

# State of the art of the sum over paths approach in quantum mechanics education

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**Abstract.** In this contribution I present an overview of the history and recent developments of the Feynman sum over paths approach for teaching introductory quantum mechanics to high school students and University undergraduates. It is argued that sum over paths has now reached full maturity as an educational reconstruction of quantum physics, and offers several advantages with respect to other approaches in terms of leading students to develop consistent mental models of quantum objects and ultimately achieving better conceptual understanding.

## 1 Introduction

The sum over paths approach in physics education originates mainly from two sources: Feynman's path integral formulation of quantum physics [1] and his own divulgation book "QED: the strange theory of light and matter" [2]. The latter, in fact, constitutes the first, fundamental sketch of an educational reconstruction of quantum physics based on the path integral formulation. Among the milestones of the approach's development one can trace the undergraduate course on quantum physics "Demystifying quantum mechanics" held by E. F. Taylor at MIT [3] which had a profound impact in the international physics education research community; and the Advancing Physics project [4] of the British Institute of Physics, an advanced physics course for high schools, designed to attract students to physics, and to give them a good basis for their future progression in the subject at university level, in which J. Ogborn, A. Dobson and co-workers [5] proposed an innovative presentation of quantum physics based on sum over paths. After the turn of the millennium, interest on the sum over paths approach has grown, with several works of great interest, both empirical [6,7] and theoretical [8,9].

## 2 Recent developments

Recent educational research has addressed most of the open issues standing on the sum over paths approach, including devising effective educational strategies for discussing time-independent problems such as bound states and tunneling [10, 11]; improving the treatment of the uncertainty principle [11]; establishing connections with two state approaches based on spin or polarization [12]; designing and realizing tools, such as interactive simulations and tutorials, to sustain students' learning [13, 14]; pinpointing and clarifying the educational advantages of sum over paths, including reliable measures of conceptual learning outcomes [15, 16] and highlighting the importance of concepts, such as path distinguishability, which were not central in the initial educational tests of the approach, but have demonstrated extremely fecund in leading to conceptual understanding of wave particle duality, and allowing to introduce modern experimental settings and technologies [17].

## 3 Educational advantages

Based on the results of research literature, and several years of direct experience with using the sum over paths approach in teacher education, we can summarize the main educational advantages offered by the approach in the following way:

- On the mathematical level, the sum over paths approach allows to discuss quantum phenomena using a very simple formal language. At its heart, such possibility is due to the fact that, rather than finding solutions to the Schrödinger equation, Feynman's method constructs the Green function for the same equation, representing it as a sum of complex amplitudes computed over all possible paths. In educational practice, complex amplitudes associated to paths can be represented and added up as vectors or "little arrows", a strategy directly derived from the one used by Feynman himself, which greatly reduces the stress on student's cognitive resources while learning the basics of quantum theory.
- On the conceptual level, sum over paths has the unique peculiarity of offering students a very clear and unambiguous representation of one of the most profound quantum mysteries, namely wave particle duality. The central distinction between classical and quantum ways of computing probabilities, which is at the heart of the approach, allows both to clearly distinguish classical and quantum physics, and to make the classical limit (correspondence principle) completely transparent. Furthermore, modern educational reconstructions based on sum over paths can offer deep insight on the origin of energy quantization for bound systems, help

clarifying the meaning of the uncertainty principle, and pinpointing the similarities and differences in the quantum behaviour of photons and electrons. Finally, sum over paths, with the introduction of the idea of path distinguishability/indistinguishability, allows to construct a language capable to discuss modern experiments and technologies based on quantum optics.

#### 4 Conclusions

In my overview of research on the sum over paths approach for teaching introductory quantum physics, I argue that such approach, started in the late 1980's, has reached full maturity in the second decade of the XXI century. Based on its characteristics, sum over paths can help researchers and educators improve educational outcomes in terms of conceptual understanding, and be an invaluable aid in the introduction of quantum technologies, an issue which is increasingly felt as central and urgent.

#### References

- [1] Feynman, R. P. (1948) "Space-time approach to non-relativistic quantum mechanics." *Reviews of Modern Physics*, 20(2), 367.
- [2] Feynman, R. P. (1985). *QED: The strange theory of light and matter*. Princeton, NJ: Princeton University Press.
- [3] Taylor, E. F., Vokos, S., O'Meara, J. M. and Thornber, N. S. (1998). "Teaching Feynman's sum-over-paths quantum theory." *Computers in Physics*, 12, 190.
- [4] Ogborn, J. and Whitehouse, M. (eds), (2000). *Advancing Physics AS*. Bristol (UK): Institute of Physics Publishing.
- [5] Dobson, K. , Lawrence, I. and Britton, P. (2000)."The A to B of quantum physics." *Physics Education*, 35(6), 400.
- [6] Hanc, J. and Tuleja, S. (2005). The Feynman Quantum Mechanics with the help of Java applets and physlets in Slovakia." In 10th Workshop on multimedia in physics teaching and learning, October 2005.
- [7] de los Angeles Fanaro M., Otero M. R., and Arlego M. ,(2012)."Teaching Basic Quantum Mechanics in Secondary School Using Concepts of Feynman Path Integrals Method." *The Physics Teacher*, 50(3), 156{158.
- [8] Ogborn, J. and Taylor, E. F. (2005). "Quantum physics explains Newton's laws of motion." *Physics Education*, 40(1), 26.
- [9] Hanc, J. (2006). "The time-independent Schrodinger equation in the frame of Feynman's version of quantum mechanics." In *Proceedings of 11th Workshop on Multimedia in Physics Teaching and Learning*, University of Szeged, Hungary.
- [10] Malgieri, M., Onorato, P., & De Ambrosis, A. (2016). "A sum-over-paths approach to one-dimensional time-independent quantum systems." *American Journal of Physics*, 84(9), 678-689.
- [11] Malgieri, M., & Onorato, P. (2021). "Educational reconstructions of quantum physics using the sum over paths approach with energy dependent propagators." In *Journal of Physics: Conference Series* (Vol. 1929, No. 1, p. 012047). IOP Publishing.
- [12] Malgieri, M., Tenni, A., Onorato, P., & De Ambrosis, A. (2016). "What Feynman could not yet use: the generalised Hong–Ou–Mandel experiment to improve the QED explanation of the Pauli exclusion principle." *Physics Education*, 51(5), 055002.
- [13] Malgieri, M., Onorato, P., & De Ambrosis, A. (2014). "Teaching quantum physics by the sum over paths approach and GeoGebra simulations." *European Journal of Physics*, 35(5), 055024.
- [14] Malgieri, M., Onorato, P., & De Ambrosis, A. (2018)." GeoGebra simulations for Feynman's sum over paths approach." *Il nuovo cimento C*, 41(3), 1-10.
- [15] Malgieri, M., Onorato, P., & De Ambrosis, A. (2017). "Test on the effectiveness of the sum over paths approach in favoring the construction of an integrated knowledge of quantum physics in high school." *Physical Review Physics Education Research*, 13(1), 010101.
- [16] de los Angeles Fanaro, M., Arlego, M., & Otero, M. R. (2017). "A Didactic Proposed for Teaching the Concepts of Electrons and Light in Secondary School Using Feynman's Path Sum Method." *European Journal of Physics Education*, 3(2), 1-11.
- [17] Hochrainer, A., Lahiri, M., Erhard, M., Krenn, M., & Zeilinger, A. (2021). "Quantum Indistinguishability by Path Identity: The awakening of a sleeping beauty." *arXiv preprint arXiv:2101.02431*.

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