Testing General Relativity with Gravitational-Wave Observations

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On behalf of the LIGO Scientific and Virgo Collaborations

Based on:

"Tests of General Relativity with Binary Black Holes from the Second LIGO-Virgo Gravitational-Wave Transient Catalog"

R Abbott et al, arXiv:2010.14529, PRD 103 122002 (2021)

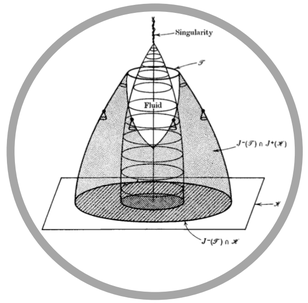
What Can We Learn From GWs?

(a snapshot anyway)

- Probe spacetime in strong-field regime: large compactness $\mathscr{C} = \dfrac{M}{R} \sim \mathcal{O}(1)$, characteristic velocities $\mathscr{V} \sim \mathcal{O}(c)$
- Complementary to other probes: cosmological, solar system, double pulsar, atom interferometry, etc
- Many fundamental questions on the nature of compact objects and gravity…

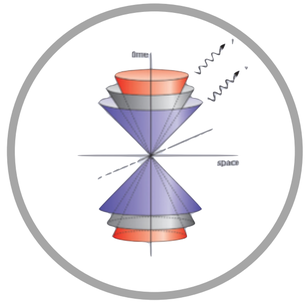
Black holes?

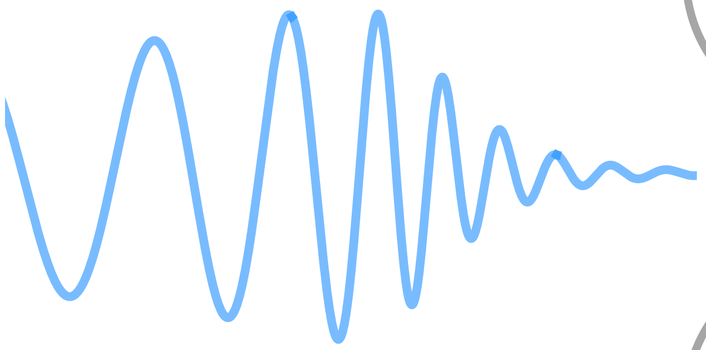
What is the fundamental nature of black holes? Do they have a horizon? Are they governed by the Kerr metric?



Lorentz invariance?

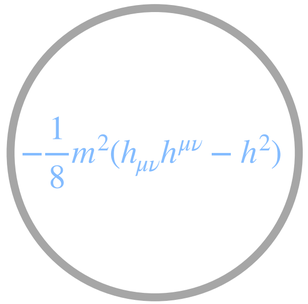
Is Lorentz invariance a fundamental symmetry of nature? Is it broken on some energy scale?





Gravitational fields?

Do we have additional gravitational fields beyond $g_{\mu\nu}$? Are gravitons massive?



Extra dimensions?

Are there large extra dimensions?

Figueras+17

[1]

How Do We Test GR With GWs…?

(a snapshot anyway)

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- Key limitation! Lack understanding of strong-field in nearly all modified theories of gravity…
- So we focus on testing *General Relativity* by using it as our *null hypothesis*...
  - Are our results consistent with the predictions of GR?
  - Use parameterized deformations of GR to constrain *degree* to which deviations agree with data
  - Split into: i) tests of GW propagation, ii) tests of GW generation, iii) GR consistency tests
- Recent advances in modelling binaries in beyond-GR theories both *analytically* and *numerically*
  - Not yet clear if full theories are mathematically well posed, physically viable, etc
  - Not yet at stage where results can be used in GW data analysis

How Do We Test GR With GWs…?

(a snapshot anyway)

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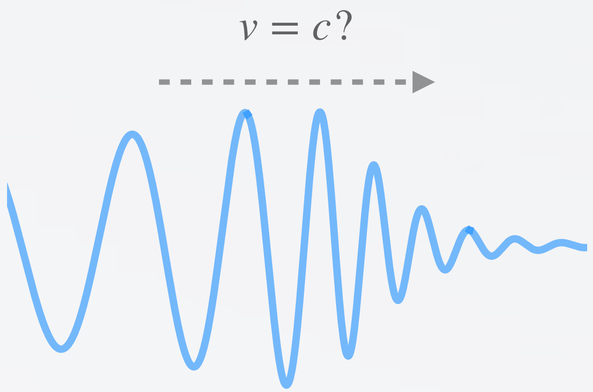
- Quick summary of GWTC-2 (see also talk by Lazzaro Claudia on Weds.)
- Use events detected by LIGO-Virgo up to 1st October 2019
- Impose more stringent cut on events, must have false-alarm-rate (FAR) $< 10^{-3}$ per year
- 24 new events in O3a + BBH from GWTC-1
- Additional cuts imposed by tests

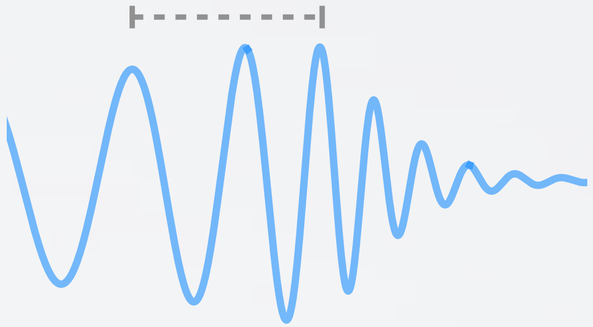
Event	Inst.	Properties					SNR	Tests performed							
		D_L [Gpc]	$(1+z)M$ [$M_\odot$]	$(1+z)\mathcal{M}$ [$M_\odot$]	$(1+z)M_f$ [$M_\odot$]	χ_f		RT	IMR	PAR	SIM	MDR	RD	ECH	POL
GW190408_181802	HLV	$1.55^{+0.40}_{-0.60}$	$55.5^{+3.5}_{-3.8}$	$23.7^{+1.4}_{-1.7}$	$53.0^{+3.2}_{-3.4}$	$0.67^{+0.06}_{-0.07}$	$15.3^{+0.2}_{-0.3}$	✓	✓	✓	✓	✓	✓	✓	✓
GW190412	HLV	$0.74^{+0.14}_{-0.17}$	$44.2^{+4.5}_{-4.6}$	$15.2^{+0.2}_{-0.2}$	$42.9^{+4.6}_{-4.7}$	$0.67^{+0.05}_{-0.06}$	$18.9^{+0.2}_{-0.3}$	✓	–	✓	✓	✓	–	✓	✓
GW190421_213856	HL	$2.88^{+1.37}_{-1.38}$	$108.7^{+15.3}_{-12.4}$	$46.6^{+6.6}_{-6.0}$	$103.9^{+14.1}_{-11.3}$	$0.67^{+0.10}_{-0.11}$	$10.7^{+0.2}_{-0.4}$	✓	✓	✓	–	✓	✓	✓	–
GW190503_185404	HLV	$1.45^{+0.69}_{-0.63}$	$91.6^{+11.2}_{-11.8}$	$38.6^{+5.3}_{-6.0}$	$87.6^{+10.2}_{-10.8}$	$0.66^{+0.09}_{-0.12}$	$12.4^{+0.2}_{-0.3}$	✓	✓	✓	–	✓	✓	✓	✓
GW190512_180714	HLV	$1.43^{+0.55}_{-0.55}$	$45.3^{+3.9}_{-2.8}$	$18.6^{+0.9}_{-0.8}$	$43.5^{+4.0}_{-2.8}$	$0.65^{+0.07}_{-0.07}$	$12.2^{+0.2}_{-0.4}$	✓	–	✓	✓	✓	✓	✓	✓
GW190513_205428	HLV	$2.06^{+0.88}_{-0.80}$	$73.6^{+12.7}_{-6.7}$	$29.5^{+5.6}_{-2.5}$	$70.6^{+11.5}_{-6.7}$	$0.68^{+0.14}_{-0.12}$	$12.9^{+0.3}_{-0.4}$	✓	✓	✓	–	✓	✓	✓	✓
GW190517_055101	HLV	$1.86^{+1.62}_{-0.84}$	$85.4^{+9.6}_{-7.3}$	$35.9^{+4.0}_{-3.4}$	$79.8^{+8.8}_{-6.4}$	$0.87^{+0.05}_{-0.07}$	$10.7^{+0.4}_{-0.6}$	✓	–	✓	–	✓	–	✓	✓
GW190519_153544	HLV	$2.53^{+1.83}_{-0.92}$	$155.1^{+16.7}_{-17.9}$	$65.1^{+7.7}_{-10.3}$	$146.8^{+14.7}_{-15.4}$	$0.79^{+0.07}_{-0.13}$	$15.6^{+0.2}_{-0.3}$	✓	✓	✓	–	✓	✓	✓	✓
GW190521	HLV	$3.92^{+2.19}_{-1.95}$	$269.4^{+39.8}_{-34.6}$	$114.8^{+15.2}_{-17.6}$	$256.6^{+36.6}_{-30.4}$	$0.71^{+0.12}_{-0.16}$	$14.2^{+0.3}_{-0.3}$	✓	–	✓	–	–	✓	✓	✓
GW190521_074359	HL	$1.24^{+0.40}_{-0.57}$	$92.6^{+4.8}_{-5.4}$	$39.8^{+2.2}_{-3.0}$	$88.0^{+4.3}_{-4.8}$	$0.72^{+0.05}_{-0.07}$	$25.8^{+0.1}_{-0.2}$	✓	✓	✓	✓	✓	✓	✓	–
GW190602_175927	HLV	$2.69^{+1.79}_{-1.12}$	$171.8^{+23.2}_{-20.6}$	$72.9^{+10.8}_{-13.7}$	$163.8^{+20.7}_{-18.3}$	$0.70^{+0.10}_{-0.14}$	$12.8^{+0.2}_{-0.3}$	✓	–	✓	–	✓	✓	✓	✓
GW190630_185205	LV	$0.89^{+0.56}_{-0.37}$	$69.6^{+4.2}_{-3.5}$	$29.4^{+1.6}_{-1.5}$	$66.3^{+4.2}_{-3.3}$	$0.70^{+0.05}_{-0.07}$	$15.6^{+0.2}_{-0.3}$	✓	✓	✓	✓	✓	–	✓	–
GW190706_222641	HLV	$4.42^{+2.59}_{-1.93}$	$180.3^{+23.3}_{-27.7}$	$75.1^{+11.0}_{-17.5}$	$171.1^{+20.0}_{-23.7}$	$0.78^{+0.09}_{-0.18}$	$12.6^{+0.2}_{-0.4}$	✓	✓	✓	–	✓	✓	✓	✓
GW190707_093326	HL	$0.77^{+0.38}_{-0.37}$	$23.1^{+1.8}_{-0.5}$	$9.89^{+0.1}_{-0.09}$	$22.1^{+1.9}_{-0.5}$	$0.66^{+0.03}_{-0.04}$	$13.3^{+0.2}_{-0.4}$	✓	–	✓	✓	✓	–	✓	–
GW190708_232457	LV	$0.88^{+0.33}_{-0.39}$	$36.1^{+2.5}_{-0.8}$	$15.5^{+0.3}_{-0.2}$	$34.4^{+2.7}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$13.1^{+0.2}_{-0.3}$	✓	–	✓	✓	✓	✓	✓	–
GW190720_000836	HLV	$0.79^{+0.69}_{-0.32}$	$24.9^{+5.0}_{-1.2}$	$10.4^{+0.2}_{-0.1}$	$23.7^{+5.2}_{-1.2}$	$0.72^{+0.06}_{-0.05}$	$11.0^{+0.3}_{-0.7}$	✓	–	✓	✓	✓	–	✓	✓
GW190727_060333	HLV	$3.30^{+1.54}_{-1.50}$	$104.4^{+11.9}_{-10.9}$	$44.7^{+5.3}_{-5.7}$	$99.2^{+10.7}_{-9.8}$	$0.73^{+0.10}_{-0.10}$	$11.9^{+0.3}_{-0.5}$	✓	✓	✓	–	✓	✓	✓	✓
GW190728_064510	HLV	$0.87^{+0.26}_{-0.37}$	$23.9^{+5.3}_{-0.7}$	$10.1^{+0.09}_{-0.08}$	$22.7^{+5.5}_{-0.7}$	$0.71^{+0.04}_{-0.04}$	$13.0^{+0.2}_{-0.4}$	✓	–	✓	✓	✓	–	✓	✓
GW190814	LV[a]	$0.24^{+0.04}_{-0.05}$	$27.1^{+1.1}_{-1.0}$	$6.41^{+0.02}_{-0.02}$	$26.9^{+1.1}_{-1.0}$	$0.28^{+0.02}_{-0.02}$	$24.9^{+0.1}_{-0.2}$	✓	✓	✓	–	✓	–	–	–
GW190828_063405	HLV	$2.13^{+0.66}_{-0.93}$	$79.9^{+6.9}_{-5.9}$	$34.5^{+2.9}_{-2.8}$	$75.7^{+6.0}_{-5.2}$	$0.75^{+0.06}_{-0.07}$	$16.2^{+0.2}_{-0.3}$	✓	✓	✓	✓	✓	✓	✓	✓
GW190828_065509	HLV	$1.60^{+0.62}_{-0.60}$	$44.4^{+6.4}_{-4.0}$	$17.4^{+0.6}_{-0.7}$	$42.7^{+6.6}_{-4.2}$	$0.65^{+0.08}_{-0.08}$	$10.0^{+0.3}_{-0.5}$	✓	–	✓	✓	✓	–	✓	✓
GW190910_112807	LV	$1.46^{+1.03}_{-0.58}$	$101.9^{+10.4}_{-7.8}$	$43.9^{+4.6}_{-3.6}$	$97.0^{+9.3}_{-7.1}$	$0.70^{+0.08}_{-0.07}$	$14.1^{+0.2}_{-0.3}$	✓	✓	✓	–	✓	✓	✓	–
GW190915_235702	HLV	$1.62^{+0.71}_{-0.61}$	$78.3^{+8.4}_{-8.1}$	$33.1^{+3.3}_{-3.9}$	$74.8^{+7.9}_{-7.4}$	$0.70^{+0.09}_{-0.11}$	$13.6^{+0.2}_{-0.3}$	✓	–	✓	–	✓	✓	✓	✓
GW190924_021846	HLV	$0.57^{+0.22}_{-0.22}$	$15.5^{+5.7}_{-0.7}$	$6.44^{+0.04}_{-0.03}$	$14.8^{+5.9}_{-0.8}$	$0.67^{+0.05}_{-0.05}$	$11.5^{+0.3}_{-0.4}$	✓	–	✓	✓	✓	–	✓	✓

