The cosmological constant in supergravity and string theory

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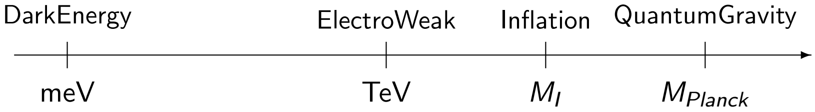
Universe evolution: based on positive cosmological constant

- Dark Energy

  simplest case: infinitesimal (tuneable) +ve cosmological constant

- Inflation (approximate de Sitter)

  describe possible accelerated expanding phase of our universe



The cosmological constant in Supergravity

Highly constrained: $\Lambda \geq -3m_{3/2}^2$

- equality $\Rightarrow$ AdS (Anti de Sitter) supergravity

  $m_{3/2} = W_0$: constant superpotential

- inequality: dynamically by minimising the scalar potential

  $\Rightarrow$ uplifting Λ and breaking supersymmetry

• Λ is not an independent parameter for arbitrary breaking scale $m_{3/2}$

What about breaking SUSY with a $<$D$>$ triggered by a constant FI-term?

standard supergravity: possible only for a gauged $U(1)_R$ symmetry:

absence of matter $\Rightarrow$ $W_0 = 0 \rightarrow$ dS vacuum   Friedman '77

• exception: non-linear supersymmetry [8]

Non-linear SUSY in supergravity

I.A.-Dudas-Ferrara-Sagnotti '14

$$K = X\bar{X} \quad ; \quad W = f\,X + W_0$$

$X \equiv X_{NL}$ nilpotent goldstino superfield [6]

$$X_{NL}^2 = 0 \;\Rightarrow\; X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\Rightarrow \quad V = |f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign   contrary to global NL SUSY
- NL SUSY in flat space $\Rightarrow f = \sqrt{3}\, m_{3/2} M_p$
- R-symmetry is broken by W_0

A new FI term   Cribiori-Farakos-Tournoy-Van Proeyen '18

gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge

Global supersymmetry:

gauge field-srength superfield

$$\mathcal{L}_{\rm FI}^{new} = \xi_1 \int d^4\theta \frac{\mathcal{W}^2\overline{\mathcal{W}}^2}{\mathcal{D}^2\mathcal{W}^2\bar{\mathcal{D}}^2\overline{\mathcal{W}}^2}\mathcal{D}\mathcal{W} = -\xi_1 \mathrm{D} + \text{fermions}$$

It makes sense only when $< \mathrm{D} > \neq 0 \Rightarrow$ SUSY broken by a D-term

Supergravity generalisation: straightforward

unitary gauge: goldstino $=$ $U(1)$ gaugino $= 0 \Rightarrow$ standard sugra $-\xi_1 \mathrm{D}$

New FI term in supergravity

Pure sugra + one vector multiplet $\Rightarrow$ [4]

$$\mathcal{L} = R + \bar{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left(-3 m_{3/2}^2 + \frac{1}{2} \xi_1^2 \right)$$

- $\xi_1 = 0 \Rightarrow$ AdS supergravity
- $\xi_1 \neq 0$ uplifts the vacuum energy and breaks SUSY

  e.g. $\xi_1 = \sqrt{6} m_{3/2} \Rightarrow$ massive gravitino in flat space

New FI-term introduces a cosmological constant in the absence of matter

Presence of matter $\Rightarrow$ non trivial scalar potential net result: $\xi_1 \to \xi_1 \, e^{K/3}$

but breaks Kähler invariance

However new FI-term in the presence of matter is not unique

Question: can one modify it to respect Kähler invariance?

Answer: yes, constant FI-term + fermions as in the absence of matter

$\Rightarrow$ constant uplift of the potential I.A.-Chatrabhuti-Isono-Knoops '18

Jang-Porrati '21

In general $\xi_1 \to \xi_1 \, f(m_{3/2}[\phi, \bar{\phi}])$ I.A.-Rondeau '99

It can also be written in $N = 2$ supergravity

I.A.-Derendinger-Farakos-Tartaglino Mazzucchelli '19

Swampland de Sitter conjecture

String theory: vacuum energy and inflation models

related to the moduli stabilisation problem

Difficulties to find dS vacua led to a conjecture:

$$\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -c' \quad \text{in Planck units}$$

