Dualities of QCD phase diagram and influence of chiral imbalance on color superconductivity phenomenon

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БАЗИС
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in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
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details can be found in

Eur.Phys.J.C 80 (2020) 10, 995 arXiv:2005.05488 [hep-ph]
JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

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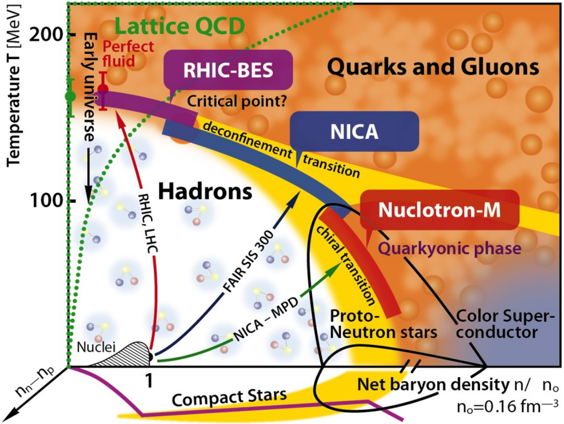
QCD Phase Diagram



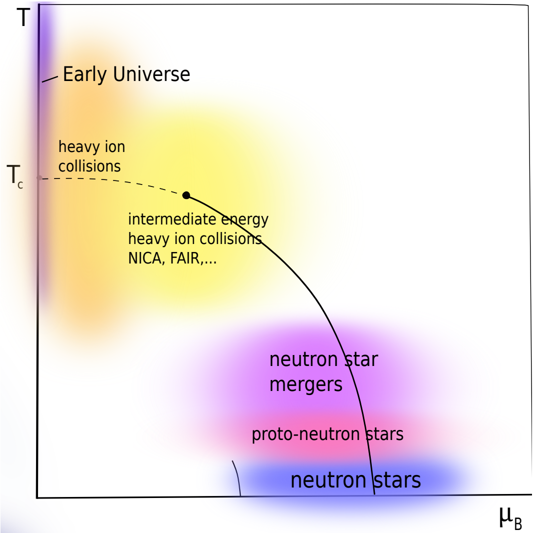
Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD Dhase Diagram

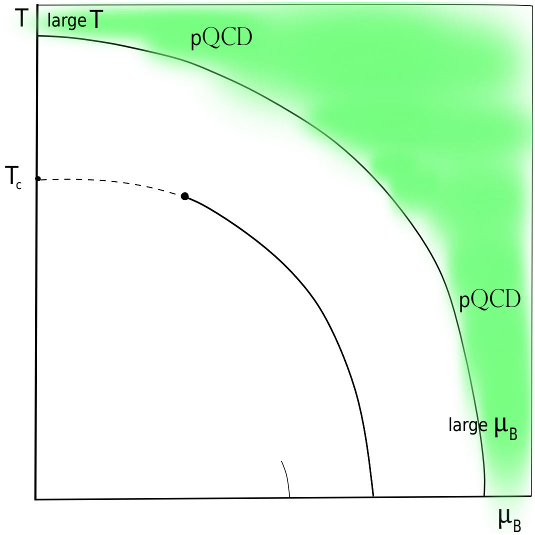
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



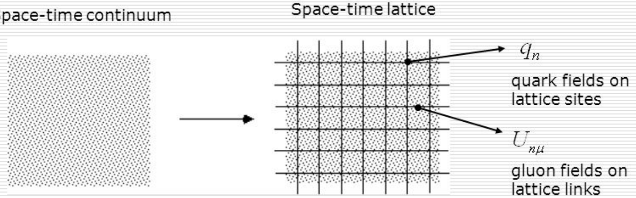
Methods of dealing with QCD

- Perturbative QCD



QCD on a space-time lattice

K. G. Wilson 1974



- Feynman path integral
  - Action $S_{QCD} = \frac{1}{g_s^2}\sum_P tr(UUUU) + \sum_f \bar{q}_f(\gamma \cdot U + m_f)q_f$
  - Physical quantities as integral averages

$$\langle O(U,\bar{q},q)\rangle = \frac{1}{Z}\int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n \, O(U,(U,\bar{q},q))e^{-S_{QCD}}$$

