

Proposal and Planning for an Electron Ion Collider in China (EicC)

Xurong Chen, Rong Wang

(on behalf of the EicC Working Group)

Institute of Modern Physics, CAS, China

*The 2021 School on the Physics of Baryons (Baryons-21_School)
18-22 October, Sevilla (videoconference)*

Outline

- **Introduction**

- **Electron-ion collider in China (EicC)**

1st stage: 3.5 ~ 5 GeV (pol. e) X 20 GeV (pol. p), $L = 2 - 4 \times 10^{33}$

EicC R&D, construction : 2026 – 2038

- **EicC physics highlights**

Spin-flavor structure (sea quark polarization)

3D structure of the nucleon (GPDs, TMDs)

Proton mass origin

π & K structures

Exotic hadronic states & partonic structure of nucleus

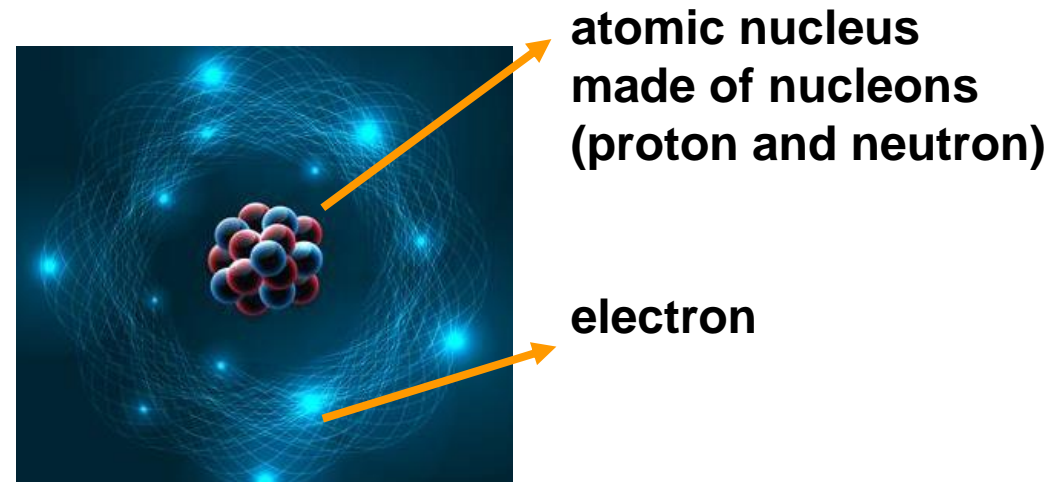
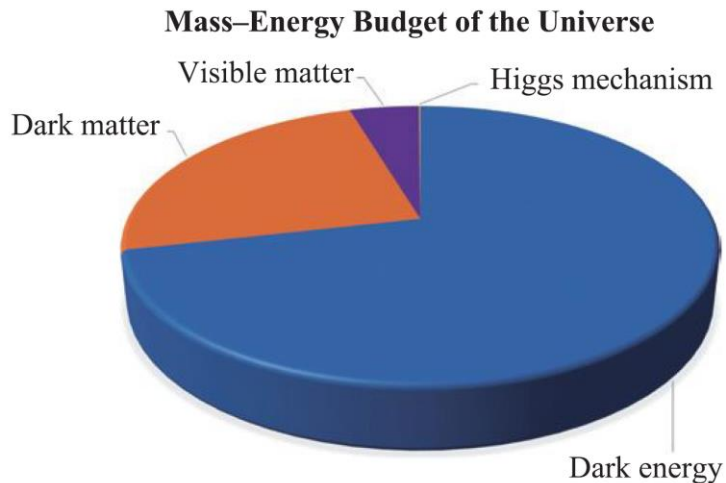
- **Conclusions**

Introduction

**Fundamental questions about the proton
& QCD theory**

The nucleons

- The baryonic matter accounts for nearly all the mass of the visible universe.
- The nucleons inside the atomic nucleus are the quite stable baryons (the proton decay has never been observed).



The model of atom

The spin of nucleon

- The proton has a big anomalous magnetic moment, which indicates that the proton is not point-like Dirac fermion.
- The anomalous magnetic moment of the nucleon has been a puzzle for a long time, until the quark model was developed.



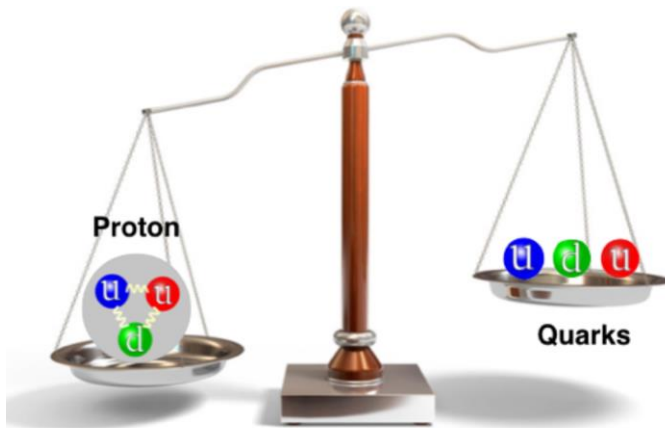
- The polarized DIS experiment of EMC@CERN found that the spin of all the quarks is far less than 0.5. The proton spin is from the complex workings of quarks and gluons in QCD theory of the standard model.

small portion

$$\frac{1}{2} = \text{Spin of all Quarks} + \text{Spin of Gluons} + \text{Angular Momentum of all Quarks} + \text{Angular Momentum of Gluons}$$

The mass of nucleon

- The mass of the light quarks are almost massless, compared to the proton mass. Most of the proton mass comes from the complicated interactions among the quarks. The QCD trace anomaly set a scale of the proton mass.



$$M_q = \frac{3}{4} \left(a - \frac{b}{1 + \gamma_m} \right) M_N, \quad \text{Quark energy}$$

$$M_g = \frac{3}{4} (1 - a) M_N, \quad \text{Gluon energy}$$

$$M_m = \frac{4 + \gamma_m}{4(1 + \gamma_m)} b M_N, \quad \text{Quark mass}$$

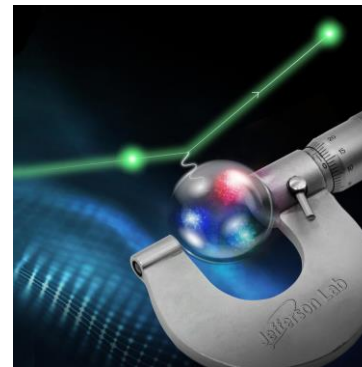
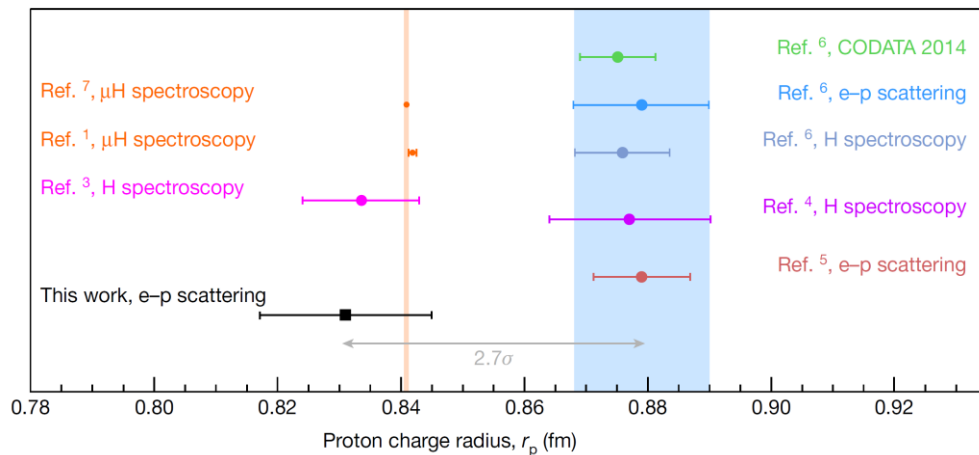
$$M_a = \frac{1}{4} (1 - b) M_N, \quad \text{anomaly energy}$$

- To understand/quantify these mass origins from nonperturbative QCD & experiments are big science questions.

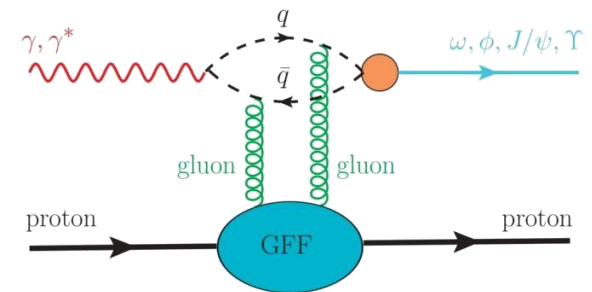
The size of nucleon

- The size of the nucleon is related to the quark confinement and the nucleon mass. Various radius of the nucleon are defined, such as the charge radius, magnetic radius, weak radius and mass radius. To answer how large the nucleon is from the different probes is also a nonperturbative QCD question.

The new result of pRad



probe the mass radius



The success of perturbative QCD

- Strong interaction, running coupling ~ 1

-- asymptotic freedom (2004 Nobel)

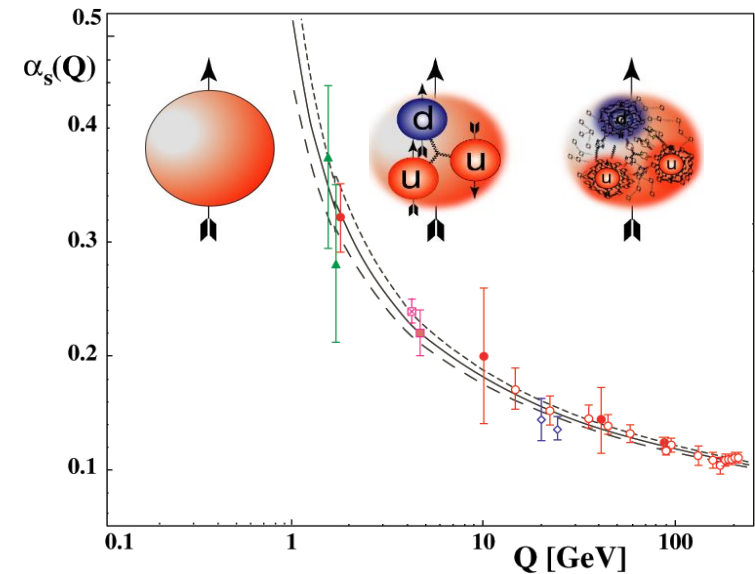
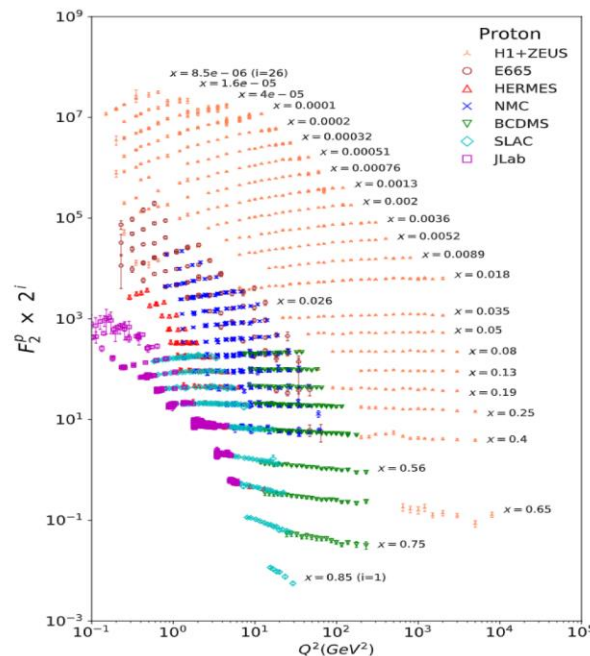
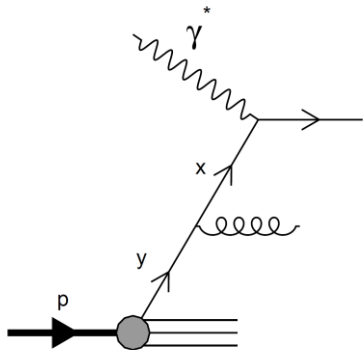
-- gluons self interacting

perturbation calculation works at high energy

-- interaction is strong at low energy



confinement



What are the challenges?

- **Success of the Standard Model**

Electro-Weak theory tested to very good level of precision

Discovery of Higgs (like) particle at LHC

QCD tested in the high energy (short distance) region

- **Major challenges**

Test QCD at long distance (non-perturbative)

Understand quark-gluon structure of the nucleon

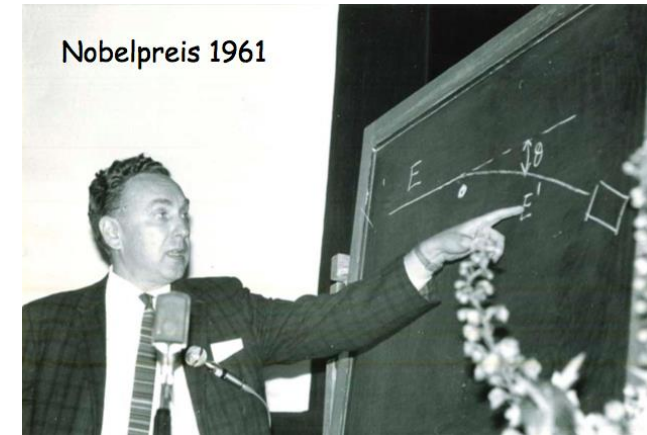
Confinement

- **Beyond Standard Model**

Intensity (precision) frontier: test Standard Model at low energy

Electron scattering and nucleon structure

- Clean probe to study nucleon structure
only electro-weak interaction, well understood
- Elastic Electron Scattering: Form Factors
→ 60s: established nucleon has structure (Nobel Prize)
electrical and magnetic distributions
- Resonance Excitations
→ internal structure, rich spectroscopy (new particle search)
constituent quark models
- Deep Inelastic Scattering
→ 70s: established quark-parton picture (Nobel Prize)
parton distribution functions (PDFs)
polarized PDFs : Spin Structure



Robert Hofstadter,
Nobel Prize 1961



J.T. Friedman



R. Taylor



H.W. Kendall

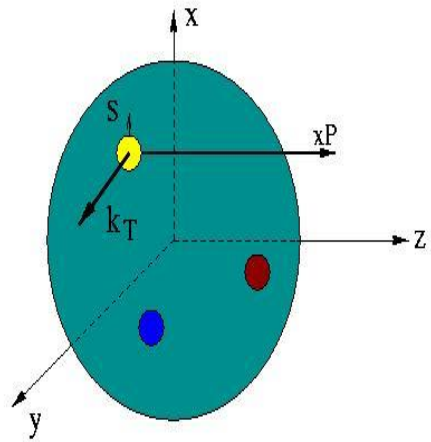
Nobel Prize 1990

One dimensional structures of nucleon have been well determined.

