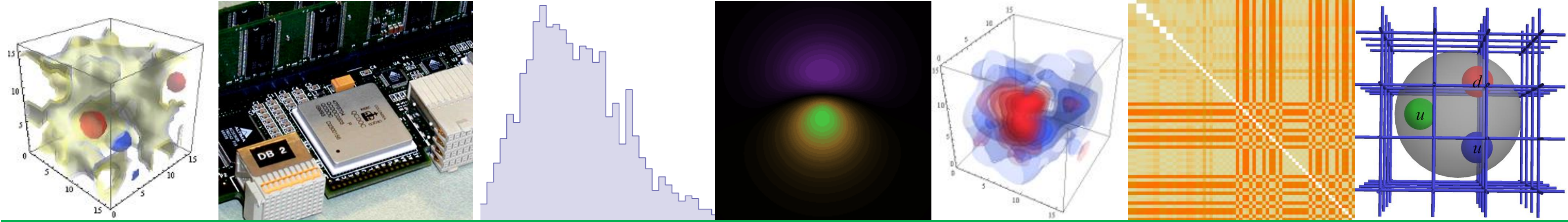
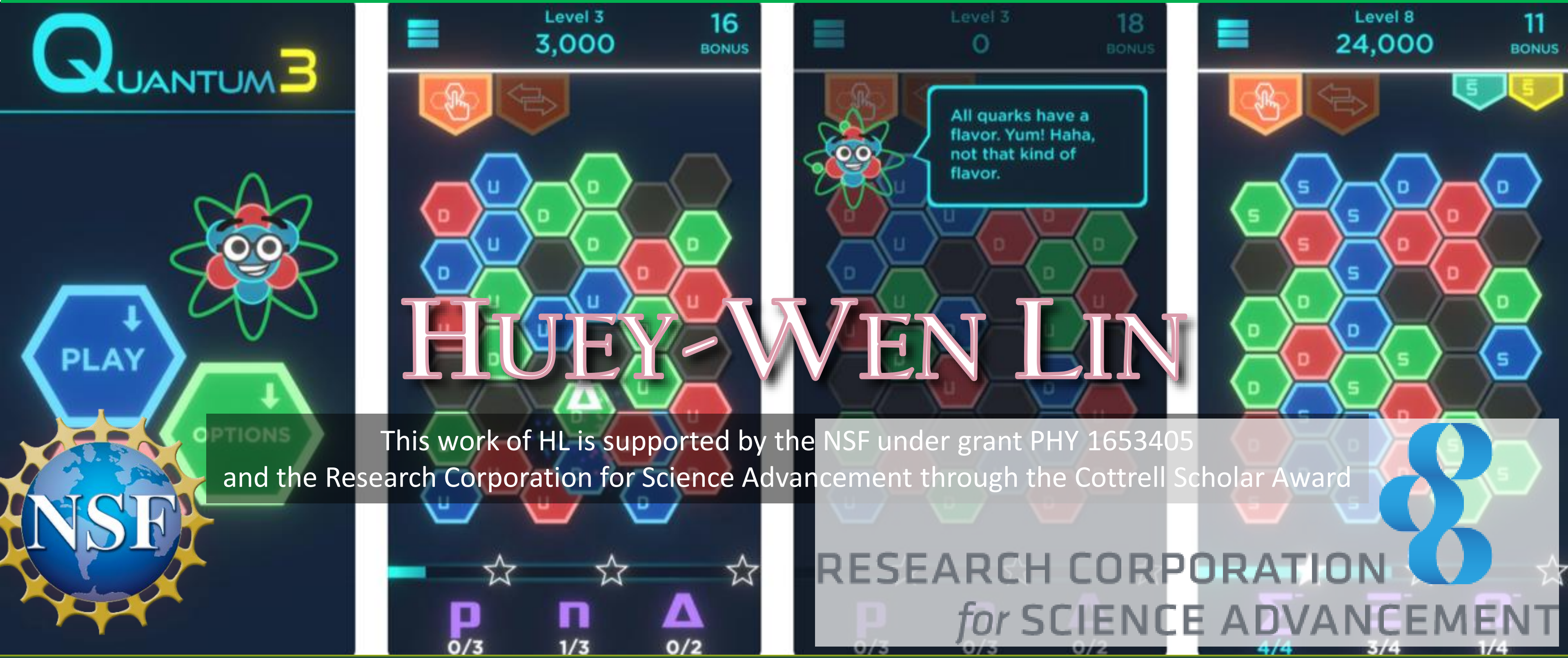


Overview of lattice results for hadron spectroscopy and structure I



Huey-Wen Lin

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

NSF

RESEARCH CORPORATION *for* SCIENCE ADVANCEMENT

Outline

§ Lattice QCD 101

§ How to do spectroscopy calculations on the lattice

- "Simple" spectroscopy
- Excited-state spectroscopy

5 mins coffee-refill break

§ Hadron Structure

Lattice QCD 101

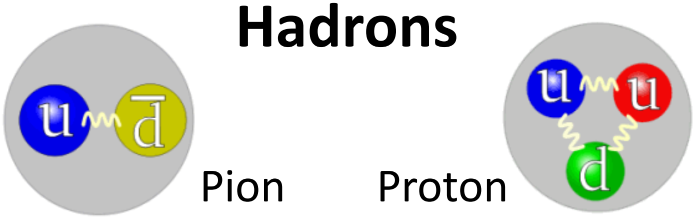
The Color Force

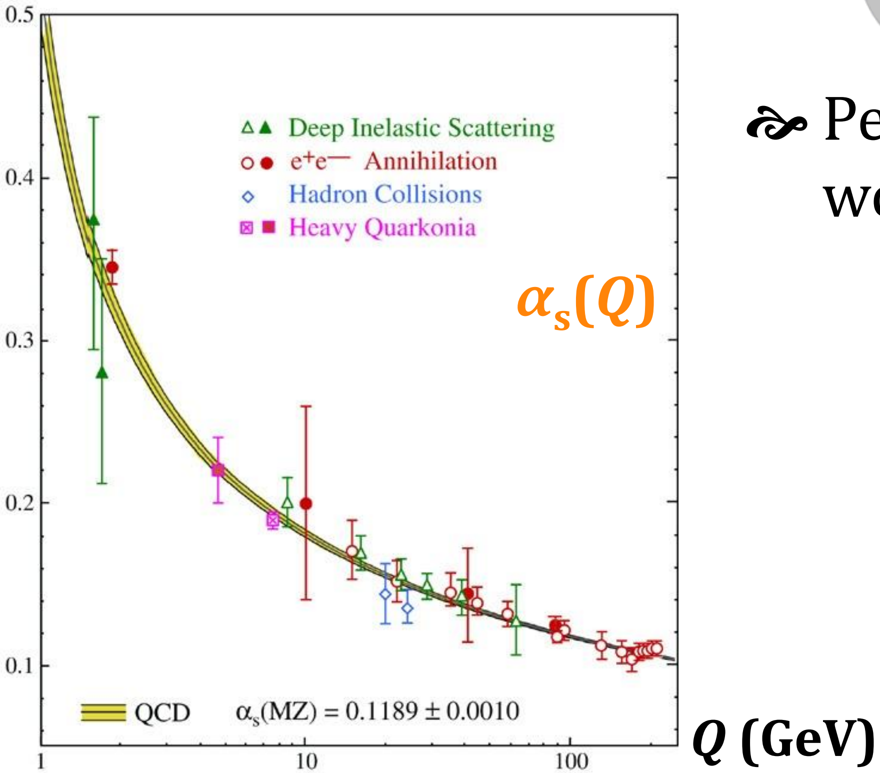
§ Quantum chromodynamics (QCD) in theoretical physics

§ The strong interactions of *quarks* and *gluons* (SU(3) gauge)

