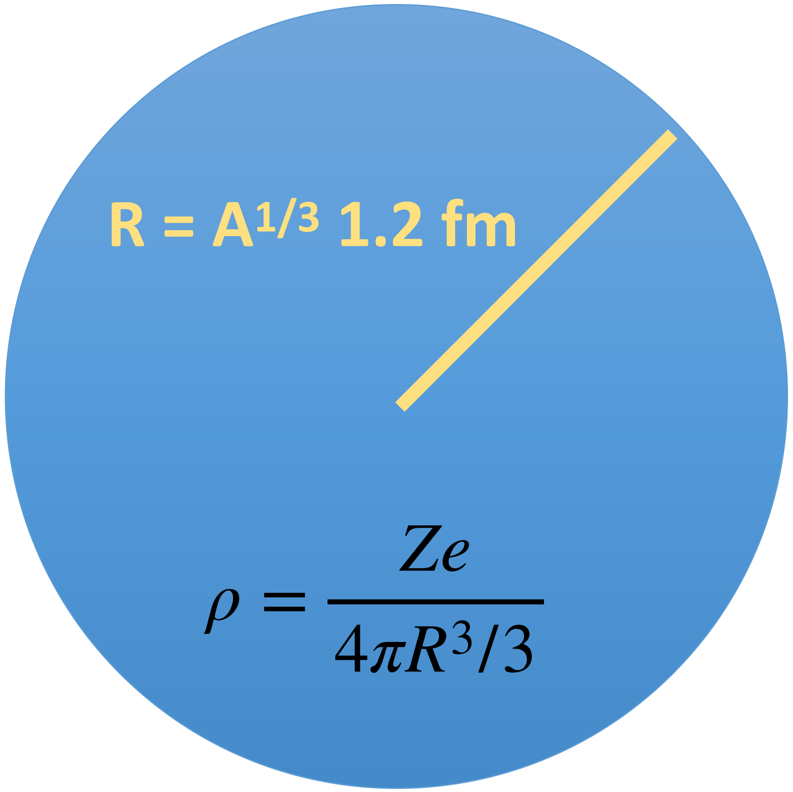


Vacuum polarization in the mass formula

$$BE = a_V A - a_s A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

- Assume uniformly charged sphere
- $U_{e^+e^-} = \frac{1}{2}\int \rho d^3r \int \rho d^3r' \, V_{e^+e^-}(\mathbf{r})$
- $U_{e^+e^-} = (0.0073 - 0.00052 \ln A) a_C \frac{Z(Z-1)}{A^{1/3}}$