with c, c' positive order 1 constants Ooguri-Palti-Shiu-Vafa '18

Dark energy: forbid dS minima but allow maxima

Inflation: forbid standard slow-roll conditions

Assumptions: heuristic arguments, no quantum corrections

$\longrightarrow$ here: explicit counter example

Moduli stabilisation in type IIB

Compactification on a Calabi-Yau manifold $\Rightarrow N = 2$ SUSY in 4 dims

Moduli: Complex structure in vector multiplets

Kähler class & dilaton in hypermultiplets

$\Rightarrow$ decoupled kinetic terms

turn on appropriate 3-form fluxes (primitive self-dual) $\Rightarrow N = 1$ SUSY

$\uparrow$ field-strengths of 2-index antisymmetric gauge potentials

$+$ orientifolds and D3/D7-branes

vectors and RR companions of geometric moduli are projected away $\Rightarrow$

all moduli in $N = 1$ chiral multiplets $+$ superpotential for the

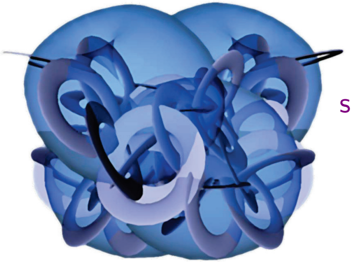
complex structure & dilaton $\rightarrow$ fixed in a SUSY way Frey-Polchinski '02

Kähler moduli: no scale structure, vanishing potential (classical level) [11]

String moduli

String compactifications from 10/11 to 4 dims $\rightarrow$ scalar moduli

arbitrary VEVs: parametrize the compactification manifold



size of cycles, shapes, . . . , string coupling

- $N = 1$ SUSY $\Rightarrow$ complexification: scalar + i pseudoscalar $\equiv \phi_i$
- Low energy couplings: functions of moduli

Stabilisation of Kähler moduli

Non perturbative superpotential from gaugino condensation on D-branes

$\Rightarrow$ stabilisation in an AdS vacuum Derendinger-Ibanez-Nilles '85

Uplifting using anti-D3 branes Kachru-Kallosh-Linde-Trivedi '03

or D-terms and perturbative string corrections to the Kähler potential

Large Volume Scenario (LVS) Conlon-Quevedo et al '05

Ongoing debate on the validity of these ingredients in full string theory

While perturbative stabilisation has the old Dine-Seiberg problem

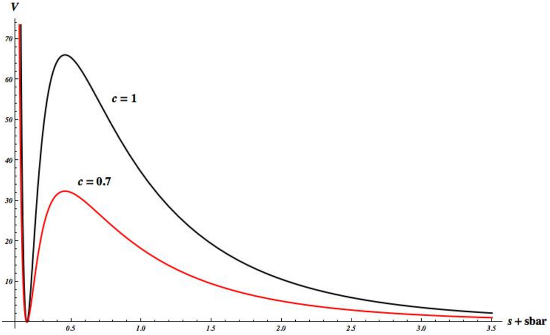
put together 2 orders of perturbation theory violating the expansion

possible exception known from filed theory:

logarithmic corrections $\rightarrow$ Coleman-Weinberg mechanism

The Dine-Seiberg problem

Runaway potential towards vanishing string coupling or large volume



$\Rightarrow$ if there is perturbative minimum, it is likely to be at strong coupling

or string size volume

Analogy with Coleman-Weinberg symmetry breaking

Effective potenial in massless $\lambda\Phi^4$

$$V = \left\{ \sum_{N>1} c_N \lambda^N(\Phi) \right\} \Phi^4 \;\Rightarrow\; \text{minimum at } \lambda = 0 \text{ or } \mathcal{O}(1)$$

C-W perturbative symmetry breaking needs 2 couplings + logs: [18]

$$V_{\rm C-W} = \left(\lambda + c_1 e^4 \ln \frac{|\Phi|^2}{\mu^2} \right) |\Phi|^4 \;\Rightarrow\; |\Phi|^2_{\rm min} \propto \mu^2 e^{-\frac{\lambda}{c_1 e^4}}$$

both λ and e are weak < 1

realising this proposal in string theory:

- replace gaugino condensation by log corrections in the F-part potential
- use D-term uplifting as in LVS

Log corrections in string theory:

localised couplings + closed string propagation in $d \leq 2$

Effective propagation of massless bulk states in $d \leq 2 \Rightarrow$ IR divergences [18]

