Electroweak transitions due to magnetic field: first-principle lattice results

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Motivating question:

Vacuum instability in strong magnetic field background?

Partially supported by grant No. 0657-2020-0015 of the Ministry of Science and Higher Education of Russia.

What happens with the vacuum in strong magnetic field?

1) QCD scale, B ~ 10^{16} T, could proceed via the ρ meson condensation

[M.Ch., PRD 80, 054503 (2009); PRL 106, 142003 (2011)]

(possible weak crossover transition via inhomogeneous condensation of composite states, difficult to see — not this talk)

3) EW scale, B ~ 10^{20} T, could proceed via the W boson condensation

[J. Ambjorn, P. Olesen, PLB 214, 565 (1988); NPB 315, 606 (1989)]

(inhomogeneous condensation, looks easy and trivial — this talk)

not true, in fact

Free relativistic particle in magnetic field

Landau levels:

$$\begin{aligned} \text{scalar:} \quad E_n^2 &= k_z^2 + (2n+1)eH + m^2 \\ \text{spinor:} \quad E_n^2 &= k_z^2 + (2n+1)eH - 2eH \cdot s + m^2 \qquad s = \pm\frac{1}{2} \\ \text{vector:} \quad E_n^2 &= k_z^2 + (2n+1)eH - 2eH \cdot s + m^2 \qquad s = \pm 1, 0 \end{aligned}$$

instability: $eH_{crit}^{(1)} = m^2$

For W bosons (if we disregard interactions):

$$B_c^{\mathrm{EW}} = \frac{M_W^2}{e} \simeq 1.1 \times 10^{20}\,\mathrm{T}$$

Electroweak vacuum should become unstable toward W condensation!

*) we use both H and B to denote magnetic field

Vacuum instability, what is the nature of the new phase?

… the one which is just about the **(first)** critical field.

1) **Condensation of W bosons**

[J. Ambjorn, P. Olesen, PLB 214, 565 (1988); NPB 315, 606 (1989)]

2) **Vacuum superconductivity**

[M.Ch., PRD 80, 054503 (2009)]

Vacuum should enter the new exotic phase which

a) is anisotropically superconducting

b) but does not possess Meissner effect

(= magnetic field screening by a charged condensate)

Superconductivity of the vacuum is interesting and nontrivial phenomenon. The first step to establish the vacuum superconductivity is to make sure that

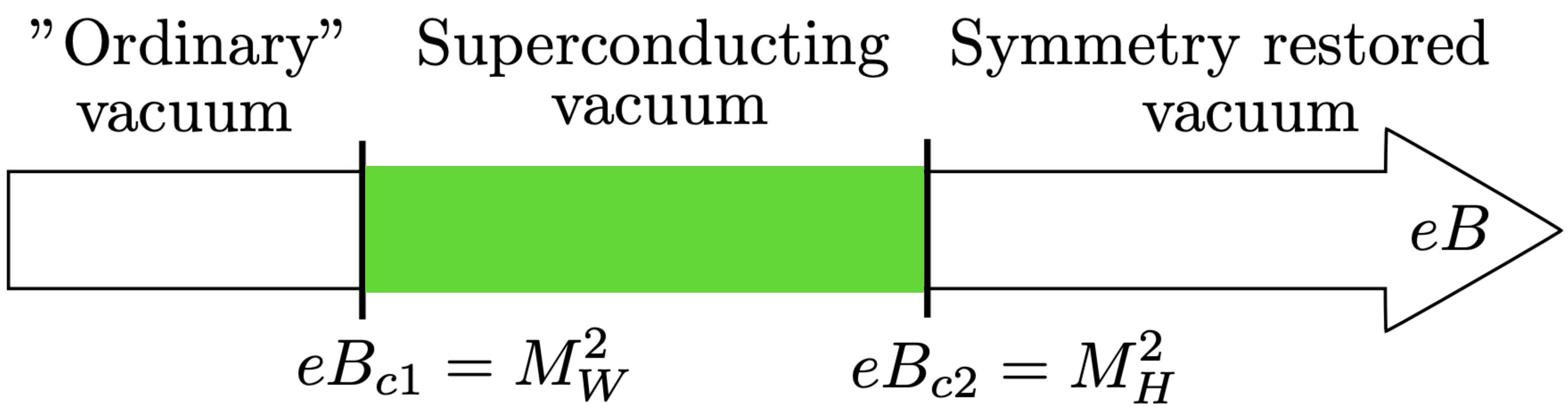
1) the vacuum instability towards the new phase exists;

2) the new phase has appropriate condensates (consistent with the theory);

→ aim of this work

What theory says about the phase structure?

(Weinberg-Salam model in strong magnetic field)



Lagrangian:

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left(|\Phi|^2 - v^2/2 \right)^2$$

$$D_\mu = \partial_\mu - ig\tau^a W^a_\mu/2 - ig' X_\mu/2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

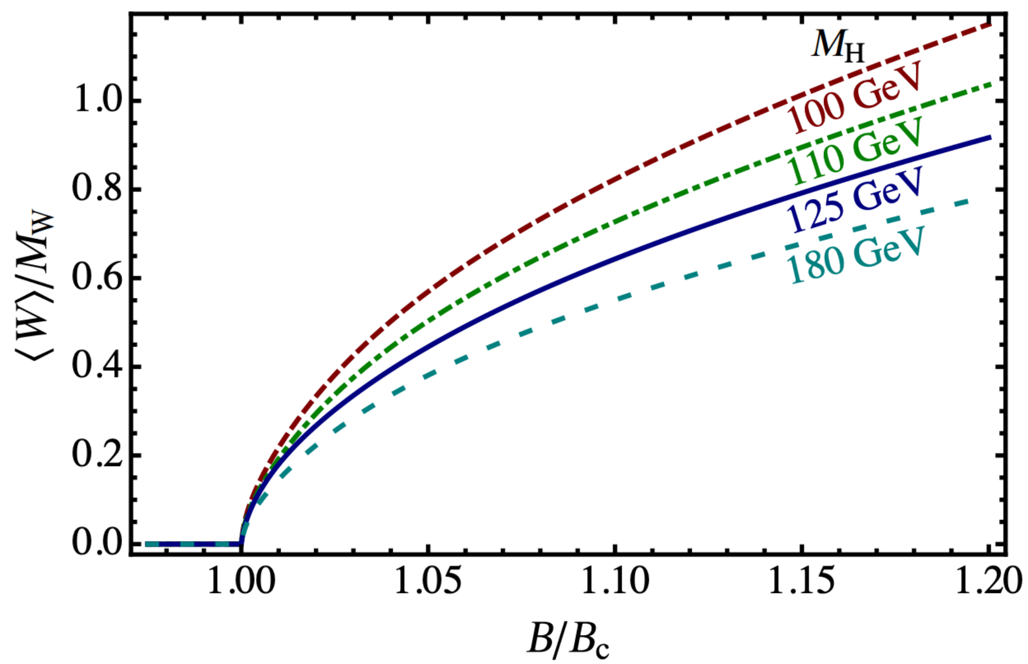
Ordinary vacuum, symmetry breaking:

$$SU(2)_L \times U(1)_X \to U(1)_{\text{em}}$$

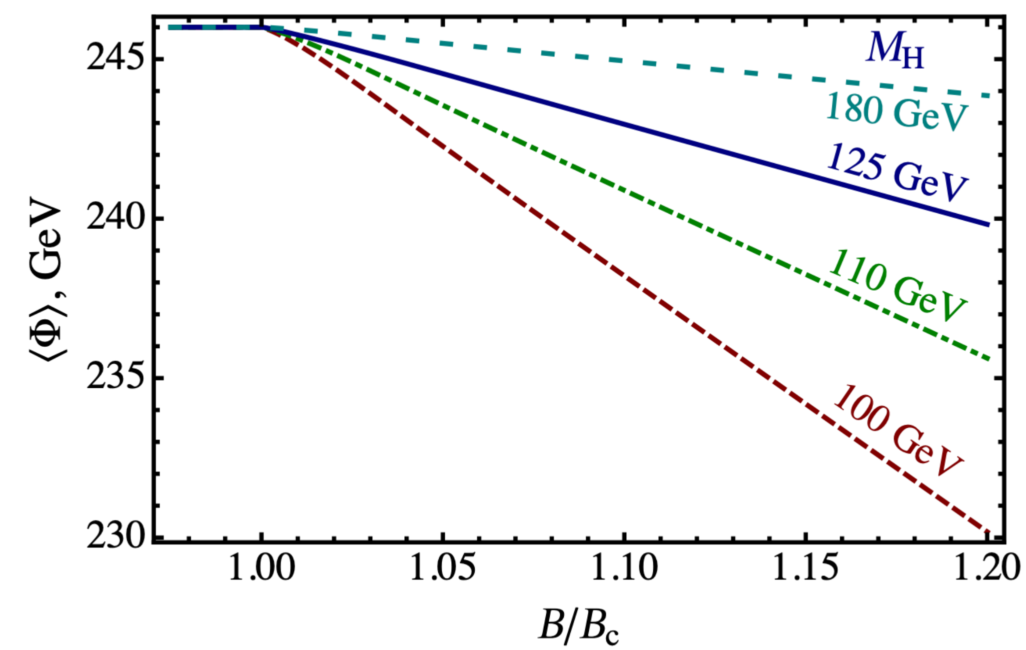
Superconducting phase, what to expect (theory)

Solution of classical equations of motion (at a set of Higgs masses)

