Medium effects on the Electrical and Hall conductivities of a hot and magnetised pion gas

Pallavi Kalikotay

Department of Physics, Kazi Nazrul University, Asansol, India

ICNFP 2021

Based on: P. Kalikotay et. al. Physical Review D 102, 076007 (2020)

Motivation

- In noncentral heavy ion collisions (HICs) at the RHIC and LHC, strong magnetic fields of the order of $\sim 10^{18}$ G or larger may be generated due to the collision geometry.
- Phenomenologically, electrical conductivity is important in the sense that if it is large, the created magnetic field in noncentral HICs persists for a longer time
- Necessary to calculate the B-dependent electrical conductivity of HM as accurately as possible taking into account finite temperature and/or density and magnetic field effects.

Formalism

Boltzmann transport equation in magnetic field

$$\frac{\partial f}{\partial t} \; + \; \vec{v}\frac{\partial f}{\partial \vec{r}} \; - \; e[\vec{E} + (\vec{v} \times \vec{H})] \; \frac{\partial f}{\partial \vec{p}} = C[f] \tag{1}$$

$$C[f] = \frac{\delta f}{\tau(\epsilon)} = -\frac{\phi}{\tau}\frac{\partial f_0}{\partial \epsilon} \tag{2}$$

where $f = f_0 \; + \; \delta f$, δf is the deviation from equilibrium distribution fucntion and τ is the relaxation time.
Since we are dealing with uniform and static medium both f and f_0 are independant of time and space. Thus,

$$e\vec{E} \cdot \vec{v}\frac{\partial f_0}{\partial \epsilon} \; - \; e(\vec{v} \times \vec{H}) \; \frac{\partial \phi}{\partial \vec{p}} \; \frac{\partial f_0}{\partial \epsilon} = -\frac{\phi}{\tau}\frac{\partial f_0}{\partial \epsilon} \tag{3}$$

In order to solve for the electrical conductivity we specify the form for ϕ

$$\phi = \vec{p} \cdot \vec{\Xi}(\epsilon) \tag{4}$$

where the vector $\vec{\Xi}$ contains information of the dissipation produced due to electric and magnetic fields and is given in mosty general form using $\hat{e} = \frac{\vec{E}}{E}$, $\hat{h} = \frac{\vec{B}}{B}$ by

$$\vec{\Xi} \;=\; \alpha\vec{e} \;+\; \beta\vec{h} \;+\; \gamma[\vec{e} \times \vec{h}]. \tag{5}$$

Using Eq.(4) and Eq.(5) in Eq.(3) the constants α, β and γ are found to be

$$\alpha \;=\; -\frac{eE}{\epsilon}\,\frac{\tau}{1+\omega_c^2\tau^2} \tag{6}$$

$$\frac{\beta}{\alpha} \;=\; -(\omega_c\tau)^2\,(\vec{h}\cdot\vec{e}) \tag{7}$$

$$\frac{\gamma}{\alpha} \;=\; -\omega_c\tau \tag{8}$$

Hence vector $\vec{\Xi}$ in Eq.(5) in terms of constants α, β and γ can be written as

$$\begin{aligned} \phi &= \alpha\epsilon\;\vec{v}\cdot[1 \;+\; (\omega_c\tau)^2\,(\hat{e}\cdot\hat{h})\hat{h} \;-\; (\omega_c\tau)(\hat{e}\times\hat{h})] \\ &= -\frac{e\tau}{1+(\omega_c\tau)^2}\;v_i[\delta_{ij} \;-\; \omega_c\tau\epsilon_{ijk}h_k \;+\; (\omega_c\tau)^2 h_i h_j]\;E_j. \end{aligned} \tag{9}$$

The current density j_i is given by

$$j_i = \sigma_{ij}\ E_j = g_\pi \int \frac{d^3p}{(2\pi)^3}\ e\ v_i\ \phi\ \frac{\partial f_0}{\partial \epsilon} \tag{10}$$

where σ_{ij} is the conducitvity tensor. Now using Eq.(9) in Eq.(10) we get the conductivity tensor as

$$j_i = \ \delta_{ij}\ \sigma_0\ -\ \epsilon_{ijk}\ h_k\ \sigma_1\ +\ h_i\ h_j\ \sigma_2 \tag{11}$$

where

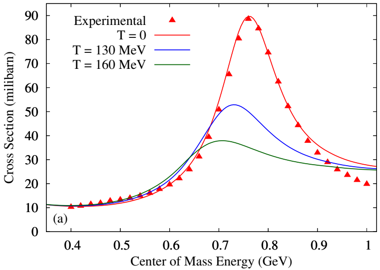
$$\sigma_n = g\frac{e^2}{3T}\int \frac{d^3p}{(2\pi)^3}\ \frac{p^2}{\epsilon^2}\ \frac{\tau(\omega_c\tau)^n}{1+(\omega_c\tau)^2}\ f_0\ (1+f_0) \qquad\qquad n=0,1,2 \tag{12}$$

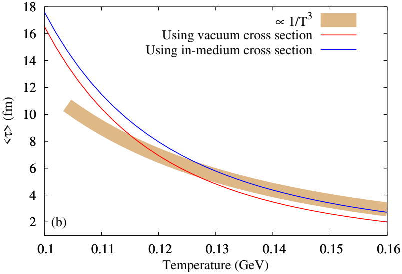
σ_0 is electrical conductivity in presence of magnetic field, σ_1 is Hall conductivity and $\sigma_0 + \sigma_2$ is the electrical conductivity in absence of magnetic field.

