Quantum fluctuation of energy and its pseudo-gauge dependence in subsystems of hot relativistic gas

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Motivation

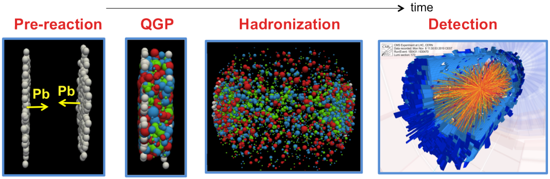


Figure: Simplified picture of heavy ion collisions [1]

- Heavy ion collision experiments: AGS at BNL:- 2 A GeV-10 A GeV, SPS at CERN:-20 A GeV-158 A GeV, RHIC at BNL:- 200 A GeV and LHC at CERN:-5.5 A TeV.
- At high temperature and/ densities the hadronic matter should under go a transition to a state of free quarks and gluons (QGP) [2].
- $(2\text{-}3)\text{GeV/fm}^3 \quad \epsilon_0 \quad (5 \quad 6)\text{GeV/fm}^3$ for SPS to RHIC energies[3].

[1] http://wl33.web.rice.edu/research.html
[2] E.V.Shuryak, Phys. Lett. B 78 (1978) 150
[3] J. D . Bjorken, Phys . Rev. D 27 (1983) 140.

- Standard model of the heavy ion collision [4; 5; 6]
  1. Modeling of the early stage.
  2. Hydrodynamic description of the space-time evolution of strongly interacting matter.
  3. Freeze-out of hadrons.
- Relativistic viscous hydrodynamics has become nowadays the basic theoretical tool for modeling relativistic heavy-ion collisions.
- Concepts used in hydrodynamics: energy density and pressure, both are defined locally – formally, the fluid cell has zero size.
- Interestingly, hydro models which are successful in explaining the experimental data can be used to conclude about the energy density attained in the collision processes.
- Is the energy density is a well defined concept for fluid cell of arbitrary size? classically?
- Does quantum fluctuation play any role?
- For fermionic systems energy momentum tensor calculated from Noether's theorem is not unique $\Rightarrow$ possible pseudo gauge dependence??

[4] P. Romatschke and U. Romatschke, PRL 99, 172301 (2007)
[5] C. Gale, S. Jeon, B. Schenke, Int.J.Mod.Phys.A 28 (2013) 1340011
[6] S. Jeon, U. Heinz, Int.J.Mod.Phys.E 24 (2015) 10, 1530010.

Variance of energy-momentum tensor

- For a real scalar field the canonical energy-momentum tensor is :

$$\hat{T}^{00} = \pi\phi \quad L \quad H. \tag{1}$$

In the Natural units, $\hbar = c = 1$, $[\hat{T}^{00}] = [L] = [M^d] = [H]$.

- We define a smeared operator,

$$A(a) = (a^{p}\overline{\pi})^{1 \quad d} \int d^{d \quad 1}\mathbf{x}\, H(0, \mathbf{x}) e^{\quad \mathbf{x}^2 = a^2}. \tag{2}$$

- Variance of the operator $A(a)$,

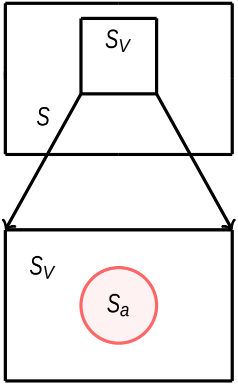
$$\text{var}\, A = hA^2 i \quad hAi^2 \quad =) \quad [\text{var}\, A] = [A]^2 = [H]^2 = [M]^{2d}. \tag{3}$$

- If we set,

$$\text{var}\, A(a) \quad a \quad =) \quad \beta = \quad 2d \tag{4}$$

- Therefore the fluctuations of the energy density grow rapidly at small distances [7].