Monte Carlo Evaluation of the path integral

lattice QCD at non-zero baryon chemical potential μ_B

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

$$Z = \int D[\text{gluons}]\, Det\, \mathbb{D}(\mu)\, e^{-S^E_{\text{gluons}}}$$

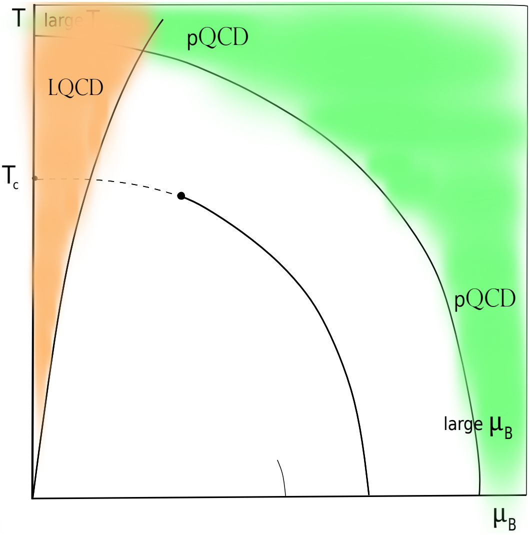
It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

QCD Dhase Diagram and Approaches

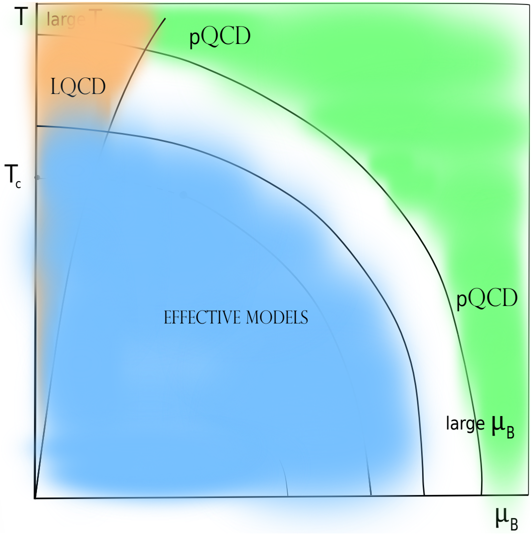
Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD



Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

$$\mu, T < 600\,\mathrm{MeV}$$

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

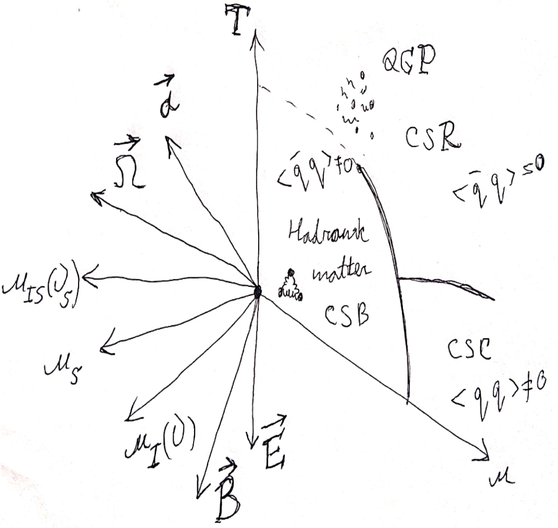
$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More external conditions to QCD

More than just QCD at (μ, T)

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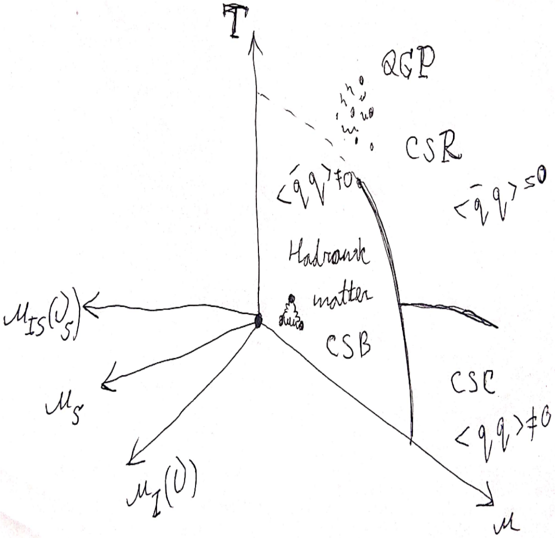


Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

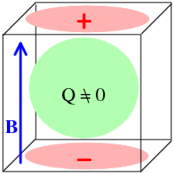
chiral (axial) chemical potential

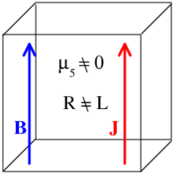
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

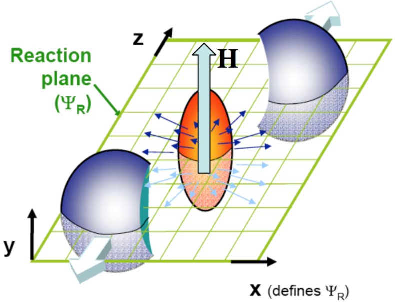
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

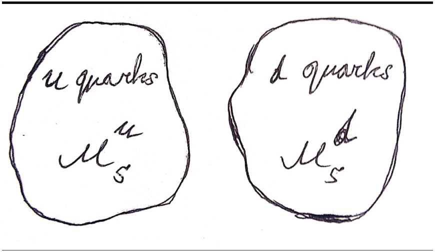






$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$

- Chiral isospin imbalance and chiral imbalance μ_{I5} and μ_5 can be generated in parallel magnetic and electric fileds $\vec{E} \,||\, \vec{B}$

- Chiral imbalance could appear in dense matter
  - Chiral separation effect
    (Thanks for the idea to Igor Shovkovy)
  - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

- **QCD phase diagram**

  with **different chemical potentials**

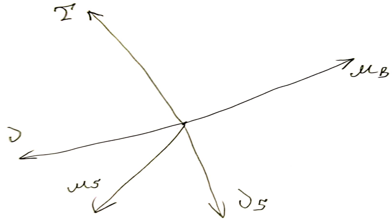
  and matter content including **chiral imbalance**

- **QC_2D phase diagram** and **diquark condensation** phenomenon

  with different chemical potentials, including μ_5

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

Dualities

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$$

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$ $\Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

Duality in the phase portrait

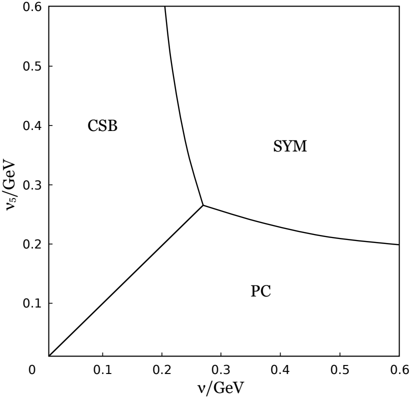


Figure: NJL model results

$$\mathcal{D}: \ M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\mathrm{PC} \longleftrightarrow \mathrm{CSB} \quad \nu \longleftrightarrow \nu_5$$

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a .$$

$$\mathcal{L}_{\mathrm{NJL}} = \sum_{f=u,d} \bar{q}_f \left[\mathrm{i}\gamma^\nu \partial_\nu - m_f \right] q_f + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 \vec{\tau} q)^2 \right]$$

m_f is current quark masses

In the chiral limit $m_f = 0$ the Duality $\mathcal{D}$ is exact

m_f : $\frac{m_u + m_d}{2} \approx 3.5$MeV
In our case typical values of $\mu, \nu, ..., T, .. \sim 10 - 100s$ MeV, for example, 200-400 MeV
One can work in the chiral limit $m_f \to 0$
$m_f = 0 \quad \to \quad m_\pi = 0$
physical m_f a few MeV $\quad \to \quad$ physical $m_\pi \sim 140$ MeV

