The cosmological constant in supergravity and string theory

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Universe evolution: based on positive cosmological constant

- Dark Energy
  simplest case: infinitesimal (tuneable) +ve cosmological constant

- Inflation (approximate de Sitter)
  describe possible accelerated expanding phase of our universe

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The cosmological constant in Supergravity

Highly constrained: \( \Lambda \geq -3m_{3/2}^2 \)

- equality \( \Rightarrow \) AdS (Anti de Sitter) supergravity
  
  \[ m_{3/2} = W_0 : \text{constant superpotential} \]

- inequality: dynamically by minimising the scalar potential
  
  \( \Rightarrow \) uplifting \( \Lambda \) and breaking supersymmetry

- \( \Lambda \) is not an independent parameter for arbitrary breaking scale \( m_{3/2} \)

What about breaking SUSY with a \( \langle D \rangle \) triggered by a constant FI-term?

standard supergravity: possible only for a gauged \( U(1)_R \) symmetry:

absence of matter \( \Rightarrow \) \( W_0 = 0 \) \( \rightarrow \) dS vacuum \hspace{1cm} Friedman '77

- exception: non-linear supersymmetry [8]
Non-linear SUSY in supergravity

\( K = X \bar{X} \; ; \; W = f X + W_0 \)

\( X \equiv X_{NL} \) nilpotent goldstino superfield \([6]\)

\[ X_{NL}^2 = 0 \Rightarrow X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F \]

\[ \Rightarrow \quad V = |f|^2 - 3|W_0|^2 \quad ; \quad m^2_{3/2} = |W_0|^2 \]

- \( V \) can have any sign contrary to global NL SUSY
- NL SUSY in flat space \( \Rightarrow f = \sqrt{3} m_{3/2} M_p \)
- R-symmetry is broken by \( W_0 \)
gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge

Global supersymmetry:

\[ \mathcal{L}_{\text{FI}}^{\text{new}} = \xi_1 \int d^4\theta \frac{W^2 \bar{W}^2}{D^2 W^2 \bar{D}^2 \bar{W}^2} \partial W = -\xi_1 \mathcal{D} + \text{fermions} \]

It makes sense only when \( <\mathcal{D}> \neq 0 \Rightarrow \) SUSY broken by a D-term

Supergravity generalisation: straightforward

unitary gauge: goldstino = \( U(1) \) gaugino = 0 \Rightarrow \) standard sugra \(-\xi_1 \mathcal{D}\)
Pure sugra + one vector multiplet $\Rightarrow$ [4]

$$\mathcal{L} = R + \bar{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left(-3m_{3/2}^2 + \frac{1}{2} \xi_1^2 \right)$$

- $\xi_1 = 0$ $\Rightarrow$ AdS supergravity
- $\xi_1 \neq 0$ uplifts the vacuum energy and breaks SUSY
  - e.g. $\xi_1 = \sqrt{6} m_{3/2}$ $\Rightarrow$ massive gravitino in flat space
New FI-term introduces a cosmological constant in the absence of matter

Presence of matter $\Rightarrow$ non trivial scalar potential  
net result: $\xi_1 \rightarrow \xi_1 e^{K/3}$

but breaks Kähler invariance

However new FI-term in the presence of matter is not unique

Question: can one modify it to respect Kähler invariance?

Answer: yes, constant FI-term $+$ fermions as in the absence of matter

$\Rightarrow$ constant uplift of the potential  
I.A.-Chatrabhuti-Isono-Knoops '18

Jang-Porrati '21

In general $\xi_1 \rightarrow \xi_1 f\left(\frac{m_3}{2}[\phi,\bar{\phi}]\right)$  
I.A.-Rondeau '99

It can also be written in $N = 2$ supergravity  
I.A.-Derendinger-Farakos-Tartaglino Mazzucchelli '19
Swampland de Sitter conjecture

String theory: vacuum energy and inflation models related to the moduli stabilisation problem