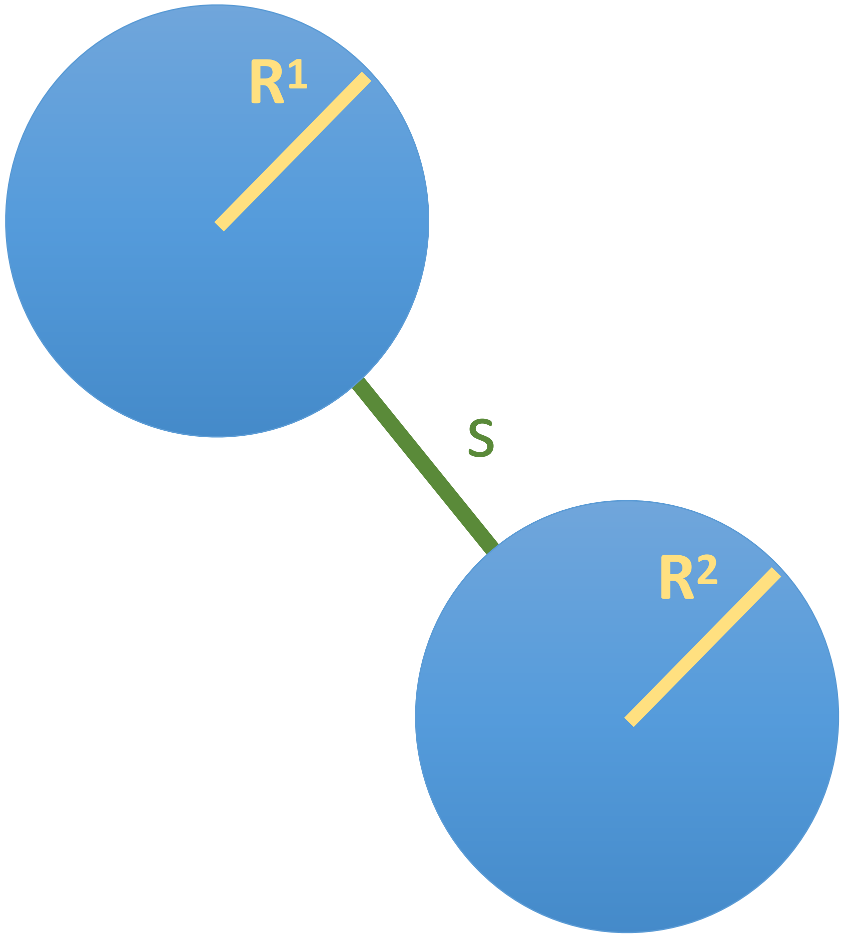


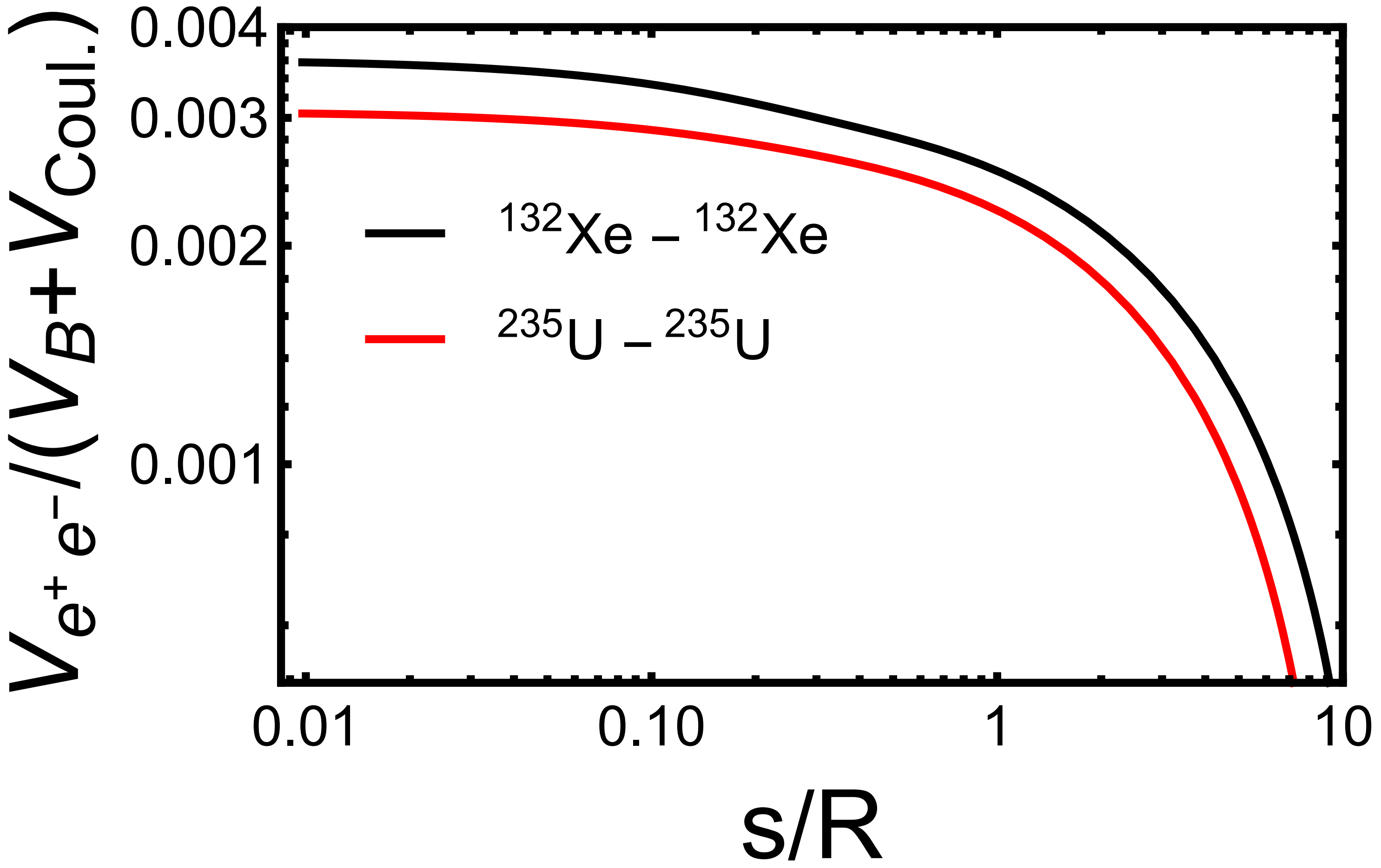


Including nuclear (Bass) potential

Bass potential is phenomenological model of nuclear