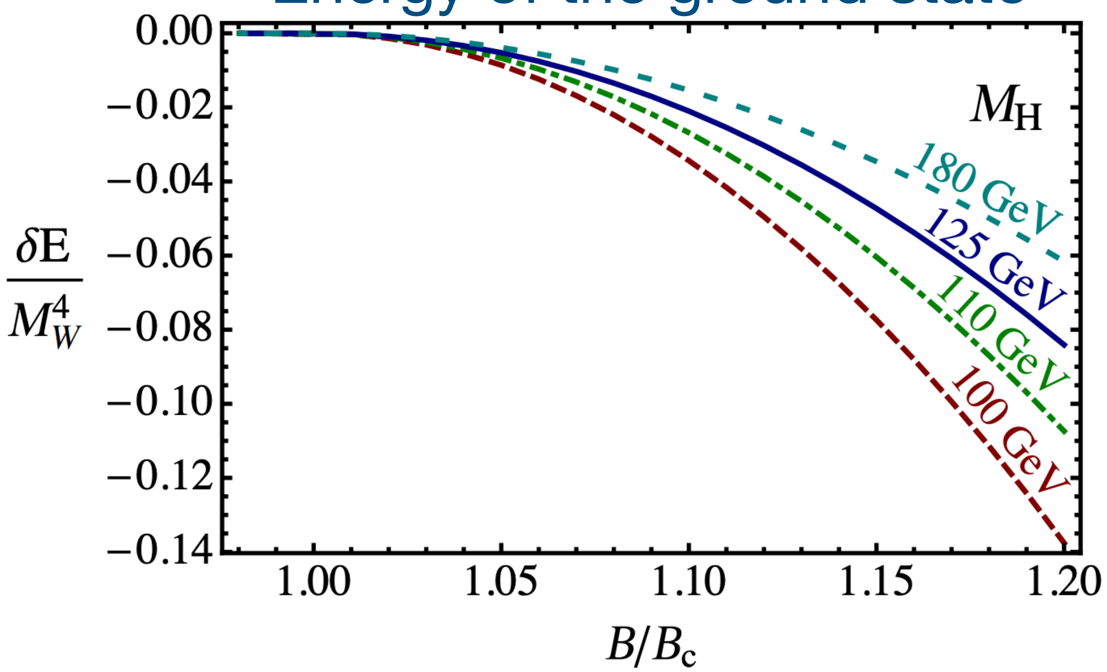
W-boson condensate



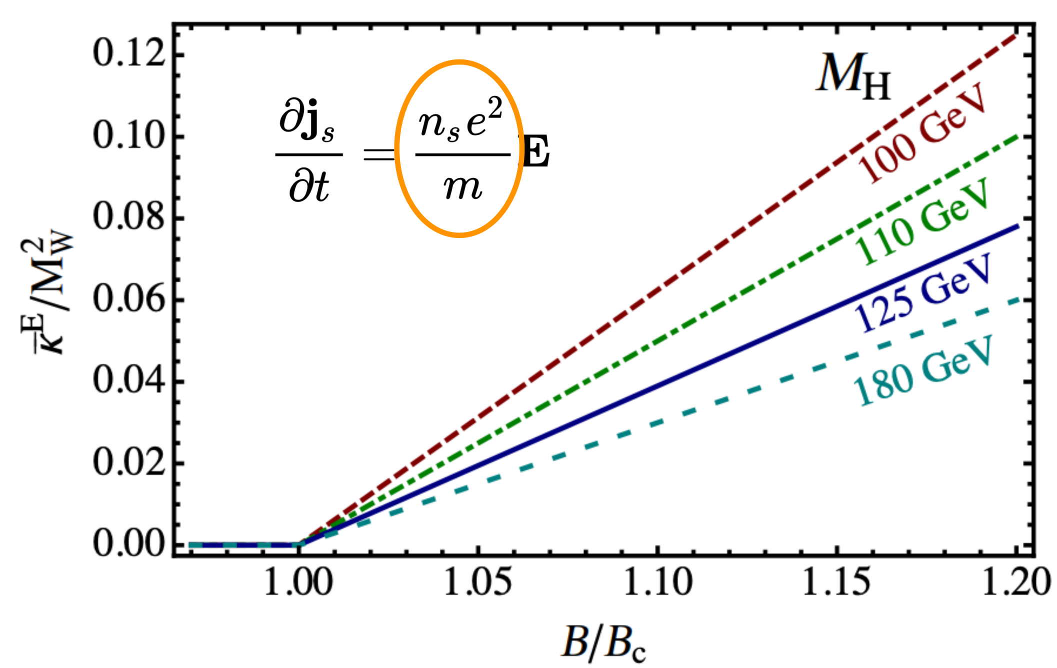
Higgs condensate



Energy of the ground state



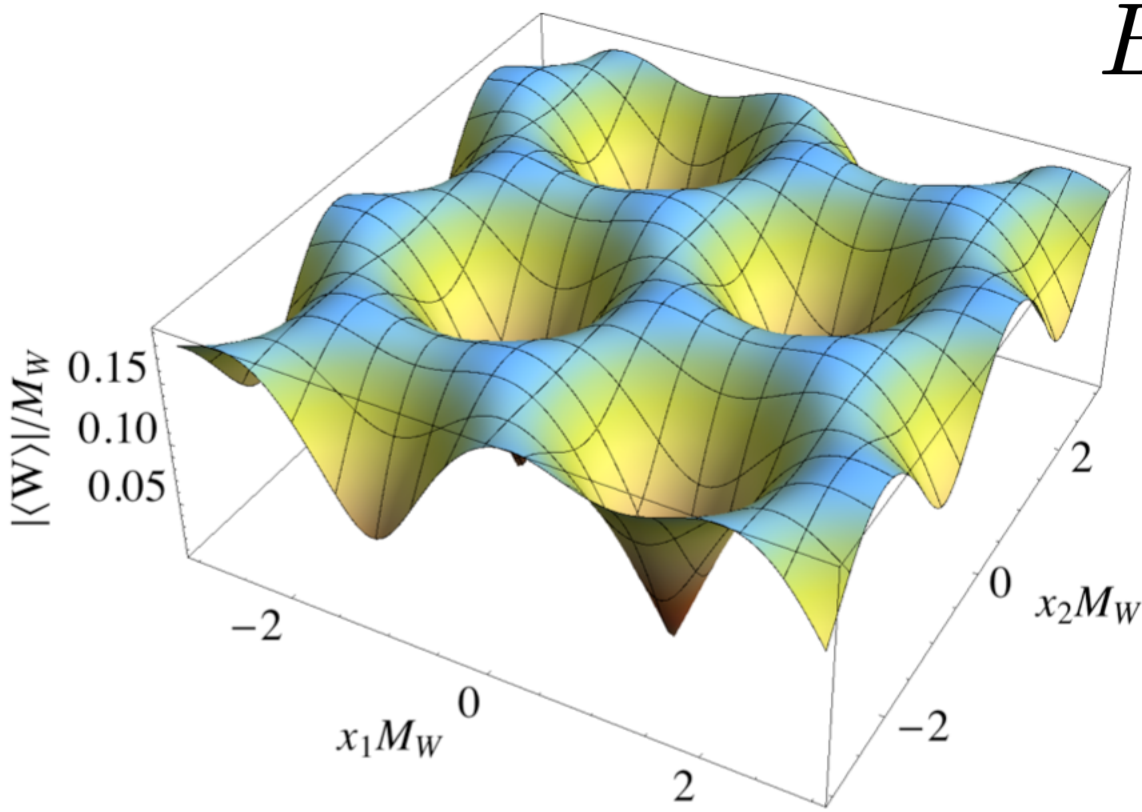
Density of superconducting “pairs”



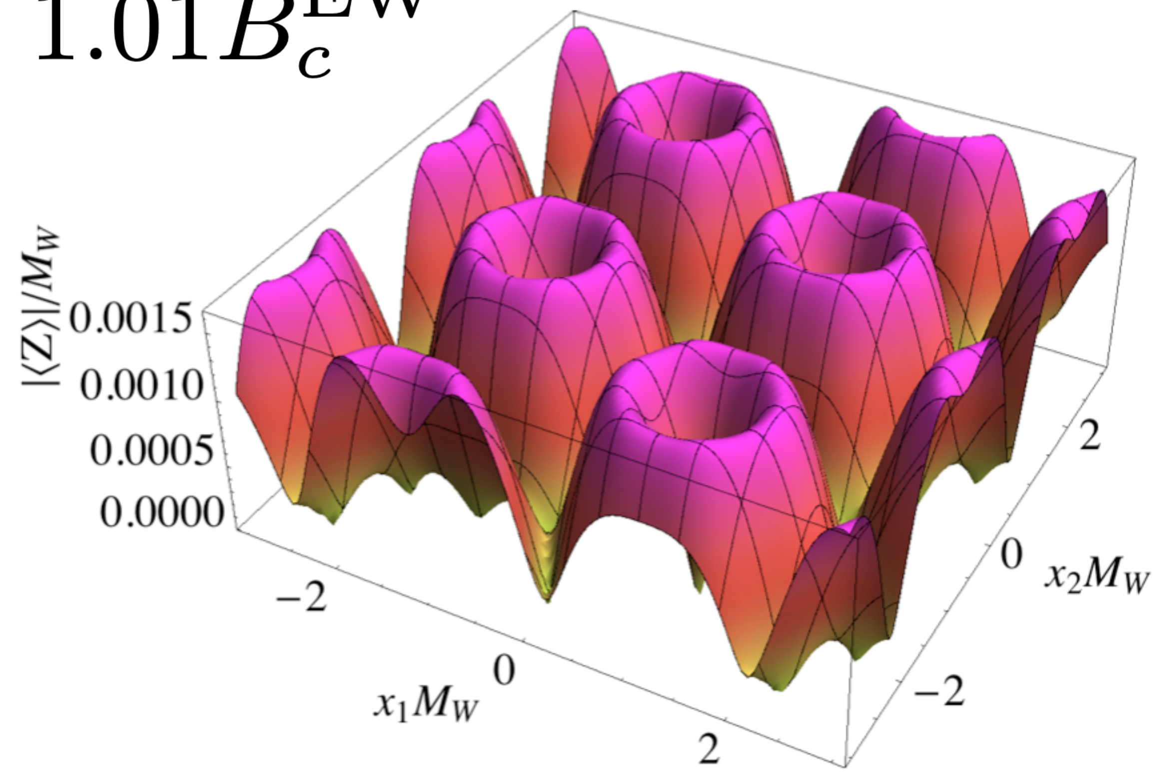
[Jos Van Doorsselaere, Henri Verschelde, M.Ch., Phys. Rev. D 88, 065006 (2013)]

Superconducting phase, inhomogeneity (theory)