- The calculation of relaxation time (τ) of the pions requires $\pi\pi$ scattering cross section as the dynamical inputs.
- We have considered the elastic $\pi\pi \to \pi\pi$ scattering mediated via explicit ρ and σ meson exchange.
- Here, the temperature dependence in the cross section enters through in-medium spectral broadening of ρ and σ.
- The details of the methodology can be found in Sukanya et. al. (Physical Review C **85**, 064917 and Physical Review D **87**, 094026).

Medium dependant Cross section and Average Relaxation time

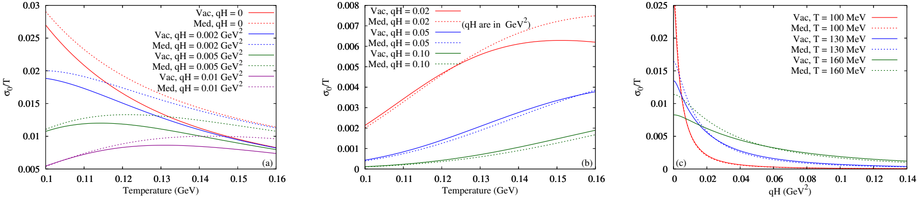
$$[\tau(p)]^{-1} = \frac{g'}{2} \int \frac{d^3k}{(2\pi)^3} (\sigma v_{\rm rel})\ f_0(\omega_k)(1 + f_0(\omega_k))$$





Variation of σ_0/T with temperature for different magnetic field strengths

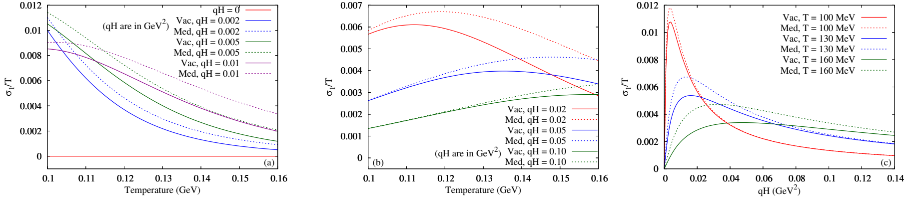
$$\sigma_0 = \frac{g_\pi e^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{\epsilon^2} \frac{\tau}{1+(\omega_c\tau)^2} f_0 \,(1+f_0)$$



$\frac{\sigma_0}{T}$	T Dependance	$\frac{\tau T}{1+(\omega_c\tau)^2}$	Low B $\rightarrow \omega_c\tau << 1$	$\sim \tau T \sim \frac{1}{T^2}$
			High B $\rightarrow \omega_c\tau >> 1$	$\sim \frac{T}{\tau} \sim T^4$
	B dependance	$\frac{1}{1+(\omega_c\tau)^2}$	As B $\uparrow$ $\omega_c \uparrow$ implying $\frac{\sigma_0}{T} \downarrow$ B	
	Medium Effects	Low B	$\sim \tau$	Medium effect $\uparrow \frac{\sigma_0}{T}$
		High B	$\sim \frac{1}{\tau}$	Medium effect $\downarrow \frac{\sigma_0}{T}$

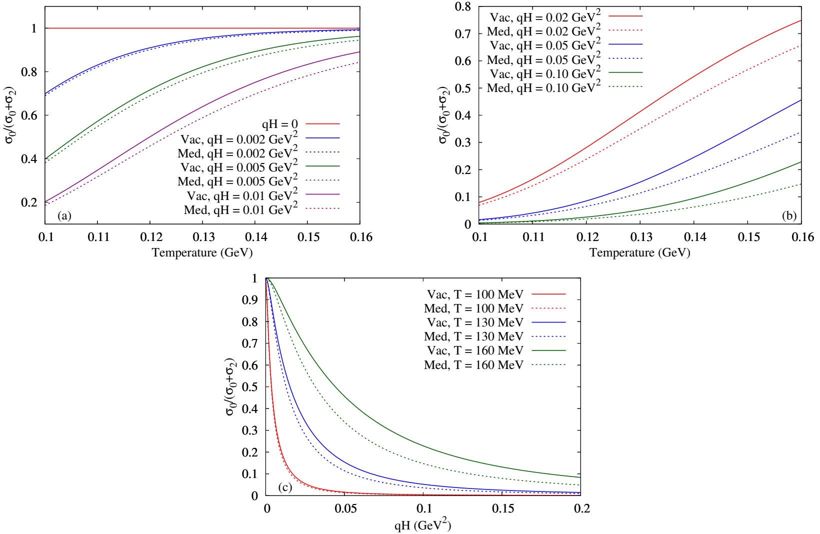
Variation of σ_1/T with temperature for different magnetic field strengths

$$\sigma_1 = \frac{g_\pi e^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{\epsilon^2} \frac{\tau(\omega_c\tau)}{1+(\omega_c\tau)^2} f_0 \,(1+f_0)$$