Difficulties to find dS vacua led to a conjecture:

\[
\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -c' \quad \text{in Planck units}
\]

with \(c, c'\) positive order 1 constants

Ooguri-Palti-Shiu-Vafa '18

Dark energy: forbid dS minima but allow maxima

Inflation: forbid standard slow-roll conditions

Assumptions: heuristic arguments, no quantum corrections

\[\rightarrow\] here: explicit counter example
Moduli stabilisation in type IIB

Compactification on a Calabi-Yau manifold $\Rightarrow N = 2$ SUSY in 4 dims

**Moduli:** Complex structure in vector multiplets

- Kähler class & dilaton in hypermultiplets

$\Rightarrow$ decoupled kinetic terms

turn on appropriate 3-form fluxes (primitive self-dual) $\Rightarrow N = 1$ SUSY

- field-strengths of 2-index antisymmetric gauge potentials

+ orientifolds and D3/D7-branes

vectors and RR companions of geometric moduli are projected away $\Rightarrow$

all moduli in $N = 1$ chiral multiplets $+$ superpotential for the

**complex structure & dilaton** $\rightarrow$ fixed in a SUSY way Frey-Polchinski ’02

Kähler moduli: no scale structure, vanishing potential (classical level) [11]
String moduli

String compactifications from 10/11 to 4 dims → scalar moduli

arbitrary VEVs: parametrize the compactification manifold

- $N = 1$ SUSY $\Rightarrow$ complexification: scalar $+ i$ pseudoscalar $\equiv \phi_i$

- Low energy couplings: functions of moduli

size of cycles, shapes, . . . , string coupling
Stabilisation of Kähler moduli

Non perturbative superpotential from gaugino condensation on D-branes
⇒ stabilisation in an AdS vacuum  Derendinger-Ibanez-Nilles ’85

Uplifting using anti-D3 branes  Kachru-Kallosh-Linde-Trivedi ’03

or D-terms and perturbative string corrections to the Kähler potential

Large Volume Scenario (LVS)  Conlon-Quevedo et al ’05

Ongoing debate on the validity of these ingredients in full string theory

While perturbative stabilisation has the old Dine-Seiberg problem

put together 2 orders of perturbation theory violating the expansion

possible exception known from filed theory:

logarithmic corrections → Coleman-Weinberg mechanism
The Dine-Seiberg problem

Runaway potential towards vanishing string coupling or large volume

\[ V \]

\[ \Rightarrow \text{ if there is perturbative minimum, it is likely to be at strong coupling or string size volume} \]
Analogy with Coleman-Weinberg symmetry breaking

Effective potential in massless $\lambda \Phi^4$

$$V = \left\{ \sum_{N > 1} c_N \lambda^N(\Phi) \right\} \Phi^4 \Rightarrow \text{minimum at } \lambda = 0 \text{ or } \mathcal{O}(1)$$

C-W perturbative symmetry breaking needs 2 couplings + logs: [18]

$$V_{C-W} = \left( \lambda + c_1 e^4 \ln \frac{|\Phi|^2}{\mu^2} \right) |\Phi|^4 \Rightarrow |\Phi|^2_{\text{min}} \propto \mu^2 e^{-\frac{\lambda}{c_1 e^4}}$$

both $\lambda$ and $e$ are weak $< 1$

realising this proposal in string theory:

- replace gaugino condensation by log corrections in the F-part potential
- use D-term uplifting as in LVS
Log corrections in string theory:

**localised couplings + closed string propagation in** $d \leq 2$

Effective propagation of massless bulk states in $d \leq 2 \Rightarrow$ IR divergences \[^{[18]}\]

$d = 1$: linear, $d = 2$: logarithmic

$\Rightarrow$ corrections to (brane) localised couplings

depending on the size of the bulk due to local closed string tadpoles

I.A.-Bachas '98

e.g. threshold corrections to 4d gauge coupling

linear dilaton dependence on the 11th dim of M-theory \[^{[16]}\]