[7] Quantum Field Theory: Lectures of Sidney Coleman



- S is closed/isolated system described by microcanonical ensemble.
- S_V is a sub system of the closed system in equilibrium, described by the canonical ensemble. Fluctuation in energy in S_V (large volume limit):

$$\sigma_H^2 = \frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2} = \frac{T^2 C_V}{V\varepsilon^2} \to 0. \quad (5)$$

  H is the Hamiltonian, T is temperature, ε is energy density, C_V is specific heat.
- S_a is a subsystem of S_V which is described by the "Gaussian box": $(a\sqrt{\pi})^{-3}\exp(-\mathbf{x}^2/a^2)$.

[8]

[8] Talk given by W. Florkowski at Workshop on QGP Phenomenology, Sharif University of Technology, Teheran, Iran May 28,2021

Quantum scalar field

- We describe our system by a quantum scalar field in thermal equilibrium[9].

$$\phi(t,\boldsymbol{x}) = \int \frac{d^3k}{\sqrt{(2\pi)^3\, 2\omega_{\boldsymbol{k}}}} \left(a_{\boldsymbol{k}} e^{-ik\cdot x} + a^{\dagger}_{\boldsymbol{k}} e^{ik\cdot x} \right);\ \ [a_{\boldsymbol{k}}, a^{\dagger}_{\boldsymbol{k}'}] = \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}') \tag{6}$$

Single particle energy: $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$, $a \cdot b = a^{\mu} b_{\mu} = a^0 b^0 - \boldsymbol{a} \cdot \boldsymbol{b}$.

- Hamiltonian density:

$$\mathcal{H} = \frac{1}{2}\left(\dot{\phi}^2 + (\boldsymbol{\nabla}\phi)^2 + m^2\phi^2 \right). \tag{7}$$

- We define an operator H_a for a *finite* subsystem S_a placed at the origin of the coordinate system,

$$H_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\, \mathcal{H}(x)\, \exp\left(-\frac{\boldsymbol{x}^2}{a^2} \right). \tag{8}$$

- Our objective:

$$\sigma^2(a,m,T) = \langle H_a H_a \rangle - \langle H_a \rangle^2,\ \ \sigma_n(a,m,T) = \frac{(\langle H_a H_a \rangle - \langle H_a \rangle^2)^{1/2}}{\langle H_a \rangle}. \tag{9}$$

[9] S. Coleman, Lectures of Sidney Coleman on Quantum Field Theory. WSP, Hackensack, 12, 2018.

- Normal ordering: Composite QFT operators $\rightarrow$ Some "Normal ordering" prescription required.

$$H_a \rightarrow :H_a: \tag{10}$$
$$H_a H_a \rightarrow :H_a::H_a: \tag{11}$$

- Alternative definition of normal ordering: $H_a H_a \rightarrow :H_a H_a:$
- To perform thermal averaging, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators [10, 11, 12]

$$\langle a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}'} \rangle = \delta^{(3)}(\mathbf{k} - \mathbf{k}') f(\omega_{\mathbf{k}}), \tag{12}$$
$$\langle a^{\dagger}_{\mathbf{k}} a^{\dagger}_{\mathbf{k}'} a_{\mathbf{p}} a_{\mathbf{p}'} \rangle = \Big(\delta^{(3)}(\mathbf{k} - \mathbf{p})\, \delta^{(3)}(\mathbf{k}' - \mathbf{p}') + \delta^{(3)}(\mathbf{k} - \mathbf{p}')\, \delta^{(3)}(\mathbf{k}' - \mathbf{p}) \Big) f(\omega_{\mathbf{k}}) f(\omega_{\mathbf{k}'})$$

The Bose–Einstein distribution function, $f(\omega_{\mathbf{k}}) = 1/(\exp[\beta\, \omega_{\mathbf{k}}] - 1)$.

- Any other combinations of two and four creation and/or annihilation operators can be obtained through the commutation relation between $a_{\mathbf{k}}$ and $a^{\dagger}_{\mathbf{k}}$.