Towards the 3D structures of the nucleon

$W_p^u(x, k_T, r)$ Wigner distributions (X. Ji)

6D Dist.



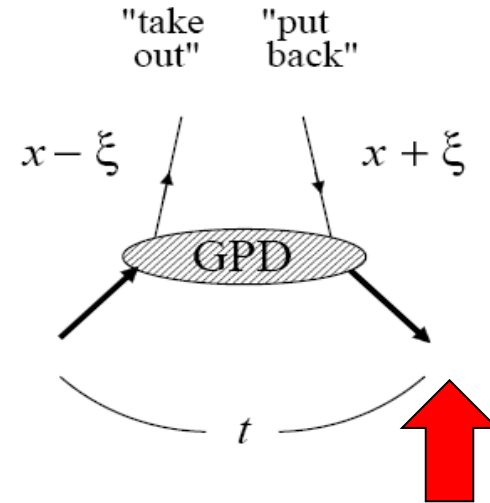
d^3r

$d^2k_T dr_z$

TMD PDFs
 $f_1^u(x, k_T), \dots$
 $h_1^u(x, k_T)$

GPDs/IPDs

3D imaging



dx & Fourier Transformation

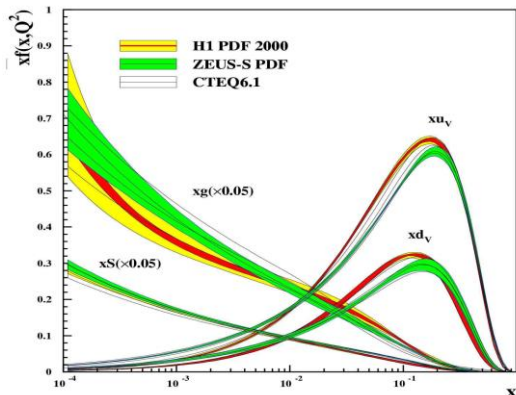
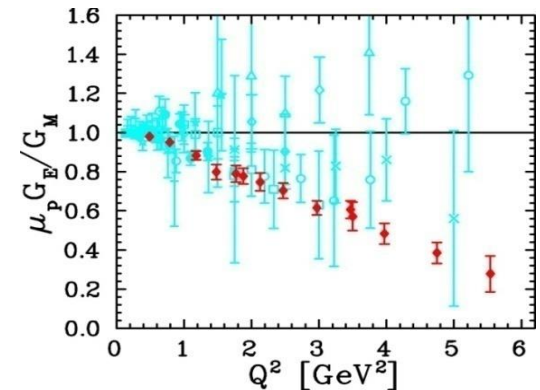
d^2k_T

d^2r_T

PDFs
 $f_1^u(x), \dots, h_1^u(x)$

1D

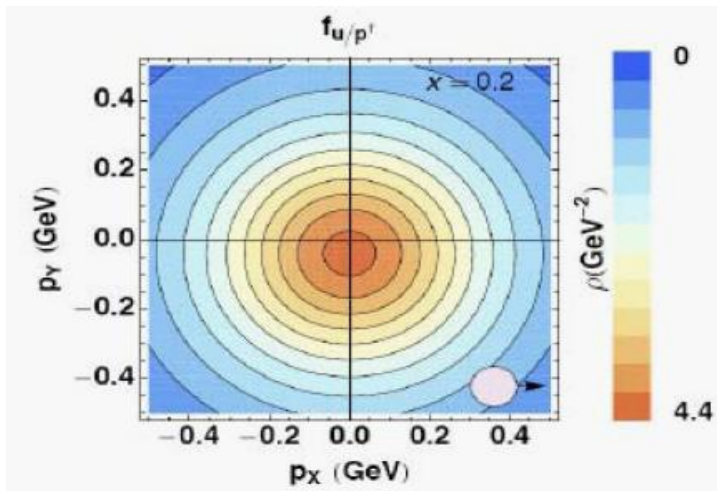
Form Factors
 $G_E(Q^2),$
 $G_M(Q^2)$



3D imaging of nucleon: two approaches

TMDs

2+1 D picture in momentum space

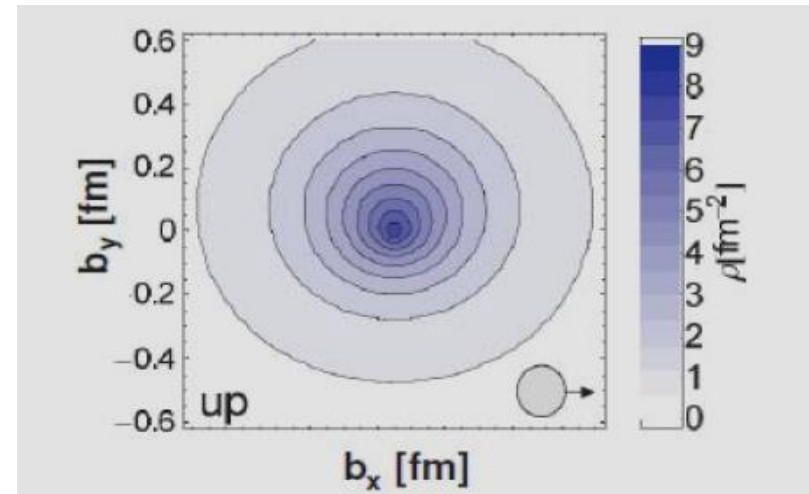


Bacchetta, Conti, Radici

- intrinsic transverse motion
- spin-orbit correlations- relate to OAM
- **accessible in SIDIS (and Drell-Yan), high energy electron-proton scattering**

GPDs

2+1 D picture in impact-parameter space



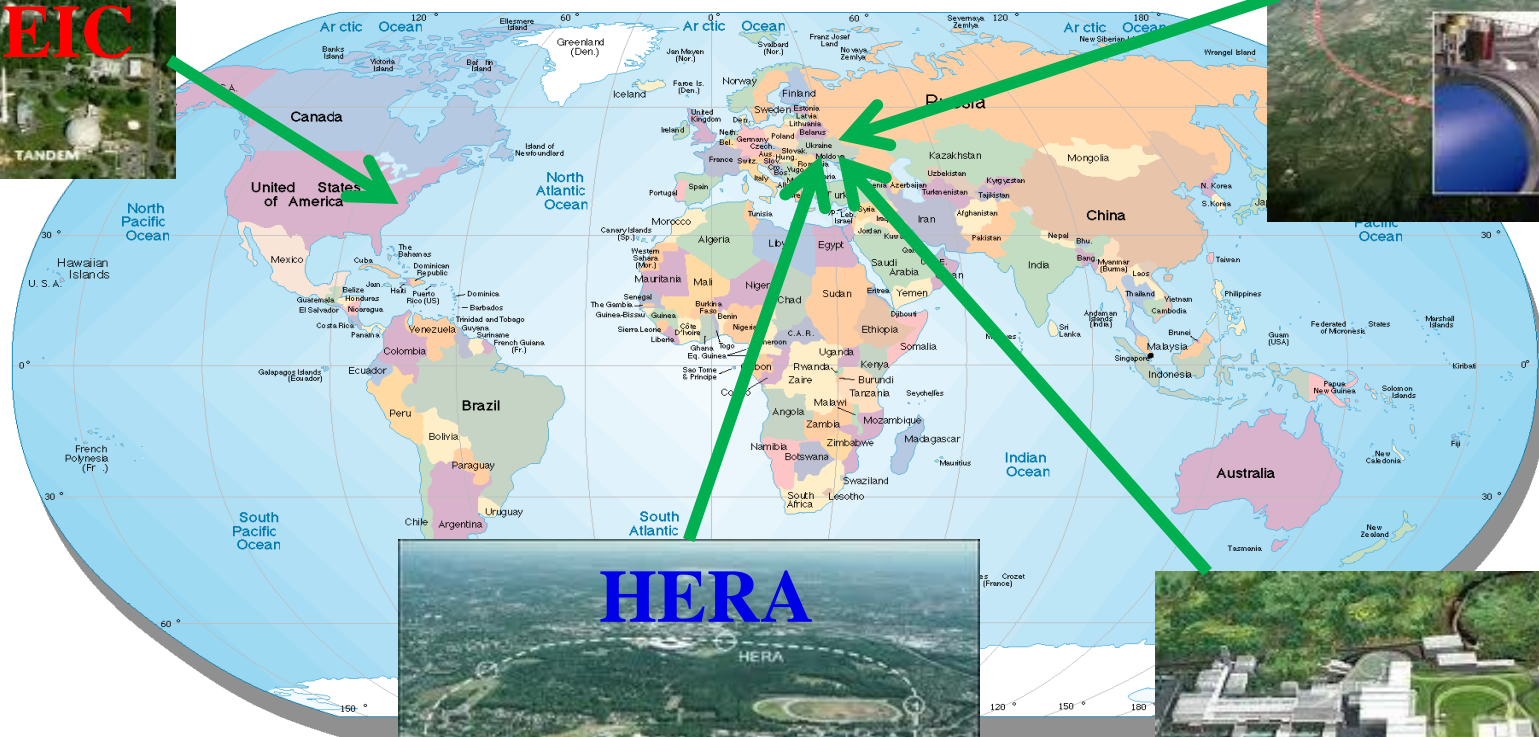
QCDSF collaboration

- collinear but long. momentum transfer
- indicator of OAM; access to Ji's total $J_{q,g}$
- **DVCS, exclusive vector-meson production, high energy electron-proton scattering**

Electron-ion collider in China (EicC)

**A new facility to study sea quark
and nuclear physics**

Electron-Ion Colliders on the world map

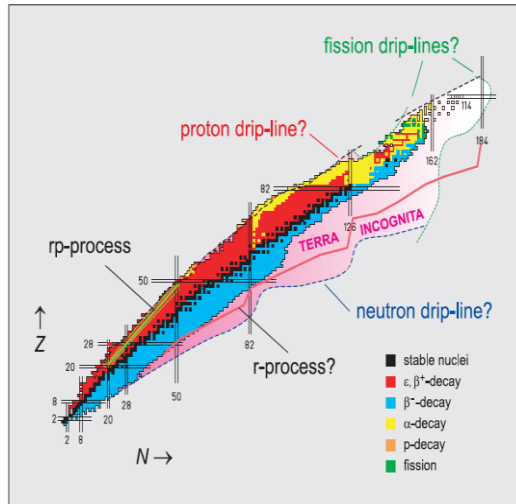


LHC → LHeC



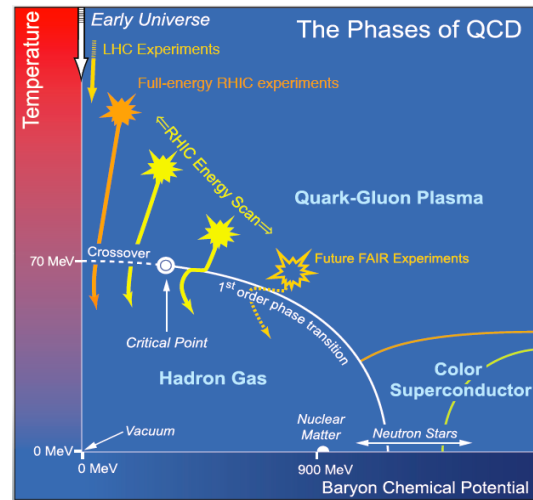
Nuclear Physics Facilities at IMP, China

Nuclear Structure



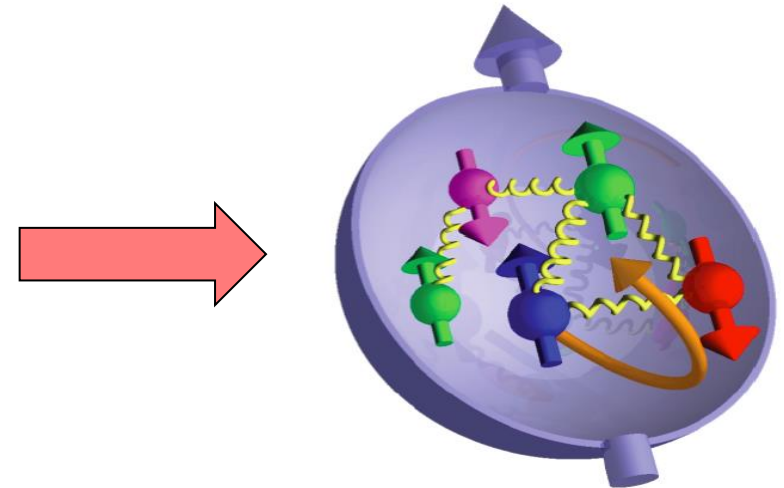
CSR

QCD Phase diagram



CEE (CSR External Target)
HIAF

Nucleon Structure



*EicC: Eic in China
(planning)*

High Intensity heavy-ion Accelerator Facility (HIAF)

EIC: science motivations

A High Luminosity, High Energy Electron-Ion Collider:

A New Experimental Quest to Study the Sea and Glue

*How do we understand the visible matter in our universe
in terms of the fundamental quarks and gluons of QCD?*

Precisely image the sea-quarks and gluons in the nucleon:

- How do the gluons and sea-quarks contribute to the spin structure of the nucleon?
- What is the spatial distribution of the gluons and sea quarks in the nucleon?
- How do hadronic final-states form in QCD?

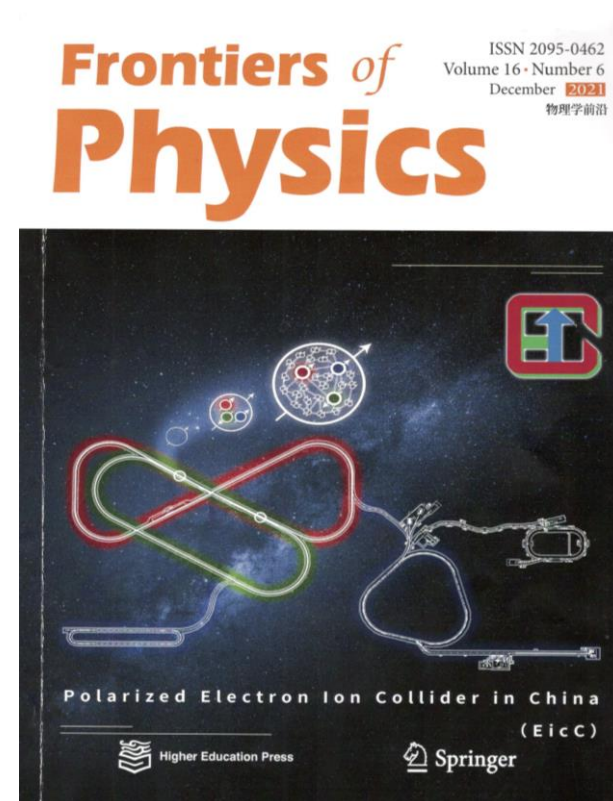
Explore the new QCD frontier: strong color fields in nuclei:

- How do the gluons contribute to the structure of the nucleus?
- What are the properties of high-density gluon matter?
- How do fast quarks or gluons interact as they traverse nuclear matter?

