Strong inhibition of convection in Dirac and Weyl semimetals

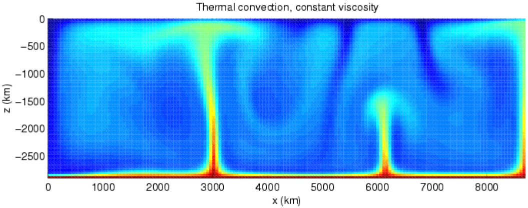
E.V. Gorbar

Taras Shevchenko National University of Kyiv
Bogolyubov Institute for Theoretical Physics

P.O. Sukhachov, E.V.G., I.A. Shovkovy, arXiv:2103.15836

ICNFP, August 31, 2021

Convective instability



Nusselt number, 1-10 for laminar and 100-1000 for turbulent flow

$$Nu = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} = \frac{Q_{conv}}{Q_{cond}}$$

Fourier's law

$$\mathbf{q} = -k\nabla T \qquad \rightarrow \qquad Q_{cond} = kA(T_R - T_L)$$

Large ∇T near boundaries, convection is a heat conveyor belt

Dirac and Weyl Hamiltonians

Dirac Hamiltonian (1928)

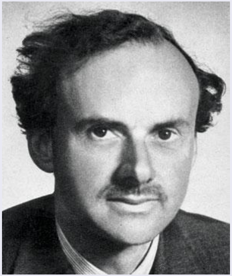


Figure: Paul Adrien Maurice Dirac (1902-1984)

$$\mathcal{H}_D = \begin{pmatrix} v\sigma\cdot\mathbf{k} & -m \\ -m & -v\sigma\cdot\mathbf{k} \end{pmatrix}$$

Weyl Hamiltonian (1929)

Figure: Hermann Klaus Hugo Weyl (1885-1955)

$$\mathcal{H}_W = \pm v\sigma\cdot\mathbf{k}$$

Band structure of Dirac and Weyl semimetals

Nielsen-Weyl theorem: Weyl nodes occur in pairs of opposite chirality

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Hamiltonian of Dirac ($\mathbf{b} = 0$, $b_0 = 0$) and Weyl semimetals ($\mathbf{b} \neq 0$ or $b_0 \neq 0$):

$$\mathcal{H} = \begin{pmatrix} v\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{b}) + b_0 & 0 \\ 0 & -v\boldsymbol{\sigma}\cdot(\mathbf{k}+\mathbf{b}) - b_0 \end{pmatrix},$$

$\mathbf{b}$ – momentum space separation,

b_0 – separation in energy

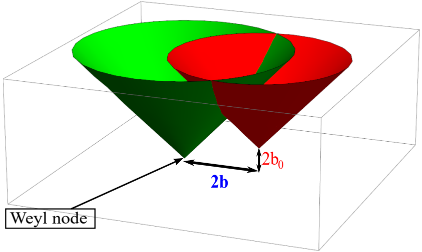
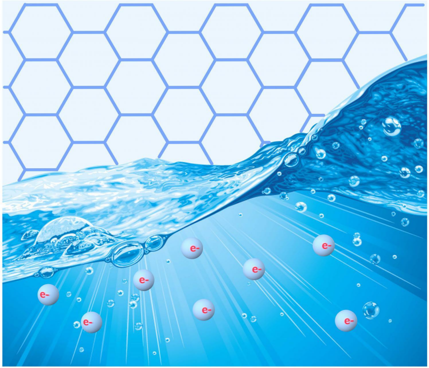


Figure: Energy spectrum of Weyl semimetal

Electron hydrodynamics in solids

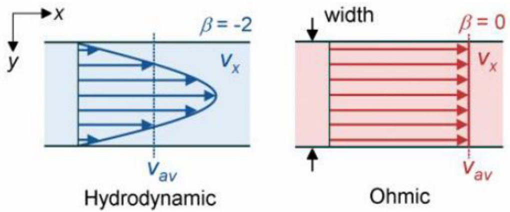


Drude regime of transport: momentum-relaxing collisions (e-impurities, e-phonons) dominate