[10] C. Cohen-Tannoudji, B. Diu, F. Laloë, and S. R. Hemley, Quantum mechanics: Vol. 3, Wiley, New York, 1977.

[11] T. Evans and D. A. Steer, Nucl. Phys. B 474 (1996) 481.

[12] C. Itzykson and J. Zuber, Quantum Field Theory. International Series In Pure and Applied Physics. McGraw-Hill, New York, 1980.

- The thermal expectation value of the operator H_a is

$$\langle : H_a : \rangle = \int \frac{d^3k}{(2\pi)^3}\, \omega_{\mathbf{k}}\, f\left(\omega_{\mathbf{k}}\right) \equiv \varepsilon(T) \quad \text{well known result} \tag{13}$$

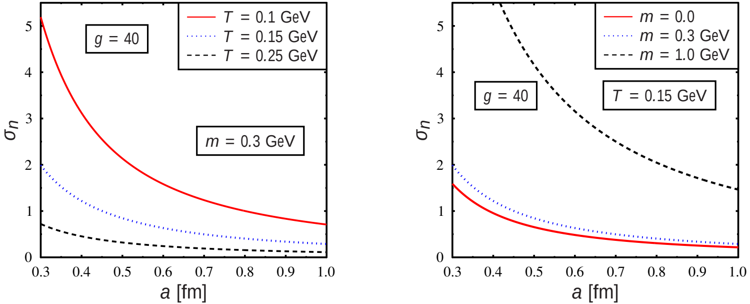
- Important new result: Fluctuation,

$$\sigma^2(a,m,T) = \int dK\, dK' f(\omega_{\mathbf{k}})(1 + f(\omega_{\mathbf{k}'})) \\ \left[(\omega_{\mathbf{k}}\omega_{\mathbf{k}'} + \mathbf{k}\cdot\mathbf{k}' + m^2)^2 e^{-\frac{a^2}{2}(\mathbf{k}-\mathbf{k}')^2} + (\omega_{\mathbf{k}}\omega_{\mathbf{k}'} + \mathbf{k}\cdot\mathbf{k}' - m^2)^2 e^{-\frac{a^2}{2}(\mathbf{k}+\mathbf{k}')^2}\right]. \tag{14}$$

here $dK = d^3k/((2\pi)^3 2\omega_{\mathbf{k}})$.

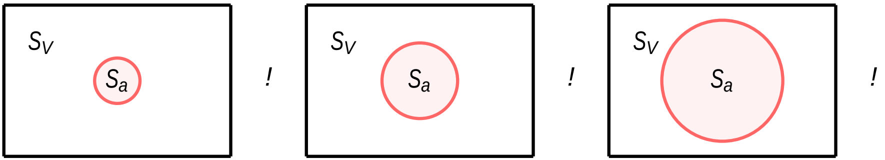
- All the vacuum energy term coming from the composite operator may not be removed by "Normal ordering".
- $\langle : H_a : \rangle$ is independent of the scale a, but the fluctuation $\sigma^2(a,m,T)$ depends on the scale.
- Degeneracy factor: $\varepsilon \rightarrow g\varepsilon, \quad \sigma^2 \rightarrow g\sigma^2$.

Temperature and mass dependence of $_n$



- With a possible interpretation of heavy-ion data in mind, we consider temperatures in the range 0.1 GeV $< T <$ 0.25 GeV, and particle masses: $m = 0, 0.3$ and 1.0 GeV.
- The value of g varies between 37 (for two quark flavor QGP) and 47.5 (for three quark flavor QGP). g 400 for Hadron gas, but high mass hadronic contribution is thermally suppressed.
- For high T and m, σ_n is small, but the fluctuation σ is large.