EicC (a polarized low-energy collider) will focus on the sea quark region and the transition from nonperturbative to perturbative. In terms of the collision energy, EicC will bridge JLab-12GeV and EIC.

Electron-ion collider in China (EicC)

EicC white paper



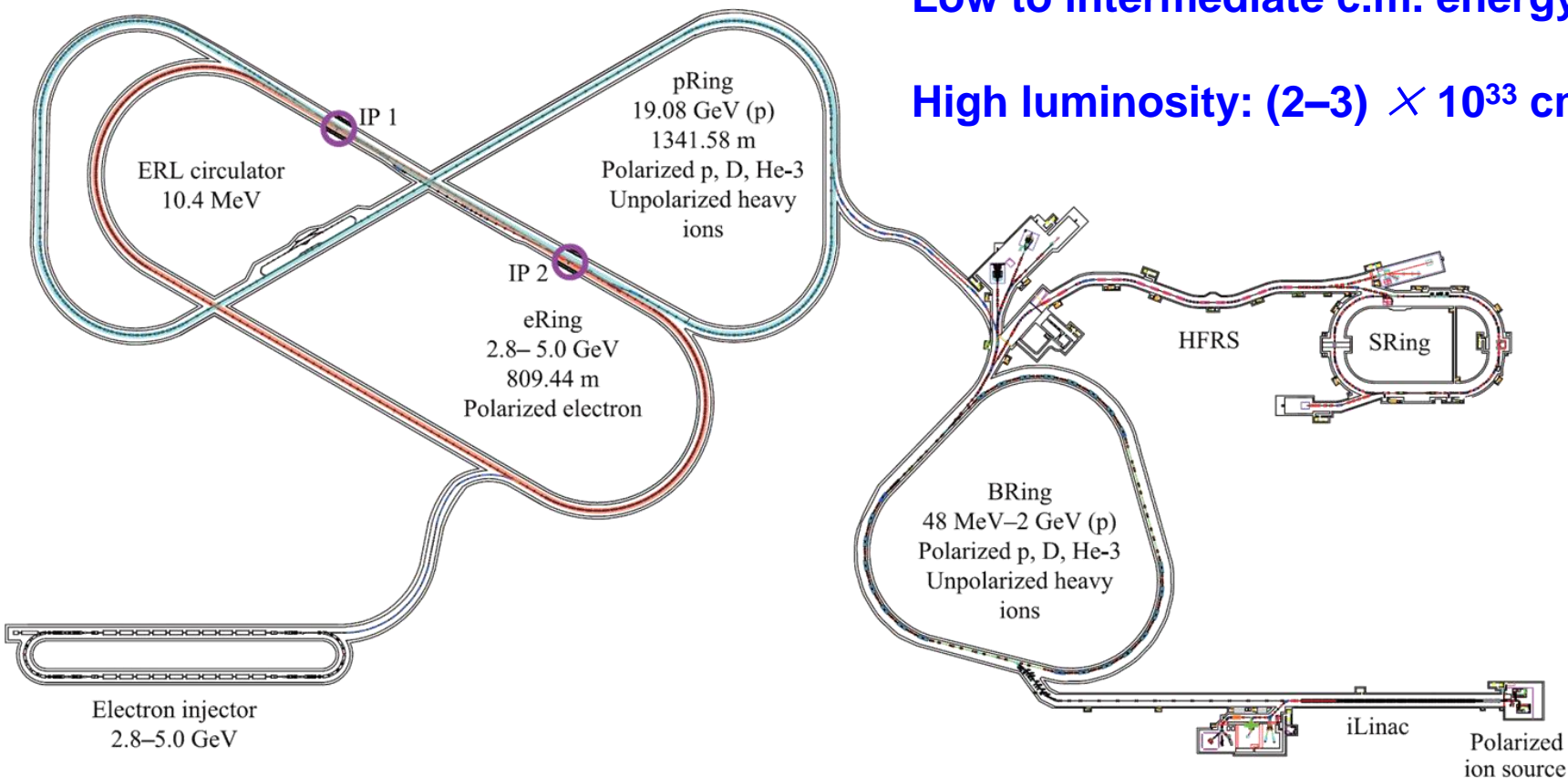
Frontiers of Physics, 16 (6), 64701, 2021, arXiv:2102.09222

EicC accelerator complex overview

The proposed collider will provide highly polarized electrons (a polarization of $\sim 80\%$) and protons (a polarization of $\sim 70\%$).

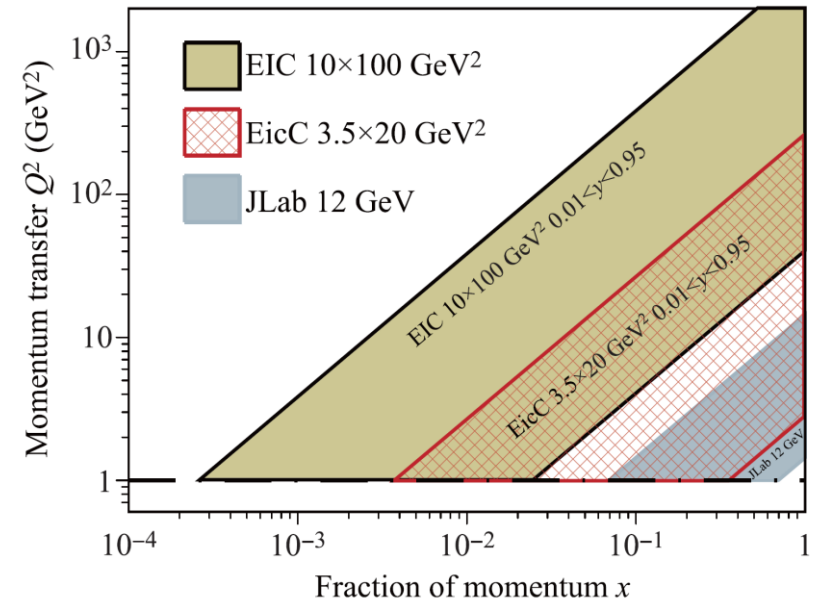
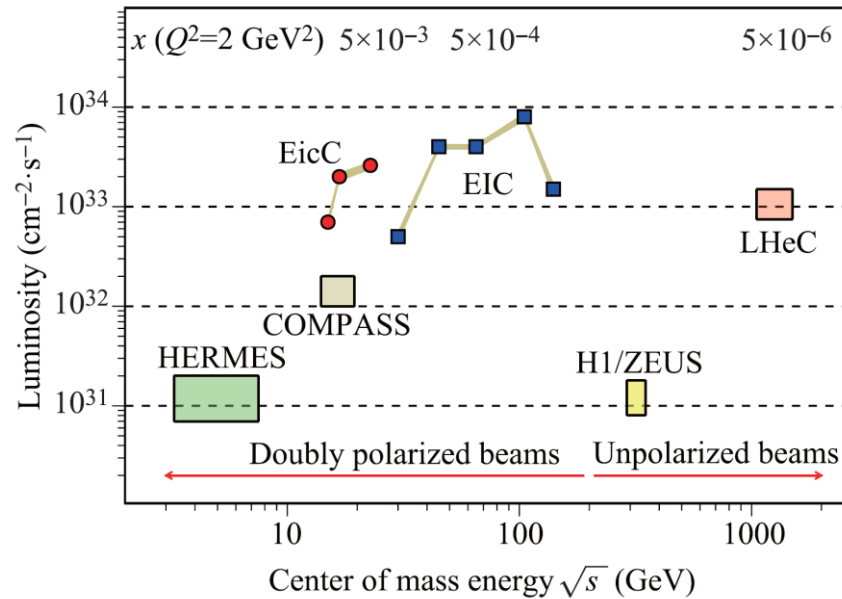
Low to intermediate c.m. energy: 15 to 20 GeV

High luminosity: $(2-3) \times 10^{33} \text{ cm}^{-2} \cdot \text{s}^{-1}$



The layout of EicC accelerator facility.

The kinematical domain of EicC



EicC will bridge JLab-12GeV and EIC.

EicC physics highlights

**Hadron structure, exotic hadron state,
& partonic structure of nucleus**

*Case-1: spin-flavor structure
(sea quark polarization)*

Spin milestones (Nature)

- 1896: Zeeman effect (milestone 1)
- 1922: Stern-Gerlach experiment (2)
- 1925: Spinning electron (Uhlenbeck/Goudsmit)(3)
- 1928: Dirac equation (4)
- Quantum magnetism (5)
- 1932: Isospin(6)
- 1935: Proton anomalous magnetic moment
- 1940: Spin–statistics connection(7)
- 1946: Nuclear magnetic resonance (NMR)(8)
- 1971: Supersymmetry(13)
- 1973: Magnetic resonance imaging(15)
- 1980s: “Proton spin crisis”
- 1990: Functional MRI (19)
- 1997: Semiconductor spintronics (23)
- 2000s: Breakthrough in nucleon spin physics?
- 2000s: Application of nucleon spin physics?

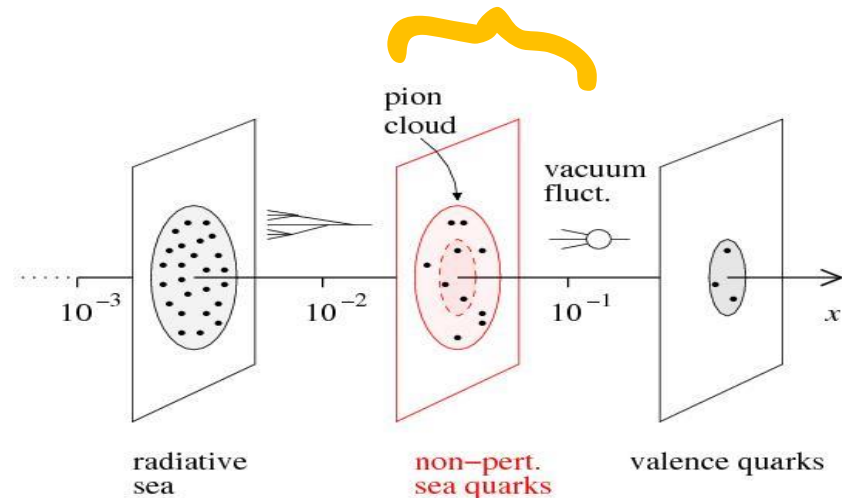


Pauli and Bohr watch a spinning top

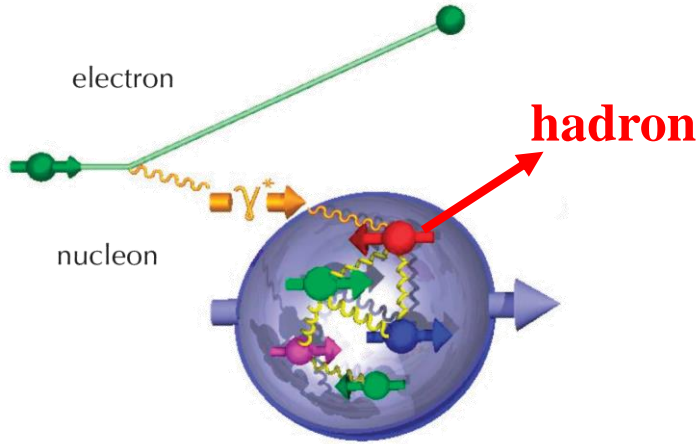
Case-1: spin-flavor structure

- Sea quarks are poorly known!
 - Without EIC: large uncertainties in nuclear sea quarks and gluons
 - With EIC: significantly reduces uncertainties: wide coverage in x, Q^2
- EicC, combination of energy and luminosity
Significant improvement for D ubar, D dbar from SIDIS

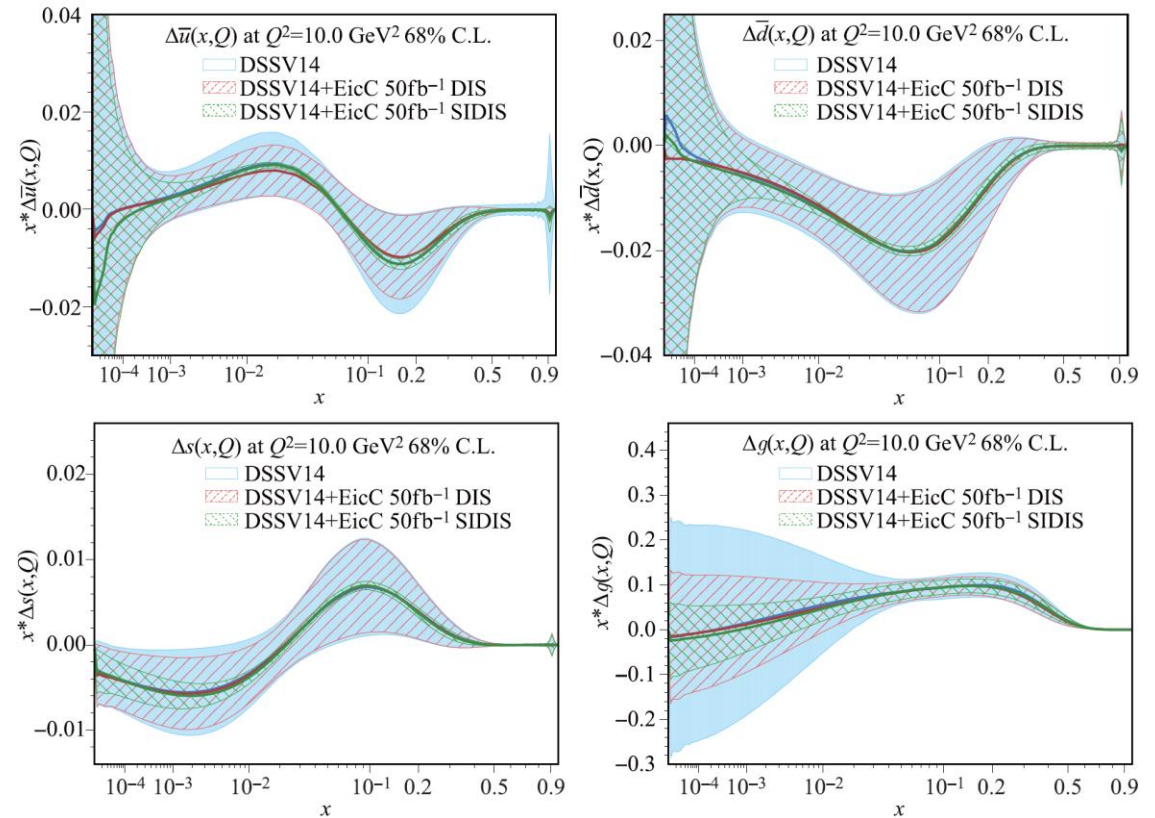
EicC kinematic:
Unique opportunity for Δs , nonperturbative sea



Case-1: spin-flavor structure




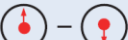
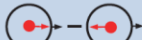
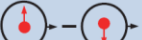
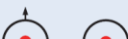
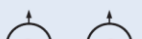

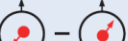
Polarized semi-inclusive DIS process to study the helicity distributions of different flavors.