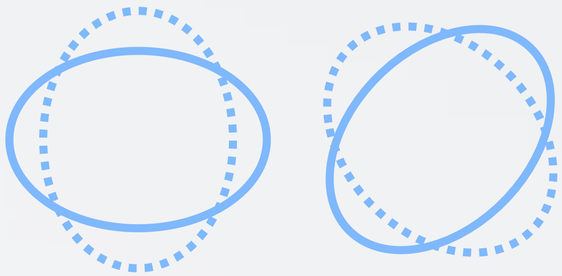
Abbott+20 [arXiv:2010.14529]

Gravitational Wave Propagation

Gravitational Wave Propagation







Speed

Speed of GWs is a fundamental property of gravity, GR predicts that $v = c$. In modified theories of gravity, GWs do not have to travel on null geodesics of background metric…

Dispersion

Modifications, such as Lorentz violating terms, can lead to a frequency-dependent modification of the dispersion relations… unravelling of wave packet!

Polarization

Generic metric theories of gravity can have up to 6 GW polarizations: two tensor modes (helicity ±2), two vector modes (helicity ±1), and two scalar modes (helicity 0)

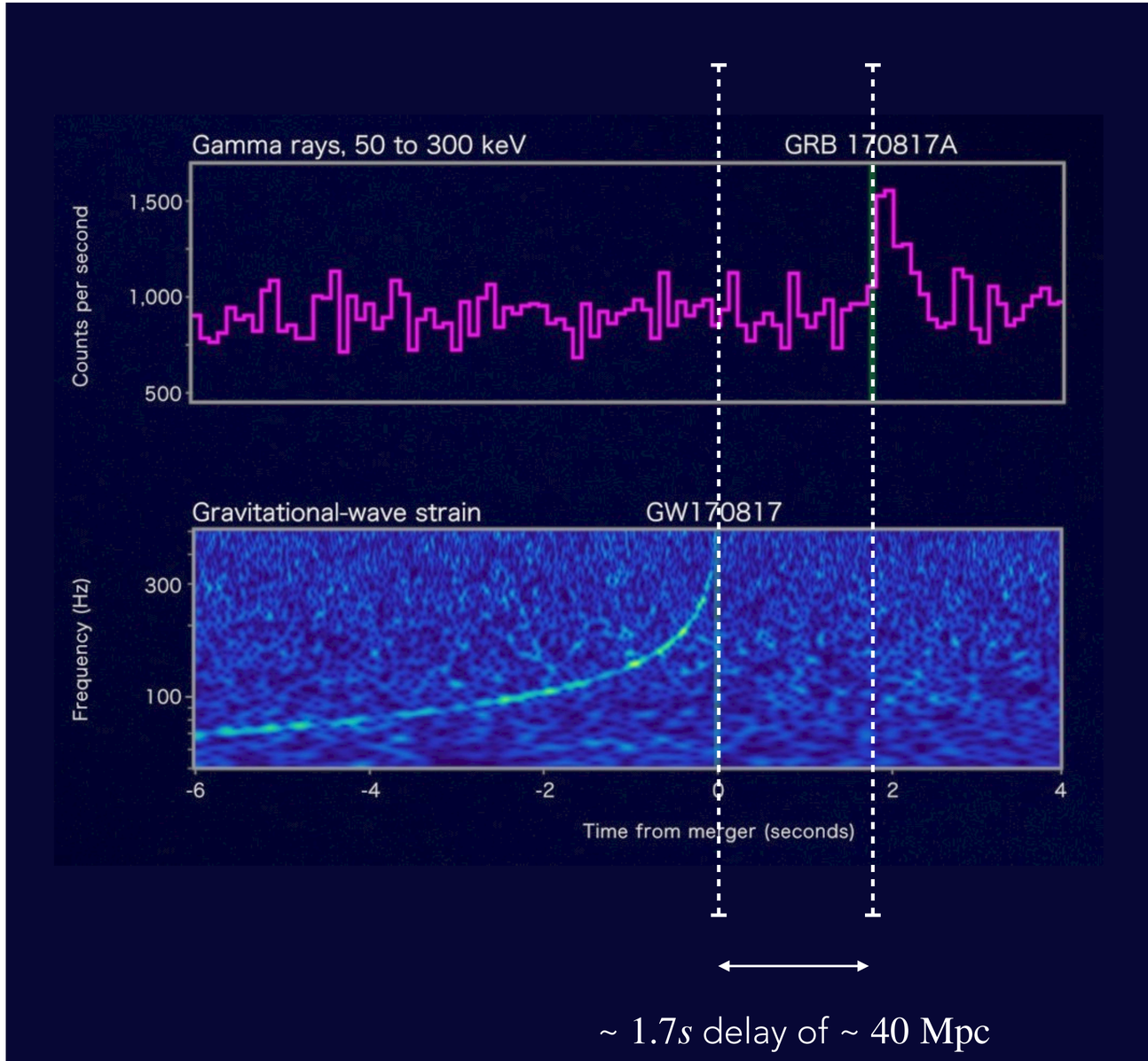
Gravitational Wave Propagation: Speed

- Measure arrival time of GWs at detectors
- Measure arrival time of EM from counterpart
- GW170817 + GRB powerful example

$$-3 \times 10^{-15} \leq \frac{c_{\rm gw}}{c} - 1 \leq 7 \times 10^{-16}$$

Abbott+17 [arXiv:1710.05834]

- Stringent constraints on modified theories of gravity and severely reduces viable range of cosmological models: [Baker+17, Creminelli+17, Ezquiaga+17, Sakstein+17]



NASA's Goddard Space Flight Center, Caltech/MIT/LIGO Lab and ESA

Gravitational Wave Propagation: Dispersion

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- Modified dispersion relation [Mirshekari+12]

Speed of Light

Phenomenological coefficients
($\alpha = 3,4$ in some quantum gravity models)

$$E^2 = p^2 c^2 + \underbrace{A_\alpha p^\alpha c^\alpha}_{\text{Lorentz violating term}}$$

Energy Momentum

- Unravelling of wave-packet grows with distance! BBH provide excellent constraints due to cosmological distances ✓