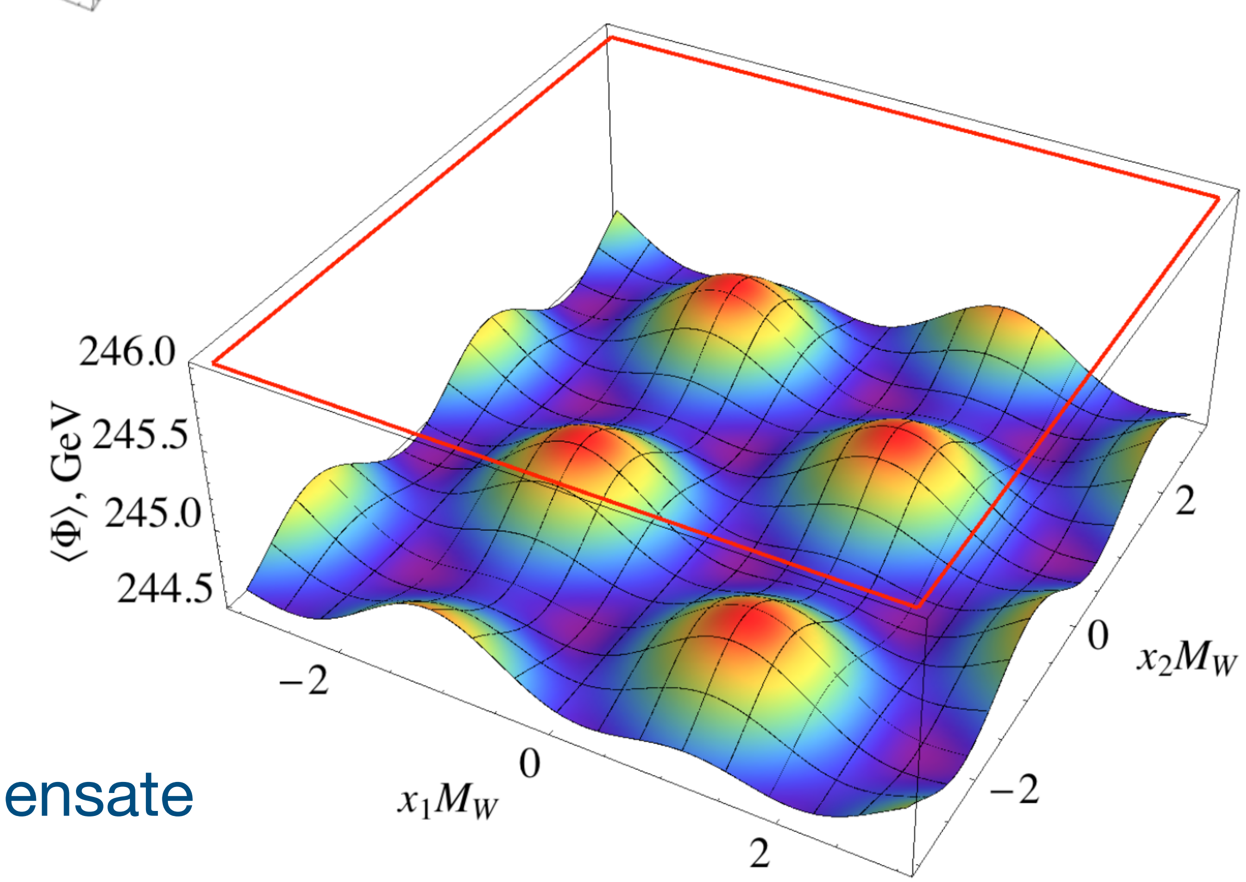
$$B = 1.01 B_c^{\mathrm{EW}}$$



W-boson condensate



Z-boson condensate

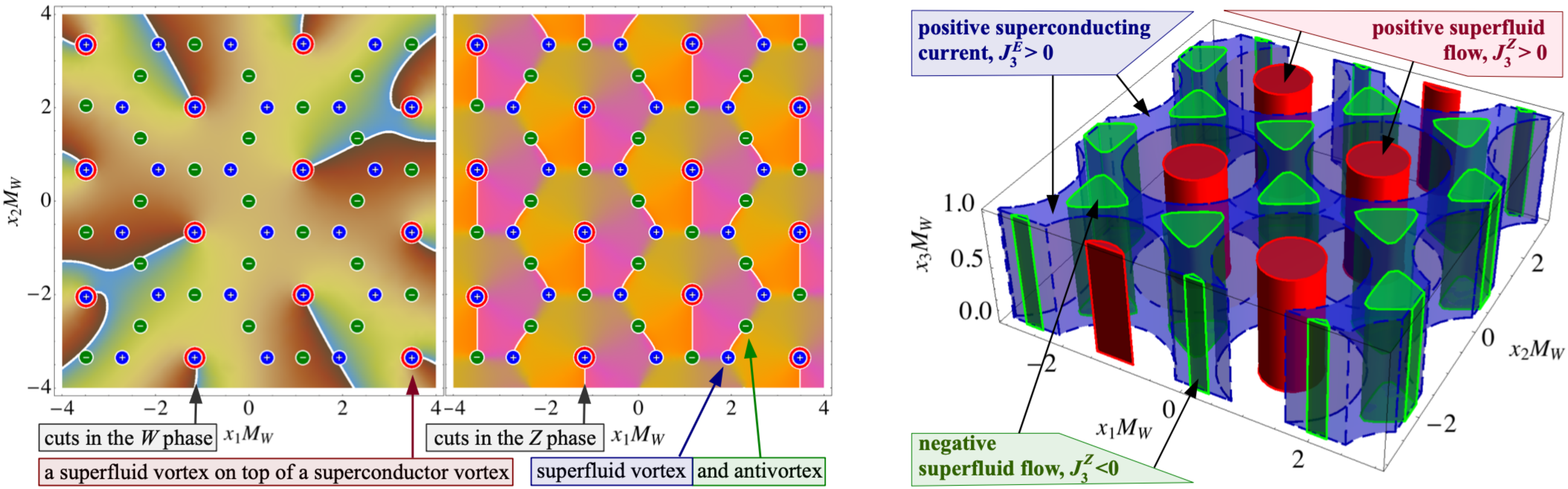


Higgs condensate

[Jos Van Doorsselaere, Henri Verschelde, M.Ch., Phys. Rev. D 88, 065006 (2013)]

Superconducting phase, inhomogeneity (theory)

Vortex structure in superconducting (W) and superfluid (Z) condensates



[Jos Van Doorsselaere, Henri Verschelde, M.Ch., Phys. Rev. D 88, 065006 (2013)]

Visually (and distantly) similar but physically very different from the Abrikosov lattice in type-2 superconductors

Theoretical expectations based on classical equations of motion:

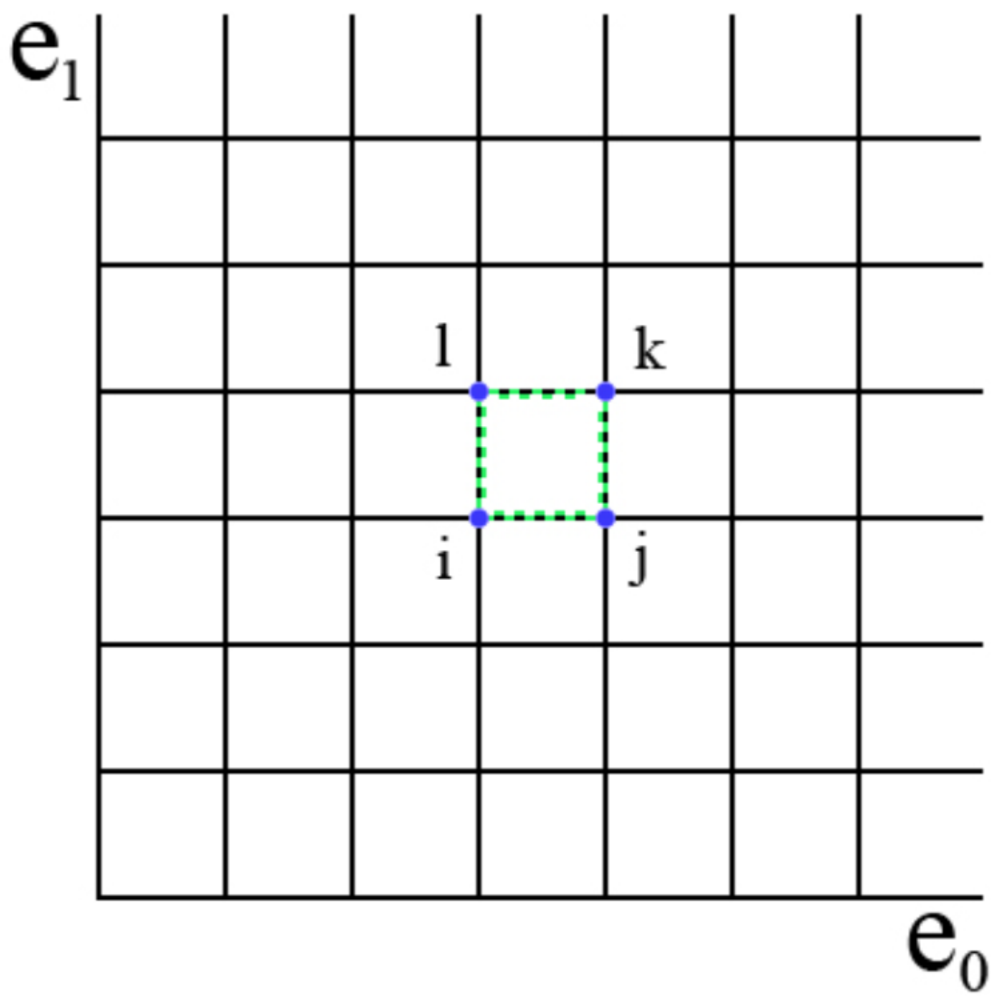
— Magnetic field leads to condensation of charged W bosons
— Condensation of the W's leads to a condensation of neutral Z bosons

→ **Coexisting superconducting and superfluid condensates**

Reality = classical picture + quantum fluctuations

(+ magnetic-field-induced vortex lattice will vibrate and generate phonon modes!)

