Understanding the relation between classical and quantum mechanics: prospects for undergraduate teaching

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Abstract. Classical and quantum mechanics are two very different theories, each one describing the world within its own range of validity. According to Planck's version of the correspondence principle, classical mechanics is recovered when the limit in which Planck's constant h goes to zero is taken, while within Bohr's version the limit of large quantum numbers is taken. However, despite what suggested by many textbooks, the relation between the two theories is much more complex to state and understand. Here we deal with this issue by analysing some key examples. Implications for quantum mechanics teaching at undergraduate level are carefully discussed.

1 Introduction

Classical and quantum mechanics successfully describe the world within their specific range of validity. While the first one is a reliable description of phenomena at a macroscopic scale, the second one concerns the microscopic level and is characterized by a bunch of counterintuitive features. As a matter of fact, it is a widespread state of affairs that the relation between classical and quantum mechanics is expressed in terms of Planck's correspondence principle [1], according to which classical physics is recovered when Planck's constant h goes to zero. This formulation was originally set up in order to show how to get the correct classical Raileigh-Jeans energy density for black body radiation from quantum Planck's formula. Indeed there exists another formulation of this relation by Bohr, in which the limit of large quantum numbers is taken [2]. As a consequence of these limits, classical mechanics can be considered only as an approximation of a more refined and more general theory, which is thought as fundamental: quantum mechanics. In principle this should point to a reduction relation between the two theories, but (see, for instance, Ref. [3] and references therein) the situation is much more involved as claimed by some authors. In this contribution we show, by a critical analysis of some concrete examples, why to establish the relation between classical and quantum mechanics is a challenging task. Indeed the limit operation is no more than a heuristic tool which in several cases gives valid results, while in other cases it doesn't work. Accordingly, undergraduate teaching should take into account these features and moreover suggest also the view of a plurilinear history of theoretical physics [4].

2 Quantum mechanics vs classical mechanics

Planck's and Boh's formulations of the correspondence principle are not universally equivalent [5], so that in order to get a meaningful classical limit of quantum mechanical eigenvalues both limits ($h \to 0$, $n \to \infty$) have to be taken while constraining the product nh to be fixed at the appropriate classical action. This is the case of quantum mechanical systems with a discrete energy spectrum such as the harmonic oscillator, the particle in a box and the hydrogen atom. The limit relation $h \to 0$ between classical and quantum mechanics turns out to be highly non trivial because it is singular for many physical systems. This singularity implies that, when approaching the limit, the behavior is different from the corresponding behavior at the limit [3]. A singular limit arises, for instance, when considering a particle with energy E in the presence of a step barrier $V(x)$ characterized by a smoothness L, such that $E > V(x)$ [6]. While the classical reflection coefficient is always zero as expected, quantum mechanics predicts that there is a small probability for the particle to be reflected by the barrier, resulting in a non vanishing reflection coefficient. Although it disappears for $h = 0$, the quantum description of particle's behavior when it approaches the limit $h \to 0$ shows a singularity. The situation is even worse for a sharp step barrier ($L = 0$), whose reflection coefficient is independent of h and can never give rise to the expected classical limit. Further problems arise when dealing with systems whose classical counterpart is chaotic. Since chaos shows up for long times, both $t \to \infty$ and $h \to 0$ limits are required in order to get the correct classical behavior. In addition to the singularity of $h \to 0$, the above limits do not commute, which produce highly non trivial features [7]. Another stumbling stone against reduction is the failure of Ehrenfest's theorem, which says that, under certain conditions, the average of position and momentum of a quantum system follow a classical trajectory. In fact it has been shown that Ehrenfest substitution (the replacement of the average values of the functions with the functions of the average values) holds on only in the case of negligible fluctuations of the canonical coordinates [8]. There exist systems of physical interest for which it doesn't work, such as for instance a particle scattering off a potential step [9].

3 Conclusion

The relation between classical and quantum mechanics is highly non trivial: the limit $h \to 0$ is singular for a number of physical systems, the Ehrenfest theorem has a restricted validity, classically chaotic systems show a puzzling behavior. Therefore quantum mechanics teaching should shift from a strict reductionism based approach, as that of the present time, to a new one also open to a theoretical pluralism.

References

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