- For General Relativity: no frequency dependent corrections, massless graviton

$$A_\alpha = 0 \; \forall \; \alpha, \quad m_g = 0$$

- For massive gravity, we find modified dispersion relations [Will 98]

$$\alpha = 0, \quad A_\alpha > 0, \quad m_g = A_0^{1/2} \, c^{-2}$$

Gravitational Wave Propagation: Dispersion

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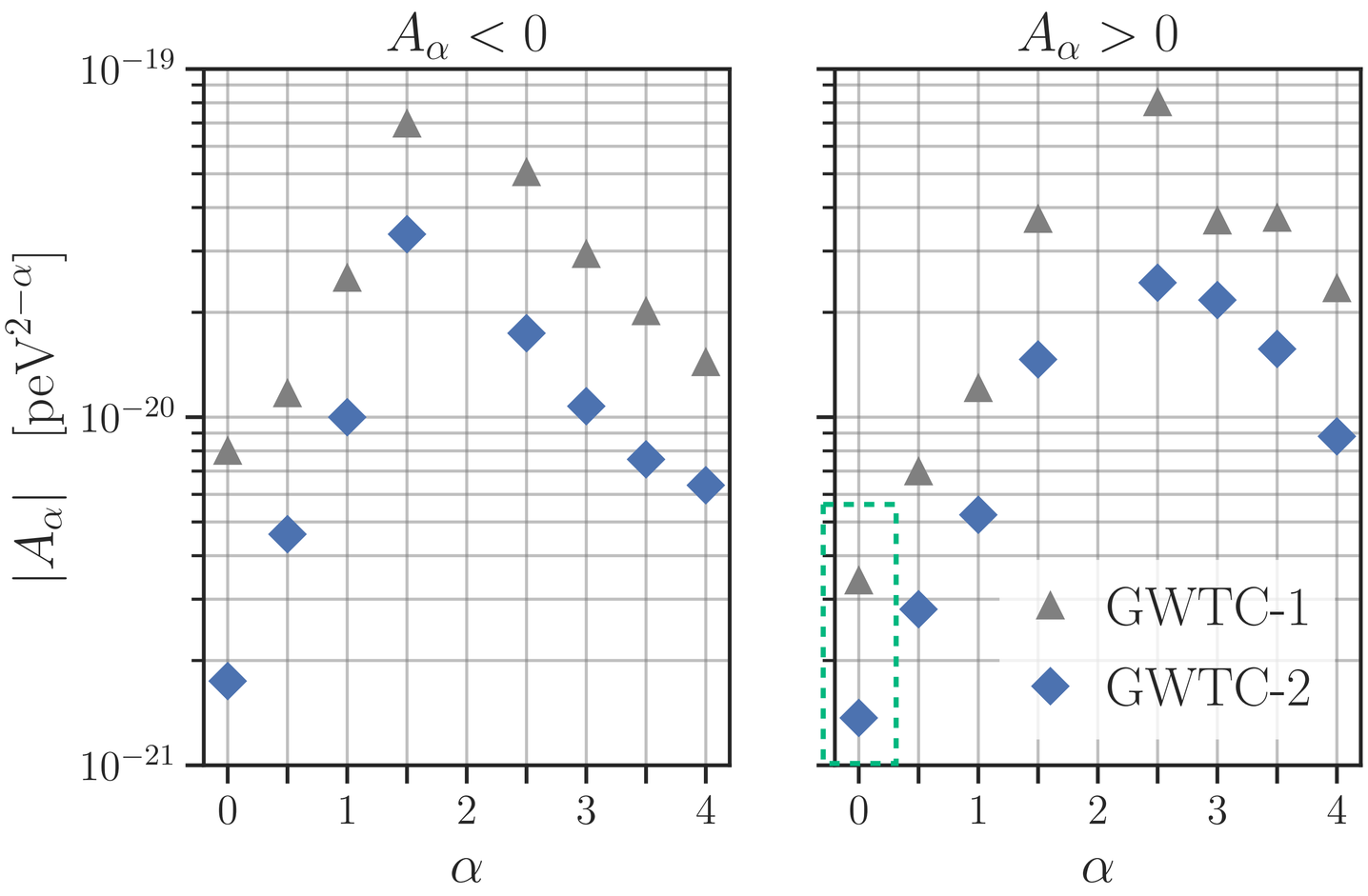
- Combine results from many detections

  GWTC-1 Abbott+19 [arXiv:1903.04467]

  GWTC-2 Abbott+20 [arXiv:2010.14529]

- Constraints on graviton mass ~ 1.8x more stringent than Solar System bounds [Bernus+20]

$$m_g = \frac{\sqrt{A_0}}{c^2} \leq 1.76 \times 10^{-23}\,\mathrm{eV}/c^2$$



Massive graviton case

Abbott+20 [arXiv:2010.14529]

Gravitational Wave Propagation: Polarization

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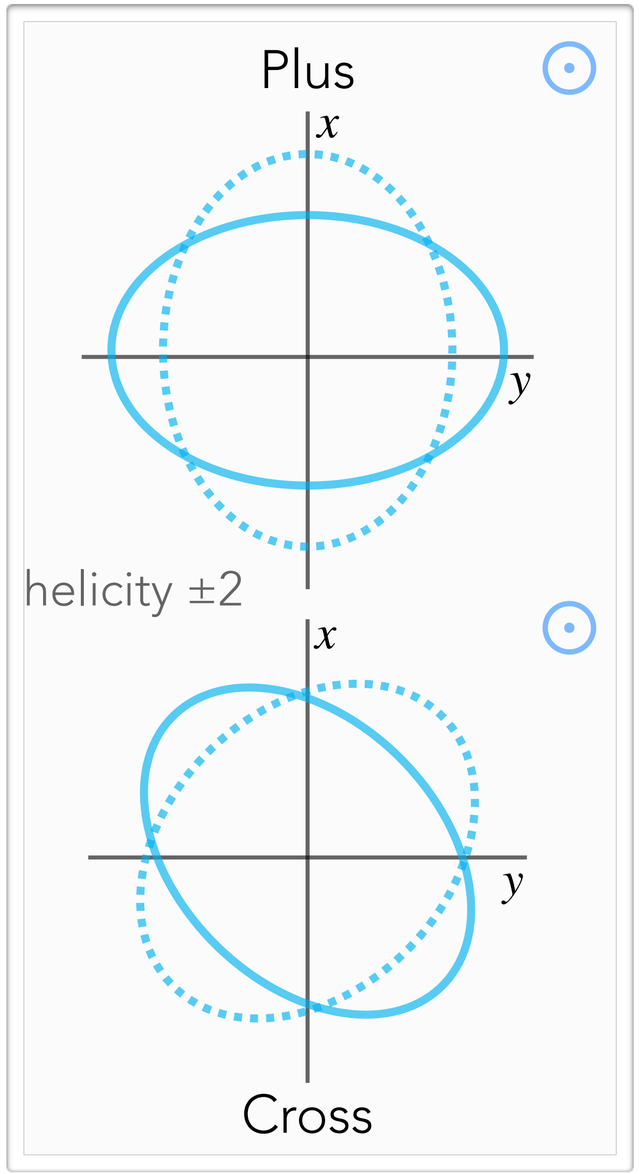
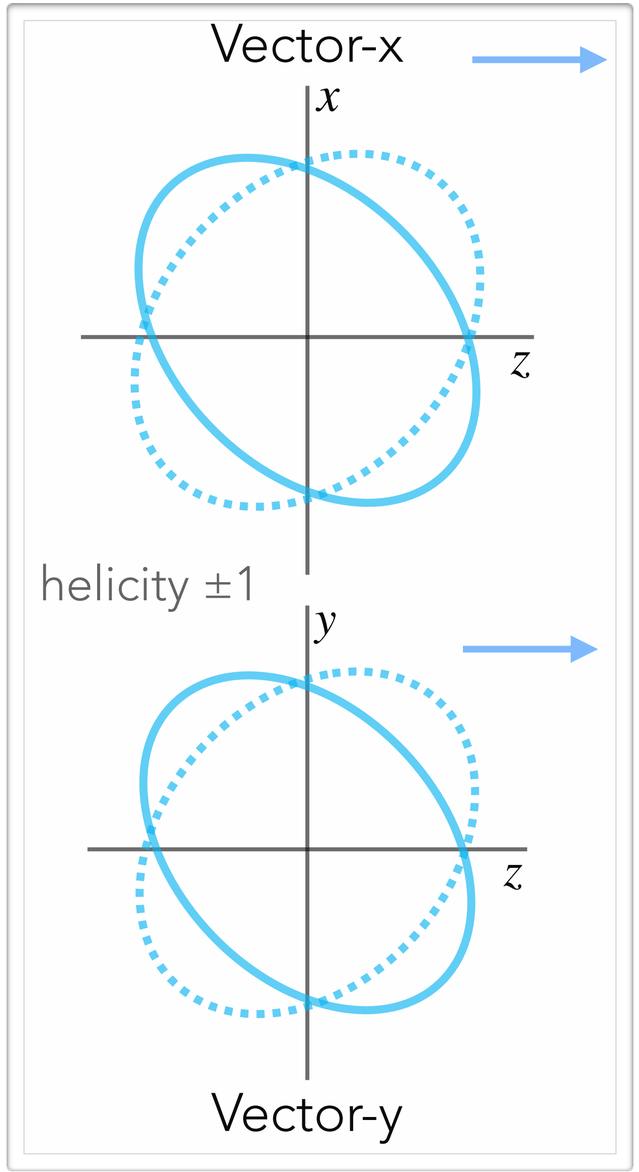
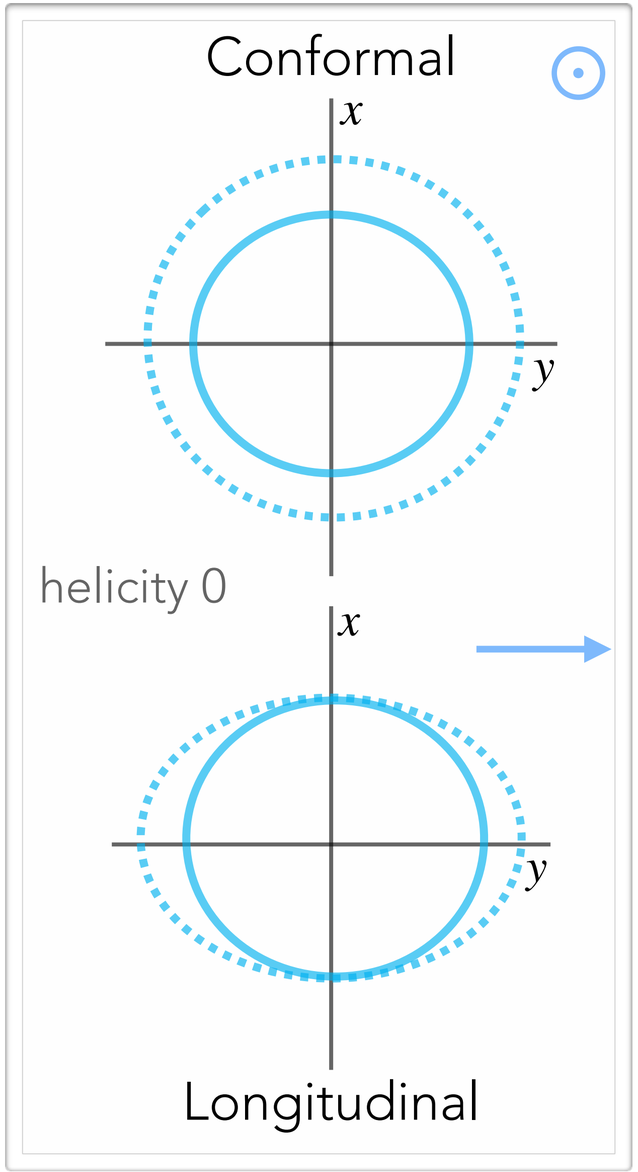
General Relativity (Tensor)



