Ice in the Fire puzzle:
Dynamical cluster and hypernuclei production in heavy-ion collisions

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&
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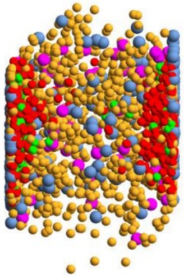
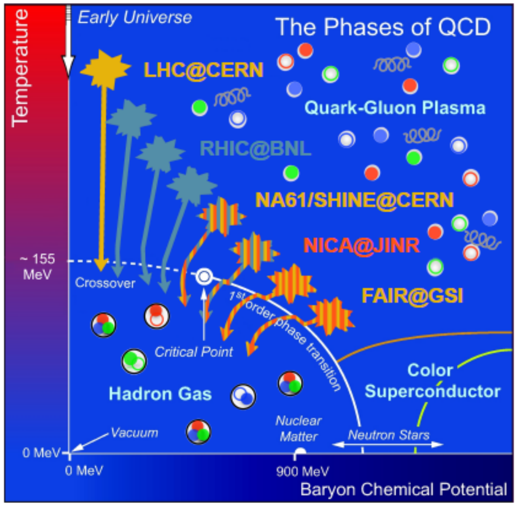
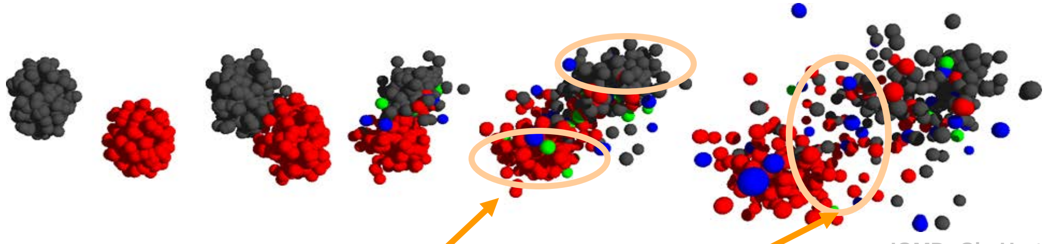


The ‚holy grail‘ of heavy-ion physics: Phase diagramm of QCD

The phase diagram of QCD



Experimental observables:
... Clusters and (anti-) hypernuclei



IQMD: Ch. Hartnack

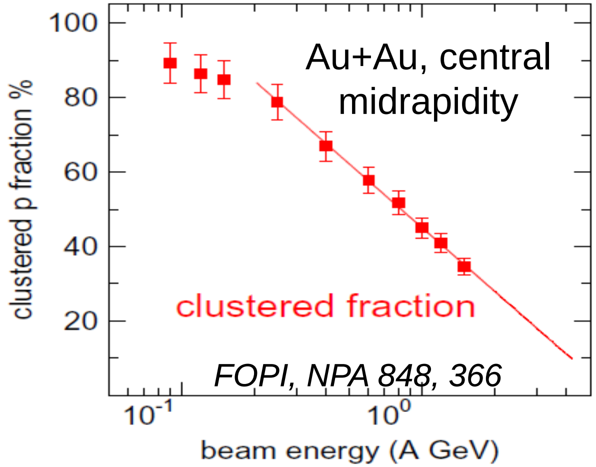
- projectile/target spectators➔ heavy cluster formation
- midrapidity➔ light clusters

! Hyperons are created in participant zone

(Anti-) hypernuclei production:

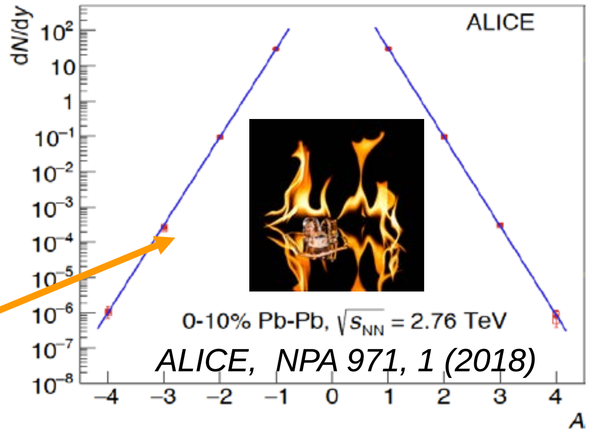
- at mid-rapidity by coalescence of Λ with nucleons during expansion
- at projectile/target rapidity by rescattering/absorption of Λ by spectators

- ❑ Clusters are very abundant at low energy



FOPI, NPA 848, 366

High energy HIC:
‚Ice in a fire‘ puzzle:
how the weakly bound objects can be formed and survive in a hot enviroment ?!



ALICE, NPA 971, 1 (2018)

Modeling of cluster and hypernuclei formation

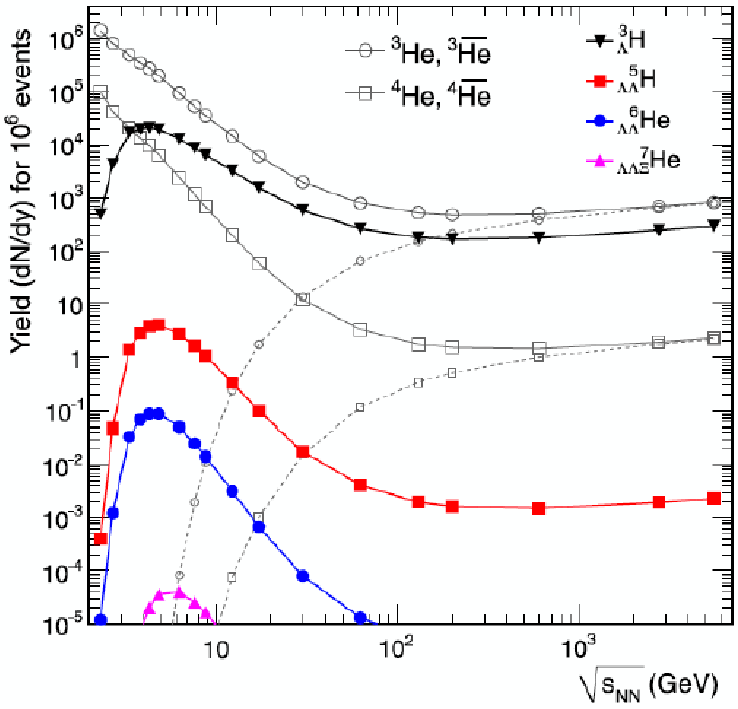
Existing models for clusters formation:

☐ **statistical model:**
- **assumption of thermal equilibrium**

☐ **coalescence model:**
- **determination of clusters at a given time by coalescence radii in coordinate and momentum space**

➔ **don‘t provide information on the dynamical origin of clusters formation**

A. Andronic et al., PLB 697, 203 (2011)



In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

→ **transport models:**
- **dynamical modeling of cluster formation based on interactions**

PHQMD

PHQMD: a **unified n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

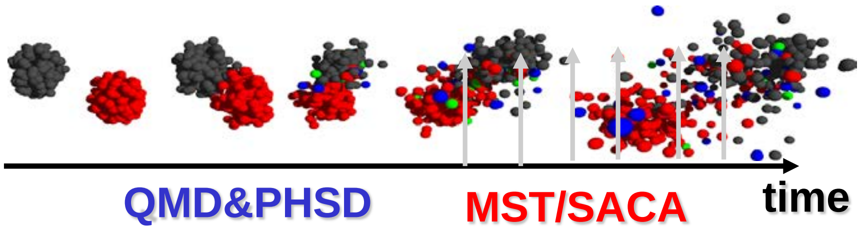
Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)



J. Aichelin et al.,
PRC 101 (2020) 044905
& arXiv:1907.03860

QMD propagation

- **Generalized Ritz variational principle:** $$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$$

Assume that $\psi_N = \prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})$ **for N particles** **(neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle *“i”* :
Gaussian with width *L* centered at r_{i0}, p_{i0}

L=4.33 fm^2

$$\psi_i(q_i, q_{0i}, p_{0i}) = Cexp[-(q_i - q_{0i} - \frac{p_{0i}}{m}t)^2/4L] \cdot exp[ip_{0i}(q_i - q_{0i}) - i\frac{p_{oi}^2}{2m}t]$$

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\boxed{\dot{r}_{i0} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}}$$

Hamiltonian: $$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j\neq i} V_{i,j})$$

$$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t) = V_{\mathrm{Skyrme}} + V_{\mathrm{Coul}}$$

QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H\rangle = \langle T\rangle + \langle V\rangle = \sum_i\left(\sqrt{p_{i0}^2+m^2}-m\right)+\sum_i\langle V_{Skyrme}(\mathbf{r}_{i0},t)\rangle$$

- ❑ **Skyrme potential** **(‘static’) * :**

$$\langle V_{Skyrme}(\mathbf{r}_{i0},t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r}_{i0},t)}{\rho_0}\right)+\beta\left(\frac{\rho_{int}(\mathbf{r}_{i0},t)}{\rho_0}\right)^{\gamma}$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

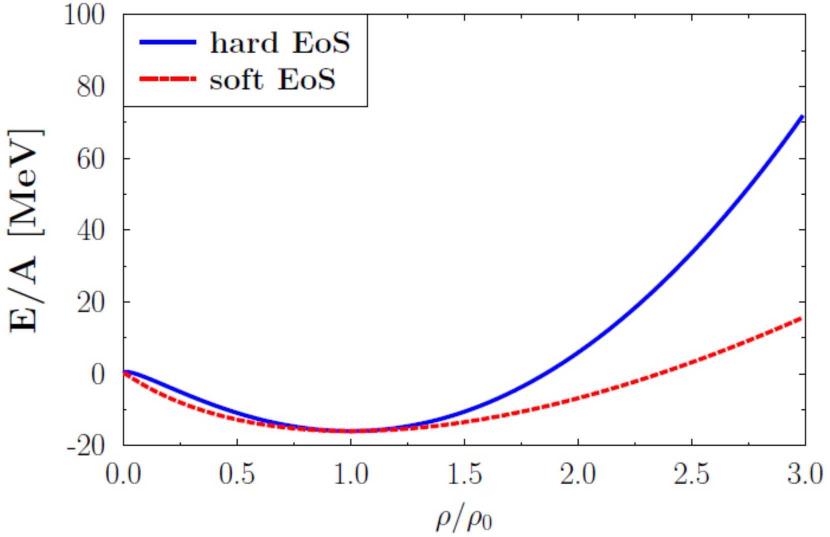
- ❑ **modifed interaction density (with relativistic extension):**

$$\rho_{int}(\mathbf{r}_{i0},t) \rightarrow C\sum_j\left(\frac{4}{\pi L}\right)^{3/2} e^{-\frac{4}{L}(\mathbf{r}^{\mathrm{T}}_{i0}(t)-\mathbf{r}^{\mathrm{T}}_{j0}(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}^{\mathrm{L}}_{i0}(t)-\mathbf{r}^{\mathrm{L}}_{j0}(t))^2},$$

- ❖ **HIC ←→ EoS for infinite matter at rest**
- o **compression modulus K of nuclear matter:**

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}\Big|_{\rho=\rho_0}$$

EoS for infinite matter at rest



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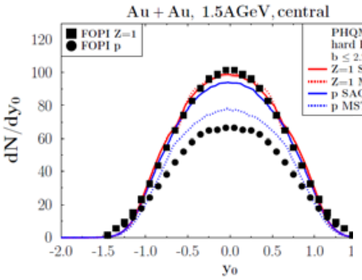
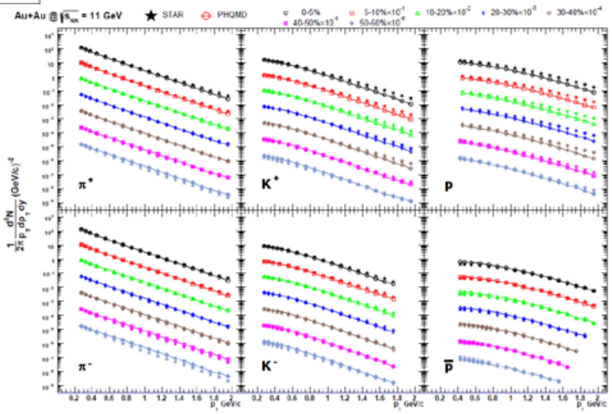
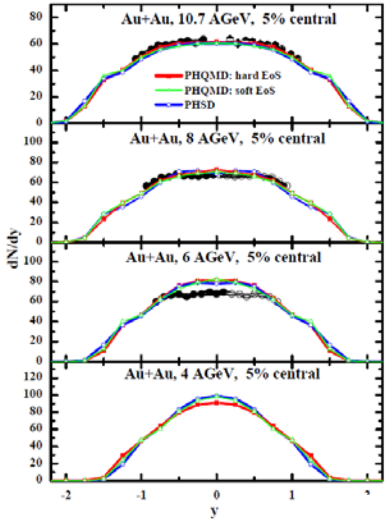
*Work in progress: implementation of momentum dependent potential (M. Winn)

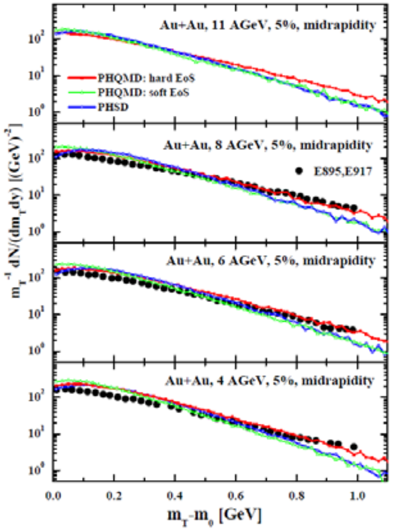
Highlights: PHQMD ‚bulk‘ dynamics from SIS to RHIC

