Entropy wave instability in Dirac and Weyl semimetals

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P. O. Sukhachov, E. V. Gorbar, and I. A. Shovkovy, Entropy wave instability in Dirac and Weyl semimetals, arXiv:2106.11992

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$H_0(k) = \begin{pmatrix} v_F \sigma \cdot (k - b) + b_0 & 0 \\ 0 & -v_F \sigma \cdot (k + b) - b_0 \end{pmatrix}$. 

Chiral shift parameter $-b \cdot \vec{\gamma} \gamma_5$
Electron hydrodynamics

- Two types of collisions: momentum-relaxing ($l_{MR}$) and momentum-conserving ($l_{MC}$).

Nonhydrodynamic regimes: $l_{MR} \ll l_{MC}, L$, where $L$ is the sample size.

Hydrodynamic regime: $l_{MC} \ll L \ll l_{MR}$.


[J. Zaanen, Science 351, 1058 (2016)]
Experimental observations

❖ **Gurzhi effect** in 2D electron gas of (Al,Ga)As heterostructures

❖ Viscous contribution to the resistance of 2D metal PdCoO$_2$
   (Poiseuille flow) [P.J.W. Moll et al., Science **351**, 1061 (2016)]

   - Higher than ballistic transport in constrictions [H. Guo et al., PNAS **114**, 3068 (2017); R. Krishna Kumar et al., Nat. Phys. **13**, 1182 (2017)]
   - Visualization of the Poiseuille flow via the Hall field profile
     [J.A. Sulpizio et al., Nature **576**, 75 (2019)]
Backflows in graphene and GaAs

Whirlpools

Negative potential regions

\[ \eta = 0 \]

\[ \eta \neq 0 \]


[D.A. Bandurin et al., Science 351, 1055 (2016)]

Dyakonov-Shur instability

\[ \partial_t u + u \partial_x u = -\frac{e}{m} \partial_x \varphi \]

\[ \partial_t \varphi + \partial_x (\varphi u) = 0 \]

Gradual channel approximation: \( n \propto \varphi \)

\[ \delta \varphi(x = 0) = 0 \]

\[ \delta J(x = L) = 0 \]

\[ \text{Re} [\omega] = \frac{|v_p^2 - u_0^2|}{2Lv_p} \ln \left| \frac{v_p + u_0}{v_p - u_0} \right| \]

\[ \text{Im} [\omega] = \frac{v_p^2 - u_0^2}{2Lv_p} \ln \left| \frac{v_p + u_0}{v_p - u_0} \right| \]

Heuristic argument

[Plasma wave reflection:]

\[ R = \begin{cases} 1 & \text{if } v_p + u_0 > v_p - u_0 \\ \frac{v_p + u_0}{v_p - u_0} & \text{otherwise} \end{cases} \]

\[ t_0 = \frac{L}{v_p + u_0} + \frac{L}{v_p - u_0} \]

\[ n(t) \propto \left( \frac{v_p + u_0}{v_p - u_0} \right)^{t/t_0} \]

\[ \text{Im} \left[ \omega \right] = \frac{v_p^2 - u_0^2}{2Lv_p} \ln \left| \frac{v_p + u_0}{v_p - u_0} \right| \]

\[ \text{Gate} \quad \varphi \]

\[ \text{2D electron gas} \]

Source \[ J_0 \]

\[ L \]

Drain \[ J_0 \]

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Application of the instability

Experimental observation:

- Emission mode [x]
- Detection mode [v]

