



EMERGENT PHENOMENA IN STRONG DYNAMICS 1

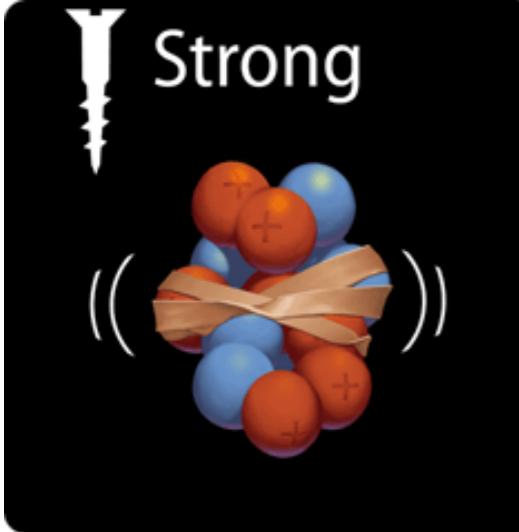
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Baryons 21 Sevilla
OCTOBER 18 - 22

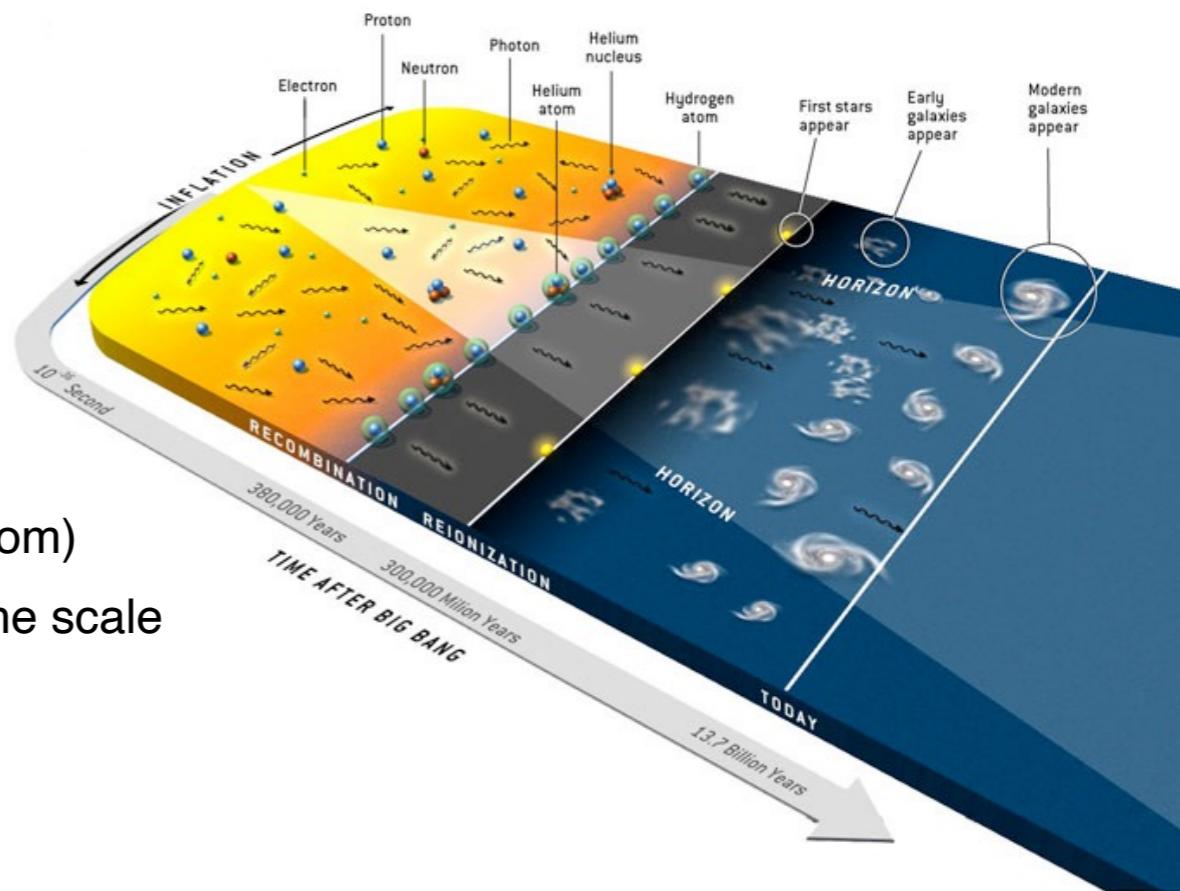


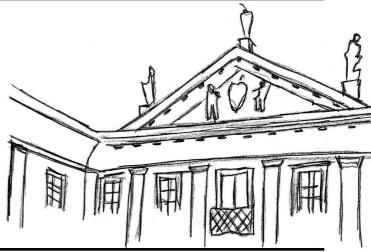


Standard Model: QCD

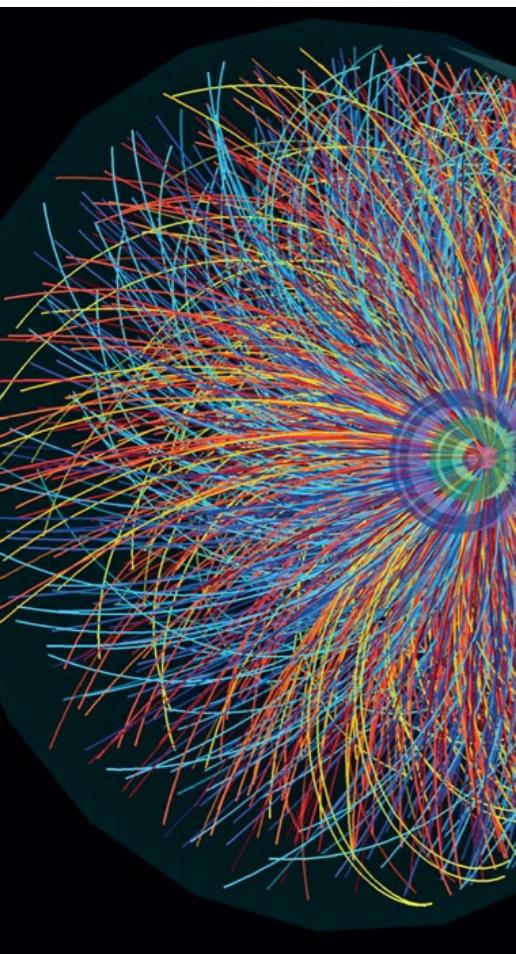
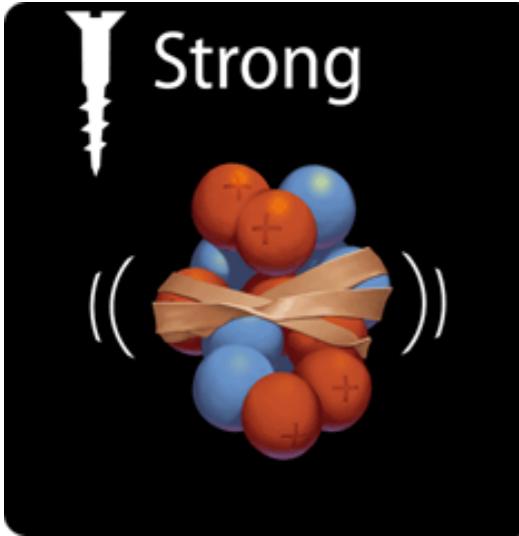


- **QCD started 10^{-6} secs. after the Big-Bang:** it is the glue that binds us all, and understanding its dynamics has profound implications
 - Explain how massless gluons and light quarks are confined and bind together to form hadrons...
 - ...thereby explain the origin of ~98% of the mass in the visible Universe
- **QCD is *likely* a perfect theory**
nothing needs to be added or changed
 - **Validated** over an incredible energy range: $0 \lesssim E \leq 8000$ GeV
 - **Unlikely to break down** at any energy scale (asymptotic freedom)
 - **No intrinsic parameters**, just need one observable to define the scale
 $\Lambda_{\text{QCD}} \simeq 200 - 300$ MeV QCD's "standard kilogram"
- **QCD is a *theory* not an effective theory**
- **However, it is innately nonperturbative**
a priori no idea what such theory can produce





QCD: degrees of freedom



- **QCD basic degrees of freedom:**
matter (quarks); gauge (gluons)

$$q_f^i \begin{cases} \text{color} & i = 1, 2, 3 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

$$A_\mu^a \begin{cases} \text{color} & a = 1, \dots, 8 \\ \text{spin} & \varepsilon_\mu^{\pm, 0} \end{cases}$$

- **QCD action:**
encodes all the dynamics

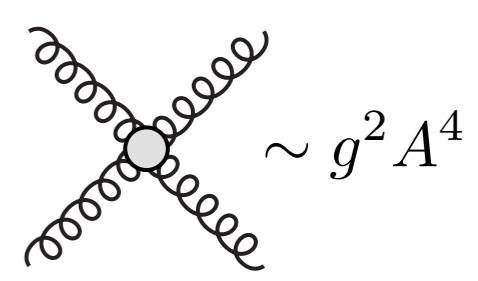
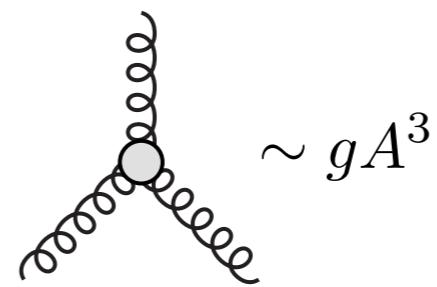
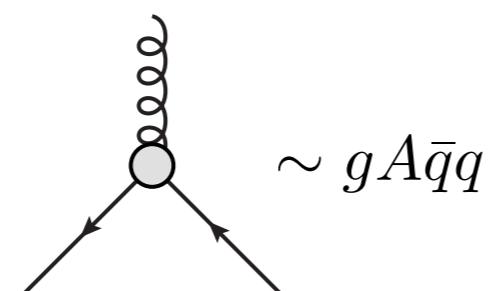
$$S_{\text{QCD}} = \int d^4x (\mathcal{L}_I + \mathcal{L}_{\text{GF+FPG}})$$

$$\mathcal{L}_I = \bar{q}_f^i (i\gamma^\mu D_\mu - m)_{ij} q_f^j - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

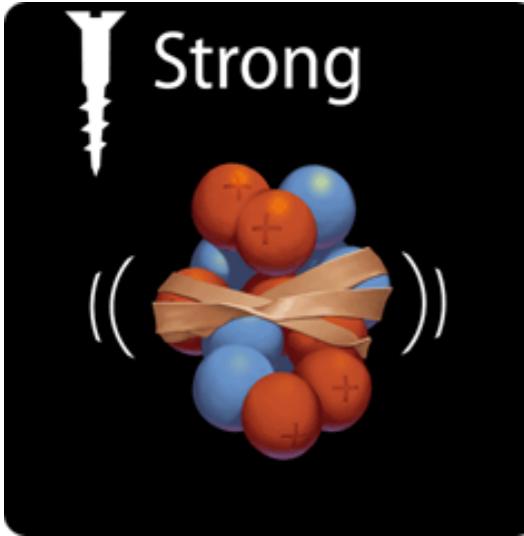
$$D_\mu = \partial_\mu - ig A_\mu^a T^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

- **QCD (self-)interactions**
dictate the theory's behavior



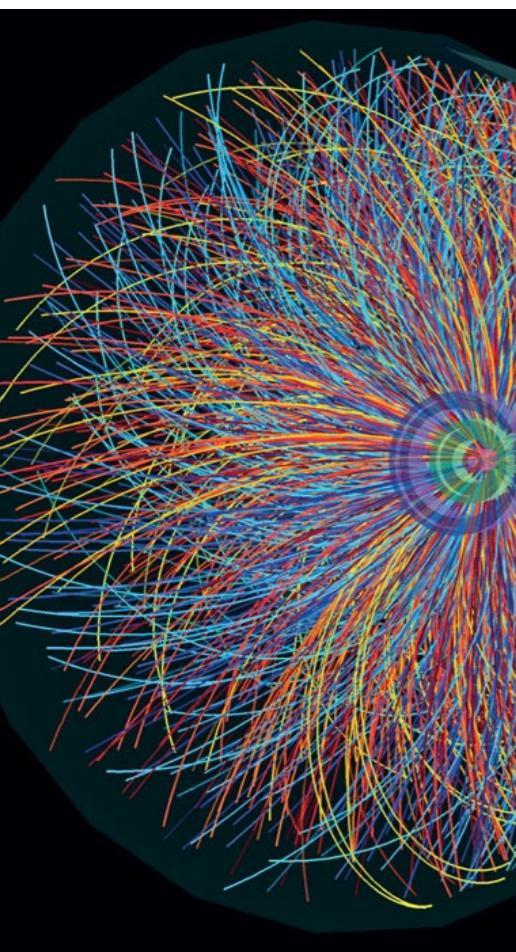
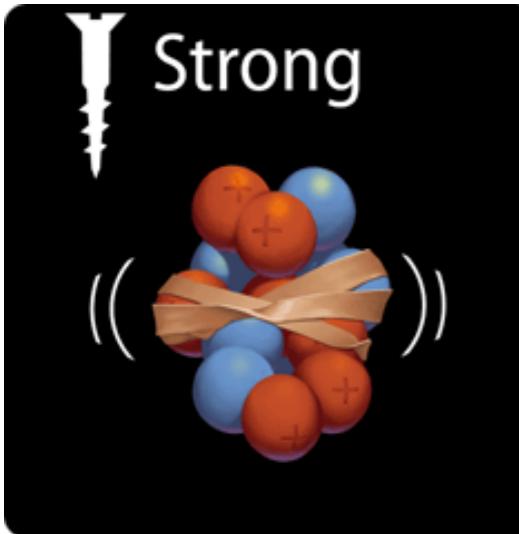
QCD: trace anomaly