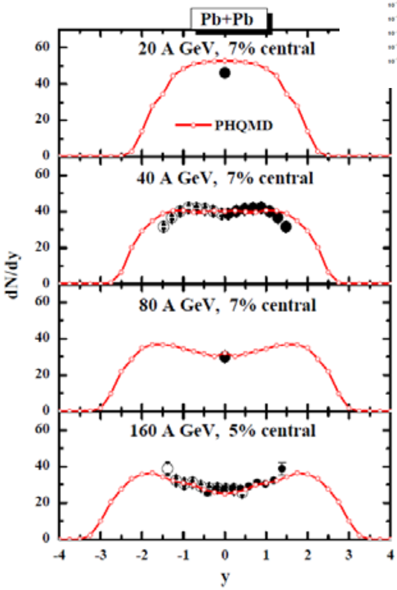


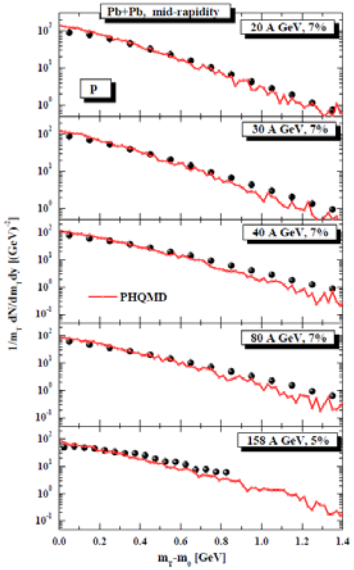
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905

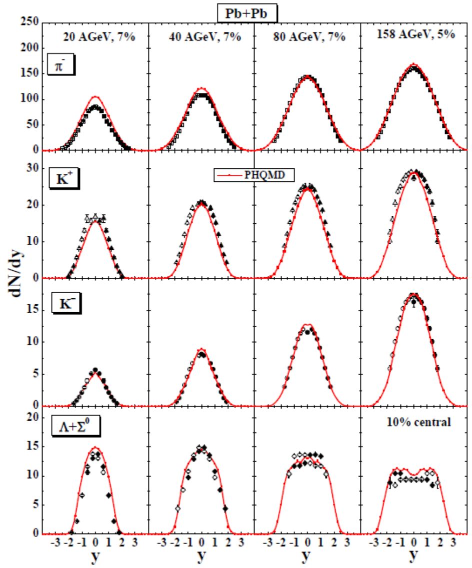


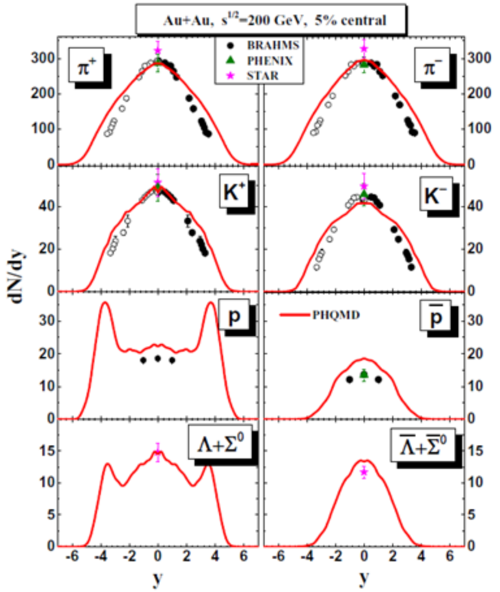












PHQMD provides a good description of hadronic ‘bulk’ observables from SIS to RHIC energies

Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

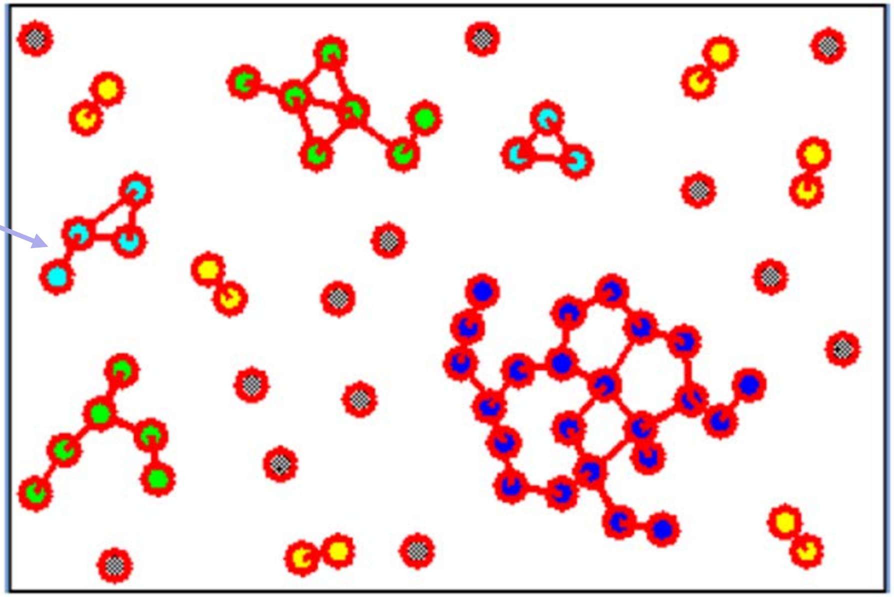
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are 'bound' if their distance in the cluster rest frame fulfills

$$|\vec{r_i} - \vec{r_j}| \leq 4\ \text{fm}$$

2. Particle is bound to a cluster if it bounds with at least one particle of the cluster.



* Remark:
inclusion of an additional momentum cuts (coalescence) lead to a small changes: particles with large relative momentum are mostly not at the same position
(V. Kireyeu, 2103.10542)

Cluster stability in semi-classical models

Limitation of semi-classical models (as QMD):

- ☐ **Clusters in QMD are semiclassical bound objects (with a binding energy close to the Weizsäcker mass formula) but not quantum system with a defined ground state**
- ☐ **In bound QMD clusters kinetic energy can be therefore accumulated by one of the nucleons which may escape (what is not possible in a quantum cluster)**

➔ **We have therefore to fix a time at which we analyse the clusters. This choice influences the multiplicity. We verified that it does not influence the form of**
- **the rapidity distribution**
- **the pt distribution**
- **ratio of particles**

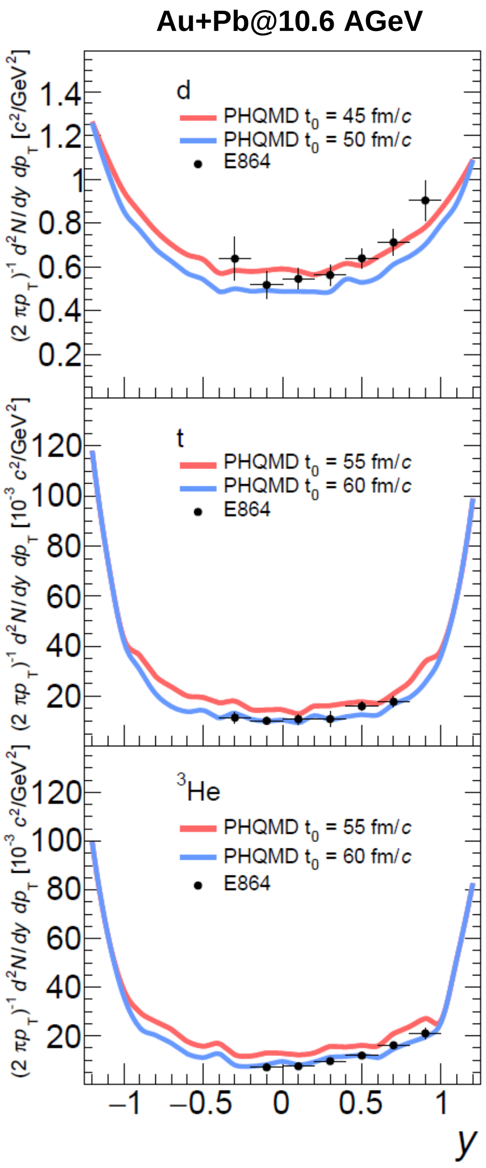
⇨ **To compare the PHQMD results at different rapidities we have to chose the same 'physical time' :**

$$t = t_0 \cosh(y)$$

where t_0 is the time taken at y=0 (in the center-of-mass system) to compensate for the time dilatation

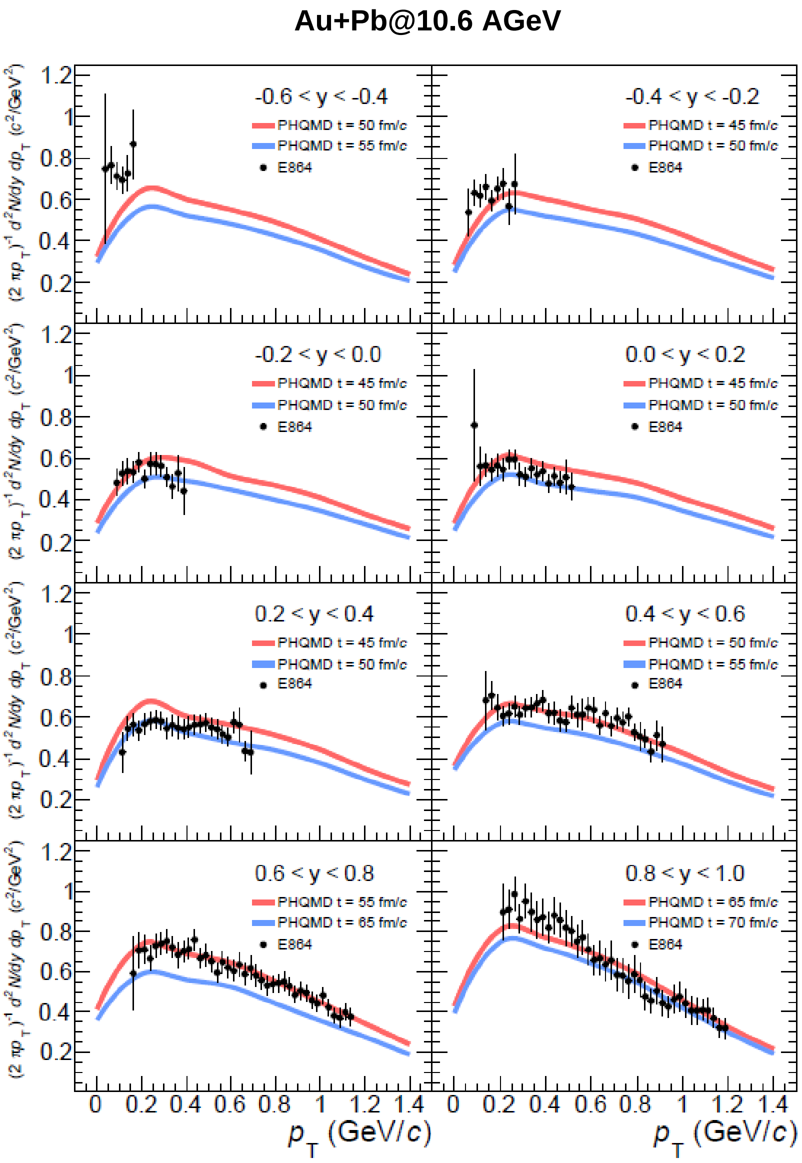
Cluster production in HIC at AGS energies