force with parameters fit from scattering data.

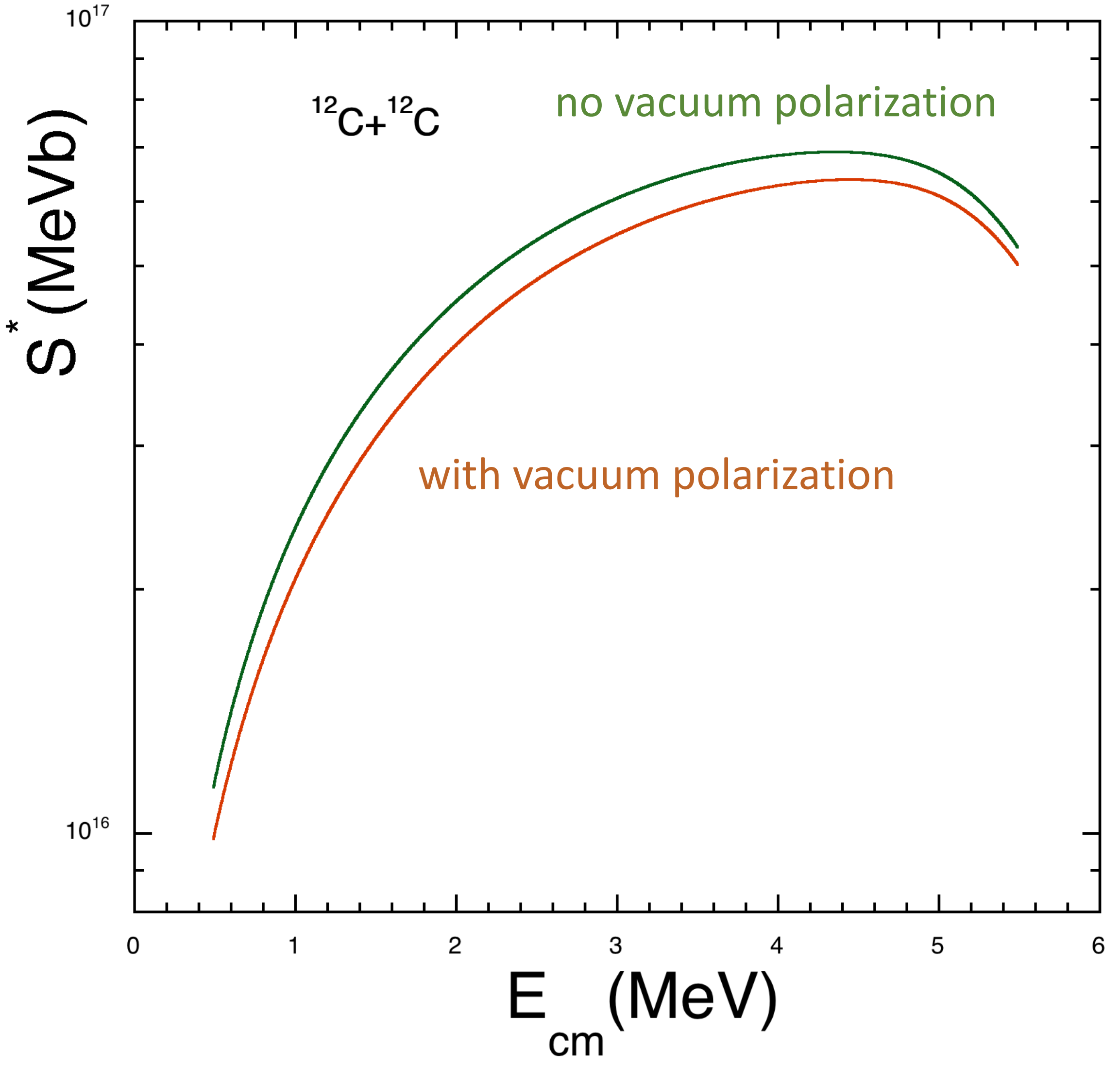
$$V_B(s) = -\frac{R_1 R_2}{R_1 + R_2}[\alpha \exp(s/d_1) + \beta \exp(s/d_2)]^{-1}$$