Hydro regime: momentum-conserving collisions (e-e) dominate

Electron hydrodynamics is realized for $l_{ee} \ll L \ll l_{imp}$

Experimental observations



Gurzhi effect in 2D electron gas of (Al,Ga)As heterostructures

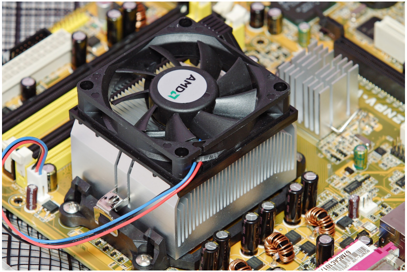
[L.W. Molenkamp and M.J.M. de Jong, Solid-State Electron. 37, 551 (1994)]

Graphene

- negative nonlocal resistance and whirpools [D.A. Bandurin et al., Science 351, 1055 (2016); F.M.D. Pellegrino et al., Phys. Rev. B 94, 155414 (2016); L. Levitov and G. Falkovich, Nat. Phys. 12, 672 (2016)]
- higher than ballistic transport in constrictions [H. Guo et al., PNAS 114, 3068 (2017); R. Krishna Kumar et al., Nat. Phys. 13, 1182 (2017)]

Motivation

Efficient heat transfer in electronics due to convection of electron fluid?!



Hydrodynamical equations

Navier-Stokes equation for hydrodynamic velocity $\mathbf{u}(t, \mathbf{x})$

$$\frac{1}{v_F^2}\left[\partial_t + (\mathbf{u}\cdot\nabla)\right](\mathbf{u}w) - \eta\Delta\mathbf{u} - \frac{\eta}{d}\nabla\left(\nabla\cdot\mathbf{u}\right) = -\nabla P - \frac{w\mathbf{u}}{v_F^2\tau} - en\mathbf{E},$$

pressure $P = \epsilon/d$, enthalpy $w = \epsilon + P$. Electric and energy currents

$$\mathbf{J} = -en\mathbf{u} + \sigma\left[\mathbf{E} + \frac{T}{e}\nabla\left(\frac{\mu}{T}\right)\right],$$

$$\mathbf{J}^{\epsilon} = w\mathbf{u} - \eta\left[(\nabla\cdot\mathbf{u})\,\mathbf{u} + u_j\nabla u_j - \frac{2}{d}\mathbf{u}\left(\nabla\cdot\mathbf{u}\right)\right]$$

Continuity relations

$$-e\partial_t n + (\nabla\cdot\mathbf{J}) = 0, \quad \partial_t\epsilon + (\nabla\cdot\mathbf{J}^{\epsilon}) = (\mathbf{E}\cdot\mathbf{J})$$

Steady-state solution and set-up

Steady-state solution with constant temperature gradient $\nabla T = (T_R - T_L)/L$. Electric field is screened to constant in-medium $\mathbf{E}_0$

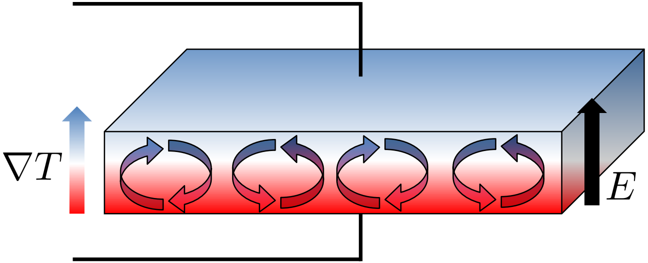


Figure: Convection set-up

Characteristic equation

Plane-wave ansatz for hydrodynamic variables u_x, μ_u, e.g.,

$$T_u = C_T e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} e^{ik_x x}$$

Characteristic equation

$$\mathrm{Ra} = L^4 \frac{(k_\perp^2 + k_x^2)\left(k_\perp^2 + k_x^2 + \lambda_G^{-2}\right)(k_\perp^2 + k_x^2 + q_{\mathrm{TF}}^2)}{k_\perp^2},$$

$$\text{Gurzhi length } \lambda_G = \sqrt{\frac{v_F^2 \tau \eta}{w_0}}, \quad \text{TF wv } q_{TF} = \sqrt{4\pi e^2 \partial_\mu n_0}$$