Check the picture in first-principle lattice simulations



Gauge action

- vertex – fields
  $\psi(x) \to \psi(x_i)$
- edge (link) – gauge fields
  $A_\mu \to U(L) = e^{\imath g_0 \int_L A_\mu \, dx^\mu}$

gauge transformation:

$$U(L) \to g^{-1}(L_{end})\, U(L)\, g(L_{begin})$$

Wilson: $S_W = \sum_{plaquettes} S_P$, where $S_P = \beta \left(1 - \frac{1}{N} Re\, Tr\, U_P\right)$

Electroweak theory on the lattice

— fermions play no essential role in the mechanism, we exclude them

— background hypermagnetic field = magnetic field in the broken phase

Dynamical fields:

- $U_{x,\mu} = \exp\left(\imath \frac{\sigma_i}{2} W^i_{x,\mu}\right) \in \mathrm{SU}(2)$
- $\theta_{x,\mu} \in \mathcal{R}$
- $\phi_x = \begin{pmatrix} \phi_{1,x} \\ \phi_{2,x} \end{pmatrix}$

$$
\begin{aligned}
S &= \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{2}\mathrm{Tr}\, U_{x,\mu\nu}\right) + \frac{\beta_Y}{2} \sum_{x,\mu<\nu} \theta^2_{x,\mu\nu} && \textit{(gauge)} \\
&+ \sum_x \left(-\kappa \phi^\dagger_x \phi_x + \lambda \left(\phi^\dagger_x \phi_x\right)^2\right) && \textit{(Higgs)} \\
&+ \sum_{x,\mu} \left|\phi_x - e^{\imath\left(\theta_{x,\mu} + \theta^B_{x,\mu}\right)} U_{x,\mu} \phi_{x+\hat{\mu}}\right|^2 && \textit{(interaction)}
\end{aligned}
$$

Boundary condition: periodic

Magnetic field : along Z direction

Lattice size: 64×48^3

Parameters: β, β_Y, κ, λ, $\theta^B_{x,\mu}$.

Where is physical point?

Finding a physical point

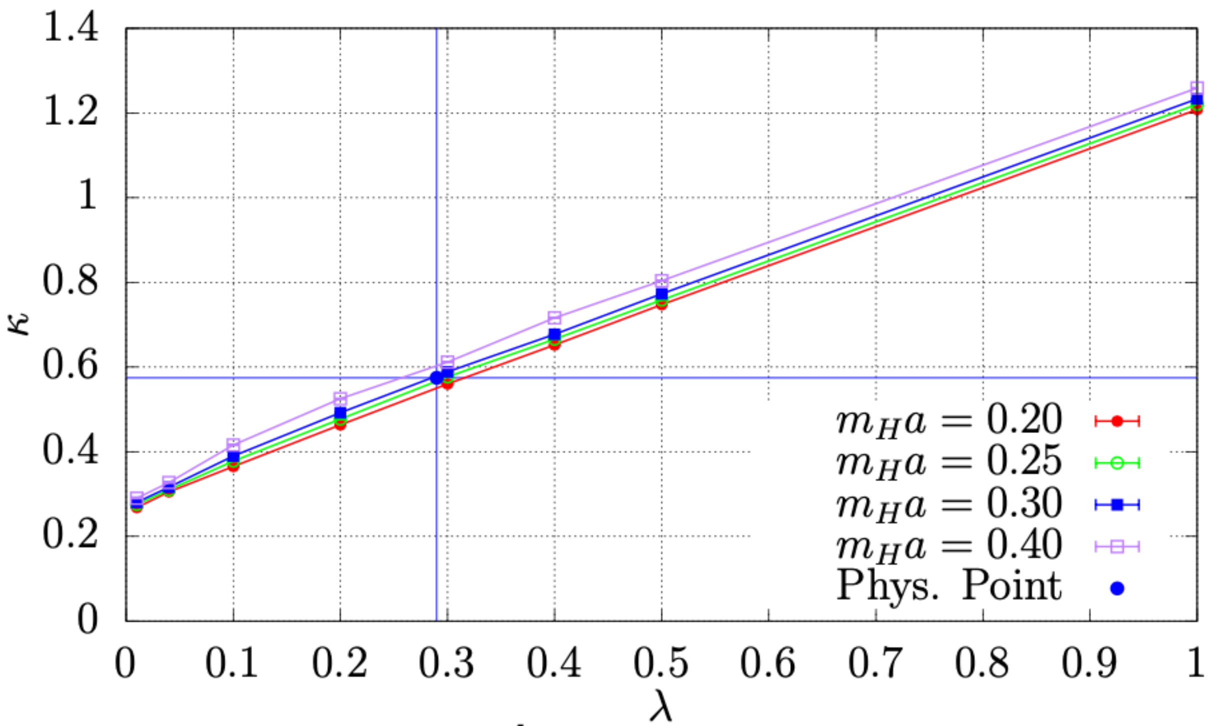
$e \approx 0.303 \quad m_H \approx 125.3 \text{ GeV}$
$g \approx 0.642 \quad m_Z \approx 91.2 \text{ GeV}$
$g' \approx 0.344 \quad m_W \approx 80.4 \text{ GeV}$
$\sin^2\theta_W \approx 0.223$

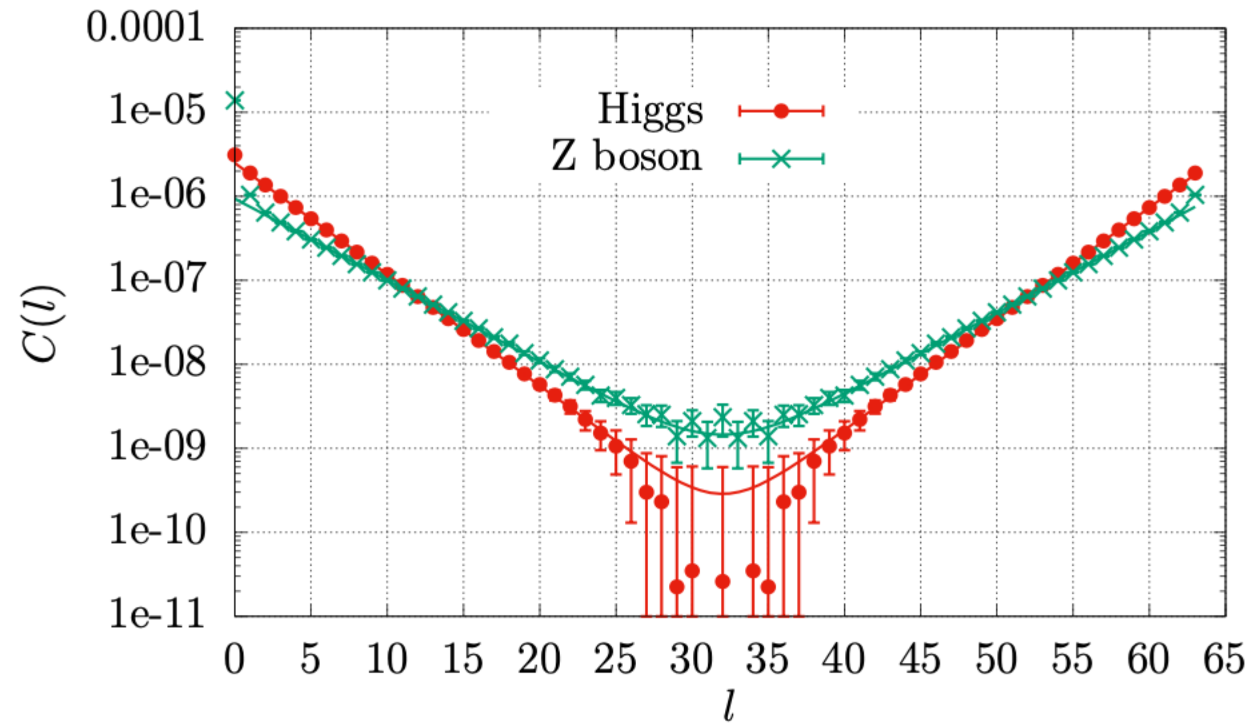
$$\beta = \frac{4}{g^2}, \quad \beta_Y = \frac{1}{g'^2} \equiv \frac{1}{g^2 \tan^2 \theta_W}$$

$$\Rightarrow \beta = 4\beta_Y \tan^2\theta_W$$

Our values: $\beta_Y = 7$, $\beta = 8$.

[Phys. Lett. B284 (1992) 371; Nucl.Phys. B544 (1999) 357]





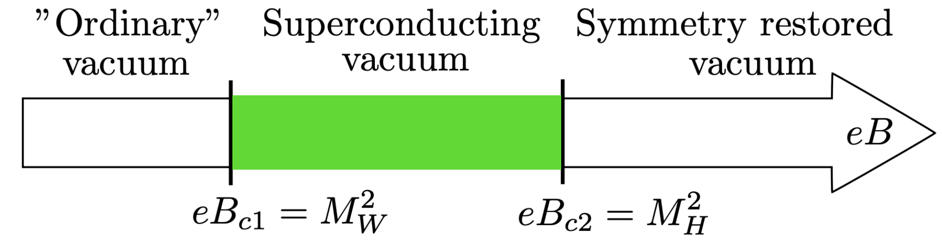
$$\frac{m_Z^{ph.}}{m_H^{ph.}} = 0.7280$$