Sub-barrier fusion

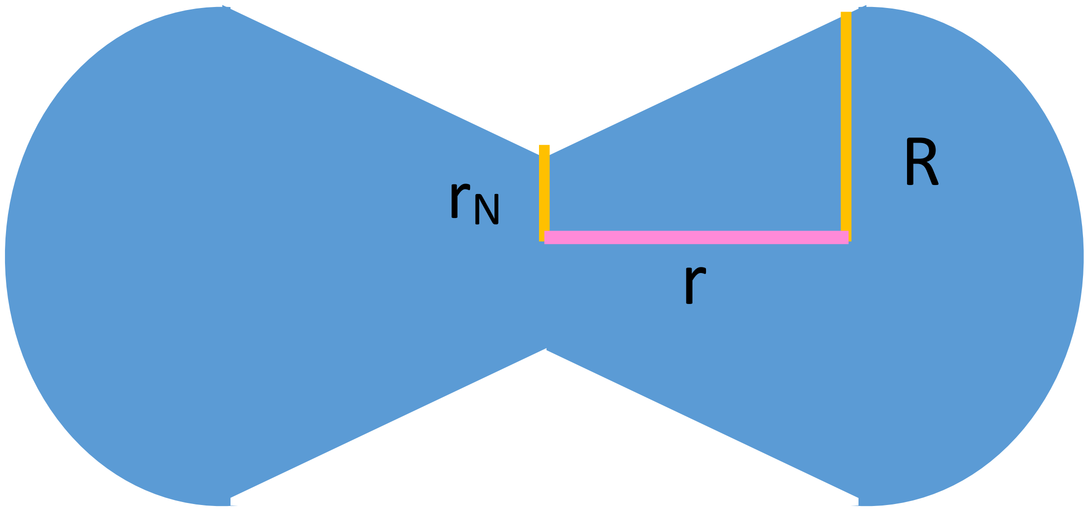
- The two nuclei approach each other under Coulomb plus the nuclear (Bass) potential.
- Classically → stop and bounce back.
- Quantum mechanically → finite probability of tunneling
- Probability and cross section found by integrating imaginary action between turning points.
- Vacuum polarization decreases fusion cross section by as much as 15% for energies ~ MeV.