The simulation result shows that SIDIS discriminate the flavors of the quarks very well and reduce the uncertainties greatly.

***Case-2: 3D structure of the nucleon
(GPDs, TMDs)***

Case-2: 3D imaging (TMD study at EicC)

TMDs		Quark polarization		
		Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon polarization	U	f_1  Unpolarized		h_1^\perp  Boer-Mulders
	L		g_{1L}  Helicity	h_{1L}^\perp  Longi-transversity
	T	f_{1T}^\perp  Sivers	g_{1T}  Trans-helicity	h_1  Transversity h_{1T}^\perp  Pretzelosity

 Nucleon spin
  Quark spin

The leading-twist quark TMD distributions.

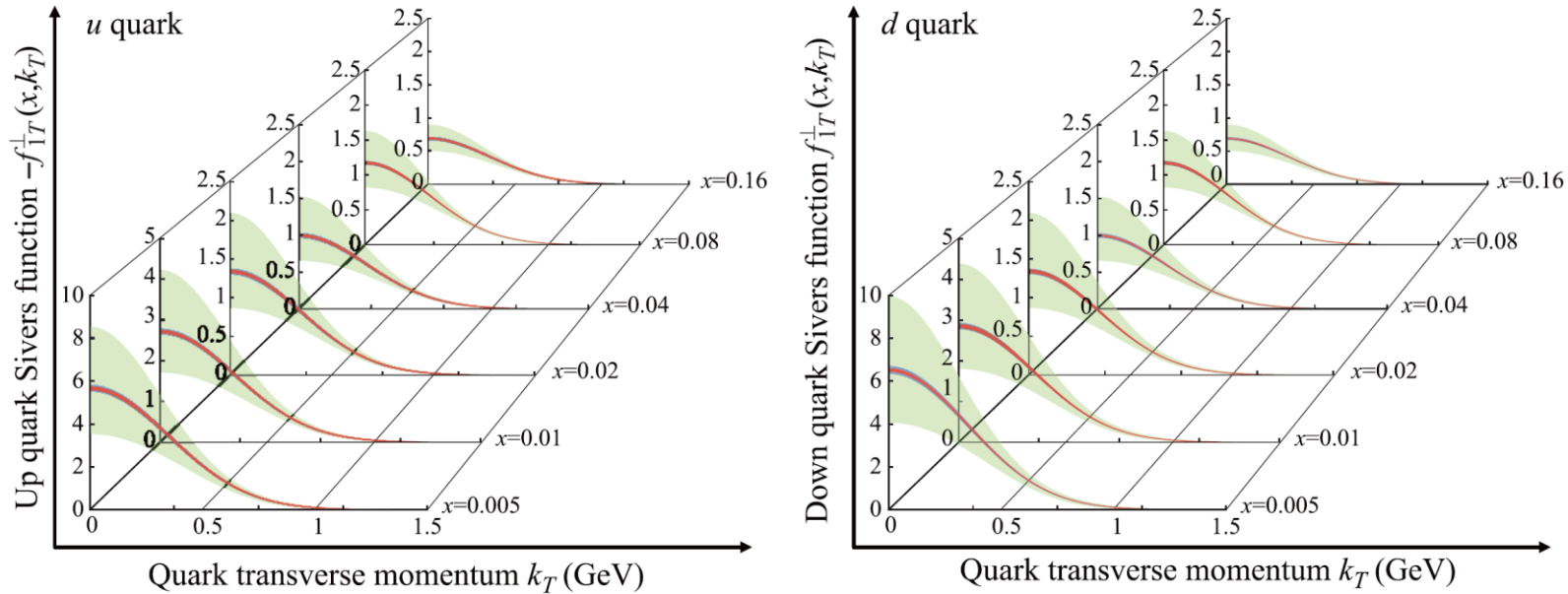
- **JLab12: Semi-inclusive DIS in valence region**

Precise observables, but limited phase space Upsilon & our simulation & our works

- **EicC: Wide kinematic range for SIDIS**

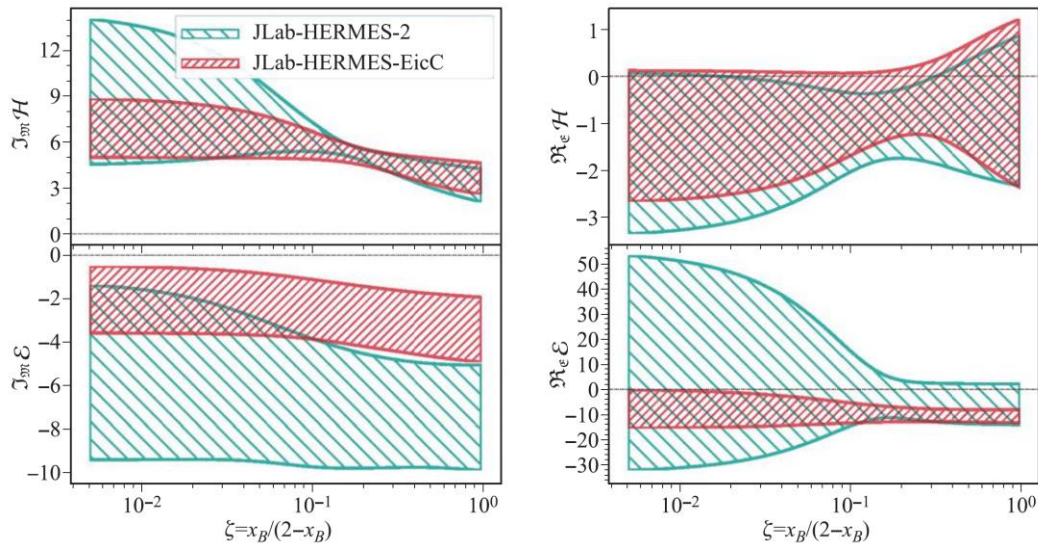
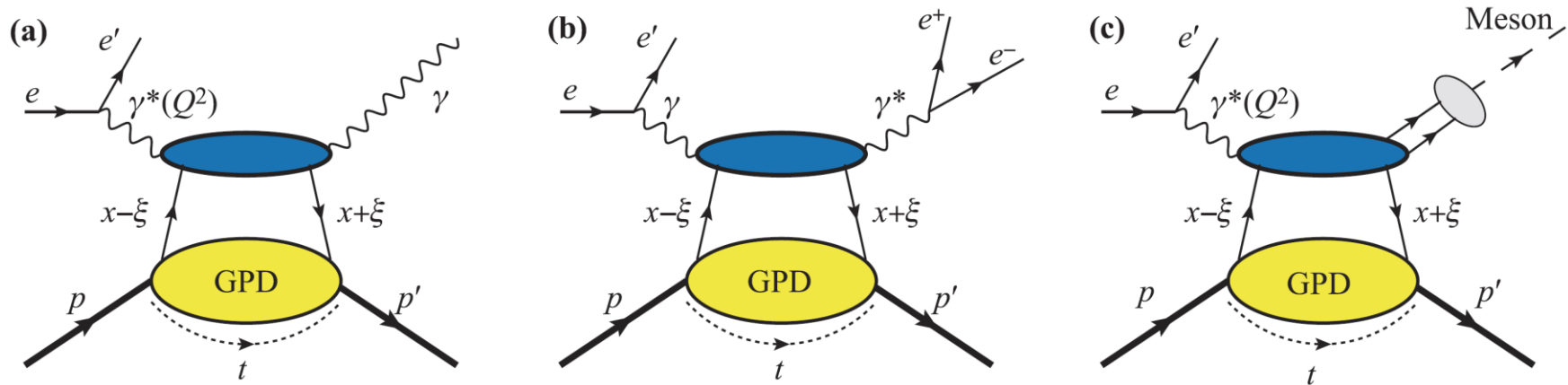
1. High precision quantitative measurements of all the quark TMDs in the valence region
2. Significant increase in Q^2 range for valence region: energy reach $Q^2 \sim 40 \text{ GeV}^2$ at $x \sim 0.4$
3. Unique opportunity for TMD in “sea quark” region: reach $x \sim 0.01$

Case-2: 3D imaging (TMD study at EicC)



The simulation implies that the EicC data would reduce the statistical uncertainties of Sivers function significantly in the sea quark region.

Case-2: 3D imaging (GPD study at EicC)



The simulation implies that the EicC data would improve the current mapping of the Compton form factors (integrals of GPDs). The transverse polarized data would help us understand the least-known GPD E.

Case-3: proton mass origin

Case-3: explore the origins of proton Mass

- *Ji's decomposition of proton mass: parameters a and b*

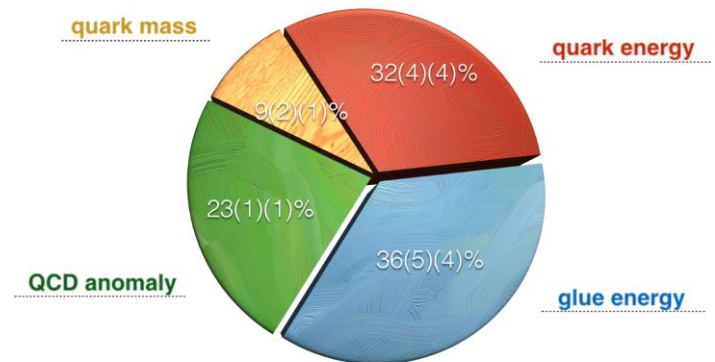
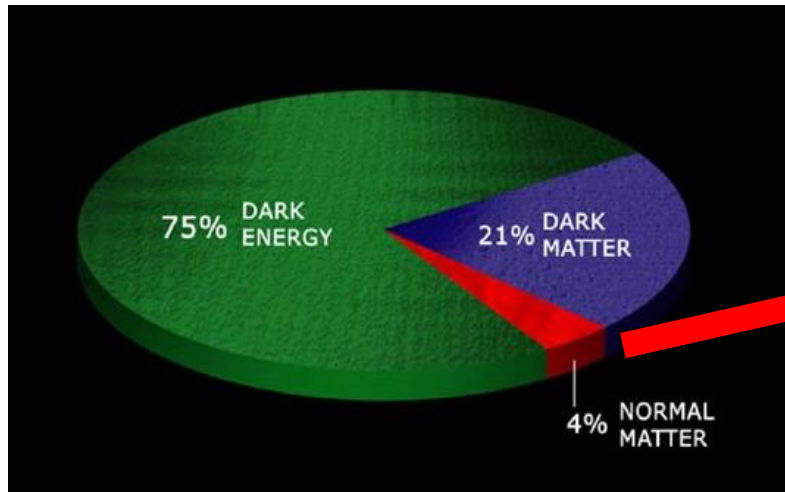
Viral theorem

X. Ji, PRL 74, 1071 (1995), PRD 52, 271 (1995),
& Sci. China Phys. Mech. Astron. 64, 281012 (2021), arXiv:2101.04483

→ *Proton mass budget: about 22% come from trace anomaly*

→ *We know very little about it*

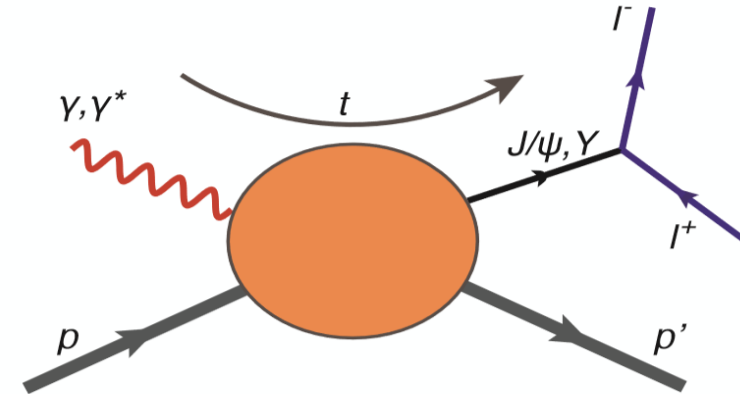
$$\begin{aligned}M_q &= \frac{3}{4} \left(a - \frac{b}{1 + \gamma_m} \right) M, \\M_g &= \frac{3}{4} (1 - a) M, \\M_m &= \frac{4 + \gamma_m}{4(1 + \gamma_m)} b M, \\M_a &= \frac{1}{4} (1 - b) M,\end{aligned}$$



Yi-Bo Yang et al

Case-3: explore the origins of proton Mass

- *Parameter a*: related to PDFs, well constrained
- *Parameter b*: related to quarkonium-proton scattering amplitude $M_{\psi p}$ near-threshold
- **Quarkonium as a probe to study the gluonic structure of the nucleon**



VMD questionable?

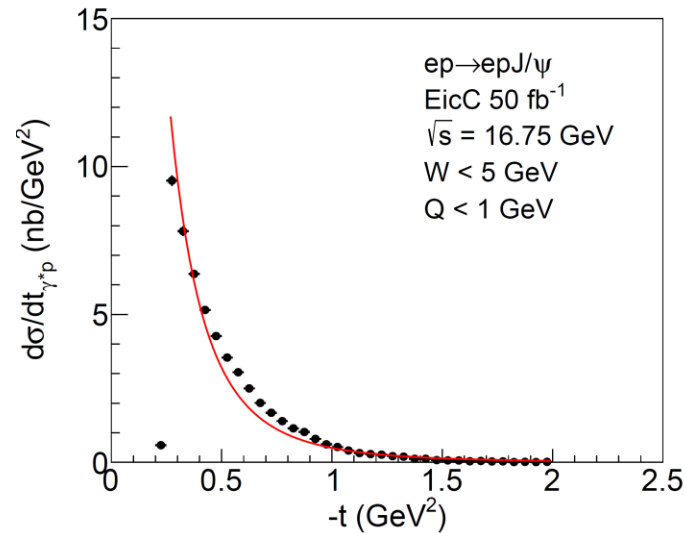
arXiv:2107.03488

VMD relates photo-production cross section to quarkonium-nucleon scattering amplitude