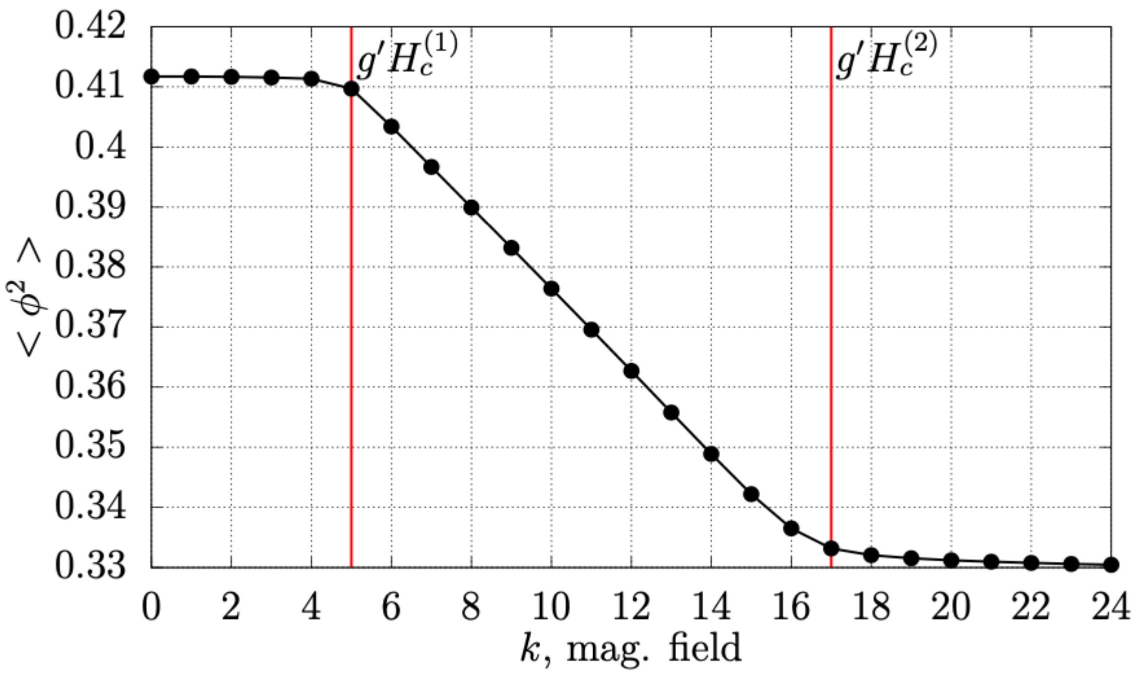
$m_H a = 0.3049(2) \quad m_Z a = 0.2237(3)$

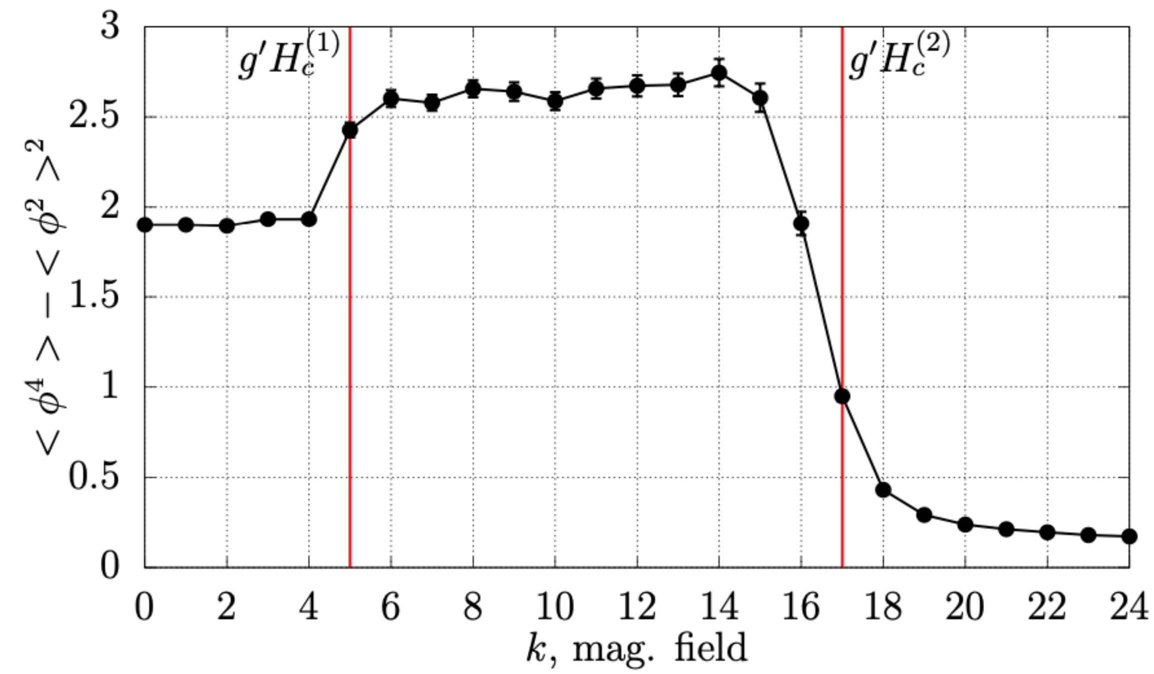
$m_Z = 91.88 \pm 0.12$ (err. $< 1\%$)

Mean Higgs condensate in (hyper)magnetic field

$$m_H a = 0.3049(2)$$







1. We see two transitions: $3.3(4) \times 10^{19}$ Tesla and $11.4(4) \times 10^{19}$ Tesla.
2. Transitions are smooth.

$$\sqrt{g' H_c^{(1)}} = 48.0 \pm 2.4 \ \text{GeV} \qquad \sim (48.8 \pm 2.5)\% \cdot m_W^{(our)}$$

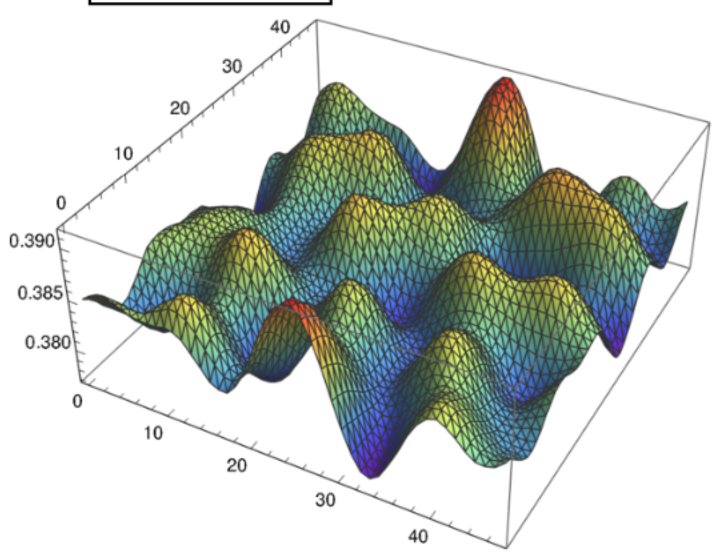
$$\sqrt{g' H_c^{(2)}} = 88.4 \pm 1.3 \ \text{GeV} \qquad \sim (70.6 \pm 1.0)\% \cdot m_H$$

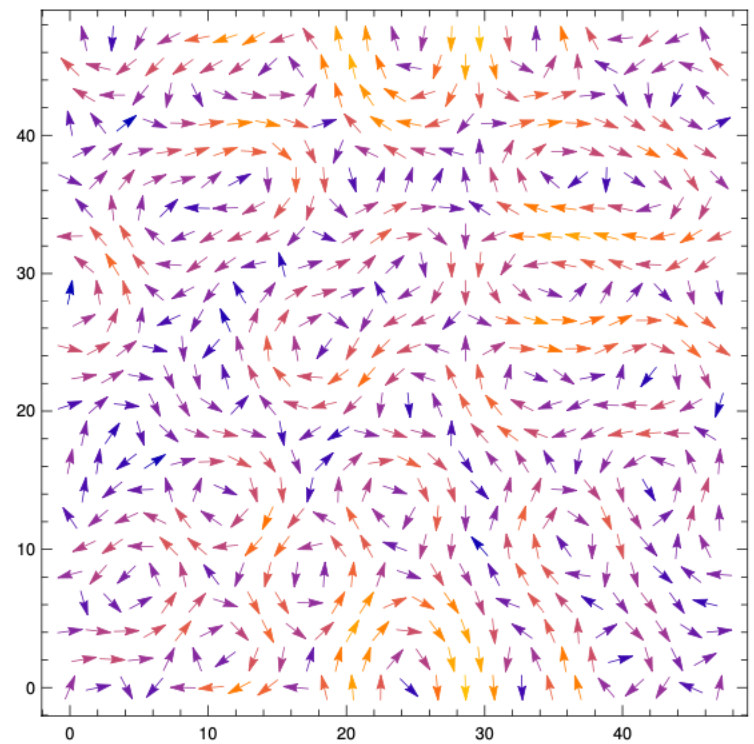
Crossovers! (no real thermodynamical singularity)