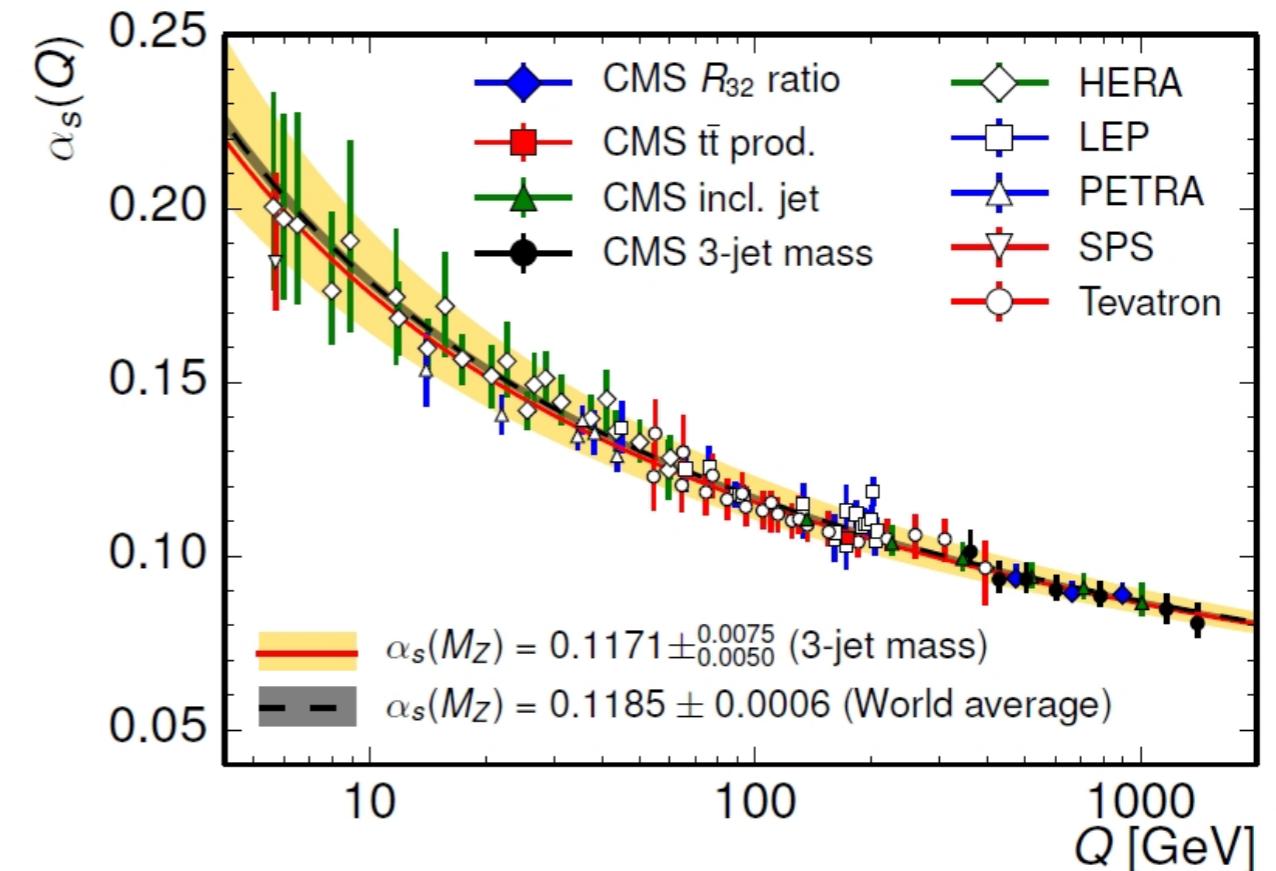
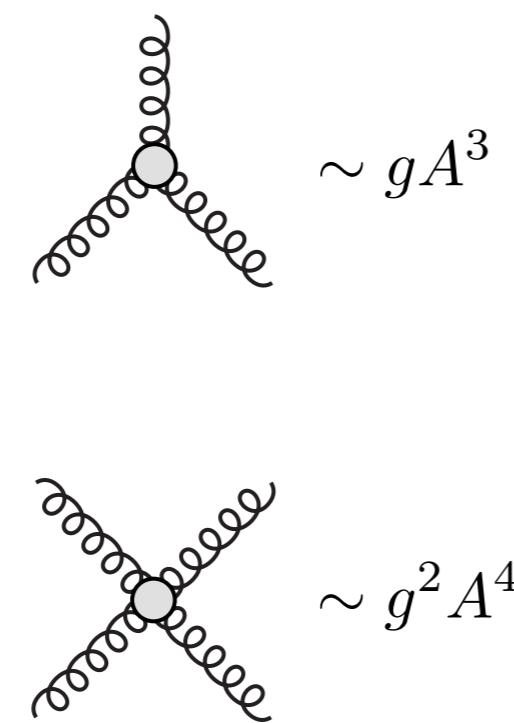
- Remove current mass from classical Lagrangian
no energy scale is left
$$\mathcal{L}_I = \bar{q}_f^i (i\gamma^\mu D_\mu - \cancel{m})_{ij} q_f^j - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$
- No dynamics in a scale invariant theory
theory looks the same at all length scales
 - Bound states are impossible...
- Our Universe cannot exist
Higgs boson cannot help
 - Mass of the proton is 100 times bigger than anything the Higgs can produce
- Classical QCD is meaningless
must be quantized
- Regularization and renormalization of (UV) divergences
lead to dimensional transmutation
 - Trace anomaly
non-zero value of the trace of the stress energy tensor: $\Theta_0 = \frac{1}{4}\beta(\alpha)F_{\mu\nu}^a F_a^{\mu\nu}$



QCD: asymptotic freedom



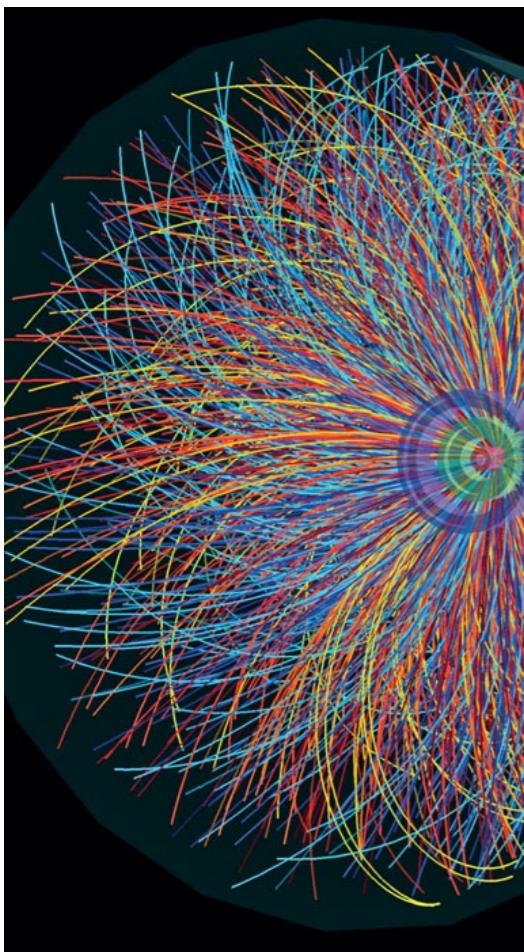
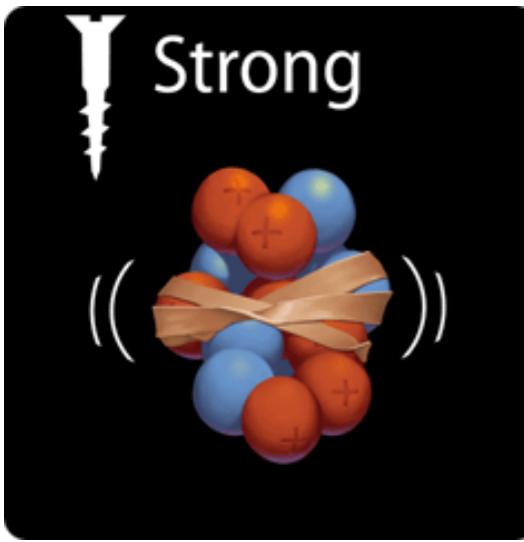
- **Gluon self-interaction runs**
implies QCD is non perturbative and quark/gluons are confined



- **QCD interaction strength:**
grows with separation between gluons and quarks
- **Typical scale of hadron physics**
 $r \sim 2 \text{ fm}$ $\alpha \sim 0.5$
 - **Perturbation theory breaks down at this distance**
QCD is entirely nonperturbative across almost the entire proton's volume



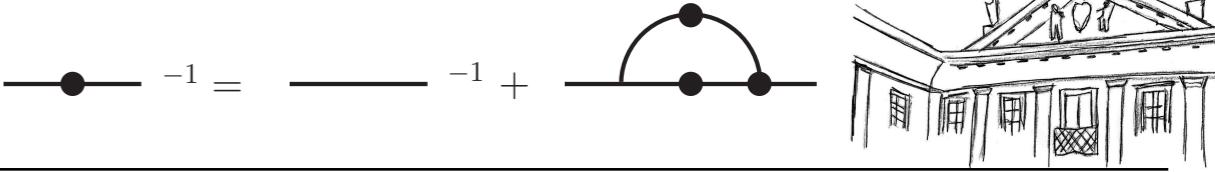
QCD: Dyson-Schwinger equations



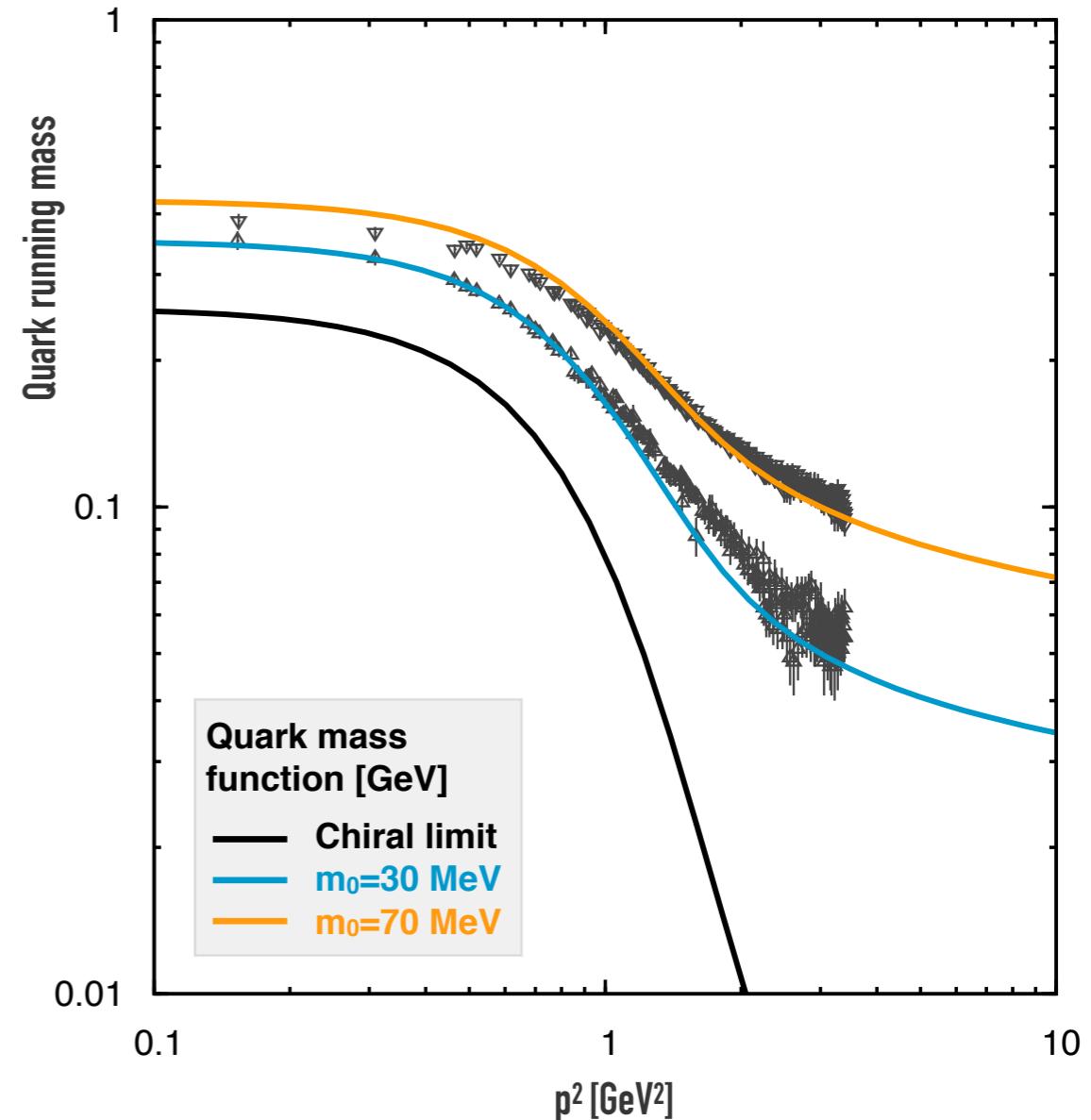
- **Understanding the origin of mass in QCD** is quite likely inseparable from understanding confinement
- **One possible way of addressing this are Schwinger-Dyson eqs** which are quantum eom of Green's functions
- **SDEs: nonperturbative, covariant, IR/UV, light/heavy quarks; but:** infinite system of coupled integral equations
 - **Needs reliable truncation schemes** plus requires a gauge to be chosen (Landau)
- **Easiest SDE: 2-point functions** 
 - **Three equations to be considered** quarks, gluons and ghosts
 - **Gauge fixing + FP ghost:** BRST exact, does not appear in the spectrum
- **Capture two emergent phenomena**
 - Dynamical mass generation
 - Confinement (?)

$$\mathcal{L}_{\text{GF+FPG}} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$

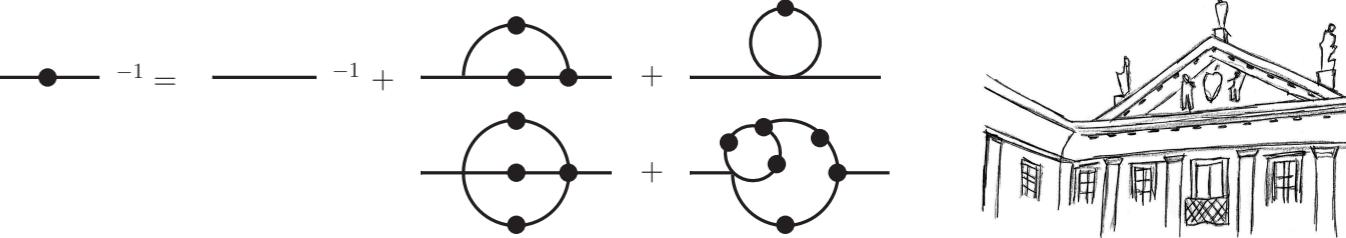
Dynamical quark mass



- **Dynamical chiral symmetry breaking:** generates “constituent-quark” masses
- **Add nothing to QCD:** effect achieved purely through the theory’s dynamics
- **Most important mass generating mechanism:** responsible for ~98% of the proton’s mass (Higgs mechanism almost irrelevant for light quarks)

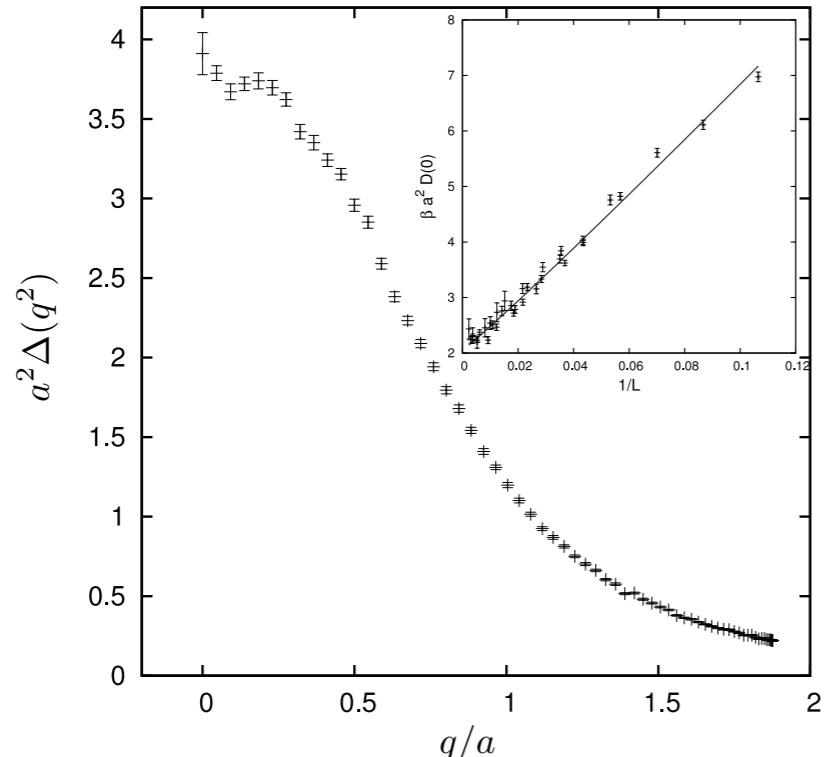


What about gluons?