- "Confinement"
  no free quarks allowed

Hadrons





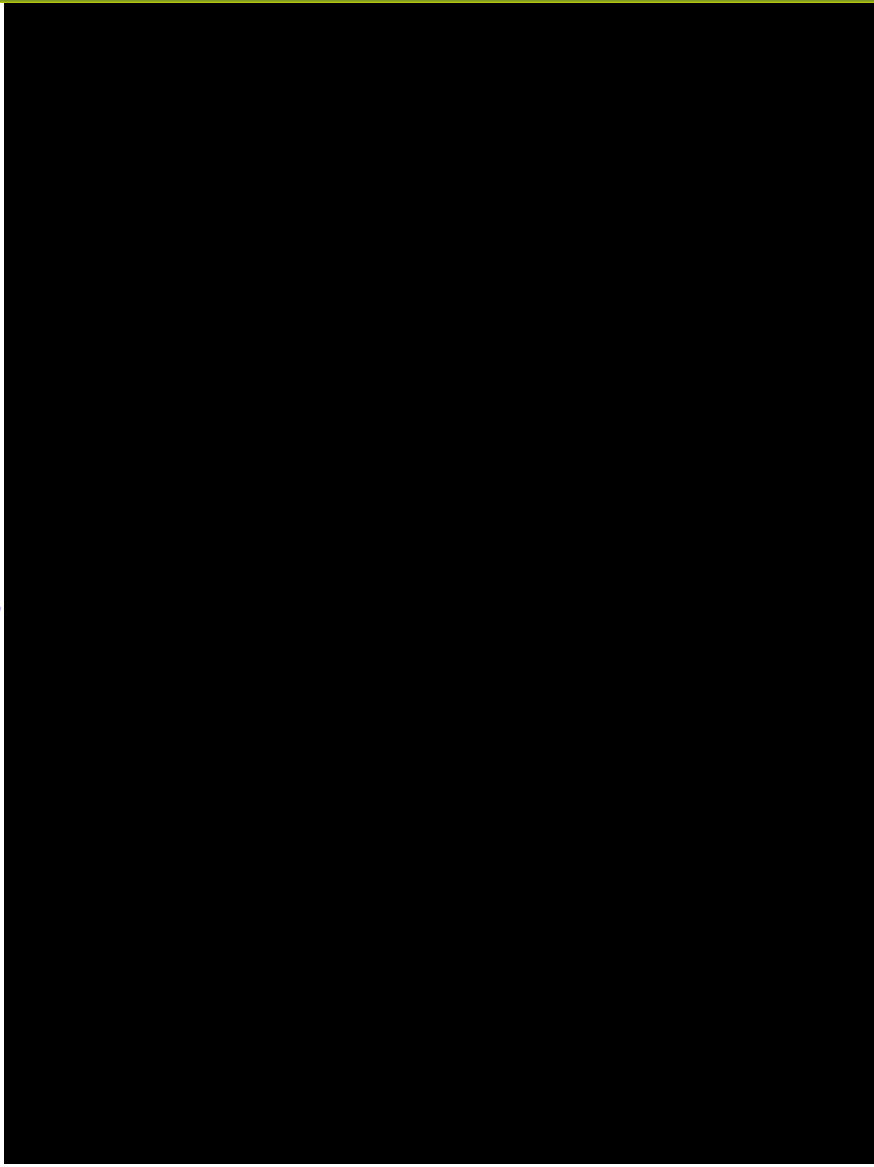
- Perturbation theory (like QED) works well at high energies

The Nobel Prize in Physics 2004

Learn QCD on Your Phone!

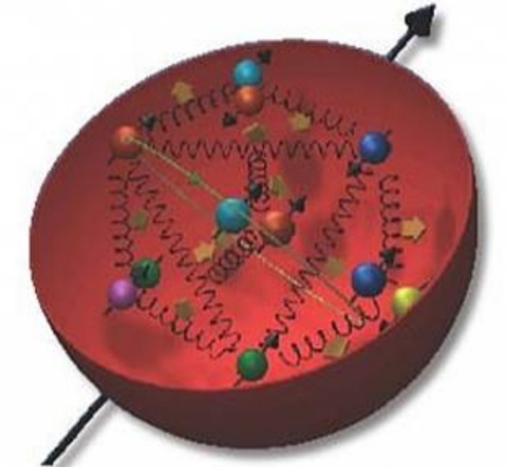
- QCD is well established but remains little publicized
- We developed a game to teach kids about it
  - Learn QCD on your phone

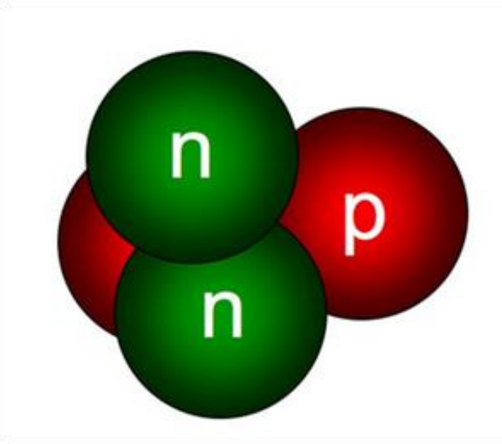
Google Play Store   Apple Appstore

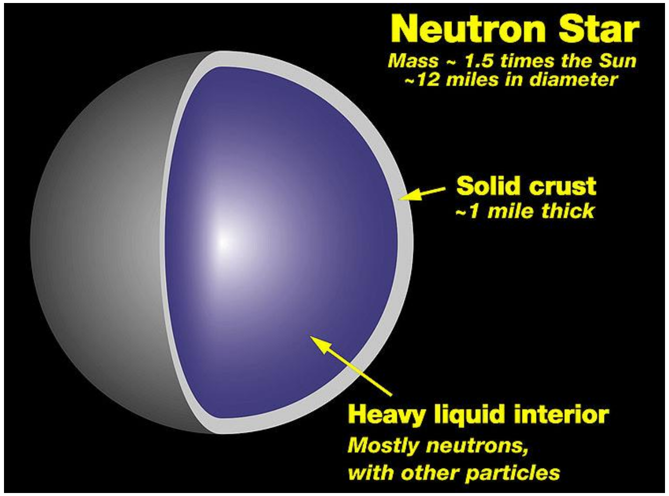


Wide-Scale Applications

§ What can we learn from it?







10^{-15} m

10^{4} m

Parton distribution functions
Properties for
new-physics searches

HWL et al, 1402.1462, 1506.06411; 1506.04196 …

Nuclei and why we exist

HWL et al, 1409.3556, 1206.5219, 1109.2889, 1012.3812 …

Neutron matter
How they evolve

HWL et al, 1204.3606

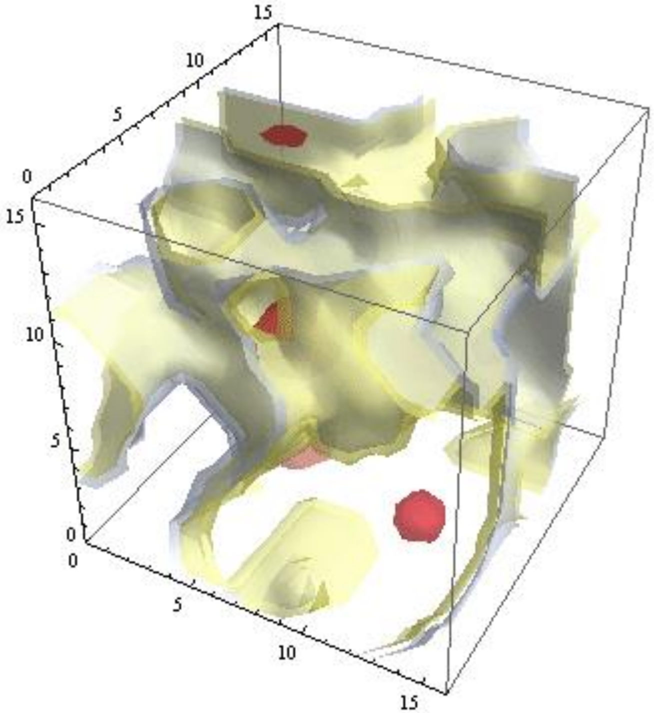
§ HOWEVER…

Difficulties at Low Energy

§ Even just the vacuum of QCD is complicated

Classical

QCD

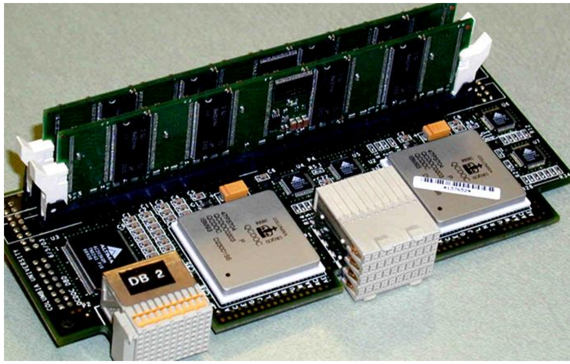


Difficulties at Low Energy

- Strong interactions make analytic calculation impossible

- Direct QCD calculation is desired
  → Lattice QCD

→ **0100101010 10111010...** →



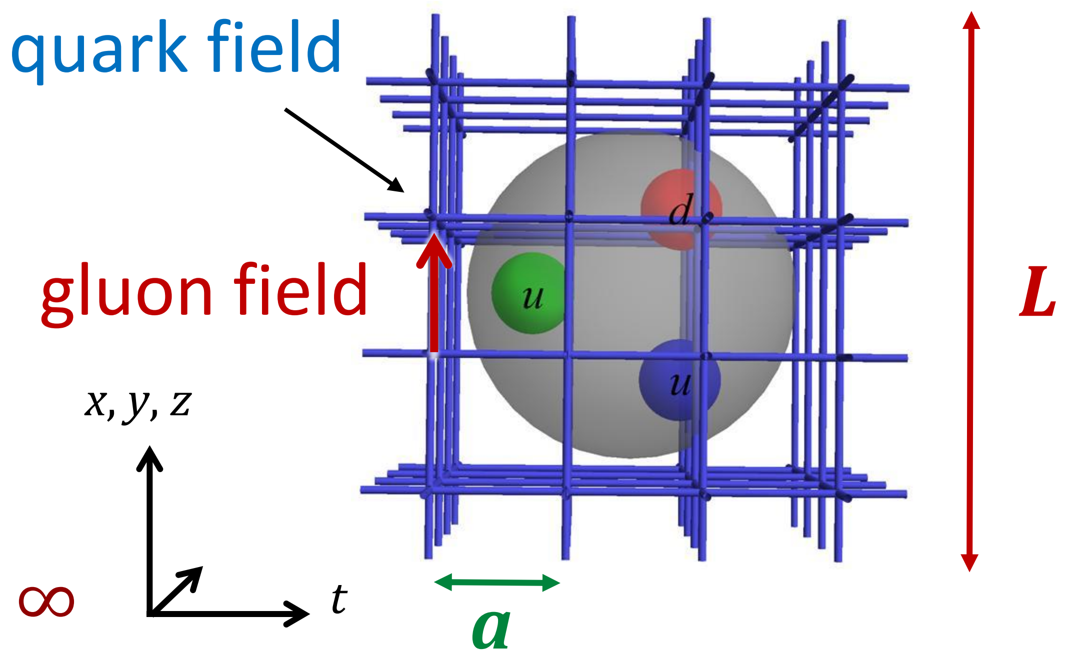
Introducing the Lattice

- Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\left(\bar{\psi},\psi,A\right)$$

in **Euclidian** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Are We There Yet?

- § Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- § Progress is limited by computational resources

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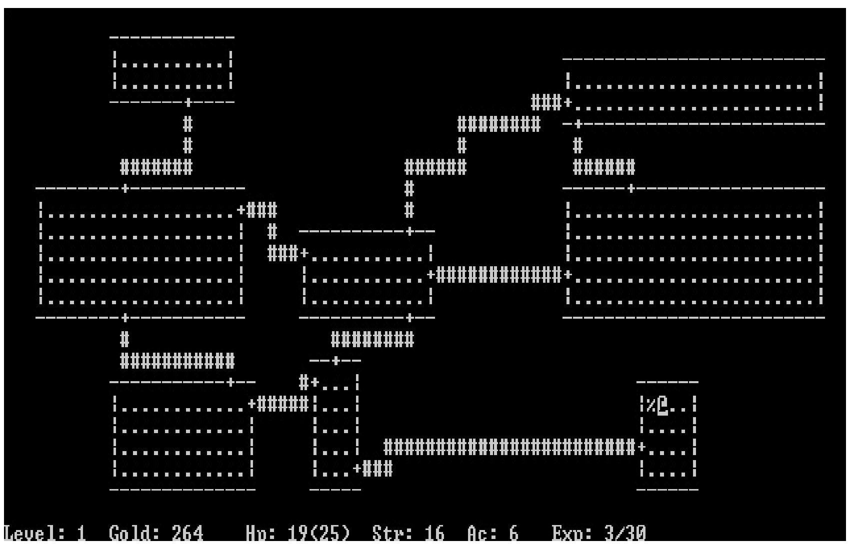
1980s



Today

- § Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

FNAL pi0 Cluster

Oak Ridge National Lab

Hopper@NERSC

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

If you are young and energetic…


QCDOC


FNAL pi0 Cluster

Oak Ridge National Lab


Hopper@NERSC

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

Online tutorials available:

http://www.int.washington.edu/PROGRAMS/12-2c/

Week 3 (Aug. 20–24, 2012)

- "Hadron Structure", James Zanotti (University of Adelaide)
- "Lattice QCD+QED", Taku Izubuchi (BNL)
- "Computational Lattice QCD", Balint Joo (Thomas Jefferson Lab)
  - Exercises: seattle_tut.tar.gz (2012 Aug 24)
  - Code: package-int.tar.gz (110 MB)
  - Lecture 1: Slides Video
  - Lecture 2: Slides Video
  - Lecture 3: Slides Video
  - Lecture 4: Slides Video
  - Lecture 5: Slides Video
- "Extreme Computing Trilogy: Nuclear Physics", Martin Savage (University of Washington)
- "Cold Atoms and Unitary Fermi Gas", Michael M. Forbes (INT)
- "Introduction to GPU Computing", Mike Clark (NVIDIA)
- "Introduction to QUDA — GPU Computing for LQCD", Mike Clark (NVIDIA)
- "Extreme Computing Trilogy: Infrastructure", Kenneth Roche (PNNL)

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)

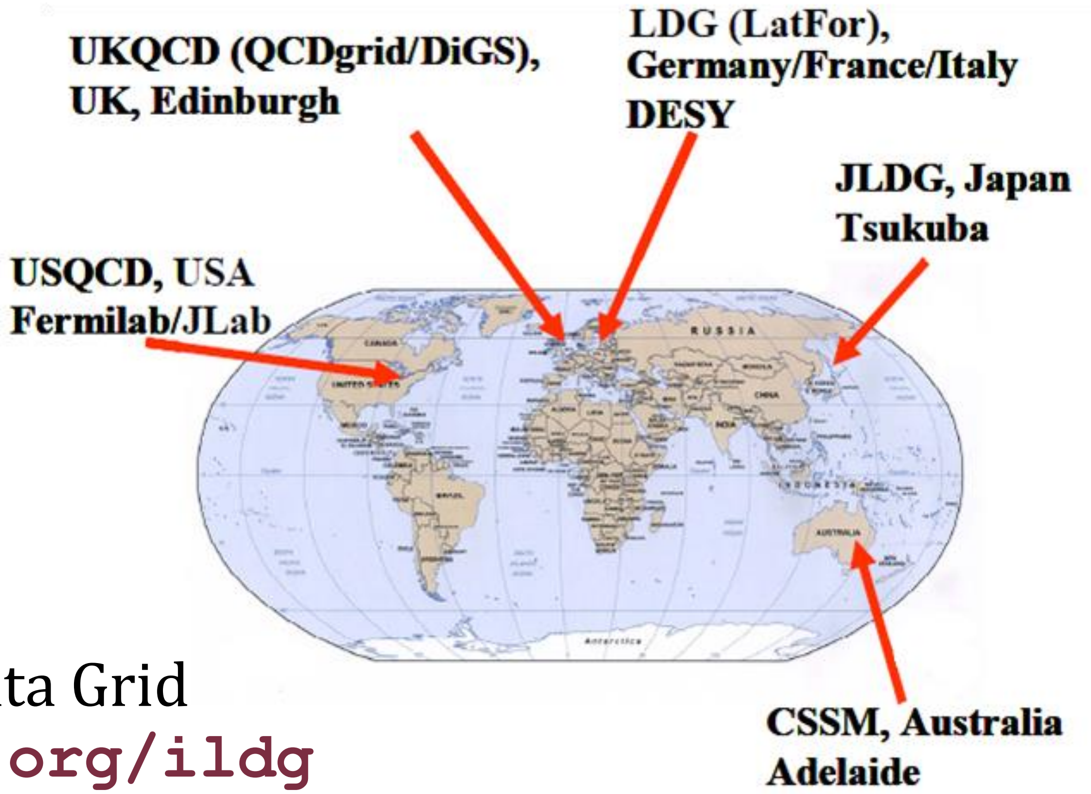
Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

2. Some QCD Vacuum (gauge configurations)

- WWW (ILDG)

International Lattice Data Grid
`http://www.usqcd.org/ildg`

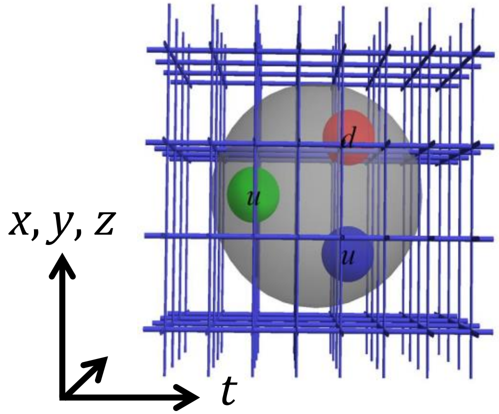


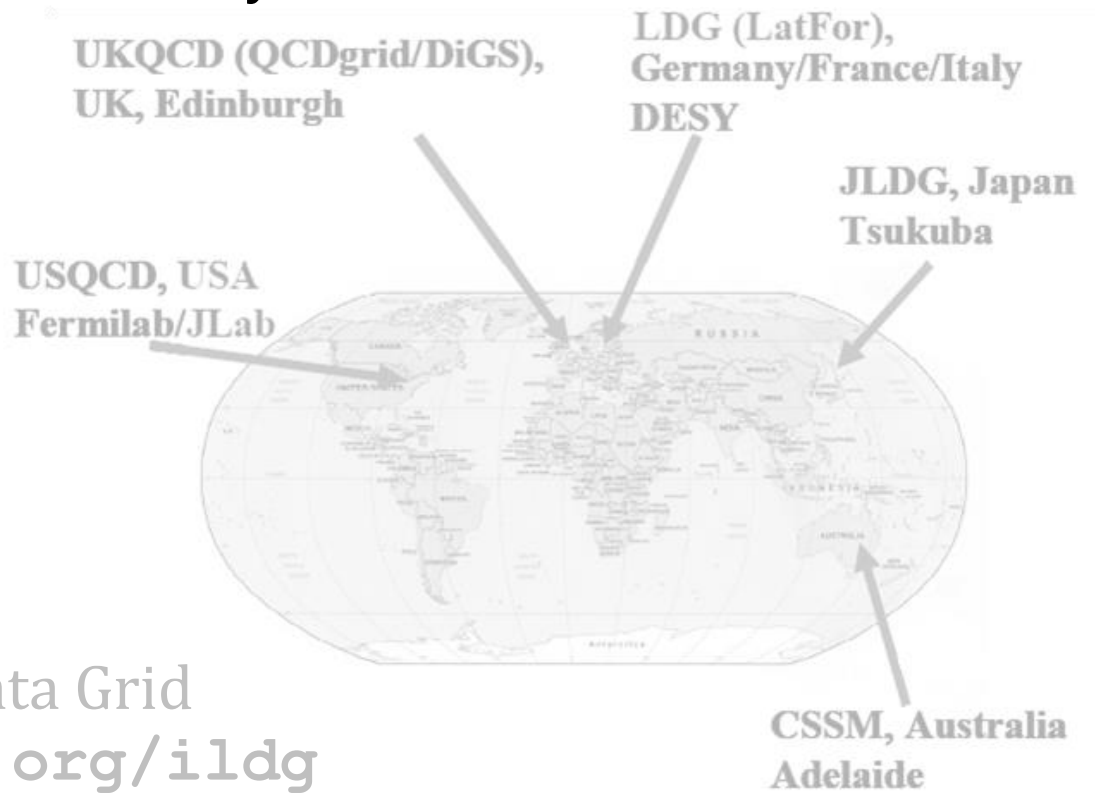
Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