Important role of quantum fluctuations

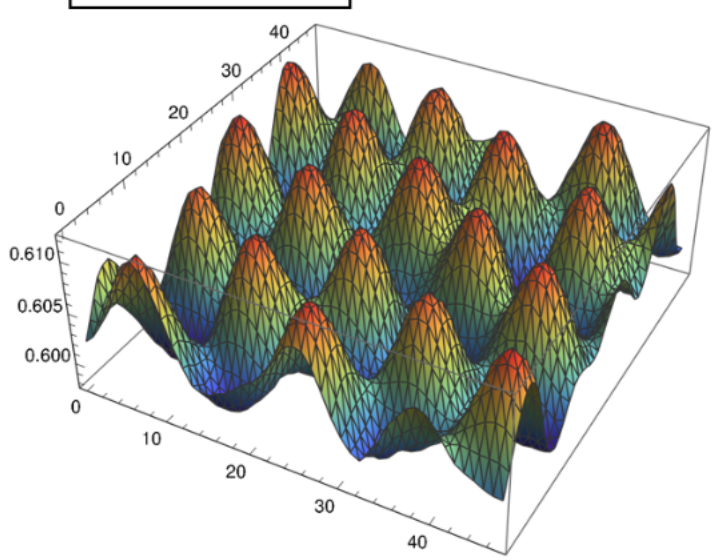
physical point

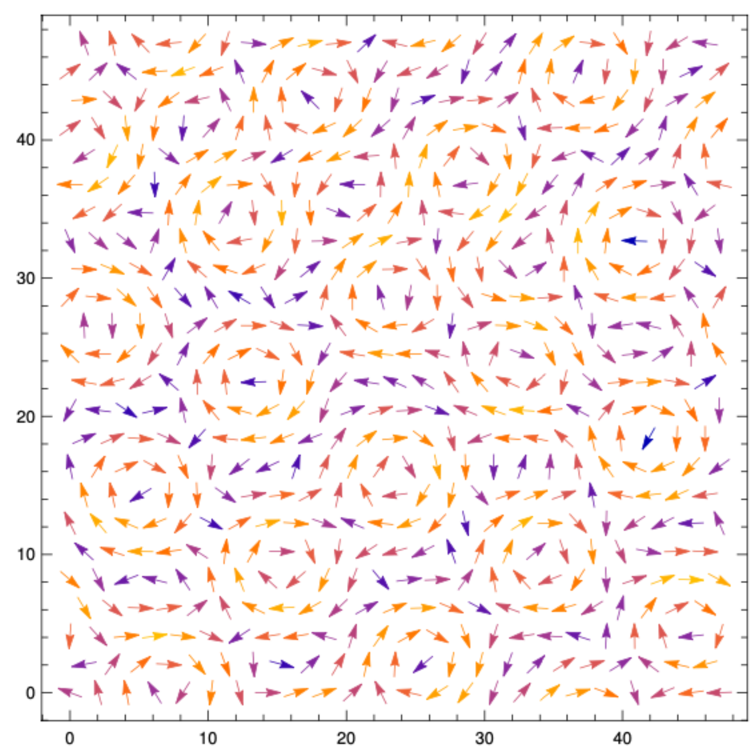
$\beta = 8$





$\beta = 20$

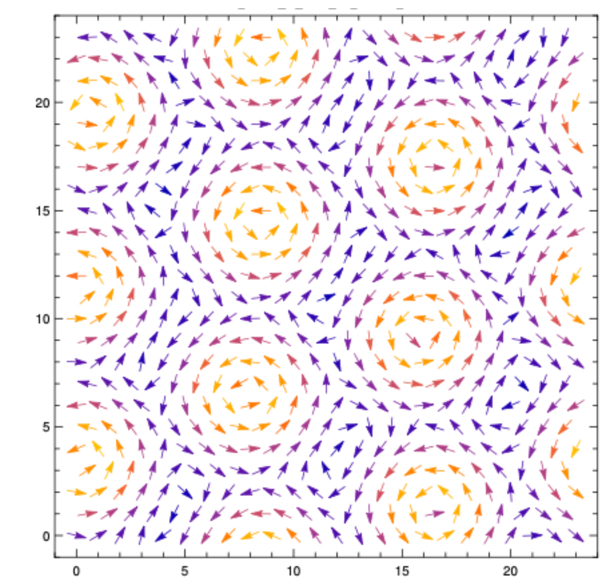




unphysical point (quantum fluctuations suppressed)

usual superconductor

VS



$\Leftarrow$ Do not form a lattice

Number of vortices: $\neq k \searrow 2k$

Hill VS pit

Deeper in phase with condensation exist lattice. $\nearrow 2k$

β	: 8	$\to$	20
$\sin^2\theta_W$	: 0.223	$\to$	0.417
$\frac{m_Z}{m_H}$	: 0.7335(9)	$\to$	0.609(3)
	physical		unphysical

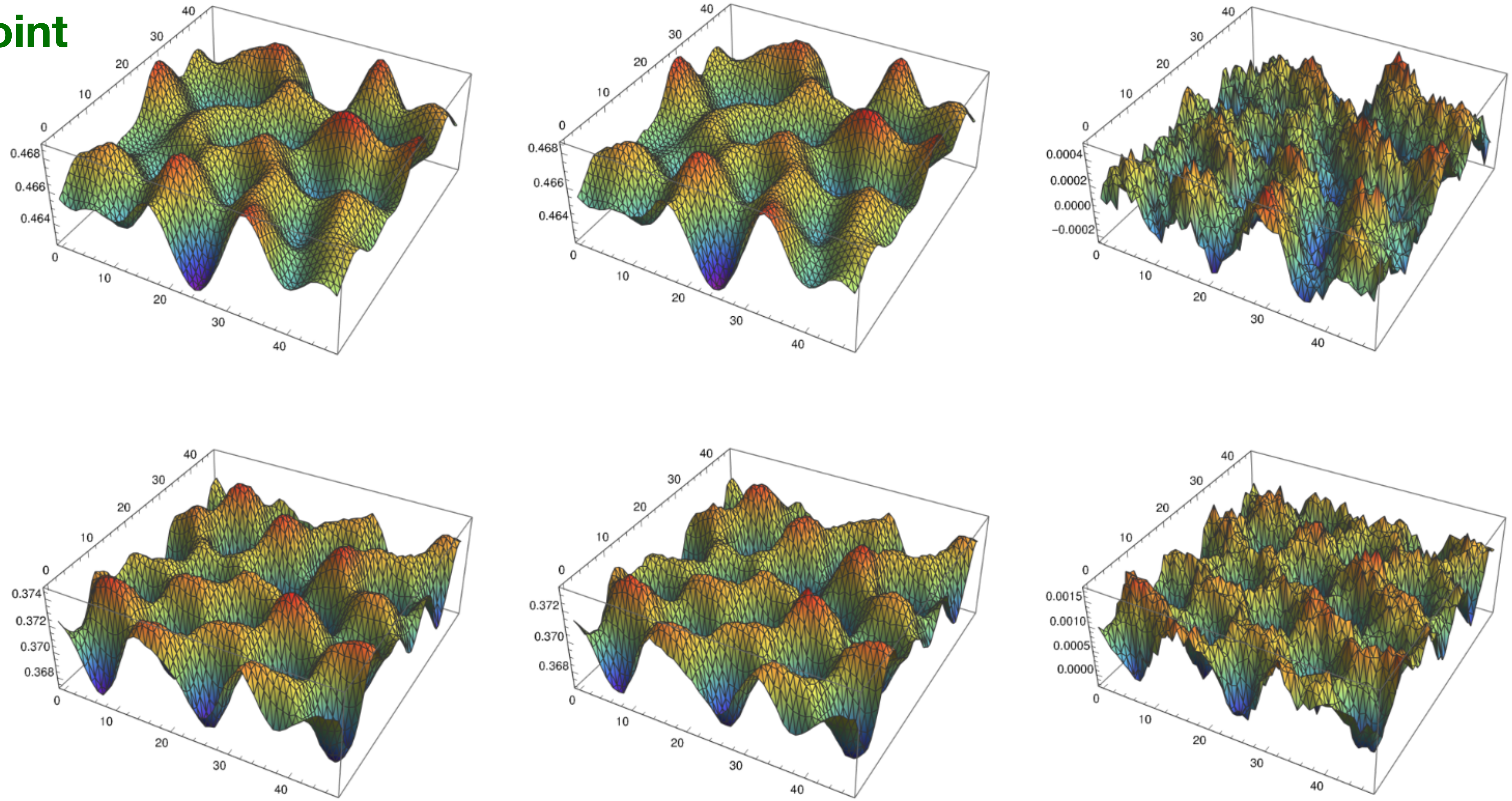
Charged W condensate

$\frac{1}{2}(W_x + W_y)$ | W_z | $\frac{1}{2}(W_x + W_y) - W_z$

physical point

$\beta = 8$

$\beta = 20$

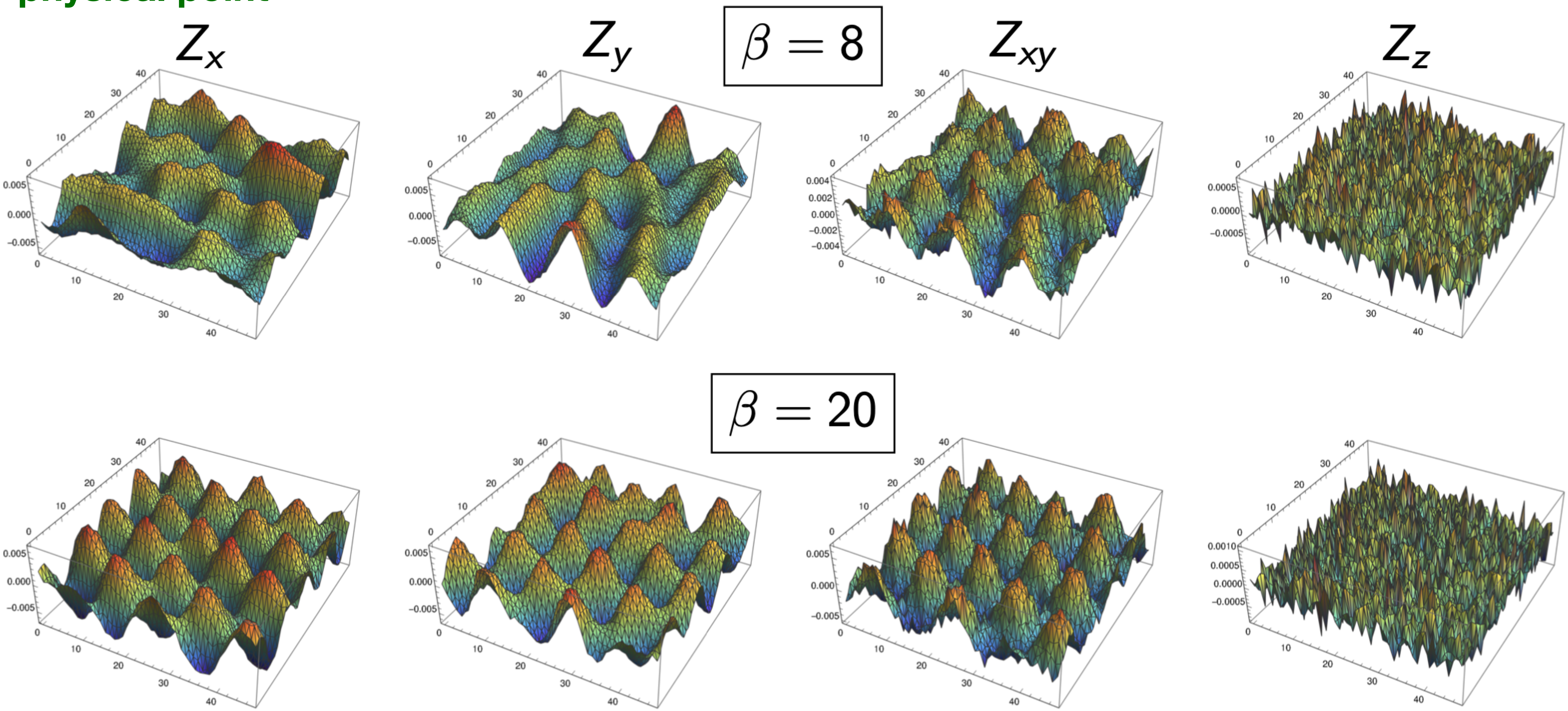


unphysical point

- the new phase does appear indeed
- vortices do emerge in the vacuum
- the charged condensates do emerge (superconductivity?)
- but the vortices form a liquid (or gas) rather than the crystal lattice
- and the vortices are not of the Ambjorn-Olesen type