y- distributions of d, t, ^{3}He



Au+Pb@10.6 AGeV

p_T - distribution of deuterons

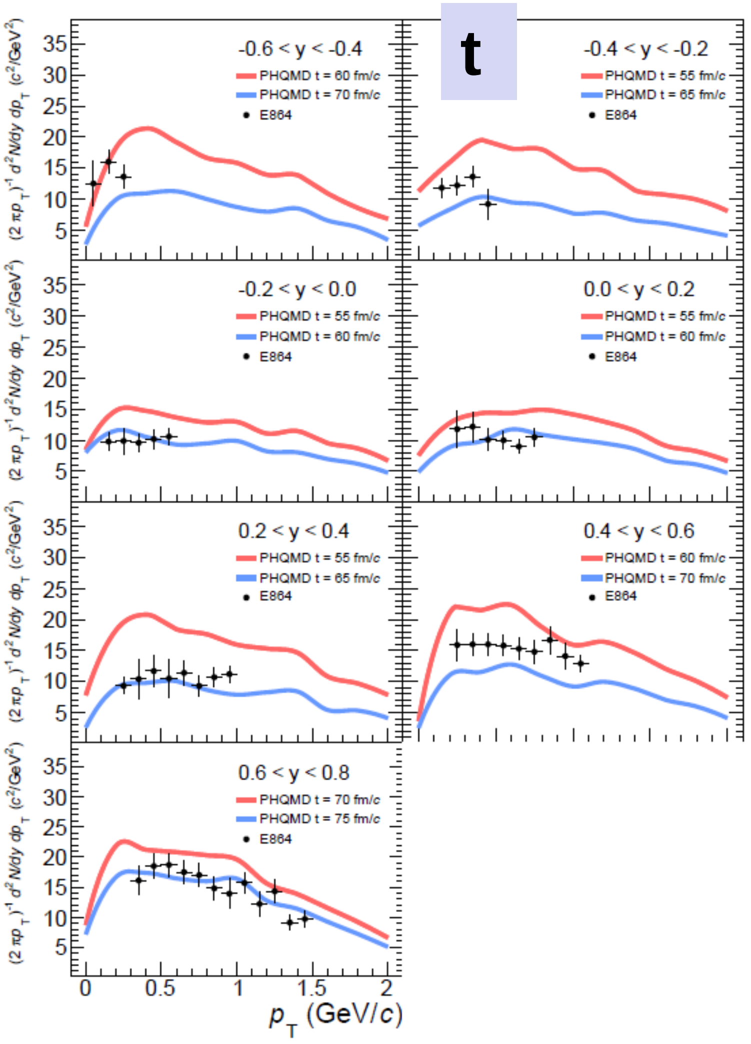


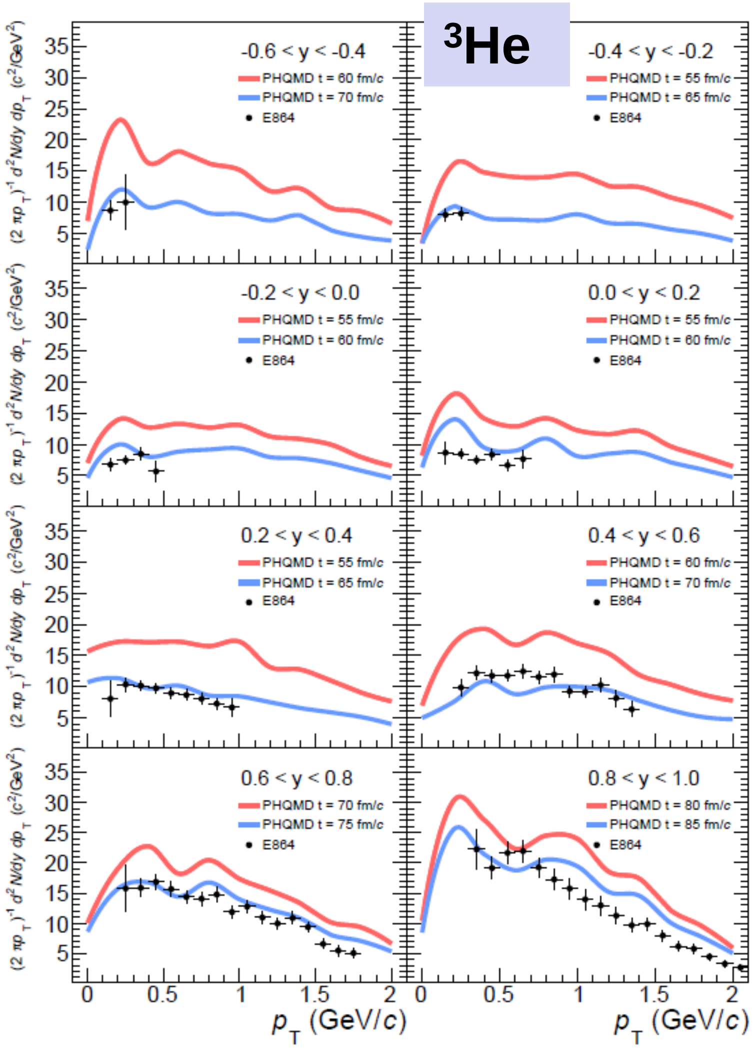
Au+Pb@10.6 AGeV

The PHQMD results are taken at
$t = t_0 \cosh(y)$,
where t_0 is the time at y=0

Cluster production in HIC at AGS energies

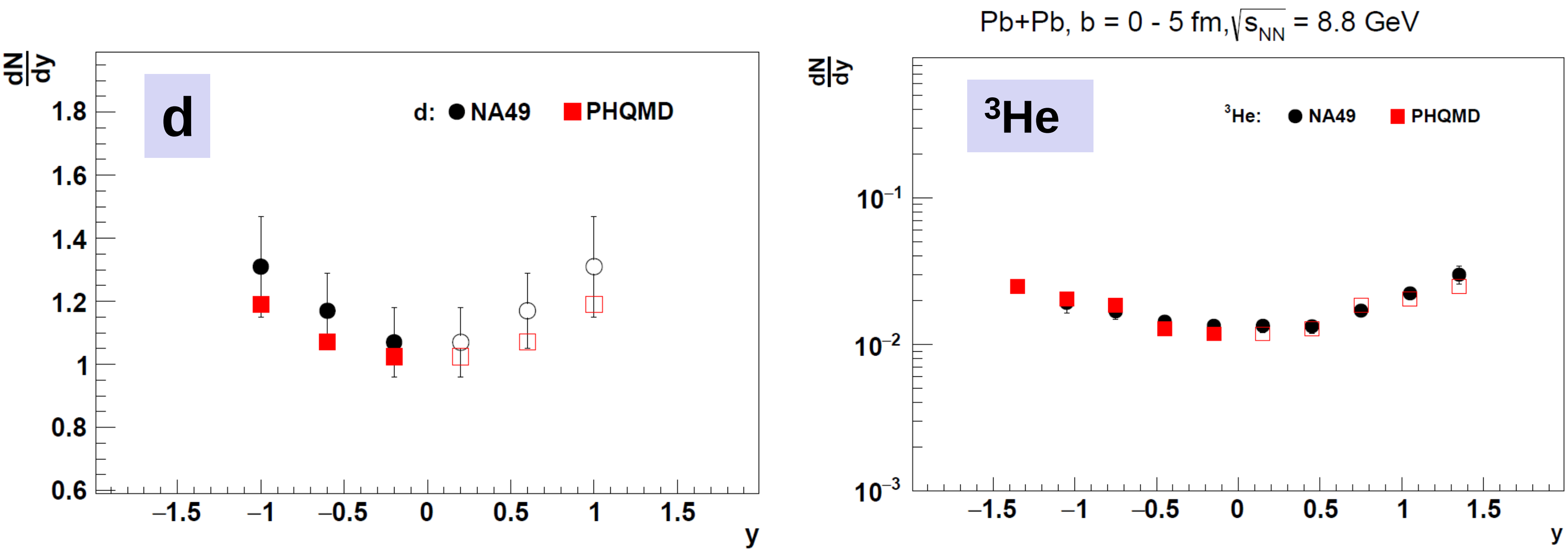
The p_T - distributions of t and ^{3}He from Au+Pb at 10.6 A GeV





Cluster production in HIC at SPS energies

The rapidity distributions of d and ^{3}He from Pb+Pb at 30 A GeV

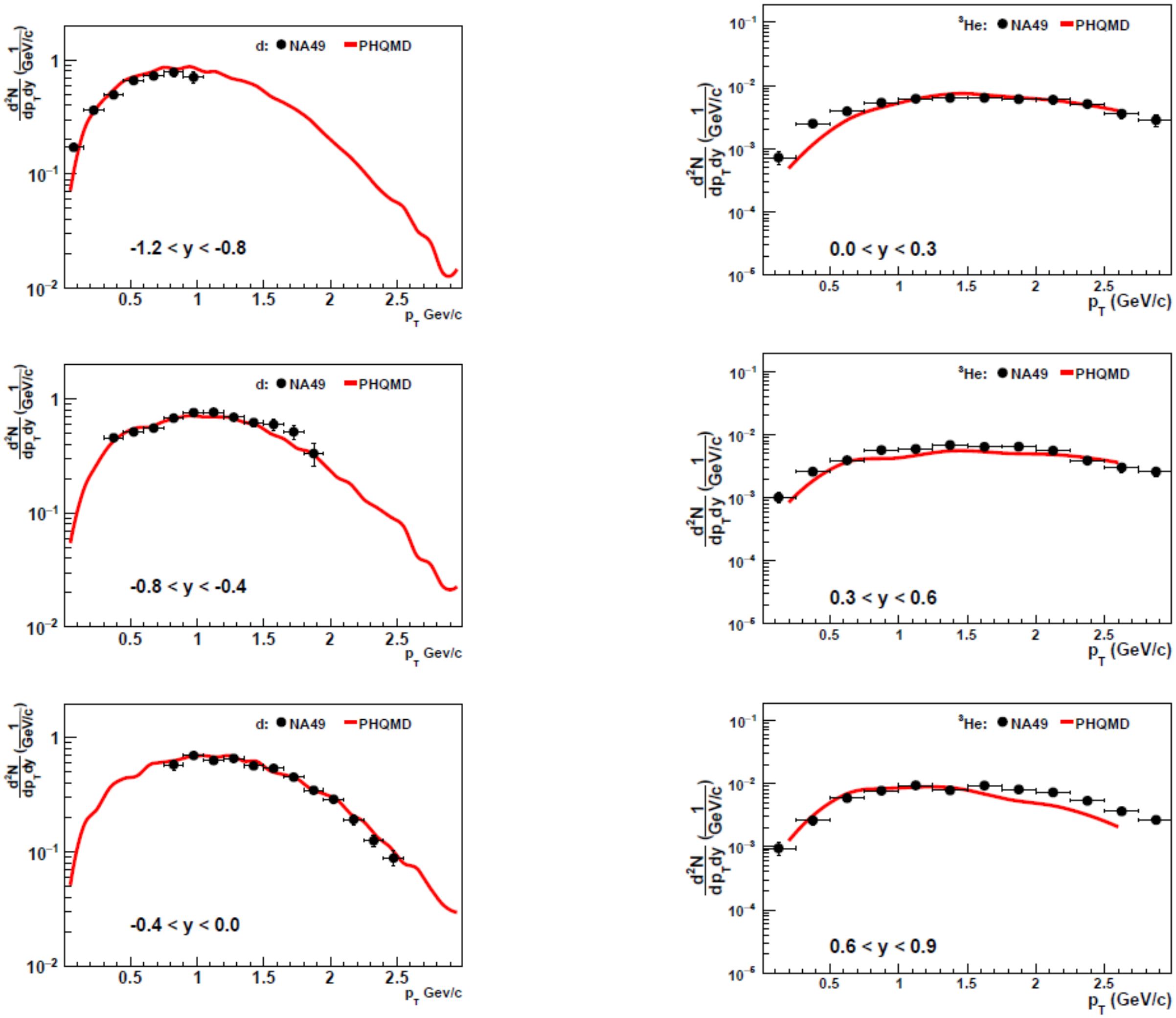


The PHQMD results for d and ^{3}He agree with NA49 data

Cluster production in HIC at SPS energies

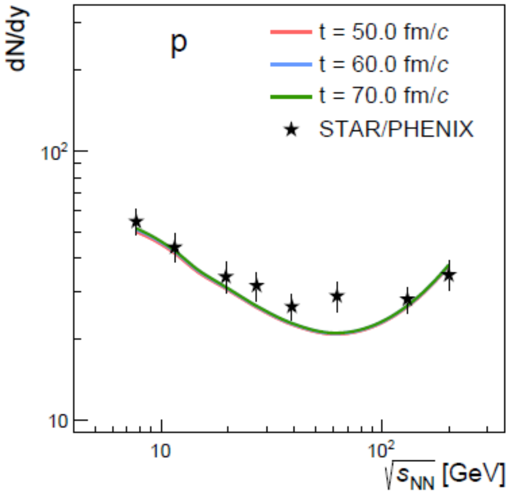
The p_T - distributions of d and ^{3}He from Pb+Pb at 30 A GeV

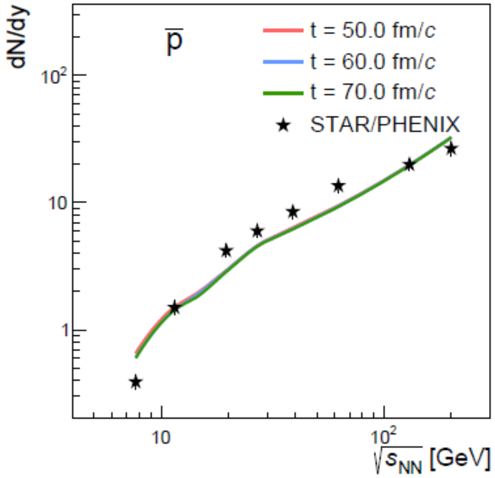
d

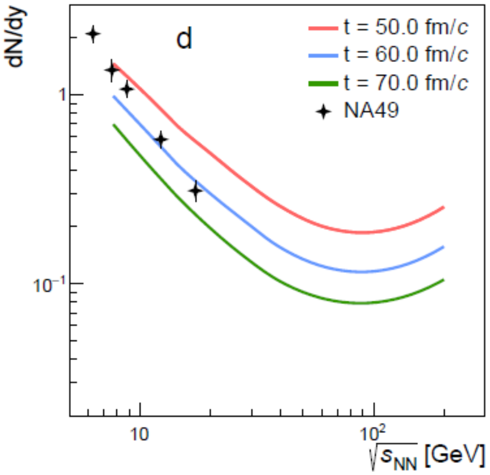


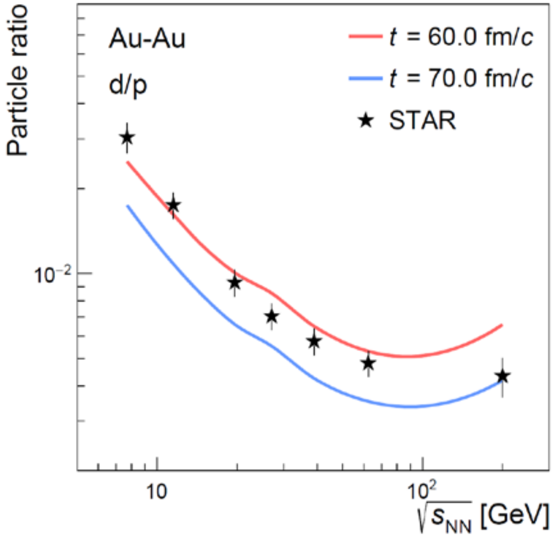
^{3}He

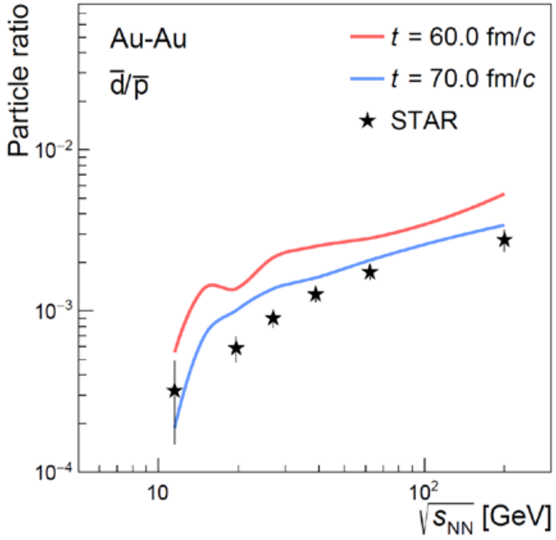
Excitation function of multiplicity of p, $\bar{p}$, d, $\bar{d}$