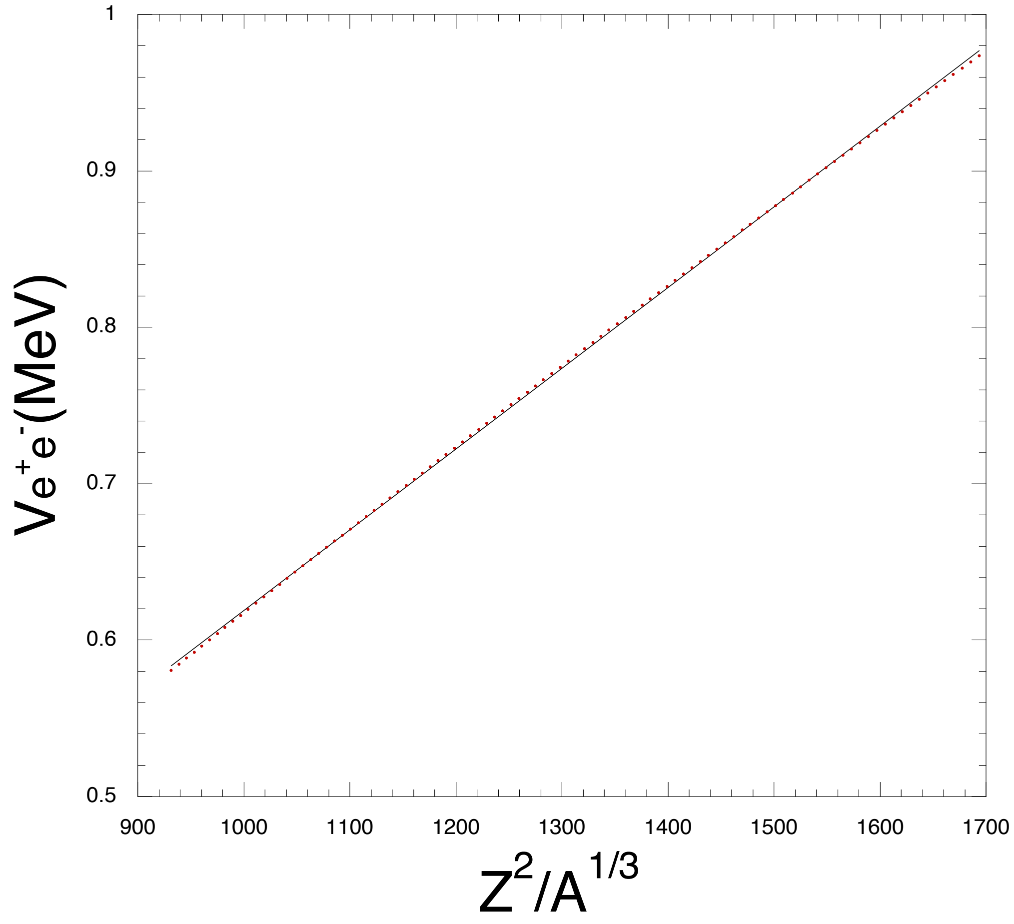


$$S^*(E_{c.m.}) = E_{c.m.}\sigma(E_{c.m.}) \times \exp(87.12E_{c.m.}^{-1/2} + 0.46Ec\,.\,m.)$$

Fission

Neck model: $$r_N = \frac{\sqrt{r^2R^2 - 4r(rR^2 - 4R^3)} - Rr}{2r}$$





Fission occurs when $r_N < 1$ fm.

Vacuum Polarization correction is less than 1% of fragment energy from Viola Systematics:

$$\langle E_k \rangle = 0.1189 Z^2/A^{1/3} + 7.3 \text{ MeV}$$

Constrained Molecular Dynamics (CoMD)

In Constrained Molecular Dynamics (CoMD) [2], each nucleon has a Gaussian distribution in phase space.