Neutral Z condensate

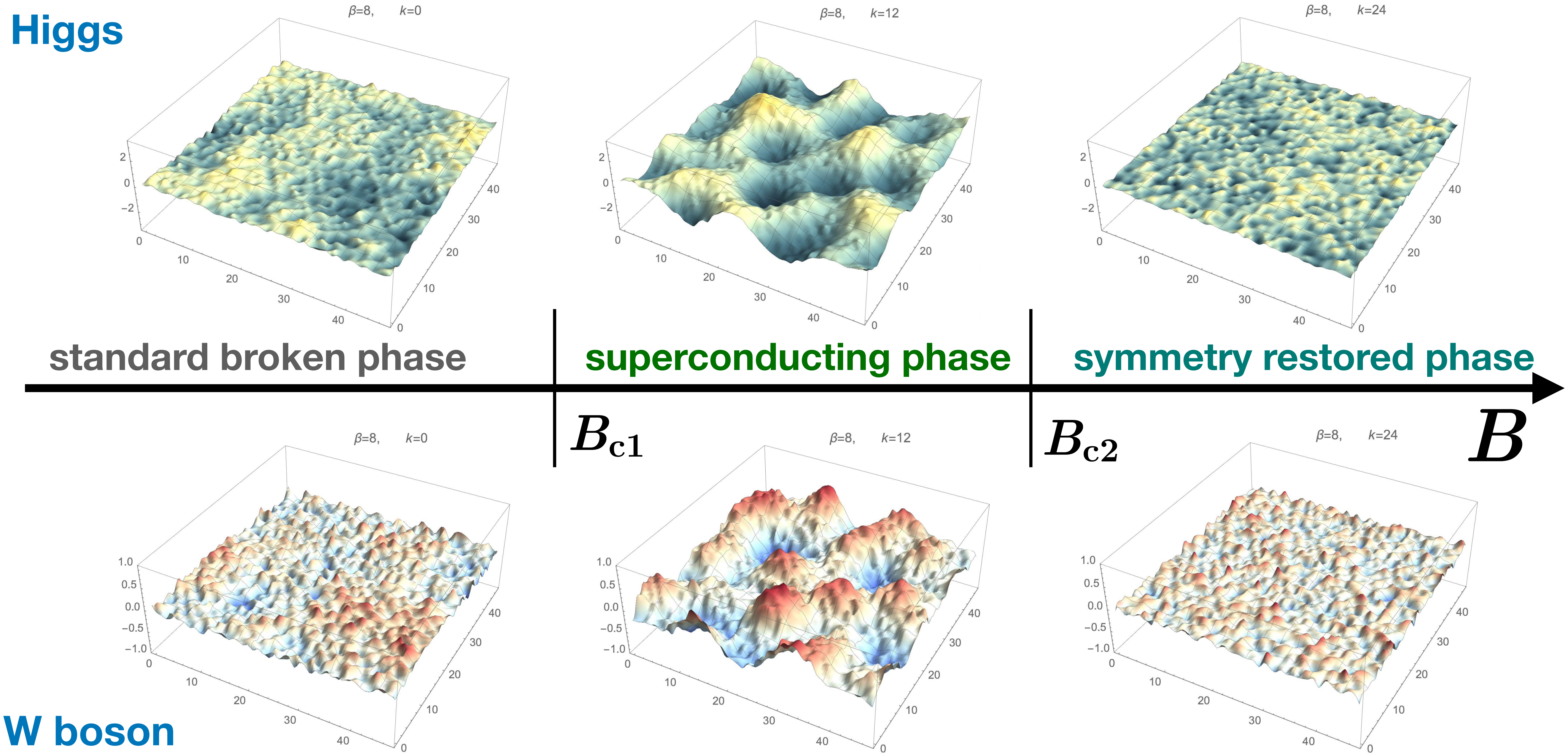
physical point



unphysical point

at the unphysical point we do see good lattice structure

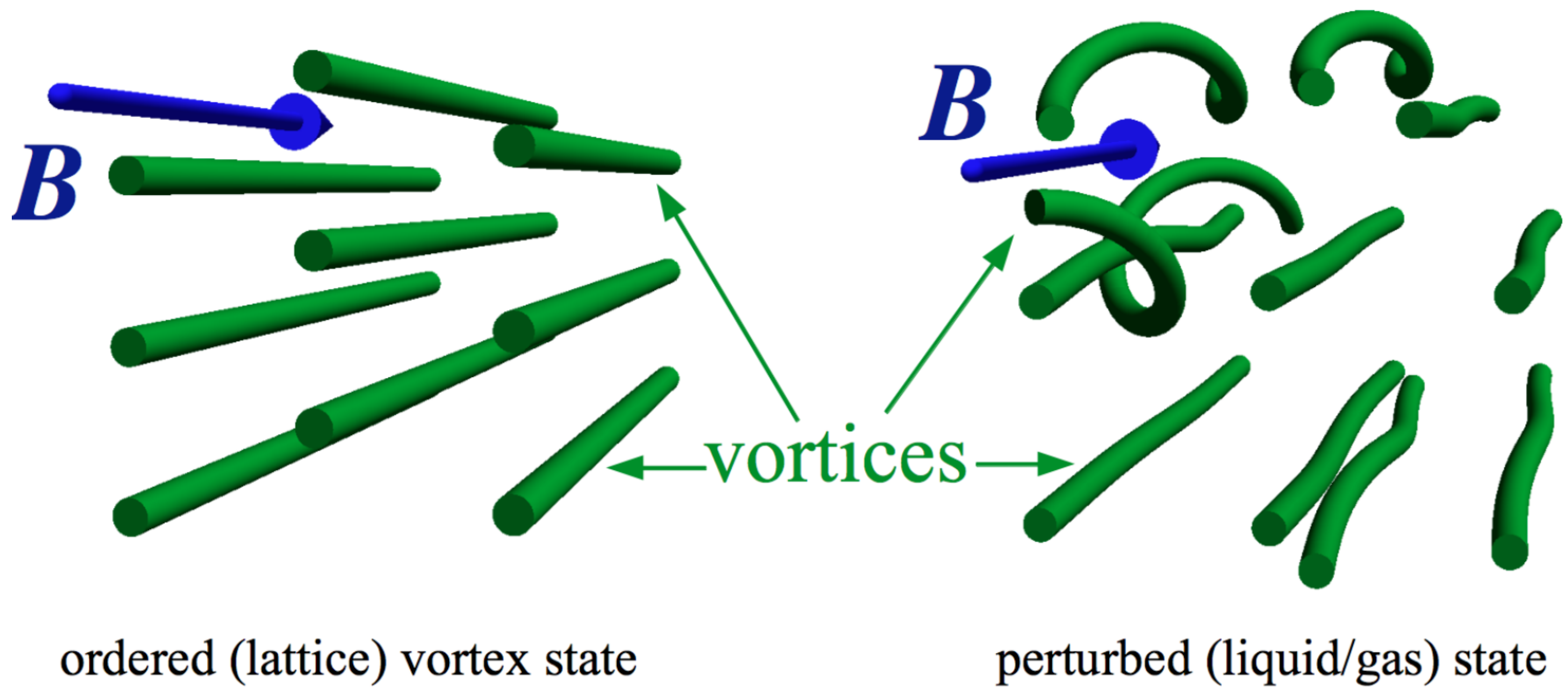
Vacuum structure: configurations



no lattice smearing, cooling or any other types of smoothening of configurations!