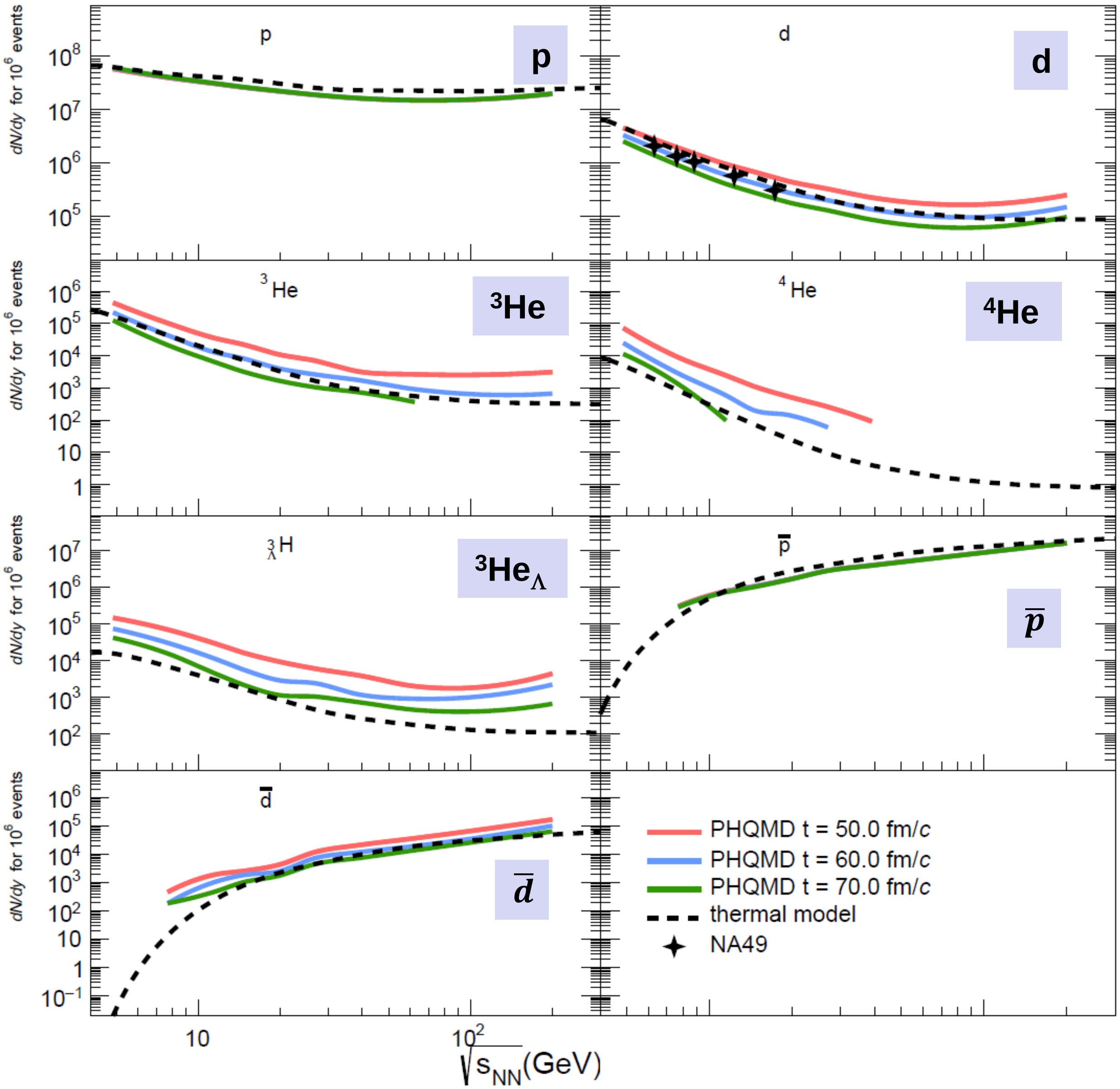






The p, $\bar{p}$ yields at y~0 are stable, the d, $\bar{d}$ yields are better described at *t= 60-70* fm/c

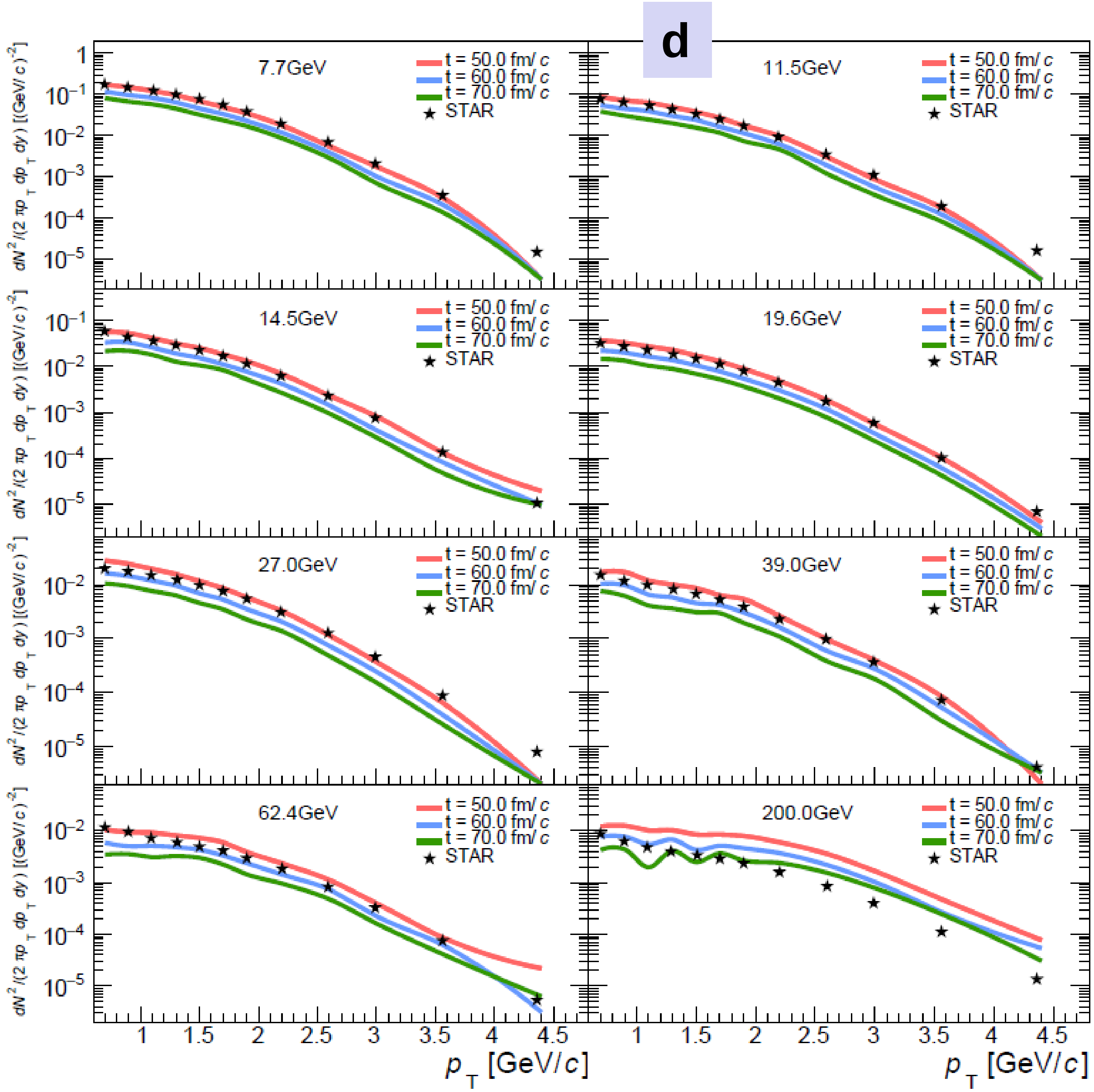
The PHQMD excitation function of cluster production versus thermal model



Comparison of the PHQMD results for Cluster and hypernuclei $^3H_\Lambda$ with thermal model and NA49 data

Thermal model:
A. Andronic et al., PLB 697 (2011) 203

Deuteron p_T spectra from 7.7GeV to 200 GeV



Comparison of the PHQMD results for the deuteron p_T-spectra at midrapidity with STAR data

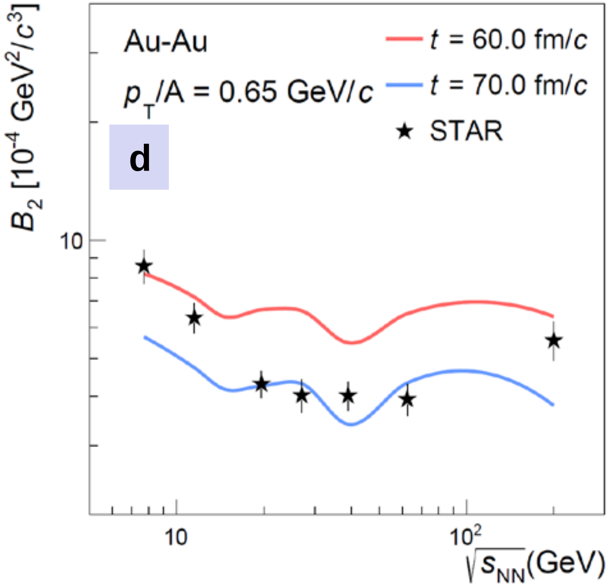
Coalescence parameter B_2 for deuterons

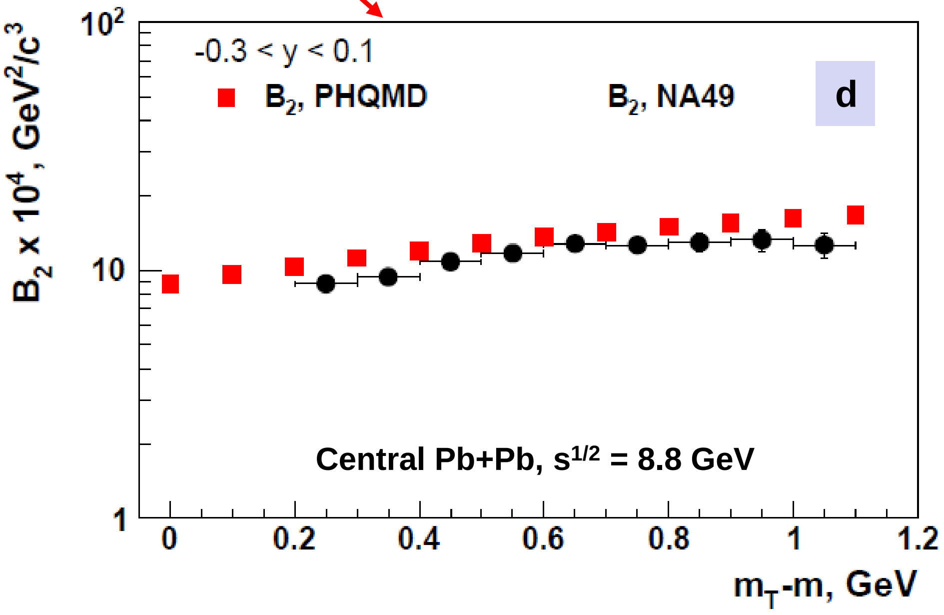
Coalescence parameter B_2:

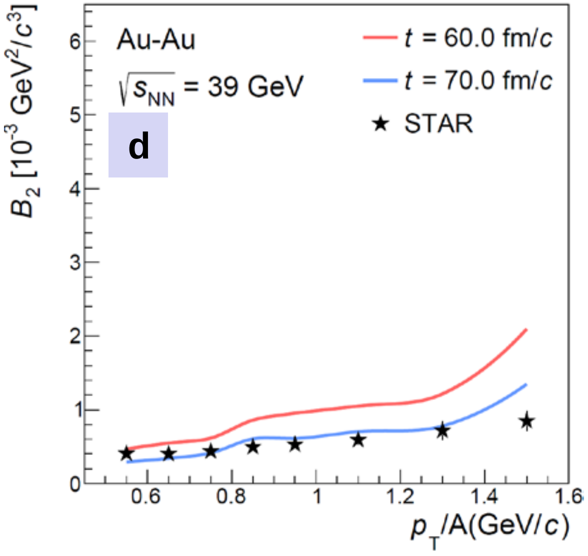
$$B_2 = \frac{E_d \frac{d^3 N_d}{d^3 P_d}}{\left(E_p \frac{d^3 N_p}{d^3 p_p} |_{p_p = P_d/2} \right)^2}$$

Comparison of the PHQMD results with NA49 and STAR data

central Au+Au collisions



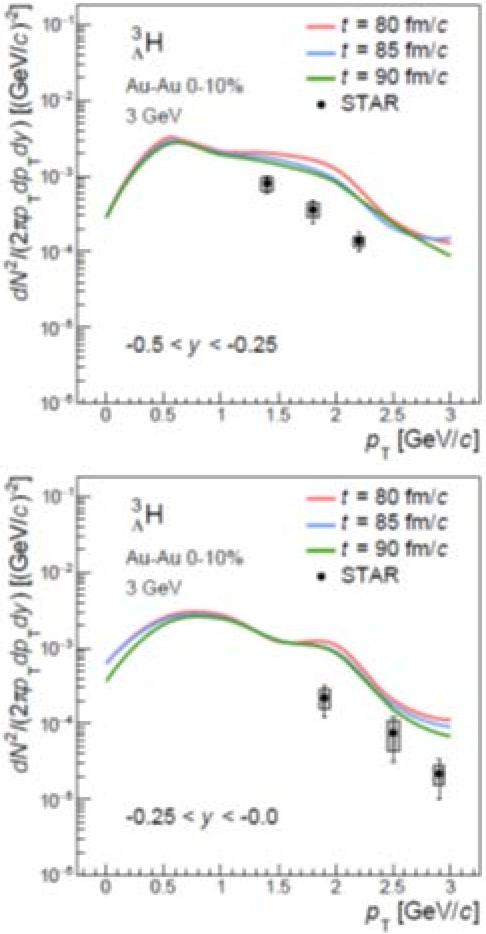




Hypernuclei production at $s^{1/2}$ = 3 GeV

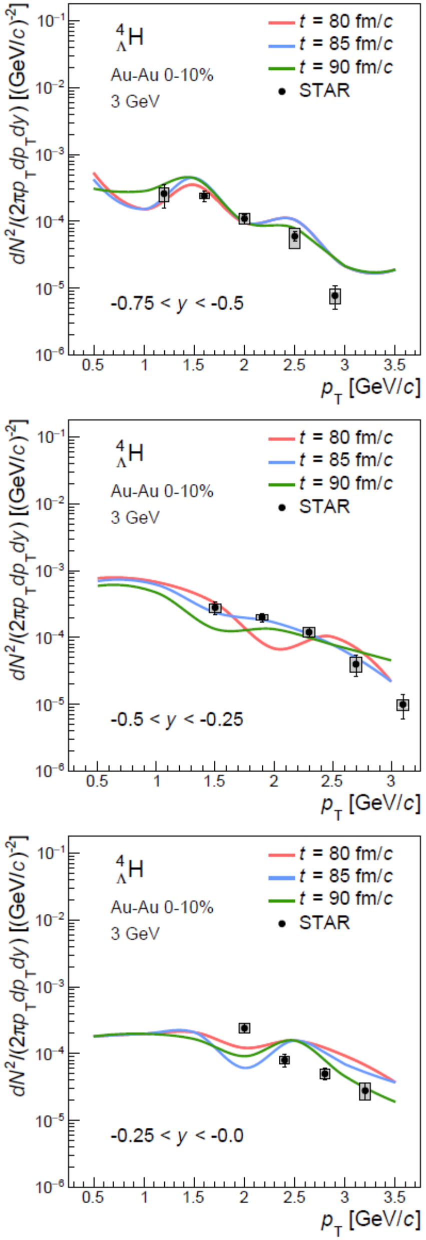
The PHQMD comparison with most recent STAR fixed target p_T distribution of ${}^3H_\Lambda$, ${}^4H_\Lambda$ from Au+Au central collisions at $\sqrt{s}=3$ GeV