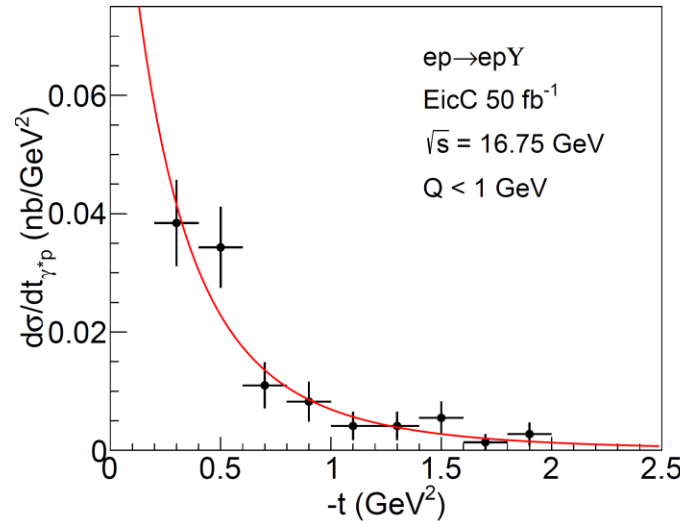


Case-3: explore the origins of proton Mass

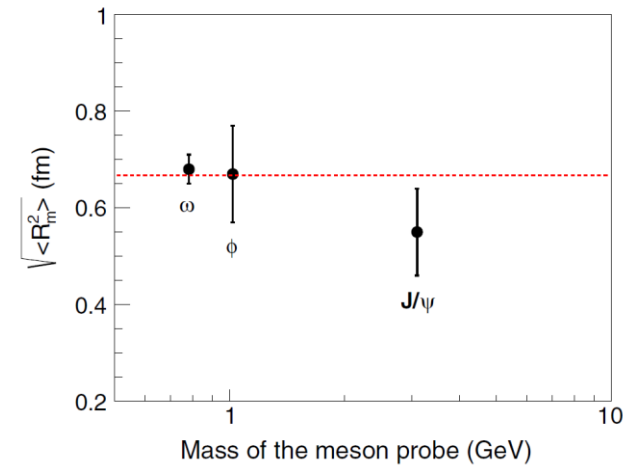
Kharzeev, Phys. Rev. D 104, 054015 (2021);
Wang et al, Phys. Rev. D 103, L091501 (2021)



Simulation of J/psi near-threshold production at EicC



Simulation of Upsilon near-threshold production at EicC



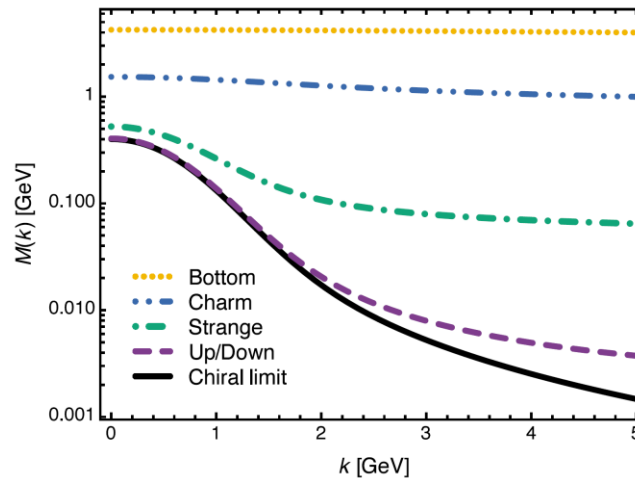
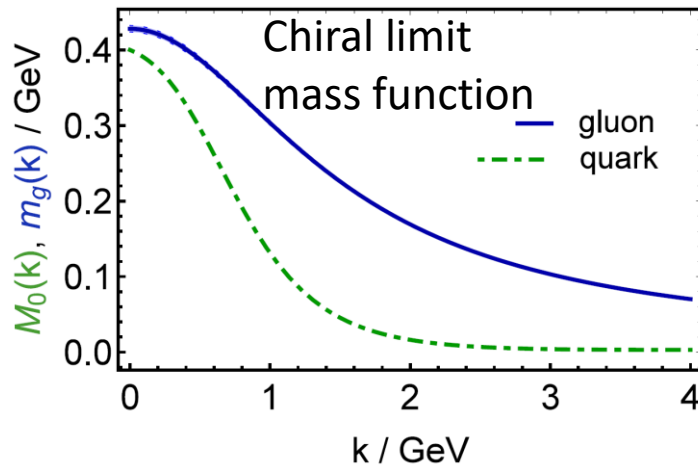
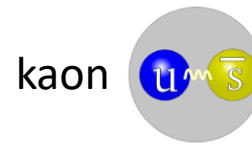
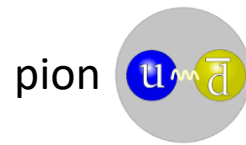
Extractions of the proton mass radius

The forward limit and the t -dependence of the differential cross section of heavy quarkonia productions would be important to check the QCD trace anomaly and the mass distribution of the proton.

Case-4: π & K structures

Case-4: π & K structures

- The π & K mesons are the quasi Nambu-Goldstone boson. They are viewed as the two-body bound states in the quark model, which can be easily studied in theory in comparison to the baryons.



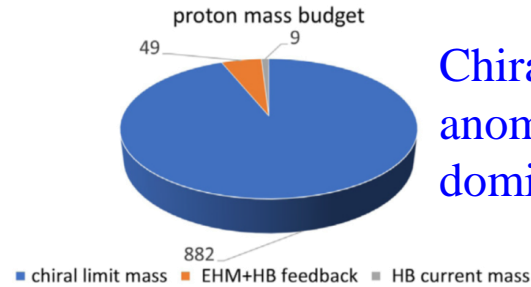
Reintroducing the Higgs couplings, the mass function becomes flavour dependent.

[Progress in Particle and Nuclear Physics 120 (2021) 103883]

The nonperturbative dynamics are well given by the continuum QCD phenomenology.

Case-4: π & K structures

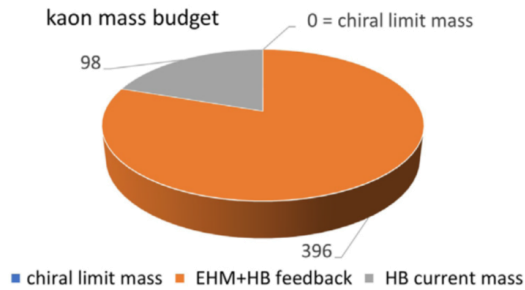
A



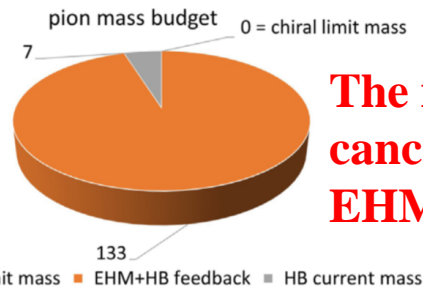
Chiral limit quantum
anomalous energy
dominate.

HB: Higgs Boson mechanism
EHM: Emergent Hadron Mass mechanism

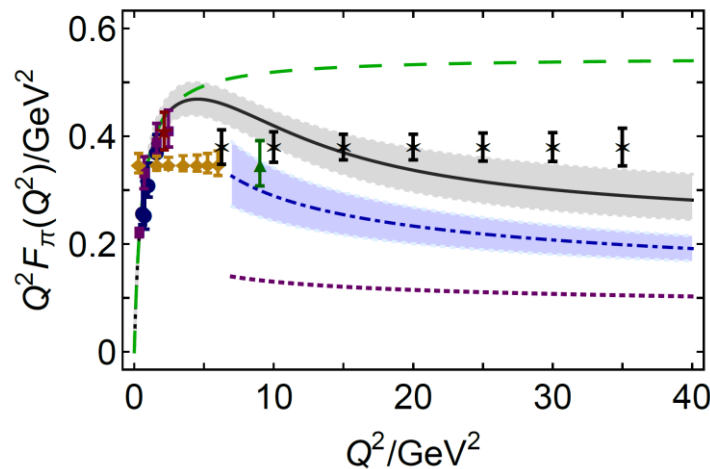
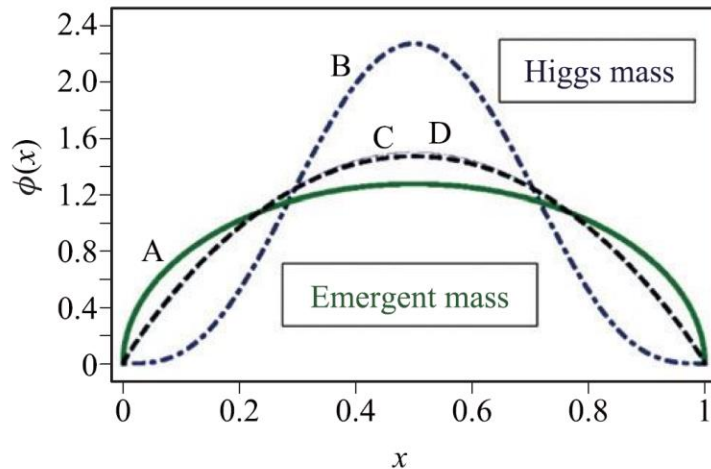
B



C

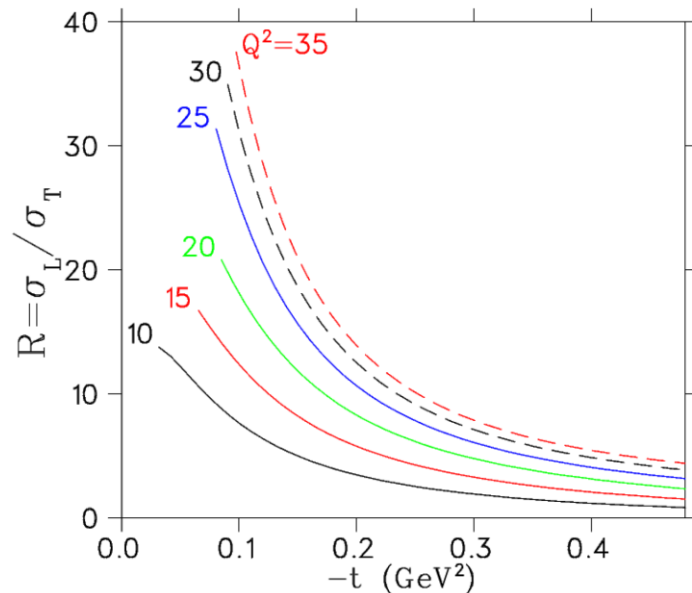


The masses of dressed quasiparticles are
canceled by the attraction potential.
EHM+HB feedback dominate for π & K.



Case-4: π & K structures

In the hard scattering regime, QCD scalling predicts $\sigma_L \propto 1/Q^6$ and $\sigma_T \propto 1/Q^8$.



- T. Vranckx, J. Ryckebusch, PRC **89**(2014)025203.
- Predictions are for $\epsilon > 0.995$ Q^2, W kinematics shown earlier.

At small t & high Q^2 , $\sigma_L \gg \sigma_T$.

No need for L-T separation!

Transverse part is just a small correction.

L-T separation method:

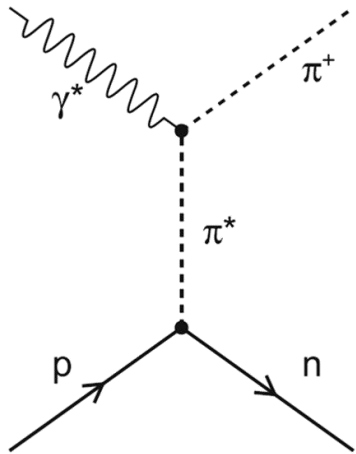
Measuring $(\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt})$ at two energies

$Q^2, \text{ GeV}^2$	$W, \text{ GeV}$	$\sqrt{s}, \text{ GeV}$	ϵ
10	10	12	0.449
10	10	16	0.862
10	10	20	0.951
15	10	12	0.388
15	10	16	0.846
15	10	20	0.946
20	10	12	0.324
20	10	16	0.829
20	10	20	0.940

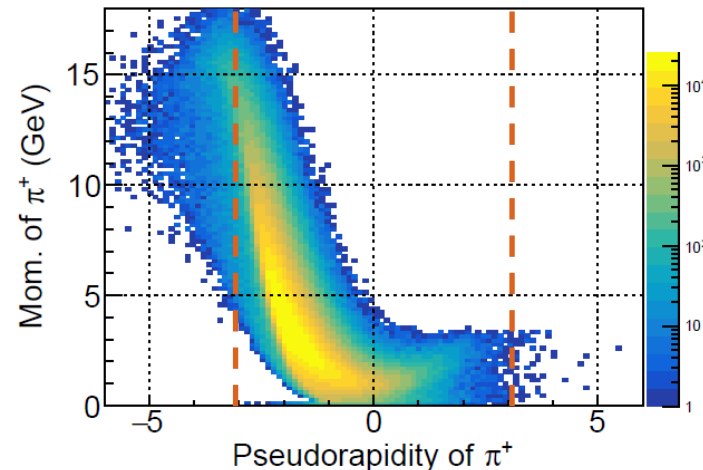
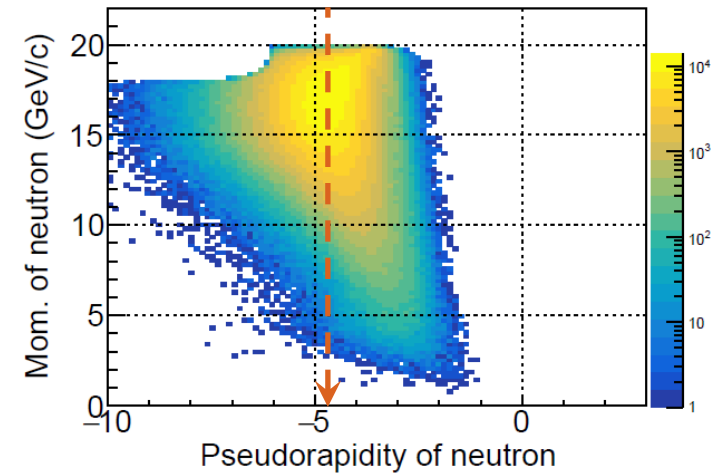
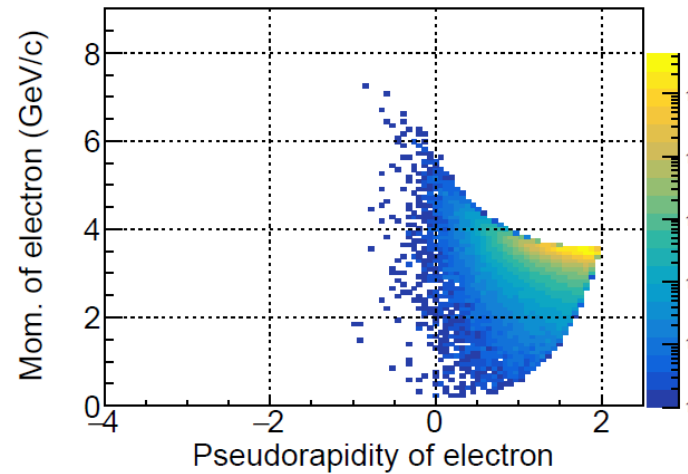
L-T separation is possible at the future EicC!

Case-4: π & K structures

A MC simulation of the exclusive channel $ep \rightarrow e\pi^+n$ at EIC



Sullivan process:
the elastic scattering
between electron probe
and the virtual pion.