2. Some QCD Vacuum (gauge configurations)

- WWW (ILDG) or generate it yourself 0810.3588 200+



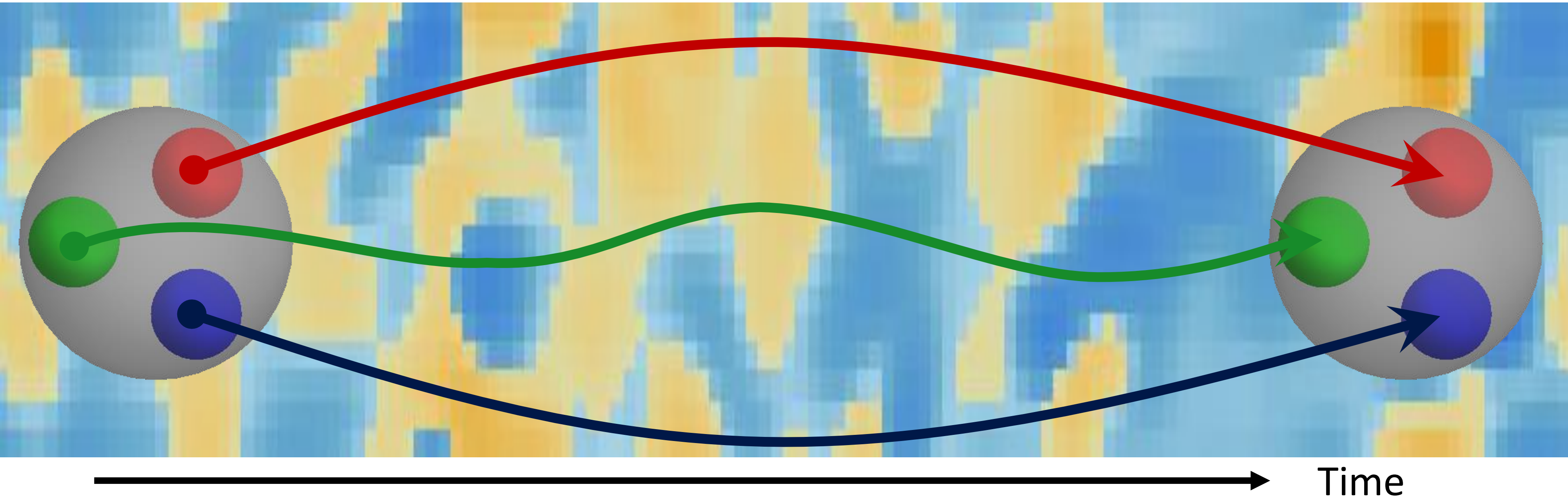


ILDG

International Lattice Data Grid
http://www.usqcd.org/ildg

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
   - Invert Dirac operator matrix (rank 10^{12})
   - Combine using color, spin and momentum into hadrons

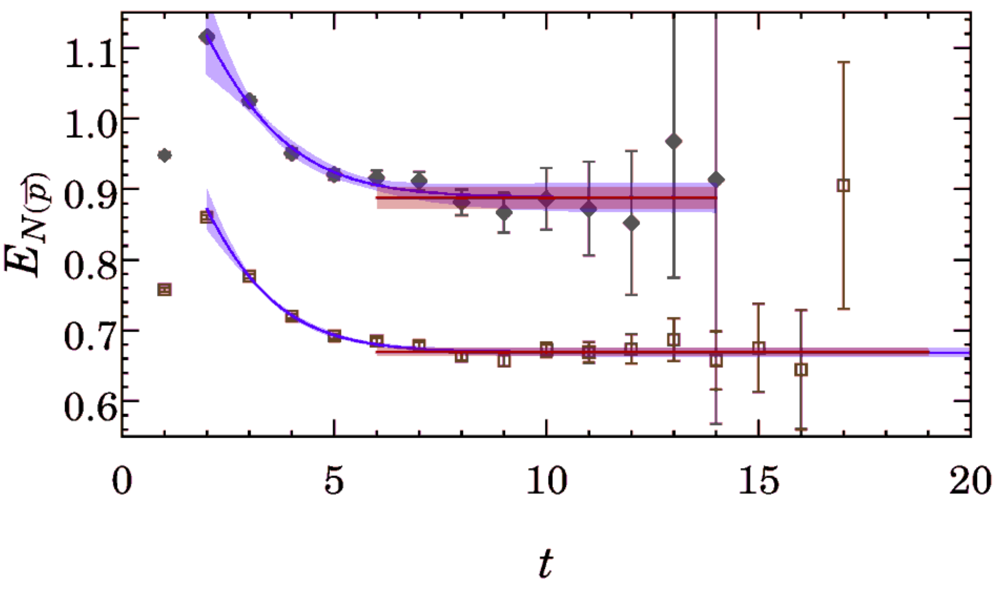


Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
4. **Analysis** (extraction of masses or couplings)

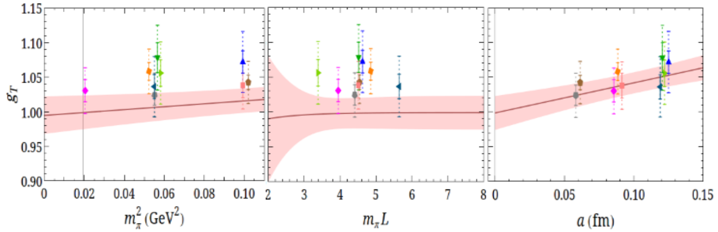
$$\langle J_N J_N \rangle = \sum_n \langle J_N | n \rangle \langle n | J_N \rangle e^{-E_n t}$$

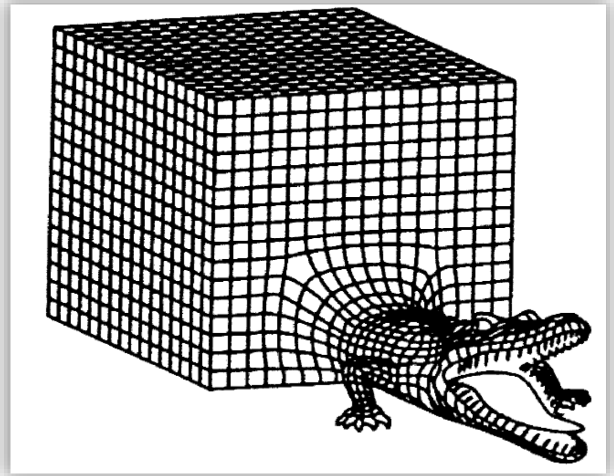
$$M_{\text{eff}} = -\log\left(\frac{\langle J_N J_N \rangle (t+1)}{\langle J_N J_N \rangle (t)}\right)$$



Anatomy of a Lattice Calculation

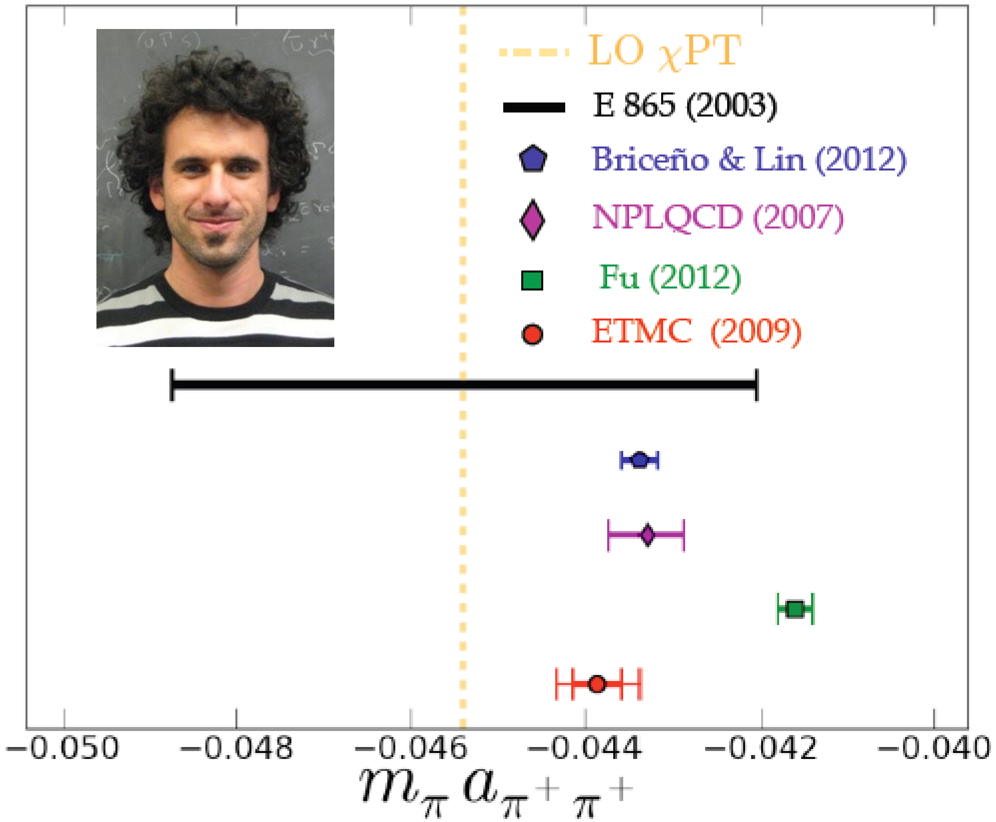
1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
4. Analysis (extraction of masses or couplings)
5. Systematic Uncertainty (nonzero *a*, finite *L*, etc.)
   - Contamination from excited states
   - Nonperturbative renormalization e.g. RI/SMOM scheme
   - Extrapolation to the continuum limit $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$





Successful Examples

- § Provide higher precision for known quantities
- § Make a lot of mass predictions



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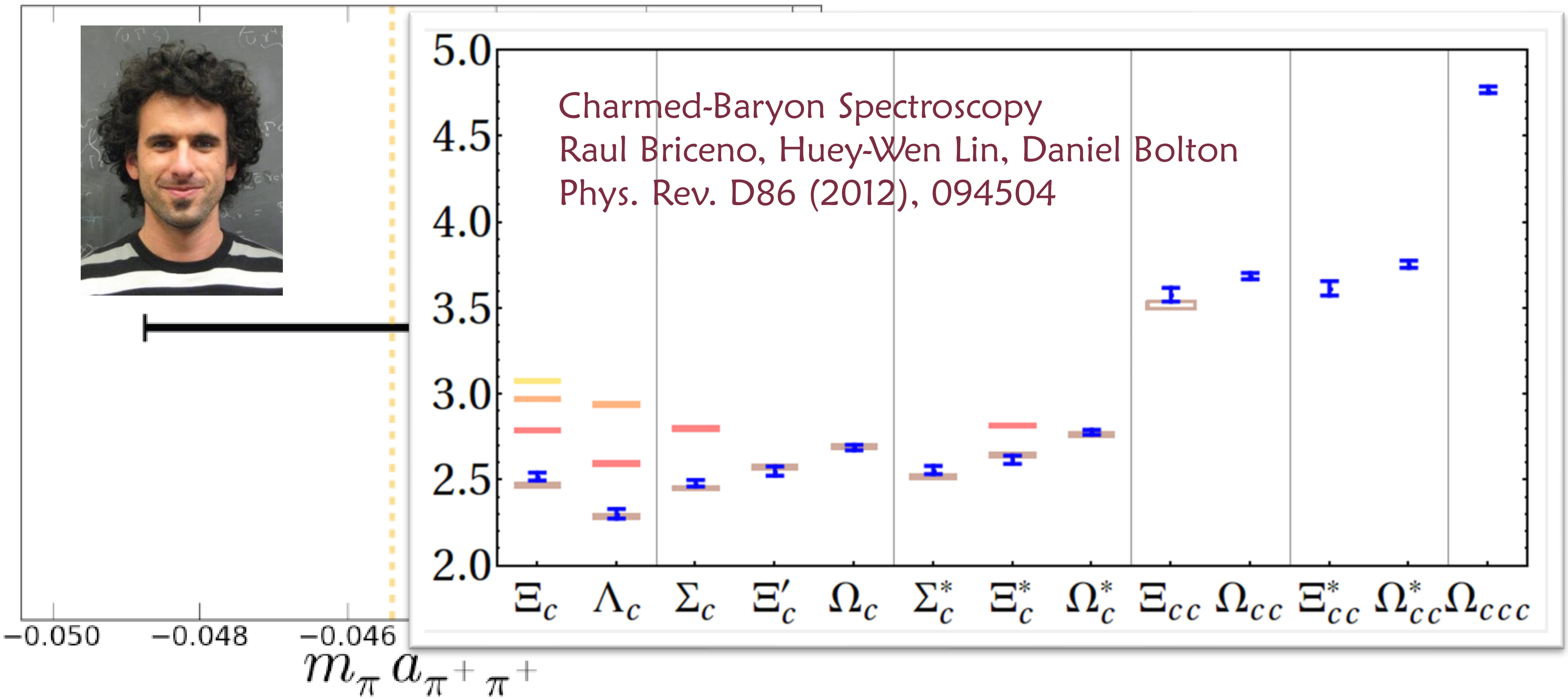


Explore the Unknown

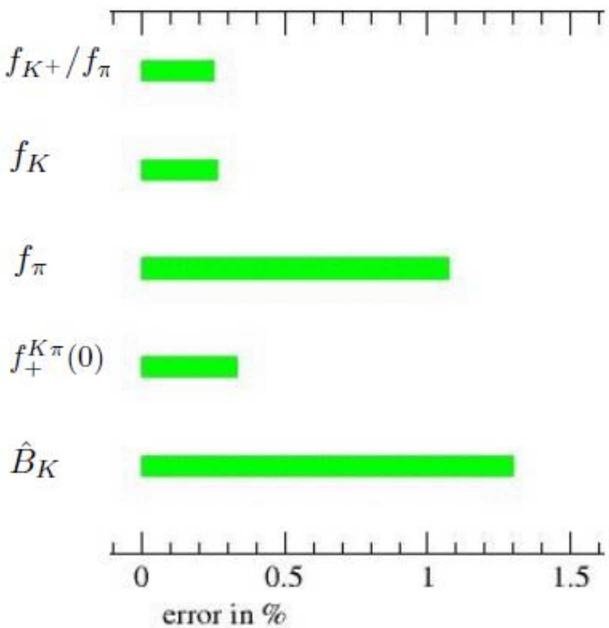
§ To discern new physics, we need to know the SM well

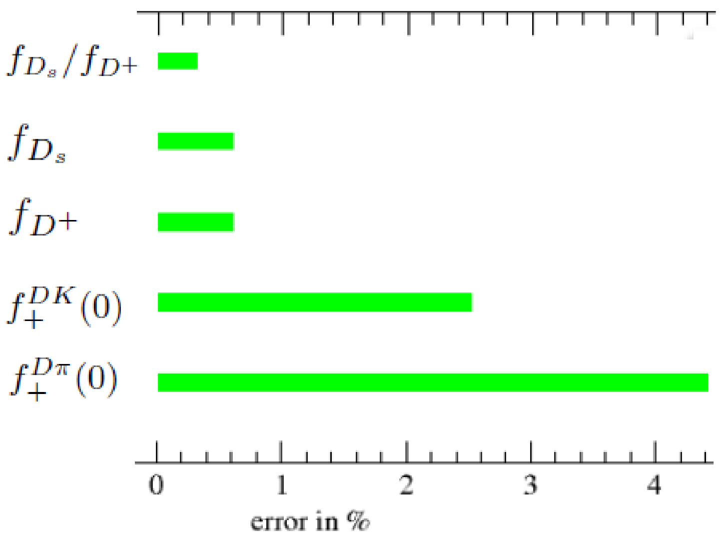
§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

- Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages





We are beginning to do precision calculations in nucleons

Topics in QCD Spectroscopy

Operators and Correlators

§ Simple hadron operators

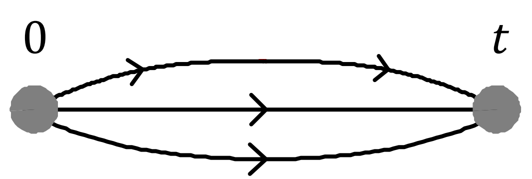
- $\pi^+(x) = \bar{u}^a(x)\gamma_5 d^a(x)$
- $\rho^+(x) = \bar{u}^a(x)\gamma_\mu d^a(x)$
- $p_\delta(x) = \left(\bar{u}^a(x) C\gamma_5 d^b(x)\right) u^c_\delta \epsilon_{abc}$
- $\Delta^{++}_{\mu,\delta}(x) = \left(\bar{u}^a(x) C\gamma_\mu u^b(x)\right) u^c_\delta \epsilon_{abc}$

§ Correlators

- Study hadron properties by looking at 2-point function

$$C(t) = \sum_x \left\langle O_h\left(\vec{0},0\right) O_h^\dagger(\vec{x},t)\right\rangle$$

$$= \sum_n \underbrace{\langle O_h|n\rangle \left\langle n\middle|O_h^\dagger\right\rangle}_{A_n} e^{-E_n t}$$



Effective Mass

§ Time-series math problem

- $C(t) = \sum_n A_n e^{-E_n t}$
  - $y_n = C(t)$
  - $\alpha_n = e^{-E_n t}$
  - $a_n = A_n$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_K \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_K^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^N & \alpha_2^N & \cdots & \alpha_K^N \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{pmatrix}$$

Effective Mass

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§ Assuming there is only one state

- $\alpha_1 = y_{n+1}/y_n$
- Thus, "effective mass": $M_{\text{eff}} = \ln(y_{n+1}/y_n)$