$d = 1$: linear, $d = 2$: logarithmic

$\Rightarrow$ corrections to (brane) localised couplings

depending on the size of the bulk due to local closed string tadpoles

I.A.-Bachas '98

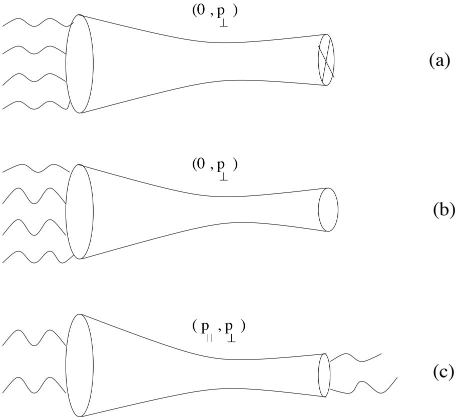
e.g. threshold corrections to 4d gauge coupling

linear dilaton dependence on the 11th dim of M-theory [16]

Type II strings: correction to the Kähler potential $\leftrightarrow$ Planck mass

I.A.-Ferrara-Minasian-Narain '97

decompactification limit in the presence of branes



$$\mathcal{A} \sim \frac{1}{V_\perp} \sum_{|p_\perp|<M_s} \frac{1}{p_\perp^2} F(\vec{p}_\perp)$$

$$V_\perp = R^d \quad \vec{p}_\perp = \vec{n}/R$$

$$R >> l_s \Rightarrow$$

$$\mathcal{A} \sim \begin{cases} \mathcal{O}(R) & \textit{for } d{=}1 \\ \mathcal{O}(\log R) & \textit{for } d{=}2 \\ \text{finite} & \textit{for } \mathrm{d}>2 \end{cases}$$

local tadpoles: $F(\vec{p}_\perp) \sim \left(2^{5-d} \prod_{i=1}^d (1+(-)^{n_i}) - 2\sum_{a=1}^{16} \cos(\vec{p}_\perp \vec{y}_a)\right)$

Localised gravity kinetic terms

Corrections to the 4d Planck mass in type II strings

Large volume limit: localised Einstein-Hilbert term in the 6d internal space

I.A.-Minasian-Vanhove '02 [18]

10d: $R \wedge R \wedge R \wedge R \quad \rightarrow \quad$ in 4d: $\chi \mathcal{R}_{(4)}$

Euler number $= 4(n_H - n_V)$ [21]

$$S_{\rm grav}^{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)} + \frac{\chi}{(2\pi)^4 \alpha'} \int_{M_4} \left(2\zeta(3) e^{-2\phi} + \frac{2\pi^2}{3} \right) \mathcal{R}_{(4)}$$

4-loop σ-model $\nearrow$ vanishes for orbifolds

localisation width $w \sim |\chi| l_s = l_p^{(4)}$

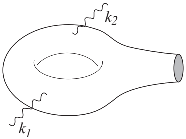
in agreement with general arguments of localised gravity

Dvali-Gabadadze-Porrati '00

perturbative moduli stabilisation I.A.-Chen-Leontaris '18, '19

localised vertices from $\mathcal{R}_{(4)}$ can emit massless closed strings

$\Rightarrow$ local tadpoles in the presence of distinct 7-brane sources

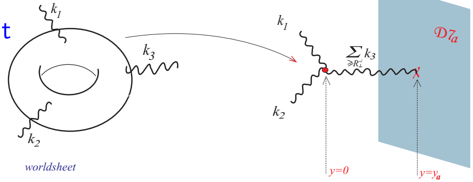


propagation in 2d transverse bulk $\rightarrow$ $\log R_\perp$ corrections

exact computation: difficult either in CY or in orbifolds - genus 3/2

computation in the degeneration limit

for Z_N orbifold ($\chi \sim N$)



$$\sim -\sum_{q_\perp \neq 0} g_s^2 T N e^{-w^2 q_\perp^2/2} \frac{1}{q_\perp^2 R_\perp^2} = -N g_s^2 T \log(R_\perp / w) + \cdots$$