Thermodynamic limit



- $S_V : V = L^3$, S_a is characterised by scale a, $a < L$. L is large. Thermodynamic limit corresponds to $a \to \infty$ limit.
- Gaussian representation of the three dimensional Dirac delta function

$$\delta^{(3)}(\boldsymbol{k} - \boldsymbol{p}) = \lim_{a \to \infty} \frac{a^3}{(2\pi)^{3/2}} e^{-\frac{a^2}{2}(\boldsymbol{k} - \boldsymbol{p})^2}. \quad (15)$$

- This leads us to the formula valid in the large a limit

$$\sigma^2 \simeq \frac{g}{(2\pi)^{3/2} a^3} \int \frac{d^3k}{(2\pi)^3} \omega_{\boldsymbol{k}}^2 f(\omega_{\boldsymbol{k}})(1 + f(\omega_{\boldsymbol{k}})) \propto T^5/a^3 (\text{Massless limit}).$$

- For $a \to \infty$ limit σ^2 can be expressed in terms of the specific heat at constant volume,

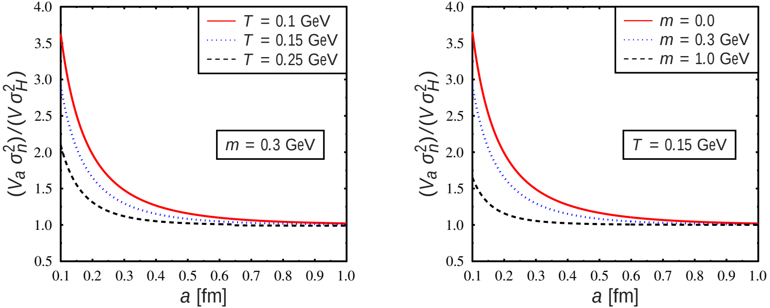
$$c_V = \frac{d\varepsilon}{dT} = \frac{g}{T^2}\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}^2 f(\omega_{\boldsymbol{k}})(1+f(\omega_{\boldsymbol{k}})). \tag{16}$$

- One obtains,

$$V_a\sigma_n^2 = \frac{T^2 c_V}{\varepsilon^2} = V\frac{\langle H^2\rangle - \langle H\rangle^2}{\langle H\rangle^2} \equiv V\sigma_H^2, \tag{17}$$

- $V_a = a^3(2\pi)^{3/2}$ can be considered as the volume of the subsystem S_a — a nontrivial factor of $(2\pi)^{3/2}$ is an artifact of using the “Gaussian” box.
- $V\sigma_H^2$ can be identified as the normalized energy fluctuation in the system S_V.
- This result can also be shown analytically but for mass less case and for Boltzmann statistics.

Approach to the thermodynamic limit



- Variation of $V_a\sigma_n^2/V\sigma_H^2$ with the size of the subsystem S_a in the case where particles have a non vanishing mass and they obey Bose-Einstein statistics.
- One expects that in the thermodynamic limit $V_a\sigma_n^2/V\sigma_H^2$ should approach unity.
- Quantum fluctuations agree with the thermodynamic ones already for $a > 1$ fm.
- Quantum fluctuations become very important at the scale of 0.1 fm.

System of fermions

- We describe our system by a spin-$1/2$ field in thermal equilibrium. The field operator has the standard form [13]

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}}\left(U_r(\boldsymbol{k})a_r(\boldsymbol{k})e^{-ik\cdot x} + V_r(\boldsymbol{k})b_r^{\dagger}(\boldsymbol{k})e^{ik\cdot x}\right), \tag{18}$$

- The canonical anti-commutation relations,

$$\{a_r(\boldsymbol{k}), a_s^{\dagger}(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \tag{19}$$

$$\{b_r(\boldsymbol{k}), b_s^{\dagger}(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') \tag{20}$$