Vector



Scalar



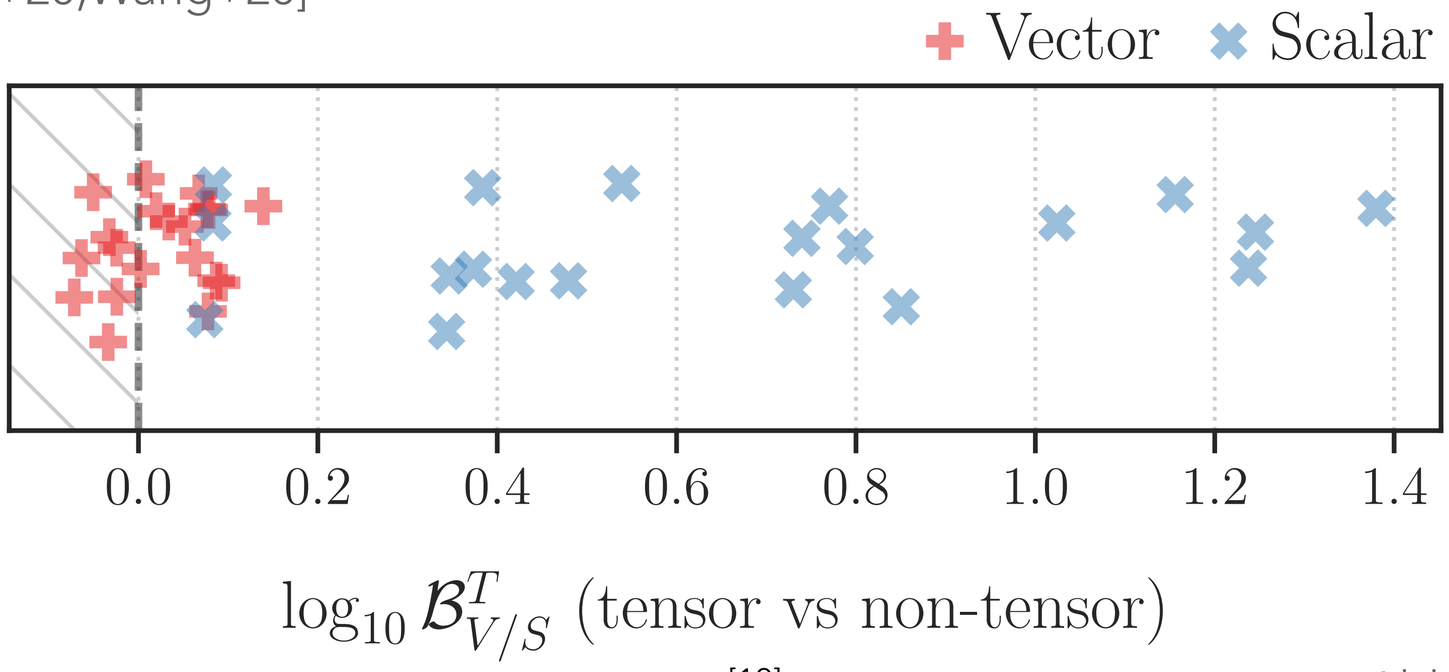
[9]

Gravitational Wave Propagation: Polarizations

- Polarization constraints from relative amplitudes and phases at different detectors
- For BBHs, need 5 detectors to break all possible degeneracies
- Tightest constraints from GW170817
- Limited studies to date, disfavour pure-vector or pure-scalar modes: [Abbott+17, Abbott+19, Abbott+20, Isi+17, Pang+20, …]
- Not discussed here: birefringence, where one polarization state suppressed relative to other [Okounkova+21,Shao+20,Wang+20]

Preference for non-tensor hypothesis if

$\log_{10} \mathcal{B}^{T}_{V/S} < 0$



Abbott+20 [arXiv:2010.14529]

Gravitational Wave Source Generation

Gravitational Wave Generation: Parameterized Tests

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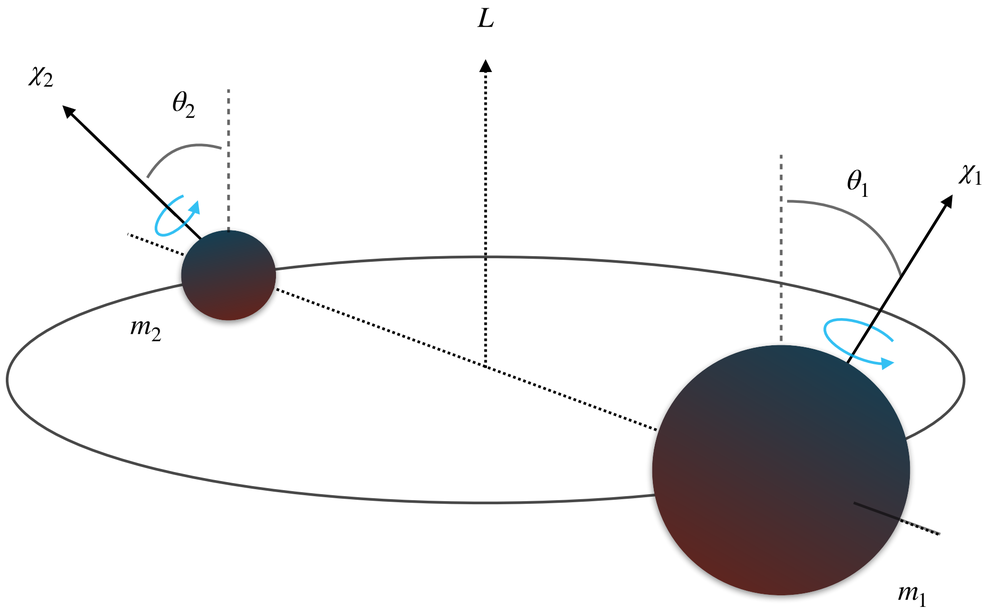
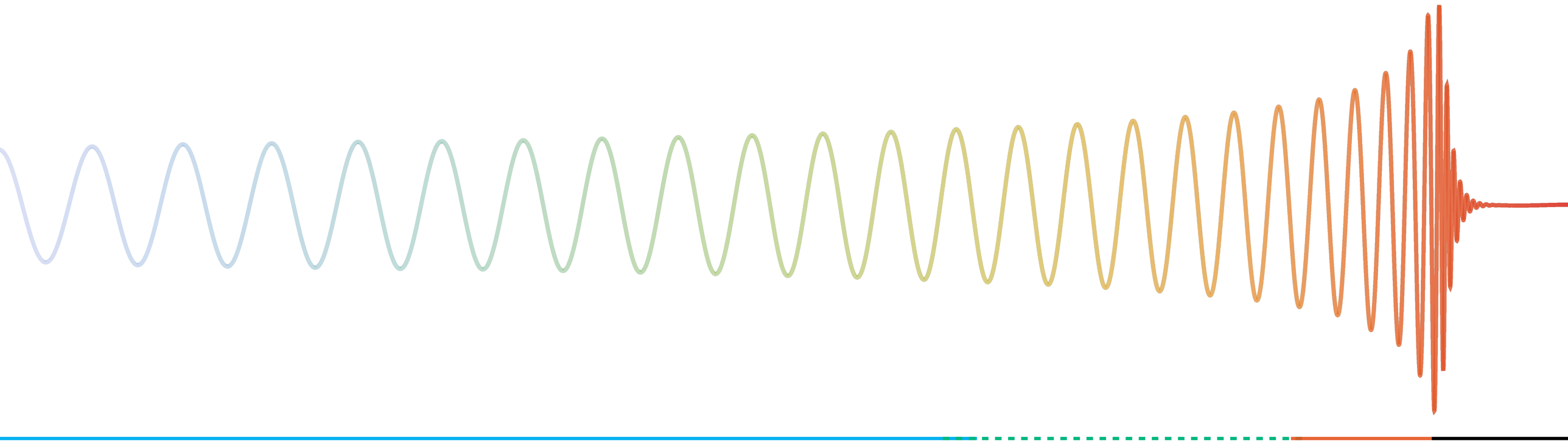
- GR hypothesis? We have a non-eccentric (quasi-circular) binary black hole
- From no hair theorem, BBH parameterized by:
  - mass ratio + two spin vectors
- + extrinsic parameters (sky location, distance, etc)
- GW signal as sum over spin-weighted spherical harmonics

$$h_+ - ih_\times = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} h_{\ell m} \, {}_{-2}Y_{\ell m}(\theta, \varphi)$$

- We use two waveform models (helps gauge systematics)
  - Precessing phenomenological model, PhenomPv2 [Hannam+14, Husa+16, Khan+16]
  - Aligned-spin effective-one-body model, SEOBNRv4 [Bohe+16]



Gravitational Wave Generation: Parameterized Tests



Inspiral:
Post-Newtonian Theory
Effective One Body

Analytical approximations begin to break down

Merger:
No analytical model

Ringdown:
Black Hole Perturbation Theory

Flux Balance:

$$\frac{dE_{\mathrm{orbital}}}{dt} = \mathcal{L}_{\mathrm{GW}} \approx \frac{32}{5}\frac{c^5}{G}\frac{(m_1 m_2)^2}{M^4}\left(\frac{v}{c}\right)^5$$

[13]

Gravitational Wave Generation: Parameterized Tests

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- Precision models for phase evolution allows us to place tight constraints on GR!
- Inspiral described by post-Newtonian expansion in the orbital velocity

$$\varphi(v) = \left(\frac{v}{c}\right)^{-5}\left[\varphi_0 + \varphi_1\left(\frac{v}{c}\right)^1 + \varphi_2\left(\frac{v}{c}\right)^2 + \cdots + \varphi_{5l}\ln\left(\frac{v}{c}\right)\left(\frac{v}{c}\right)^5 + \cdots + \varphi_7\left(\frac{v}{c}\right)^7\right]$$

0PN 0.5PN 1PN 2.5PN(l) 3.5PN

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \qquad q = \frac{m_2}{m_1}$$