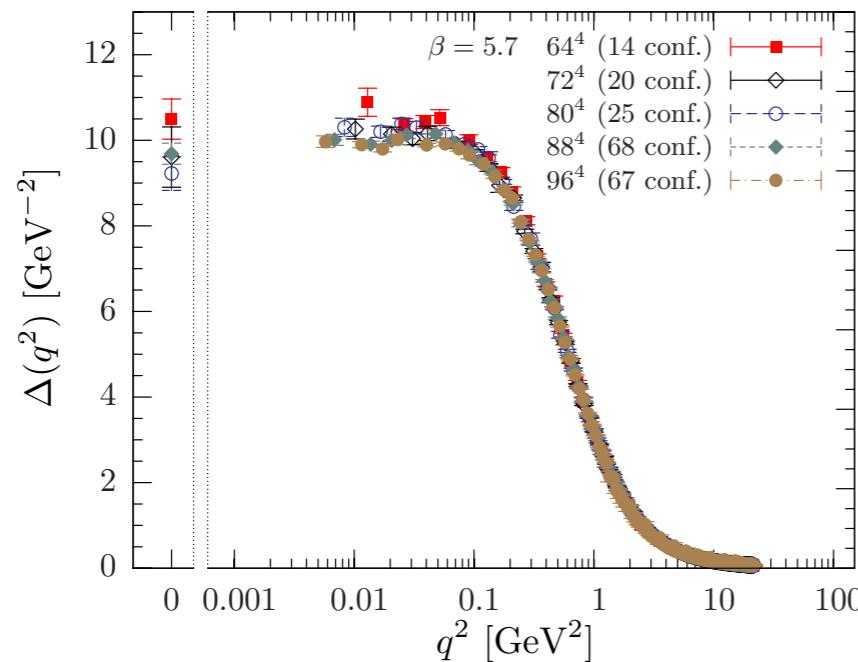


- **Landau gauge (quenched)**

Cucchieri, Mendes POS LATTICE (2007)

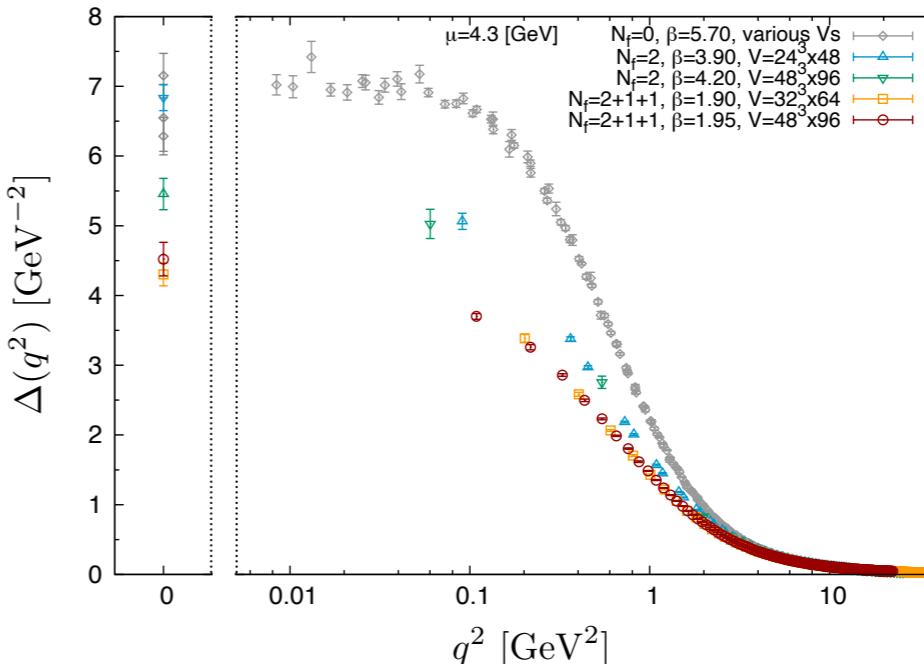


Bogolubsky, Ilgenfritz, Muller-Preussker, Sternbeck, PLB 676 (2009)



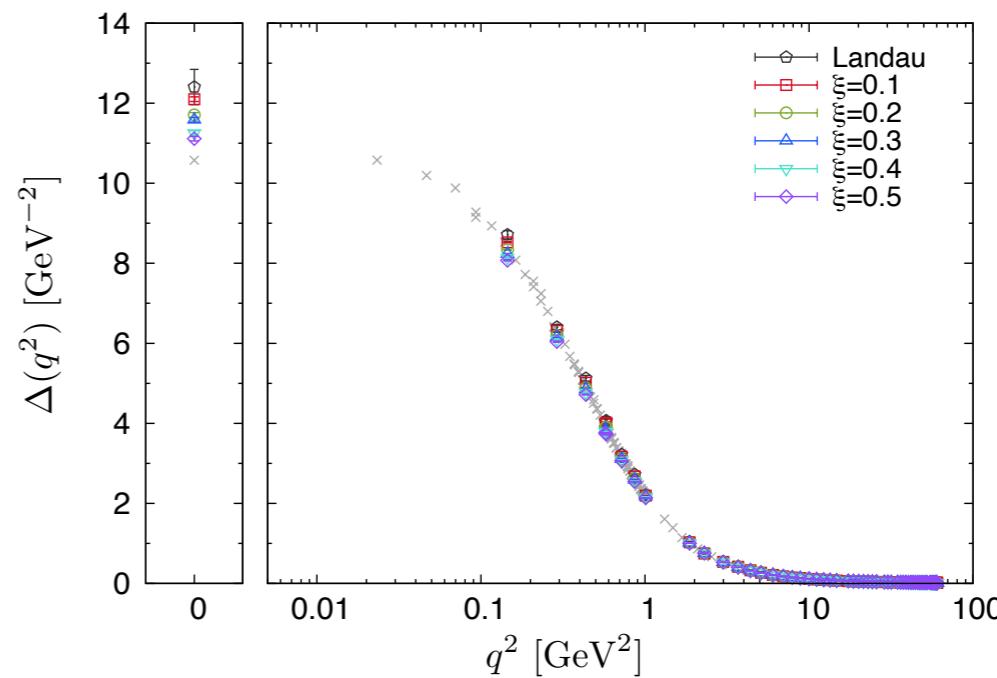
- **Landau gauge (unquenched)**

Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)

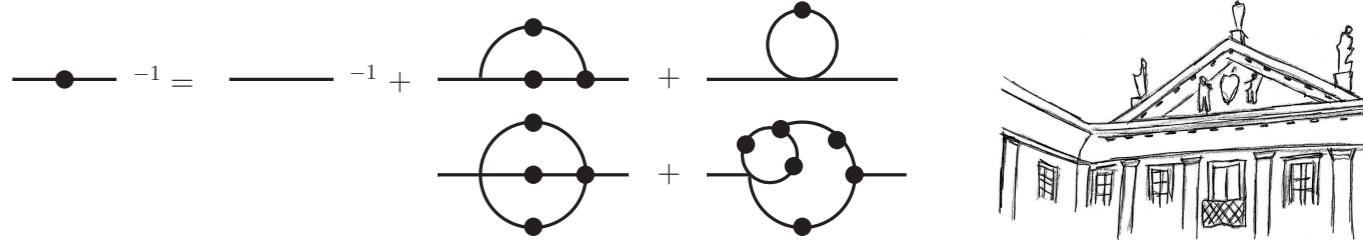


- **Linear gauges (quenched)**

Bicudo, DB, Cardoso, Oliveira, Silva, PRD 92 (2015)



PT-BFM framework



- **IR Saturating/massive gluon propagator**
challenging from a continuous perspective
- **Best understood within PT-BFM framework**
[DB, Papavassiliou, PRD 77 \(2008\); JHEP 0811 \(2008\); PR 479 \(2009\)](#)
- **How to get PT Green's functions?**
use the PT algorithm
[Cornwall, Papavassiliou, PRD 40 \(1989\)](#)
- **Resummed Green's functions**
with better truncations properties

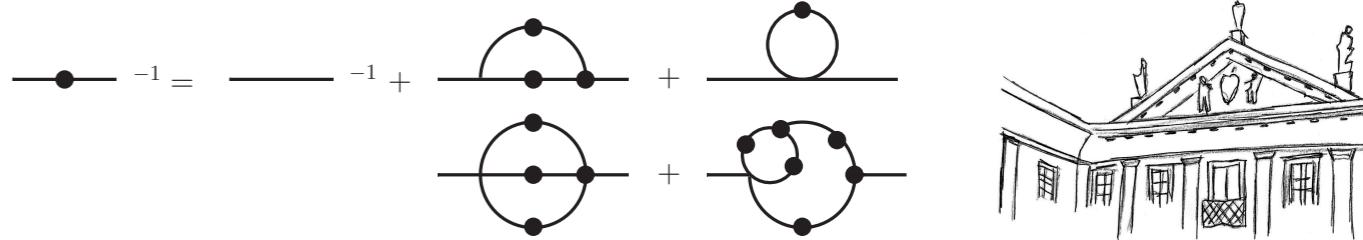
$$\hat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^\alpha g^{\mu\nu} + 2q^\nu g^{\alpha\mu} - 2q^\mu g^{\alpha\nu}$$

$$\Gamma_P^{\alpha\mu\nu} = k_1^\mu g^{\alpha\nu} - k_2^\nu g^{\alpha\mu}$$

- **longitudinal momenta**
trigger elementary Ward identities

- **Apply the PT to the quark-gluon vertex**
one loop result:

PT-BFM framework



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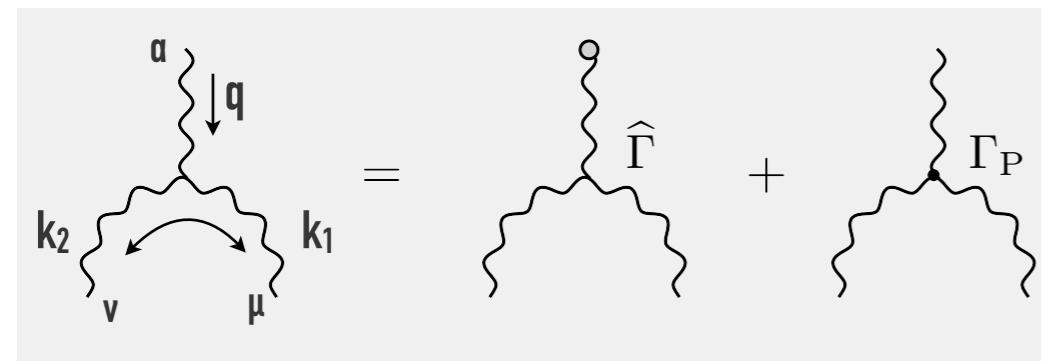
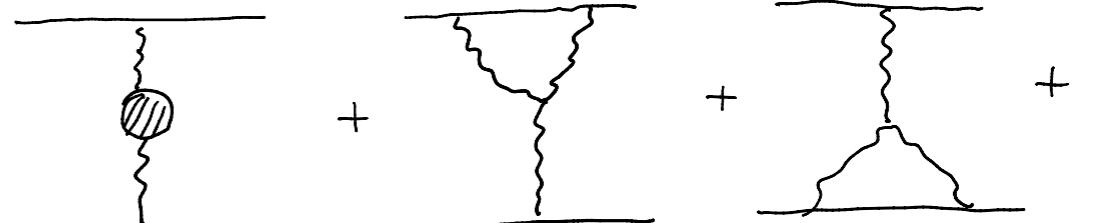
DB, Papavassiliou, PRD 77 (2008); JHEP 0811 (2008); PR 479 (2009)

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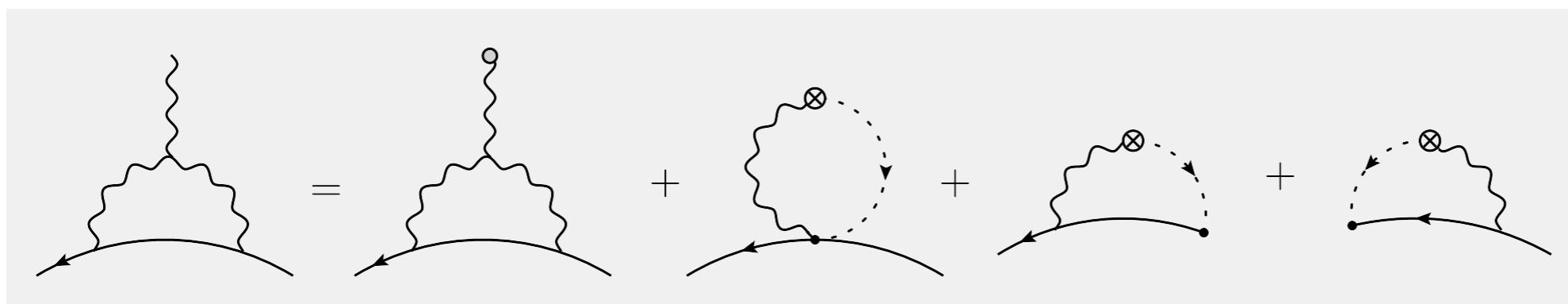


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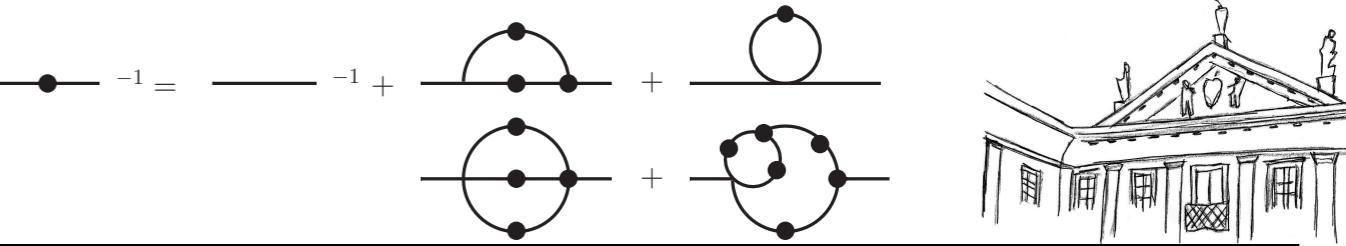
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one loop result:



PT-BFM framework



- **Allot pieces to different Green's functions**
construct $\hat{\Delta}$ and $\hat{\Gamma}_\mu$

A Feynman diagram identity showing the decomposition of a bare propagator into a dressed propagator and loop corrections. On the left, a horizontal line with a dot at one end is labeled $\hat{\Delta}^{-1}$. This is followed by an equals sign. To the right of the equals sign is another horizontal line with a dot at one end, followed by a plus sign. To the right of the plus sign is a loop with two external lines and a crossed line through it, labeled with a circled \otimes .