Duality between CSB and PC is **approximate** in **physical point**

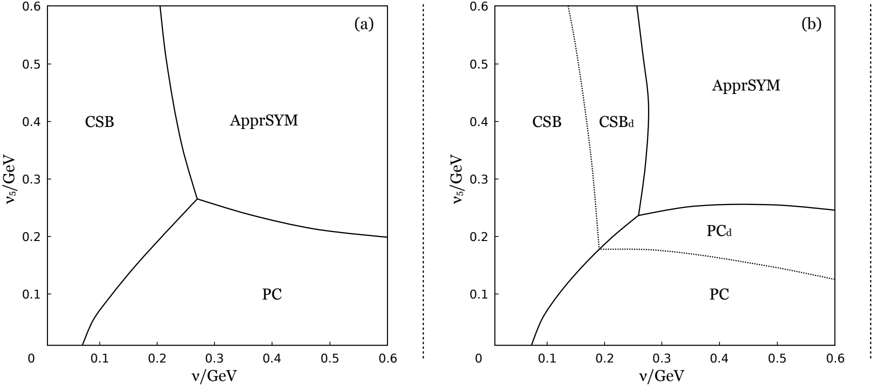


Figure: (ν, ν_5) phase diagram

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

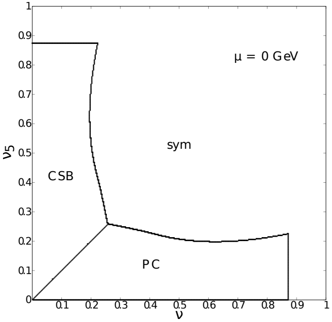
Duality is shown to take place in particular cases in lattice QCD

- **QCD at** μ_5 —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()



Uses of Dualities

A few rather interesting uses of dualities

discussed in Particles 2020, 3(1), 62-79

Two colour QCD case

QC_2D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T *and* μ

- A lot of physical quantities coincide up to few dozens percent

  Critical temperature $T_c/\sqrt{\sigma}$*, topological susceptibility* $\chi^{\frac{1}{4}}/\sqrt{\sigma}$ *shear viscosity* η/s

There are **no sign problem** in SU(2) case

$$(Det(D(\mu)))^\dagger = Det(D(\mu))$$

and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

Phase diagram of QC_2D

Condensates and phases

$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5 \tau_1 q \rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle,$ BSF phase: $\Delta \neq 0.$

Catalysis of chiral symmetry beaking

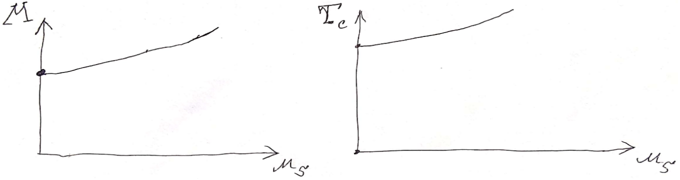
A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

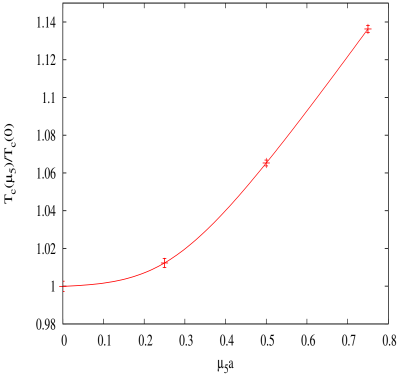
QCD at non-zero μ_5



catalysis of CSB by chiral imbalance:

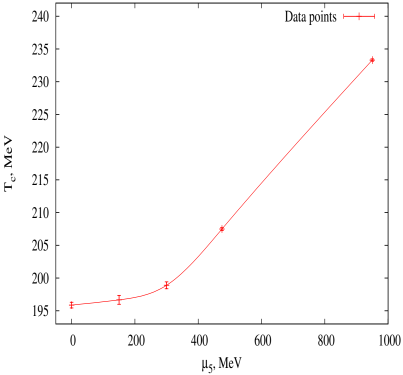
- increase of $\langle \bar{q} q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)