Effective Mass

§ Time-series math problem

- $C(t) = \sum_n A_n e^{-E_n t}$
  - $y_n = C(t)$
  - $\alpha_n = e^{-E_n t}$
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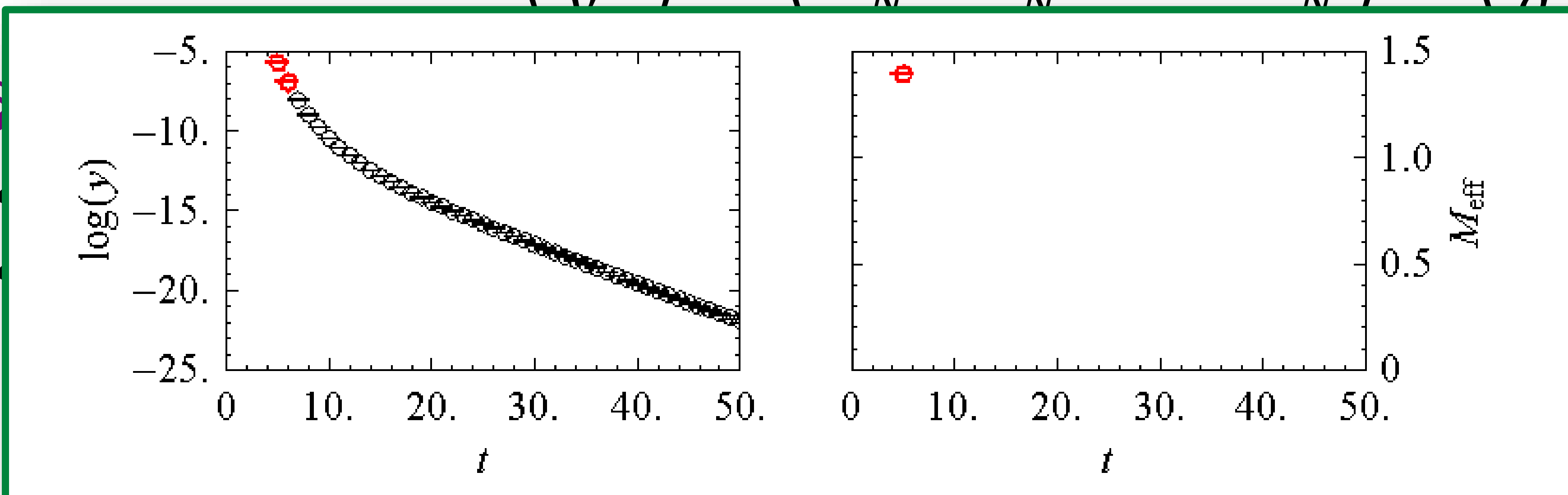
$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_K \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_K^2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1^N & \alpha_2^N & & \alpha_K^N \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{pmatrix}$$



Excited-State Effective Mass

§ Continue and solve for 2 states as

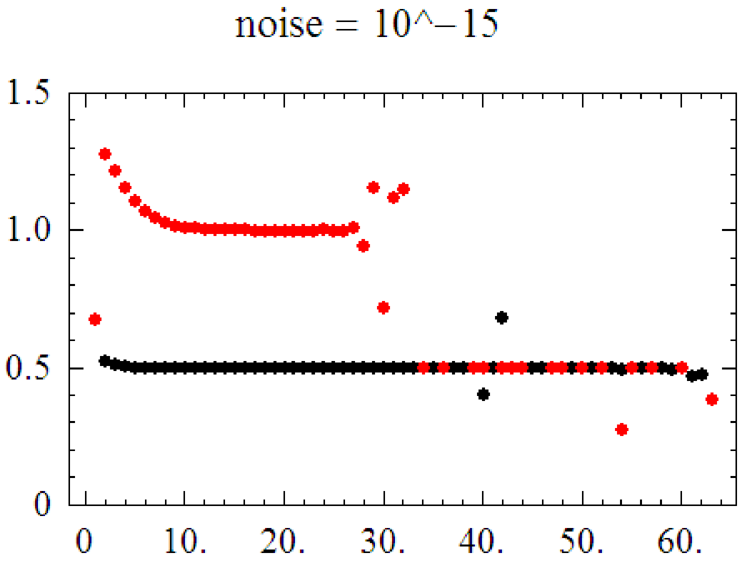
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_1^2 & \alpha_2^2 \\ \alpha_1^3 & \alpha_2^3 \\ \alpha_1^4 & \alpha_2^4 \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

- With the solution

$$\alpha_{1,2} = \frac{y_1 y_4 - y_2 y_3 \pm \sqrt{(y_2 y_3 - y_1 y_4)^2 + 4\left(y_2^2 - y_1 y_3\right)\left(y_2 y_4 - y_3^2\right)}}{2\left(y_1 y_3 - y_2^2\right)}$$

§ Toy model

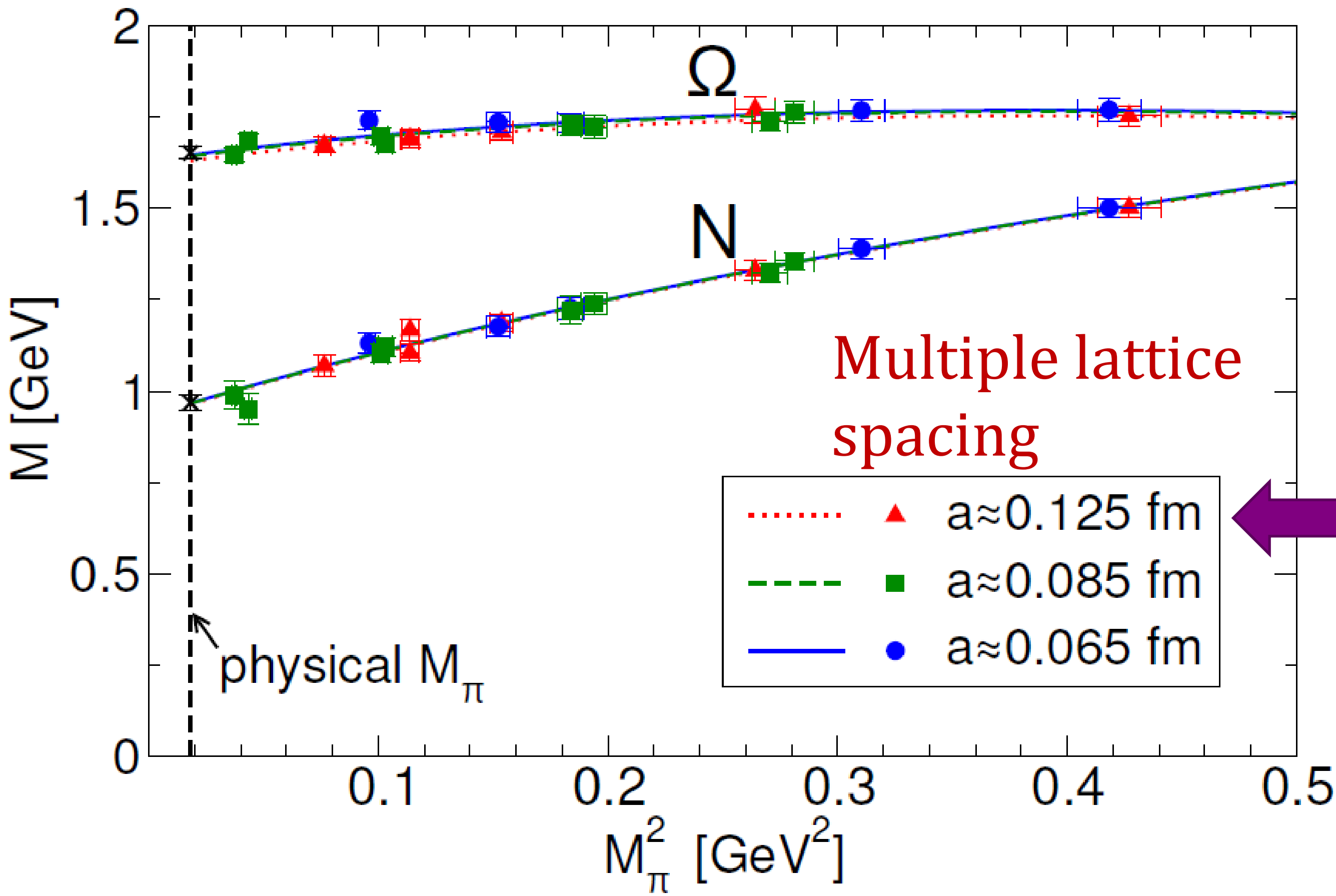
- Consider three states with masses 0.5, 1.0, 1.5 and with the same amplitude



Extrapolation and Finite-a Effect

§ Currently, not running with the physical pion mass

- Example: BMW Collaboration, Science (2008)



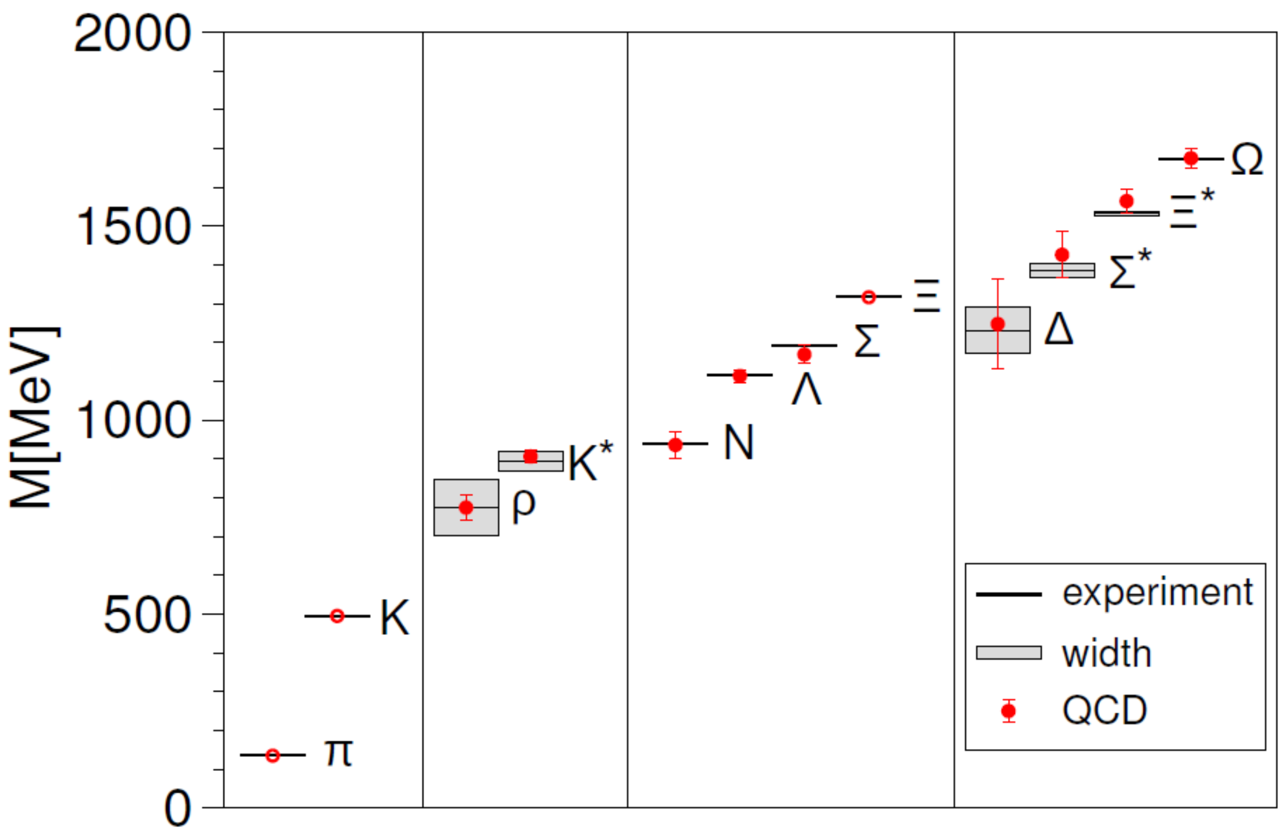
Multiple lattice spacing

Typical starting *a*

Lattice in the News

§ Post-dictions of well known quantities

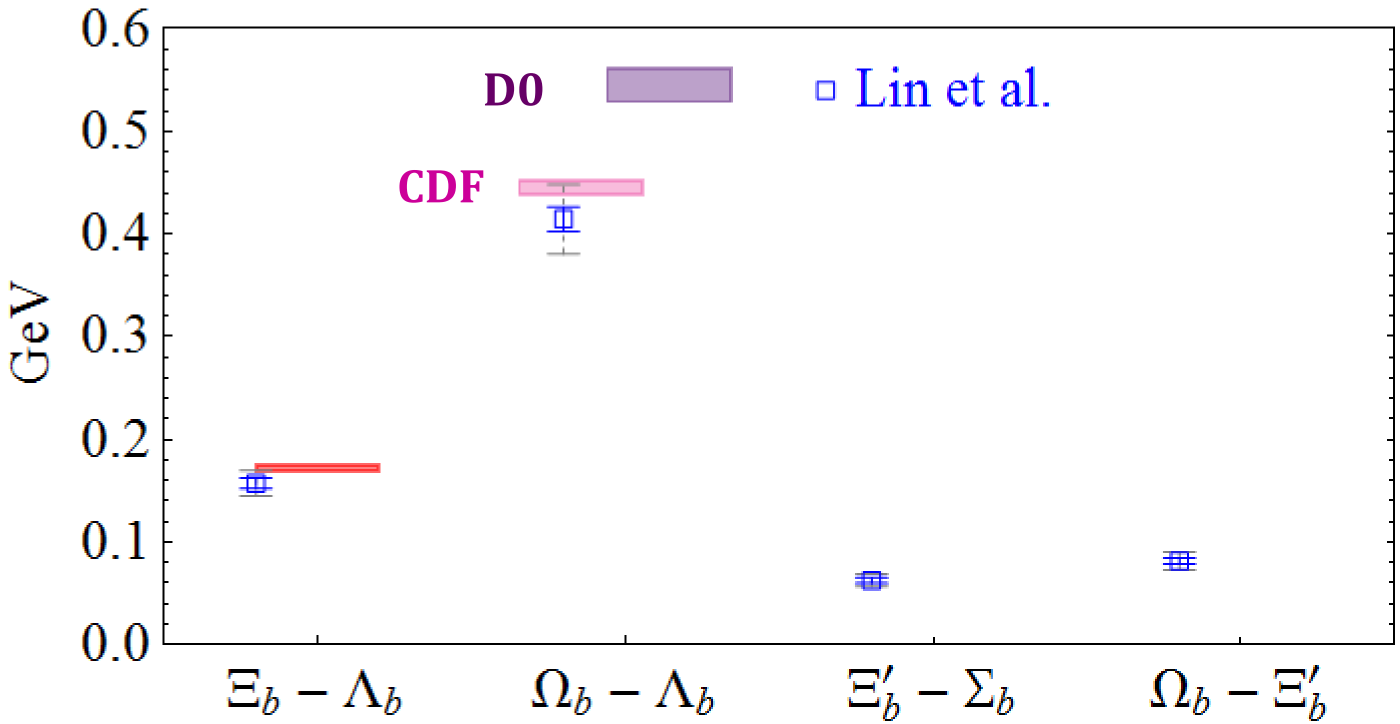
- Example: BMW Collaboration, *Science* 2008



Bottom Baryons

§ Inconsistency in the CDF and DØ results for Ω_b mass

- Our Ω_b agrees with the CDF result



HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009)

Bottom Baryons

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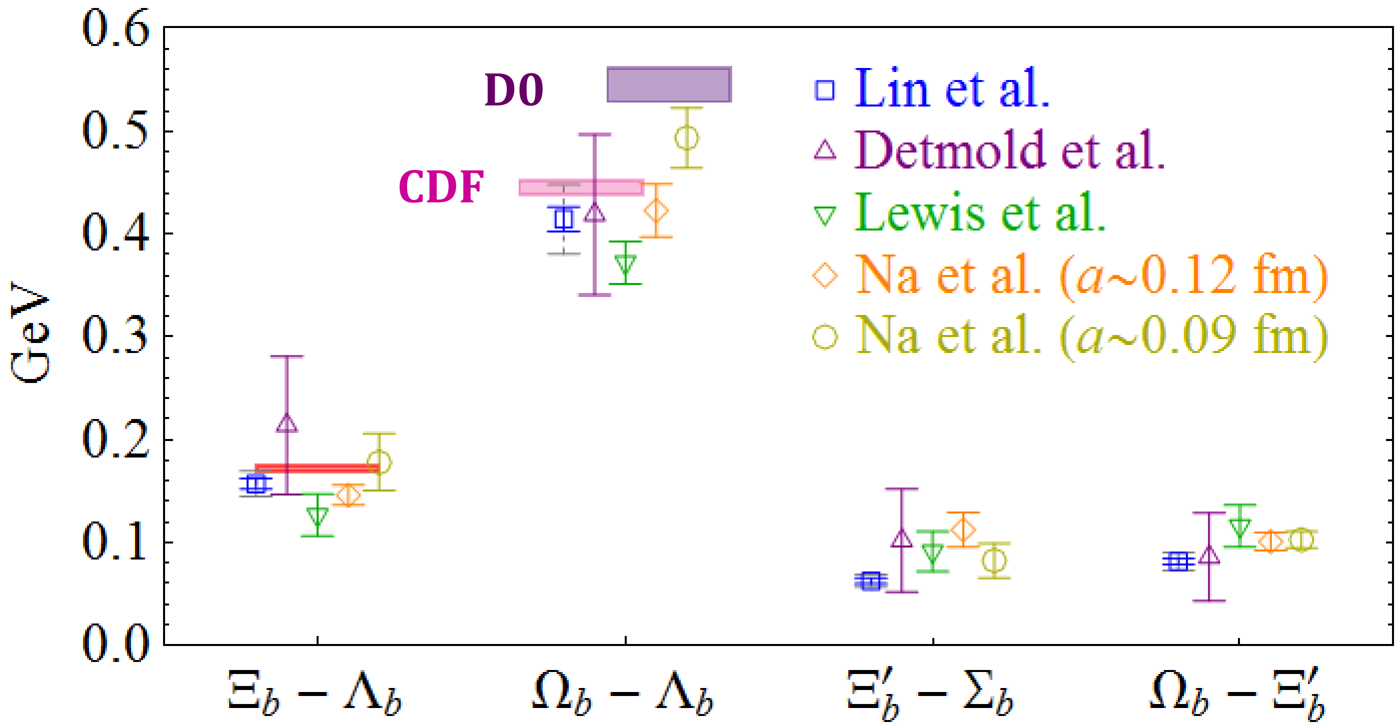
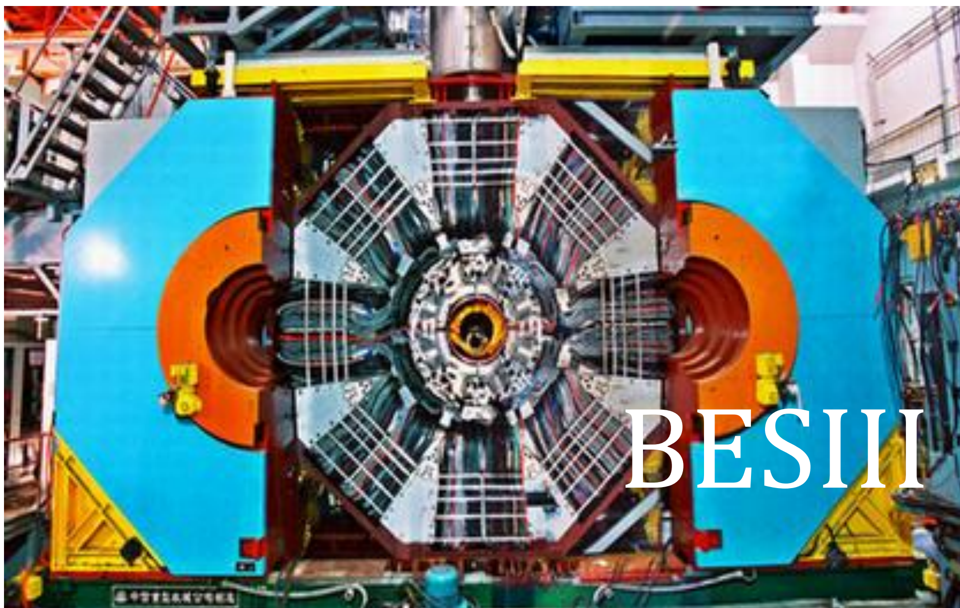