- Spin effects enter phase at 1.5PN
- PN coefficients encode a range of physical phenomena: tails of radiation due to backscattering, spin-orbit and spin-spin couplings, BH absorption, …
- PN coefficients would take different values in modified theories of gravity compared to GR

Gravitational Wave Generation: Parameterized Tests

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- For late-inspiral and merger-ringdown: parameterize coefficients in **phenom**enological waveform

$$\varphi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3}\right)$$

Caution: Coefficients calibrated against NR but are *not* expressed in parameters relevant to GR or any modified theory of gravity…

$$\varphi_{\rm MR} = \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}$$

- Constrain deviations using parametric deformations to GR by varying one coefficient at a time

$$p_i \to (1 + \delta\hat{p}_i)\, p_i \qquad p_i = \{\varphi_i, \beta_i, \alpha_i\}$$

Gravitational Wave Generation: Parameterized Tests

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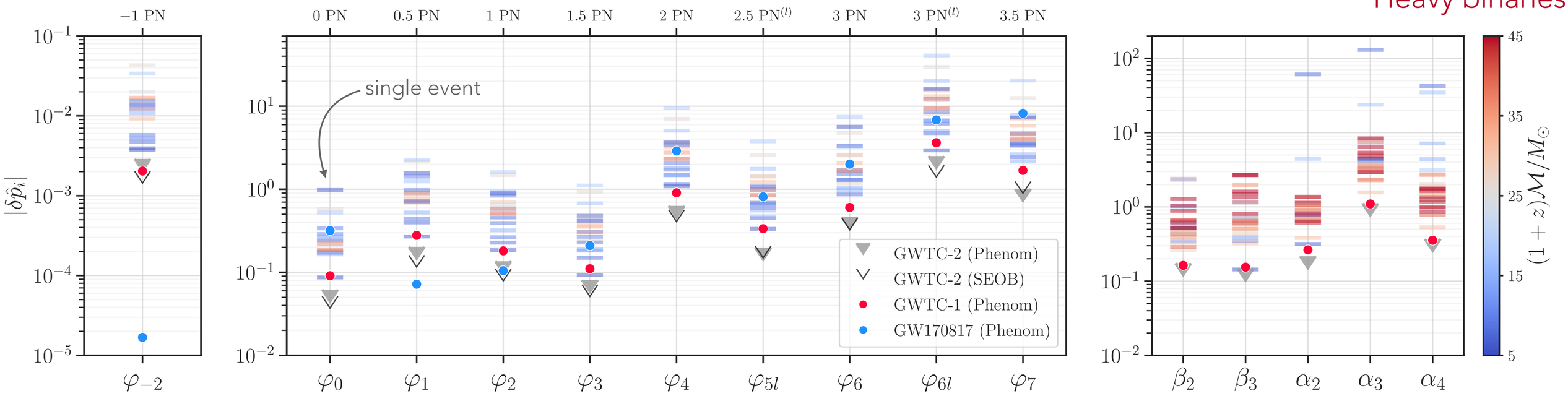
- GWTC-2: 90%-credible upper limit on each deviation parameter

Heavy binaries



Light binaries

← lower frequency — $\varphi_i \sim f^{(i-5)/3}$ inspiral — higher frequency →

- Caveat: joint results assume the same value for *all* events

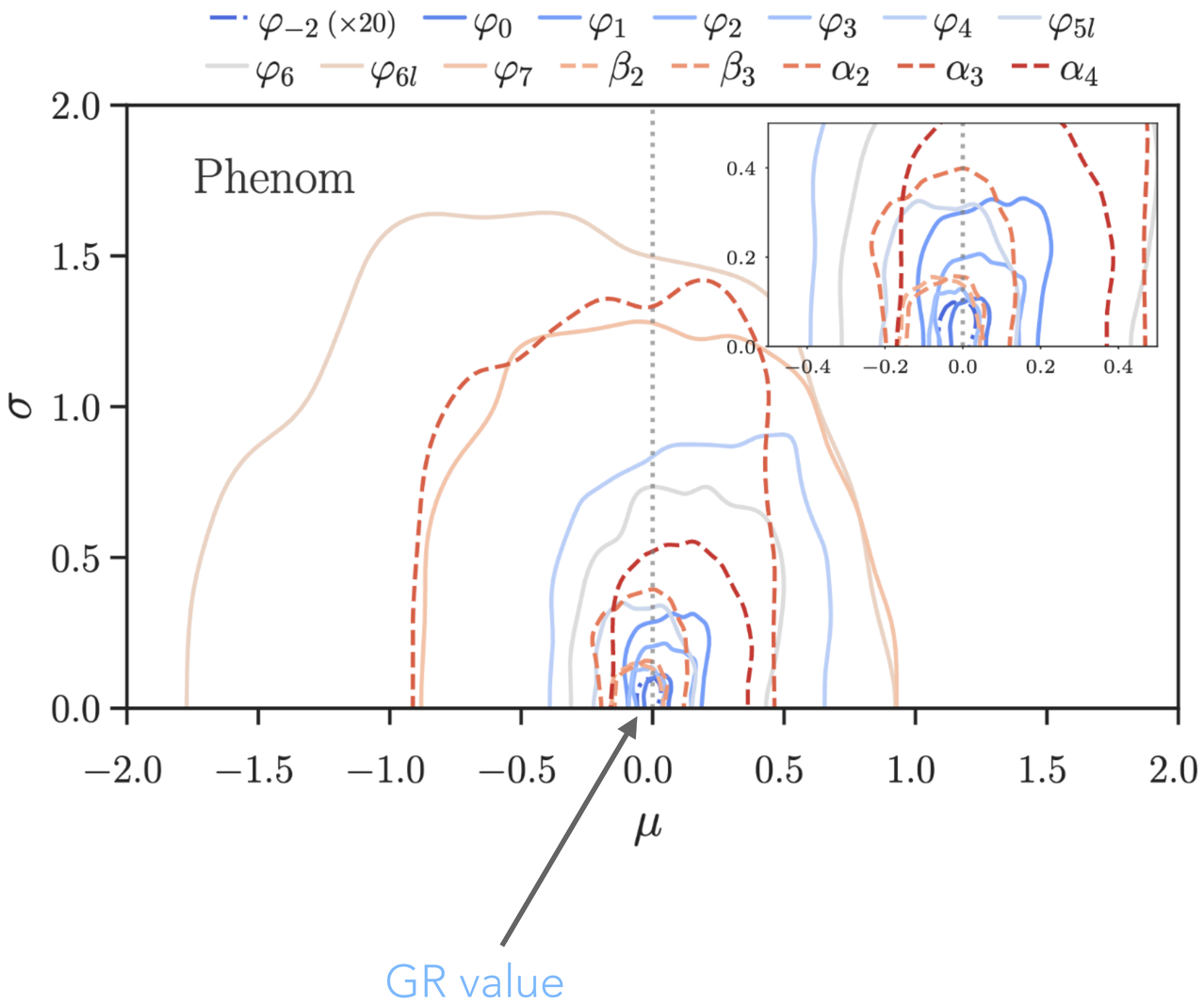
Abbott+20 [arXiv:2010.14529]

Gravitational Wave Generation: Parameterized Tests

- Deviations from GR expected to depend on binary parameters (mass, spin, etc)
- Should not simply multiply likelihoods or Bayes factors [Zimmerman+19]
- Alternative: model the observed distribution using hierarchical Bayesian inference [Isi+19]

$$p(x|d) = \int p(x|\mu,\sigma)p(\mu,\sigma|d)d\mu d\sigma$$

Infer μ_i and σ_i from all events simultaneously



Abbott+20 [arXiv:2010.14529]

Gravitational Wave Generation: Parameterized Tests

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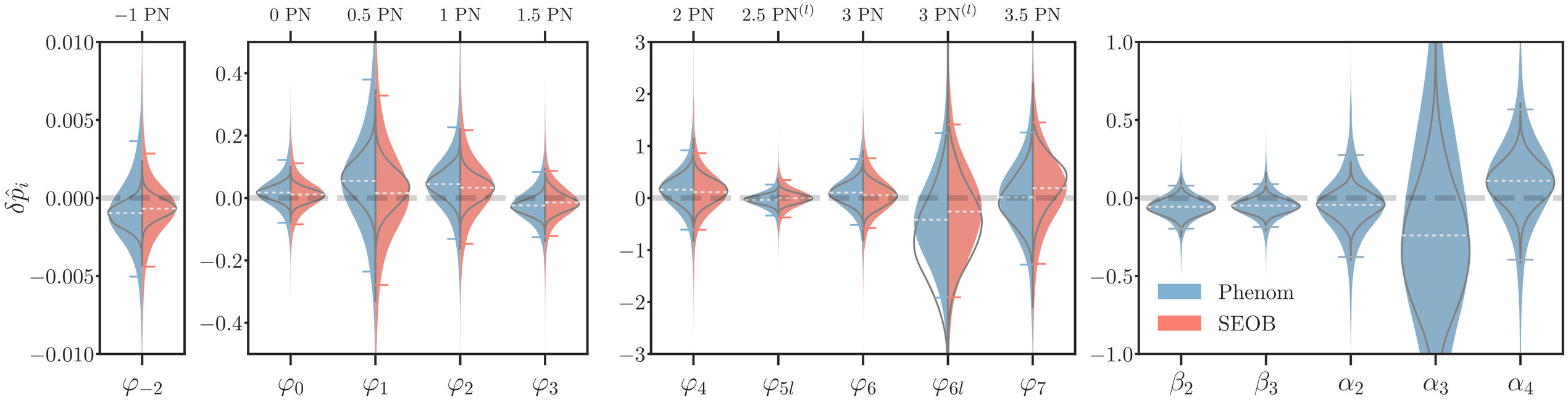
- Results using Hierarchical Bayesian inference…



Empty = shared deviation; Filled = non-shared deviation (hierarchical)

Abbott+20 [arXiv:2010.14529]

