Testing General Relativity with Gravitational-Wave Observations

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On behalf of the LIGO Scientific and Virgo Collaborations

Based on:
“Tests of General Relativity with Binary Black Holes from the Second LIGO-Virgo Gravitational-Wave Transient Catalog”
What Can We Learn From GWs?
(a snapshot anyway)

- Probe spacetime in strong-field regime: large compactness $\mathcal{C} = \frac{M}{R} \sim \mathcal{O}(1)$, characteristic velocities $\mathcal{V} \sim \mathcal{O}(c)$

- Complementary to other probes: cosmological, solar system, double pulsar, atom interferometry, etc

- Many fundamental questions on the nature of compact objects and gravity…

**Black holes?**

What is the fundamental nature of black holes? Do they have a horizon? Are they governed by the Kerr metric?

**Lorentz invariance?**

Is Lorentz invariance a fundamental symmetry of nature? Is it broken on some energy scale?

**Gravitational fields?**

Do we have additional gravitational fields beyond $g_{\mu\nu}$? Are gravitons massive?

**Extra dimensions?**

Are there large extra dimensions?

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Figueras+17

$$\mathcal{C} = \frac{M}{R} \sim \mathcal{O}(1)$$

$$\mathcal{V} \sim \mathcal{O}(c)$$

$$-\frac{1}{8}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$
How Do We Test GR With GWs…?

(a snapshot anyway)

• Key limitation! Lack understanding of strong-field in nearly all modified theories of gravity…

• So we focus on testing *General Relativity* by using it as our *null hypothesis*…

  • Are our results consistent with the predictions of GR?
  
  • Use parameterized deformations of GR to constrain *degree* to which deviations agree with data
  
  • Split into: i) tests of GW propagation, ii) tests of GW generation, iii) GR consistency tests

• Recent advances in modelling binaries in beyond-GR theories both *analytically* and *numerically*
  
  • Not yet clear if full theories are mathematically well posed, physically viable, etc
  
  • Not yet at stage where results can be used in GW data analysis
How Do We Test GR With GWs…?
(a snapshot anyway)

- Quick summary of GWTC-2 (see also talk by Lazzaro Claudia on Weds.)
- Use events detected by LIGO-Virgo up to 1st October 2019
- Impose more stringent cut on events, must have false-alarm-rate (FAR) \(< 10^{-3}\) per year
- 24 new events in O3a + BBH from GWTC-1
- Additional cuts imposed by tests

<table>
<thead>
<tr>
<th>Event</th>
<th>Inst.</th>
<th>Delta t [Gpc]</th>
<th>(1+z)M</th>
<th>(1+z)M</th>
<th>(1+z)M</th>
<th>Chi</th>
<th>SNR</th>
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<tr>
<td>GW190408.181802</td>
<td>HLV</td>
<td>1.55 ± 0.06</td>
<td>55.5 ± 3.5</td>
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<td>HLV</td>
<td>0.74 ± 0.14</td>
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<tr>
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<td>91.6 ± 11.2</td>
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<td>43.5 ± 4.0</td>
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<td>HL</td>
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<td>70.6 ± 11.5</td>
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<td>GW190519.153544</td>
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<td>LV*</td>
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<td>14.8 ± 0.8</td>
<td>0.67 ± 0.05</td>
<td>11.5 ± 0.4</td>
</tr>
</tbody>
</table>

Gravitational Wave Propagation
Modifications, such as Lorentz violating terms, can lead to a frequency-dependent modification of the dispersion relations…

\[ v = c^{?} \]

### Speed
Speed of GWs is a fundamental property of gravity, GR predicts that \( v = c \). In modified theories of gravity, GWs do not have to travel on null geodesics of background metric…

### Dispersion
Modifications, such as Lorentz violating terms, can lead to a frequency-dependent modification of the dispersion relations… unravelling of wave packet!