Type II strings: correction to the Kähler potential $\leftrightarrow$ Planck mass

I.A.-Ferrara-Minasian-Narain '97
decompactification limit in the presence of branes

\[
\mathcal{A} \sim \frac{1}{V_\perp} \sum_{|p_\perp| < M_s} \frac{1}{p_\perp^2} F(\vec{p}_\perp)
\]

\[
V_\perp = R^d \quad \vec{p}_\perp = \vec{n}/R
\]

\[
R >> l_s \Rightarrow
\]

\[
\mathcal{A} \sim \begin{cases} 
O(R) & \text{for } d=1 \\
O(\log R) & \text{for } d=2 \\
\text{finite} & \text{for } d \geq 2
\end{cases}
\]

local tadpoles: \[ F(\vec{p}_\perp) \sim \left( 2^{5-d} \prod_{i=1}^{d} (1 + (-1)^{n_i}) - 2 \sum_{a=1}^{16} \cos(\vec{p}_\perp \vec{y}_a) \right) \]
Localised gravity kinetic terms

Corrections to the 4d Planck mass in type II strings

Large volume limit: localised Einstein-Hilbert term in the 6d internal space

10d: $R \wedge R \wedge R \wedge R \rightarrow$ in 4d: $\chi \mathcal{R}_{(4)}$

Euler number $= 4(n_H - n_V)$ \[^{[21]}\]

$$S^\text{IIB}_{\text{grav}} = \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times X_6} e^{-2\phi} \mathcal{R}_{(10)} + \frac{\chi}{(2\pi)^4 \alpha'} \int_{M_4} \left(2\zeta(3)e^{-2\phi} + \frac{2\pi^2}{3}\right) \mathcal{R}_{(4)}$$

4-loop $\sigma$-model vanishes for orbifolds

localisation width $w \sim |\chi| l_s = l_p^{(4)}$

in agreement with general arguments of localised gravity

Dvali-Gabadadze-Porrati '00
localised vertices from $\mathcal{R}_{(4)}$ can emit massless closed strings

$\Rightarrow$ local tadpoles in the presence of distinct 7-brane sources

propagation in 2d transverse bulk $\rightarrow \log R_\perp$ corrections

exact computation: difficult either in CY or in orbifolds - genus 3/2

computation in the degeneration limit

for $Z_N$ orbifold ($\chi \sim N$)

$$\sim - \sum_{q_\perp \neq 0} g_s^2 T N e^{-w^2 q_\perp^2 / 2} \frac{1}{q_\perp^2 R_\perp^2} = -Ng_s^2 T \log (R_\perp / w) + \cdots$$

$T = T_0 / g_s$: brane tension
Kähler potential:

\[
\mathcal{K} = -2 \ln \left( \mathcal{V} + \xi + \eta \ln \frac{\mathcal{V} \perp}{w^2} + \mathcal{O}\left( \frac{1}{\mathcal{V}} \right) \right) = -2 \ln \left( \mathcal{V} + \eta \ln \mu^2 \mathcal{V} \perp \right)
\]

\[
\xi = -\frac{1}{4} \chi f(g_s); \quad f(g_s) = \begin{cases} 
\zeta(3) \simeq 1.2 & \text{smooth CY} \\
\frac{\pi^2}{3} g_s^2 & \text{orbifolds}
\end{cases} \quad \eta = -\frac{1}{2} g_s T_0 \xi \quad [16]
\]