A Feynman diagram identity showing the decomposition of a bare vertex into a dressed vertex and loop corrections. On the left, a vertex with a blue circle and a wavy line attached is labeled $\hat{\Gamma}$. This is followed by an equals sign. To the right of the equals sign is another vertex with a wavy line attached, followed by a plus sign. To the right of the plus sign are three diagrams: a loop with two external lines and a crossed line through it, a loop with three external lines and a crossed line through it, and a loop with four external lines and a crossed line through it.

$$q^\mu \hat{\Gamma}_\mu = S^{-1}(p_1) - S^{-1}(p_2)$$

vanish on-shell

- **New propagator has special properties**
gives rise to an effective charge like in QED

$$\hat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \quad b = 11C_A/48\pi^2$$

- **Absorbs all the RG logs** as the photon in QED
- **Renormalizes as** Z_g^{-2}
- **Yields Feynman rules** for systematic calculations

- **Best understood within PT=BFM identity...**

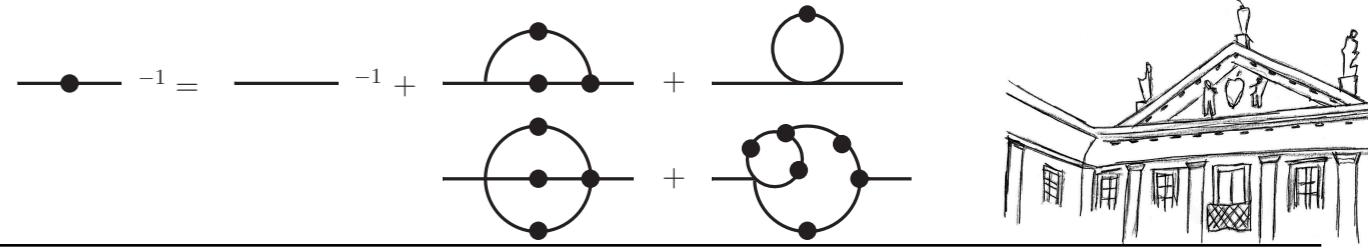
DB, Papavassiliou, PRD 77 (2008); JHEP 0811 (2008); PR 479 (2009)

- **...and BRST+antiBRST=BFM**

plethora of symmetry identities, in particular so-called BQ identities

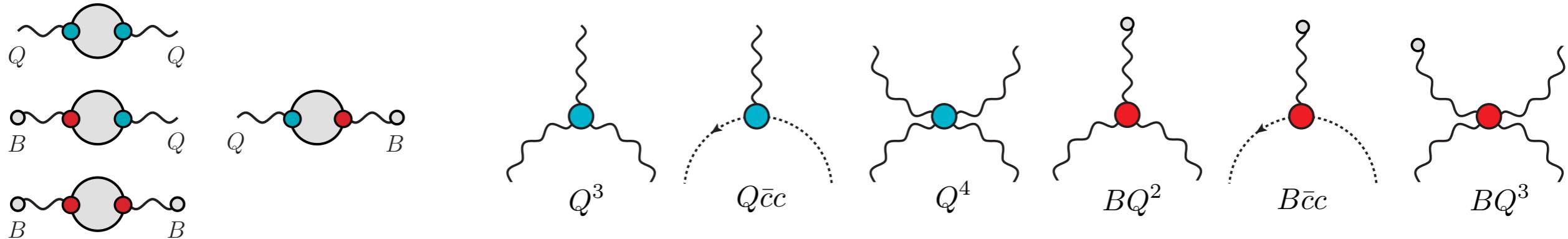
DB, Quadri, PRD 88 (2013)

PT-BFM framework



- **Split gauge field**
into background (B) and quantum fluctuating (Q) parts
[Abbott, NPB 185 \(1981\)](#)

- **Proliferation of Green's functions**
three possibilities in two-point gluon sector



- **Symmetry induced identities**
relate B and Q functions; in 2-point sector
[DB, Papavassiliou, PRD 66 \(2002\); DB, Quadri, PRD 88 \(2013\)](#)

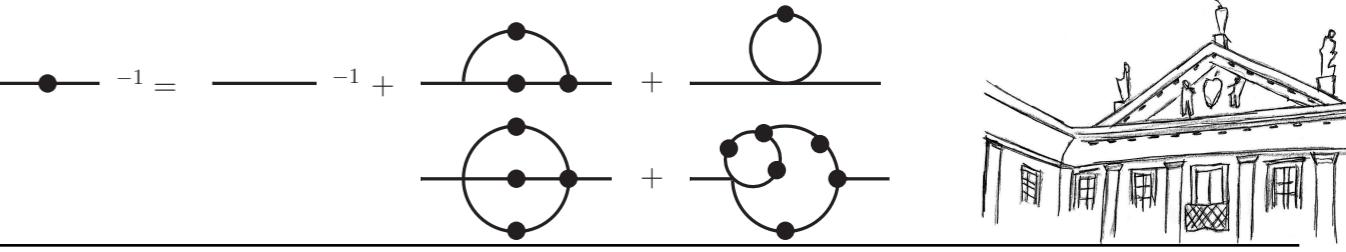
$$[1 + G(q^2)]^{-1} \times \begin{array}{c} q \\ \xrightarrow{\quad} \\ B \end{array} = \begin{array}{c} q \\ \xrightarrow{\quad} \\ Q \end{array}$$

$$\begin{aligned} \Lambda_{\mu\nu}(q) &= \begin{array}{c} \text{Diagram with ghost loop} \\ \mu \dots \nu \end{array} + \begin{array}{c} \text{Diagram with ghost loop and ghost-gluon vertex} \\ \mu \dots \nu \end{array} \\ &= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2} \end{aligned}$$

- **G special PT-BFM function:**
determined by ghost-gluon dynamics
- **Combination $1+G$ appears in all BQIs**
fundamental non-Abelian quantity
- **G is related (Landau gauge) to the ghost dressing:**
use ghost gap equation to constrain $1+G, L$

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

PT-BFM framework



- **Resummed gluon SDE**
expressed in terms of QB self-energy

$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\tilde{\Pi}_{\mu\nu}(q)}{1 + G(q^2)}$$

- **Stronger version of transversality**
allows gauge invariant truncation

$$q^\nu \times \begin{array}{c} \text{Diagram 1: } \alpha, m \\ \text{Diagram 2: } \beta, n \\ \text{Diagram 3: } \gamma, r \end{array} + \begin{array}{c} \text{Diagram 1': } \rho, m' \\ \text{Diagram 2': } \sigma, n' \\ \text{Diagram 3': } \tau, r' \end{array} = 0$$

where the diagrams are:

- Diagram 1: A loop with two external gluons. Top-left: μ, a , top-right: ν, b , bottom-left: β, n , bottom-right: σ, n' . Internal lines: α, m (top), ρ, m' (right), k (left), $k+q$ (bottom).
- Diagram 2: A loop with two external gluons. Top-left: μ, a , top-right: ν, b , bottom-left: n , bottom-right: n' . Internal lines: m (top), m' (right), k (left), $k+q$ (bottom).
- Diagram 3: A loop with two external gluons. Top-left: μ, a , top-right: ν, b , bottom-left: γ, r , bottom-right: τ, r' . Internal lines: α, m (top), ρ, m' (right), $k+\ell$ (left), $k+q$ (bottom).
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- **G function known**
calculable in QCD and on the lattice too

$$\Lambda_{\mu\nu}(q) = \begin{array}{c} \text{Diagram 1: } \mu \\ \text{Diagram 2: } \nu \end{array} + \begin{array}{c} \text{Diagram 1': } \mu \\ \text{Diagram 2': } \nu \end{array}$$

- **Divergence of B legs**
gives rise to Abelian STIs

$$q^\nu \tilde{\Gamma}_{\nu\alpha\beta}(q, r, p) = i\Delta_{\alpha\beta}^{-1}(r) - i\Delta_{\alpha\beta}^{-1}(p)$$

$$q^\nu \tilde{\Gamma}_\nu(q, r, p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

$$q^\nu \tilde{\Gamma}_{\nu\alpha\beta\gamma}^{mnrs}(q, r, p, t) = f^{mse} f^{ern} \Gamma_{\alpha\beta\gamma}(r, p, q+t) + f^{mne} f^{esr} \Gamma_{\beta\gamma\alpha}(p, t, q+r) + f^{mre} f^{ens} \Gamma_{\gamma\alpha\beta}(t, r, q+p)$$

$\Delta(0)$ in the absence of poles



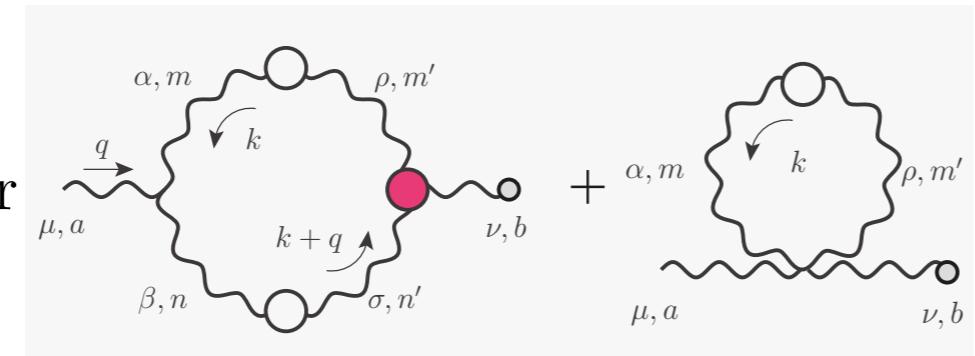
- **Abelian STI for BQ^2 and BQ^3 vertices:**

$$q^\mu \tilde{\Gamma}_{\mu\alpha\beta}(q, r, p) = i\Delta_{\alpha\beta}^{-1}(r) - i\Delta_{\alpha\beta}^{-1}(p)$$

$$\begin{aligned} q^\nu \tilde{\Gamma}_{\nu\alpha\beta\gamma}^{mnrs}(q, r, p, t) &= f^{mse} f^{ern} \Gamma_{\alpha\beta\gamma}(r, p, q + t) \\ &\quad + f^{mne} f^{esr} \Gamma_{\beta\gamma\alpha}(p, t, q + r) \\ &\quad + f^{mre} f^{ens} \Gamma_{\gamma\alpha\beta}(t, r, q + p) \end{aligned}$$

- **Plug into BQ gluon self-energy:**

$$\Delta^{-1}(0) = \lim_{q \rightarrow 0} \text{Tr}$$



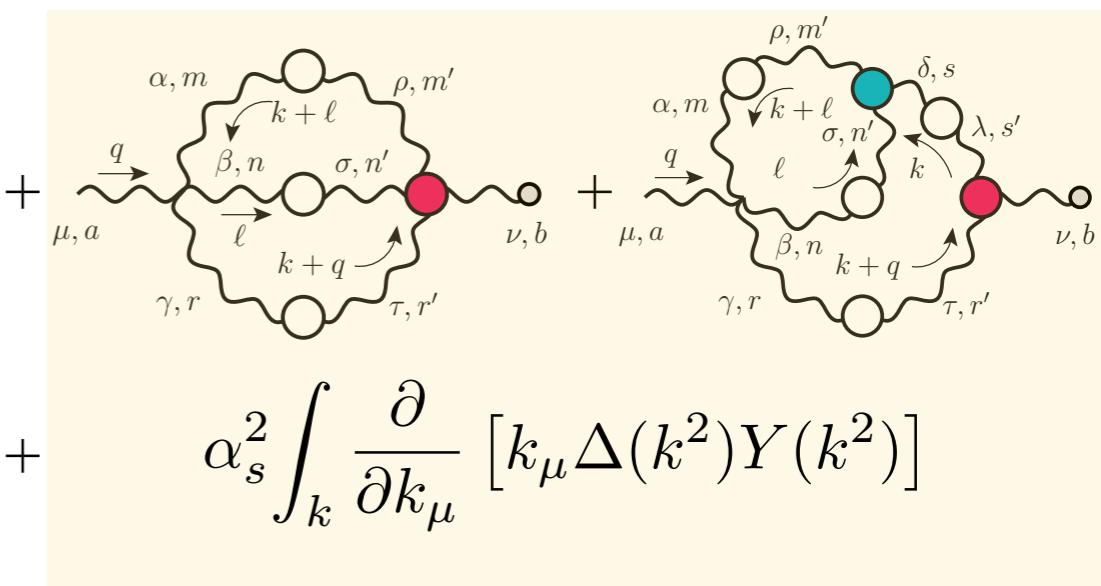
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$$\alpha_s \int_k \frac{\partial}{\partial k_\mu} [k_\mu \Delta(k^2)]$$

- **Taylor expand around $q=0$ assuming no $1/q^2$ poles are present**

$$\tilde{\Gamma}_{\mu\alpha\beta}(0, r, -r) = -i \frac{\partial}{\partial r^\mu} \Delta_{\alpha\beta}^{-1}(r)$$

$$\begin{aligned} \tilde{\Gamma}_{\mu\alpha\beta\gamma}^{mnrs}(0, -r, -p, r + p) &= -f^{mne} f^{esr} \frac{\partial}{\partial r^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p) \\ &\quad + f^{mre} f^{ens} \frac{\partial}{\partial p^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p) \end{aligned}$$



- **Seagull identity $\Delta^{-1}(0) = 0$ valid independently for each diagrams set**

Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

- **No gluon mass scale generation**
need to relax one of the underlying assumptions...

$\Delta(0)$ in the absence of poles



- **Abelian STI for BQ^2 and BQ^3 vertices:**

$$q^\mu \tilde{\Gamma}_{\mu\alpha\beta}(q, r, p) = i\Delta_{\alpha\beta}^{-1}(r) - i\Delta_{\alpha\beta}^{-1}(p)$$

$$\begin{aligned} q^\nu \tilde{\Gamma}_{\nu\alpha\beta\gamma}^{mnrs}(q, r, p, t) &= f \\ &\quad + f \\ &\quad + f \end{aligned}$$

- **Plug into BQ gluon**

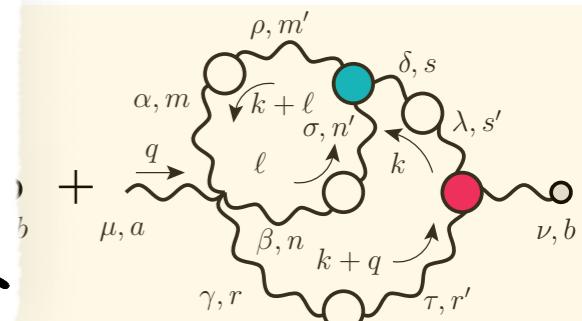
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$$i \frac{\partial}{\partial r^\mu} \Delta_{\alpha\beta}^{-1}(r)$$

$$f^{mne} f^{esr} \frac{\partial}{\partial r^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p)$$

$$ire f^{ens} \frac{\partial}{\partial p^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p)$$



$$\sim \alpha_s \int_k \frac{\partial}{\partial k_\mu} [k_\mu \Delta(k^2)] + \alpha_s^2 \int_k \frac{\partial}{\partial k_\mu} [k_\mu \Delta(k^2) Y(k^2)]$$

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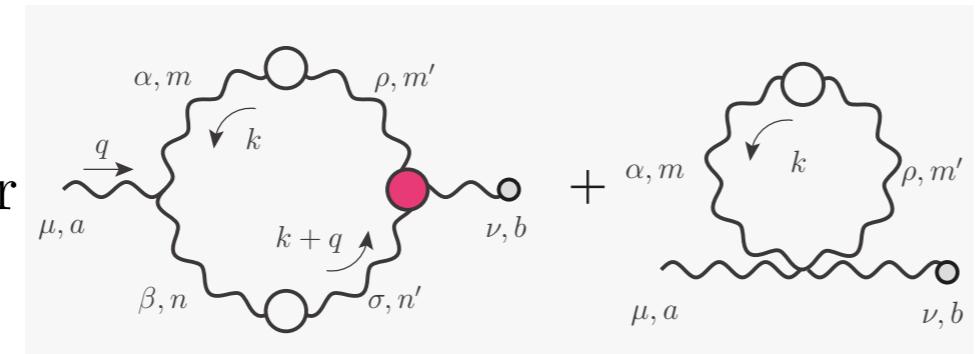
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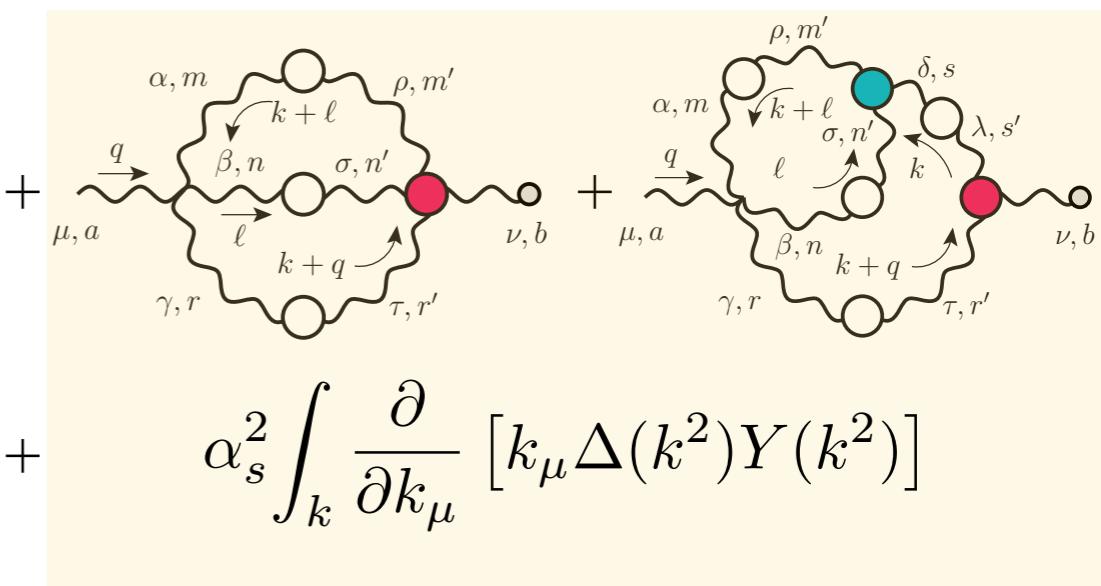
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- **Taylor expand around $q=0$ assuming no $1/q^2$ poles are present**

$$\tilde{\Gamma}_{\mu\alpha\beta}(0, r, -r) = -i \frac{\partial}{\partial r^\mu} \Delta_{\alpha\beta}^{-1}(r)$$

$$\begin{aligned} \tilde{\Gamma}_{\mu\alpha\beta\gamma}^{mnrs}(0, -r, -p, r + p) &= -f^{mne} f^{esr} \frac{\partial}{\partial r^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p) \\ &\quad + f^{mre} f^{ens} \frac{\partial}{\partial p^\mu} \Gamma_{\alpha\beta\gamma}(-r, -p, r + p) \end{aligned}$$



- **Seagull identity $\Delta^{-1}(0) = 0$ valid independently for each diagrams set**

Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

- **No gluon mass scale generation**
need to relax one of the underlying assumptions...