$T = T_0/g_s$: brane tension

perturbative moduli stabilisation I.A.-Chen-Leontaris '18, '19

Kähler potential:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \xi + \eta\ln\frac{\mathcal{V}_\perp}{w^2} + \mathcal{O}(\frac{1}{\mathcal{V}})\right) = -2\ln\left(\mathcal{V} + \eta\ln\mu^2\mathcal{V}_\perp\right)$$

$$\xi = -\frac{1}{4}\chi f(g_s)\,; \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \qquad \eta = -\frac{1}{2} g_s T_0 \xi$$ [16]

Using 3 mutual orthogonal 7-brane stacks with D-terms (magnetic fluxes)

and minimising with respect to transverse volume ratios [13]

$$\Rightarrow\ V \simeq \frac{3\eta\mathcal{W}_0^2}{\mathcal{V}^3}\left(\ln\mu^6\mathcal{V} - 4\right) + 3\frac{d}{\mathcal{V}^2}$$ $\mathcal{W}_0$: constant superpotential, d: D-term

dS minimum: $-0.007242 < \dfrac{d}{\eta\mathcal{W}_0^2\mu^6} \equiv \rho < -0.006738$ with $\mathcal{V} \simeq e^5/\mu^6$ [20]

FI D-terms

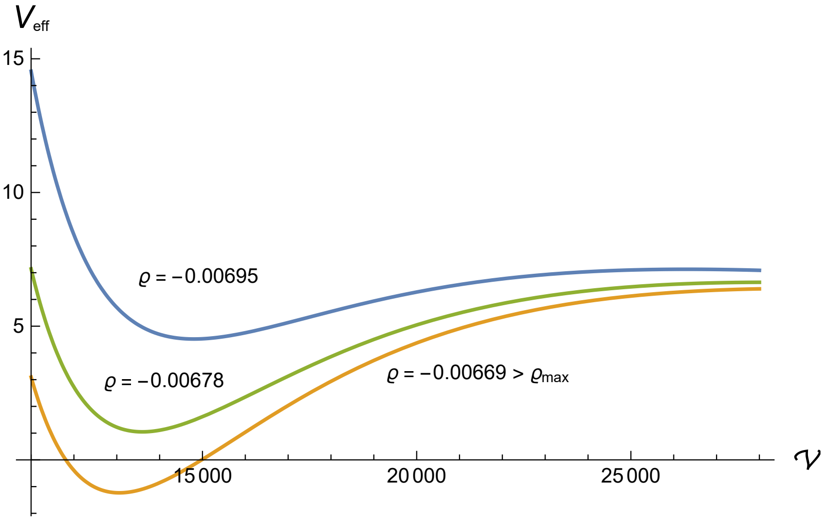
$$V_{D_i} = \frac{d_i}{\tau_i}\left(\frac{\partial K}{\partial_{\tau_i}}\right)^2 = \frac{d_i}{\tau_i^3} + \mathcal{O}(\eta_j)$$

τ_i: world-volume modulus of D7$_i$-brane stack with $\mathcal{V} = (\tau_1\tau_2\tau_3)^{1/2}$

$$\eta_i \equiv \eta \Rightarrow \quad V_{tot} = \frac{3\eta\mathcal{W}_0^2}{\mathcal{V}^3}\left(\ln(\mathcal{V}\mu^6) - 4\right) + \frac{d_1}{\tau_1^3} + \frac{d_2}{\tau_2^3} + \frac{d_3\tau_1^3\tau_2^3}{\mathcal{V}^6}$$

minimising with respect to τ_1 and $\tau_2 \Rightarrow \frac{\tau_i}{\tau_j} = \left(\frac{d_i}{d_j}\right)^{1/3} \Rightarrow$

$$V_D = 3\frac{d}{\mathcal{V}^2} \quad \text{with} \quad d = (d_1 d_2 d_3)^{1/3}$$



2 extrema min+max $\rightarrow -0.007242 < \rho < -0.006738 \leftarrow$ +ve energy [18] [24]

perturbative moduli stabilisation I.A.-Chen-Leontaris '18, '19

$$\xi = -\frac{1}{4}\chi f(g_s)\,; \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \qquad \eta = -\frac{1}{2} g_s T_0 \xi$$

dS minimum: $-0.007242 < \dfrac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho < -0.006738$ with $\mathcal{V} \simeq e^5/\mu^6$

exponentially large volume:

$$\mu = \frac{e^{\xi/6\eta}}{w} = \sqrt{|\chi|}\, e^{-\frac{1}{3 g_s T_0}} \to 0 \quad \Rightarrow$$

weak coupling and

large χ or/and $\mathcal{W}_0$ from 3-form flux to keep ρ fixed

requirement: negative χ ($\eta < 0$) [16] and surplus of D7-branes ($T_0 > 0$)