### Polarization
Generic metric theories of gravity can have up to 6 GW polarizations: two tensor modes (helicity ±2), two vector modes (helicity ±1), and two scalar modes (helicity 0)
Gravitational Wave Propagation: Speed

- Measure arrival time of GWs at detectors
- Measure arrival time of EM from counterpart
- GW170817 + GRB powerful example

\[-3 \times 10^{-15} \leq \frac{c_{gw}}{c} - 1 \leq 7 \times 10^{-16}\]

Abbott+17 [arXiv:1710.05834]

- Stringent constraints on modified theories of gravity and severely reduces viable range of cosmological models: [Baker+17, Creminelli+17, Ezquiaga+17, Sakstein+17]
Gravitational Wave Propagation: Dispersion

- Modified dispersion relation [Mirshekari+12]

\[ E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha \]

Phenomenological coefficients

\( (\alpha = 3, 4 \text{ in some quantum gravity models}) \)

- Unravelling of wave-packet grows with distance! BBH provide excellent constraints due to cosmological distances ✓

- For General Relativity: no frequency dependent corrections, massless graviton

\[ A_\alpha = 0 \ \forall \ \alpha, \quad m_g = 0 \]

- For massive gravity, we find modified dispersion relations [Will 98]

\[ \alpha = 0, \quad A_\alpha > 0, \quad m_g = A_0^{1/2} c^{-2} \]

[7]
Gravitational Wave Propagation: Dispersion

• Combine results from many detections
  GWTC-1 Abbott+19 [arXiv:1903.04467]

• Constraints on graviton mass \( \sim 1.8 \times \) more stringent than Solar System bounds
  [Bernus+20]

\[
m_g = \frac{\sqrt{A_0}}{c^2} \leq 1.76 \times 10^{-23} \text{ eV}/c^2
\]
Gravitational Wave Propagation: Polarization

General Relativity (Tensor)

- Plus
  - Helicity ±2
  - Cross

Vector

- Vector-x
  - Helicity ±1
- Vector-y

Scalar

- Conformal
  - Helicity 0
- Longitudinal
Gravitational Wave Propagation: Polarizations

- Polarization constraints from relative amplitudes and phases at different detectors
- For BBHs, need 5 detectors to break all possible degeneracies
- Tightest constraints from GW170817
- Limited studies to date, disfavour pure-vector or pure-scalar modes: [Abbott+17, Abbott+19, Abbott+20, Isi+17, Pang+20, …]
- Not discussed here: birefringence, where one polarization state suppressed relative to other [Okounkova+21, Shao+20, Wang+20]

Preference for non-tensor hypothesis if
\[ \log_{10} \frac{B^T_{V/S}}{\log_{10} B^T_{V/S} < 0} \]

Gravitational Wave Source Generation
Gravitational Wave Generation: Parameterized Tests

- GR hypothesis? We have a non-eccentric (quasi-circular) binary black hole
- From no hair theorem, BBH parameterized by:
  - mass ratio + two spin vectors
  - + extrinsic parameters (sky location, distance, etc)
- GW signal as sum over spin-weighted spherical harmonics
  \[ h_+ - ih_\times = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} h_{\ell m} Y_{\ell m}(\theta, \varphi) \]
- We use two waveform models (helps gauge systematics)
  - Precessing phenomenological model, PhenomPv2
    [Hannam+14, Husa+16, Khan+16]
  - Aligned-spin effective-one-body model, SEOBNRv4
    [Bohe+16]
Gravitational Wave Generation: Parameterized Tests

\[
\frac{dE_{\text{orbital}}}{dt} = \mathcal{L}_{GW} \approx \frac{32}{5} \frac{c^5}{G} \frac{(m_1 m_2)^2}{M^4} \left(\frac{v}{c}\right)^5
\]

Flux Balance:

- **Inspiral:** Post-Newtonian Theory
- **Effective One Body**
- **Analytical approximations begin to break down**
- **Merger:** No analytical model
- **Ringdown:** Black Hole Perturbation Theory

[13]
Gravitational Wave Generation: Parameterized Tests

- Precision models for phase evolution allows us to place tight constraints on GR!
- Inspiral described by post-Newtonian expansion in the orbital velocity

\[
\varphi(v) = \left(\frac{v}{c}\right)^{-5} \left[ \varphi_0 + \varphi_1 \left(\frac{v}{c}\right)^1 + \varphi_2 \left(\frac{v}{c}\right)^2 + \cdots + \varphi_{5l} \ln \left(\frac{v}{c}\right) \left(\frac{v}{c}\right)^5 + \cdots + \varphi_7 \left(\frac{v}{c}\right)^7 \right]
\]

- Spin effects enter phase at 1.5PN
- PN coefficients encode a range of physical phenomena: tails of radiation due to backscattering, spin-orbit and spin-spin couplings, BH absorption, …
- PN coefficients would take different values in modified theories of gravity compared to GR
Gravitational Wave Generation: Parameterized Tests