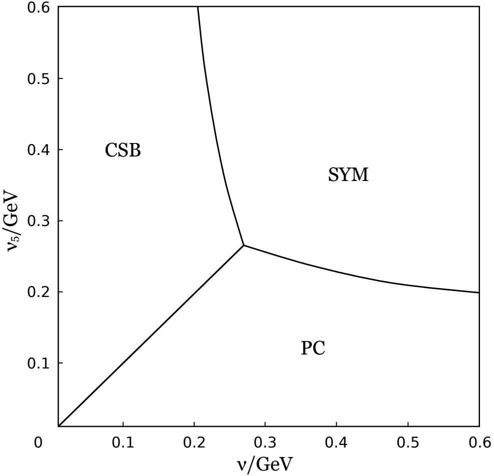
V. Braguta, A. Kotov et al, JHEP 1506, 094 (2015),PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, Phys. Rev. D 93, 034509 (2016), arXiv:1512.05873 [hep-lat]

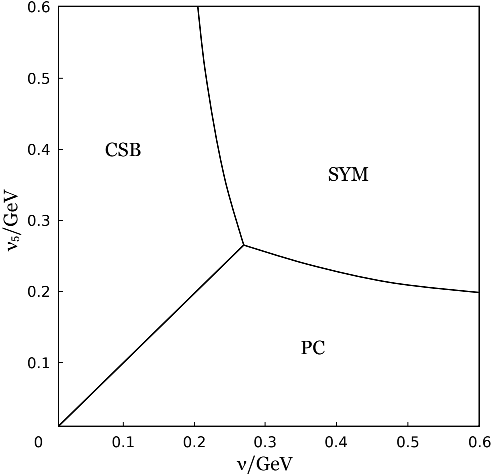
Phase structure: (ν, ν_5)-phase diagram



Isospin imbalance ν generates pion condensation

Chiral imbalance ν_5 generates chiral symmetry breaking

Phase structure: (ν, ν_5)-phase diagram



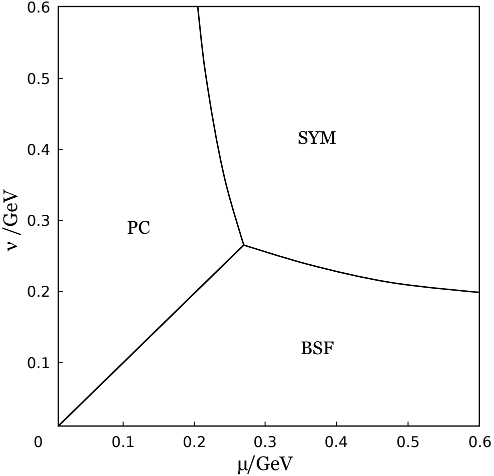
Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

$$\mathcal{D}_3 : \quad M \longleftrightarrow \pi_1, \;\; \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, ...) = \Omega(\pi, M, ...)$$

Phase structure: (μ, ν)-phase diagram

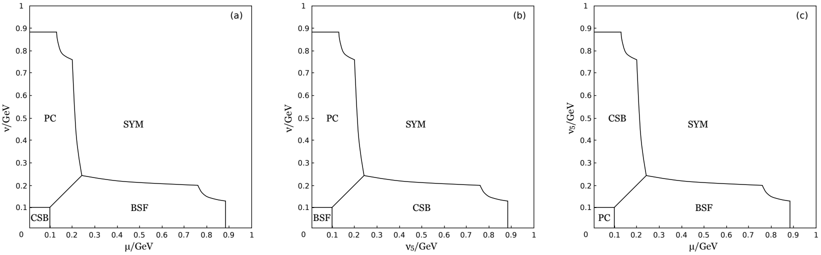


Baryon density μ generates color superconductivity

$$\text{PC} \longleftrightarrow \text{BSF} \quad \nu \longleftrightarrow \nu_5$$

$$\mathcal{D}_1 : \quad \pi_1 \longleftrightarrow |\Delta|, \quad \mu \longleftrightarrow \nu$$

$$\Omega(\pi, \Delta, ...) = \Omega(\Delta, \pi, ...)$$



$$(a) \qquad \mathcal{D}_1 : \qquad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

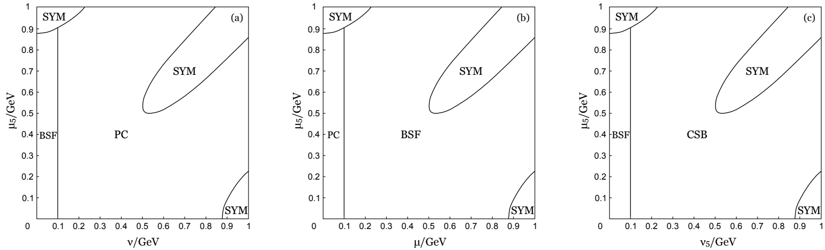
$$(b) \qquad \mathcal{D}_3 : \qquad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

$$(c) \qquad \mathcal{D}_2 : \qquad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