- Normalization of Dirac spinors,

$$U_r(\boldsymbol{k})U_s(\boldsymbol{k}) = 2m\delta_{rs} \tag{21}$$

$$V_r(\boldsymbol{k})V_s(\boldsymbol{k}) = -2m\delta_{rs} \tag{22}$$

[13] L. Tinti and W. Florkowski, arXiv:2007.04029

- To perform thermal averaging:

$$\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}), \tag{23}$$

$$\begin{aligned}
&\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle \\
&= (2\pi)^6 \Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}) \\
&\quad - \delta_{rr'}\delta_{ss'}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}')\Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}).
\end{aligned} \tag{24}$$

  Here $f(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta(\omega_{\boldsymbol{k}} - \mu)] + 1)$ is the Fermi–Dirac distribution function for particles.
- Anti particle operators also satisfies similar relation.
- For antiparticles, the Fermi–Dirac distribution function differs by the sign of the chemical potential μ, i.e. $f(\omega_{\boldsymbol{k}}) = 1/(\exp[\beta(\omega_{\boldsymbol{k}} + \mu)] + 1)$
- We consider a case with zero baryon chemical potential.

- Contrary to the real scalar field, the canonical energy momentum tensor operator is not symmetric,

$$\hat{T}_{\text{Can}} = \frac{i}{2}\psi\gamma \quad \overleftrightarrow{\partial} \quad \psi \quad g \quad L_{\text{D}} = \frac{i}{2}\psi\gamma \quad \overleftrightarrow{\partial} \quad \psi, \quad \overleftrightarrow{\partial} \qquad \overleftrightarrow{\partial} \qquad \partial \tag{25}$$

Here L_{D} denotes the Lagrangian density of a spin-1/2 field, which can be expressed as

$$L_{\text{D}} = \frac{i}{2}\psi\gamma \quad \overleftrightarrow{\partial} \quad \psi \qquad m\psi\psi, \tag{26}$$

- Mathematically, for any original energy-momentum tensor $\hat{T}$ satisfying $\partial \; \hat{T} = 0$ we can construct a different one by adding the divergence of an antisymmetric object, namely [14]

$$\hat{T}^{0} = \hat{T} + \partial \hat{A} \quad ; \quad \hat{A} = \hat{A} \tag{27}$$

- By construction, the new tensor is also conserved, i.e., $\partial \; \hat{T}^{0} = 0$.
- For spin $1/2$ field the energy momentum tensor is "Pseudo-gauge" dependent.

[14] E. Speranza and N. Weickgenannt, “Spin tensor and pseudo-gauges: from nuclear collisions to gravitational physics,” arXiv:2007.00138 [nucl-th].

- Belinfante-Rosenfeld framework (BR):

$$\hat{T}_{\rm BR} = \frac{i}{2}\psi\gamma \quad \overleftrightarrow{\partial} \quad \psi \qquad \frac{i}{16}\partial \left(\psi\left\{\gamma \;, \left[\gamma \;, \gamma \;\right]\right\}\psi\right). \tag{28}$$

- de Groot-van Leeuwen-van Weert framework (GLW)[15]:

$$\begin{aligned} \hat{T}_{\rm GLW} &= \quad \frac{1}{4m}\psi \overleftrightarrow{\partial} \quad \overleftrightarrow{\partial} \quad \psi \quad g \quad L_{\rm D} \\ &= \frac{1}{4m}\Big[\quad \psi(\partial \;\; \partial \;\; \psi) + (\partial \;\; \psi)(\partial \;\; \psi) + (\partial \;\; \psi)(\partial \;\; \psi) \\ &\qquad (\partial \;\; \partial \;\; \psi)\psi\Big]. \end{aligned} \tag{29}$$

- Hilgevoord-Wouthuysen framework (HW)[16]:

$$\begin{aligned} \hat{T}_{\rm HW} &= \hat{T}_{\rm Can} + \frac{i}{2m}\left(\partial \;\; \psi\sigma \quad \partial \;\; \psi + \partial \;\; \psi\sigma \quad \partial \;\; \psi\right) \\ &\quad \frac{i}{4m}g \quad \partial \;\; \left(\psi\sigma \quad \overleftrightarrow{\partial} \quad \psi\right), \end{aligned} \tag{30}$$

[15] S. R. De Groot, Relativistic Kinetic Theory. Principles and Applications. 1980.