Inflation possibilities

I.A.-Lacombe-Leontaris '20

- Inflaton: canonically normalised $\phi = \sqrt{2/3}\,\ln\mathcal{V}$ (in Planck units)
- one relevant parameter: ρ or $x = -\ln(-4\rho/3) - 16/3$

$0 < x < 0.072$ for dS minimum

- extrema $V'(\phi_\pm) = 0$

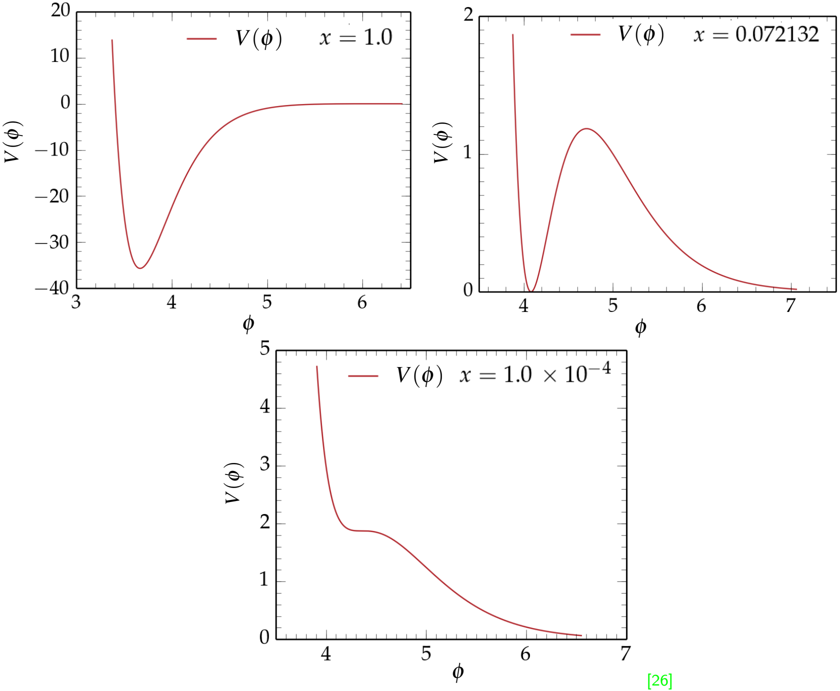
$$\phi_+ - \phi_- = \sqrt{2/3}\left(W_0(-e^{-x-1}) - W_{-1}(-e^{-x-1})\right)$$

$W_{0/-1}$: Lambert functions satisfying $W(xe^x) = x$

$$\frac{V(\phi_+)}{V(\phi_-)} = \frac{\left(W_0(-e^{-x-1})\right)^3\left(2+3W_{-1}(-e^{-x-1})\right)}{\left(W_{-1}(-e^{-x-1})\right)^3\left(2+3W_0(-e^{-x-1})\right)}$$

- slow roll parameter $\eta(\phi_{-/+}) = \frac{V''(\phi_{-/+})}{V(\phi_{-/+})} = -9\,\frac{1+W_{0/-1}(-e^{-x-1})}{\frac{2}{3}+W_{0/-1}(-e^{-x-1})}$ [25]

successful inflation possible around the minimum from the inflection point


$V(\phi)$ $x = 1.0$
20
10
0
−10
−20
−30
−40
3
4
5
6
ϕ
$V(\phi)$
$V(\phi)$ $x = 0.072132$
2
1
0
4
5
6
7
ϕ
$V(\phi)$
$V(\phi)$ $x = 1.0 \times 10^{-4}$
5
4
3
2
1
0
4
5
6
7
ϕ
$V(\phi)$

[26]

Inflation possibilities

- Friedmann equations with time replaced by the inflaton $\Rightarrow$

  Hubble parameter $\to$ $H'(\phi) = \mp\frac{1}{\sqrt{2}}\sqrt{3H^2(\phi) - V(\phi)}$