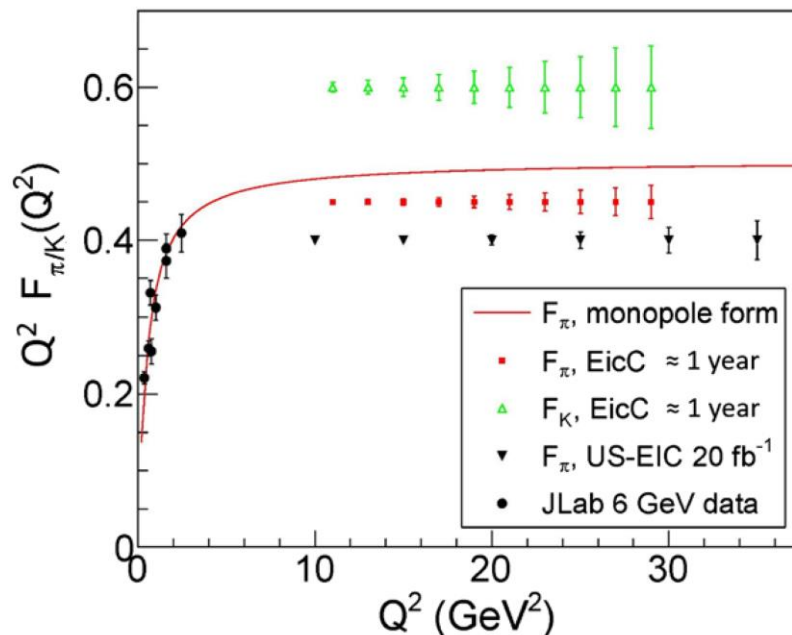
The majority of final electron
and π^+ go to the central
detectors $|\eta| < 3$.

The final neutron goes to the
extreme forward region and
should be collected with ZDC.

Case-4: π & K structures

Assuming an integrated luminosity of 50 fb^{-1} at EicC, the estimated statistical errors of the pion and kaon form factors are shown below. The Q^2 can approach around 30 GeV^2 .

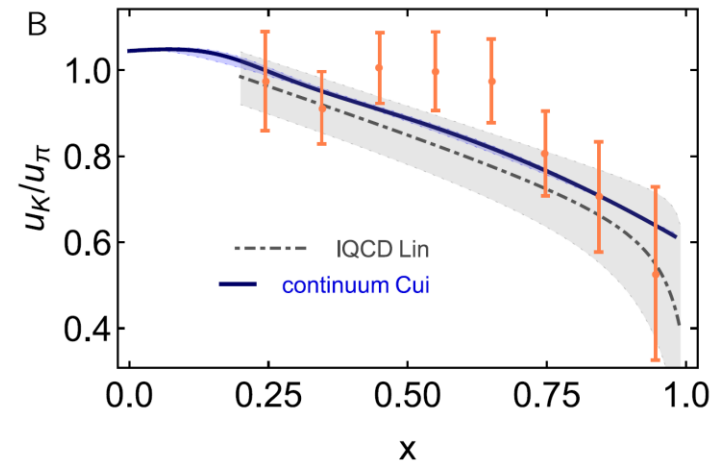
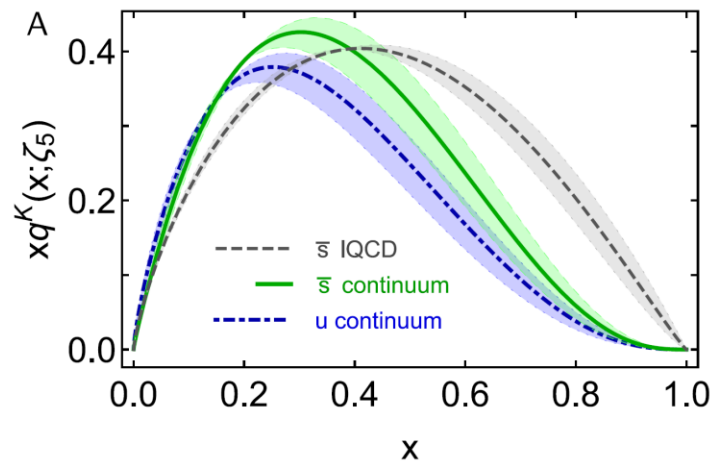
For forward neutrons, we assume a 100% acceptance. For forward Λ , we will collect the charged decays (p, π^-), and the branching ratio is 64%. For the acceptance of forward p and π^- , we require $|\eta| < 5$. These shown errors can be regarded as the ideal case for the future experimental running.



Case-4: π & K structures

- Measuring the pion and kaon structure function provide a way to see the interplay between HB and EHM, and to test fruitful calculations from the nonperturbative approaches (IQCD and the continuum DSE).

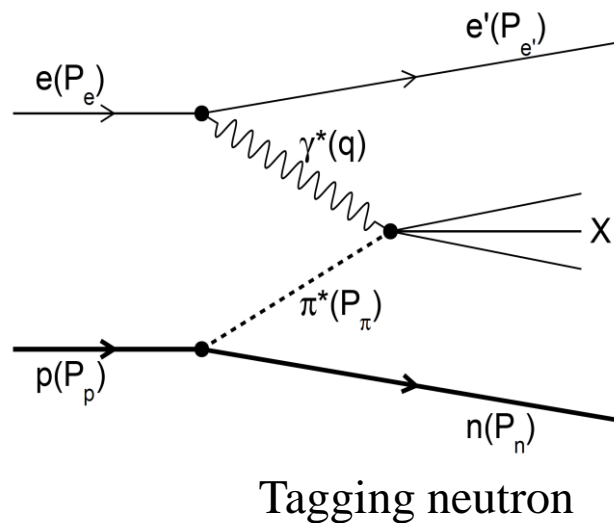
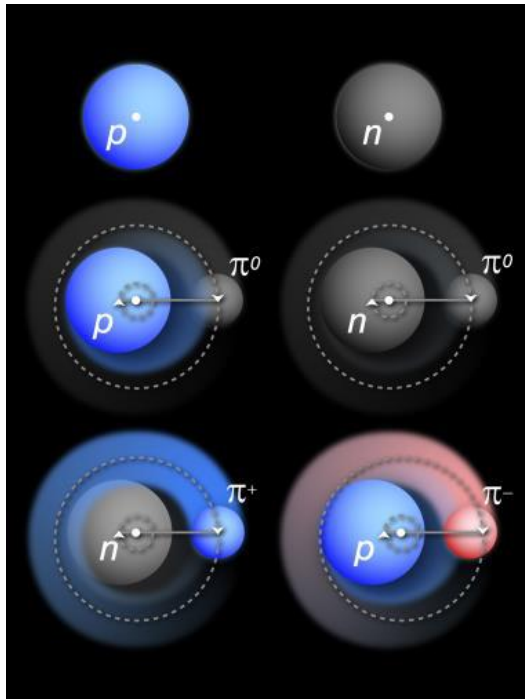
$$q^{\Pi}(x; \zeta_H) \stackrel{x \simeq 1}{=} c(\zeta_H)(1-x)^{\beta_{\Pi}(\zeta_H)}, \quad \beta_{\Pi}(\zeta_H) = 2$$



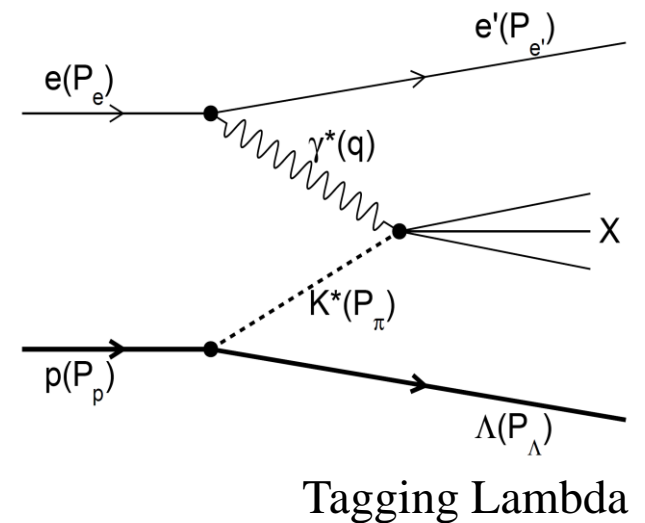
[Progress in Particle and Nuclear
Physics 120 (2021) 103883]

Case-4: π & K structures

The pion and kaon structures can be accessed by electron scattering from the 'meson cloud' outside the proton through the Sullivan process, by tagging the neutrons and Lambdas.



Leading neutron DIS



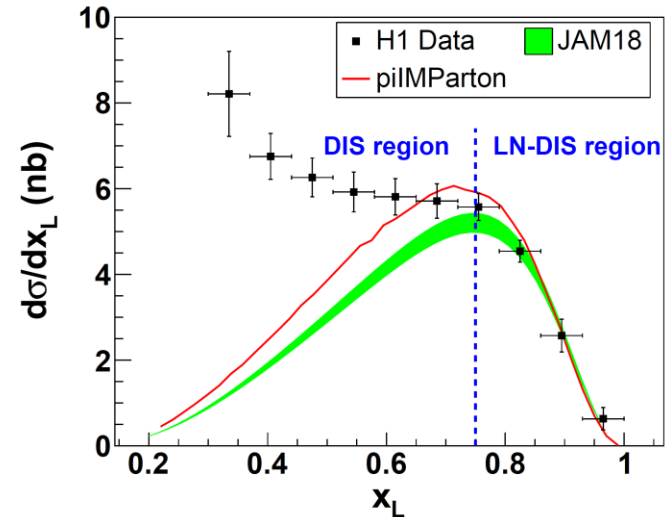
Leading Lambda baryon DIS

Case-4: π & K structures

We have constructed the **leading-neutron and leading-Lambda DIS event generators** (TaggedN_DIS.cpp & TaggedLambda_DIS.cpp) based on the pion/kaon pole flux, pion PDF from JAM analysis, and kaon PDF from IMParton analysis.

$$\begin{aligned}\frac{d^4\sigma(ep \rightarrow enX)}{dx_B dQ^2 dx_L dt} &= \frac{4\pi\alpha^2}{x_B Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2^{\text{LN}(4)}(Q^2, x_B, x_L, t) \\ &= \frac{4\pi\alpha^2}{x_B Q^4} \left(1 - y + \frac{y^2}{2}\right) F_2^\pi\left(\frac{x_B}{1 - x_L}, Q^2\right) f_{\pi^+/p}(x_L, t). \\ f_{\pi^+/p}(x_L, t) &= \frac{1}{2\pi} \frac{g_{pn\pi}^2}{4\pi} (1 - x_L) \frac{-t}{(m_\pi^2 - t)^2} \exp\left(R_{n\pi}^2 \frac{t - m_\pi^2}{1 - x_L}\right)\end{aligned}$$

Using p-Lambda-K coupling and the kaon mass for the kaon flux around the proton

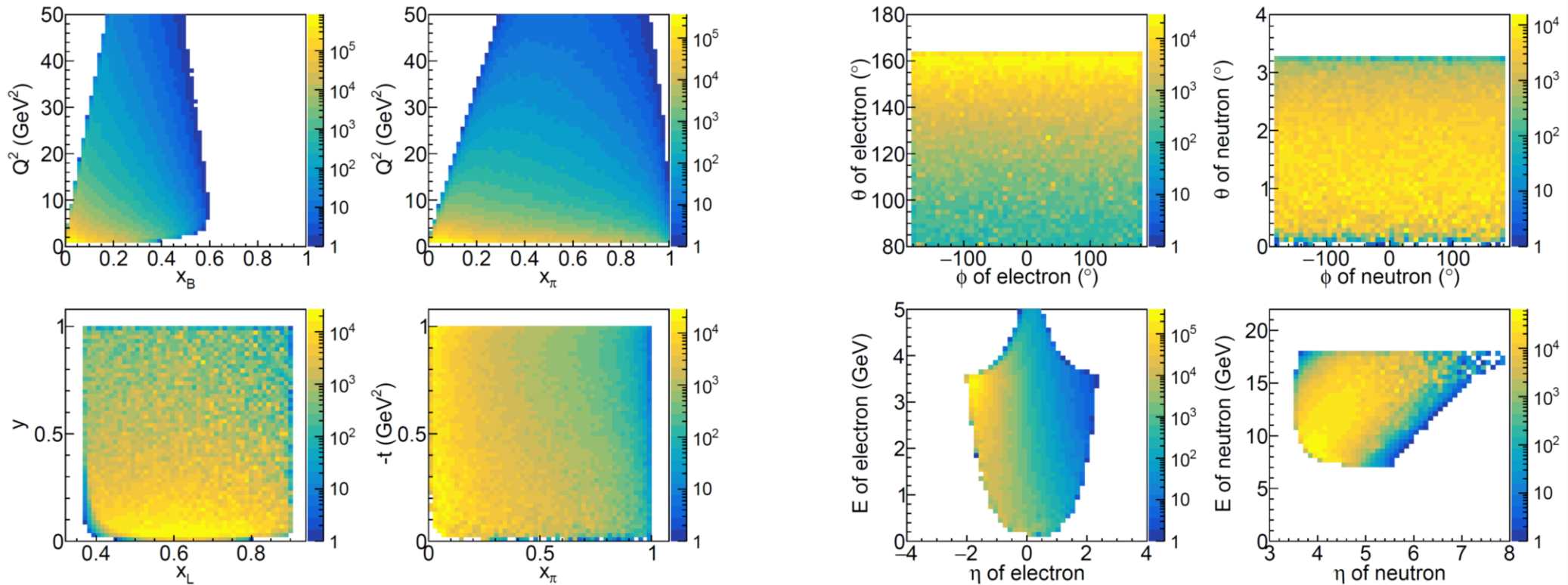


[Chinese Physics C 45 (2021) 053002]

The model for the simulation is checked with previous HERA data.

Case-4: π & K structures

$$e p \rightarrow e n X$$



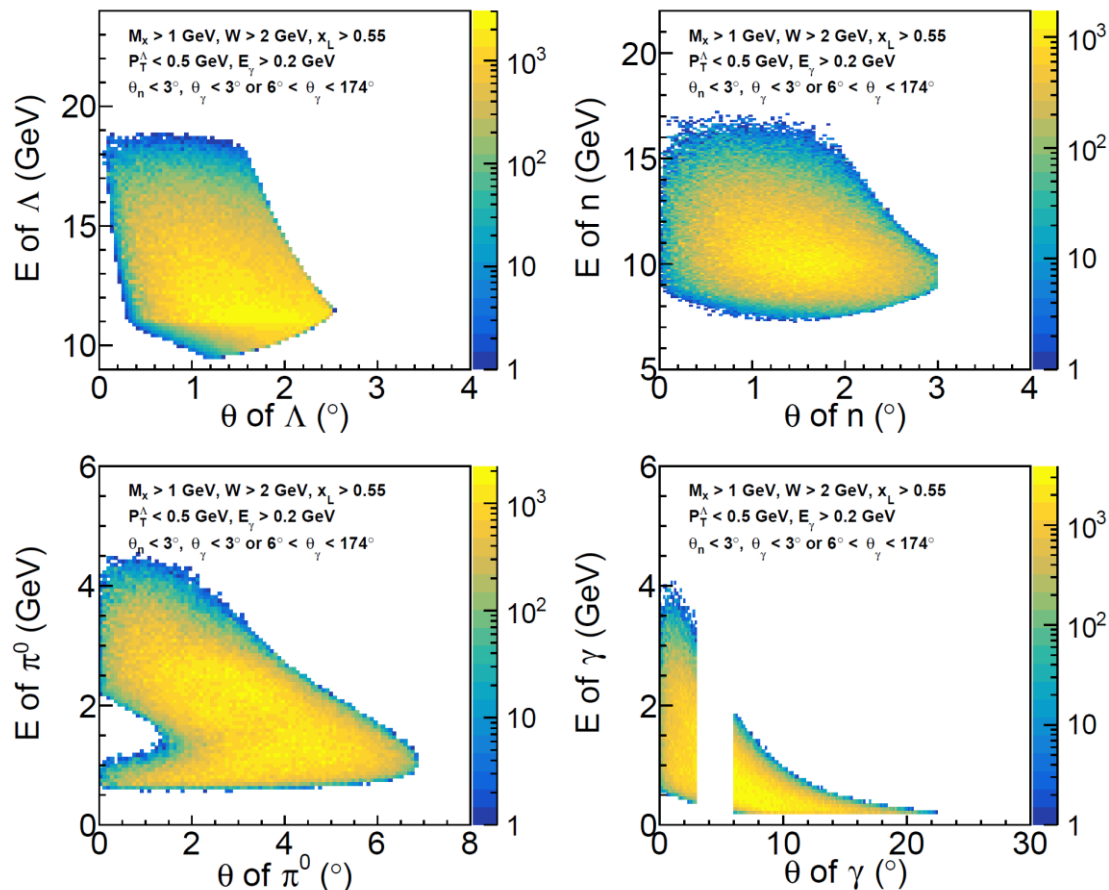
[Chinese Physics C 45 (2021) 053002]

Invariant kinematic distributions of the neutron-tagged DIS events at EicC energy.