In the limit $\lambda_G \to \infty$, $q_{TF} \to 0$, the characteristic equation coincides with that for conventional fluids

Rayleigh number

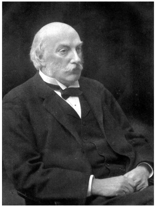


Figure: John William Strutt, 3rd Baron Rayleigh

Rayleigh number

$$\mathrm{Ra} = \frac{\textit{time scale for diffusion}}{\textit{time scale due to convection}}$$

For usual fluid convection due to gravity

$$\mathrm{Ra} \sim \frac{\rho g L^3 \Delta T}{\eta}$$

Convection instability in 3D

Rayleigh number in 3D semimetals

$$\mathrm{Ra} = \frac{4\delta T}{T_0} E_0 \left[\frac{V}{m}\right] L^3[cm] \times 10^9$$

Convection is realized for $\mathrm{Ra} \geq \mathrm{Ra}_{min}$, where Ra_{min} is determined by k_x, $\mathbf{k}_\perp$ which satisfy the boundary conditions

$$T_u(x = 0, L) = 0, \quad u_x(x = 0, L) = 0, \ \mathrm{free-surface}\ b.c.\ \mathrm{for}\ \mathbf{u}_\perp$$

In conventional fluids

$$\mathrm{Ra}_{conv} = 27\pi^2/4 \approx 657.5$$

Convection instability: results

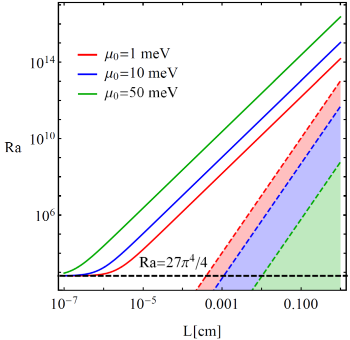


Figure: Minimal Rayleigh number (solid lines) and Rayleigh numbers achievable in Dirac and Weyl semimetals (shaded regions)

In semimetal with $\mu_0 = 20\, meV,\ T_0 = 25\, K,\ v_F = 1.4 \times 10^7 cm/s$

$$\mathrm{Ra}_{min} \approx 6.5 \times 10^{22} L^4 [cm]$$

Convection instability in graphene

In 2D semimetal (graphene), using the gradual channel approximation

$$\mathbf{E}_u = \frac{e}{C}\nabla n_u, \quad C = \epsilon/(4\pi L_g),$$

the characteristic equation

$$\mathrm{Ra} = L^4 \frac{(k_\perp^2 + k_x^2)^2 (k_\perp^2 + k_x^2 + \lambda_G^{-2})(1 + Q^2)}{k_\perp^2}, \quad Q = \sqrt{\frac{e^2 \partial_\mu n_0}{C}}$$

Rayleigh number

$$\mathrm{Ra} = 44 \times L^3 [0.1\, mm]$$

Minimal value at $L = 0.1\, mm$

$$\mathrm{Ra}_{min} = 2.6 \times 10^6$$

Summary

- Convective instability is strongly inhibited in 2D and 3D semimetals.
- Main inhibitors: (i) Coulomb forces, (ii) momentum relaxation due to scattering on impurities and phonons.
- In 3D semimetals, Coulomb forces dominate and lead to an extremely large convection threshold.
- Momentum relaxation plays the key role in 2D semimetals. Yet the threshold values of convection are a few orders of magnitude larger than in convectional fluids.

Thank you for attention!

Steady-state solution

Charge density and electric field