The massive gluon



- **Schwinger mechanism**
propagator Dyson resums to

Schwinger, PR 125 (1962)
Schwinger, PR 128 (1962)

$$\Delta(q^2) = \frac{1}{q^2 [1 + \Pi(q^2)]}$$

- If $\Pi(q^2)$ has a pole at $q^2 = 0$ the gauge boson becomes massive even if it was massless in the absence of interactions

- **Yes, but in QCD?**

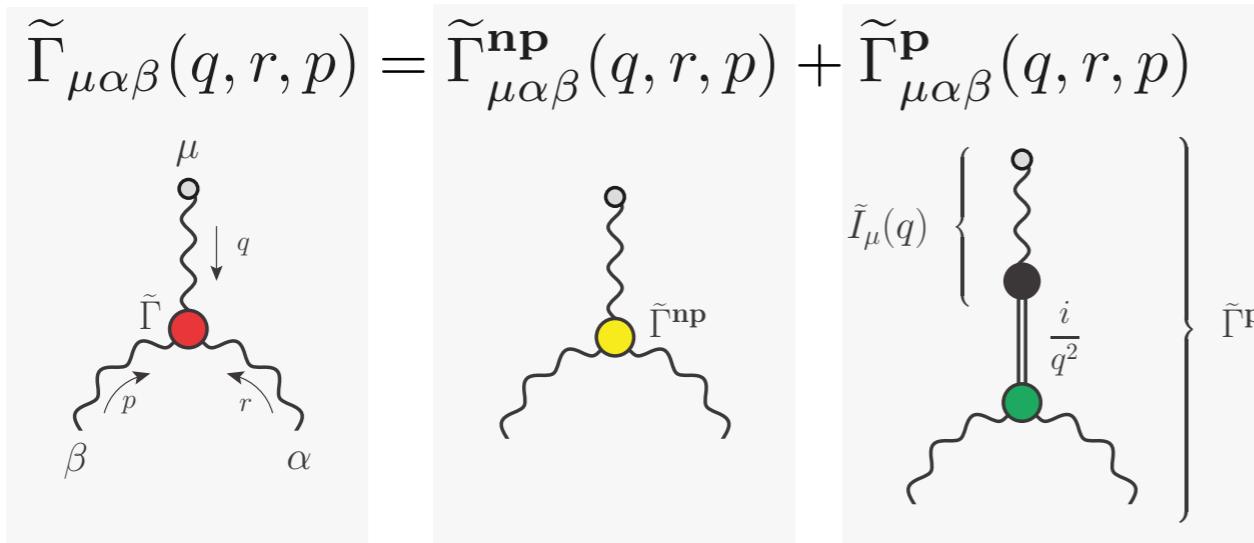
- **PT-BFM theorem:** *in any covariant gauge $\Delta(0) = 0$ in the absence of vertex non-analyticities*

Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

- **Way out:** require **massless, longitudinally coupled** Goldstone like **poles** $1/q^2$

- **Occur dynamically** (even in the absence of canonical **scalar fields**) as **composite (colored) excitations** in a **strongly coupled** gauge theory

Jackiw, Johnson, PRD 8 (1973)
Cornwall, Norton, PRD 8 (1973)
Eichten, Feinberg, PRD 10 (1974)



- **3-gluon vertex**
contains non-analyticities
- **Not kinematic singularities**
composite excitations produced by strong dynamics
- **Do not appear in the S-matrix**
(longitudinally coupled)

Evading the seagull identity



- **Vertex satisfies the same Abelian STI**

$$q^\mu \Gamma_{\mu\alpha\beta}^{\text{np}}(q, r, p) + \tilde{C}_{\alpha\beta}(q, r, p) = i\Delta_{\alpha\beta}^{-1}(r) - i\Delta_{\alpha\beta}^{-1}(p)$$

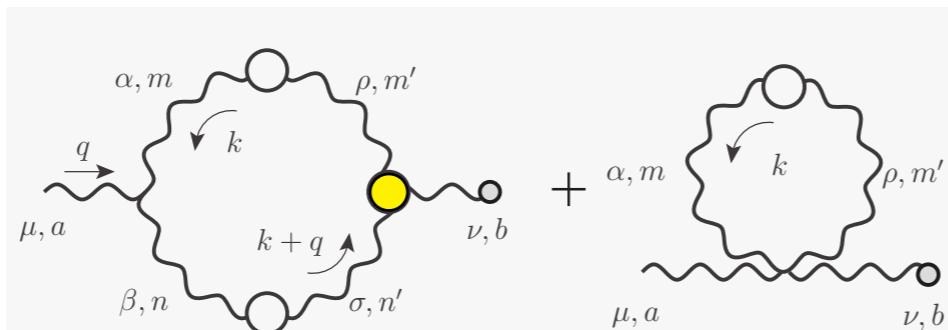
- **Expand around $q=0$**
match orders in q

$$\Gamma_{\mu\alpha\beta}^{\text{np}}(0, r, -r) = -i \frac{\partial}{\partial r^\mu} \Delta_{\alpha\beta}^{-1}(r) - \left\{ \frac{\partial}{\partial q^\mu} \tilde{C}_{\alpha\beta}(q, r, -r - q) \right\}_{q=0}$$

- **Plug into BQ gluon self-energy again**
only no-pole part participates in the seagull identity

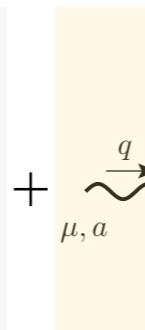
Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

$$\Delta^{-1}(0) = \lim_{q \rightarrow 0} \text{Tr}$$



\sim

$$\alpha_s \int_k k^2 \Delta^2(k^2) \tilde{C}'_1(k^2)$$



$-$

$$\int_k k^2 \Delta^2(k^2) Y(k^2) \tilde{C}'_1(k^2)$$

- If $\tilde{C}'_1 \neq 0$ a gluon mass scale can be generated



Gluon mass

- **Physically motivated parametrization**

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

- **Insert into STI**

$$\begin{aligned} q^\mu \tilde{\Gamma}_{\mu\alpha\beta}^{\text{np}}(q, r, p) &= p^2 J(p^2) P_{\alpha\beta}(p) - r^2 J(r^2) P_{\alpha\beta}(r) \\ + &= + \\ \tilde{C}_{\alpha\beta}(q, r, p) &= m^2(p^2) P_{\alpha\beta}(p) - m^2(r^2) P_{\alpha\beta}(r) \end{aligned}$$

- **Kinetic term**
associated with np part of the STI
- **Mass term**
associated with massless poles amplitude

- **Focus on the $g_{\alpha\beta}$ part**

take limit as $q \rightarrow 0$

$$\tilde{C}'_1(r^2) = \frac{dm^2(r^2)}{dr^2} \quad \Rightarrow \quad m^2(x) = \Delta^{-1}(0) + \int_0^x dy \tilde{C}'_1(y)$$

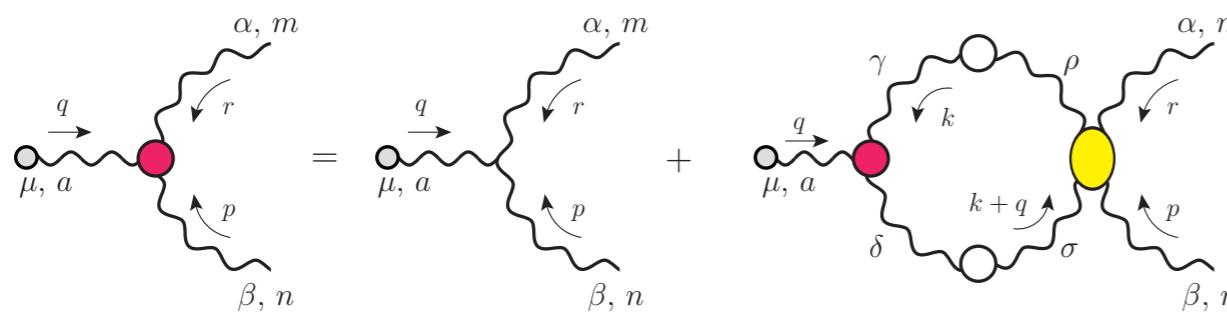
- **Not yet a running mass**
must be vanishing in the UV

$$m^2(\infty) = 0 \quad \Rightarrow \quad m^2(x) = - \int_x^\infty dy \tilde{C}'_1(y)$$

BSE for massless poles

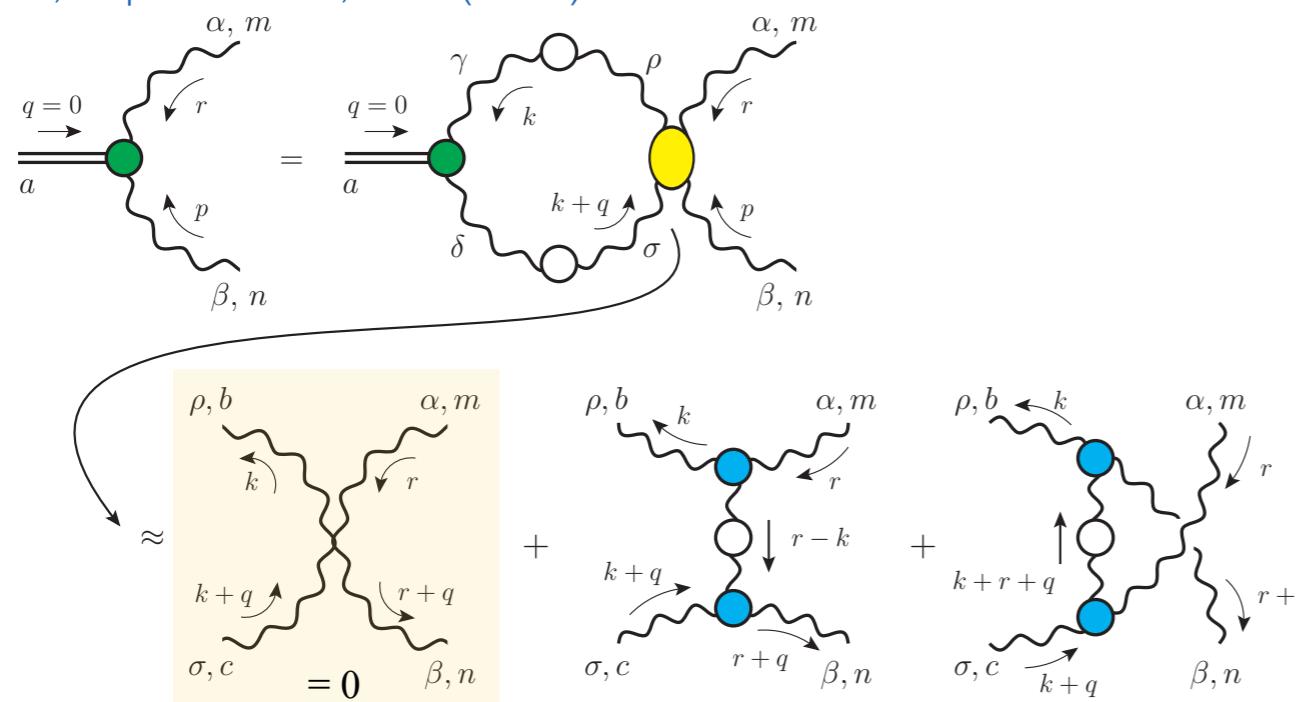


- Consider BSE for the full vertex



- Replace vertex: $\Gamma \rightarrow \Gamma^{\text{np}} + \Gamma^{\text{p}}$
expand and equate terms linear in q

DB, Papavassiliou, PRD (2018)



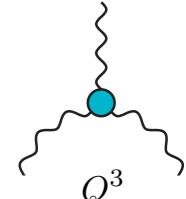
- Homogeneous BSE
eigenvalue proportional to the coupling

$$\tilde{C}'_1(x) = \alpha_s \int_0^\pi d\theta \int_0^\infty dy \mathcal{K}(x, y, \theta) \tilde{C}'_1(y)$$

- Four gluon kernel
use one-loop dressed approximation

- (quantum) Three gluon vertex

$$\Gamma_{\mu\alpha\beta}(k_1, k_2, k_3) = f(k_2) \Gamma_{\mu\alpha\beta}^{(0)}(k_1, k_2, k_3)$$



- Ensures RGI-ness of the BSE
and self consistency

- Vertex form factor
motivated by continuum/lattice studies

Cucchieri, Maas, Mendes, PRD 74 (2006)

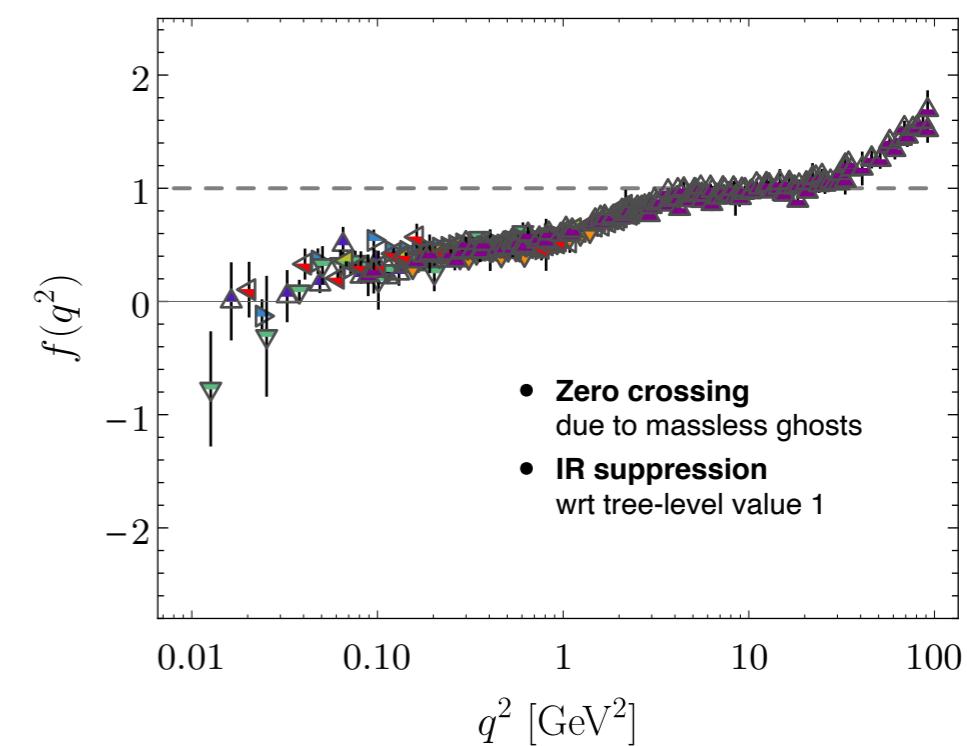
Pelaez, Tissier, Wschebor, PRD 88 (2013)