Using 3 mutual orthogonal 7-brane stacks with D-terms (magnetic fluxes) and minimising with respect to transverse volume ratios [13]

\[
\Rightarrow \mathcal{V} \simeq \frac{3 \eta \mathcal{W}_0^2}{\mathcal{V}^3} (\ln \mu^6 \mathcal{V} - 4) + 3 \frac{d}{\mathcal{V}^2} \quad \mathcal{W}_0: \text{constant superpotential, } d: \text{D-term}
\]

dS minimum: \(-0.007242 < \frac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho \equiv -0.006738 \) with \( \mathcal{V} \simeq e^5 / \mu^6 \) [20]
\[ V_{D_i} = \frac{d_i}{\tau_i} \left( \frac{\partial K}{\partial \tau_i} \right)^2 = \frac{d_i}{\tau_i^3} + O(\eta_j) \]

\( \tau_i \): world-volume modulus of D7\(_i\)-brane stack with \( V = (\tau_1 \tau_2 \tau_3)^{1/2} \)

\[ \eta_i \equiv \eta \Rightarrow V_{tot} = \frac{3\eta V_0^2}{V^3} (\ln(V \mu^6) - 4) + \frac{d_1}{\tau_1^3} + \frac{d_2}{\tau_2^3} + \frac{d_3 \tau_1^3 \tau_2^3}{V^6} \]

minimising with respect to \( \tau_1 \) and \( \tau_2 \) \( \Rightarrow \frac{\tau_i}{\tau_j} = \left( \frac{d_i}{d_j} \right)^{1/3} \Rightarrow \)

\[ V_D = 3 \frac{d}{V^2} \text{ with } d = (d_1 d_2 d_3)^{1/3} \]
\[ \rho = -0.00695 \]
\[ \rho = -0.00678 \]
\[ \rho = -0.00669 > \rho_{\text{max}} \]

2 extrema min+max \( \rightarrow -0.007242 < \rho < -0.006738 \leftarrow +\text{ve energy} \quad [18] \quad [24] \)
\[ \xi = -\frac{1}{4} \chi f(g_s) \; ; \; \; f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \]

\[ \eta = -\frac{1}{2} g_s T_0 \xi \]

dS minimum: \(-0.007242 < \frac{d}{\eta \mathcal{V}_0^2 \mu^6} \equiv \rho < -0.006738\) with \(\mathcal{V} \simeq e^5 / \mu^6\)

exponentially large volume:

\[ \mu = \frac{e^{\xi/6\eta}}{\mathcal{W}} = \sqrt{|\chi|} e^{-\frac{1}{3g_s T_0}} \rightarrow 0 \quad \Rightarrow \]

weak coupling and

large \(\chi\) or/and \(\mathcal{W}_0\) from 3-form flux to keep \(\rho\) fixed

requirement: negative \(\chi\) \((\eta < 0)\) \([16]\) and surplus of D7-branes \((T_0 > 0)\)
Inflation possibilities

- Inflaton: canonically normalised \( \phi = \sqrt{2/3} \ln V \) (in Planck units)
- one relevant parameter: \( \rho \) or \( x = -\ln (-4\rho/3) - 16/3 \)
  
  \[
  0 < x < 0.072 \text{ for dS minimum}
  \]
- extrema \( V'(\phi_{\pm}) = 0 \)
  
  \[
  \phi_+ - \phi_- = \sqrt{2/3} \left( W_0(-e^{-x-1}) - W_{-1}(-e^{-x-1}) \right)
  \]
  
  \( W_{0/-1}: \) Lambert functions satisfying \( W(xe^x) = x \)

\[
\frac{V(\phi_+)}{V(\phi_-)} = \frac{(W_0(-e^{-x-1}))^3(2+3W_{-1}(-e^{-x-1}))}{(W_{-1}(-e^{-x-1}))^3(2+3W_0(-e^{-x-1}))}
\]

- slow roll parameter \( \eta(\phi_{-/+}) = \frac{V''(\phi_{-/+})}{V(\phi_{-/+})} = -9\frac{1+W_{0/-1}(-e^{-x-1})}{3+W_{0/-1}(-e^{-x-1})} \) \[^{[25]}\]

successful inflation possible around the minimum from the inflection point
In the following sections we will study the inflationary potential coming from the D7-branes moduli stabilisation.