- **Assumption on nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$**



Star data preliminary

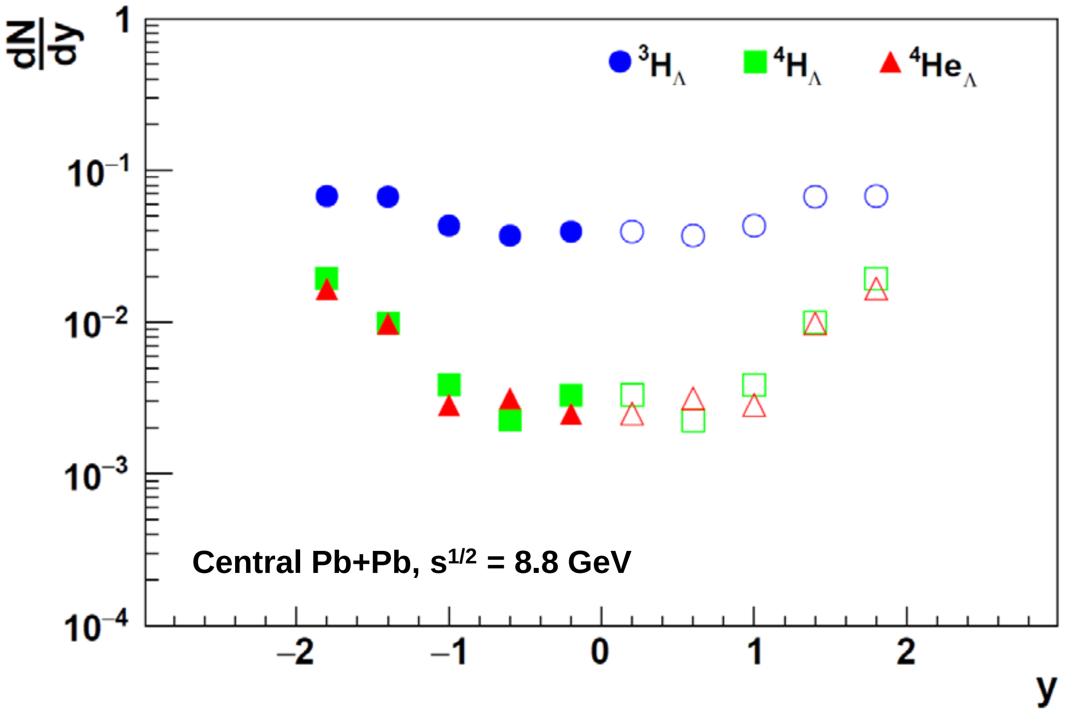
Good description in view of these very complex hypernuclei



Hypernuclei production at $s^{1/2}$ = 8.8 GeV

The PHQMD predictions on the rapidity distribution of $^{3}H_{\Lambda}$, $^{4}H_{\Lambda}$ and $^{4}He_{\Lambda}$ from Pb+Pb central collisions at 30 A GeV ($s^{1/2}$ = 8.8 GeV)

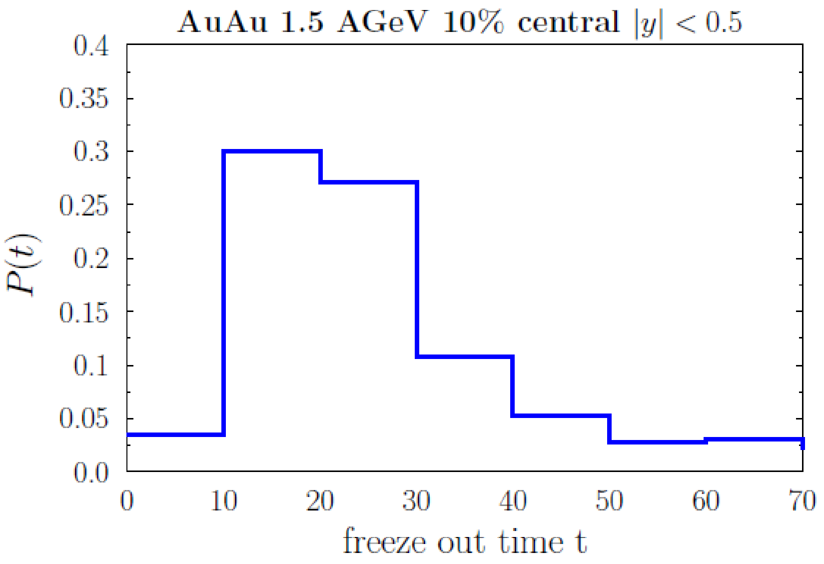
- **Assumption on nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$**

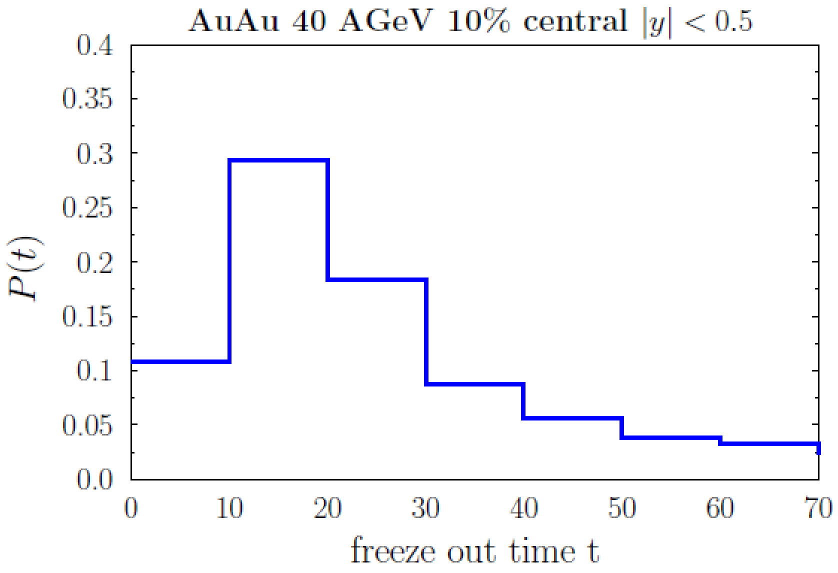


When does the system freeze out?

- ❑ **The normalized distribution of the freeze-out time of baryons (nucleons and hyperons) which are finally observed at mid-rapidity |y|<0.5**

*** Here freeze-out time as defined as a last elastic or inelastic collision, after that only potential interaction between baryons occurs**



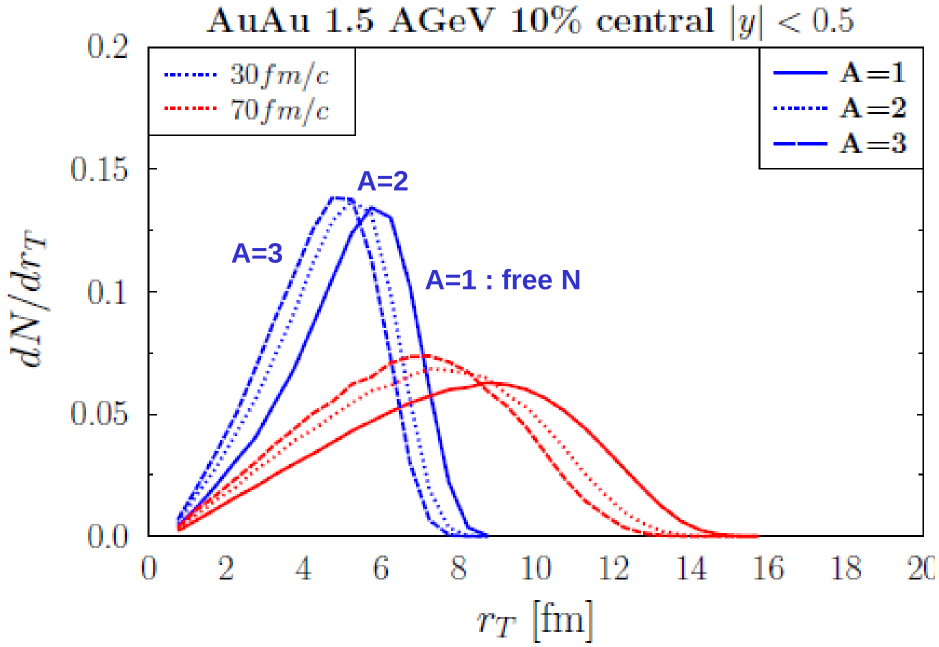


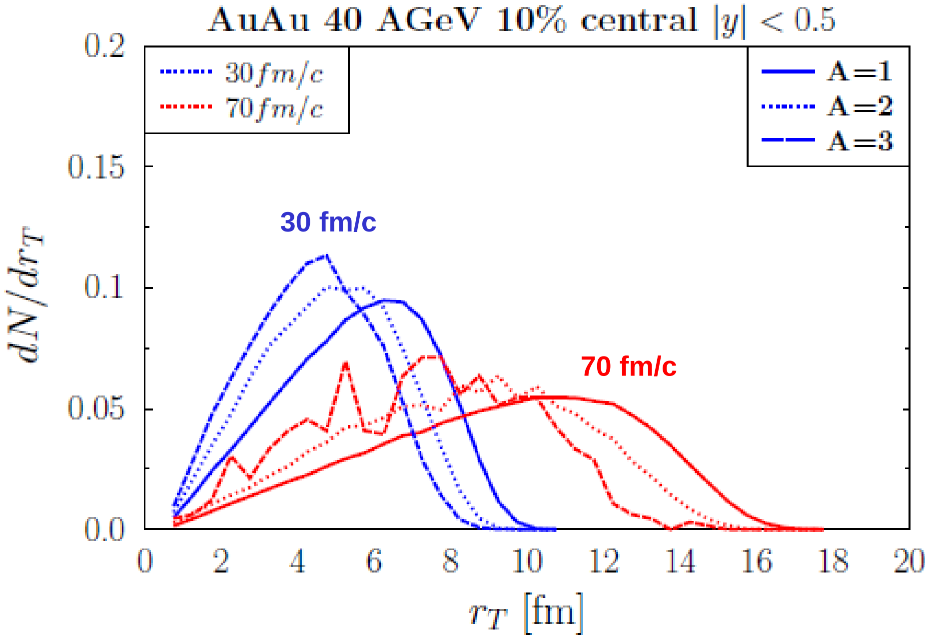
➔ Freeze-out time of baryons in Au+Au at 1.5 AGeV and 40 AGeV:

- **similar profile since expansion velocity of mid-rapidity fireball is roughly independent of the beam energy**

Where are the clusters formed?

- The snapshot (taken at time 30 and 70 fm/c) of the normalized distribution of the transverse distance r_T of the nucleons to the center of the fireball.
- It is shown for A=1 (free nucleons) and for the nucleons in A=2 and A=3 clusters

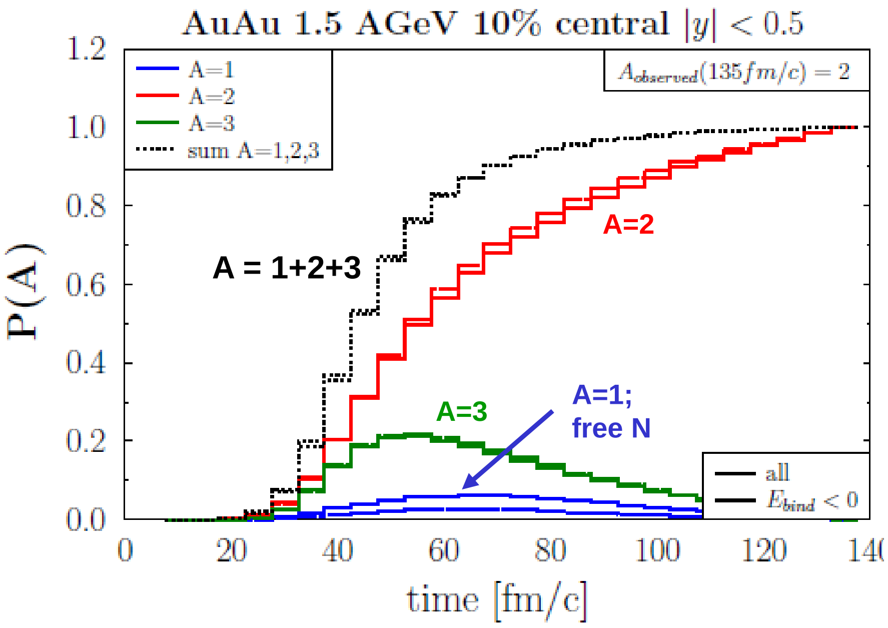


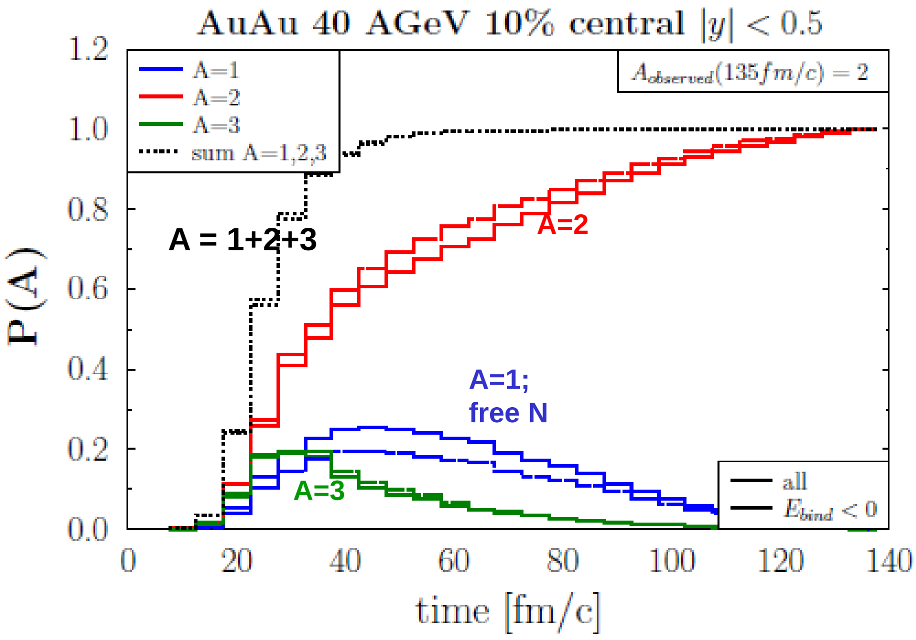


➔ Transverse distance profile of free nucleons and clusters are different! Clusters are mainly formed behind the "front" of free nucleons of the expanding fireball

Where are the clusters formed?