Each chemical potential is connected in one-to-one correspondence with some phenomenon (condensation)

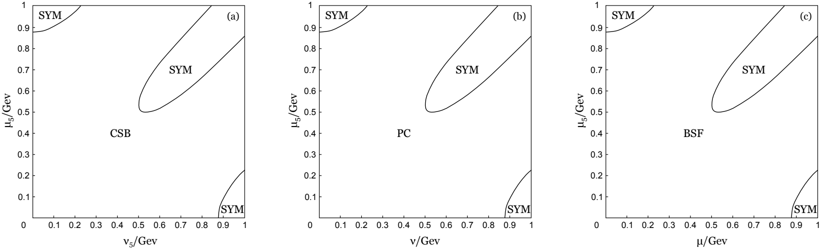
- Baryon density $\mu \iff$ diquark condensation
- Isospin imbalance $\nu \iff$ pion condensation
- Chiral imbalance $\nu_5 \iff$ chiral symmetry breaking

Universal catalizer effect of chiral imbalance



Chiral imbalance μ_5 catalyzes all the phenomena

Chameleon property of chiral imbalance

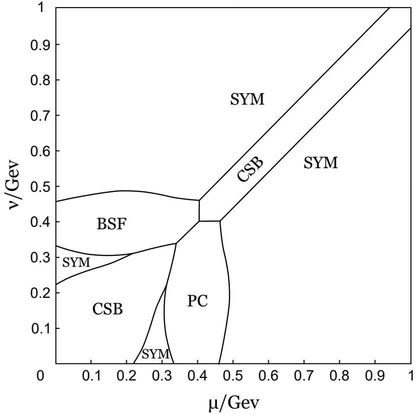


Chameleon nature of chiral imbalance μ_5

μ_5 mimics other chemical potentials μ, ν, ν_5

Diquark condensation at $\mu = 0$

Chiral imbalance μ_5 leads to several rather peculiar phases in the system, e. g. the **diquark condensation** in the region of the phase diagram at $\mu = 0$



(μ, ν, ν_5) phase diagram is highly symmetric due to dualities

and intermingled by dualities at $\mu_5 \neq 0$

μ_5 deforms the (μ, ν, ν_5) phase diagram

The influence of μ_5 is constrained by dual properties

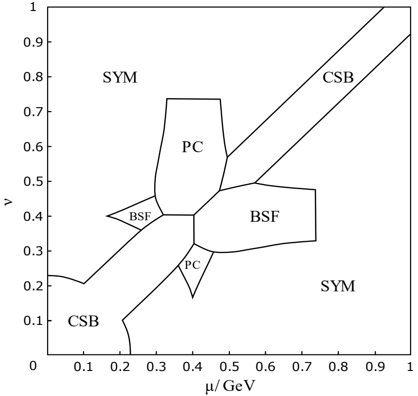
- Chameleon nature of chiral imbalance μ_5 is also a consequence of dualities
- The feature of μ_5 of being universal catalizer is a consequence of dualities as well

There are a lot of features of QC_2D phase diagram that remains the same in the case of QCD

Including the behaviour of diquark condensation in quark matter with different imbalances

PC_d phase and diquark condensation

- PC_d phase has been predicted without possibility of diquark condensation
- Diquark condensation can take over the PC_d phase
- In two colour case diquark condensation is in a sense even stronger than in three colour case and starts from $\mu > 0$



PC_d phase is unaffected by BSF phase in two color case. Maybe one can infer that it is the case also for 3 color QCD

PC_d phase and diquark condensation

- PC_d phase has been predicted without possibility of diquark condensation
- Diquark condensation can take over the PC_d phase

PC_d phase is unaffected by CSC phase in three color case.

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation

- In QCD non-pertubartively (at the level of Lagrangian)

Conclusions and Thanks for the Attention

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD