

# EMERGENT PHENOMENA

# IN STRONG DYNAMICS

# 3

DANIELE BINOSI

ECT\* - FONDAZIONE BRUNO KESSLER

**Baryons 21 Sevilla**

OCTOBER 18 - 22



**ECT\***  
EUROPEAN CENTRE  
FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS  
FONDAZIONE BRUNO KESSLER

**FBK**  
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# SPM AND SMOOTHING

$$D = \{(x_i, y_i = f(x_i)), i = 1, \dots, N\}$$

$$C_N(x) = \frac{y_1}{1+} \frac{a_1(x - x_1)}{1+} \frac{a_2(x - x_2)}{1+} \dots \frac{a_{N-1}(x - x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

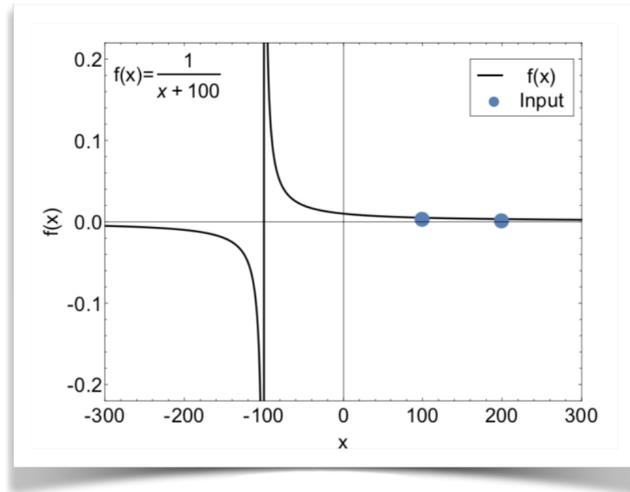
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elementary (functions) examples



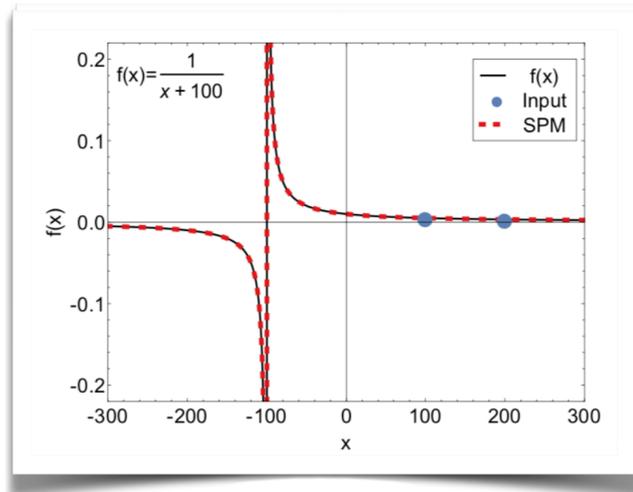
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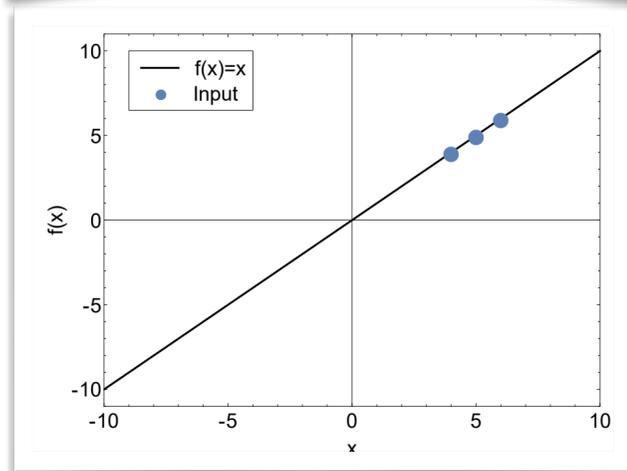
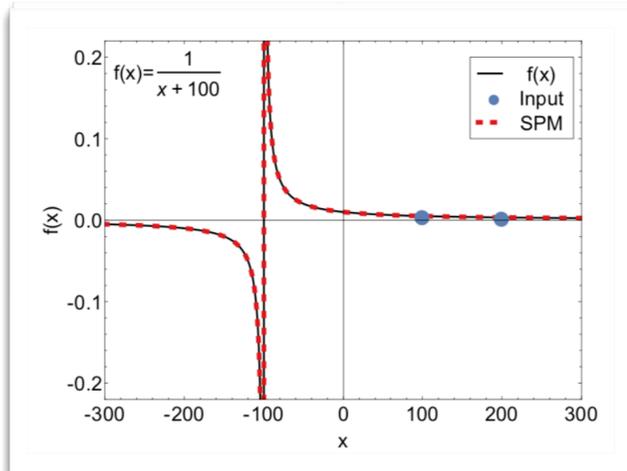
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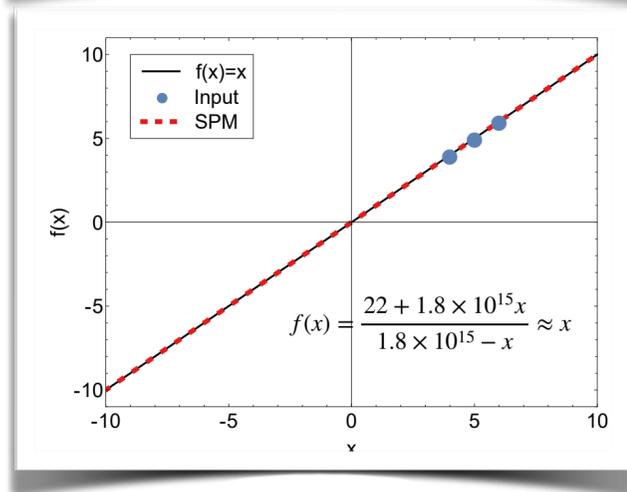
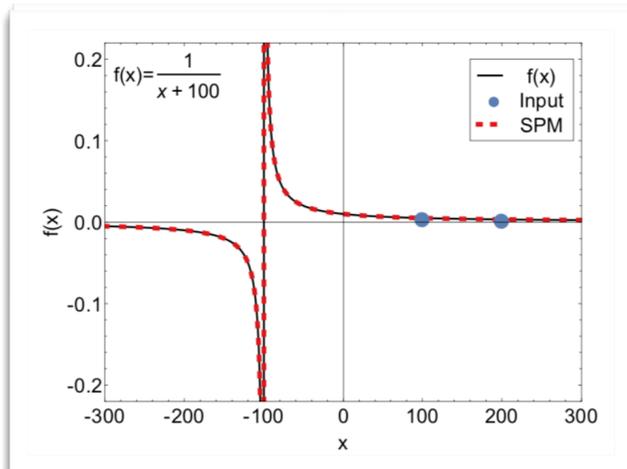
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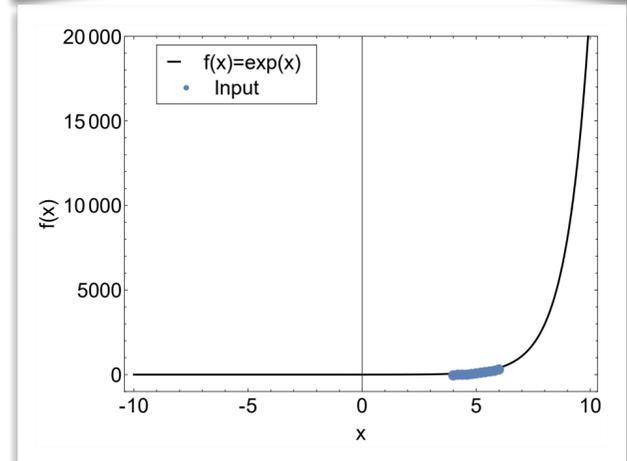
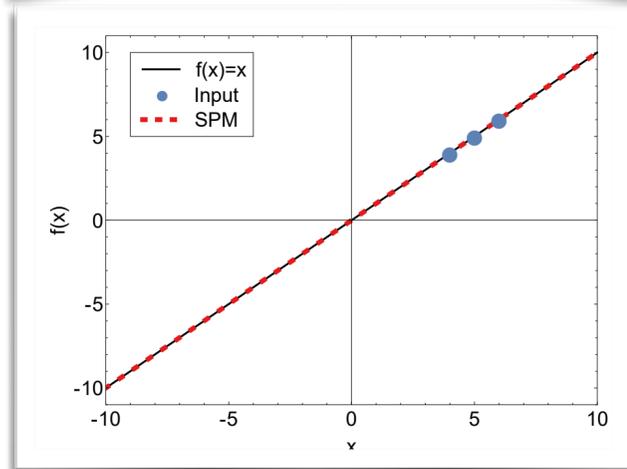
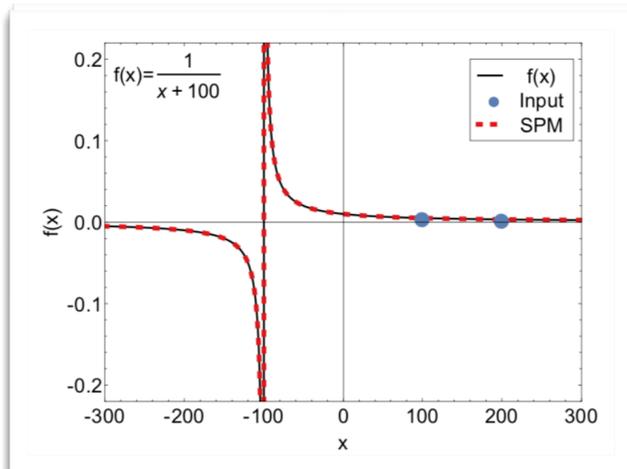
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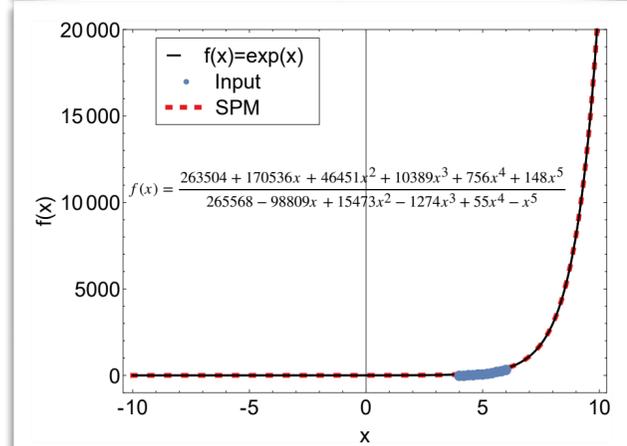
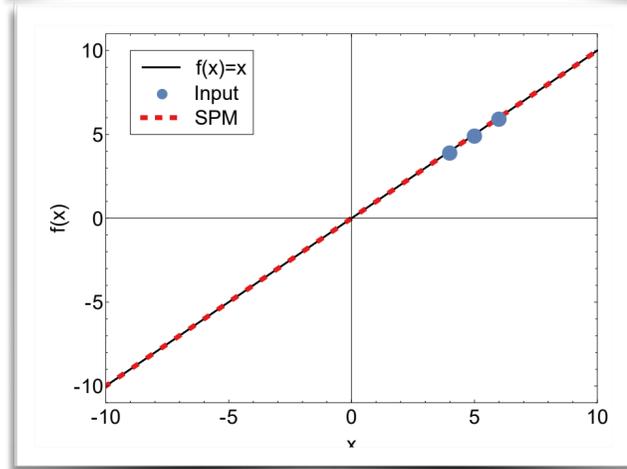
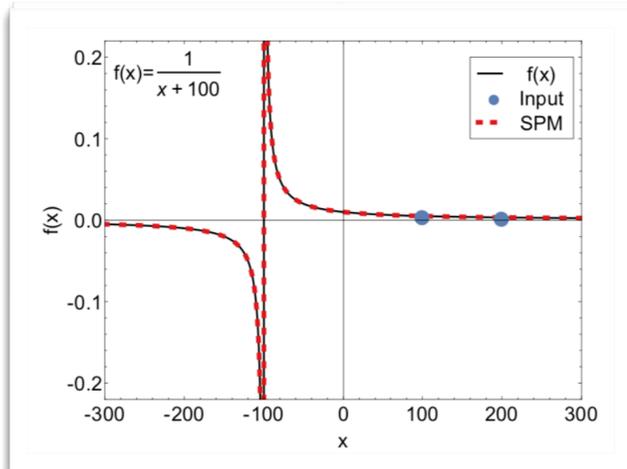
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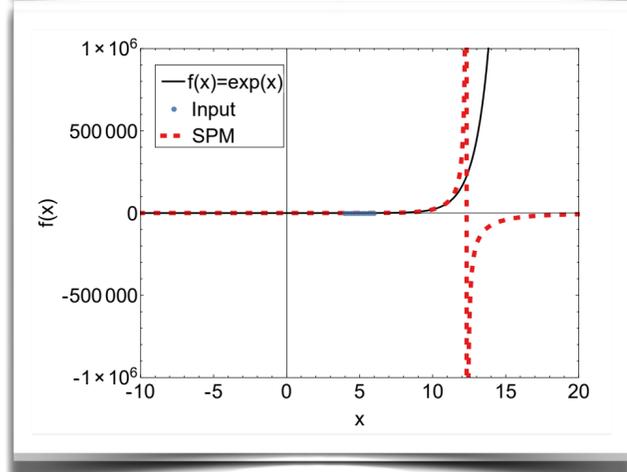
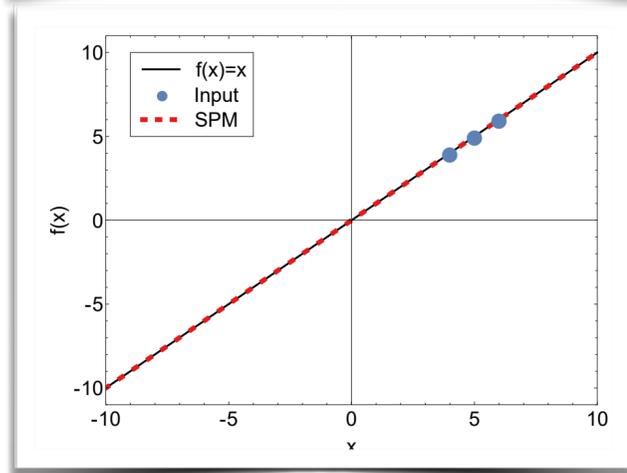
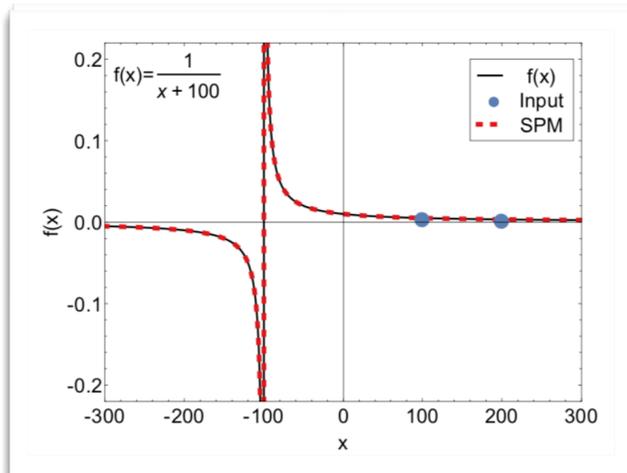
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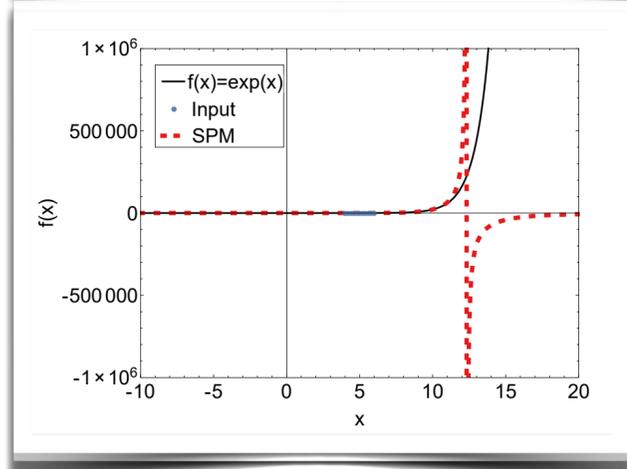
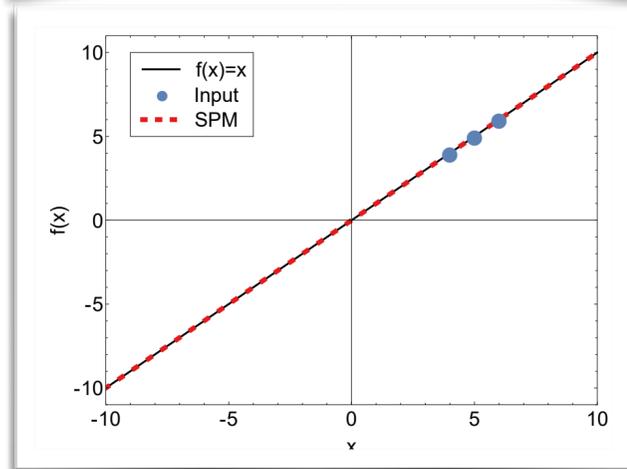
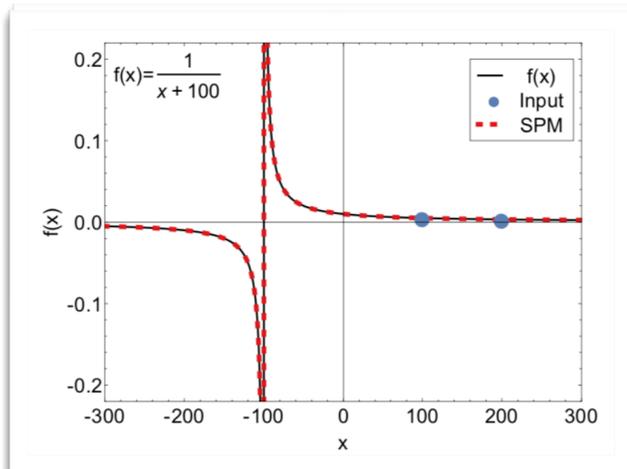
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## LARGE DATASETS

randomly choose  $4 < M \lesssim N/2$  points  
reduce (binomial) number of interpolators  
introducing **physical constraints**  
(absence of poles)

Chen et al., PRD 99 (2019)

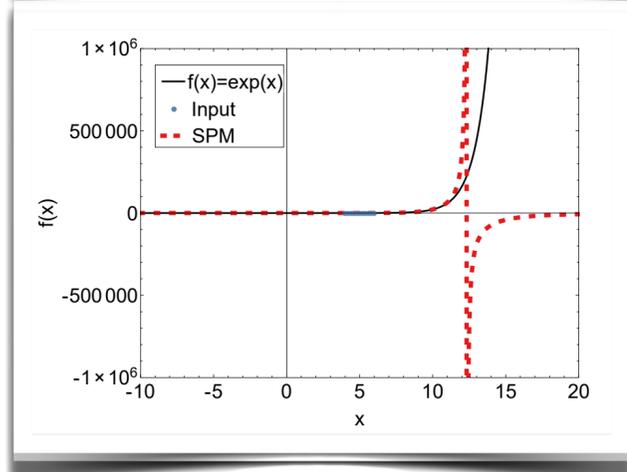
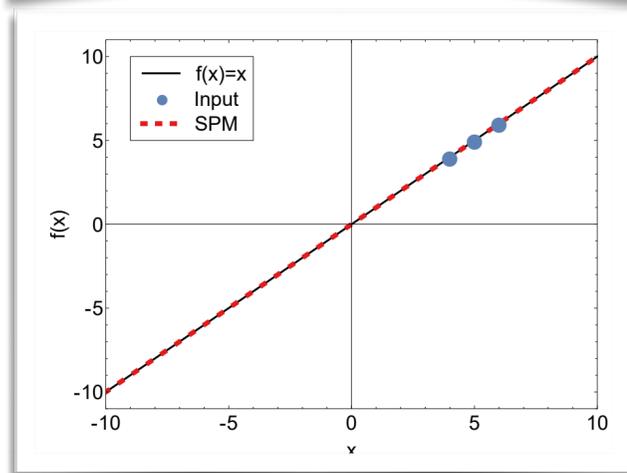
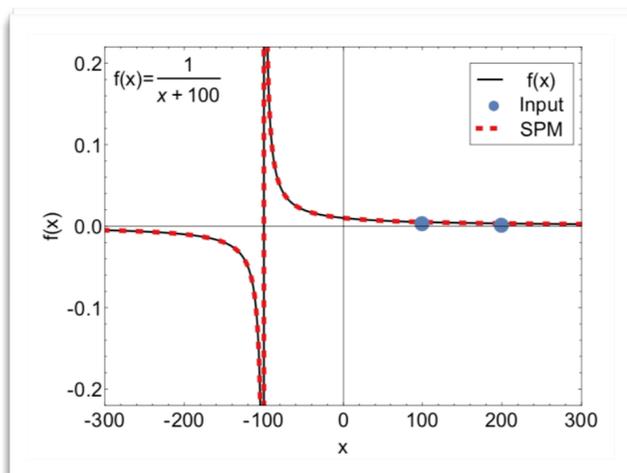
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requires **smoothing** with **roughness penalty**:

seek  $g \in \mathcal{S}$  minimising

$$P(g, \lambda) = \lambda \sum_{i=1}^{\ell} [y_i - g(x_i)]^2 + (1 - \lambda) \int_a^b dx [g''(x)]^2$$

smoothing par.                      data fidelity                      roughness penalty

**THEOREM:**  $g$  is the *natural spline* interpolant  
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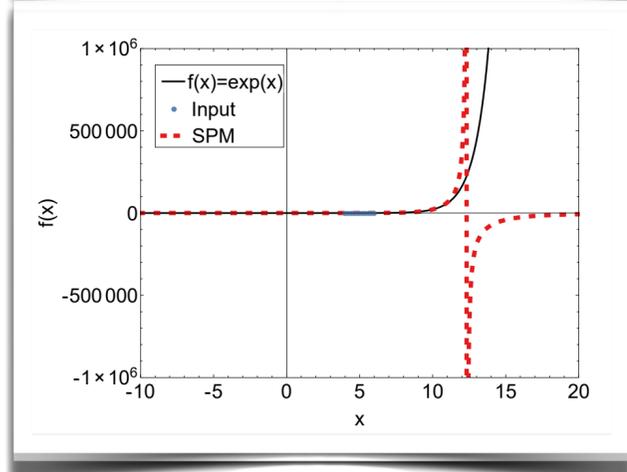
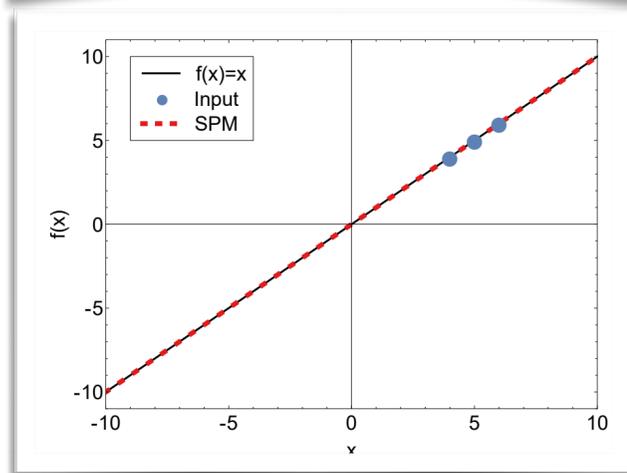
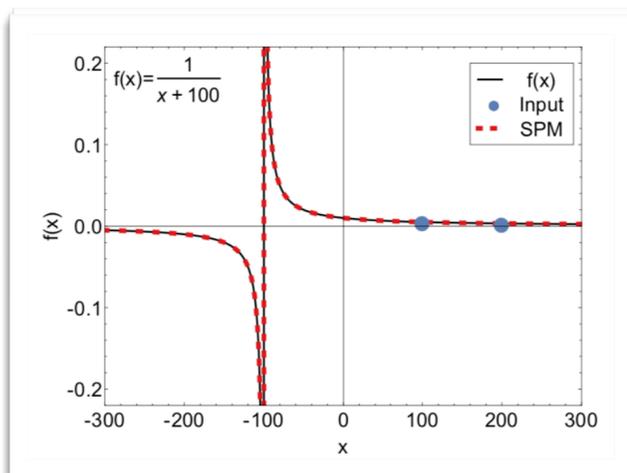
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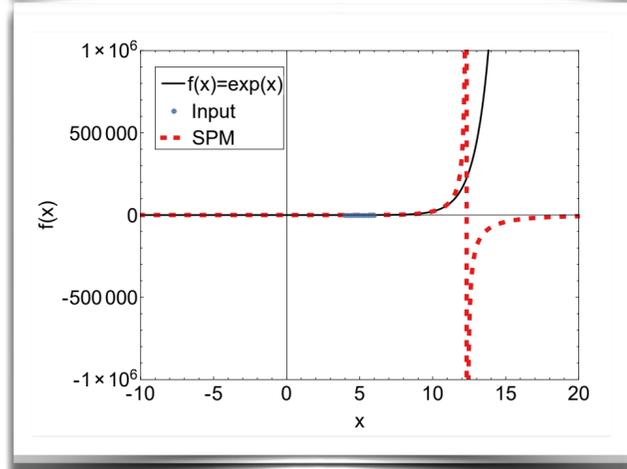
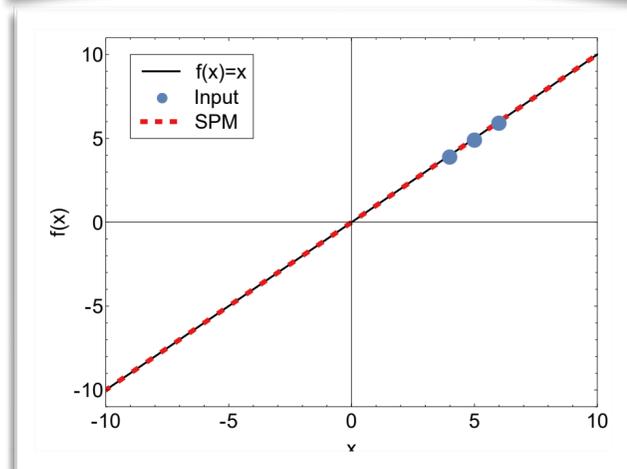
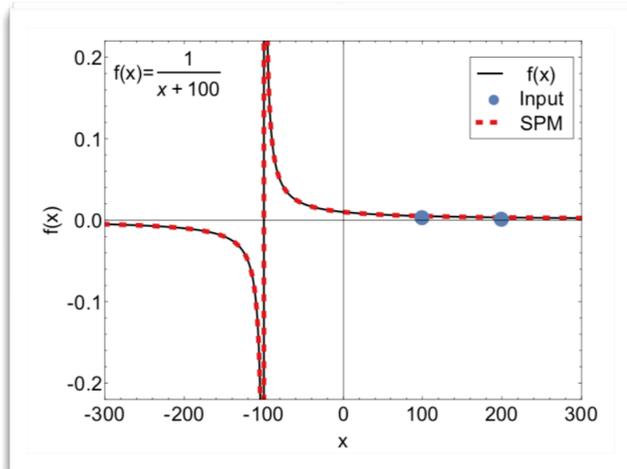
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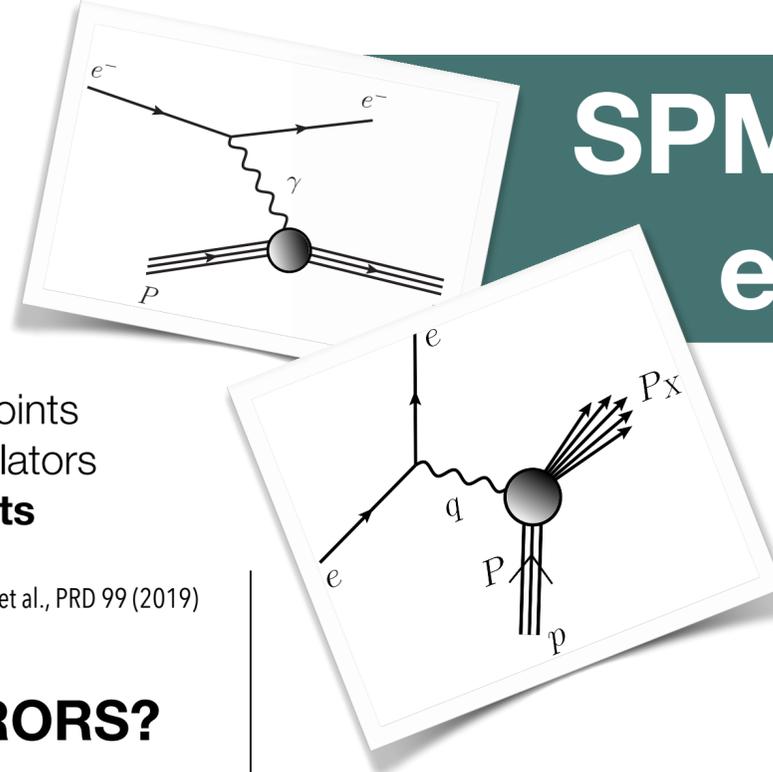
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# SPM parameter extraction



General algorithm to extrapolate a certain parameter from given noisy experimental datasets.

generate ( $10^3$ ) replicas for the given experimental central values and error  
smooth each replica with associated optimal  $\lambda$   
set  $\{M_j = 5 + j \mid j = 1, \dots, n_M\}$  for a suitable  $n_M$   
fix  $M_j$  and get a number of monotonic SPM interpolators for each replica  
determine the replicas' parameter value (averaging over the obtained curves)  
construct the (normal) distribution of the replicas' ( $10^3$ ) extracted parameter;  
extract the mean  $p^{M_j}$  and standard deviation  $\sigma_p^{M_j}$   
final result

$$p \pm \sigma; \quad p = \sum_{j=1}^{n_M} \frac{p^{M_j}}{n_M}; \quad \sigma_p = \left[ \sum_{j=1}^{n_M} \frac{(\sigma_p^{M_j})^2}{n_M^2} + \sigma_{\delta M}^2 \right]^{\frac{1}{2}}$$

standard deviation of  $p^{M_j}$  distribution

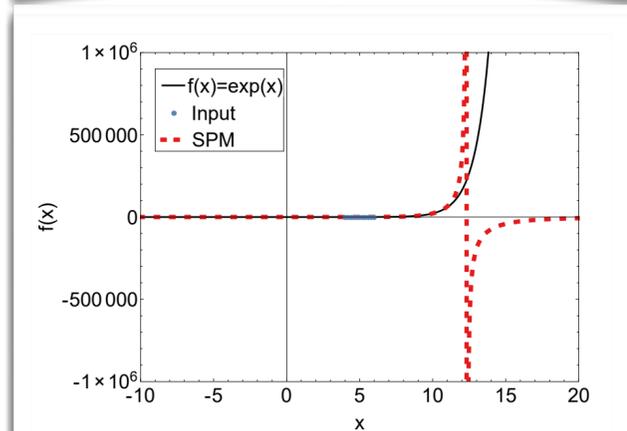
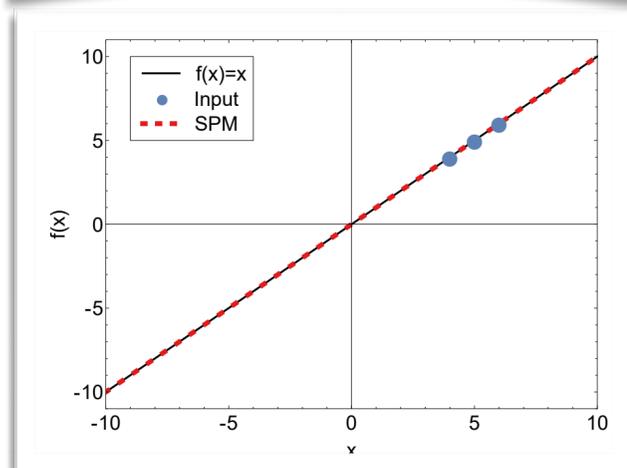
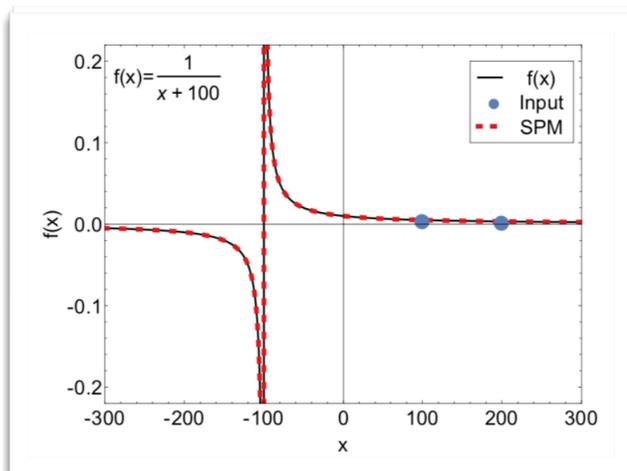
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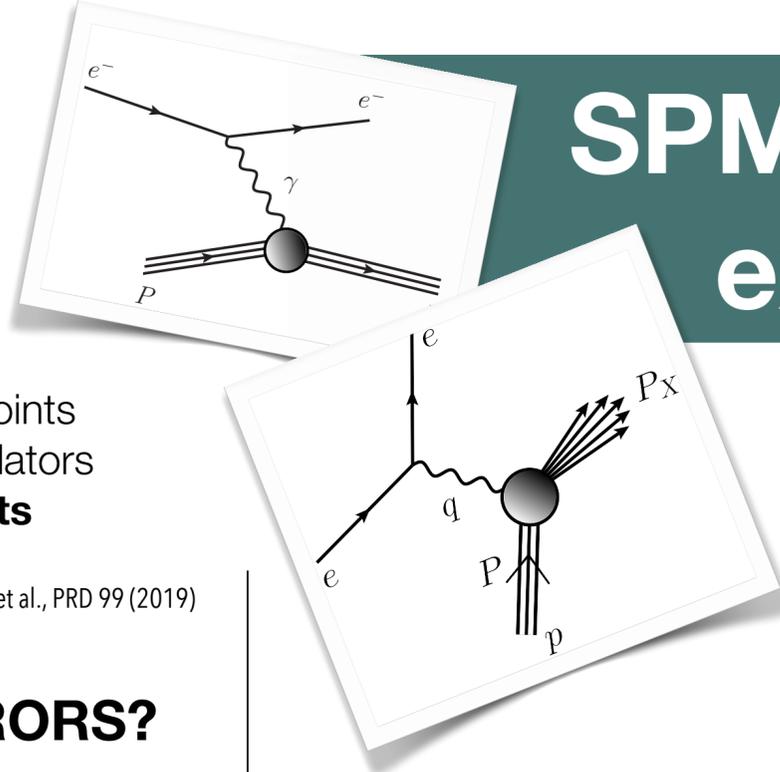
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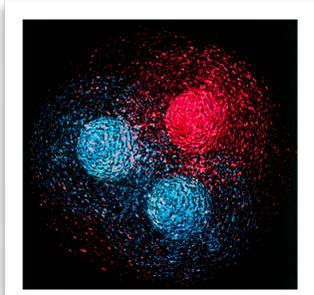
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# DOES IT WORK?

# Proton

PARTICLE



Nature's most fundamental **bound-state**

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**Quantum ChromoDynamics**  
describes the proton structure

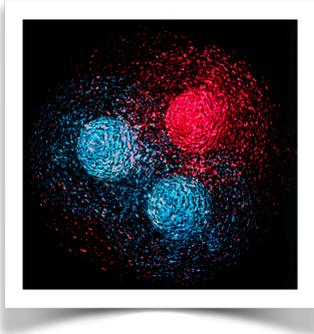
$m_p, r_p$

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if it is composite it must have a size

**HOW BIG IS IT?**

# Proton PARTICLE



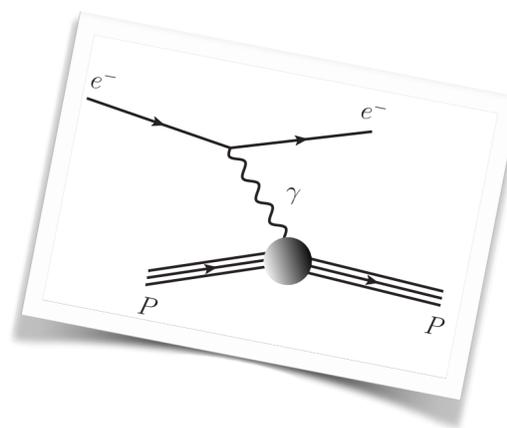
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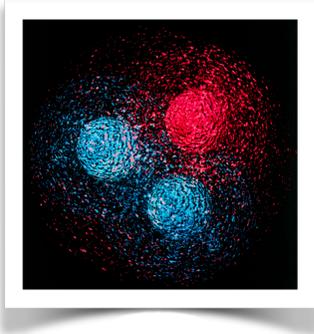
point-like probe, QED only

$$\frac{d\sigma}{d\Omega} \propto \varepsilon [G_E^p(Q^2)]^2 + \tau [G_M^p(Q^2)]^2$$

$$r_p^2 = -6 \left. \frac{d}{dQ^2} G_E^p(Q^2) \right|_{Q^2=0}$$

electric and magnetic form factor encode the shape  
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# Proton PARTICLE



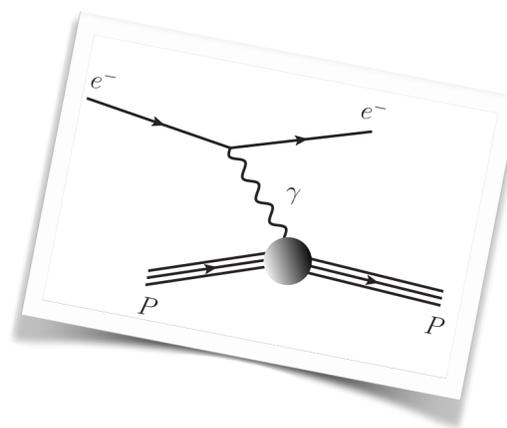
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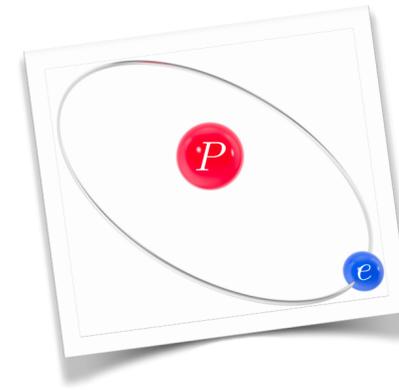
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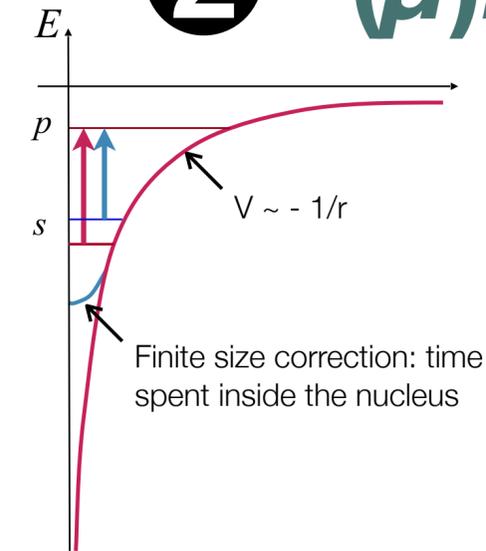
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# 2

## (μ)H SPECTROSCOPY

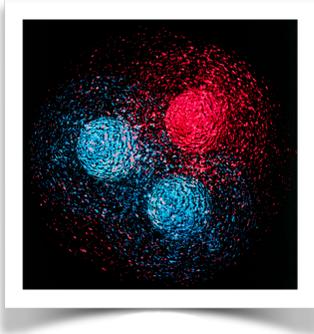


probability of lepton inside proton

$$\sim (r_p \alpha)^3 m^3$$

$$m \simeq m_e$$

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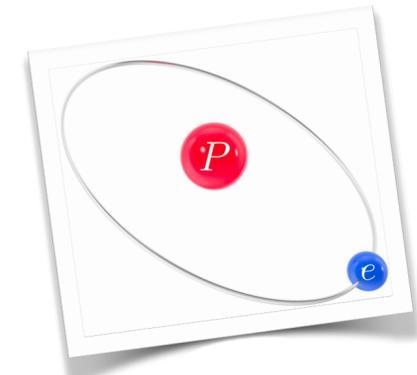
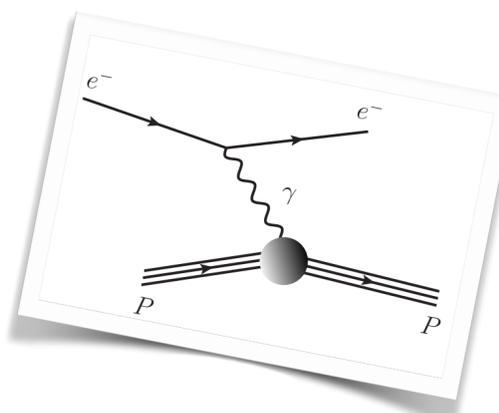
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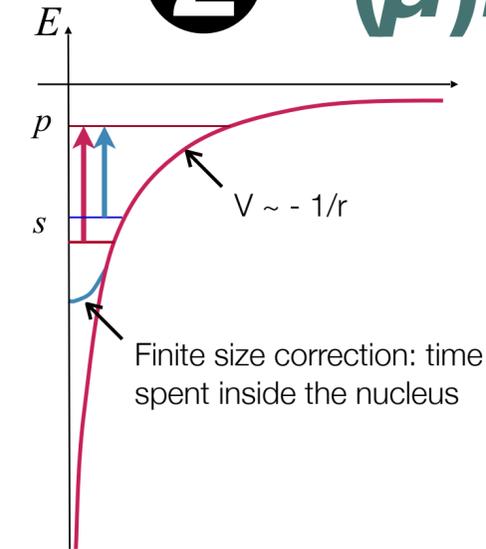
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$$r_p^2 = -6 \left. \frac{d}{dQ^2} G_E^p(Q^2) \right|_{Q^2=0}$$

electric and magnetic form factor encode the shape  
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②

## (μ)H SPECTROSCOPY



probability of lepton inside proton

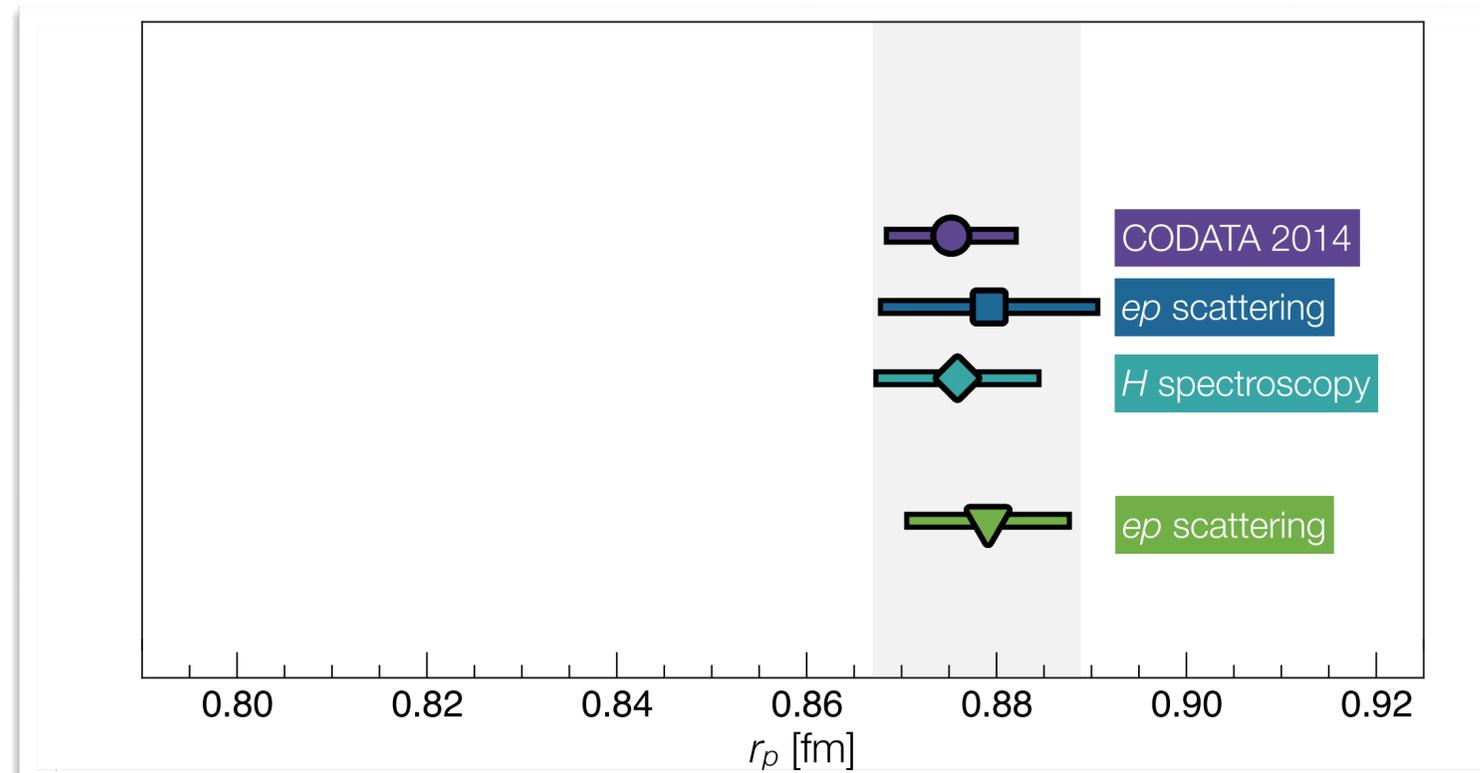
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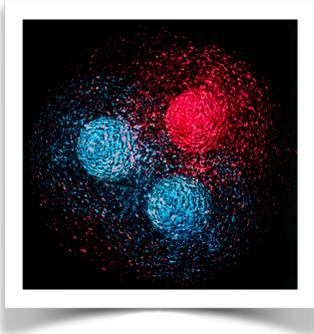
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# Proton PARTICLE



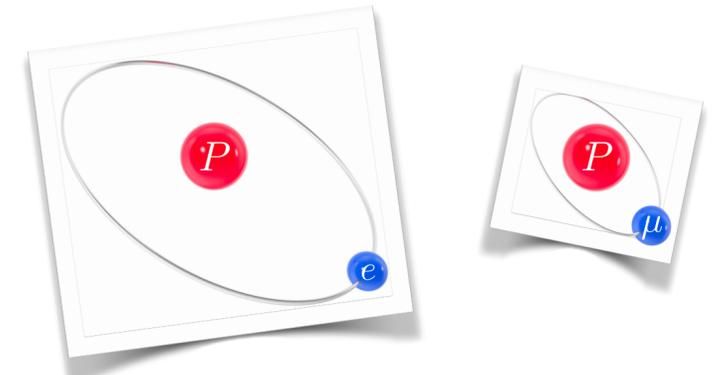
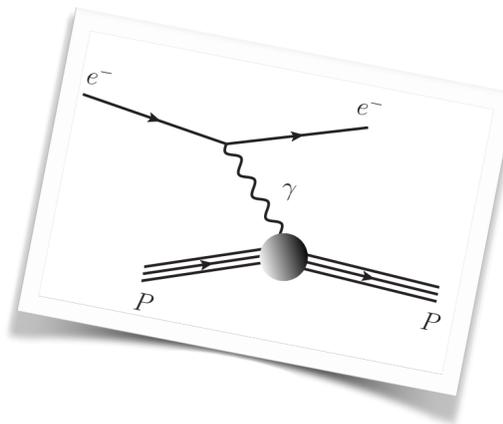
Nature's most fundamental **bound-state**

**Quantum ChromoDynamics**  
describes the proton structure

$$m_p, r_p$$

if it is composite it must have a size

**HOW BIG IS IT?**



①

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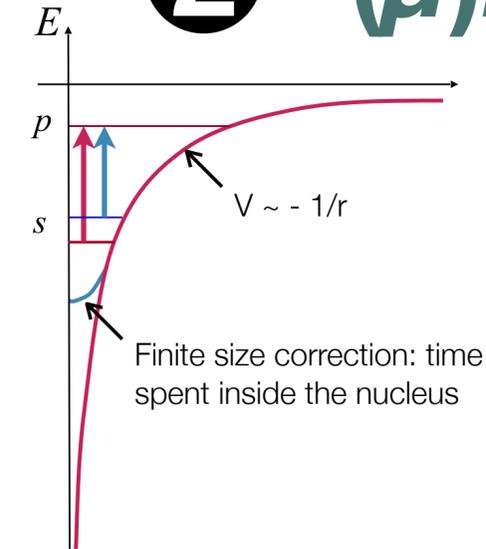
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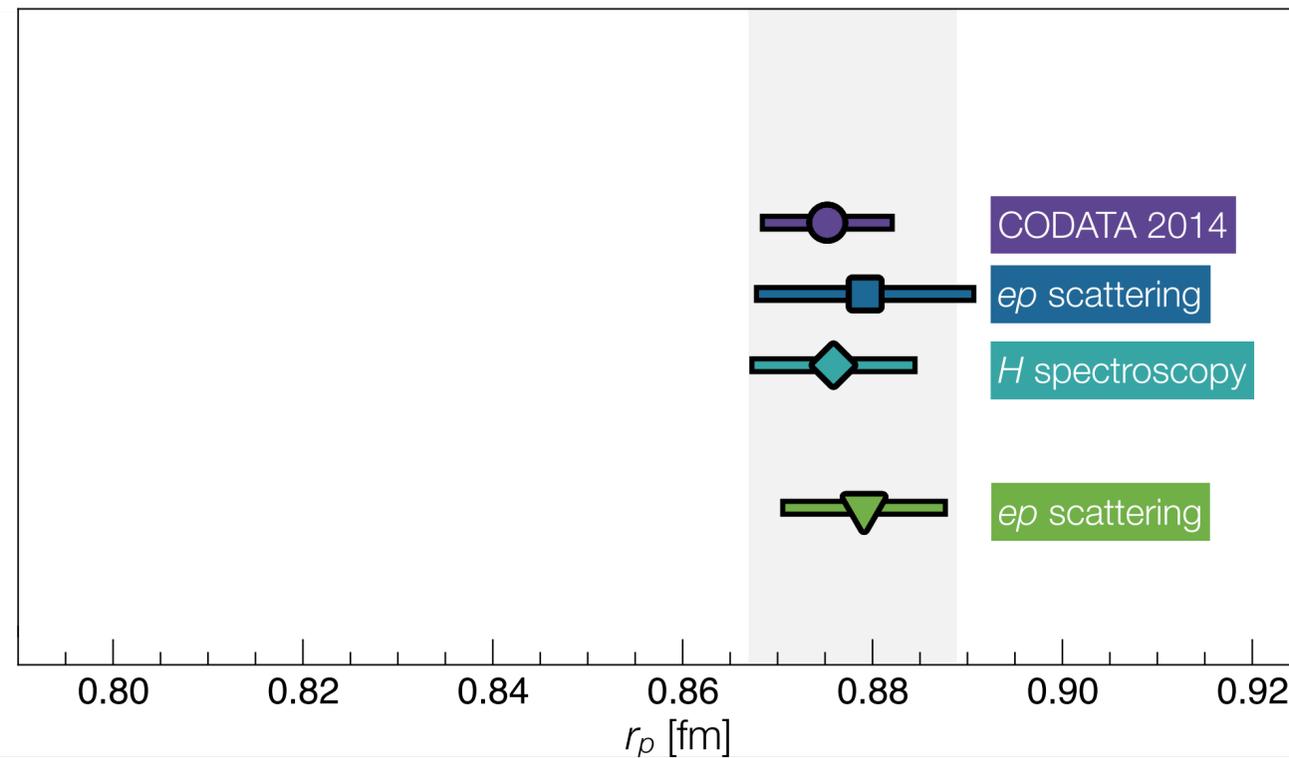
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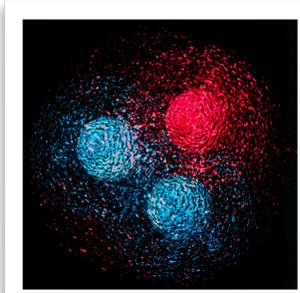
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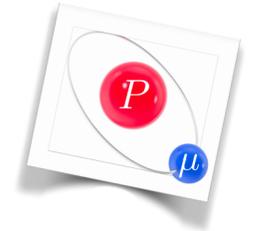
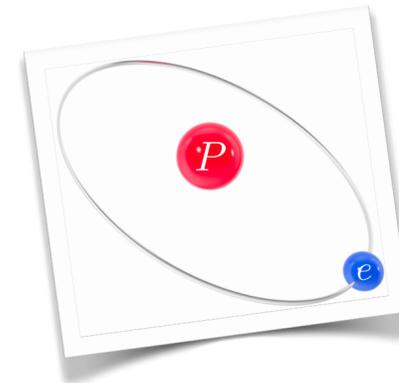
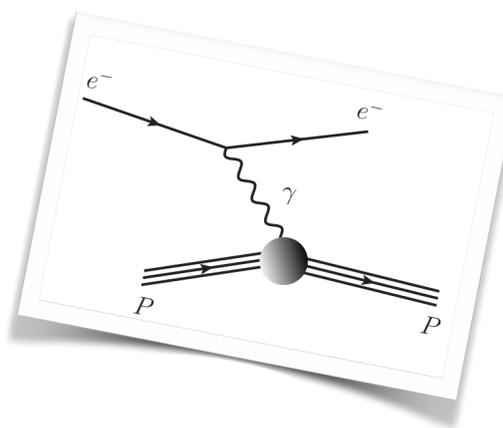
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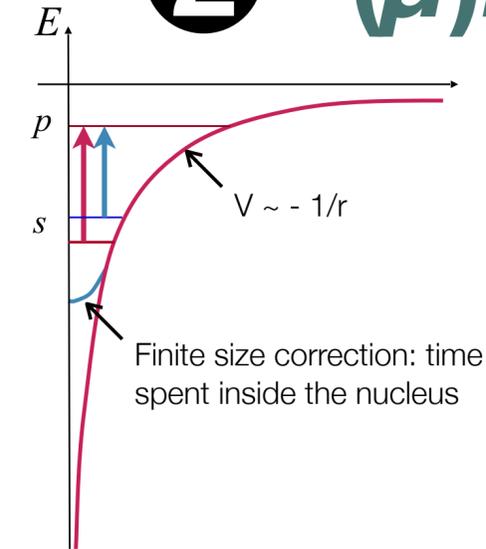
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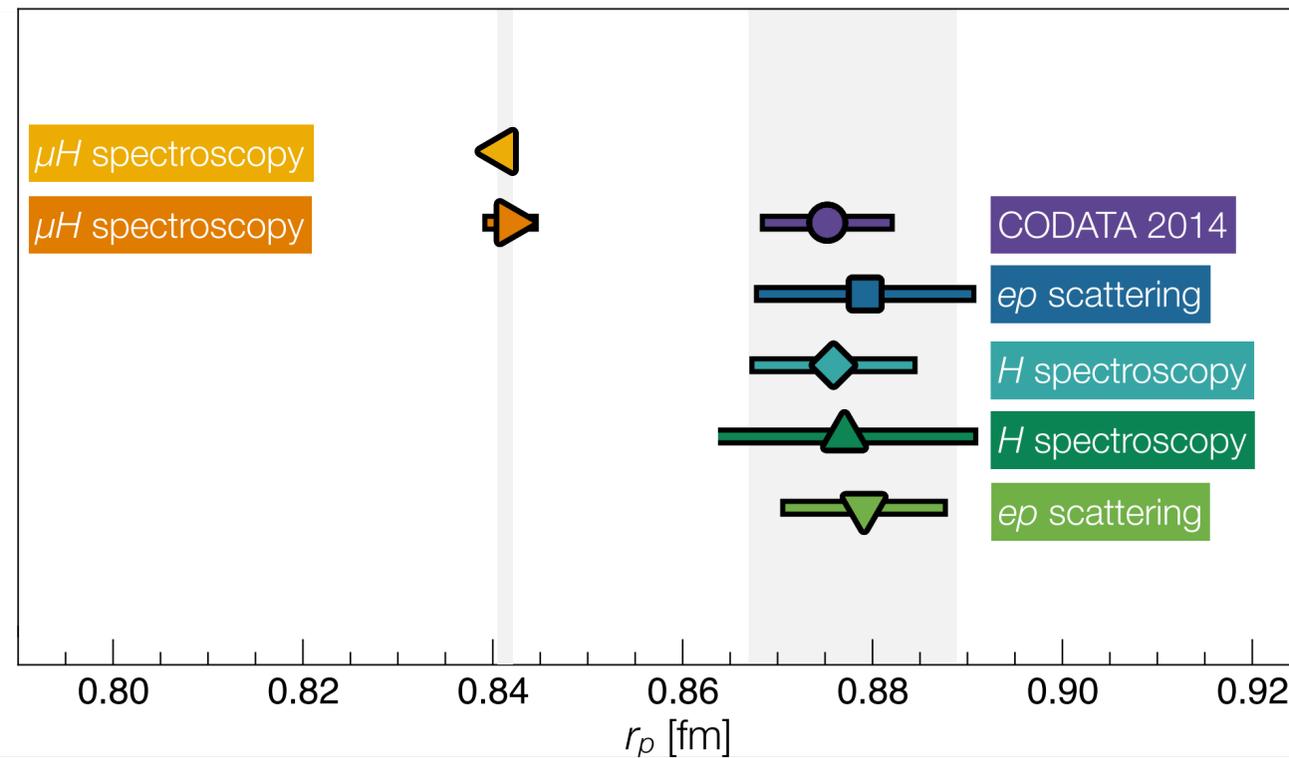
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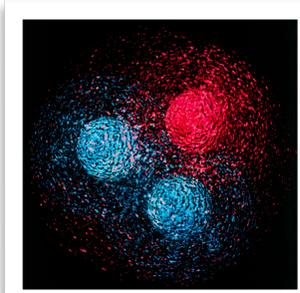
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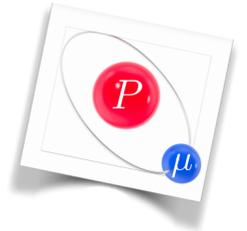
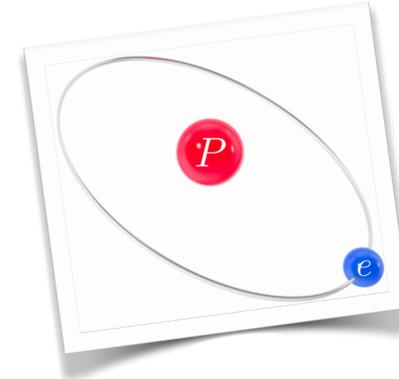
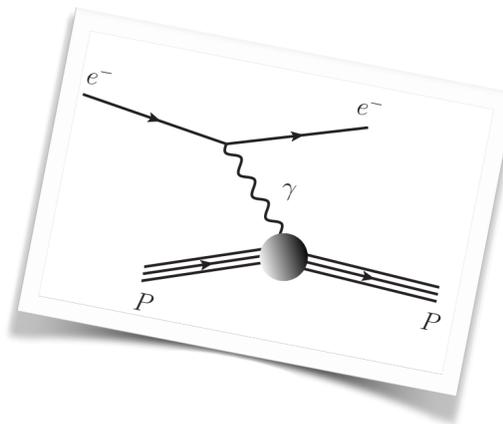
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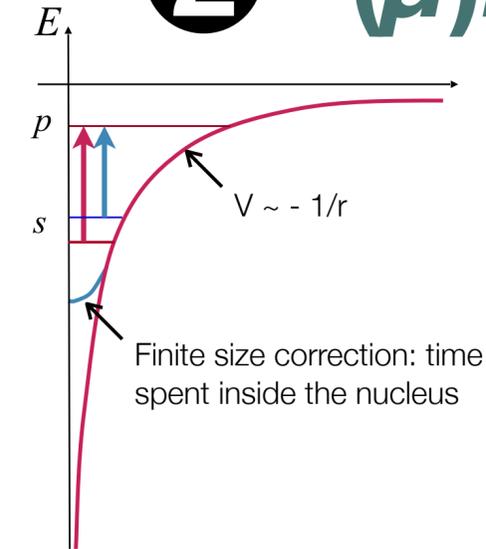
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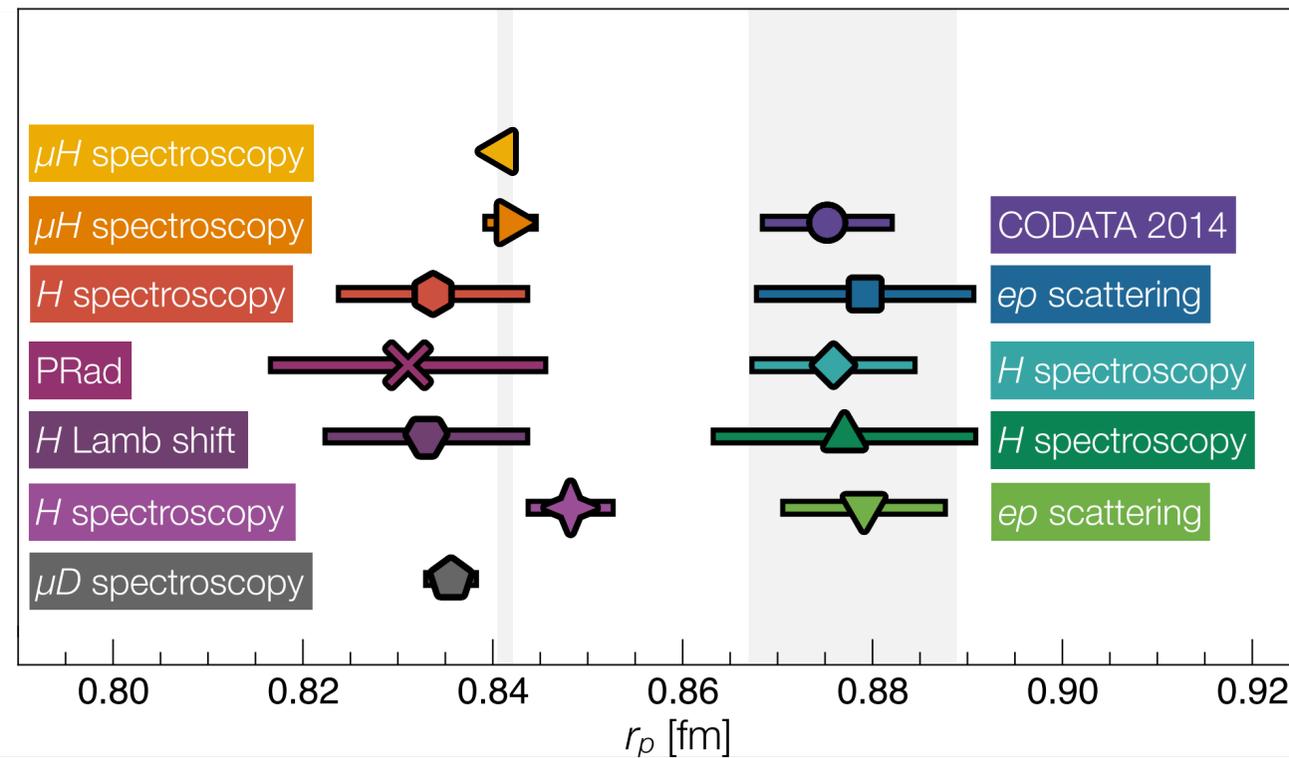
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A. Beyer *et al.*, Science 358, 79 (2017)

W. Xiong *et al.*, Nature 575, 147 (2019)

N. Bezginov *et al.*, Science 365, 1007 (2019)

A. Grinin *et al.*, Science 370, 1061 (2020)

R. Pohl *et al.*, Science 353, 669 (2016)

**SPM**  
AND  
**SMOOTHING**  
**VALIDATION**

does it really work?  
is it robust?

*If you want to disprove large radius,  
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# SPM AND SMOOTHING VALIDATION

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build elastic form factor **replicas** of **known radius**  $r_p^*$

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## $GE_p$ GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian

parametrisations of experimental data (5)

“real-world” calculations (1)

Yan et al., PRC 98 (2018)

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$\forall M$ /generators/kinematics:

Gaussianity of  $r_p$  distribution  
robustness of  $r_p$  extraction

bias

Root Mean Square Error

$$\delta r_p = r_p - r_p^*$$

$$\sigma_r$$

$$\text{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

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PRad (3), A1 low- $Q^2$  (1)

generators

replicas/kinematic

interpolators/replica

$M_j$

total interpolators

4 ×

9 ×

1,000 ×

5,000 ×

12 =

2,160,000,000

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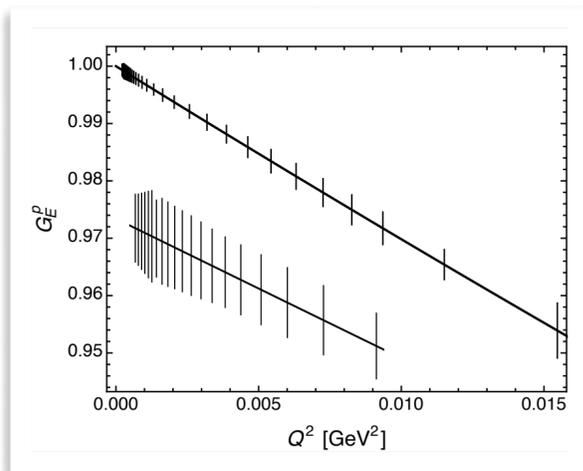
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EXAMPLE: 1.1 GeV kinematics

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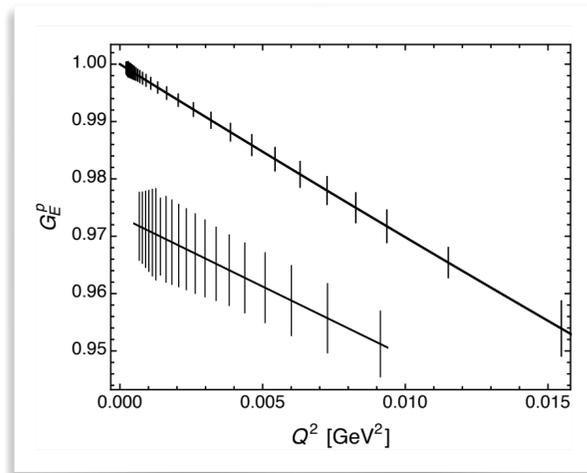
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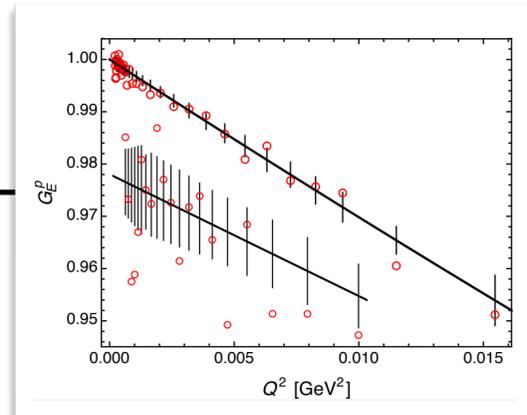
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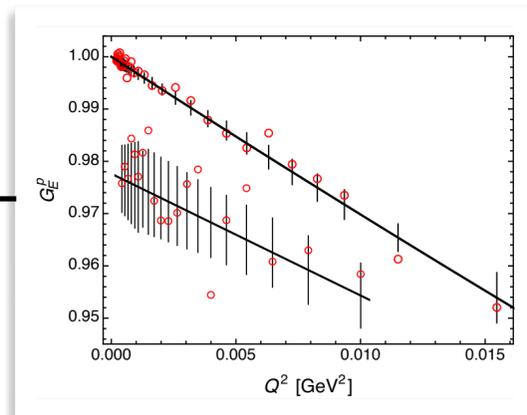
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Generate replicas



•  
• 1k  
•



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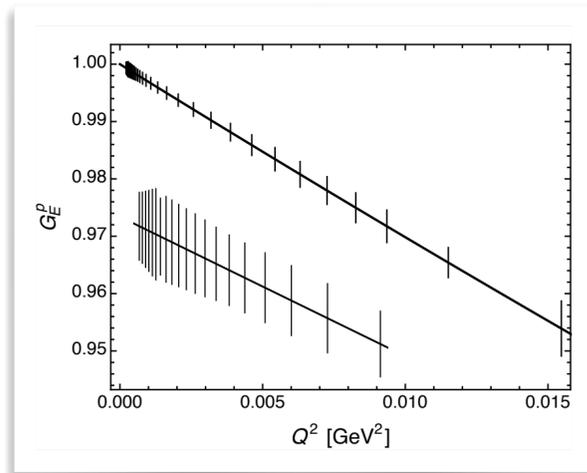
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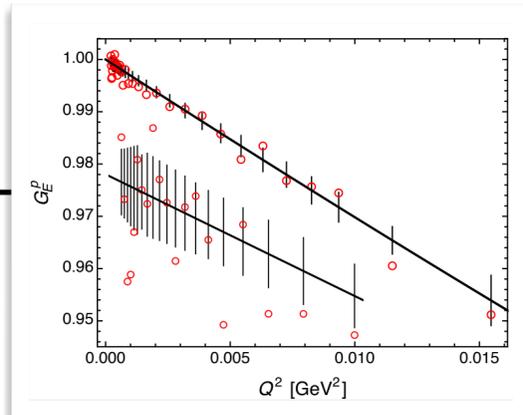
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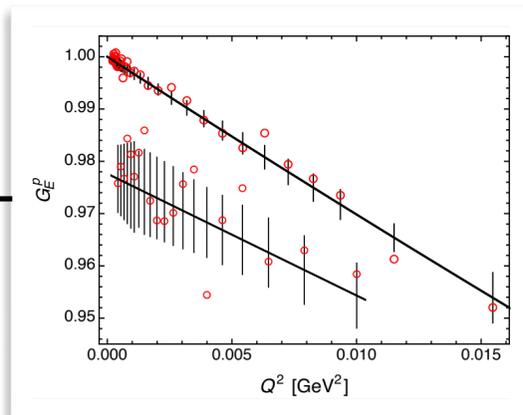
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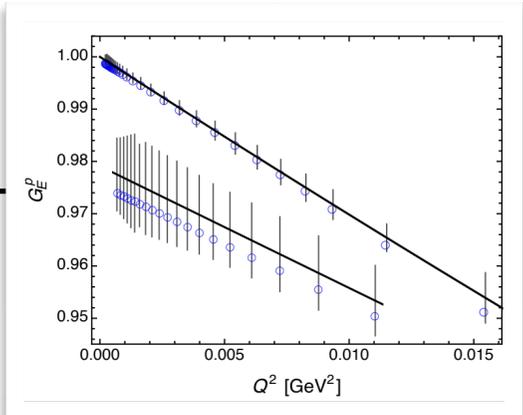


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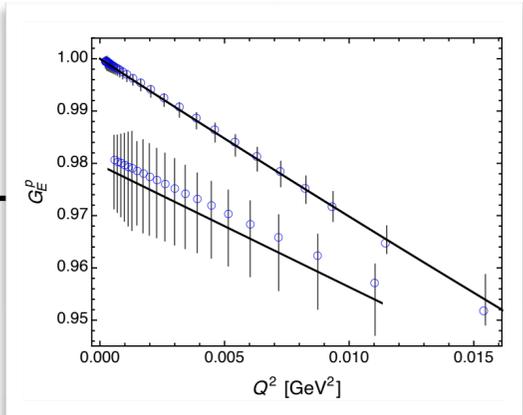


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• 1k  
•

Determine/apply optimal smoothing parameters



•  
• 1k  
•



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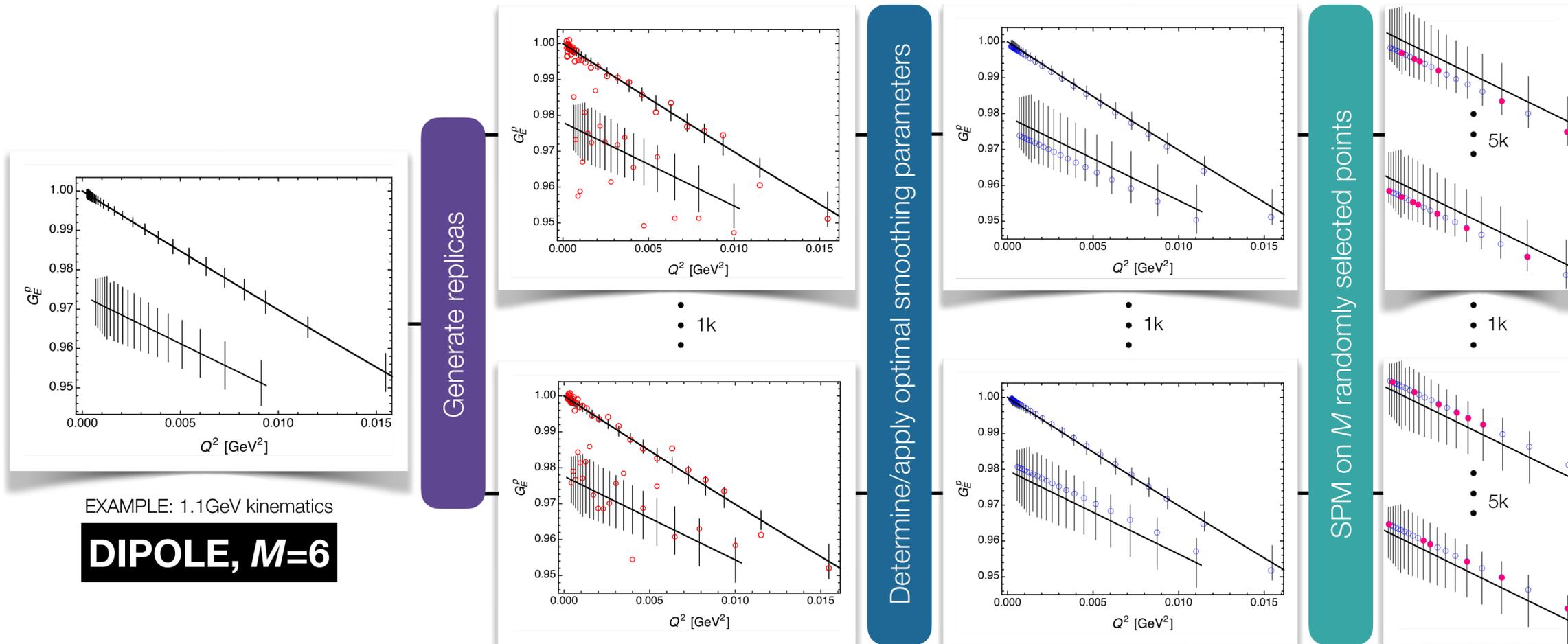
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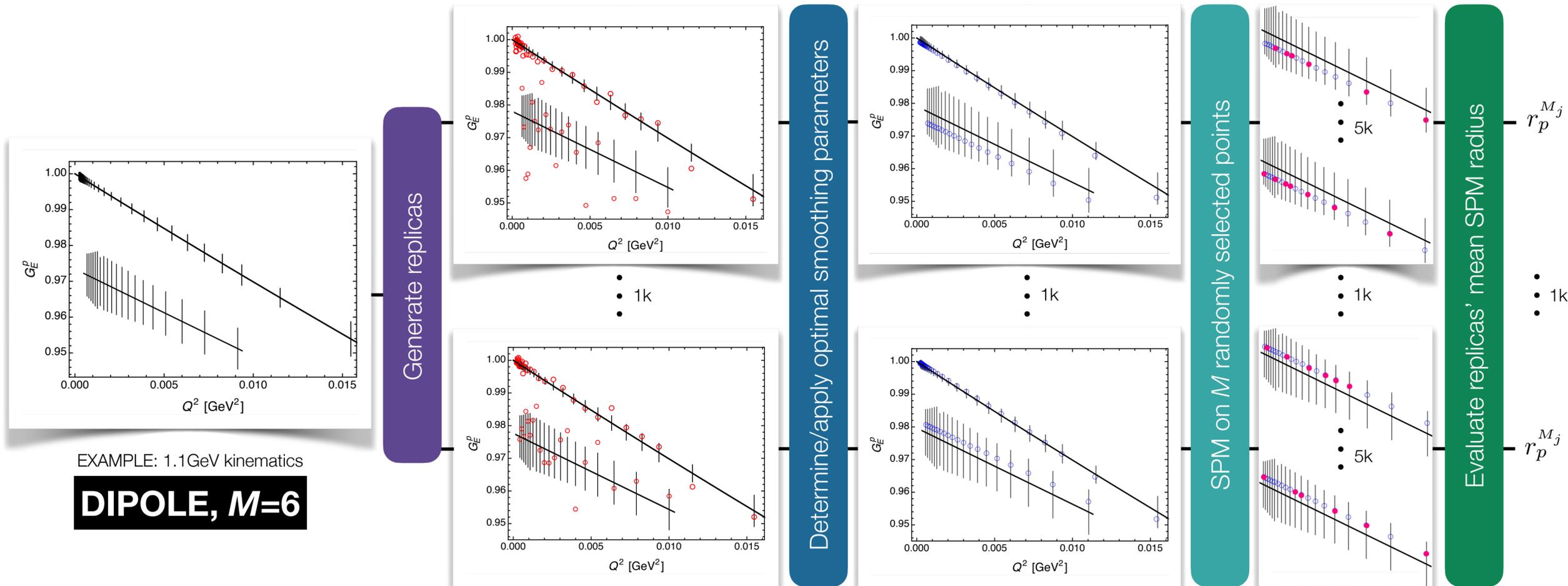
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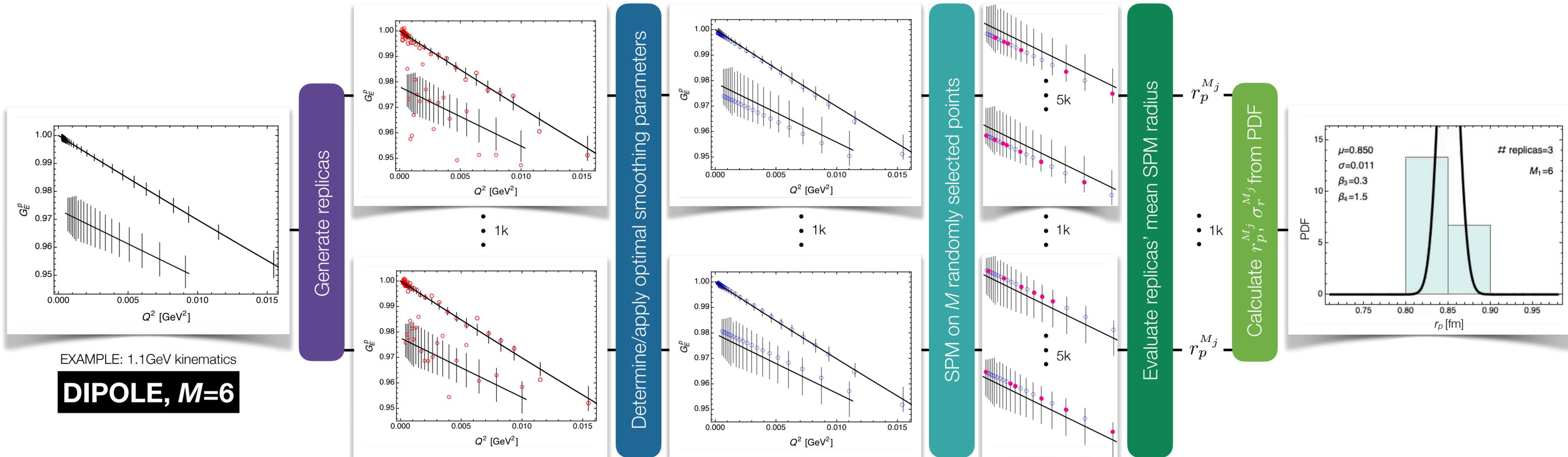
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total interpolators

4 ×

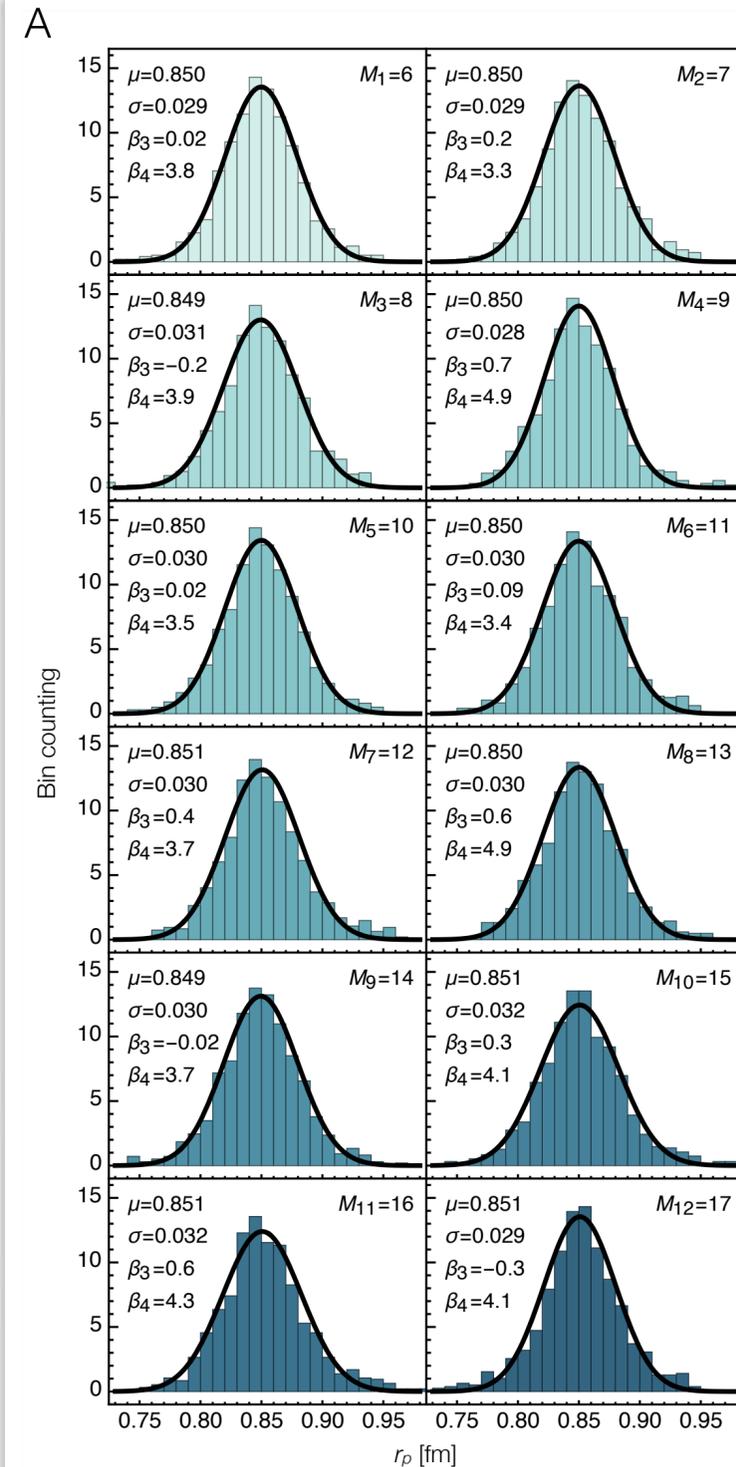
9 ×

1,000 ×

5,000 ×

12 =

**2,160,000,000**



# SPM AND SMOOTHING VALIDATION

does it really work?  
is it robust?  
*If you want to disprove large radius,  
show you can replicate it*

build elastic form factor **replicas** of **known radius**  $r_p^*$

## GE<sub>p</sub> GENERATORS

Use generators from a variety of models  
functional forms (3): monopole, dipole, Gaussian  
parametrisations of experimental data (5)  
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

## CHECKS

∀ M/generators/kinematics:

Gaussianity of  $r_p$  distribution  
robustness of  $r_p$  extraction

$$\delta r_p = r_p - r_p^* \quad \sigma_r \quad \text{Root Mean Square Error}$$

$$\text{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

standard deviation

$r_p$  extraction robust if  
 $|\delta r_p| < \sigma_r$

RMSE independent from generator

experimental kinematics:  
PRad (3), A1 low-Q<sup>2</sup> (1)

generators

replicas/kinematic

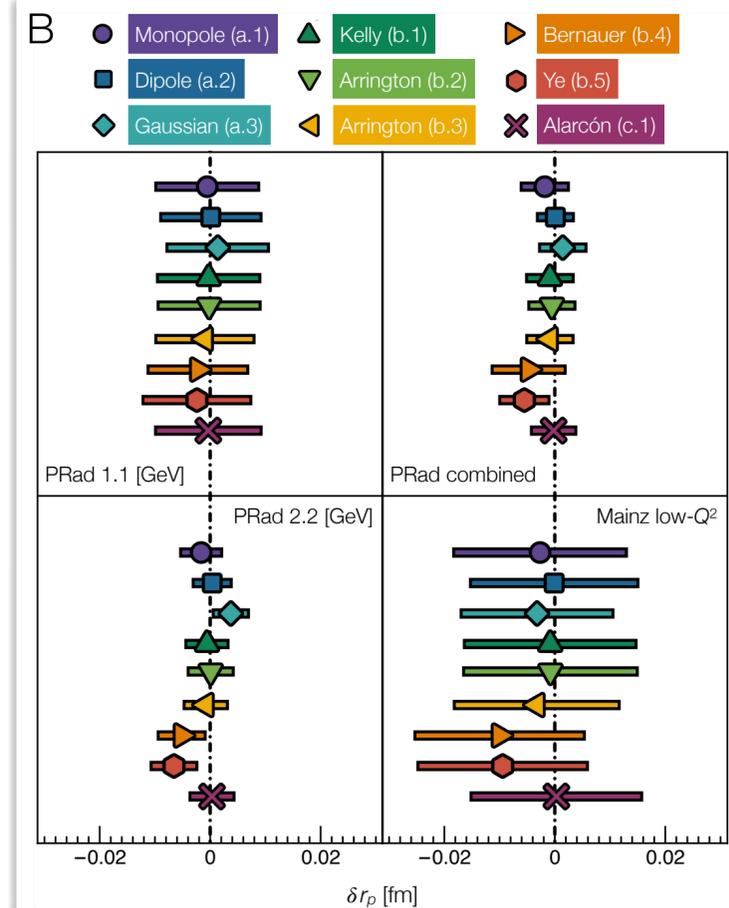
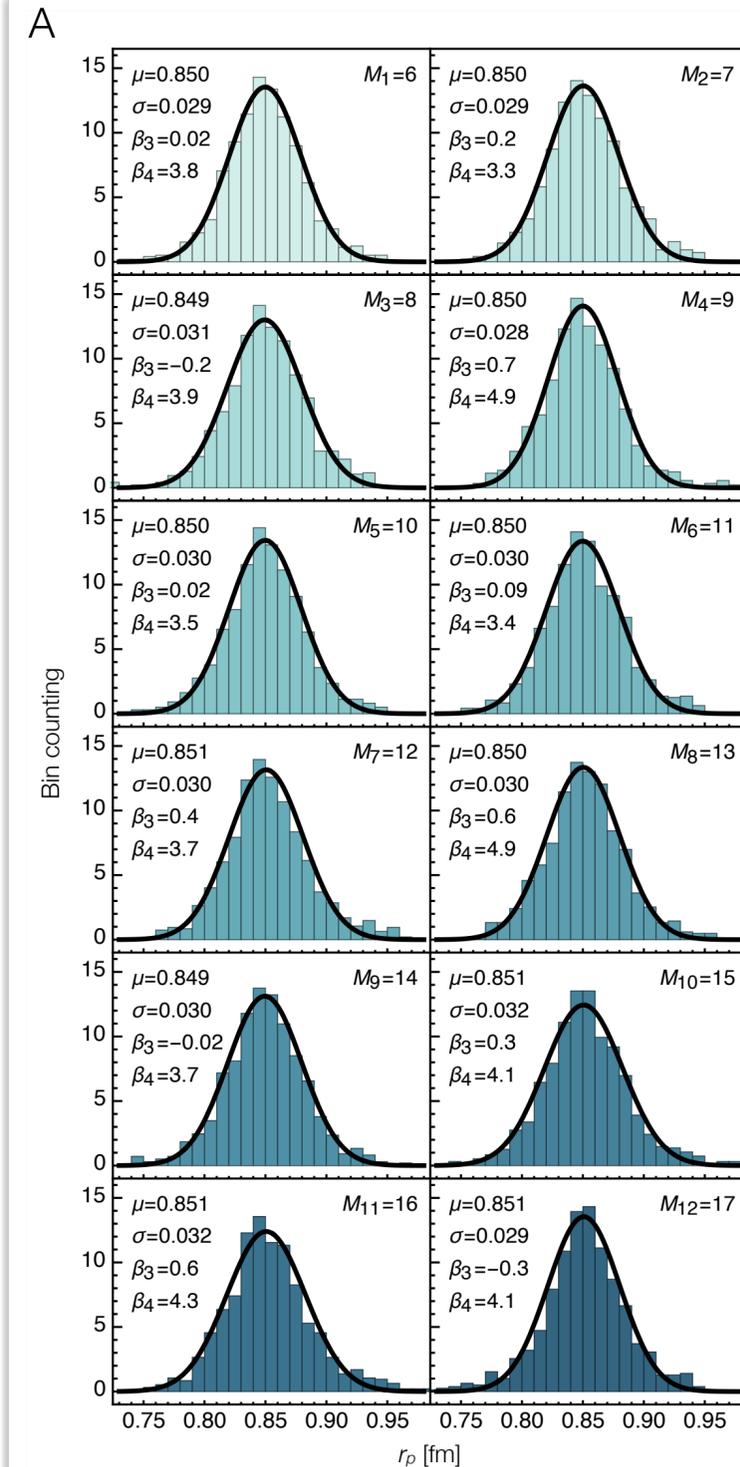
interpolators/replica

$M_j$

total interpolators

2,160,000,000

4 ×  
9 ×  
1,000 ×  
5,000 ×  
12 =



# SPM AND SMOOTHING VALIDATION

does it really work?  
is it robust?  
*If you want to disprove large radius, show you can replicate it*

build elastic form factor **replicas** of **known radius**  $r_p^*$

## GE<sub>p</sub> GENERATORS

Use generators from a variety of models  
functional forms (3): monopole, dipole, Gaussian  
parametrisations of experimental data (5)  
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

## CHECKS

$\forall M$ /generators/kinematics:  
Gaussianity of  $r_p$  distribution  
robustness of  $r_p$  extraction

bias Root Mean Square Error

$$\delta r_p = r_p - r_p^* \quad \sigma_r \quad \text{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

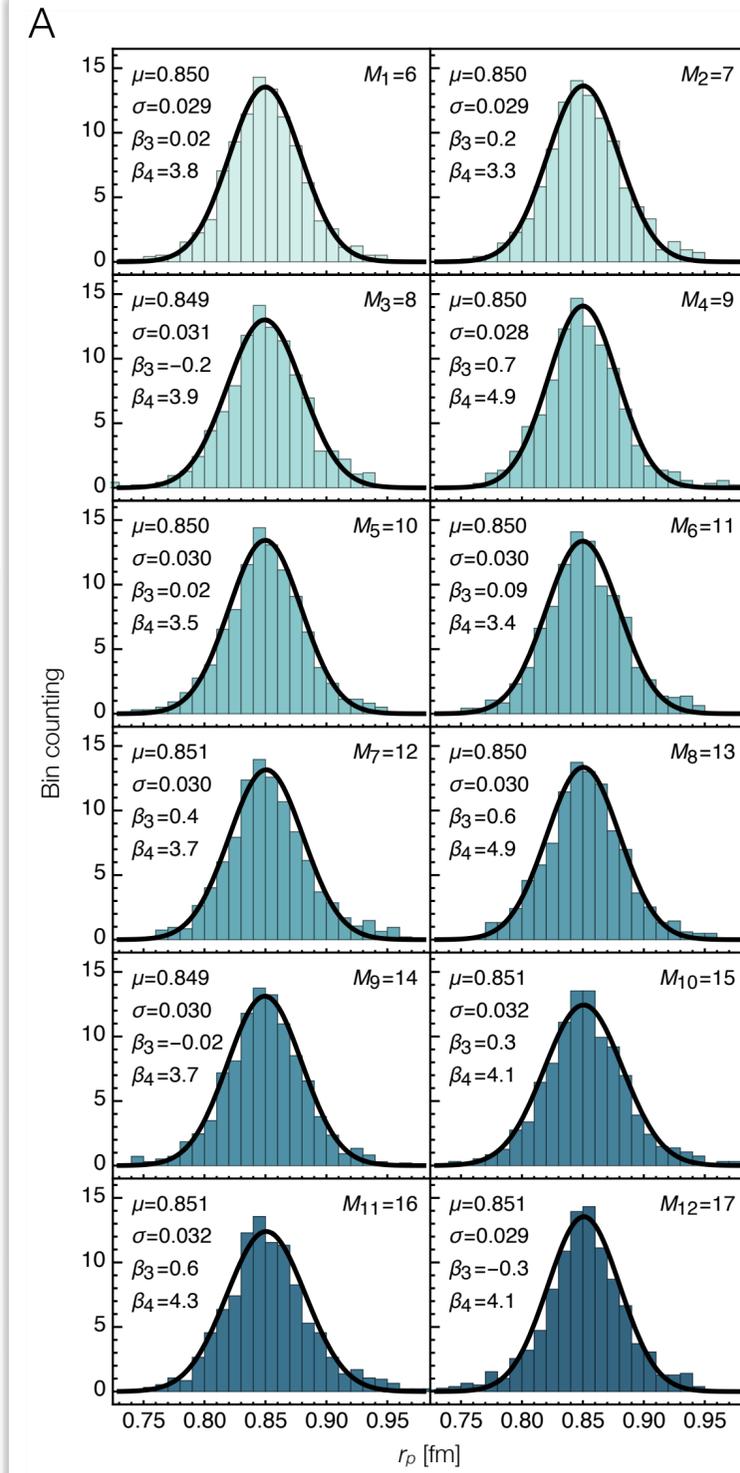
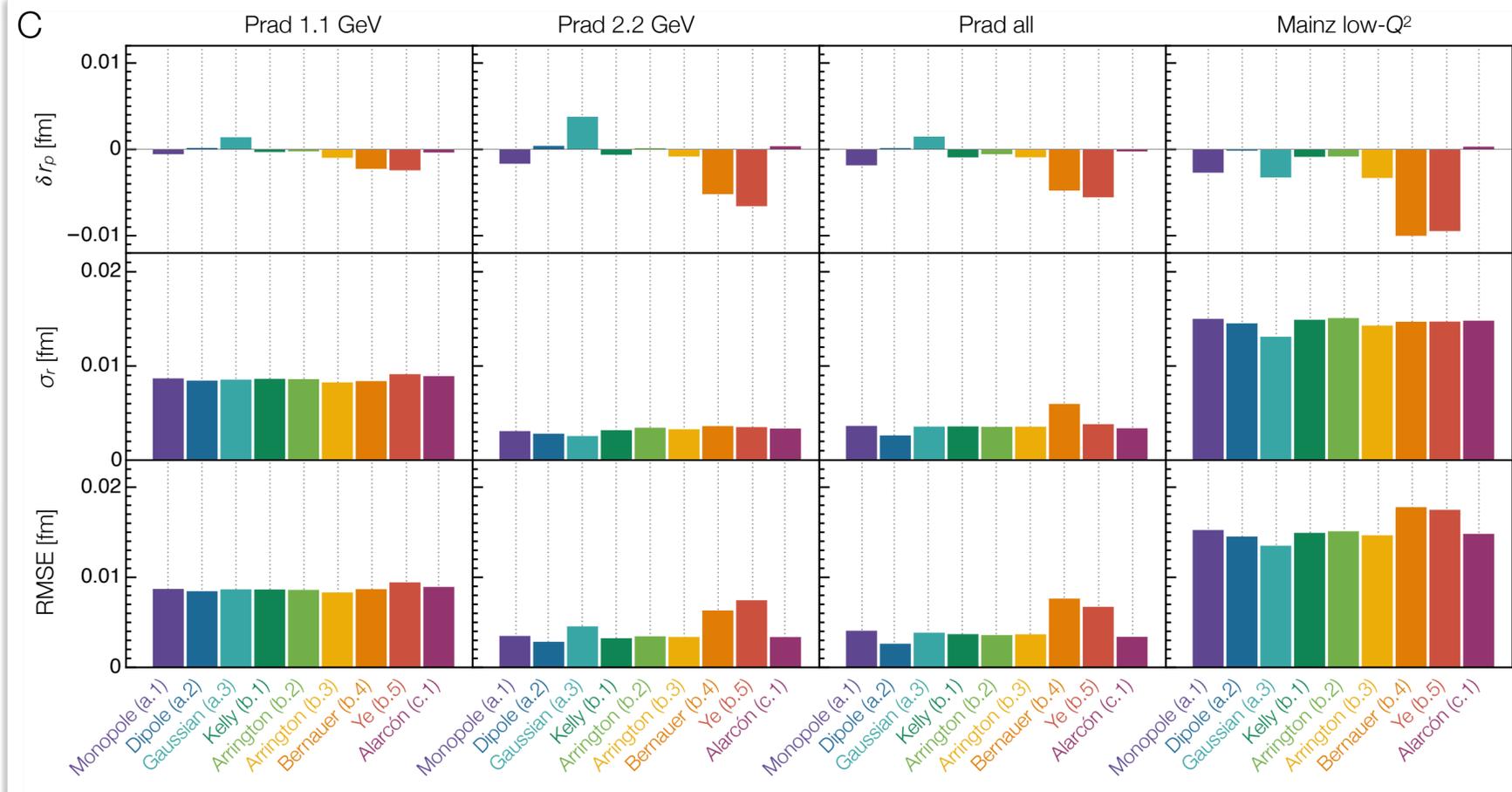
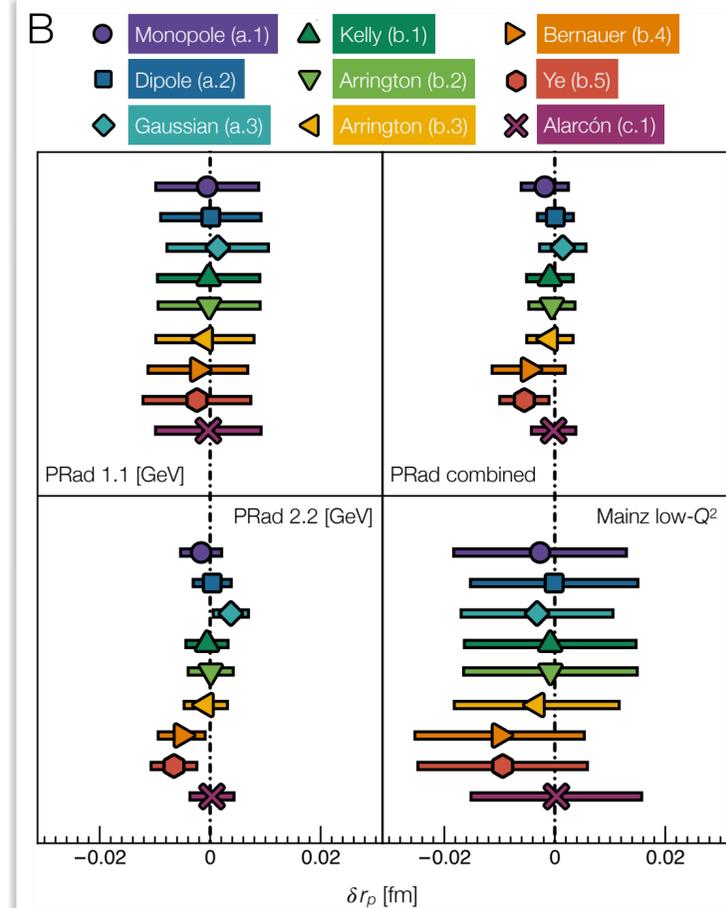
standard deviation

$r_p$  extraction robust if  
 $|\delta r_p| < \sigma_r$

RMSE independent from generator

experimental kinematics:  
PRad (3), A1 low-Q<sup>2</sup> (1)

generators 4 ×  
replicas/kinematic 9 ×  
interpolators/replica 1,000 ×  
 $M_j$  5,000 ×  
12 =  
total interpolators 2,160,000,000



# SPM AND SMOOTHING VALIDATION

does it really work? ✓  
is it robust? ✓  
If you want to disprove large radius, show you can replicate it

## CHECKS

∀  $M$ /generators/kinematics:  
Gaussianity of  $r_p$  distribution  
robustness of  $r_p$  extraction

# SPM is ROBUST

build elastic form factor **replicas** of **known radius**  $r_p^*$

## $G E_p$ GENERATORS

Use generators from a variety of models  
functional forms (3): monopole, dipole, Gaussian  
parametrisations of experimental data (5)  
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

bias Root Mean Square Error

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standard deviation

$r_p$  extraction robust if  
 $|\delta r_p| < \sigma_r$

RMSE independent from generator

experimental kinematics:  
PRad (3), A1 low- $Q^2$  (1)

generators 4 ×

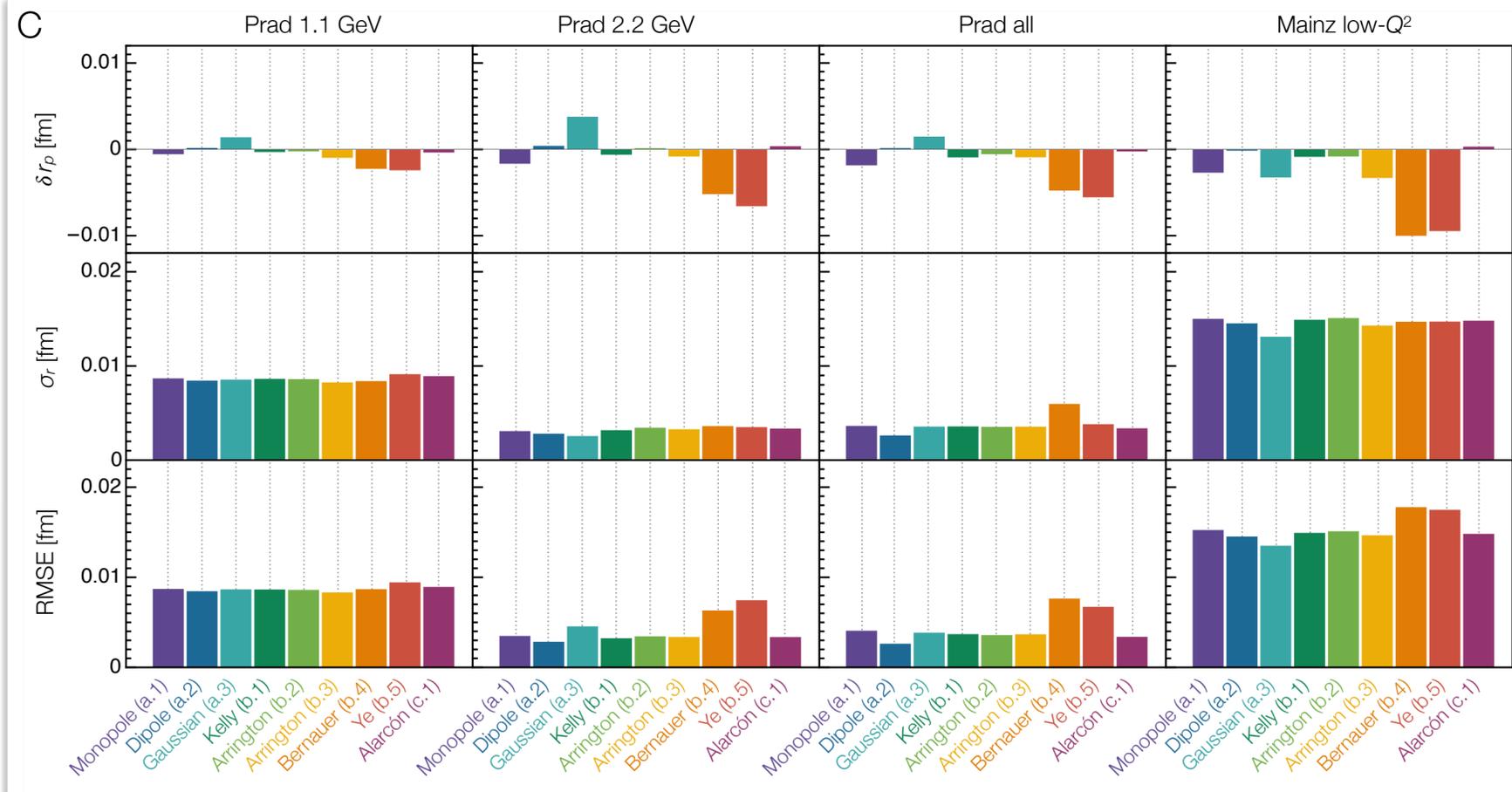
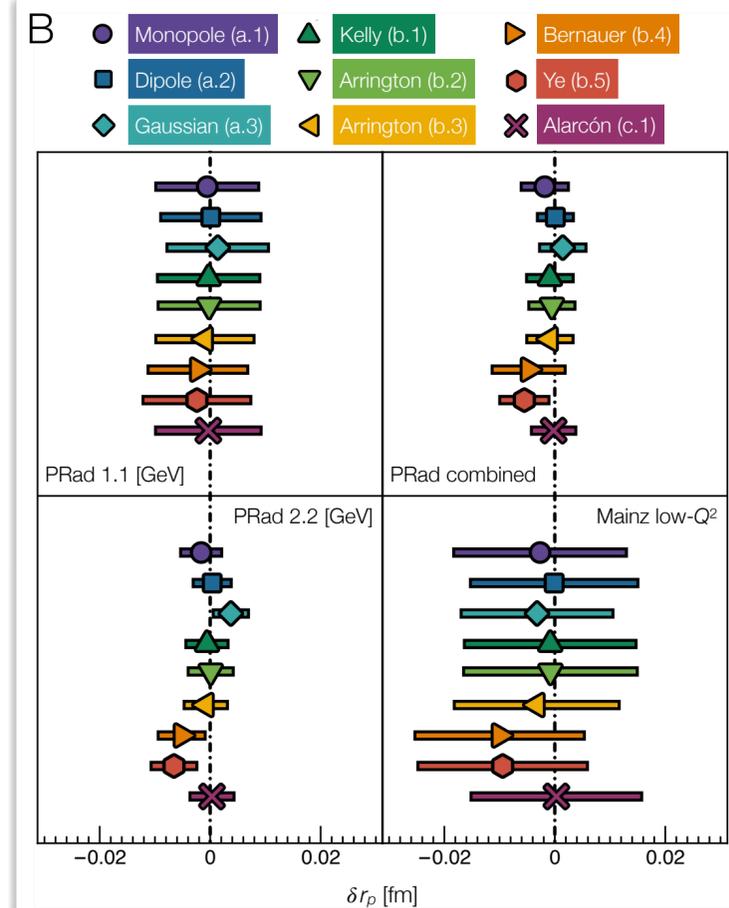
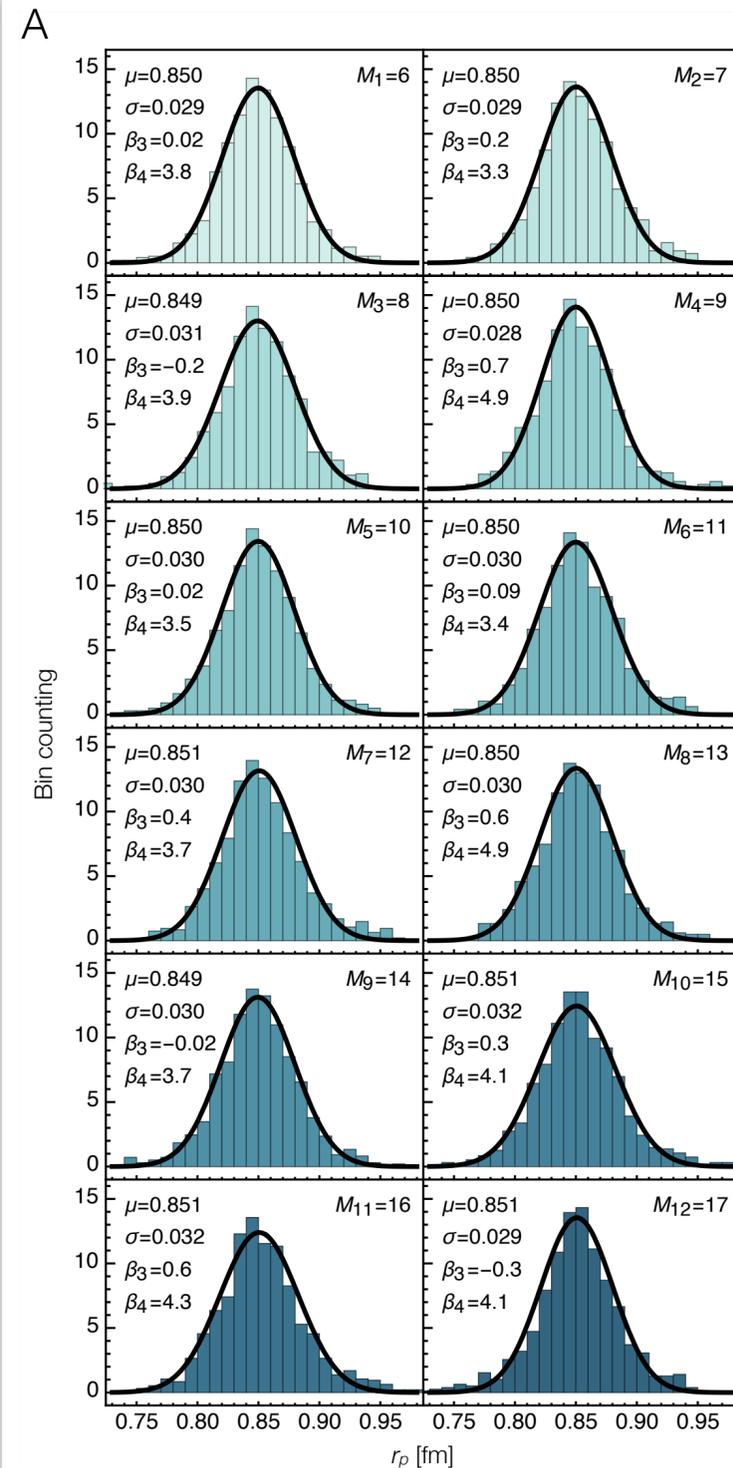
replicas/kinematic 9 ×

interpolators/replica 1,000 ×

$M_j$  5,000 ×

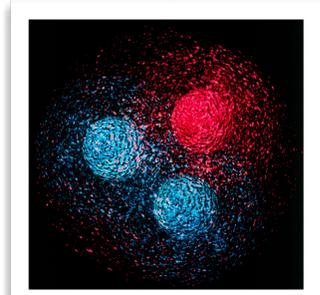
12 =

total interpolators 2,160,000,000



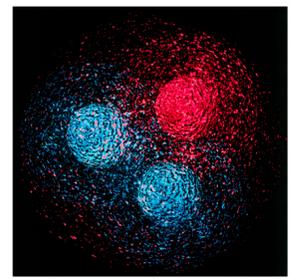
# Proton

SPM RADIUS



# Proton

## SPM RADIUS



# 1 PRad DATA

lowest yet achieved  
momentum transferred

$$2.1 \times 10^{-4} \leq Q^2 / [\text{GeV}^2] \leq 6 \times 10^{-2}$$

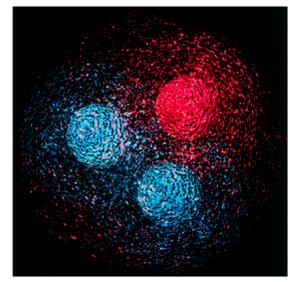
two datasets at different energy beams  
1.1 and 2.2 [GeV]

$$r_p^{1.1} = 0.842 \pm 0.008_{\text{stat}} \text{ [fm]}$$

$$r_p^{2.2} = 0.824 \pm 0.003_{\text{stat}} \text{ [fm]}$$

$$r_p^{\text{PRad}} = 0.838 \pm 0.005_{\text{stat}} \text{ [fm]}$$

# Proton SPM RADIUS



## ① PRad DATA

lowest yet achieved  
momentum transferred

$$2.1 \times 10^{-4} \leq Q^2 / [\text{GeV}^2] \leq 6 \times 10^{-2}$$

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$$r_p^{1.1} = 0.842 \pm 0.008_{\text{stat}} \text{ [fm]}$$

$$r_p^{2.2} = 0.824 \pm 0.003_{\text{stat}} \text{ [fm]}$$

$$r_p^{\text{PRad}} = 0.838 \pm 0.005_{\text{stat}} \text{ [fm]}$$

## ② A1 DATA

extends toward low- $Q^2$

$$3.8 \times 10^{-3} \leq Q^2 / [\text{GeV}^2] \leq 1$$

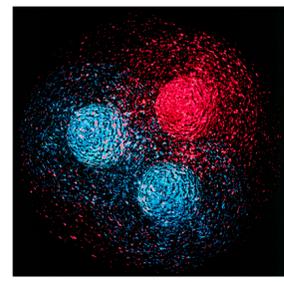
use first 40 low- $Q^2$  data

$$r_p^{\text{A1-low}Q^2} = 0.856 \pm 0.014_{\text{stat}} \text{ [fm]}$$

all data yield

$$r_p^{\text{A1}} = 0.857 \pm 0.021_{\text{stat}} \text{ [fm]}$$

# Proton SPM RADIUS



# PROTON RADIUS PUZZLE SETTLED?

## 1 PRad DATA

lowest yet achieved  
momentum transferred

$$2.1 \times 10^{-4} \leq Q^2 / [\text{GeV}^2] \leq 6 \times 10^{-2}$$

two datasets at different energy beams  
1.1 and 2.2 [GeV]

$$r_p^{1.1} = 0.842 \pm 0.008_{\text{stat}} \text{ [fm]}$$

$$r_p^{2.2} = 0.824 \pm 0.003_{\text{stat}} \text{ [fm]}$$

$$r_p^{\text{PRad}} = 0.838 \pm 0.005_{\text{stat}} \text{ [fm]}$$

## 2 A1 DATA

extends toward low- $Q^2$

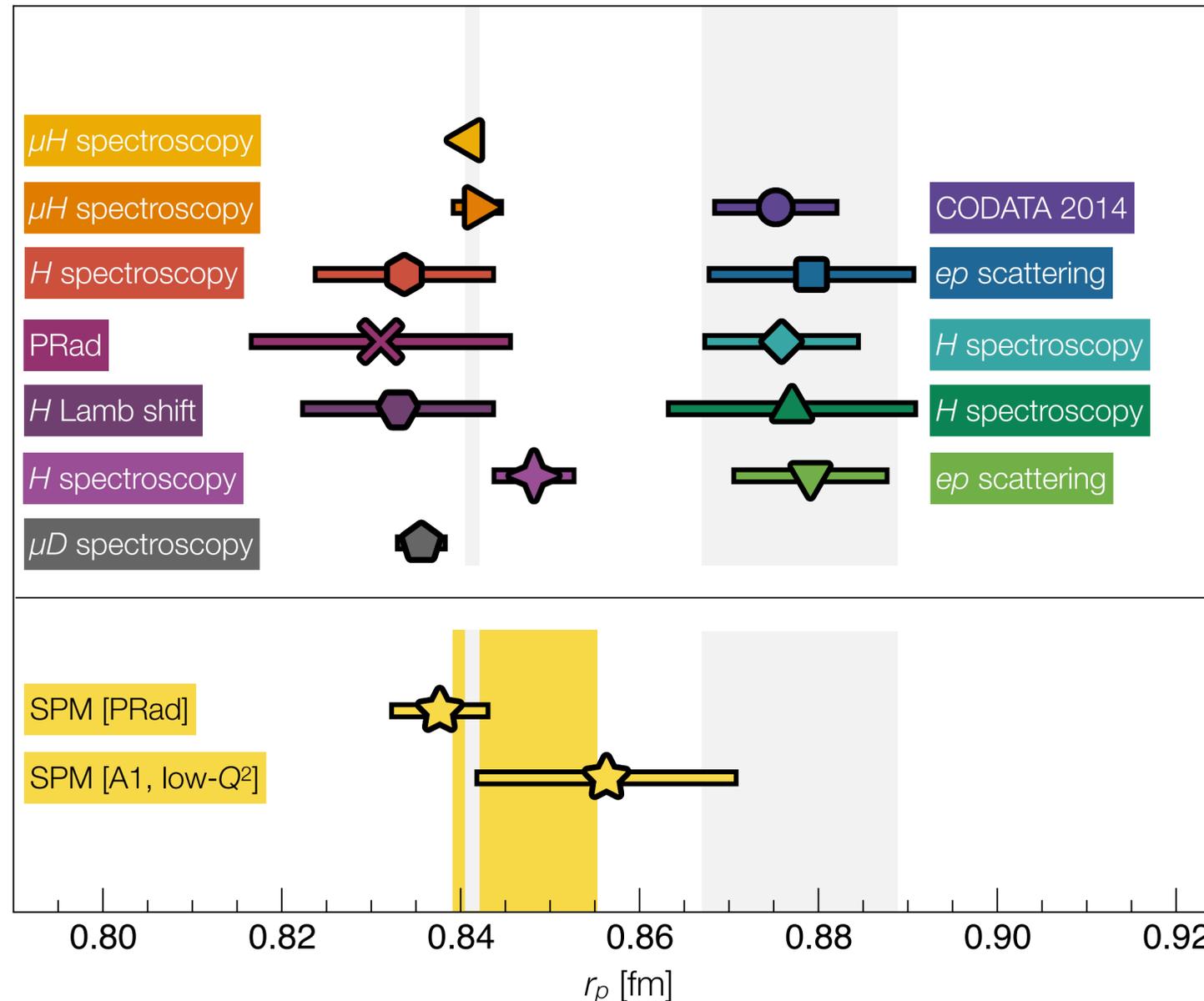
$$3.8 \times 10^{-3} \leq Q^2 / [\text{GeV}^2] \leq 1$$

use first 40 low- $Q^2$  data

$$r_p^{\text{A1-low}Q^2} = 0.856 \pm 0.014_{\text{stat}} \text{ [fm]}$$

all data yield

$$r_p^{\text{A1}} = 0.857 \pm 0.021_{\text{stat}} \text{ [fm]}$$



P. J. Mohr *et al.*, Rev. Mod. Phys. 88, 035009 (2016)

$ep$  average from P. J. Mohr *et al.*, Rev. Mod. Phys. 88, 035009 (2016)

$H$  spectroscopy average from P. J. Mohr *et al.*, Rev. Mod. Phys. 88, 035009 (2016)

H. Fleurbaey *et al.*, Phys. Rev. Lett. 120, 183001 (2018)

J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)

A. Antognini *et al.*, Science 339, 417 (2013)

R. Pohl *et al.*, Nature 466, 213 (2010)

A. Beyer *et al.*, Science 358, 79 (2017)

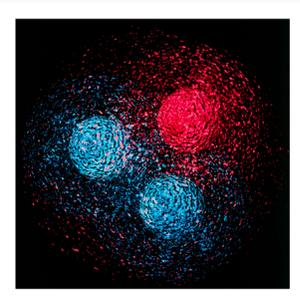
W. Xiong *et al.*, Nature 575, 147 (2019)