- ❑ **The conditional probability P(A) that the nucleons, which are finally observed in A=2 clusters at time 135 fm/c, were at time *t* the members of A=1 (free nucleons), A=2 or A=3 clusters**





➔ **Stable clusters (observed at 135 fm/c) are formed shortly after the dynamical freeze-out**

PHQMD

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation
Clusters are identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

- **provides the good description of ‘bulk’ observables from SIS to RHIC energies**

- **predicts the dynamical formation of clusters from SIS to RHIC energies due to the interactions among the nucleons**

- **reproduces cluster data on dN/dy and dN/dp_T as well as ratios d/p and $\bar{d}/\bar{p}$ for HI collisions from AGS to top RHIC energies.**

A detailed analysis reveals that clusters are formed

- **shortly after elastic and inelastic collisions have ceased**
- **behind the front of the expanding energetic hadrons**
- **since the ‘fire’ is not at the same place as the ‘ice’, cluster can survive.**

Outlook:

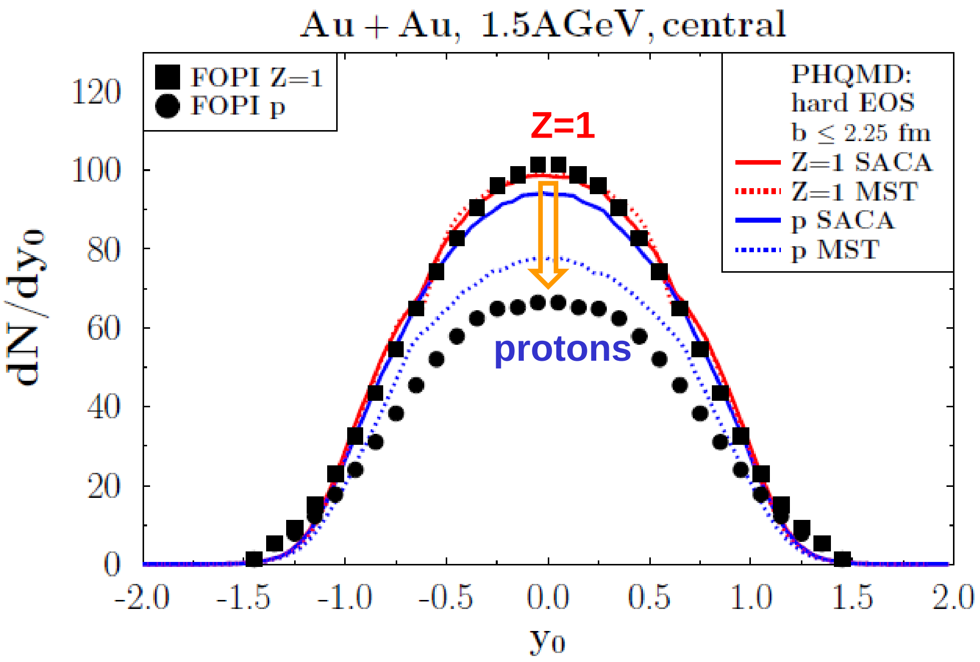
- **extension to LHC energies and study of hyper-nuclei with more realistic potentials**

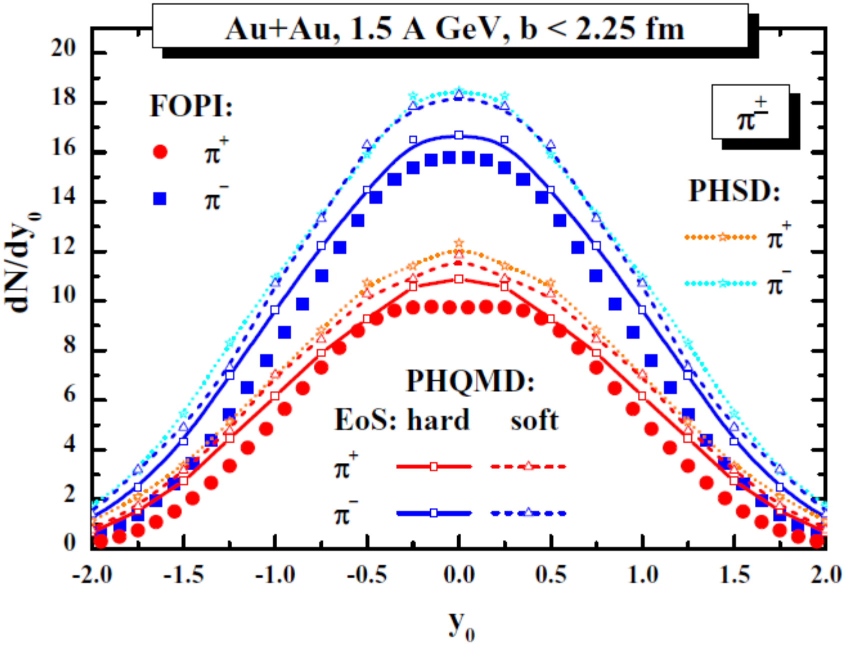
Thank you for your attention !

Thanks to the Organizers !

PHQMD: light clusters and ‚bulk‘ dynamics at SIS

Scaled rapidity distribution $y_0 = y/y_{proj}$ in central Au+Au reactions at 1.5 AGeV





- **30% of protons are bound in clusters at 1.5 A GeV**
- **Presently MST is better identifying light clusters than SACA**
  ➔ To improve in SACA: more realistic potentials for small clusters, quantum effects

- **Pion spectra are sensitive to EoS: better reproduced by PHQMD with a ‘hard’ EoS**
- **PHQMD with soft EoS is consistent with PHSD (default – soft EoS)**
  *** To improve in PHQMD: momentum dependent potentials**

Cluster formation: QMD vs MF

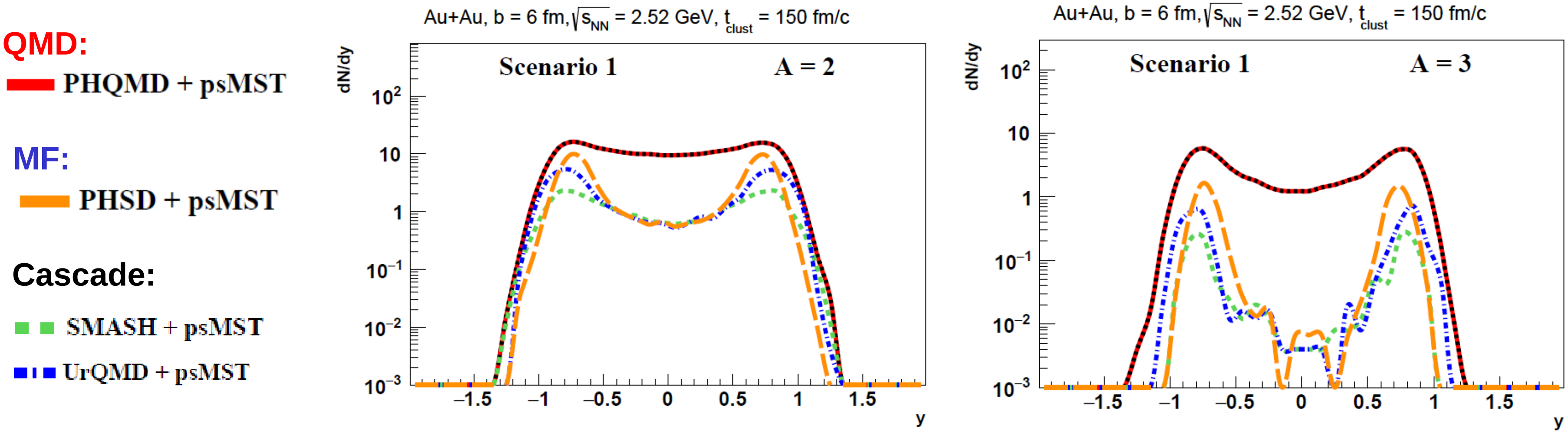
- ☐ **Cluster formation is sensitive to nucleon dynamics**

➔ One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:

- **QMD (quantum-molecular dynamics) – allows to keep correlations**
- **MF (mean-field based models) – correlations are smeared out**
- **Cascade – no correlations by potential interactions**

Example: Cluster stability over time:

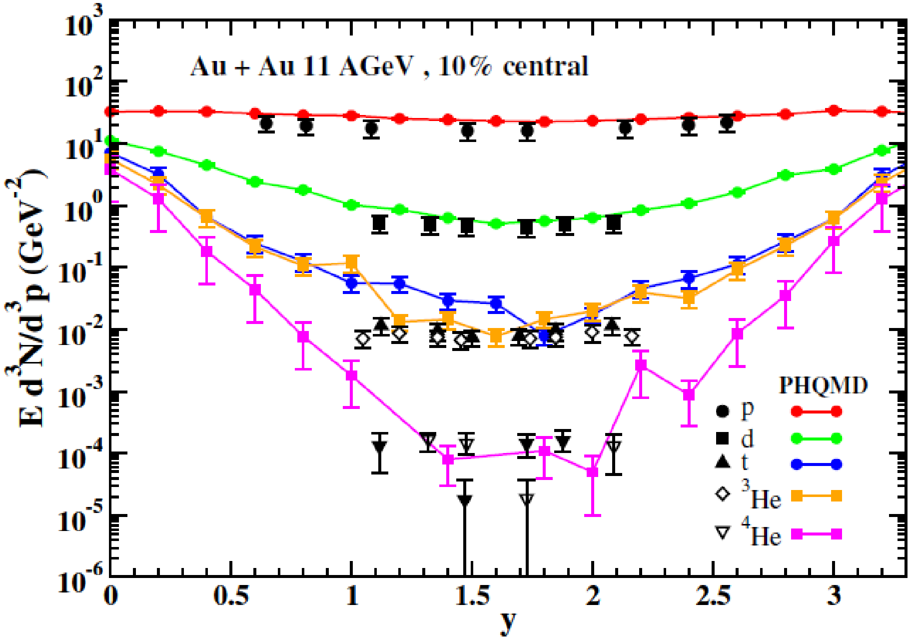
V. Kireyeu, 2103.10542



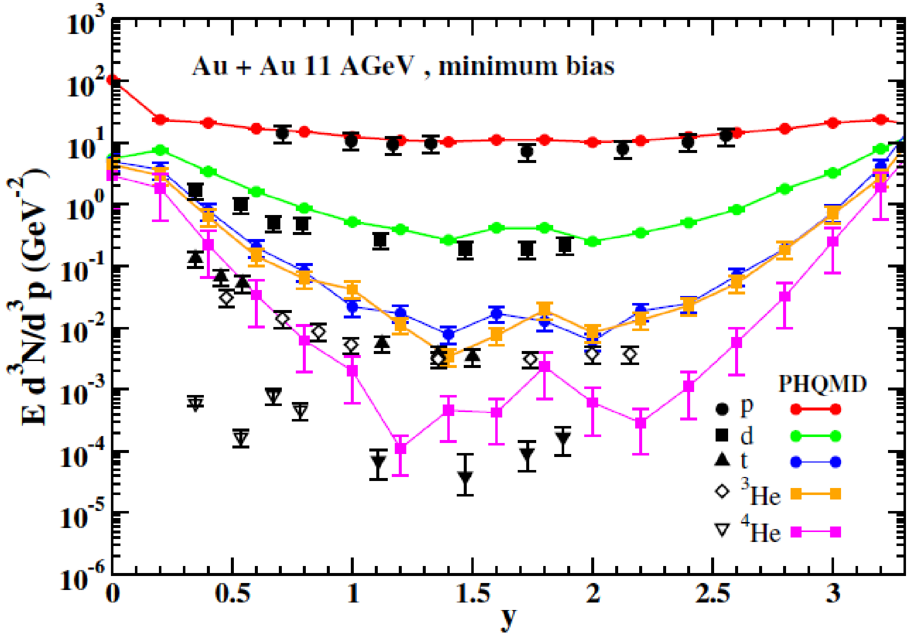
PHQMD: light clusters at AGS energies

The invariant multiplicities for p, d, t, ^{3}He, ^{4}He at p_T <0.1 GeV versus rapidity

Au+Au, 11 AGeV, 10% central



Au+Au, 11 AGeV, minimal bias



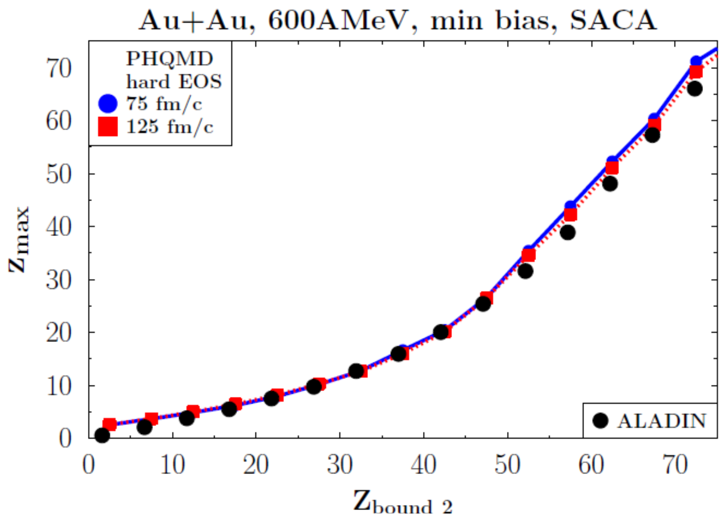
PHQMD: clusters recognition by **MST** provides a reasonable description of exp. data on light clusters at AGS energies