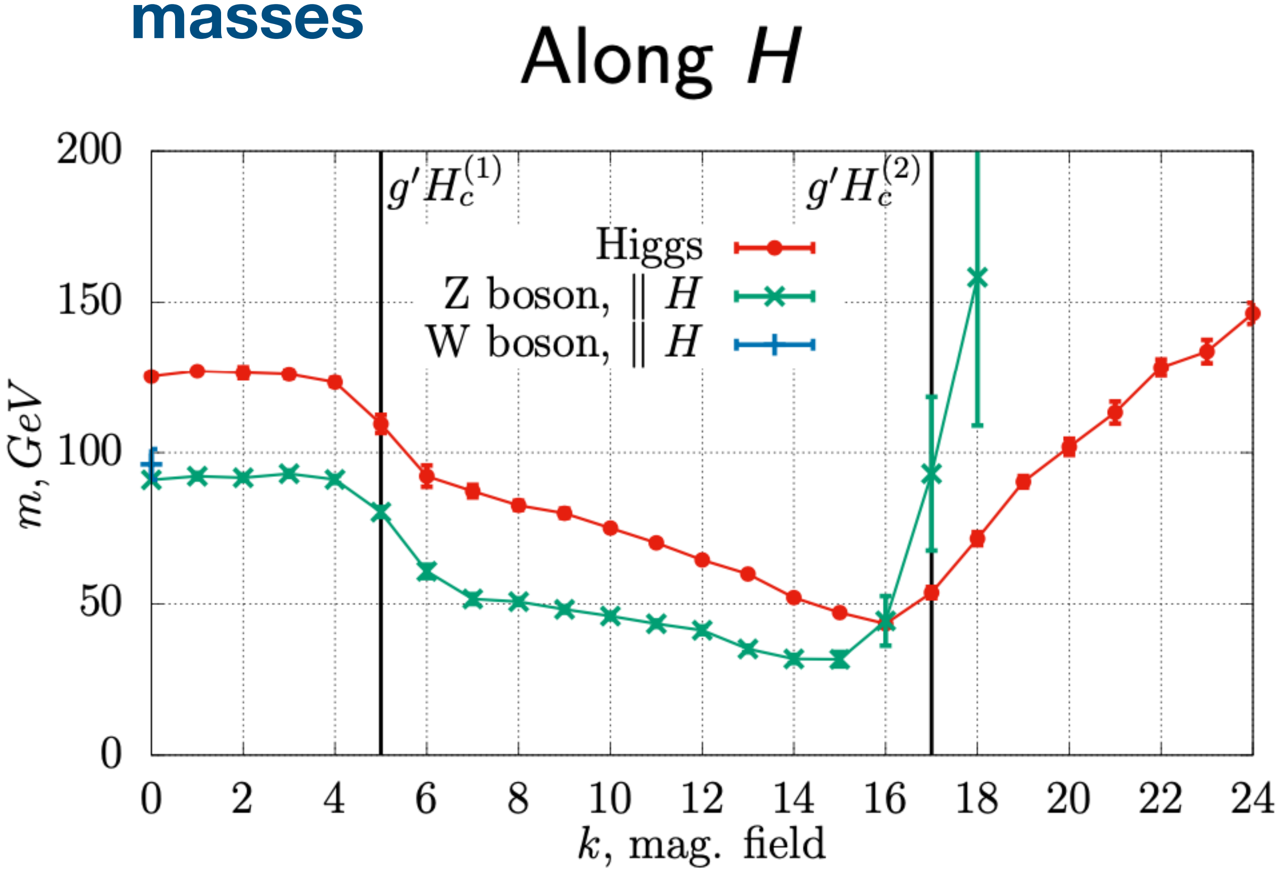
No vortex lattice

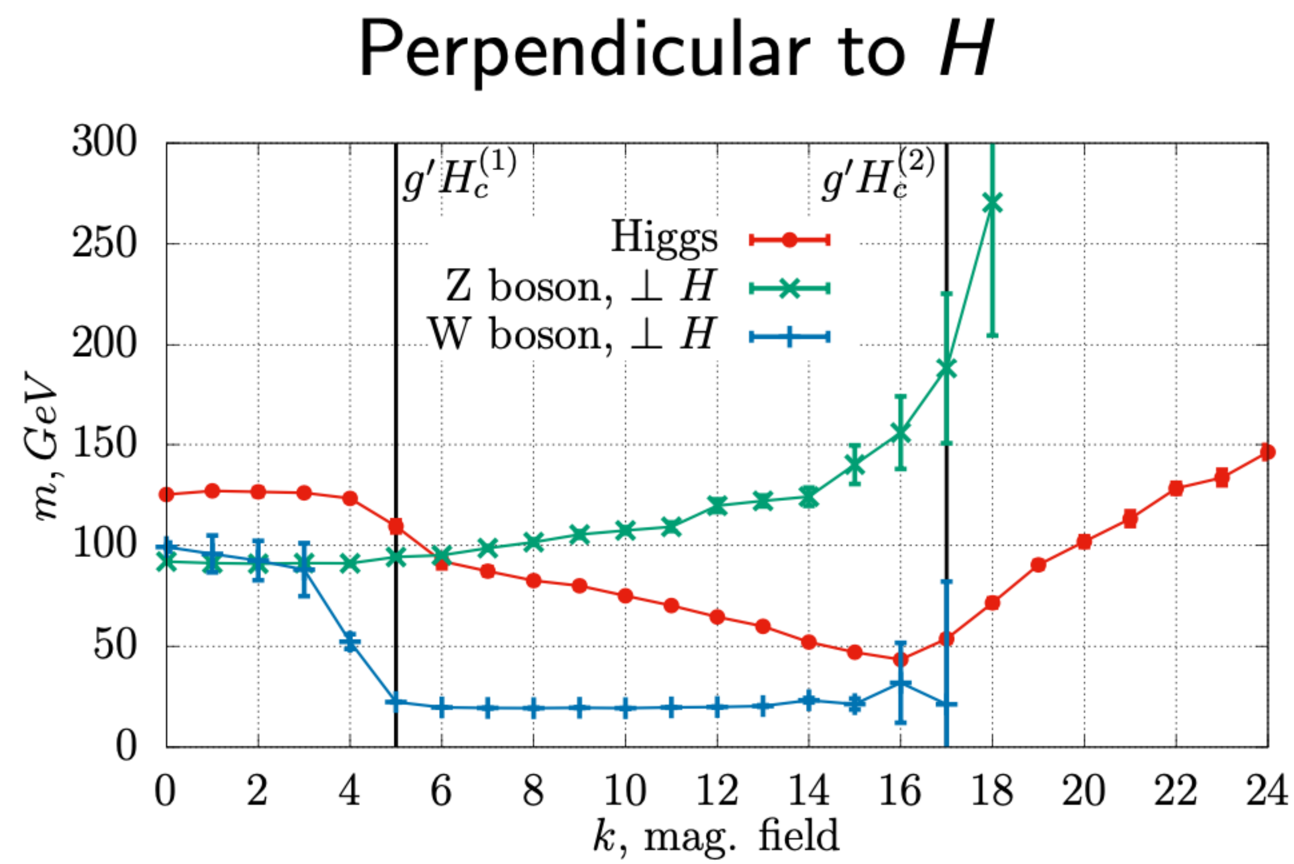
No clear vortex lattice at the physical point (at physical parameters)



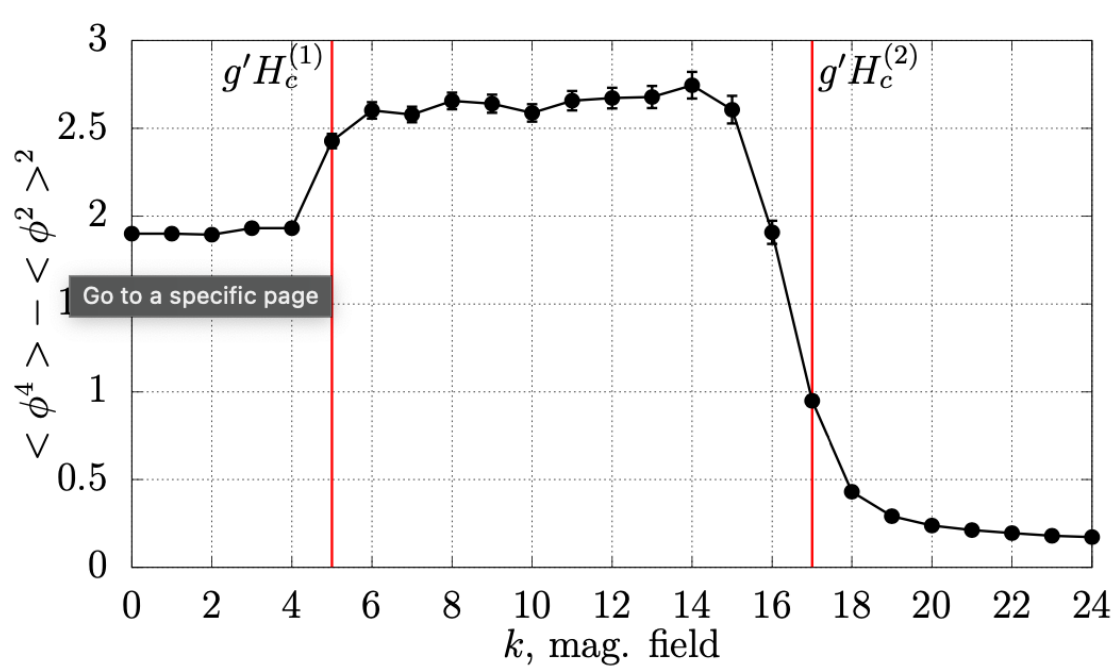
No clear thermodynamic phase transition(s)

masses





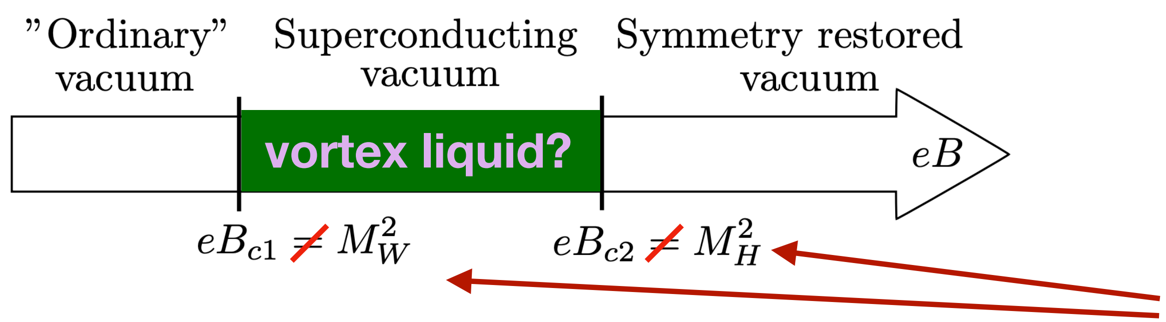
Higgs susceptibility

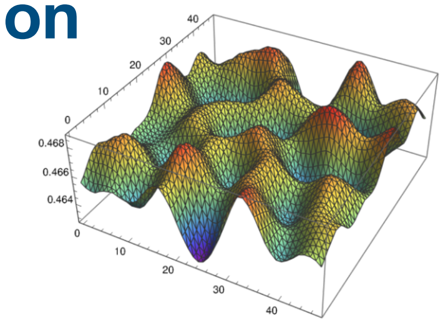


Conclusions

1. **We found the phase structure of zero-temperature electroweak theory in the magnetic-field background from first-principle lattice simulations**

2. **The phase structure is (partially) consistent with theoretical predictions based on solutions of classical equations of motion**





smooth crossovers

3. **Many differences with the theory, the role of quantum fluctuations is crucial:**

   — vortices are not of the Ambjorn-Olesen type
   — no crystal lattice formation (of the Abrikosov type)
   — the vortices form either gas or liquid (fluctuating vortex medium)
   — the transitions are not phase transitions but the smooth crossovers
   (difficult/impossible to see from thermodynamics)

4. **A similar phase in QCD at strong magnetic field? (no phase transition, a smooth appearance of the inhomogeneous phase).**

[Braguta et al. *PoS* LATTICE2013 (2014) 362]

quenched QCD