N. Bezginov *et al.*, Science 365, 1007 (2019)

A. Grinin *et al.*, Science 370, 1061 (2020)

R. Pohl *et al.*, Science 353, 669 (2016)

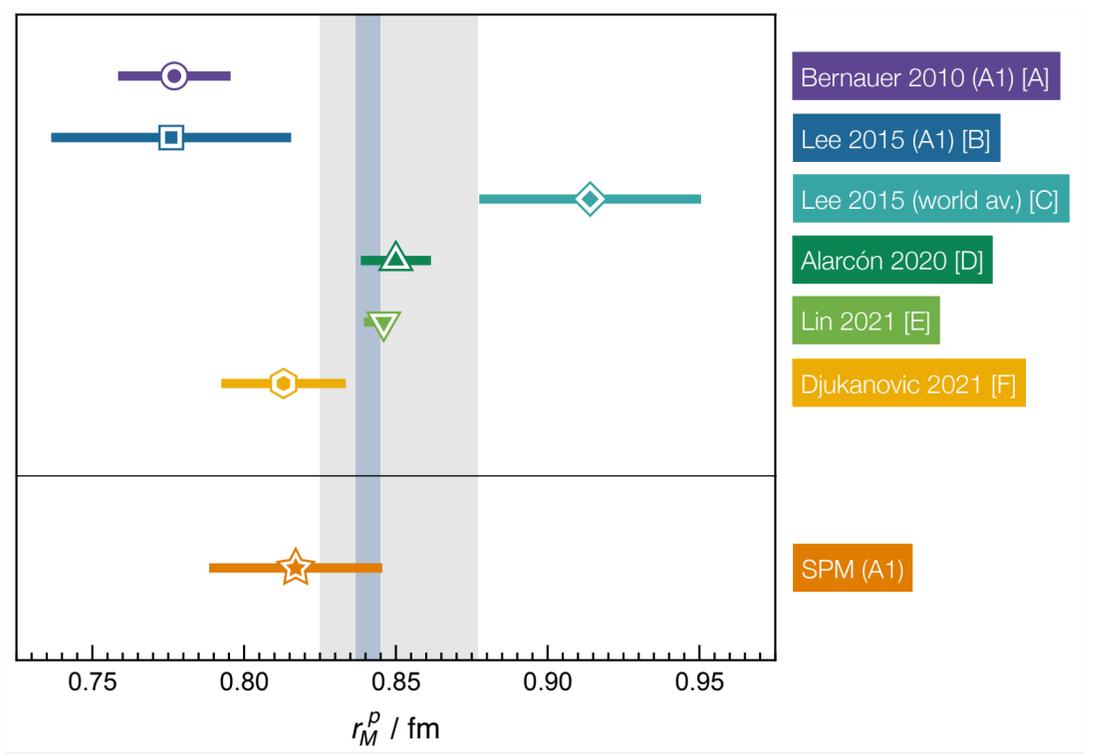
# Proton PAULI RADIUS



## 1 A1 DATA

$$r_M^{A1} = 0.817 \pm 0.027_{\text{stat}} \text{ [fm]}$$

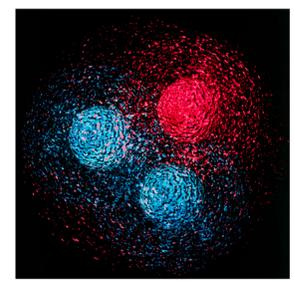
Cui *et al*, 2109.08768



$$\frac{F_2'(0)}{\kappa_p} \simeq F_1'(0) - \frac{\mu_p}{4m_p^2}$$

# Proton

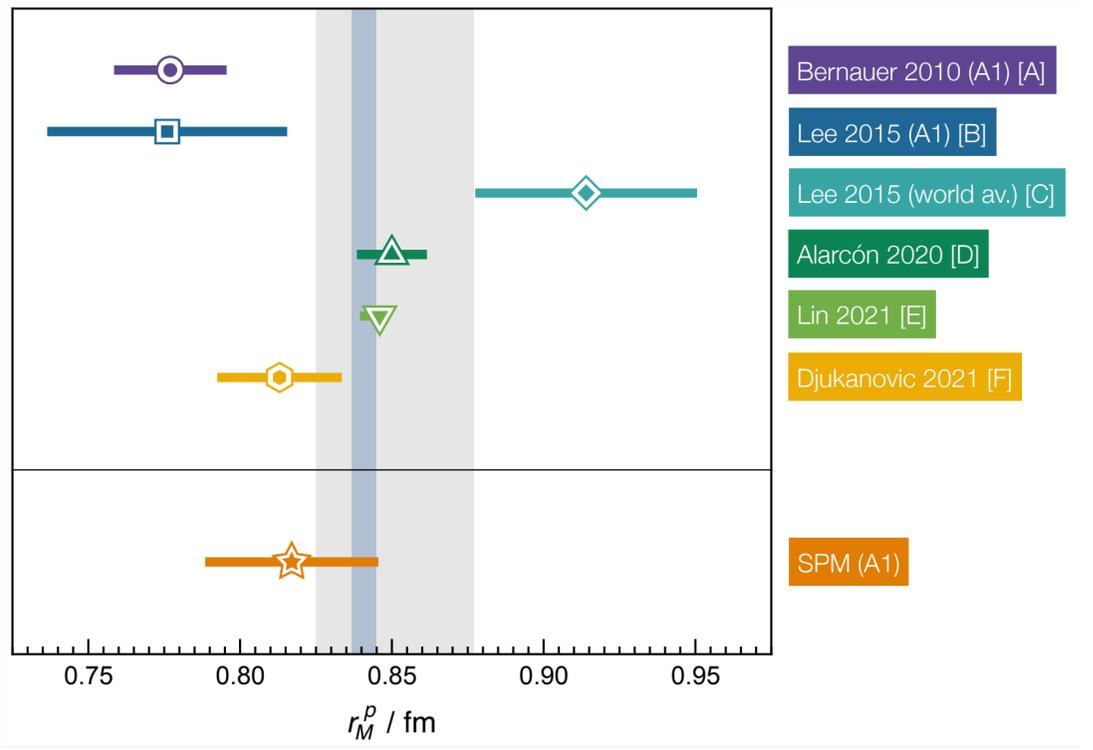
## PAULI RADIUS



### 1 A1 DATA

$$r_M^{A1} = 0.817 \pm 0.027_{\text{stat}} \text{ [fm]}$$

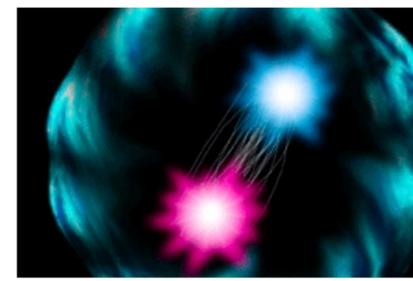
Cui et al, 2109.08768



$$\frac{F_2'(0)}{\kappa_p} \simeq F_1'(0) - \frac{\mu_p}{4m_p^2}$$

# Pion

## CHARGE RADIUS

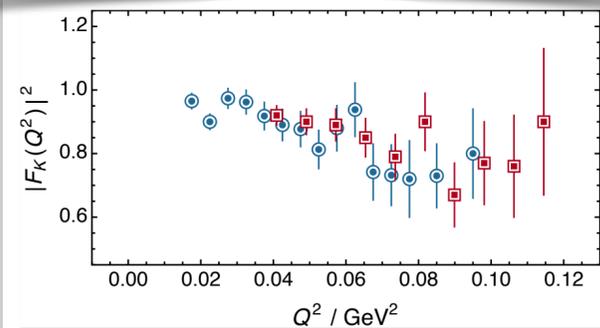
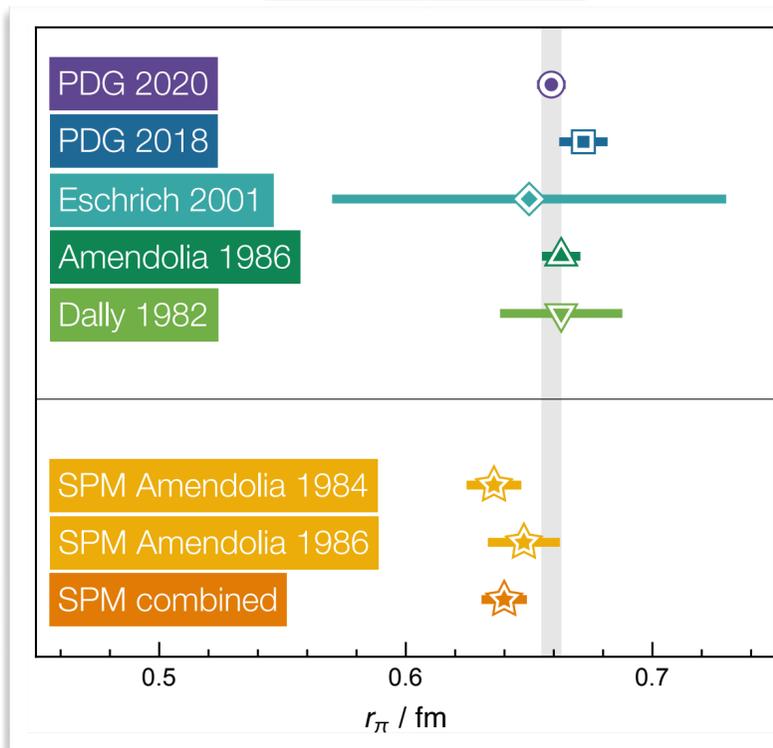


### 1 NA7 DATA

$$r_\pi^{\text{NA7-86}} = 0.648 \pm 0.013_{\text{stat}} \text{ [fm]}$$

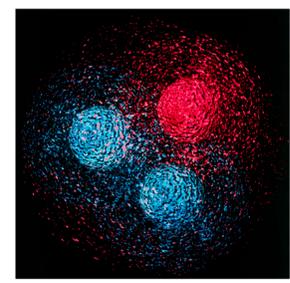
$$r_\pi^{\text{NA7-84}} = 0.636 \pm 0.009_{\text{stat}} \text{ [fm]}$$

Cui et al, PLB 822 (2021)



# Proton

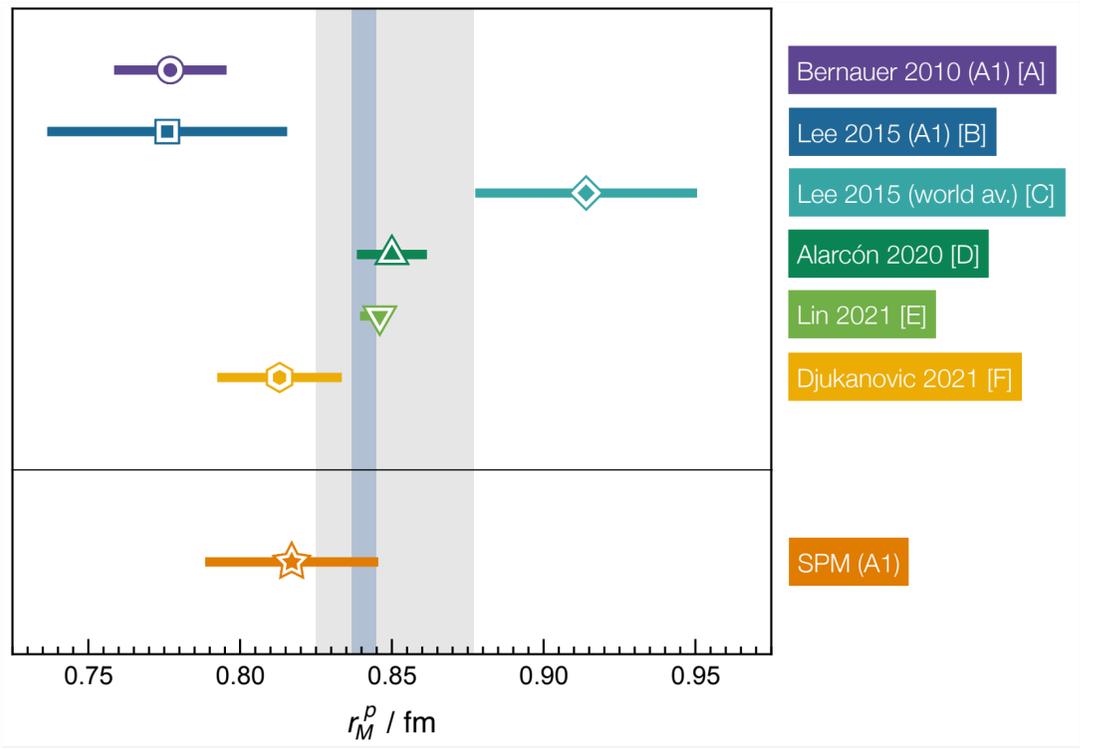
## PAULI RADIUS



### 1 A1 DATA

$$r_M^{A1} = 0.817 \pm 0.027_{\text{stat}} \text{ [fm]}$$

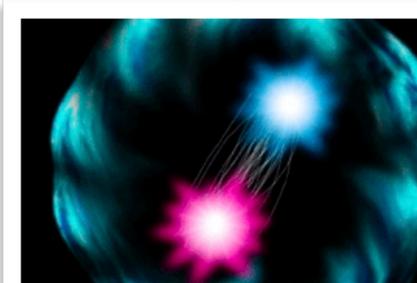
Cui et al, 2109.08768



$$\frac{F_2'(0)}{\kappa_p} \simeq F_1'(0) - \frac{\mu_p}{4m_p^2}$$

# Pion

## CHARGE RADIUS

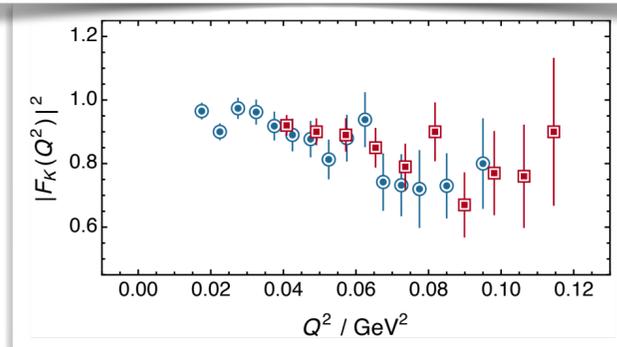
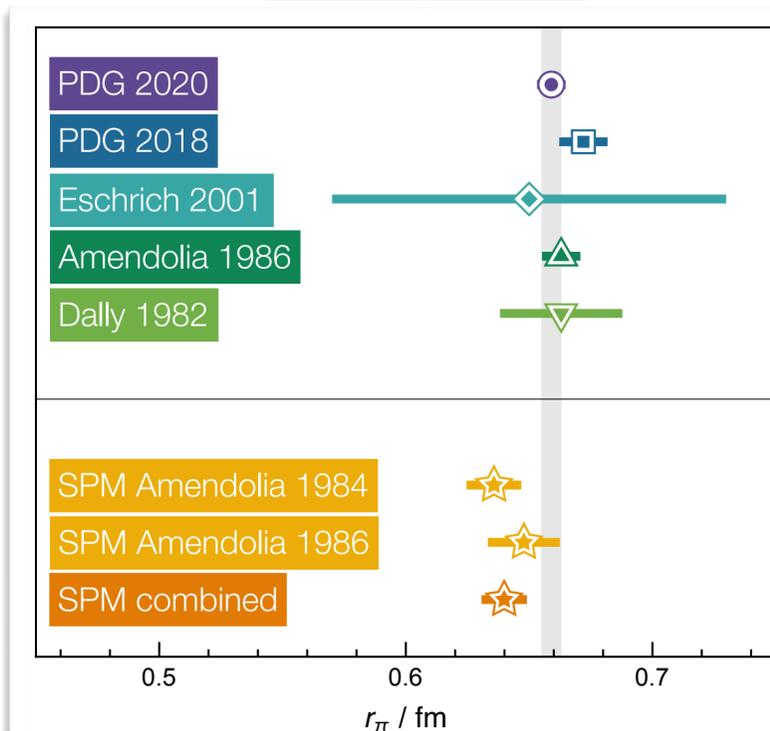


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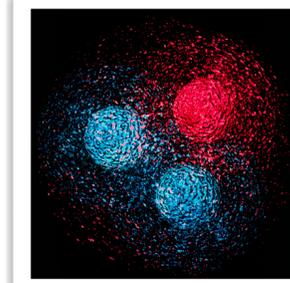
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Cui et al, PLB 822 (2021)

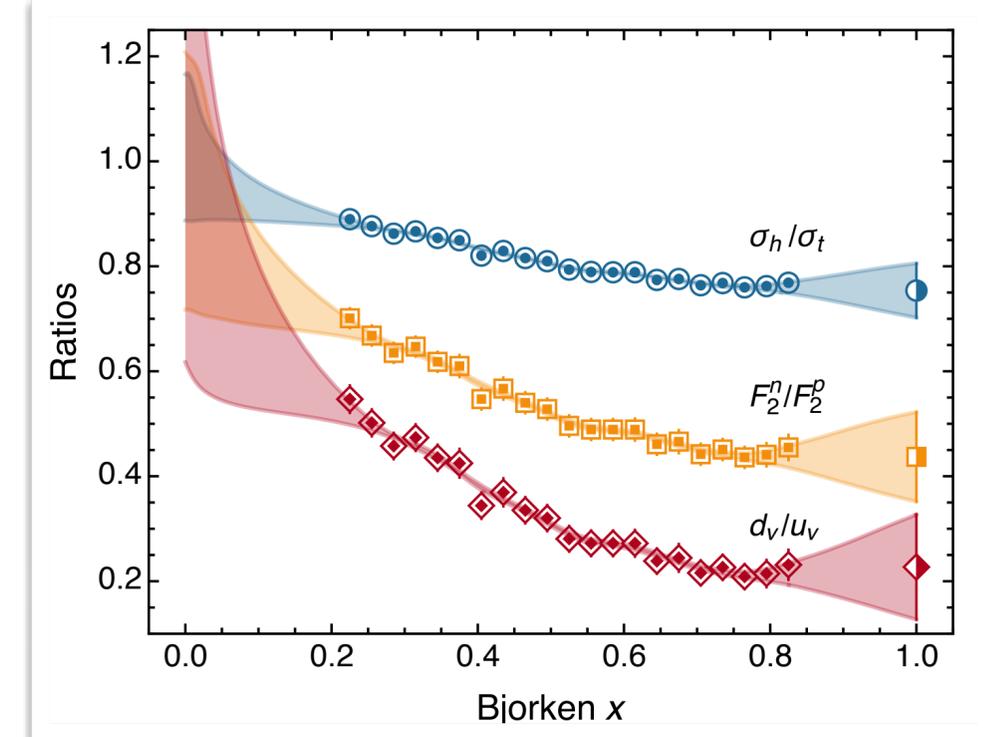


# Proton

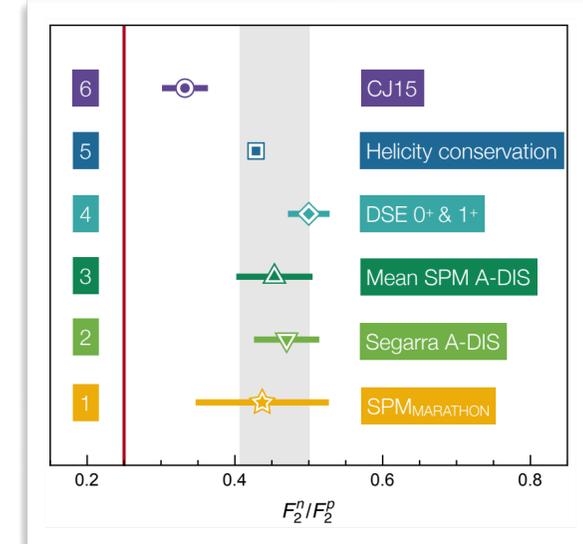
## STRUCTURE



### 1 MARATHON DATA



$$\lim_{x \rightarrow 1} \frac{d_v(x)}{u_v(x)} = 0.230 \pm 0.057$$



# EMERGENT PHENOMENA

# IN STRONG DYNAMICS

# 3

DANIELE BINOSI

ECT\* - FONDAZIONE BRUNO KESSLER

**Baryons 21 Sevilla**

OCTOBER 18 - 22



**ECT\***  
EUROPEAN CENTRE  
FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS  
FONDAZIONE BRUNO KESSLER

**FBK**  
FONDAZIONE  
BRUNO KESSLER

