A mathematical model of peer-instruction including stochastic uncertainty

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Abstract. As a class of statistical transition processes, a mathematical model of peer-instruction including stochastic uncertainty is presented. Expectation and variance are shown by using direct numerical simulations of the resultant model and moment equations with a simple closure. The resultant model will provide insights to the real data beyond the standard statistical analysis.

1 Short introduction

Peer-instruction[1] is a well-recognized method for active learning, in which students' learning gain (peer instruction efficiency; PIE[2]) is evaluated by using pre/post tests (questions before/after discussions). Nitta[2] proposed a minimal mathematical model of peer-instruction which learning strongly depends on prior knowledge of students. In Nitta[2], by using the master equation for the (normalized) number of students choosing correct answers, a simple deterministic model is derived. The purpose of this study is to give a theoretical base of stochastic uncertainty of PIE through the stochastically modification of Nitta's model.

2 Result

Peer-instruction induces learning processes including the stochastic transition between students choosing correct answers and those choosing incorrect answers. Such a stochastic transition process also appears in different fields such as chemical reaction[3,4], a population system[5], and voter dynamics[6]. In this study, we start from two state diffusion model[5] which includes the normalized number of students choosing correct answers (x_c) and incorrect answers (x_i=1-x_c) as state variables. If each incorrect answer should be divided, we should apply multi-state model[6].

The transition probabilities in the master equation of the propagability distribution $P(x_c, x_i, t)$ are given as[5,6]

$$T_c = T(x_c + \Delta x, x_i - \Delta x | x_c, x_i) = r_{ic} x_c x_i + \epsilon_{ic} x_i \quad (1)$$

$$T_i = T(x_c - \Delta x, x_i + \Delta x | x_c, x_i) = r_{ci} x_c x_i + \epsilon_{ci} x_c \quad (2)$$

where $N = 1/\Delta x$ corresponds to the system size[5]. Coefficients r_{ic}, r_{ci}, ϵ_{ic}, ϵ_{ci} correspond to transition rates of $x_i + x_c \to 2x_c$, $x_c + x_i \to 2x_i$, $x_i \to x_c$, and $x_c \to x_i$, respectively. By expanding the master equation to Δx, we obtain the Fokker-Planck equation and the stochastic differential equation (SDE)[5,6]. When the outgoing process[1] is negligible ($r_{ci} = 0$), we obtain

$$dx_c = \Delta x(r x_c(1 - x_c) - 2\epsilon x_c + \epsilon)dt + \Delta x \sqrt{r x_c(1 - x_c) + \epsilon} dW \quad (3)$$

where dW is the standard Wiener process, $r_{ic} = r$, and $\epsilon_{ic} = \epsilon_{ci} = \epsilon$. Terms including ϵ in the drift term have similar roles to the terms related to memory and tutoring in the deterministic model[7]. In the present model, terms including ϵ indicate the random transition (mutation) with the equal probability. Note that the deterministic model in Nitta[1] is derived from the master equation of x_c itself and we can obtain Nitta's deterministic model from eq.(3) in the limit $N \to \infty$ and $\epsilon = 0$. Although we can directly obtain a simple stochastically modified version of Nitta's model by taking $dt = T = 1/(r\Delta x)$, in this study we calculate $x_c(t{=}T)$ by using the SDE (3) with $dt \ll T$.

Filled-circles in Fig. 1 shows the (a)expectation and (b)variance of x_c at $t{=}T$ obtained by the numerical integration of (3) with a simplified second order Taylor scheme[8]. The horizontal axes indicate the initial condition $x_c(t{=}0)$ (x_c before discussion). Solid lines indicate the solutions of moment equations of (3) with a simple closure[9] (limit of the small second and third cumulants). Figure 1(b) shows that the variance has the maximum value at $x_c(t{=}0){=}0.1$.

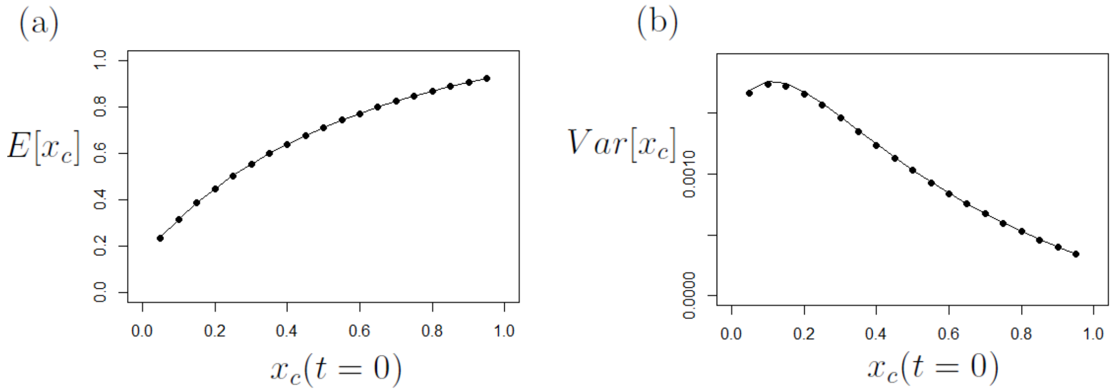

Fig. 1 Dependence of (a)expectation and (b)variance of $x_c(t{=}T)$ on $x_c(t{=}0)$ with $N = 200$, $dt = 10^{-4}$, $\epsilon = 0.1r$, and $T = 1$. Filled circles and solid lines indicate numerical solutions of (3) and solutions of moment equations, respectively.

3 Short summary and notice

In this study, we discuss the mathematical model of the peer-instruction including stochastic uncertainty. Such a mathematical model will give insights to the real data beyond the standard statistical analysis. On the other hand, it is noteworthy that while the present model treats the transition among states as random processes, interactions among students in the peer-instruction are not random but structured[10].

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