[16]. Hilgevoord and S. Wouthuysen, Nucl. Phys. 40 (1963) 1-12; J. Hilgevoord and E. De Kerf, Physica 31 No.7 (1965) 1002-1016

Energy density is pseudo-gauge independent

- We define an operator, $\hat{T}^{00}_a$:

$$\hat{T}^{00}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\, \hat{T}^{00}(x)\, \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right). \tag{31}$$

- We consider the variance

$$\sigma^2(a,m,T) = \langle :\hat{T}^{00}_a::\hat{T}^{00}_a: \rangle - \langle :\hat{T}^{00}_a: \rangle^2 \tag{32}$$

and the normalized standard deviation

$$\sigma_n(a,m,T) = \frac{\left(\langle :\hat{T}^{00}_a::\hat{T}^{00}_a: \rangle - \langle :\hat{T}^{00}_a: \rangle^2\right)^{1/2}}{\langle :\hat{T}^{00}_a: \rangle}. \tag{33}$$

- Mean/thermal averaged $\hat{T}^{00}_a$:

$$\langle :\hat{T}^{00}_{\text{Can},a}: \rangle = 4 \int \frac{d^3k}{(2\pi)^3}\, \omega_{\boldsymbol{k}}\, f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\text{Can}}(T) \tag{34}$$

$$= \langle :\hat{T}^{00}_{\text{BR},a}: \rangle = \langle :\hat{T}^{00}_{\text{GLW},a}: \rangle = \langle :\hat{T}^{00}_{\text{HW},a}: \rangle \tag{35}$$

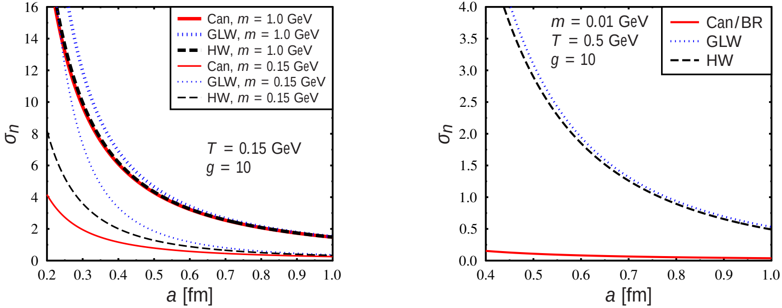
Energy density fluctuation– pseudo-gauge dependent

- Contrary to energy density the energy density fluctuation is pseudo-gauge dependent, e.g.
- For the Canonical framework:

$$\sigma^2_{\rm Can}(a,m,T) = 2\int dK\, dK^\prime f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}^\prime}))$$
$$\Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}^\prime})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}^\prime}+\boldsymbol{k}\cdot\boldsymbol{k}^\prime+m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}^\prime)^2}$$
$$-(\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}^\prime})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}^\prime}+\boldsymbol{k}\cdot\boldsymbol{k}^\prime-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}^\prime)^2}\Big], \qquad (36)$$

- For the de Groot-van Leeuwen-van Weert framework:

$$\sigma^2_{\rm GLW}(a,m,T) = \frac{1}{2m^2}\int dK\, dK^\prime f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}^\prime}))$$
$$\Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}^\prime})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}^\prime}-\boldsymbol{k}\cdot\boldsymbol{k}^\prime+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}^\prime)^2}$$
$$-(\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}^\prime})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}^\prime}-\boldsymbol{k}\cdot\boldsymbol{k}^\prime-m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}^\prime)^2}\Big]. \qquad (37)$$



- A comparison of the normalized standard deviation of fluctuations obtained for three different pseudo-gauges (Can=BR, GLW, HW).
- For $a < 0.5$ fm, we observe that the results obtained with various pseudo-gauges differ, with differences growing as a decreases.
- Irrespective of the choice of pseudo-gauges with growing system size the normalized standard deviation of fluctuations (σ_n) decreases.