The scattered electrons are collected with the central detectors. ZDC should cover the angle from 0 to 3 Deg.

Case-4: π & K structures

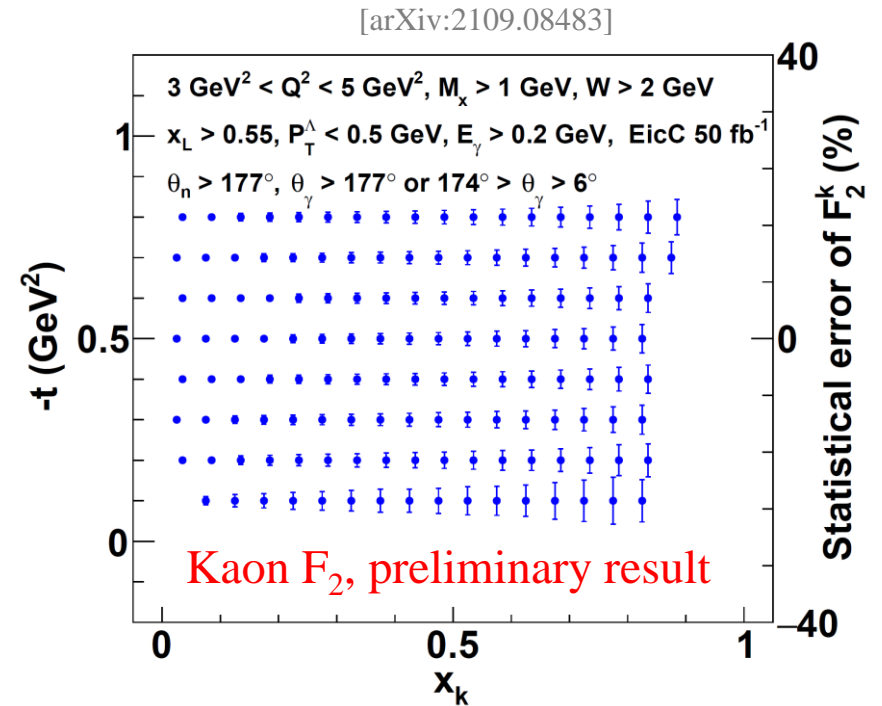
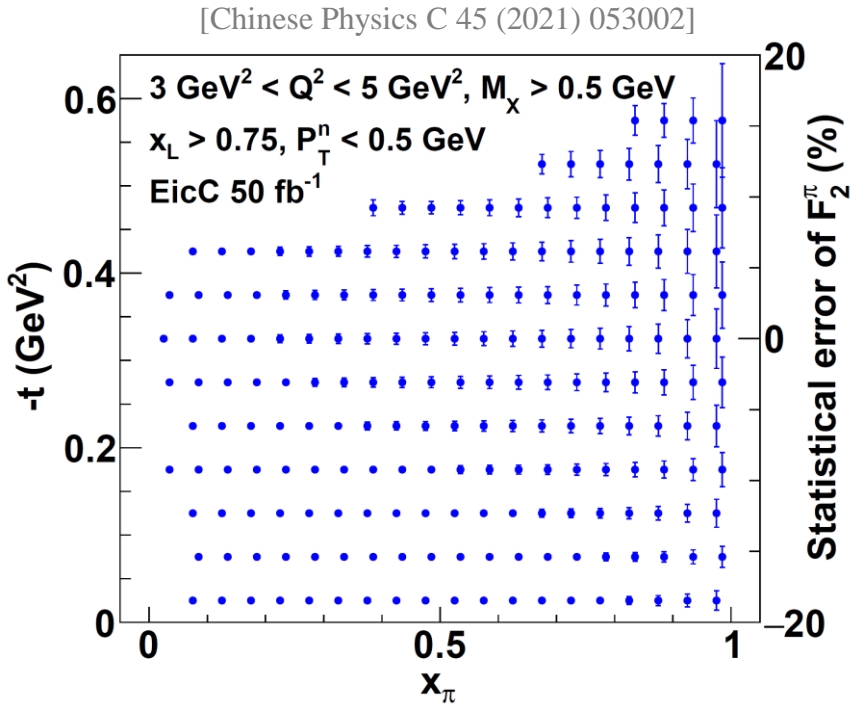
$$e p \rightarrow e \Lambda X$$



To measure the kaon structure, we tag the neutron and two gammas from Lambda decay, with the far-forward detector ZDC of EicC and the central ECal.

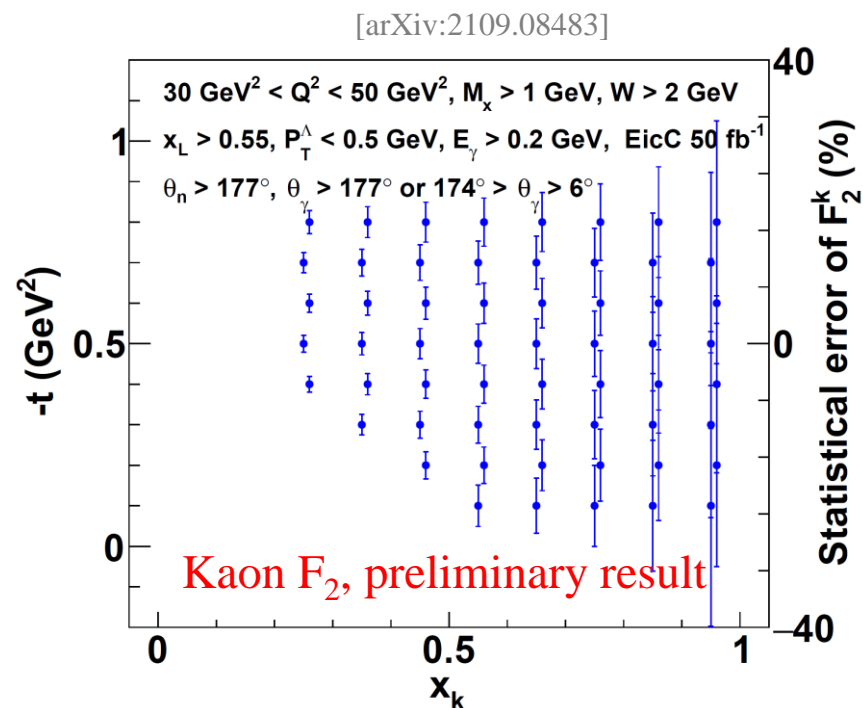
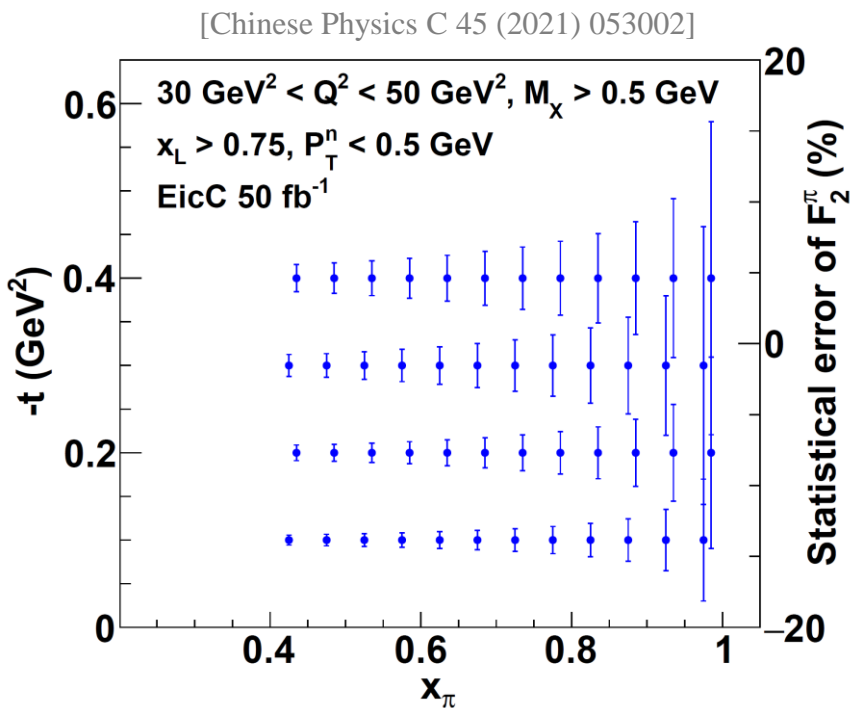
We assume ZDC covers the angle from 0 to 3 Deg. The gammas are supposed to be detected with ZDC and Endcap Ecal.

Case-4: π & K structures



For Lambda-tagged events, we use the wider bins. Under the 50 fb^{-1} luminosity, the uncertainties on the kaon structure are small, which will play an important role in the synergy to JLab12GeV, Amber, EIC measurements and the future global QCD analysis.

Case-4: π & K structures



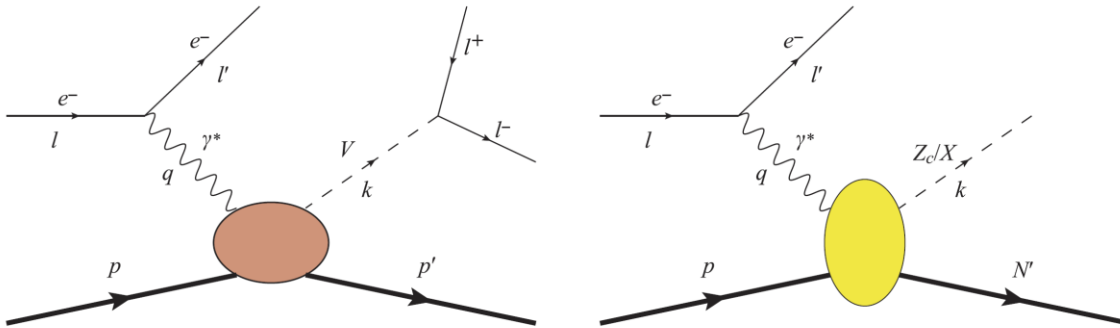
EicC also will provide the meson structure function data at high Q^2 , though with relatively larger uncertainties. This is vital for the extraction of the gluon distribution via scaling violation.

*Case-5: exotic hadron states
& partonic structure of nucleus*

Case-4: exotic hadron state & partons in nucleus

Event number estimation at EicC

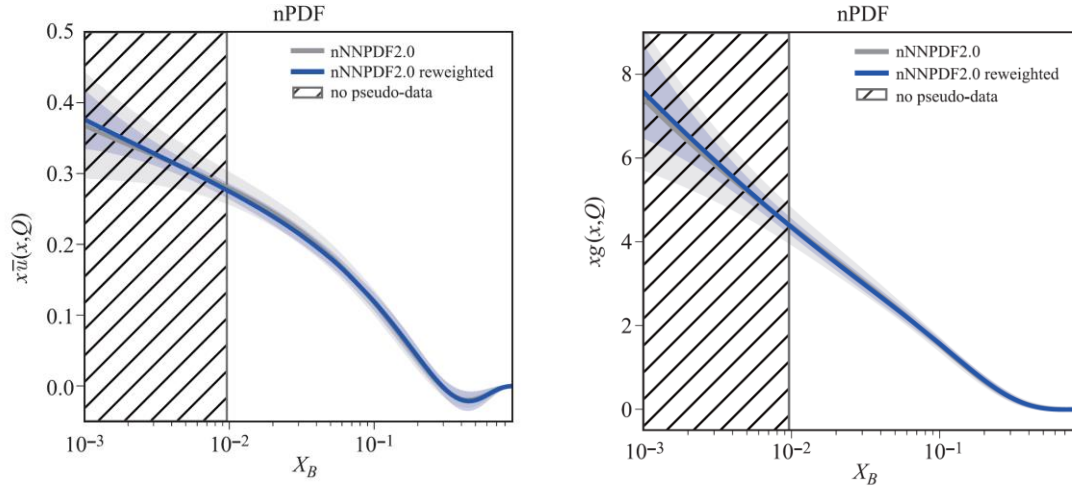
Exploring the exotic baryon (five-quark state) and exotic meson (four-quark state) in the photoproduction process.



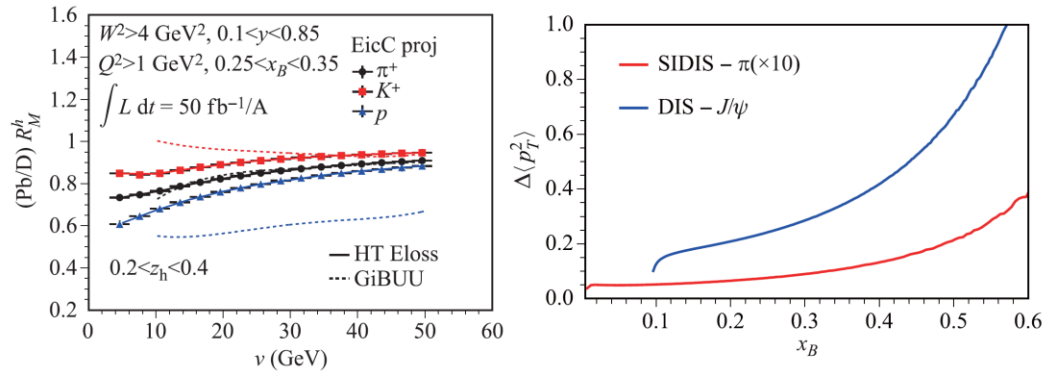
Left: The $ep \rightarrow epV \rightarrow epl^+l^-$ process. Right: The $ep \rightarrow e\chi_{c1}(3872)p$ and $ep \rightarrow eZ_c^+(3900)n$ process.