Aguilar, DB, Ibañez, Papavassiliou, PRD 89 (2014)

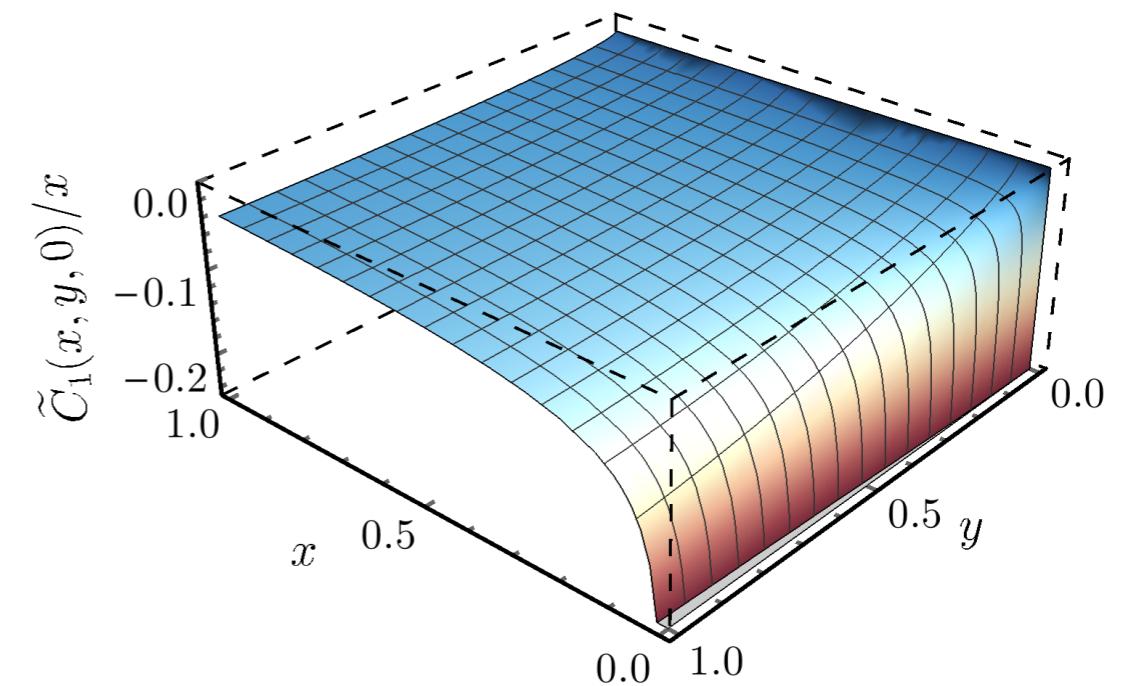
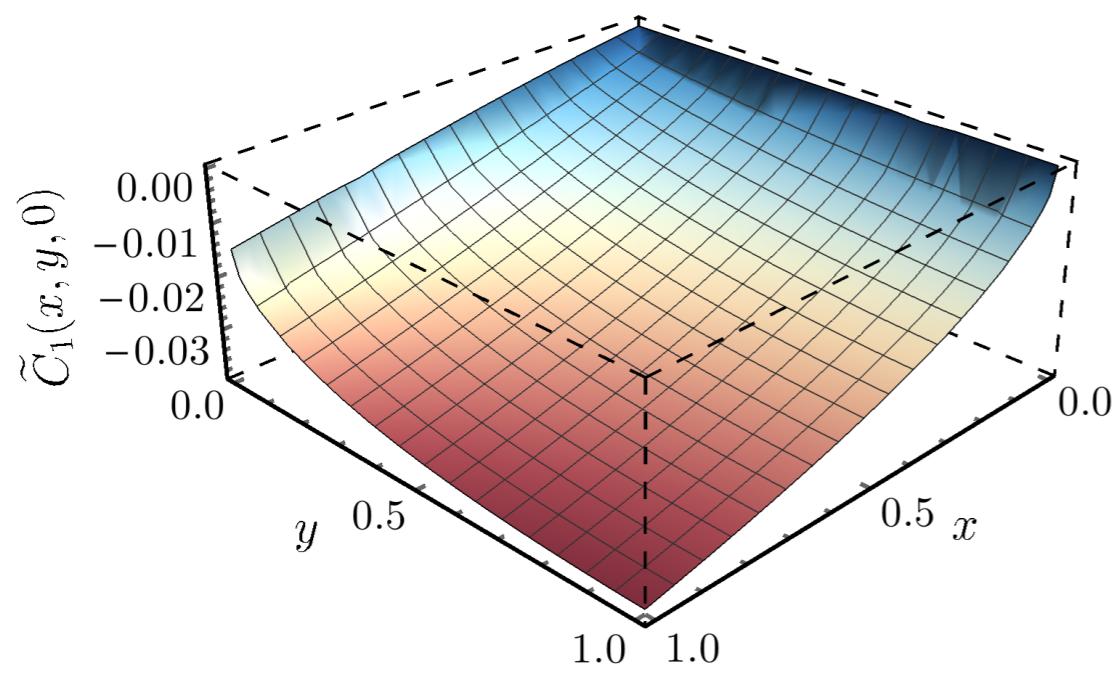
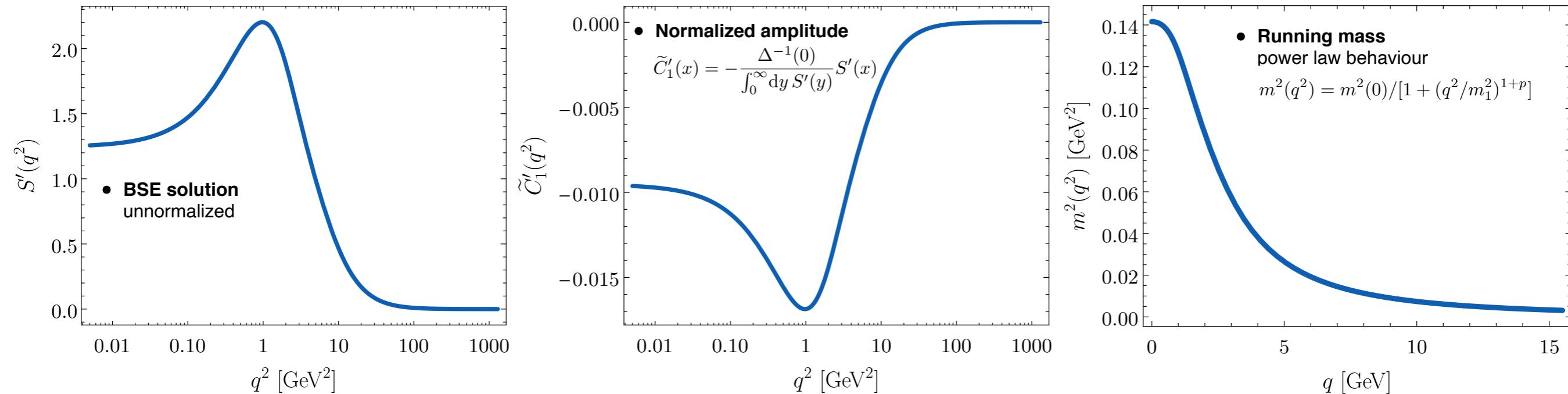
Eichmann, Williams, Alkofer, Vujinovic, PRD 89 (2014)

Blum, Huber, Mitter, von Smekal PRD 89 (2014)

Athenodorou, DB, Boucaud, De Soto, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, PLB 761 (2016)



Poles BS amplitude



Coupling the gluon DSE



- **Gluon DSE at $q = 0$**
yields quadratic equation in the coupling

DB, Papavassiliou, PRD 97 (2018)

- **Consistency condition**
for a given MOM subtraction point μ

$$\alpha_s^{\text{SDE}} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \alpha_s^{\text{BSE}}$$

- **Vertex form factor crucial**
 $f=1$ implies $\alpha_s^{\text{SDE}} = 0.42$ and $\alpha_s^{\text{BSE}} = 0.27$

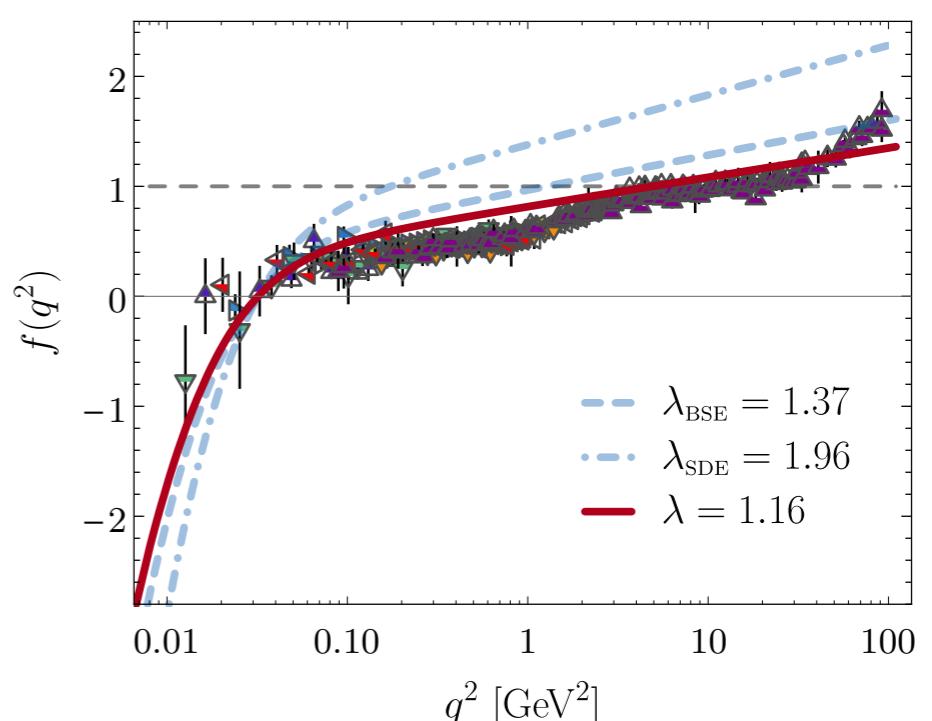
- Lattice data fit

- **Two-loop dressed diagrams fundamental**
one-loop dressed alone requires negative coupling

$$\begin{aligned} A &= \frac{3C_A^2}{32\pi^3} F(0) \int_0^\infty dy y^2 \Delta^2(y) Y(y) S'(y) \\ B &= -\frac{3C_A}{8\pi} F(0) \int_0^\infty dy y^2 \Delta^2(y) S'(y) \\ C &= - \int_0^\infty dy S'(y) \end{aligned}$$

- **Start with** $\lambda_0 = 1$
evaluate A_0, B_0, C_0 and α_0
 - **Rescale** f
track rescaling through BSE/SDE

$$C_0\lambda^3 + \alpha_0 B_0\lambda + \alpha_0^2 A_0 = 0$$
 - **Solved @** $\mu = 4.3 \text{ GeV}$ by
 $\lambda = 1.16 \quad \alpha_s^{\text{BSE}} = \alpha_s^{\text{SDE}} = 0.45$
 - **Expected value:** $\alpha_s = 0.32$
obtained if
 $\lambda_{\text{BSE}} = 1.36, \lambda_{\text{SDE}} = 1.97$



And ghosts?

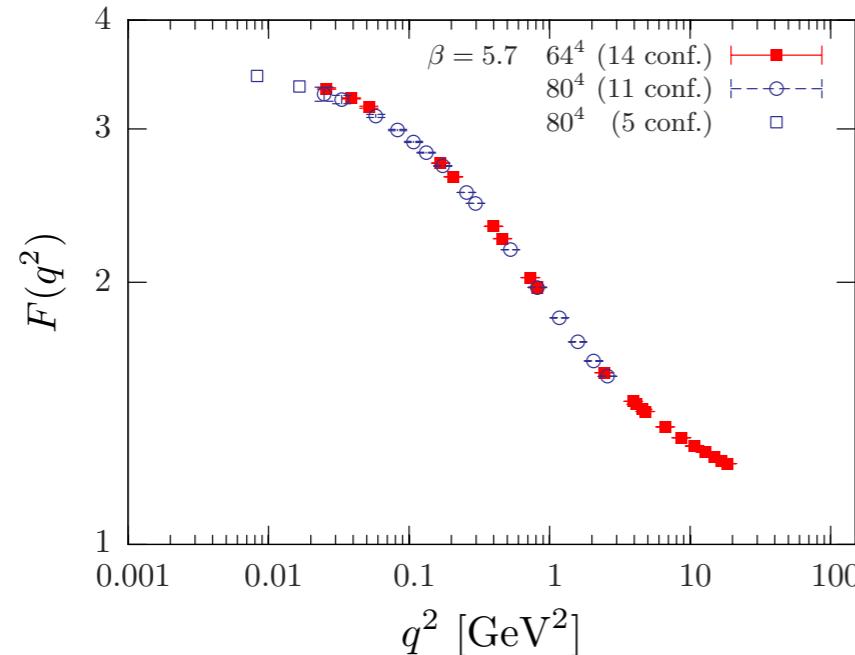
$$\text{---} \bullet \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \bullet \text{---}$$



- **Ghost remains massless** even at non-perturbative level
- **Lattice results** confirmed by continuous studies

- **Landau gauge (quenched)**

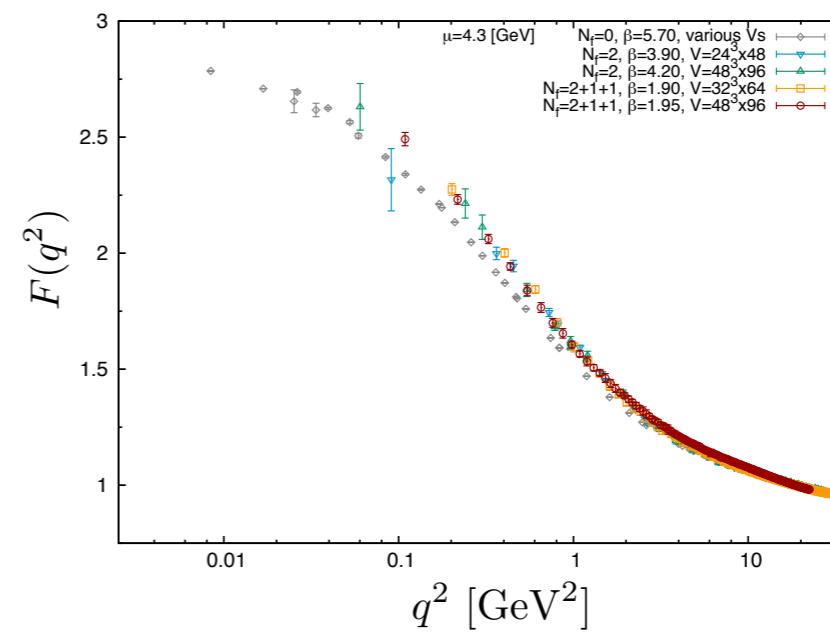
Bogolubsky, Ilgenfritz, Muller-Preussker,
Sternbeck, PLB 676 (2009)