The Kähler potential of the model is

$$K = \phi^2 \phi^4 + W_0 \phi \phi^2 \phi^4 + \phi \phi^2 \phi^4$$

(28)

$$f = \phi^2 \phi^4$$

(29)

In order to minimize and study the slow-roll parameters, we need to isolate the volume from the two other perpendicular directions. This is done by taking the logarithm of the volume and defining

$$\ln \frac{V}{V_0} = \text{constant}$$

(13)

$$\ln \frac{V}{V_0} = \text{constant}$$

(14)

In the above expression, $V$ is the total volume and $V_0$ is a reference volume. The minimisation of the scalar potential is then dictated by the condition that the derivative of the potential with respect to the volume is zero.

From (13), we obtain

$$\frac{V}{V_0} = \text{constant}$$

(15)

Hence, the volume is given by

$$V = V_0$$

(16)

And from (14), we find

$$\ln \frac{V}{V_0} = 0$$

(17)

This implies that the volume is constant and can be written as

$$V = V_0$$

(18)

The volume is thus a constant of motion, and its non-vanishing dependence on the three magnetised D7 branes is given by

$$V = V_0$$

(19)

$$V = V_0$$

(20)

In the following sections, we will study the inflationary potential coming from the D7-branes moduli stabilisation.
Inflation possibilities

- Friedmann equations with time replaced by the inflaton ⇒

  Hubble parameter → \( H'(\phi) = \mp \frac{1}{\sqrt{2}} \sqrt{3H^2(\phi) - V(\phi)} \)

- slow-roll parameters: \( \eta(\phi) = \frac{V''(\phi)}{V(\phi)} \), \( \epsilon(\phi) = \frac{1}{2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \)

- number of e-folds by the end of inflation: \( N(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}} \)

Observational constraints at the horizon exit \( \phi = \phi_* \):

1. \( N_* \simeq 50 - 60 \)
2. spectral index of power spectrum \( n_S - 1 = 2\eta_* - 6\epsilon_* \simeq -0.04 \)
3. amplitude of scalar perturbations \( A_S = \frac{V_*}{24\pi^2\epsilon_*} \simeq 2.2 \times 10^{-9} \)

⇒ inflation possible around the minimum from the inflection point [20]
\[ x = 3.3 \times 10^{-4}; \quad \eta(\phi_*) = -0.02 \]

\( \phi_* \) near the inflection point

\[ \Delta \phi \simeq 0.02 : \text{small field} \]

\[ r \simeq 4 \times 10^{-4} \] \[ [27] \]

\[ H_* \simeq 5 \times 10^{12} \text{ GeV} \]
dS vacuum metastability \textsuperscript{[23]}

- through tunnelling $H_c > H_-$ Coleman - de Luccia instanton
- over the barrier $H_c < H_-$ Hawking - Moss transition

\[ \frac{H_c^2}{H_-^2} \equiv -\frac{3V''(\phi_+)}{4V(\phi_-)} \]

HM region: $\Gamma \sim e^{-B}$; $B \sim \frac{24\pi^2}{V} \frac{\Delta V}{V}$

\[ \frac{\Delta V}{V} \sim 24\sqrt{2}x^{3/2} \Rightarrow \]

$B \sim 3 \times 10^9$ for $x \sim 3 \times 10^{-4}$
Conclusions

Novel D-terms in supergravity that do not gauge the R-symmetry allow to write a positive cosmological constant even without matter fields.

Their implementation in string theory: open problem.

New mechanism of moduli stabilisation is string theory (type IIB):

- perturbative: weak coupling, large volume
- based on log corrections in the transverse volume of 7-branes due to local tadpoles induced by localised gravity kinetic terms arising only in 4 dimensions!
- can lead to de Sitter vacua in string theory explicit counter-example to dS swampland conjecture
- inflation possible around the minimum from the inflection point