The Schrödinger equation for a non-quantized matter field: a pedagogical introduction

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Abstract. Usual presentations of Physics Education start by postulating the existence of a classical field that obeys to the equation of motion, derived from a conveniently chosen Lagrangian density. But while electromagnetic fields are given a proper physics meaning, matter fields are considered only a technical instrument, unless they are quantized. Therefore, in this work, we aim to give a pedagogical construction which allows us to assign a physical meaning also to non-quantized matter fields. This operation is particularly important since we believe that Quantum Field Theory is more suited than Quantum Mechanics to introduce quantum physics in secondary schools.

1. Introduction

Usual presentations of quantum physics start from assigning a (continuous) wave nature to the electromagnetic field, the interpretation of which is generally given within the framework of classical physics. It is only later, when the Compton effect and the photoelectric effect are discussed, that the granular aspects of the radiation are presented, considered in discrete (and thus quantum) terms. Instead, an inverse process is made with regard to matter, wherein the corpuscular/particle aspects are examined first, presenting the wave aspect only later, otherwise interpreted from a quantum point of view. Apart from this inversion of presentation (not only in terms of contents, but also from a logical point of view), the most remarkable thing is to note that, whether from a certain point of view the matter/radiation corpuscular aspects resemble one another, the wave ones have instead a different ontological nature. The electromagnetic field is considered a real “physics” object, while the wave aspects of matter are generally presented through wave functions and placed in configurations space, being turned into something much less tangible and concrete (since ψ is not a field in R^3).

On the other hand, if we analyze what happens in Quantum Field Theory (QFT), we realize that the quantization process is, in some ways, more linear and symmetric: radiation and matter are both described by a field that obeys a wave equation and they are both quantized with similar techniques, by means of specific commutation rules. But precisely because of the procedure we have previously described - which creates an ontological difference between electromagnetic fields and matter ones - no physical meaning is attributed to the latter and to their Lagrangians (to which the wave equation corresponds) [1,2].

This work is aimed to provide a pedagogical construction that allows to give a physical meaning to the non-quantized matter fields. Apart from a conceptual point of view, we believe that this operation is essential if, as we do in the Physics Education Research Group of Milan, we believe that QFT might be more suited than quantum mechanics to introduce quantum physics in secondary schools [3-6].

2. The Schrödinger equation for a non-quantized matter field

Since the matter beams experiments show the same interference and diffraction aspects of electromagnetic waves, we are induced to suppose a linear aspect of an underlying theory, but we do not know the nature of the fields involved, nor the equation they obey [7]. However, from linearity, we are able to suppose that a plane waves expansion similar to that of electromagnetic radiation can be used for a matter field. For the electromagnetic four-potential we have

$$A_\mu(\underline{x},t)=\frac{1}{(2\pi)^{3/2}}\int d^3k\left[a_\mu^+(\underline{k})e^{i(\underline{k}\cdot\underline{x}-k_0t)}+a_\mu^-(\underline{k})e^{i(\underline{k}\cdot\underline{x}+k_0t)}\right]$$
$$a_\mu^-(\underline{k})=a_\mu^{+\,*}(\underline{k}) \tag{1}$$

(the condition on a_μ arises from the request that the four-potential is a real function).

In order that (1) satisfies the electromagnetic waves equation ($c=1$), we have

$$0=\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\left[a^\mu(k)e^{i(\underline{k}\cdot\underline{x}-k_0t)}\right]=k_\nu k^\nu \tag{2}$$

holding true component by component, and in which we used the summation convention on repeated indices, with the Minkowski metric. Matter fields will obey an expansion similar to (1), but they will be described by an equation different from (2) (since they are not electromagnetic waves). Therefore, we wonder if it is possible to find a heuristic procedure that leads us to a wave equation, allowing us also to give them a physical meaning. We thus consider a scalar field ψ, which can be represented as a Fourier integral, though a plane wave expansion

$$\psi(\underline{x},t)=\frac{1}{(2\pi)^{3/2}}\int d^3k\,e^{i\underline{k}\cdot\underline{x}}\left[c^+(\underline{k})e^{-ik_0t}+c^-(\underline{k})e^{ik_0t}\right] \tag{3}$$

The relation $k_\nu k^\nu=0$ is equivalent to the electromagnetic waves equation; since $k_\nu k^\nu$ is a Lorentz scalar, its simplest generalisation leads us to assume that $k_\nu k^\nu=\mu^2$ (with μ^2 a generic constant - being a scalar - different from zero). Starting from waves equation, and generalising (2), we easily obtain the Klein-Gordon equation

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2+\mu^2\right)\psi(\underline{x},t). \tag{4}$$

Taking the limit for large wavelengths (i.e. when the central wavelength of the packet is much greater than μ^{-1}), we can both interpret μ as a mass and assume that the approximation we made is equivalent to a non-relativistic one. In this process, the Fourier development must include the c^+ term or the c^- one: thereby, equation (3) splits into two different equations, which can be obtained from one another by giving μ^2 a positive or a negative sign. One of these equations is identical to the Schrödinger equation (without $\hbar$) and interpretable as matter waves; the other, instead, as antimatter waves. Furthermore, whilst $|\psi(\underline{x},t)|^2$ varies in space and time, its integral over all space remains constant: therefore there is a continuity equation, in which $|\psi(\underline{x},t)|^2$ represents a classic field density. The existence of a continuity equation is coherent with the reasoning showed above and strengthens the interpretation of μ as a characteristic mass.

References

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