Gravitational Wave Generation: Parameterized Tests

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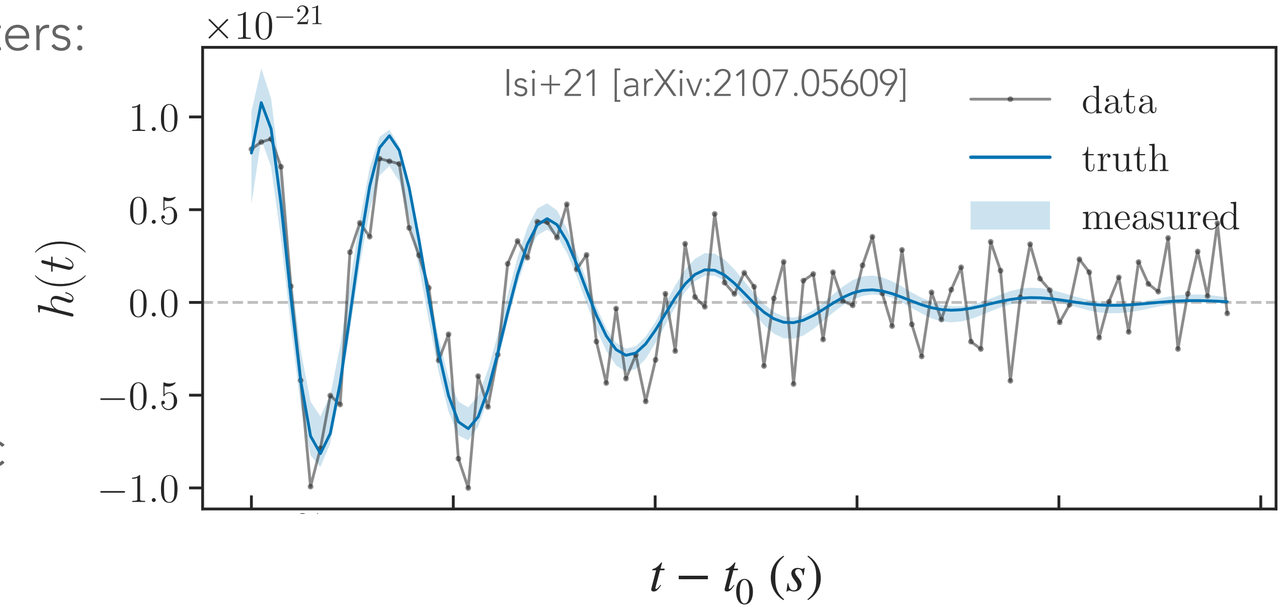
- In GR isolated BHs are characterised by 3 parameters:
  - Mass
  - Spin
  - ~~Charge~~ (No charge for astrophysical BHs)
- Infer properties of remnant BH from spectroscopic analysis of ringdown - consistency with Kerr?



$\omega_{\ell mn}$ ~ intrinsic geometry

$$h = \sum_{\ell mn} C_{\ell mn} e^{-i\tilde{\omega}_{\ell mn} t}\, {}_{-2}S_{\ell mn}(\iota, \varphi)$$

$C_{\ell mn}$ ~ initial conditions

${}_{-2}S_{\ell mn}$ ~ angular structure of radiation

Azimuthal number: $\ell \geq 2$

Magnetic number: $-\ell \leq m \leq \ell$

Overtone number: $n \geq 0$

Gravitational Wave Generation: Parameterized Tests

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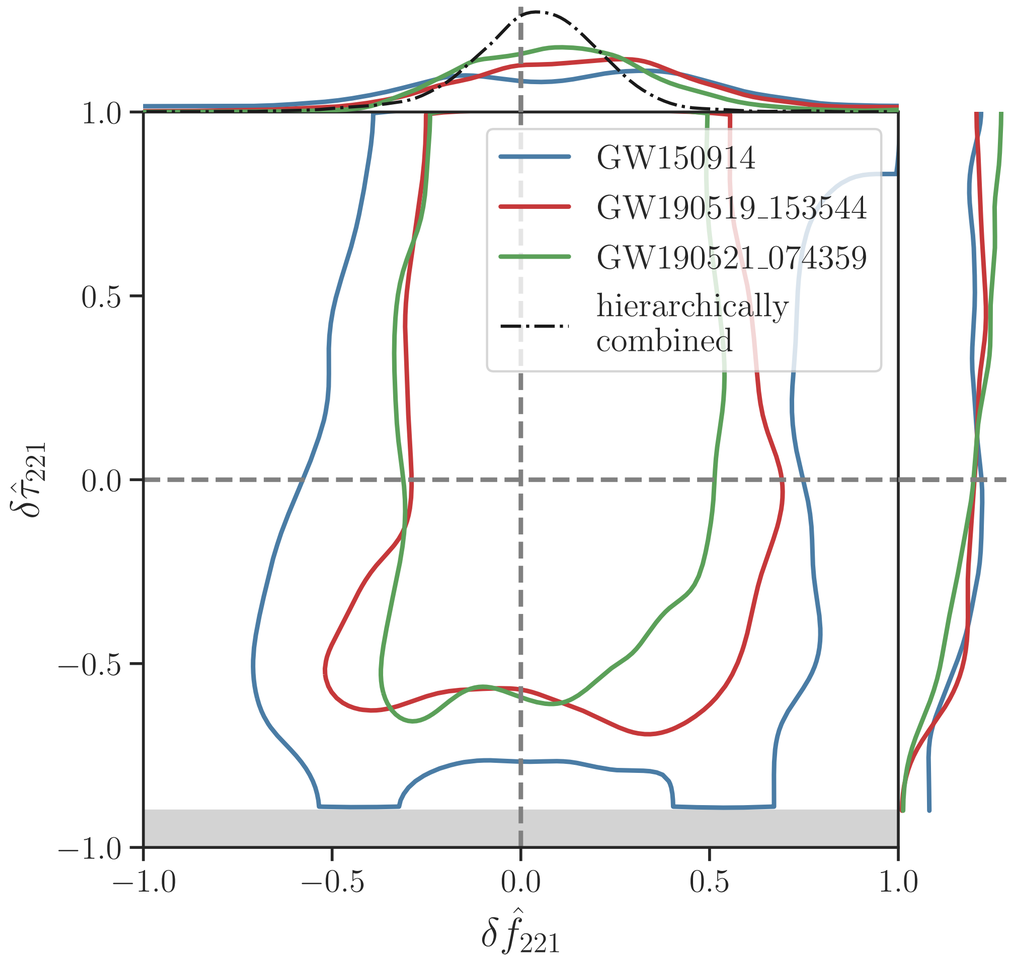
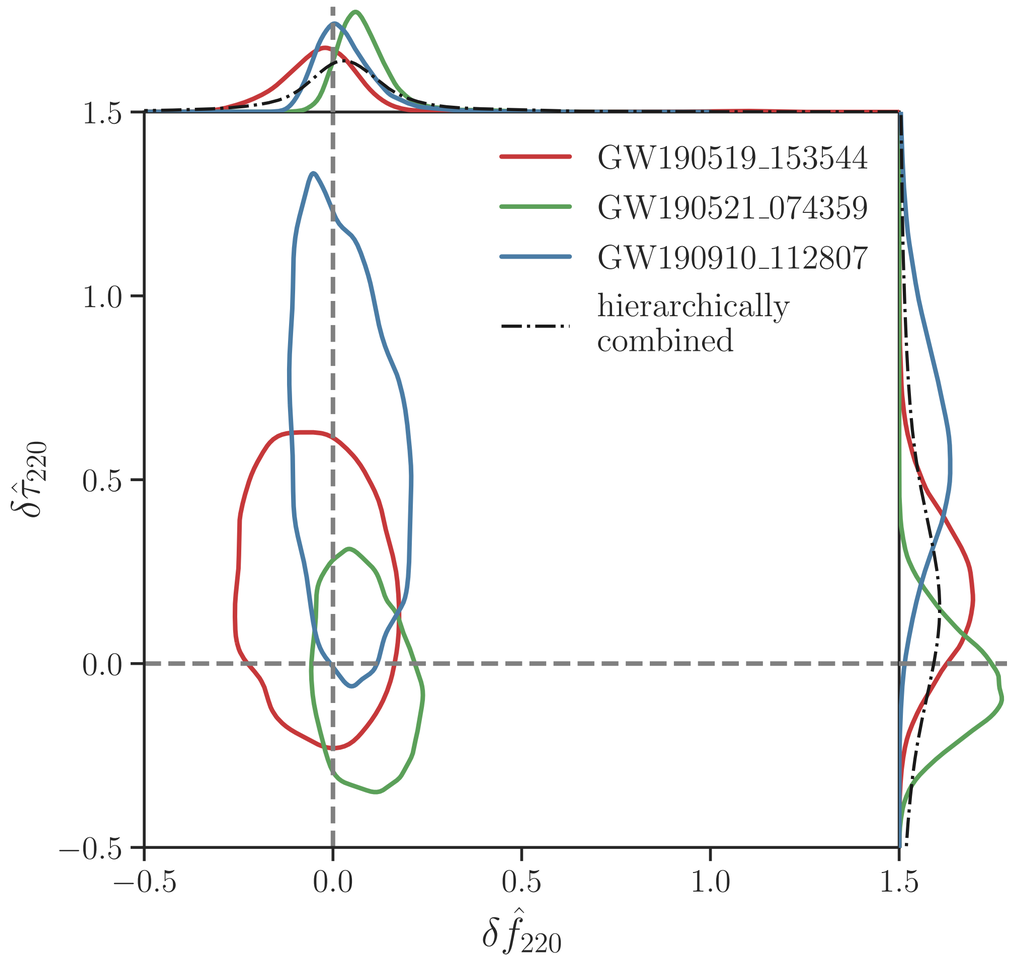
Mass & spin inferred from *fundamental mode*



Carullo+18, Isi+19

Mass & spin inferred from *inspiral*



Brito+19, Ghosh+21

Abbott+20 [arXiv:2010.14529]