- For late-inspiral and merger-ringdown: parameterize coefficients in phenomenological waveform

\[
\varphi_{\text{Int}} = \frac{1}{\eta} \left( \beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3} \right)
\]

\[
\varphi_{\text{MR}} = \frac{1}{\eta} \left\{ \alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3} \alpha_3 f^{3/4} + \alpha_4 \tan^{-1} \left( \frac{f - \alpha_5 f_{\text{RD}}}{f_{\text{damp}}} \right) \right\}
\]

- Constrain deviations using parametric deformations to GR by varying one coefficient at a time

\[
p_i \rightarrow (1 + \delta p_i) p_i \quad p_i = \{ \varphi_i, \beta_i, \alpha_i \}
\]

Caution: Coefficients calibrated against NR but are not expressed in parameters relevant to GR or any modified theory of gravity...
Gravitational Wave Generation: Parameterized Tests

• GWTC-2: 90%-credible upper limit on each deviation parameter

\[ |\phi_i| \sim f^{(i-5)/3} \text{ inspiral} \]

• Caveat: joint results assume the same value for all events [16]
Gravitational Wave Generation: Parameterized Tests

- Deviations from GR expected to depend on binary parameters (mass, spin, etc)
- Should not simply multiply likelihoods or Bayes factors [Zimmerman+19]
- Alternative: model the observed distribution using hierarchical Bayesian inference [Isi+19]

\[
p(x|d) = \int p(x|\mu, \sigma)p(\mu, \sigma|d)d\mu d\sigma
\]

Infer \(\mu_i\) and \(\sigma_i\) from all events simultaneously

Gravitational Wave Generation: Parameterized Tests

• Results using Hierarchical Bayesian inference…

Empty = shared deviation; Filled = non-shared deviation (hierarchical)
Gravitational Wave Generation: Parameterized Tests

- In GR isolated BHs are characterised by 3 parameters:
  - Mass
  - Spin
  - Charge (No charge for astrophysical BHs)
- Infer properties of remnant BH from spectroscopic analysis of ringdown - consistency with Kerr?

\[
h = \sum_{\ell m n} C_{\ell m n} e^{-i\omega_{\ell m n} t} - 2 S_{\ell m n}(\tau, \varphi)
\]

\(\omega_{\ell m n} \sim \) intrinsic geometry

\(C_{\ell m n} \sim \) initial conditions

\(-2S_{\ell m n} \sim \) angular structure of radiation

Azimuthal number: \(\ell \geq 2\)
Magnetic number: \(-\ell \leq m \leq \ell\)
Overtone number: \(n \geq 0\)
Gravitational Wave Generation: Parameterized Tests

Mass & spin inferred from fundamental mode

Carullo+18, Isi+19

Mass & spin inferred from inspiral

Brito+19, Ghosh+21

Consistency Tests
Gravitational Wave Consistency: *Inspiral-Merger-Ringdown*

- Check that observations are consistent with predictions in GR
- If GR is correct, final state of a BBH merger is a perturbed Kerr BH...
- Estimate mass and spin of remnant BH from inspiral and post-inspiral regimes independently...

\[
\frac{\Delta M_f}{\bar{M}_f} = 2 \frac{M_f^{\text{insp}} - M_f^{\text{postinsp}}}{M_f^{\text{insp}} + M_f^{\text{postinsp}}}
\]

\[
\frac{\Delta \chi_f}{\bar{\chi}_f} = 2 \frac{\chi_f^{\text{insp}} - \chi_f^{\text{postinsp}}}{\chi_f^{\text{insp}} + \chi_f^{\text{postinsp}}}
\]
Conclusions
Conclusions

- **No** statistically significant deviations from GR that cannot be accounted for by systematics
- Improve over GWTC-1 constraints by factor ~ 2-3
- New analyses, new statistical techniques
- Many topics not yet discussed:
  - Echoes, Spin-induced quadrupole, birefringence, leakage, …
- A lot of ground that still needs to be covered:
  - Understand waveform systematics
  - Need to bridge GW data analysis to theory!
  - How do we interpret results in model dependent framework?
  - Need to improve waveform models! [Pratten+21]
Thank you!

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Appendix
Gravitational Wave Propagation: Dispersion

- Combine results from many detections
  - GWTC-1 Abbott+19 [arXiv:1903.04467]
Gravitational Wave Generation: Parameterized Tests

• For GW190412 and GW190814 higher multipoles are relevant....

Empty = SEOB; Filled = Phenom
Gravitational Wave Consistency: Residuals

- Subtract best-fit template from GW data, is residual SNR consistent with noise?

\[ p = P(SNR_n \geq SNR_{\text{residual}} | \text{noise}) \]

- All p-values consistent with residual SNR produced by noise