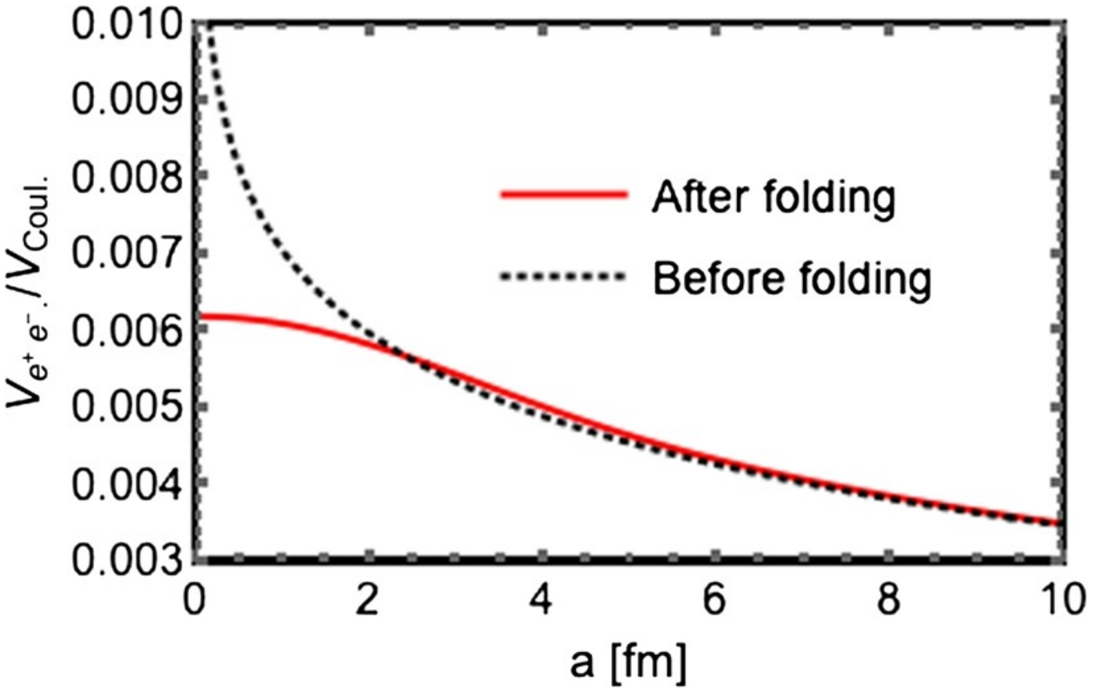
$$\rho_i \sim \exp \frac{\left(\boldsymbol{r} - \langle \boldsymbol{r}_i \rangle\right)^2}{2\sigma^2}$$

Pauli exclusion principle is enforced.

Energy is folded into this distribution with σ=1.15 fm.

$$V(a) = \int d^3\mathbf{r}_i d^3\mathbf{r}_j E(|\mathbf{r}_i - \mathbf{r}_j|)\rho_i(\mathbf{r}_i)\rho_j(\mathbf{r}_j)$$