- **Ghost dressing function saturates** $F(q^2) = q^2 D(q^2)$
- **IR propagator diverges** $D(q^2) \sim c/q^2$

- **Landau gauge (unquenched)**

Ayala, Bashir, DB, Cristoforetti,
Rodriguez-Quintero, PRD 86 (2012)



And ghosts?

$$\text{---} \bullet \text{---}^{-1} = \text{---}^{-1} + \text{---} \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \end{array}$$



- **Ghost masslessness**
phenomenologically very important

Aguilar, DB, Ibañez, Papavassiliou, PRD 89 (2014)

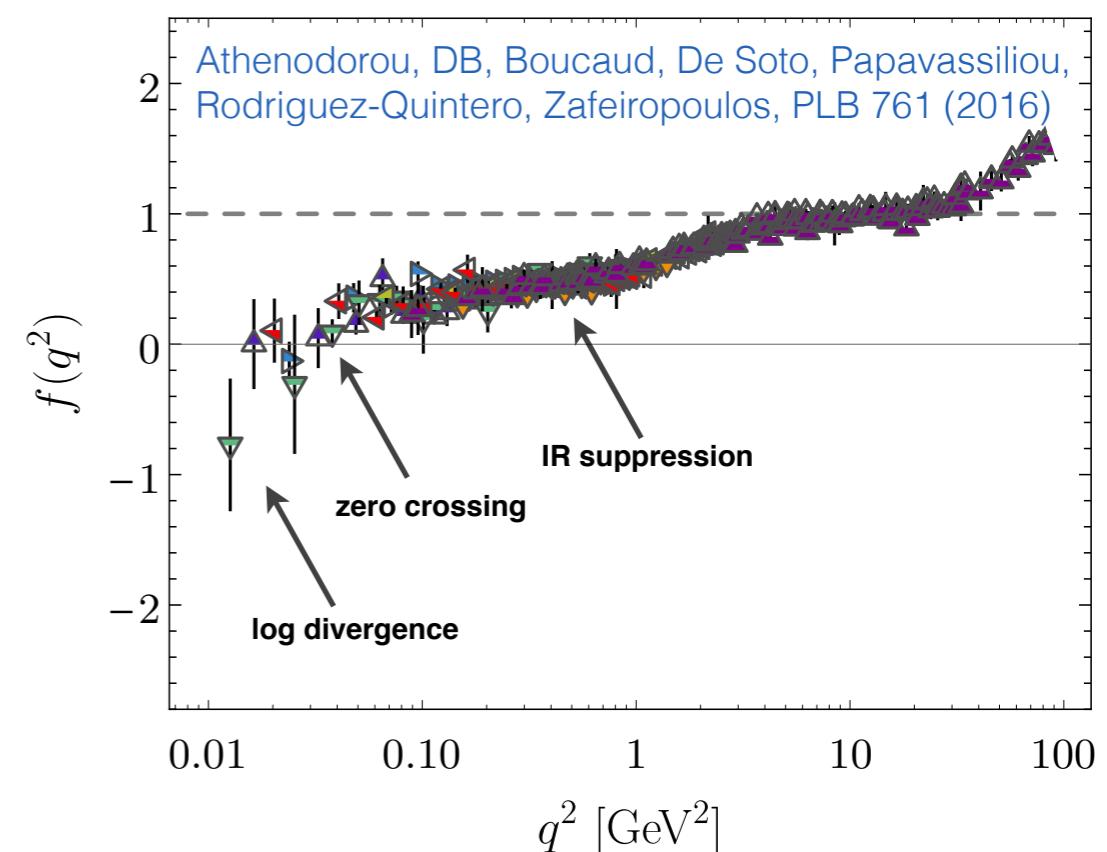
- IR behavior of derivative of the gluon propagator entirely determined by ghost loops

$$\partial_{q^2} \Delta^{-1}(q^2) \sim \log \frac{q^2}{\mu^2} \xrightarrow[q^2 \rightarrow 0]{} -\infty$$

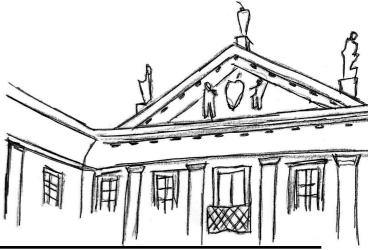
- **Related to 3-gluon vertex form factor**
proportional to tree-level tensor structure

Aguilar, DB, Ibañez, Papavassiliou, PRD 89 (2014)

- **Suppression wrt tree-level value**
vertex must drop below 1
 - **Zero crossing**
in the (deep?) IR followed by
 - **Log divergence**
as $q^2 \rightarrow 0$

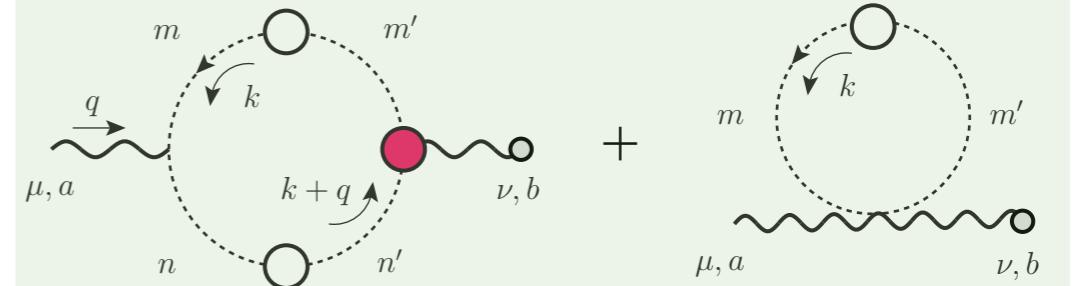


Ghost contributions to poles' BSE

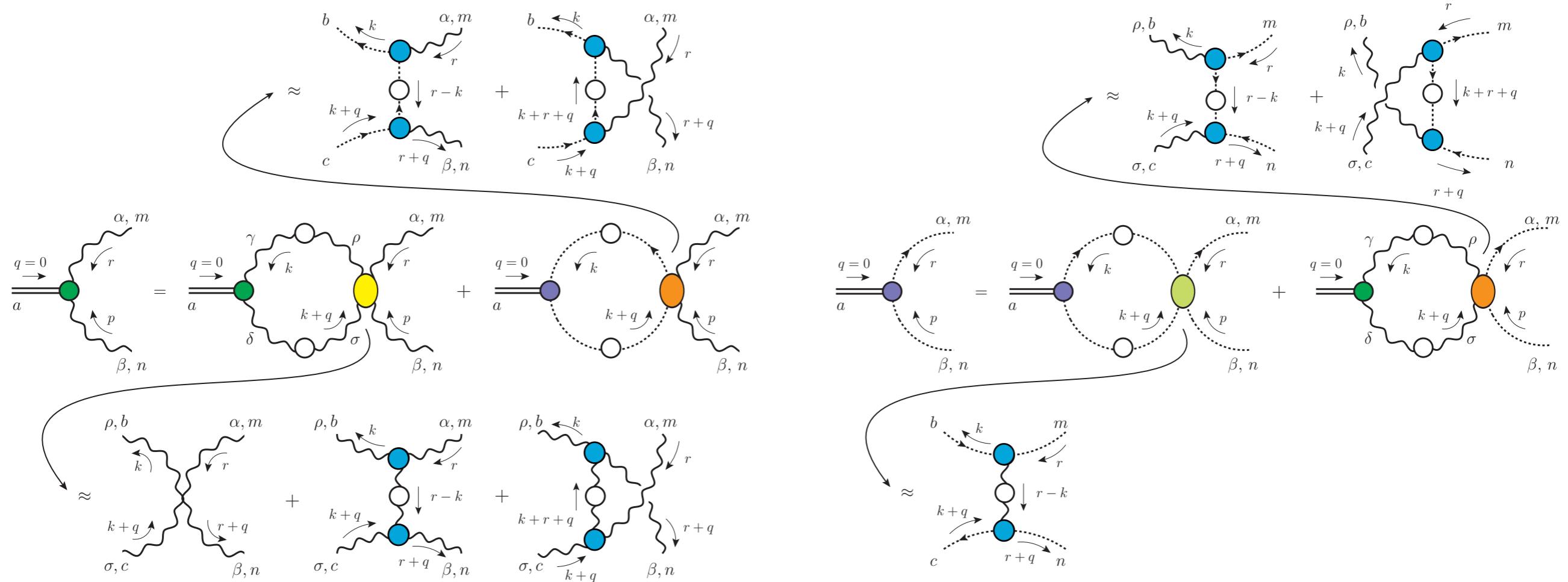


- Add ghost terms
with massless pole in $B\bar{c}c$ vertex
[Aguilar, DB, Figueiredo, Papavassiliou, EPJC 78 \(2018\)](#)

- Gluon/ghost BSEs
coupled in a system

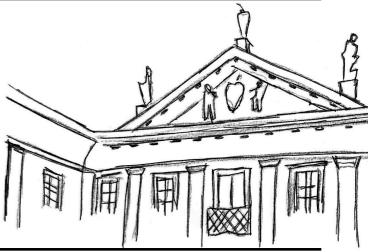


$$\alpha_s \int_k k^2 D^2(k^2) \tilde{C}'_{gh}(k^2)$$

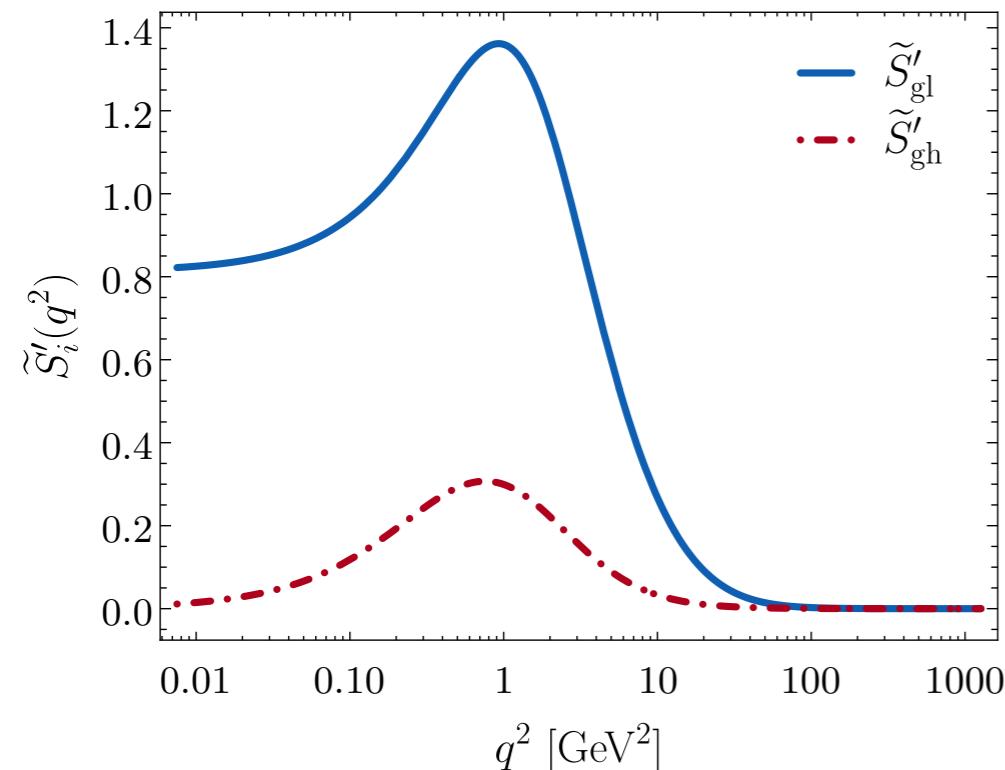


Ghost suppression

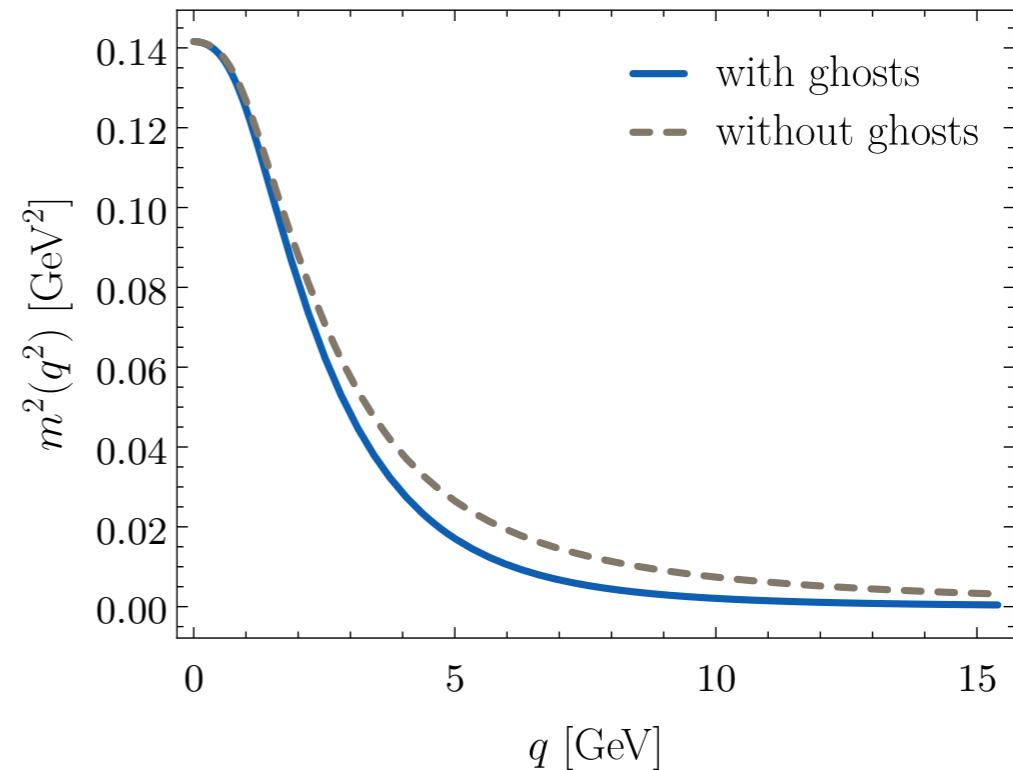
$$\bullet - \bullet = \text{---}^{-1} + \text{---}$$



- Ghost pole amplitude suppressed
~ 5 times smaller than gluon at peak



- Results almost invariate
qualitative/quantitative agreement





QCD Effective charge

- **PT-BFM vertices/propagators**

new RG invariant combination appears

$$\hat{d}(k^2) = \alpha(\mu^2) \hat{\Delta}(k^2; \mu^2) \quad \mathcal{I}(k^2) = k^2 \hat{d}(k^2)$$

- **Remarkable feature of QCD:**

$\hat{d}(k^2)$ saturates in the IR

DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\hat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