Hydrodynamics in Weyl semimetals


\[ \rho = \rho_0 + \rho_1 w^\beta \]

\[ L = \frac{\kappa \rho}{T}, \quad L_0 = \frac{\pi^2 k_B^2}{3e^2} \]
Relativistic-like electron hydrodynamics

\[
\frac{1}{v_F^2} \left[ \partial_t + (\mathbf{u} \cdot \nabla) \right] (\mathbf{u} w) + \frac{1}{v_F^2} w \mathbf{u} (\nabla \cdot \mathbf{u}) = -\nabla P + e n \nabla \varphi + \eta \Delta \mathbf{u} + \frac{\eta}{d} \nabla (\nabla \cdot \mathbf{u}) - \frac{w \mathbf{u}}{v_F^2 \tau},
\]

\[-e \partial_t n + (\nabla \cdot \mathbf{J}) = 0,\]

\[\partial_t \epsilon + (\nabla \cdot \mathbf{J}^\epsilon) = (\mathbf{E} \cdot \mathbf{J}),\]

\[\Delta \varphi = 4\pi e (n - n_0).\]

Linearization:

\[u_x(t, \mathbf{r}) = u_0 + u_1 e^{-i\omega t + i\mathbf{k} \cdot \mathbf{r}}.
\]

Collective modes in an infinite system:

3D:
\[\omega_\pm \approx \pm \sqrt{\omega_p^2 + v_s^2 k_x^2} + \frac{2}{3} u_0 k_x,\]

2D:
\[\omega_\pm \approx \pm v_p k_x + \frac{1}{2} u_0 k_x,\]

2D and 3D:
\[\omega_e \approx u_0 k_x.\]

Electric and energy currents:

\[\mathbf{J} = -en\mathbf{u},\]

\[\mathbf{J}^\epsilon = w\mathbf{u}.\]

Entropy wave

Plasmons

\[n_1/n_0 \approx - \left( \frac{v_s k_x}{\omega_p} \right)^2 \frac{\epsilon_1}{w_0}, \quad u_1 \approx 0.\]
Instability in 3D: analytical results

Boundary conditions:

\[ n_1(x = 0) = 0, \]
\[ J_x(x = L) \equiv n_0 u_1(x = L) + u_0 n_1(x = L) = 0, \]
\[ T_1(x = 0) = 0. \]

Frequencies of the collective modes:

\[ \omega^{3D}_{\pm} \approx \pm \sqrt{\omega_p^2 + \left[ v_s \frac{\pi}{L} \left( l + \frac{1}{2} \right) \right]^2 + \frac{i 2u_0}{3L} (3 - 2\Lambda_p^2)} \]

\[ \omega^{3D}_e \approx \frac{2\pi l}{L} u_0 - i \frac{u_0 \omega_p}{v_s} - i \frac{u_0}{L} \ln \left[ \frac{3}{8} \frac{v_s^2}{u_0^2 \left( 1 - \Lambda_p^2 \right)} \right] \]

\[ l = 0, \pm 1, \pm 2, \ldots \]

\[ \Lambda_p = \omega_p / (v_s q_{TF}) < 1, \quad \lim_{T \to 0} \Lambda_p = 1. \]
Instability in 3D: numerical results

Plasmons and entropy wave instabilities occur for different directions of the background current.

Growth rate for the entropy wave is much larger than that for plasmons.
Instability in 2D

\[ \omega_{\pm}^{2D} \approx \pm v_p \frac{\pi}{L} \left( l + \frac{1}{2} \right) + i \frac{u_0}{2L} \left( 4 - 3 \Lambda_p^2 \right) \]


\[ \omega_e^{2D} \approx \frac{2\pi l}{L} u_0 - i \frac{u_0}{L} \ln \left[ \frac{2}{3} \frac{v_p^2}{u_0^2 \left( 1 - \Lambda_p^2 \right)} \right] \]
1. Unlike the mass flow in conventional fluids, hydrodynamic flow in relativistic-like systems is an energy flow.

2. Under specific boundary conditions plasmons become unstable allowing for the emission of electromagnetic radiation (Dyakonov-Shur instability).

3. In systems with the relativistic-like dispersion relation, the entropy wave also becomes unstable leading to a novel entropy-wave instability.

4. Since the frequency of the entropy wave depends strongly on the background flow velocity (electric current), they might provide a tunable source of radiation.