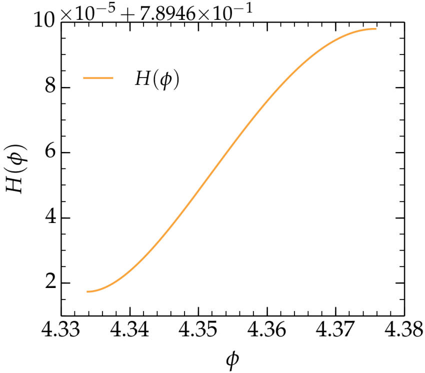
- slow-roll parameters: $\eta(\phi) = \frac{V''(\phi)}{V(\phi)}, \quad \epsilon(\phi) = \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$

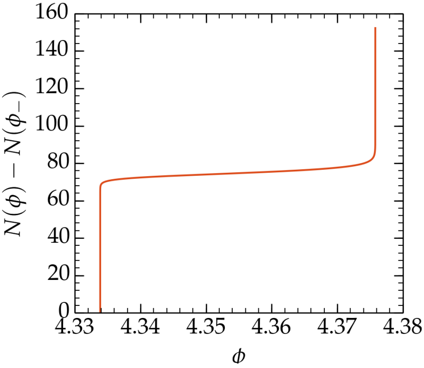
- number of e-folds by the end of inflation: $N(\phi) = \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}}$

Observational constraints at the horizon exit $\phi = \phi_*$:

1. $N_* \simeq 50 - 60$
2. spectral index of power spectrum $n_S - 1 = 2\eta_* - 6\epsilon_* \simeq -0.04$
3. amplitude of scalar perturbations $\mathcal{A}_S = \frac{V_*}{24\pi^2\epsilon_*} \simeq 2.2 \times 10^{-9}$

$\Rightarrow$ inflation possible around the minimum from the inflection point [20]





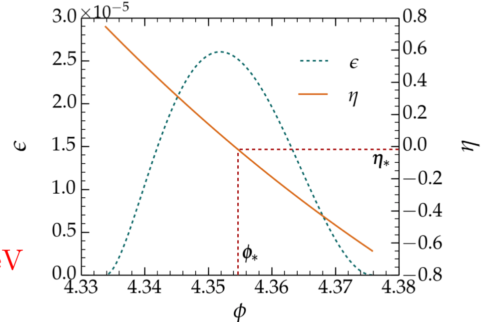
$x = 3.3 \times 10^{-4}$; $\eta(\phi_*) = -0.02$

ϕ_* near the inflection point

$\Delta\phi \simeq 0.02$: small field

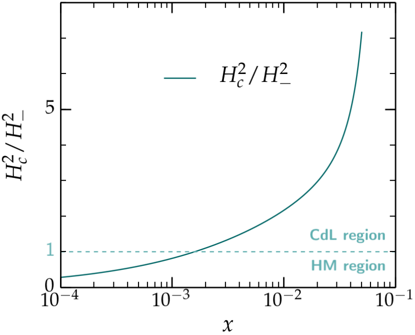
$\Rightarrow r \simeq 4 \times 10^{-4}$ [27]

$H_* \simeq 5 \times 10^{12}\,\mathrm{GeV}$



dS vacuum metastability [23]

- through tunnelling $H_c > H_-$ Coleman - de Luccia instanton
- over the barrier $H_c < H_-$ Hawking - Moss transition



$$\frac{H_c^2}{H_-^2} \equiv -\frac{3V''(\phi_+)}{4V(\phi_-)}$$

HM region: $\Gamma \sim e^{-B}$; $B \simeq \frac{24\pi^2}{V}\frac{\Delta V}{V}$

$$\frac{\Delta V}{V} \simeq 24\sqrt{2}x^{3/2} \Rightarrow$$

$$B \simeq 3 \times 10^9 \text{ for } x \simeq 3 \times 10^{-4}$$

Conclusions

Novel D-terms in supergravity that do not gauge the R-symmetry

allow to write a positive cosmological constant even without matter fields

their implementation in string theory: open problem

New mechanism of moduli stabilisation is string theory (type IIB)

- perturbative: weak coupling, large volume
- based on log corrections in the transverse volume of 7-branes

  due to local tadpoles induced by localised gravity kinetic terms

  arising only in 4 dimensions!
- can lead to de Sitter vacua in string theory

  explicit counter-example to dS swampland conjecture
- inflation possible around the minimum from the inflection point