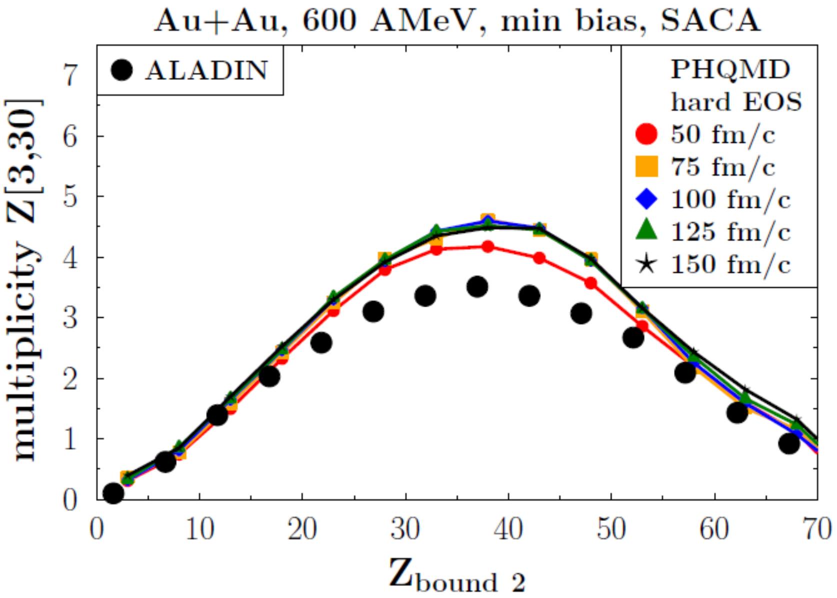
PHQMD: heavy clusters

Heavy clusters (**spectator fragments**): **experim. measured up to E_{beam} =1 AGeV (ALADIN Collab.)**

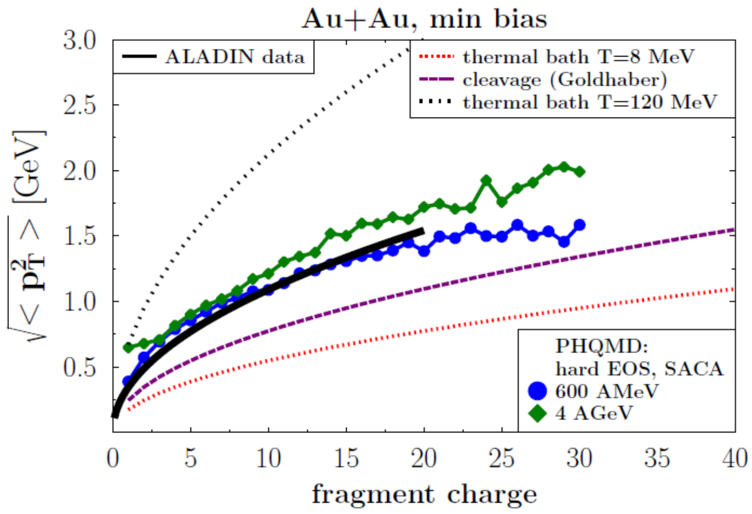
PHQMD with SACA shows an agreement with ALADIN data for very complex cluster observables as

- ☐ **Largest clusters (Z_{bound})**
- ☐ **Energy independent 'rize and fall'**
- ☐ **Rms p^2_T**





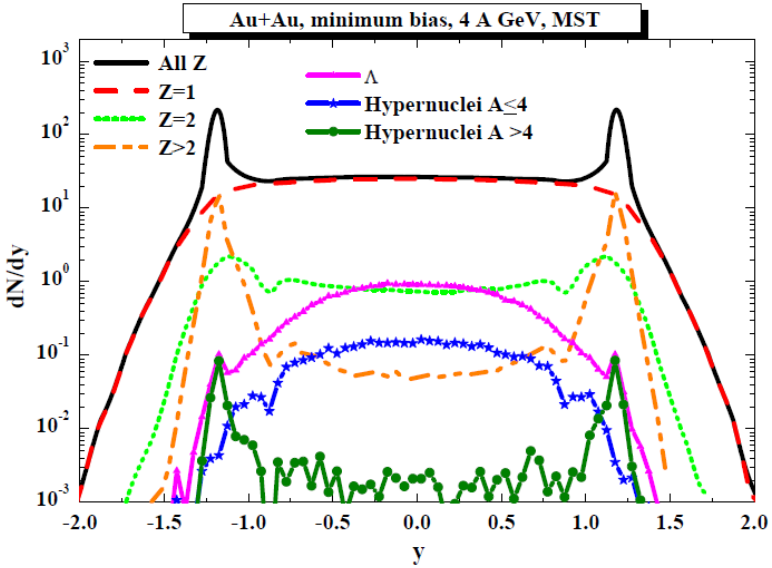
$$Z_{\text{bound 2}} = \sum_i Z_i \, \Theta(Z_i - (1+\epsilon)) \qquad (\epsilon < 1)$$

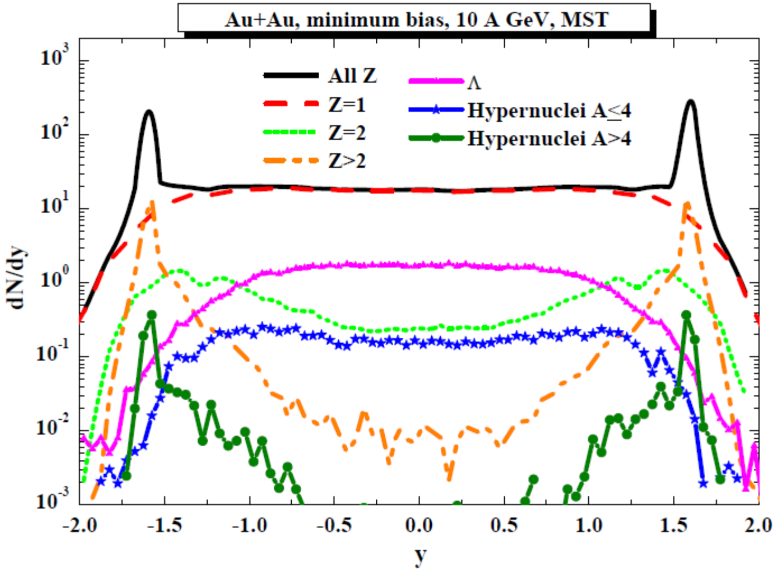


PHQMD shows $\sqrt{p_T^2(Z)} \propto \sqrt{Z}$ **dependence as exp. data**

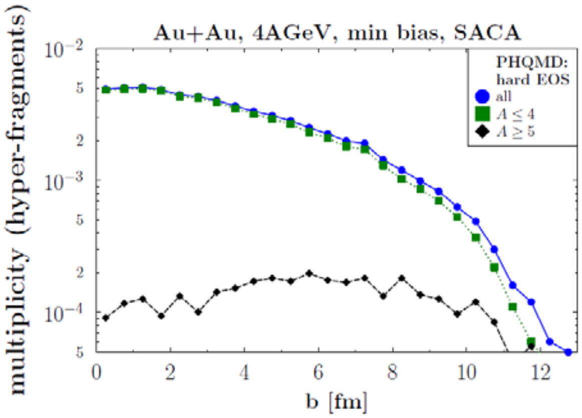
PHQMD: hypernuclei

PHQMD results (with a hard EoS and MST algorithm) for the rapidity distributions of all charges, Z = 1 particles, Z=2, Z>2, as well as Λ's, hypernuclei A$\leq$4 and A>4 for Au+Au at 4 and 10AGeV





The multiplicity of light hypercluster vs. impact parameter b for Au+Au, 4 AGeV

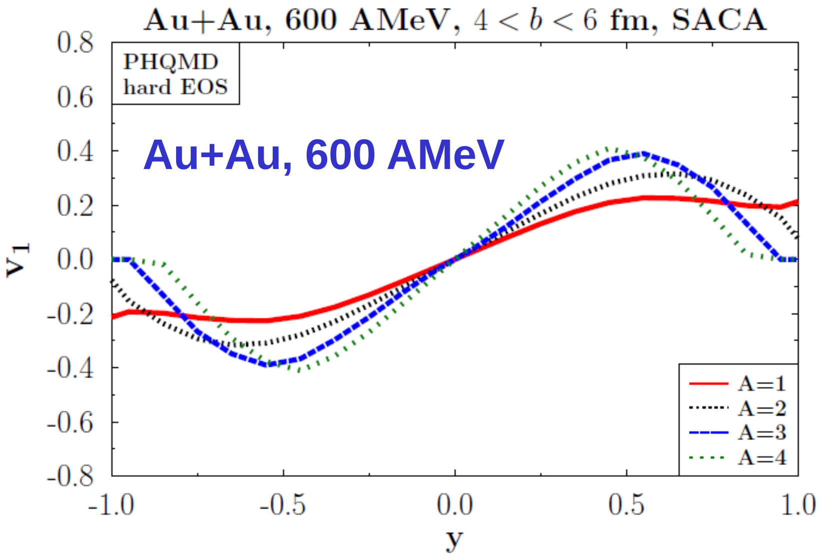


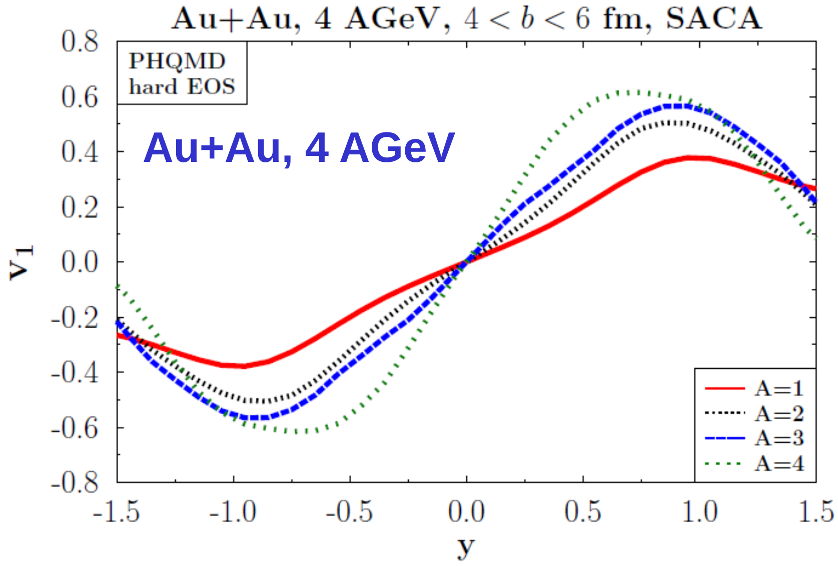
- ☐ **Central collisions → light hypernuclei**
- ☐ **Peripheral collisions → heavy hypernuclei**

Penetration of Λ's, produced at midrapidity, to target/projectile region due to rescattering

→ Possibility to study ΛN interaction

PHQMD: collectivity of clusters



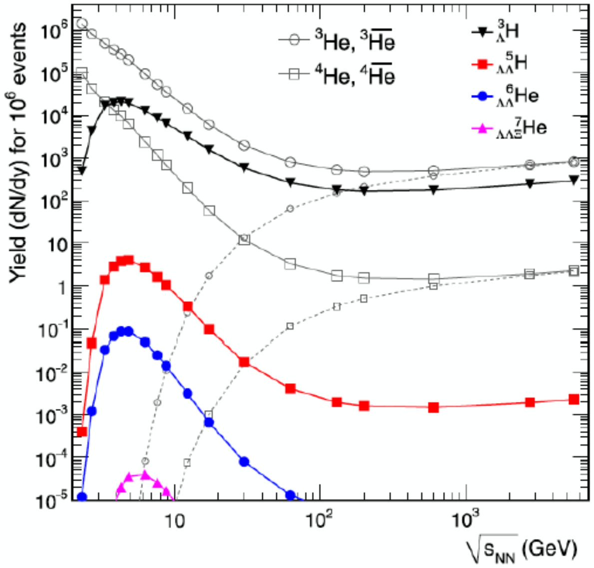


PHQMD with hard EoS, with SACA:
v_1 of light clusters (A=1,2,3,4) vs rapidity
for mid-central Au+Au at 600 AMeV, 4AGeV

- **v_1 : quite different for nucleons and clusters (as seen in experiments)**
- **Nucleons come from participant regions (→ small density gradient) while clusters from interface spectator-participant (strong density gradient)**
- **v_1 increases with E_{beam}**
- **→ larger density gradient**

Modeling of cluster and hypernuclei formation

A. Andronic et al., PLB 697, 203 (2011)



Existing models for clusters formation:

❑ **statistical model:**

- **assumption of thermal equilibrium**

❑ **coalescence model:**

- **determination of clusters at a given time by coalescence radii in coordinate and momentum spaces**

➔ **don‘t provide information on the dynamics of clusters formation**

In order to understand a microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