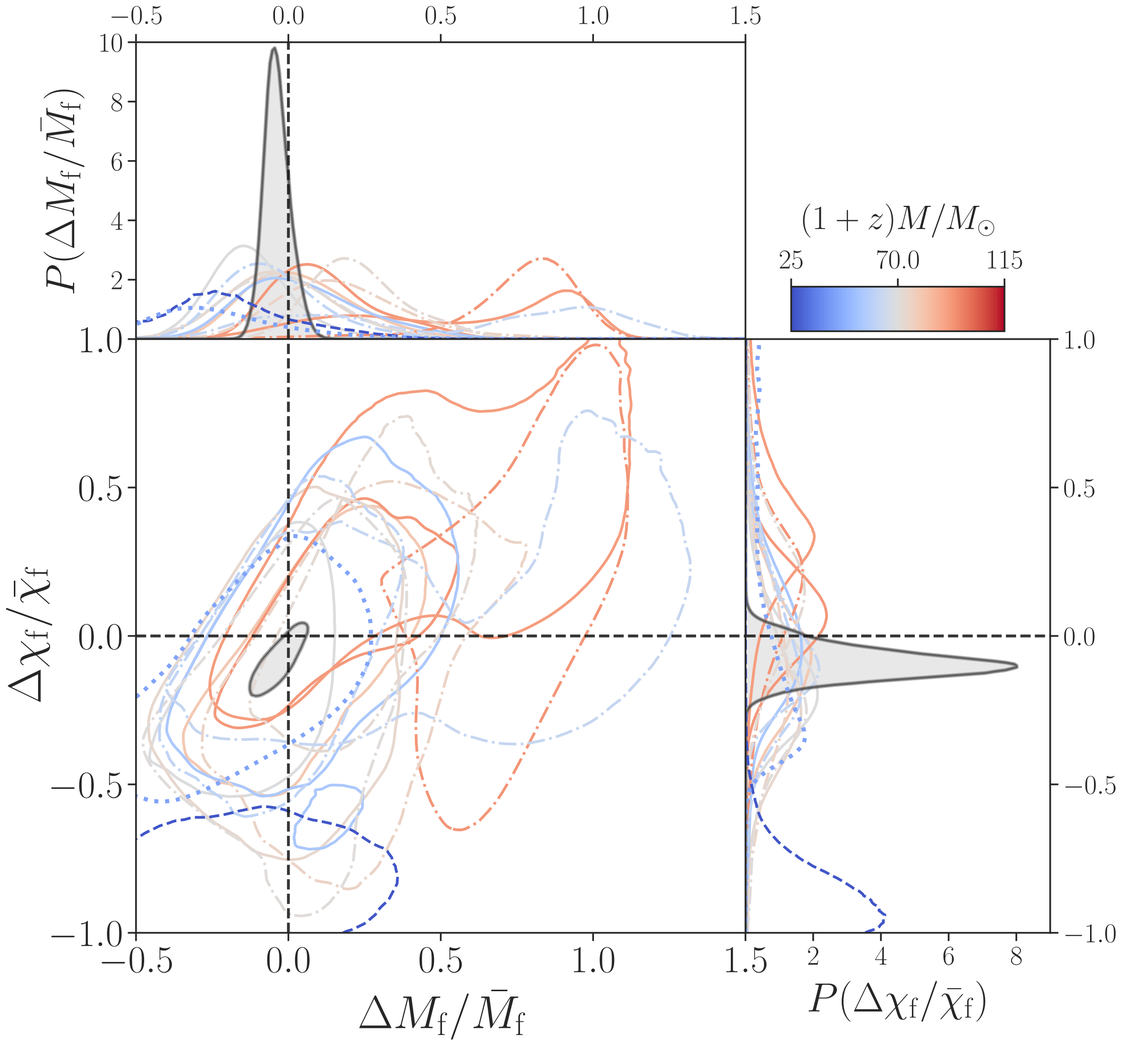
Consistency Tests

Gravitational Wave Consistency: Inspiral-Merger-Ringdown

- Check that observations are consistent with predictions in GR
- If GR is correct, final state of a BBH merger is a perturbed Kerr BH…
- Estimate mass and spin of remnant BH from inspiral and post-inspiral regimes independently…

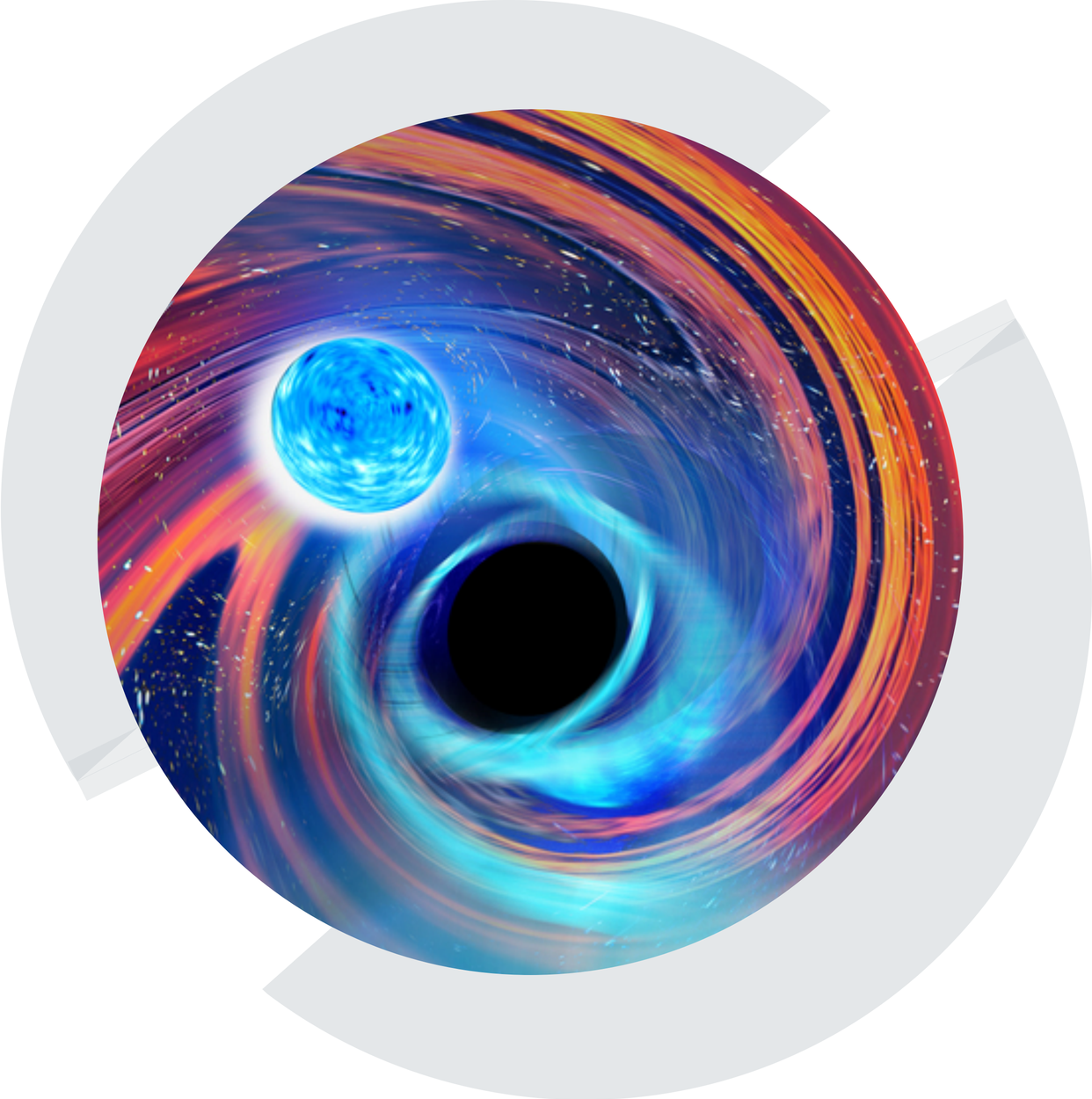
$$\frac{\Delta M_f}{\bar{M}_f} = 2\frac{M_f^{\rm insp} - M_f^{\rm postinsp}}{M_f^{\rm insp} + M_f^{\rm postinsp}}$$

$$\frac{\Delta \chi_f}{\bar{\chi}_f} = 2\frac{\chi_f^{\rm insp} - \chi_f^{\rm postinsp}}{\chi_f^{\rm insp} + \chi_f^{\rm postinsp}}$$



Abbott+20 [arXiv:2010.14529]

Conclusions



Conclusions

- **No** statistically significant deviations from GR that cannot be accounted for by systematics
- Improve over GWTC-1 constraints by factor ~ 2-3
- New analyses, new statistical techniques
- Many topics not yet discussed:
  - Echoes, Spin-induced quadrupole, birefringence, leakage, …
- A lot of ground that still needs to be covered:
  - Understand waveform systematics
  - Need to bridge GW data analysis to theory!
  - How do we interpret results in model dependent framework?
  - Need to improve waveform models! [Pratten+21]

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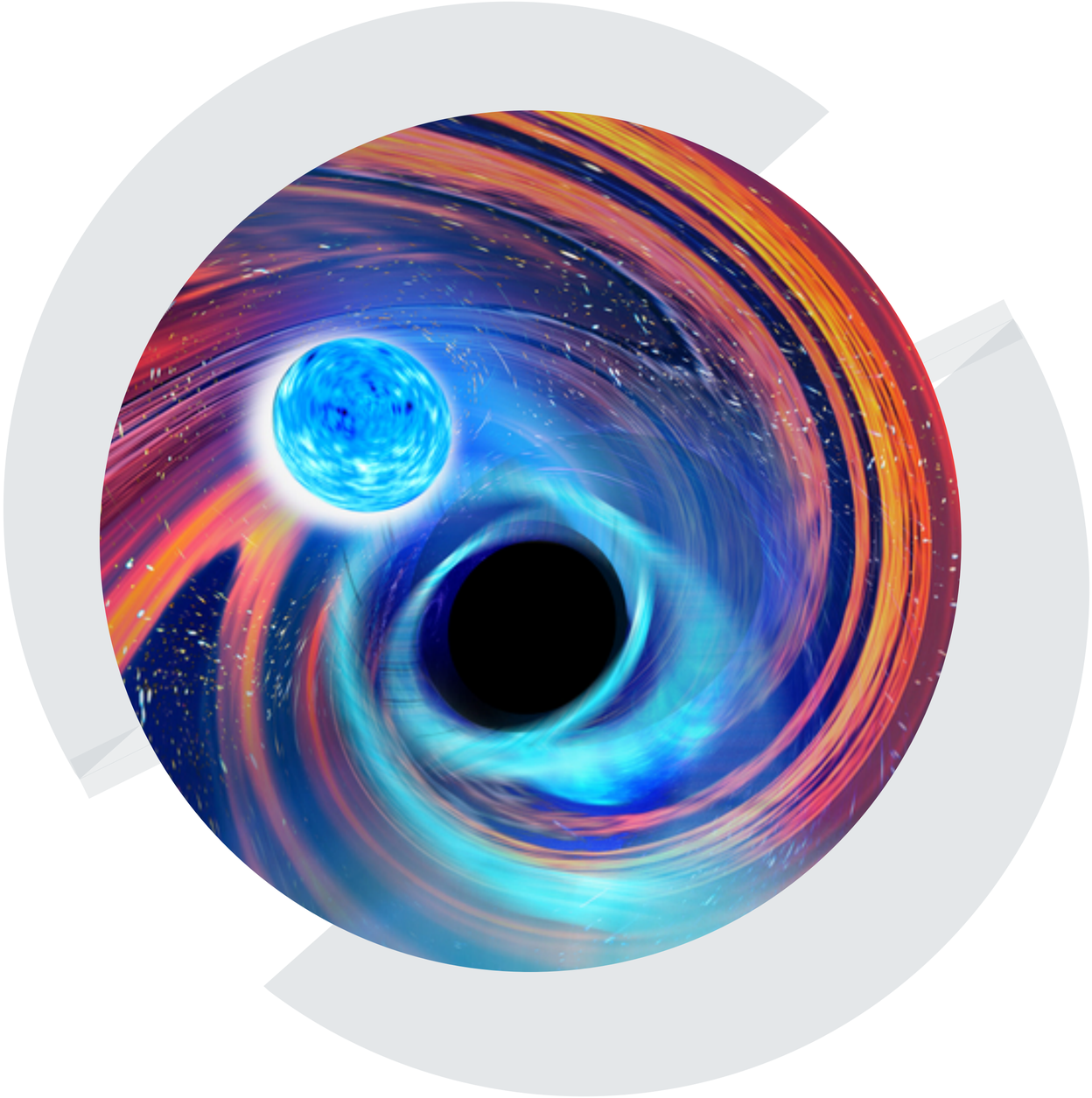

Thank you!