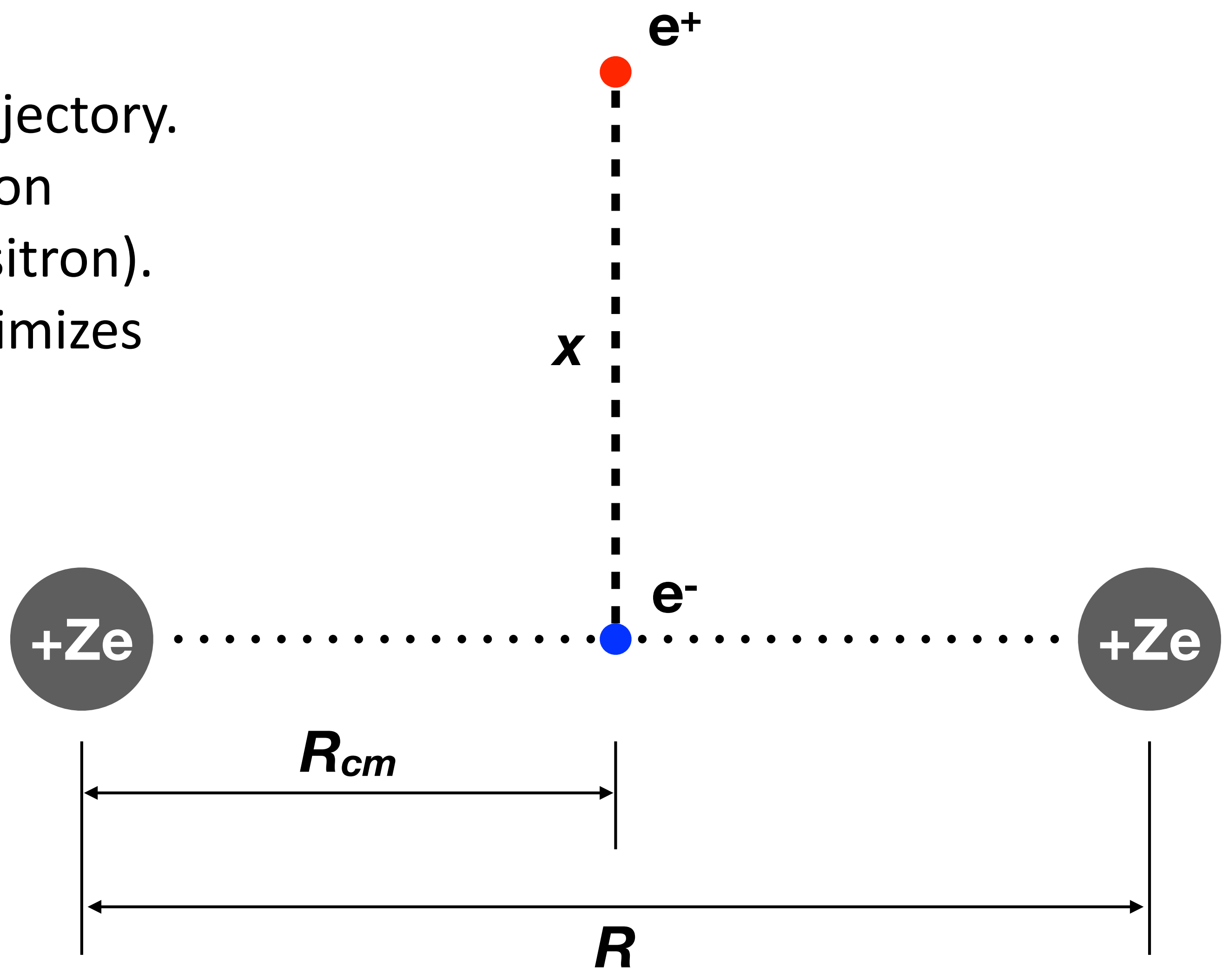
$$E(r) = -\frac{2\alpha}{3\pi}\frac{Z_1 Z_2 e^2}{r}\left(\ln\frac{r}{\lambda_0} + \gamma + \frac{5}{6}\right)$$



$$V_{e^+e^-}(a) = -\frac{2\alpha e^2}{3\pi}\left[\left(\gamma/2 + 5/6 + \ln\frac{2\sigma_r}{\lambda_0}\right)\frac{\mathrm{erf}(a/2\sigma_r)}{a} - \frac{1}{2\sqrt{\pi}\sigma_r}G^{(1)}\left(\frac{1}{2}, \frac{3}{2}; -\frac{a^2}{4\sigma_r^2}\right)\right]$$

Future work

- Solve Dirac equation along Coulomb trajectory.
- Production occurs when a virtual electron “tunnels” and leaves behind a hole (positron).
- Most probable geometry is shown: minimizes energy.



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