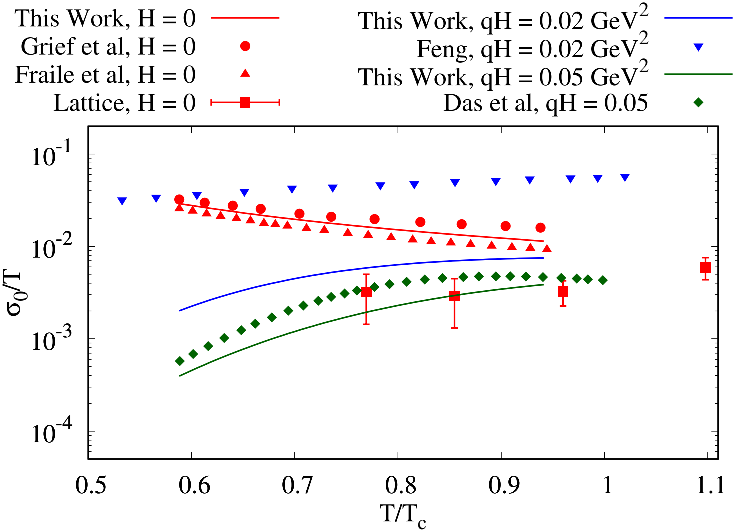


$\frac{\sigma_1}{T}$	T Dependance	$\frac{\tau^2 T}{1+(\omega_c\tau)^2}$	Low B $\rightarrow \omega_c\tau << 1$	$\sim \tau^2 \;\; T \sim \frac{1}{T^5}$
			High B $\rightarrow \omega_c\tau >> 1$	$\sim T$
	B Dependance	$\frac{\omega_c}{1+(\omega_c\tau)^2}$	Breit Wigner function of the magnetic field	
	Medium effects	$\frac{\tau^2\omega_c}{1+(\omega_c\tau)^2}$	Increases $\frac{\sigma_1}{T}$ for any value of B	

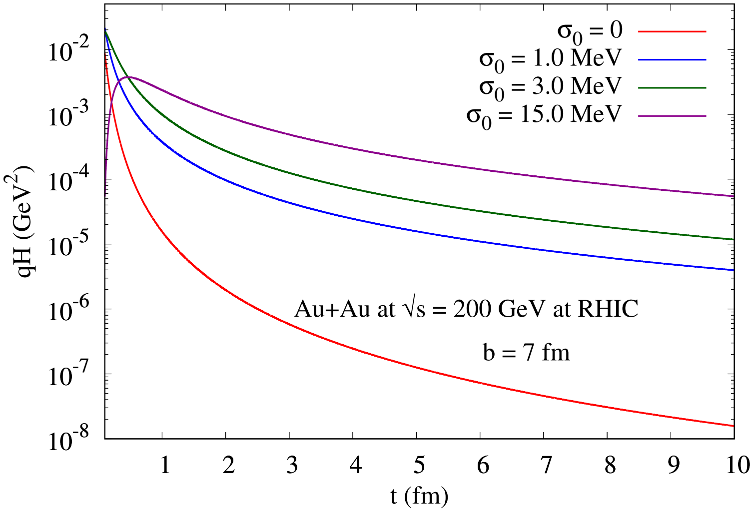
Variation of anisotropy $\frac{\sigma_0}{\sigma_0+\sigma_2}$



Comparison with previous results



Decay of magnetic field with estimated electrical conductivity



Conclusion

- As we have not considered the LQ in the dispersion relation of charged pions, our results are more accurate in the low magnetic field values ($qH \leq m^2$), which is the realistic scenario for the later stages of HICs.
- The electrical conductivity obtained in this work has been shown to have both qualitative and quantitative agreement with earlier estimates available in the literature.
- The calculated electrical conductivity has been shown to be sufficient for causing a significant delay in the decay of the external magnetic field in a HIC. This leads to the conclusion that, a weak magnetic field can be present in the later stage of a HIC (in hadronic phase) and could be phenomenologically relevant.

Collaborators

Sourav Sarkar, Varibale Energy Cyclotron Centre
Pradip Roy, Saha Institute of Nuclear Phsics
Snigdha Ghosh, Saha Institute of Nuclear Phsics
Nilanjan Chaudhuri, Varibale Energy Cyclotron Centre


Thank you

Variation of σ_2/T with temperature for different magnetic field strengths

$$\sigma_2 = \frac{g_\pi e^2}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{\epsilon^2} \frac{\tau(\omega_c\tau)^2}{1+(\omega_c\tau)^2} f_0\,(1+f_0)$$