LIGO VIRGO

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Appendix

Gravitational Wave Propagation: Dispersion

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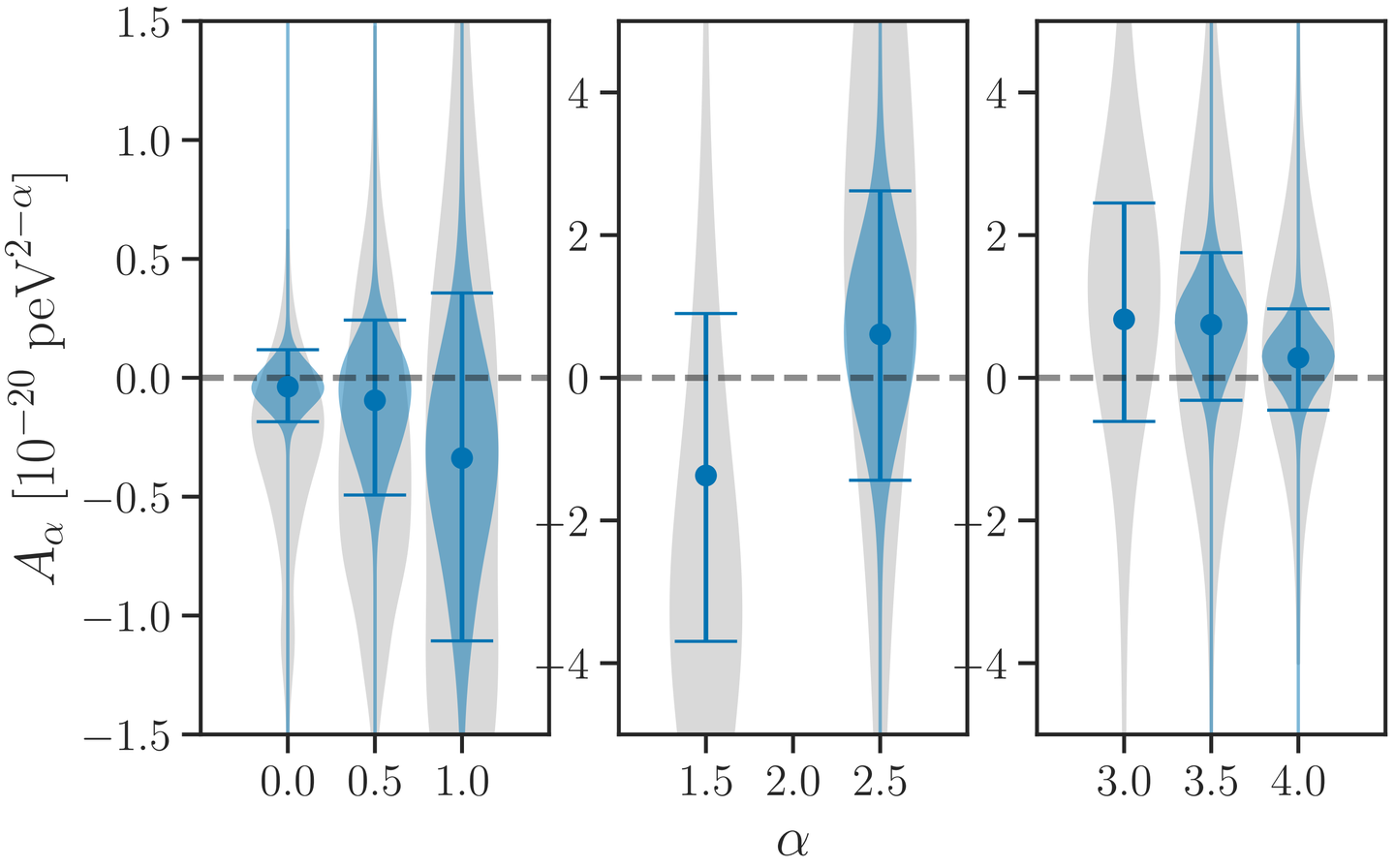
- Combine results from many detections

  GWTC-1 Abbott+19 [arXiv:1903.04467]

  GWTC-2 Abbott+20 [arXiv:2010.14529]



GWTC-1

GWTC-2

Abbott+20 [arXiv:2010.14529]

Gravitational Wave Generation: Parameterized Tests

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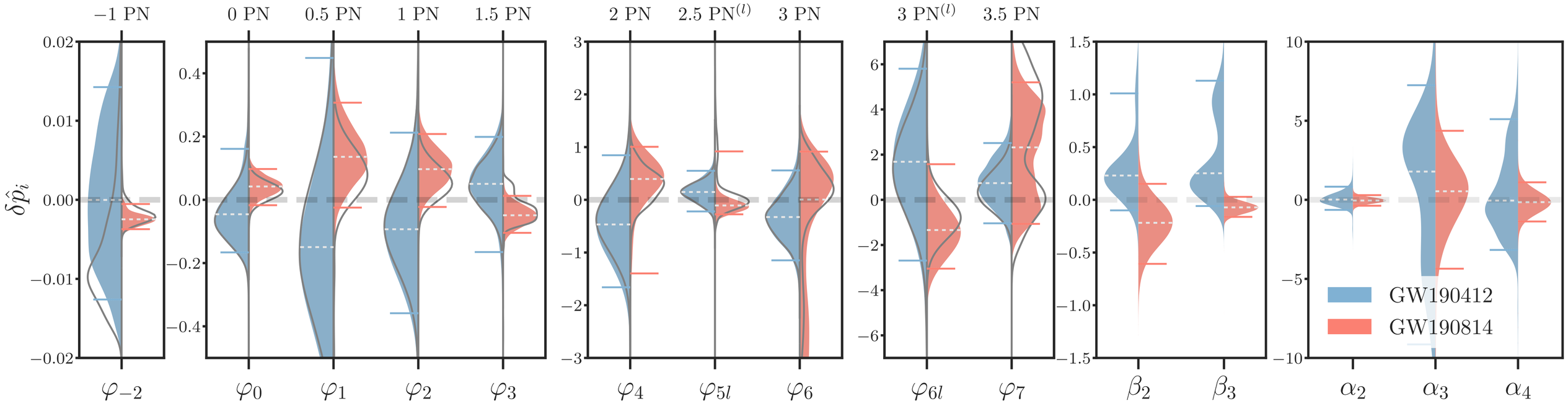
- For GW190412 and GW190814 higher multipoles are relevant….



Empty = SEOB; Filled = Phenom

Abbott+20 [arXiv:2010.14529]

Gravitational Wave Consistency: Residuals

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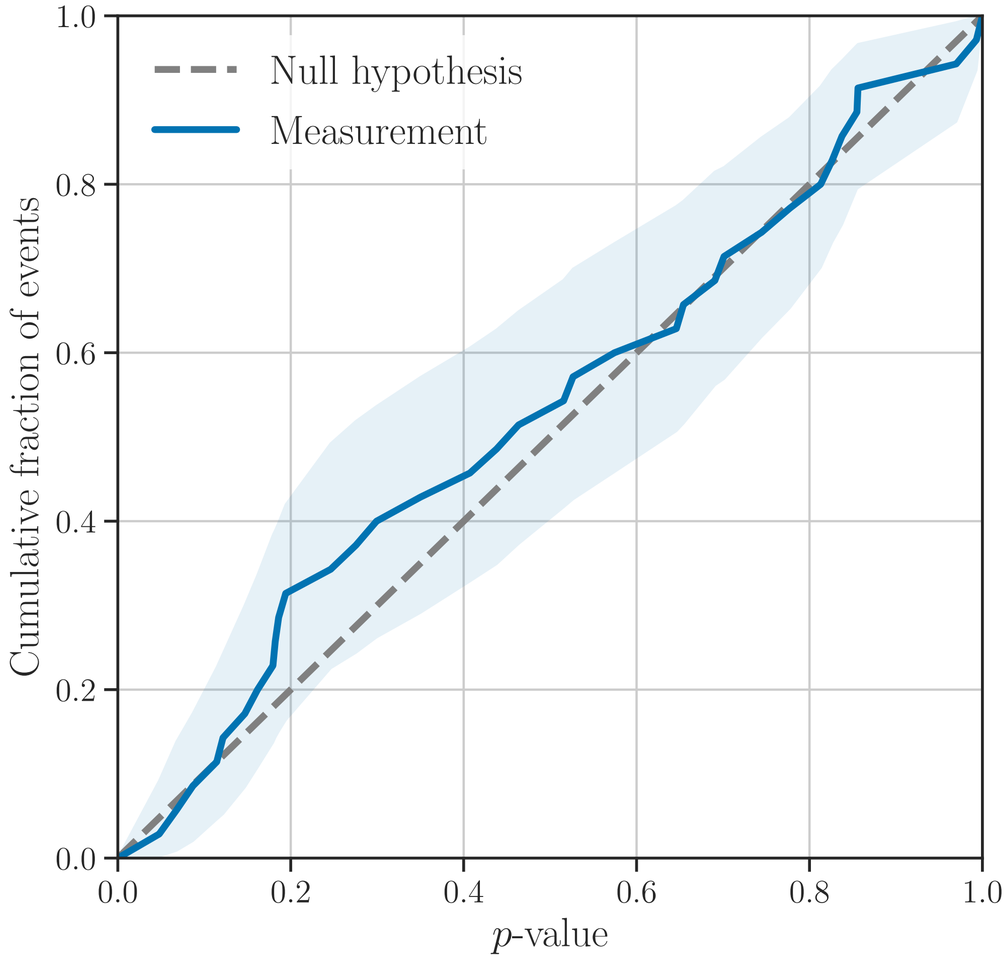
- Subtract best-fit template from GW data, is residual SNR consistent with noise?

$$p = P(\mathrm{SNR_n} \geq \mathrm{SNR_{residual}} | \mathrm{noise})$$

- All p-values consistent with residual SNR produced by noise



Abbott+20 [arXiv:2010.14529]