→ **transport models:**

- **dynamical modeling of cluster formation based on interactions**

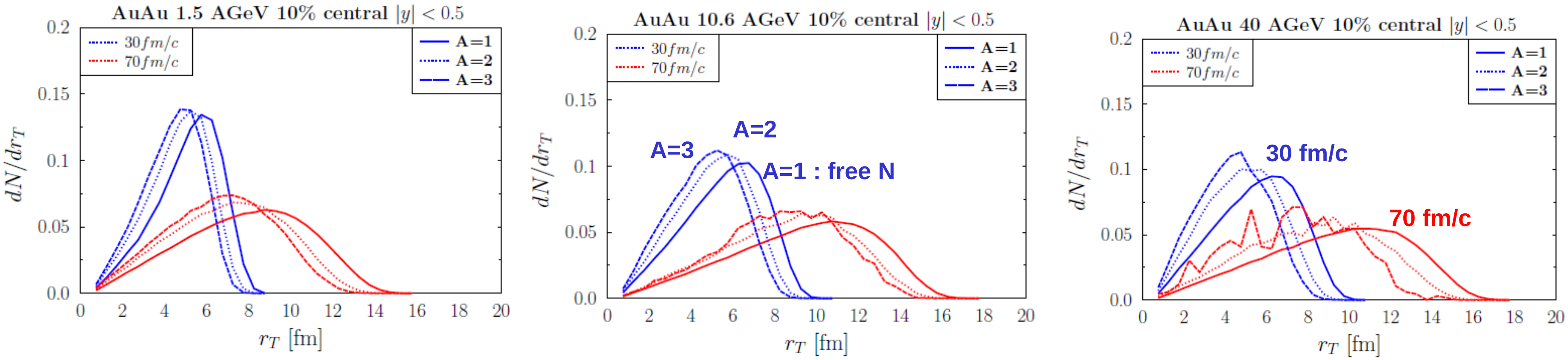
❑ **Cluster formation is sensitive to nucleon dynamics**

➔ **One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:**

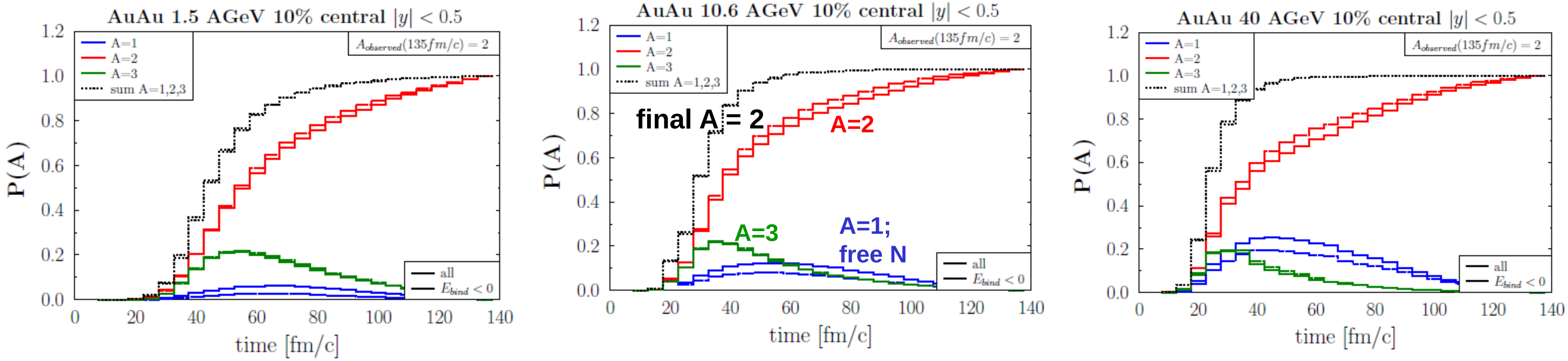
- **QMD (quantum-molecular dynamics) – allows to keep correlations**
- **MF (mean-field based models) – correlations are smeared out**

Where are the clusters formed?

- The normalized distribution of the transverse distance of the nucleons, observed at midrapidity (A=1,2,3)



- The probability distribution P(A) of the formation time of clusters at midrapidity
  - the probabilities that the finally observed A = 2 cluster has been at time *t* a part of A=1 (free nucleons), A=2 or A=3 clusters



➔ Stable clusters are formed during dynamical freeze-out

Cluster stability in semi-classical models

Problems of the semi-classical models (as QMD):

QMD cannot project the n-body density on the ground state of a cluster as a quantum system of fermions

Quantum ground state has to respect a minimal average kinetic energy of the nucleons while the semi-classical (QMD) ground state - not!

➔ nucleons may still be emitted from the clusters even if in the corresponding quantum system this is not possible anymore

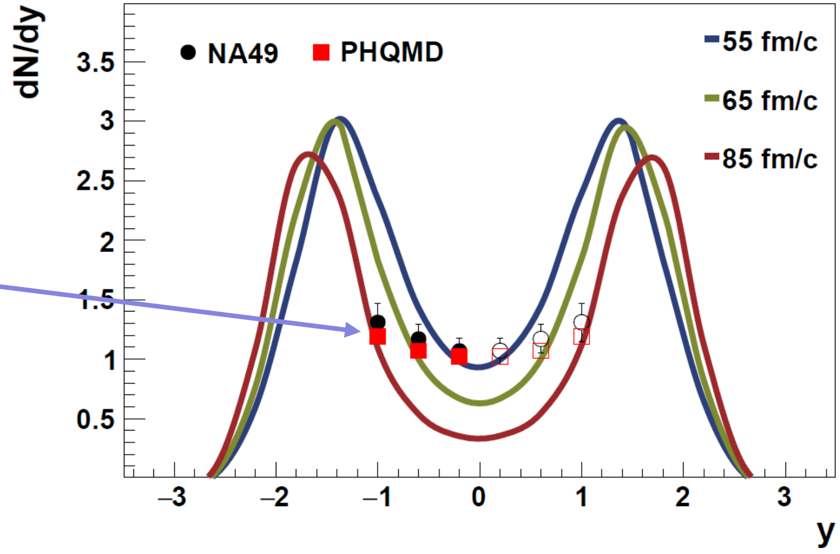
= QMD clusters are not fully stable over time

➔ the multiplicity of clusters is time dependent

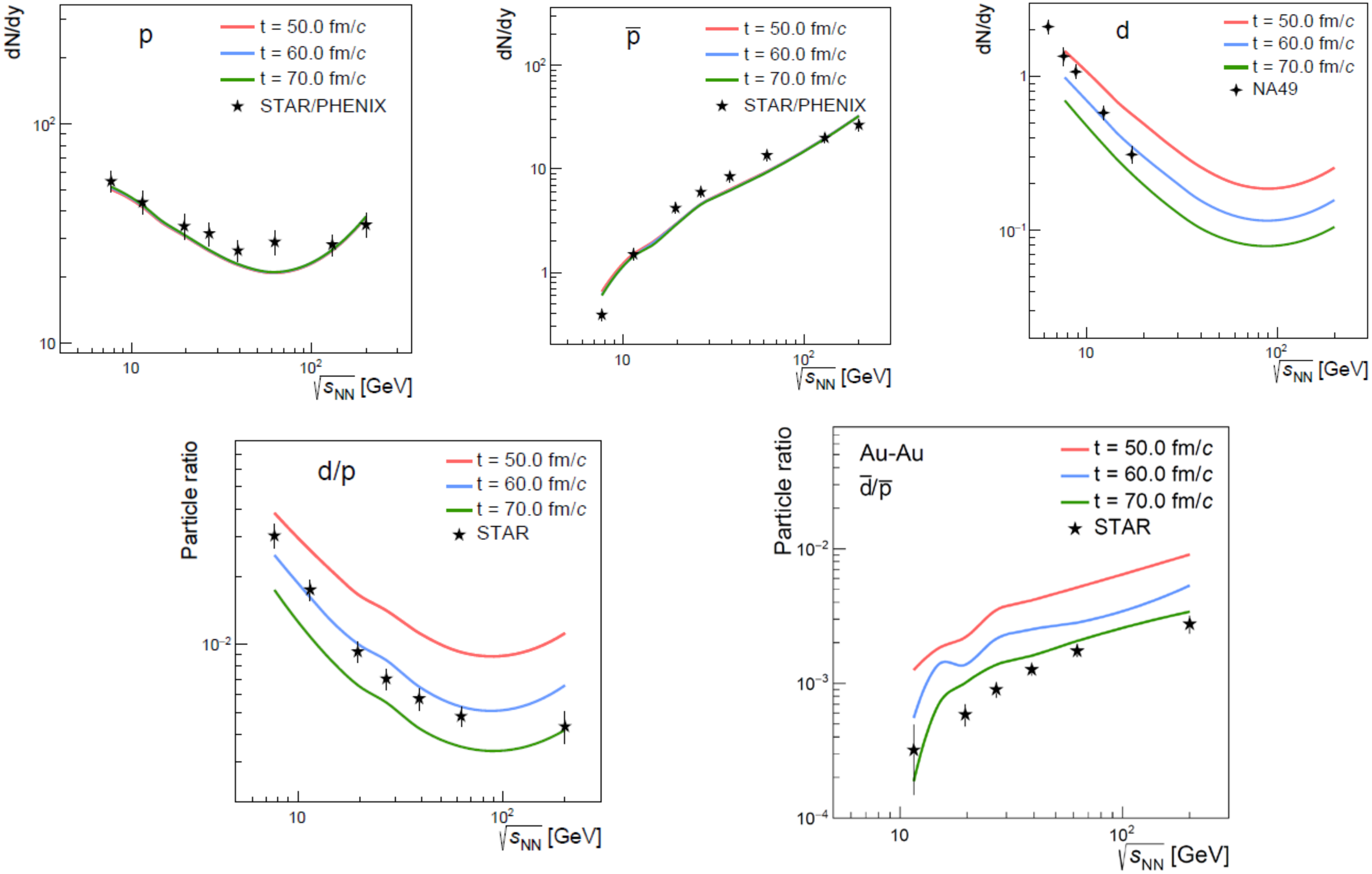
In this study the PHQMD results are taken at 'physical time' :

$$t = t_0 \cosh(y)$$

where t_0 is the time selected as a best description of the cluster multiplicity at y=0



Excitation function of multiplicity of $p, \overline{p}, d, \overline{d}$



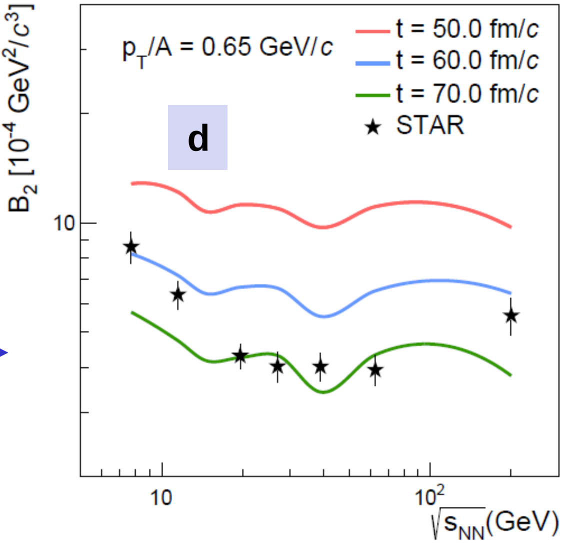
The p, $\overline{p}$ yields at y~0 are stable, the d, $\overline{d}$ yields are better described at *t= 60-70* fm/c

Coalescence parameter B_2 for deutrons

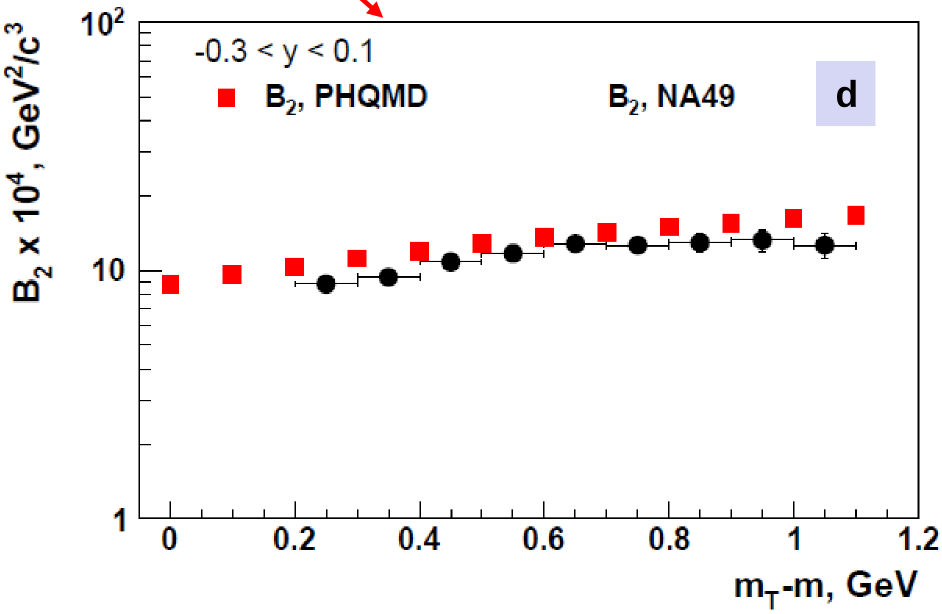
Coalescence parameter B_2:

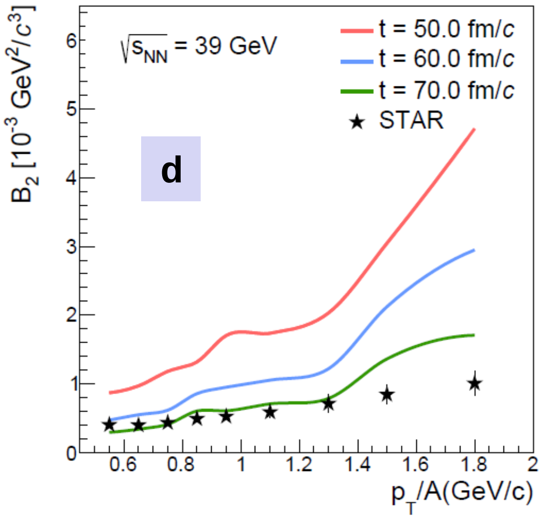
$$B_2 = \frac{E_d \frac{d^3N_d}{d^3P_d}}{\left(E_p \frac{d^3N_p}{d^3p_p}\Big|_{p_p=P_d/2}\right)^2}$$

central Au+Au collisions



Comparison of the PHQMD results with NA49 and STAR data





Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation
Clusters are identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

PHQMD

- provides the good description of hadronic 'bulk' observables from SIS to RHIC energies
- predicts the dynamical formation of clusters from low to ultra-relativistic energies due to the interactions
- allows to study the origin as well as the properties of cluster formation (rapidity and p_T spectra)
- allows to study the formation of hypernuclei originated from ΛN interactions