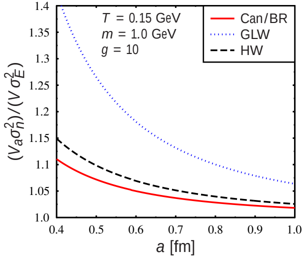
What about thermodynamic limit??

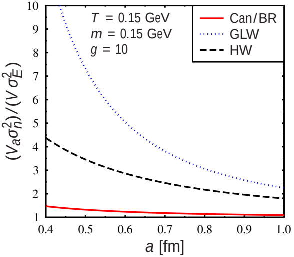
- Using the Gaussian representation of the Dirac delta function it can be shown that, in the large a limit,

$$\sigma^2_{\mathrm{Can}} = \frac{4\,g}{(2\pi)^{3/2}a^3}\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}^2\, f(\omega_{\boldsymbol{k}})(1 \quad f(\omega_{\boldsymbol{k}})) = \sigma^2_{\mathrm{BR}} = \sigma^2_{\mathrm{GLW}} = \sigma^2_{\mathrm{HW}}. \quad (38)$$

- In the large a limit we find,

$$V_a\sigma_n^2 = \frac{T^2 c_V}{\varepsilon^2} = V\frac{\langle E^2\rangle \quad \langle E\rangle^2}{\langle E\rangle^2} \quad V\sigma_E^2; \quad V_a = a^3(2\pi)^{3/2}. \quad (39)$$





Conclusions

- We have derived the formula characterizing the quantum fluctuation of energy in subsystems of a hot relativistic gas.
- It agrees with the expression for thermodynamic fluctuations, if the size of the subsystem is sufficiently large.
- For smaller sizes the effects of quantum fluctuations become relevant and the classical description with "well defined energy density" makes sense only after coarse graining over sufficiently large scale.
- For fermions quantum fluctuation of energy density does depend on the choice of the pseudo-gauge.
- On the practical side, the results of our calculations can be used to determine a scale of coarse graining for which the choice of the pseudo-gauge becomes irrelevant.
- This may be useful, in particular, in the context of hydrodynamic modeling of high-energy collisions.
- These results might be relevant for small systems.

Thank You!

Normal ordering: alternative approach

- For a composite operator we considered the following normal ordering method: $H_aH_a \to :H_a::H_a:$.
- Therefore we are first normal ordering first then then multiplying to construct the composite operator.
- Alternatively one can also argue about different method of normal ordering:

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Semiclassical gravity theory and quantum fluctuations

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$$\Delta(x) \equiv \left| \frac{\langle :T_{00}{}^2(x): \rangle - \langle :T_{00}(x): \rangle^2}{\langle :T_{00}{}^2(x): \rangle} \right| .$$

- If we consider such a normal ordering then:

$$\sigma^2(a,m,T) = \langle :H_aH_a: \rangle - \langle :H_a: \rangle^2 = \int dK\, dK' f(\omega_{\mathbf{k}}) f(\omega_{\mathbf{k}'})$$

$$\left[(\omega_{\mathbf{k}}\omega_{\mathbf{k}'} + \mathbf{k}\cdot\mathbf{k}' + m^2)^2 e^{-\frac{a^2}{2}(\mathbf{k}-\mathbf{k}')^2} + (\omega_{\mathbf{k}}\omega_{\mathbf{k}'} + \mathbf{k}\cdot\mathbf{k}' - m^2)^2 e^{-\frac{a^2}{2}(\mathbf{k}+\mathbf{k}')^2} \right]. \quad (40)$$