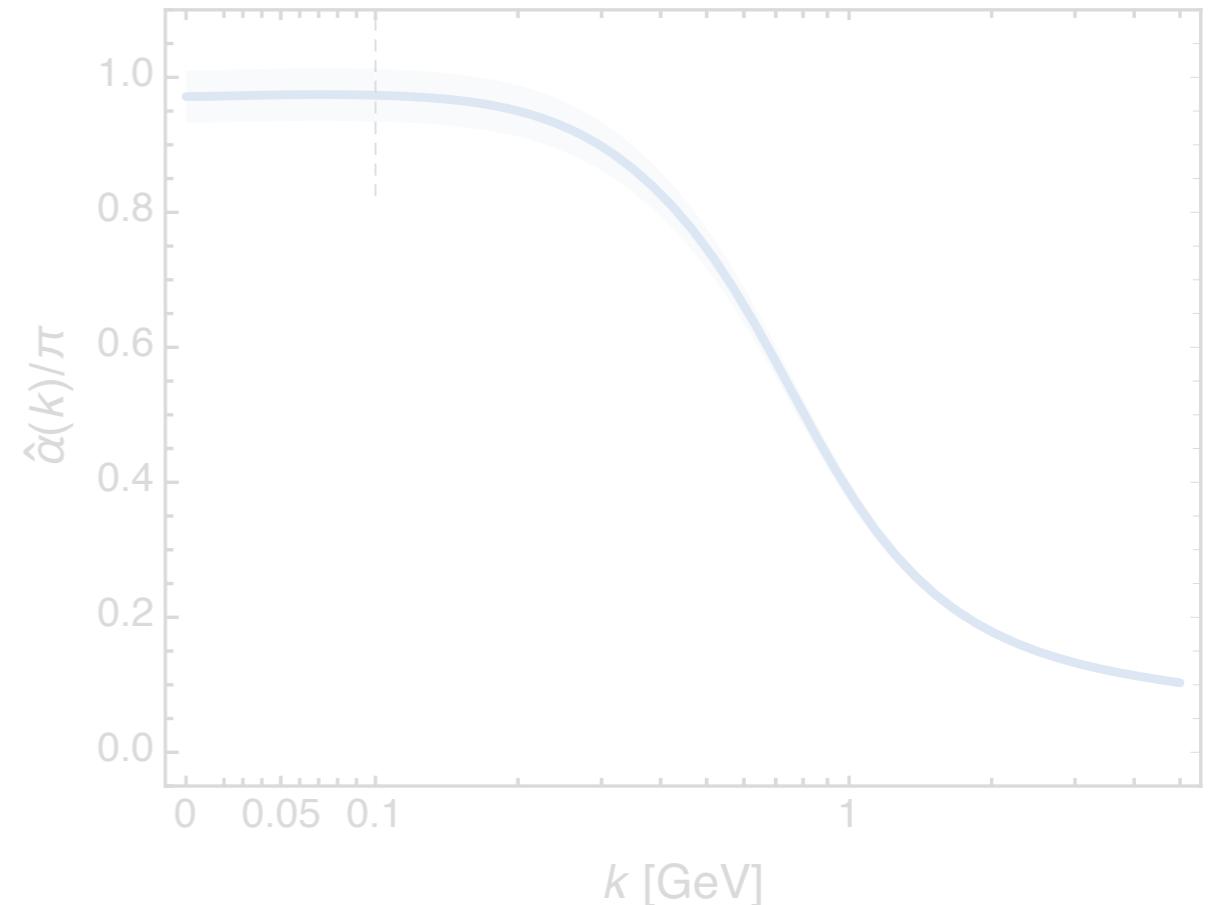
- **Define the RG invariant function**

$$\mathcal{D}(k^2) = \frac{\Delta(k^2; \mu^2)}{\Delta(0; \mu^2) m_0^2}$$

- **Extract (process independent) coupling**
using the quark gap equation

DB, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, PRD96 (2017)

$$\hat{\alpha}(k^2) = \frac{\hat{d}(k^2)}{\mathcal{D}(k^2)} \xrightarrow{k^2 \gg m_0^2} \mathcal{I}(k^2)$$



- **Parameter free**
completely determined from 2-point sector
- **No Landau pole**
physical coupling showing an IR fixed point
- **Smoothly connects IR and UV domains**
no need for matching procedures
- **Essentially non-perturbative result**
continuum/lattice results plus setting of single mass scale
- **Ghost gluon dynamics critical**
produces enhancement at intermediate momenta

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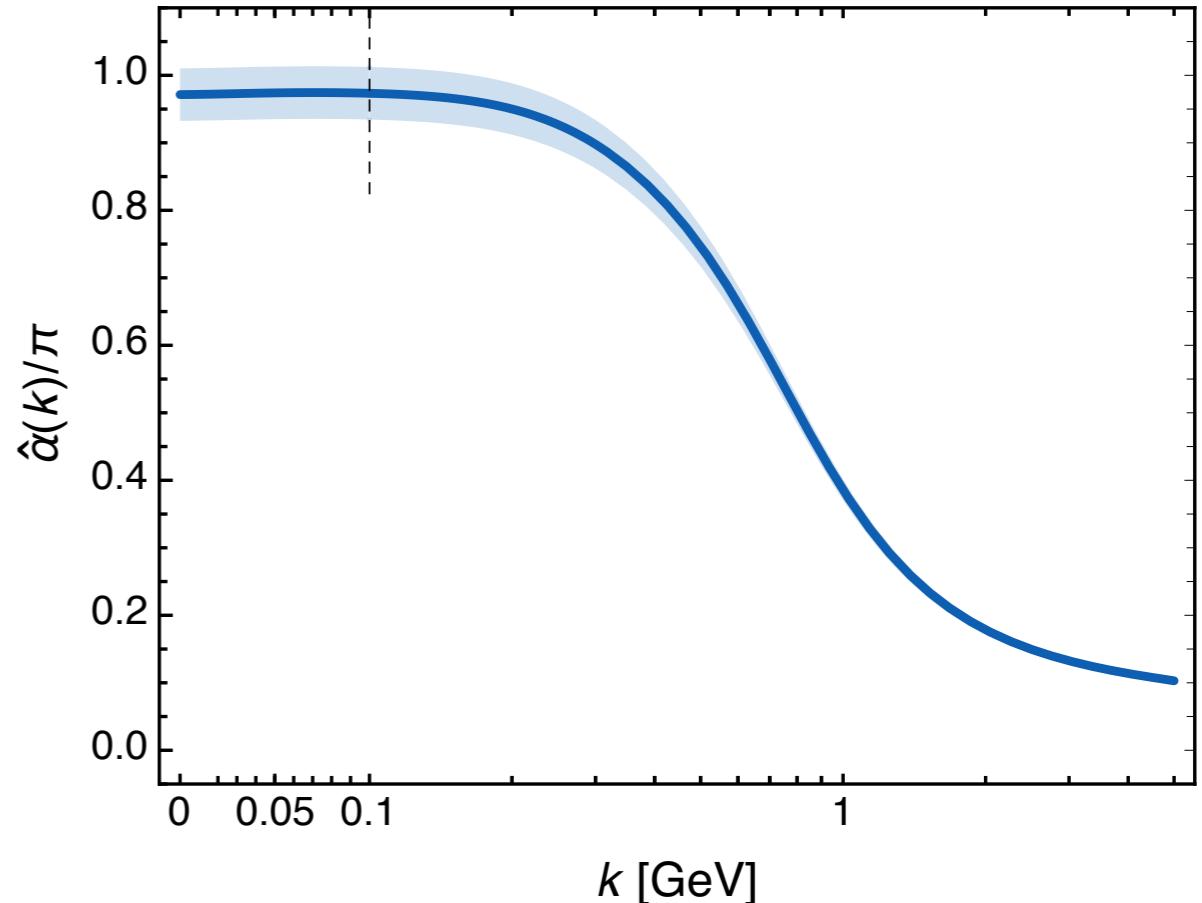
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QCD Effective charge

- **Process dependent effective charges**

fixed by the leading-order term in the expansion of a given observable

Grunberg, PRD 29 (1984)

- **Bjorken sum rule**

defines such a charge

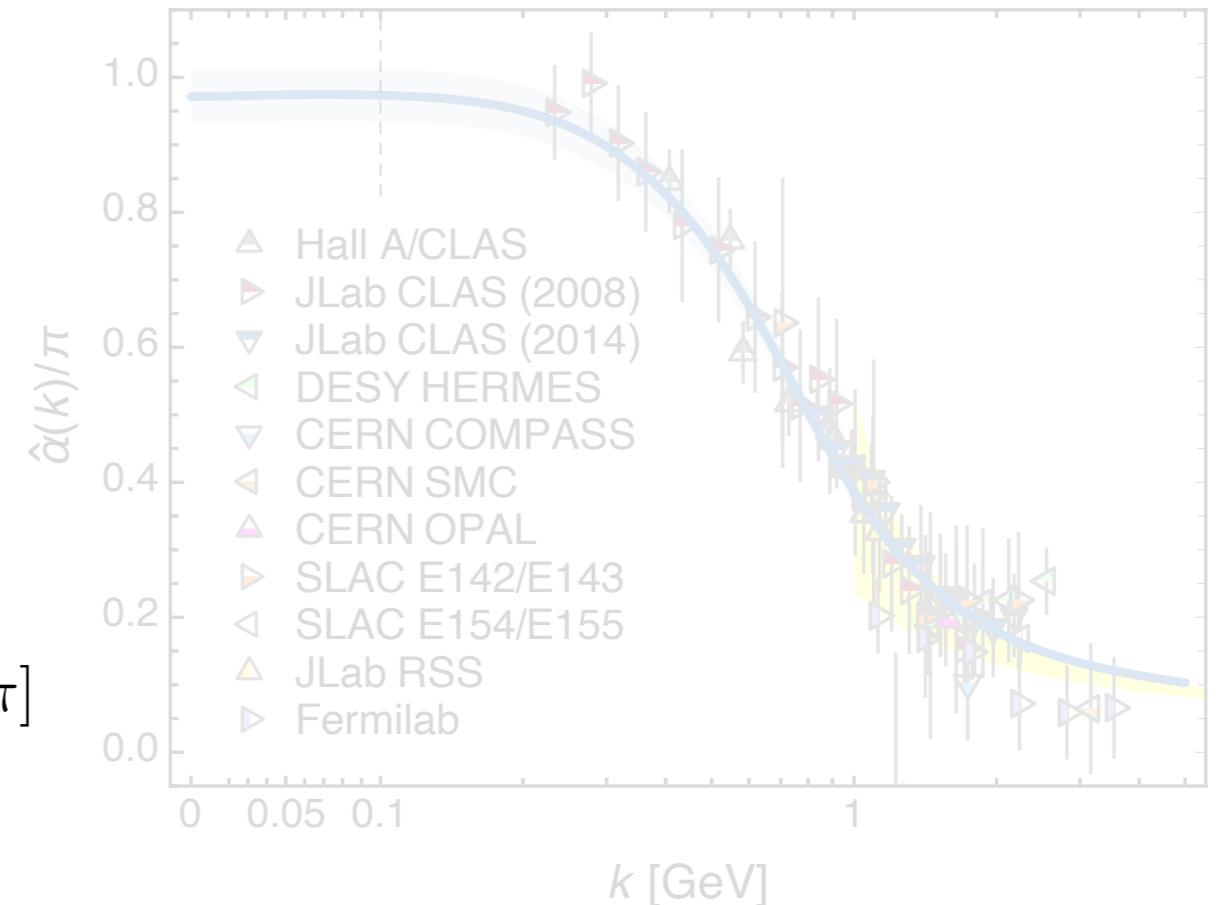
Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 dx [g_1^p(x, k^2) - g_1^n(x, k^2)] = \frac{g_A}{6} [1 - \alpha_{g_1}(k^2)/\pi]$$

- $g_1^{p,n}$ **spin dependent p/n structure functions**
extracted from measurements using unpolarized targets
- g^A **nucleon flavour-singlet axial charge**

- **Many merits**

- **Existence of data**
for a wide momentum range
- **Tight sum rules constraints on the integral**
at IR and UV extremes
- **Isospin non-singlet**
suppress contributions from hard-to-compute
processes



- **Equivalence in the perturbative domain**
reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{MS}}(k^2)[1 + 1.14\alpha_{\overline{MS}}(k^2) + \dots]$$

$$\hat{\alpha}_{PI}(k^2) = \alpha_{\overline{MS}}(k^2)[1 + 1.09\alpha_{\overline{MS}}(k^2) + \dots]$$

- **Equivalence in the non-perturbative domain**
highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
model for
Deur, Brodsky, de Teramond, PPNP 90 (2016)



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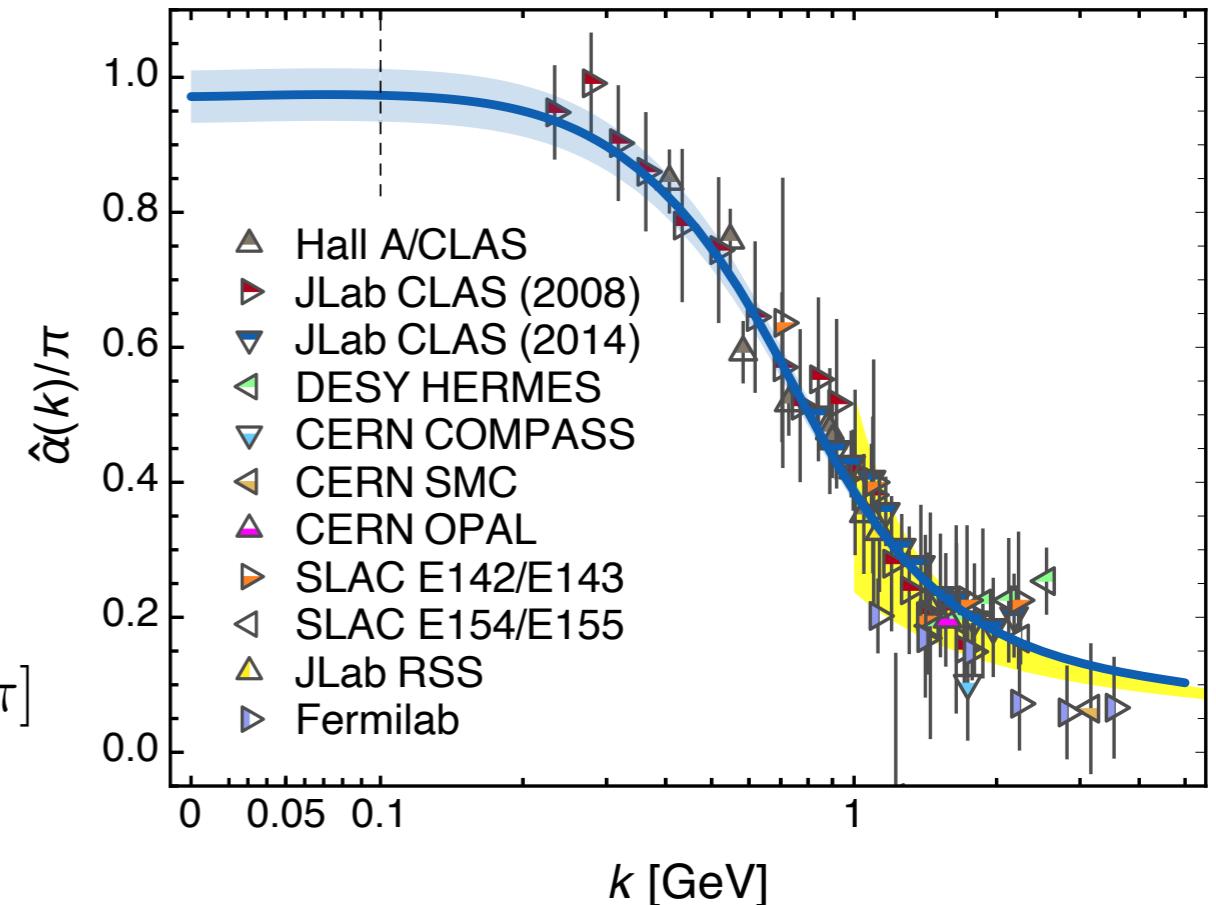
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Deur, Brodsky, de Teramond, PPNP 90 (2016)



EMERGENT PHENOMENA IN STRONG DYNAMICS 1

DANIELE BINOSI
ECT* - FONDAZIONE BRUNO KESSLER

Baryons 21 Sevilla
OCTOBER 18 - 22