Exotic states	Production/decay processes	Detection efficiency	Expected events
$P_c(4312)$	$ep \rightarrow eP_c(4312)$	$\sim 30\%$	15–1450
	$P_c(4312) \rightarrow pJ/\psi$		
	$J/\psi \rightarrow l^+l^-$		
$P_c(4440)$	$ep \rightarrow eP_c(4440)$	$\sim 30\%$	20–2200
	$P_c(4440) \rightarrow pJ/\psi$		
	$J/\psi \rightarrow l^+l^-$		
$P_c(4457)$	$ep \rightarrow eP_c(4457)$	$\sim 30\%$	10–650
	$P_c(4457) \rightarrow pJ/\psi$		
	$J/\psi \rightarrow l^+l^-$		
$P_b(\text{narrow})$	$ep \rightarrow eP_b(\text{narrow})$	$\sim 30\%$	0–20
	$P_b(\text{narrow}) \rightarrow p\Upsilon$		
	$\Upsilon \rightarrow l^+l^-$		
$P_b(\text{wide})$	$ep \rightarrow eP_b(\text{wide})$	$\sim 30\%$	0–200
	$P_b(\text{wide}) \rightarrow p\Upsilon$		
	$\Upsilon \rightarrow l^+l^-$		
$\chi_{c1}(3872)$	$ep \rightarrow e\chi_{c1}(3872)p$	$\sim 50\%$	0–90
	$\chi_{c1}(3872) \rightarrow \pi^+\pi^-J/\psi$		
	$J/\psi \rightarrow l^+l^-$		
$Z_c(3900)^+$	$ep \rightarrow eZ_c(3900)^+n$	$\sim 60\%$	90–9300
	$Z_c^+(3900) \rightarrow \pi^+J/\psi$		
	$J/\psi \rightarrow l^+l^-$		

Case-4: exotic hadron state & partons in nucleus



EicC data would help constrain the nuclear sea quark and gluon distributions.

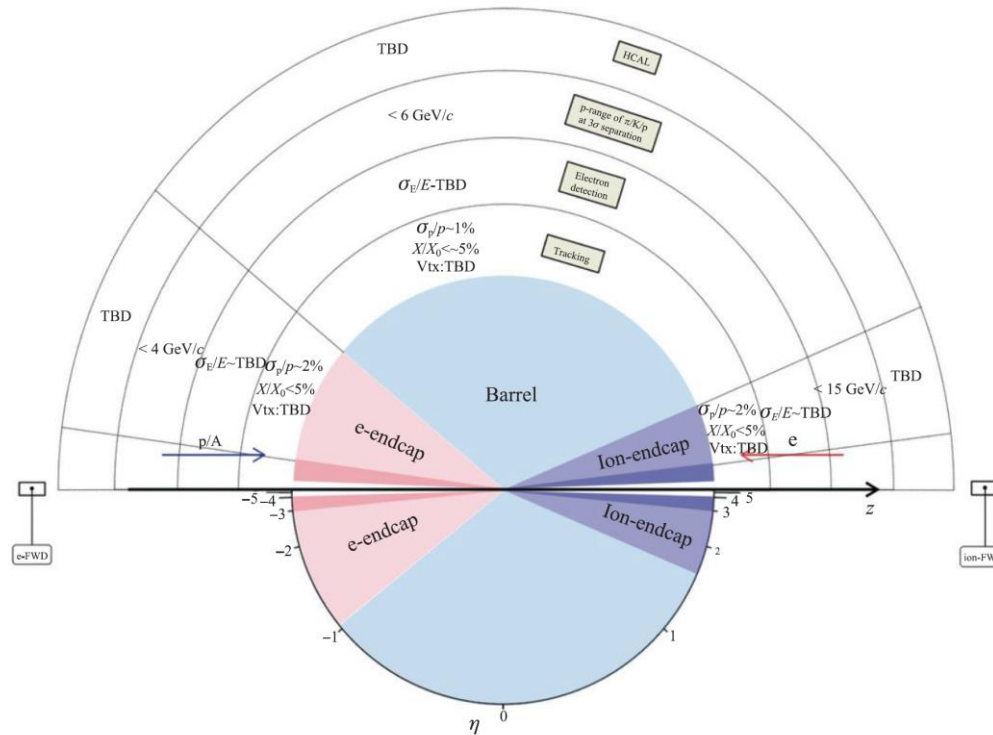


EicC data of hadron multiplicity of e-A collision will shed lights on the parton hadronization process in nuclear matter.

Conclusion

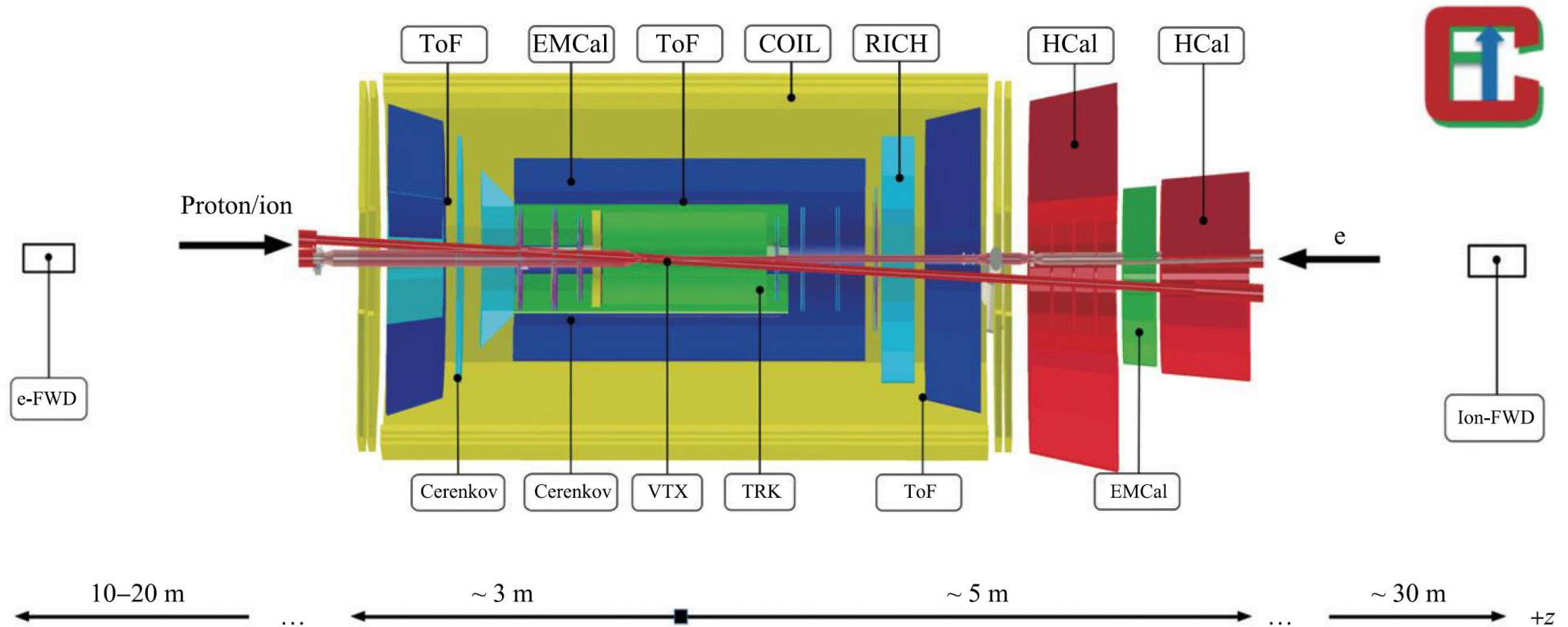
Design, R&D, Location, Summary

EicC design



Physics requirements for EicC.

EicC design



Conceptual design of the EicC detector.

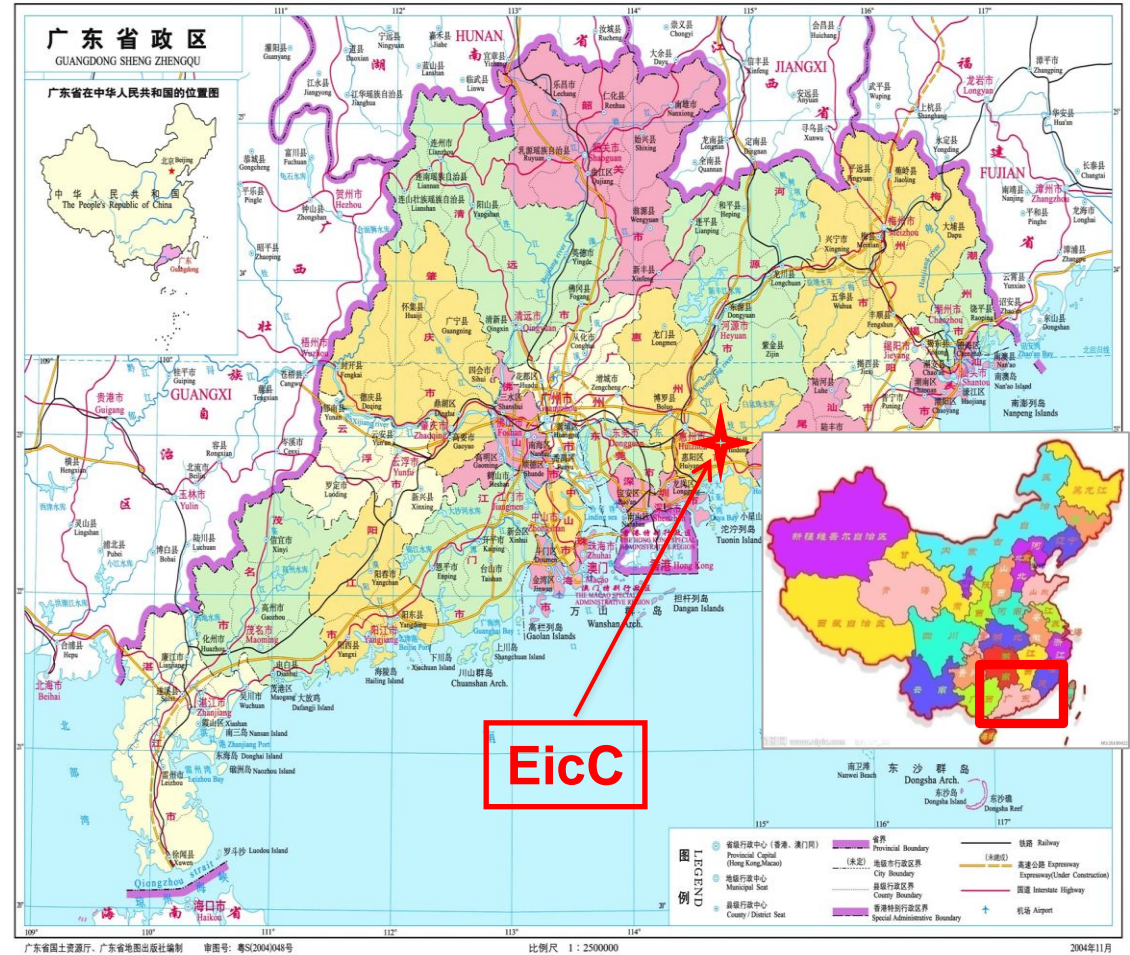
Pre R&D is ongoing! Welcome collaborations!

Location of EicC



● *Huizhou: Coast city*

● *Strong support from local government*



Summary

- EicC is the proposed facility to fully study/understand sea quark structure. It would be crucial for the proton mass origin study, the EHM phenomenon of pseudo scalar meson, mapping the 3D spin-flavor structure of the proton.
- Simulations of some “Golden Experiments” at EicC
 - Nucleon spin-flavor structure (polarized sea, Δs)
 - 3D Structure: GPDs (DVMP) and DVCS
 - 3D Structure: TMDs (sea, wide range in Q^2 , P_T)
 - Proton mass origin
 - π & K structures
 - Exotic hadronic states
 - partonic structure of nucleus
- Other interesting physics topics will be delivered as well.
- EicC focuses on fundamental questions in nuclear physics: complimentary to the EIC-US with higher c.m. energy and JLab 12 GeV
- EicC opens up a new window to study/understand nucleon structure, especially the sea and valence quarks.

*Exciting new opportunities → breakthroughs?
Please contact us at Email: EicC@impcas.ac.cn*

Thank You !

Backup: details of L-T separation at EicC

\sqrt{s} , GeV	L /cm ² /s	Q^2 , GeV ²	W, GeV	y	t, GeV ²	ε
12	1×10^{33}	10	10	0.762	0.1	0.449
16	2×10^{33}	10	10	0.428	0.1	0.862
20	1×10^{33}	10	10	0.273	0.1	0.951
12	1×10^{33}	15	10	0.797	0.1	0.388
16	2×10^{33}	15	10	0.447	0.1	0.846
20	1×10^{33}	15	10	0.286	0.1	0.946
12	1×10^{33}	20	10	0.298	0.1	0.324
16	2×10^{33}	20	10	0.467	0.1	0.829
20	1×10^{33}	20	10	0.832	0.1	0.940

To do the L-T separation, EicC is suggested to run at c.m. energy from 12 GeV to 20 GeV, which is hard to be realized at EIC of much higher energy.

Backup: exclusive pion electroproduction

To write an event generator and to estimate the statistic, we adapt a simple model for the differential cross-section:

$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi_\pi} = \Gamma(Q^2, x_B, s) \frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_\pi) \frac{d\sigma_{LT}}{dt} + \epsilon \cos(2\phi_\pi) \frac{d\sigma_{TT}}{dt} \right]$$

$$\Gamma(Q^2, x_B, s) = \frac{\alpha y^2 (1 - x_B)}{2\pi x_B (1 - \epsilon) Q^2}$$

$$\epsilon^{-1} = 1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \left(4 \frac{\nu^2}{Q^2} \frac{1 - y}{y^2} - 1 \right)$$

$$N \frac{d\sigma_L}{dt} = 4\hbar c (e g_{\pi NN})^2 \frac{-t}{(t - m_\pi^2)^2} Q^2 F_\pi^2(Q^2)$$

$$N = 32\pi (W^2 - m_p^2) \sqrt{(W^2 - m_p^2)^2 + Q^4 + 2Q^2(W^2 + m_p^2)}$$

$$g_{\pi NN}(t) = g_{\pi NN}(m_\pi^2) \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t} \right)$$

$$F_{\pi,K}(Q^2) = \frac{1}{1 + Q^2/\Lambda_{\pi,K}^2}$$

More sophisticated models:

[M. Vanderhaeghen, M. Guidal, J.-M. Laget, Phys. Rev. C 57, 1454 (1998); T.K. Choi, K.J. Kong, B.G. Yu, arXiv:1508.00969; R. Perry, A. Kizilersu, A. Thomas, Phys. Lett. B 807, 135581 (2020)...]

Pion pole contribution

Born term formula