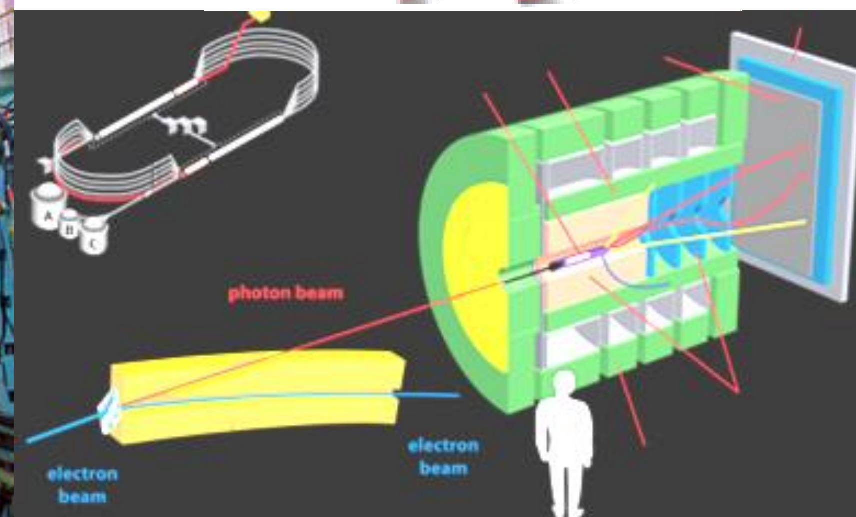
HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009);
W. Detmold et al., (2008), 0812.2583[hep-lat]; R. Lewis et al., PRD79, 014502 (2009);
H. Na et al., PoS LATTICE2008, 119 (2008).

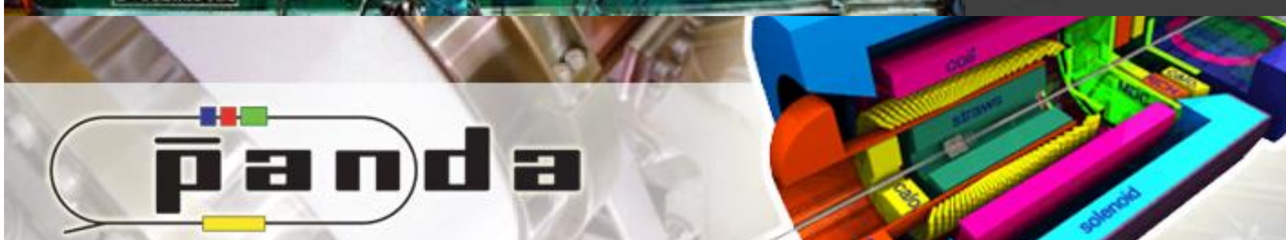
How about Higher Excited States?

Motivation

- New generations of experiments are devoted to the search for exotic & hybrid mesons, glueballs, baryons.

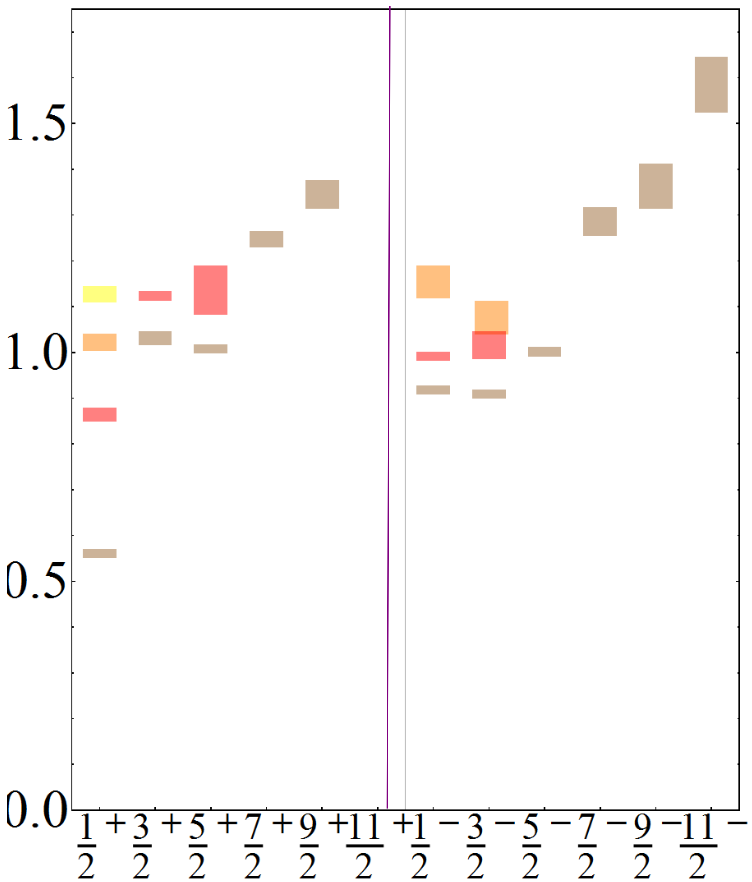




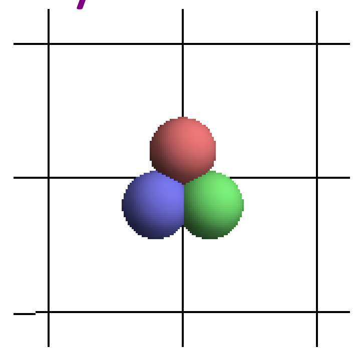


Nucleon

- All baryon spin states wanted: $|J| = 1/2, 3/2, 5/2, \ldots$
- List of 4-star states



- Only 3 distinguished states with $J = 1/2$



$$\epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)$$

$$\epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)$$

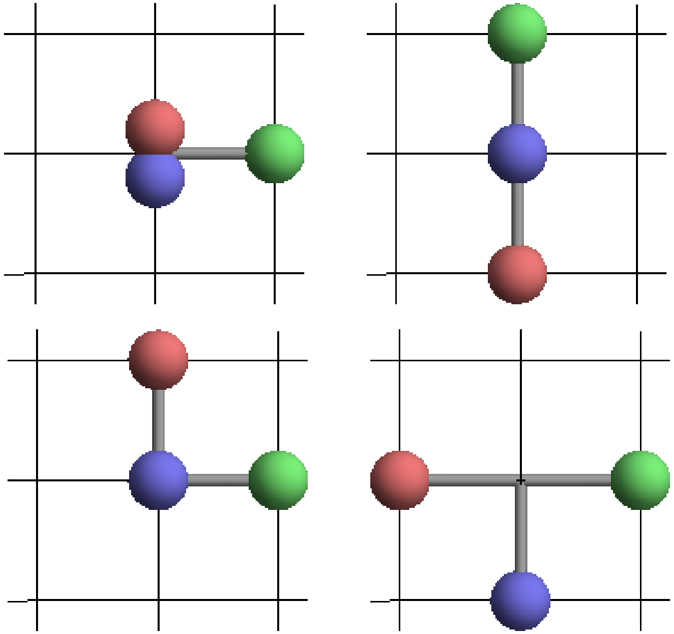
$$\epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\gamma_4\, d^b(x))\, u^c(x)$$

Nucleon

- Rotation symmetry is reduced
  rotation SO(3) ⇒ octahedral O_h group

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$

- Include more quark orientations



S. Basak et al., Phys. Rev. D72, 094506 (2005)

Nucleon

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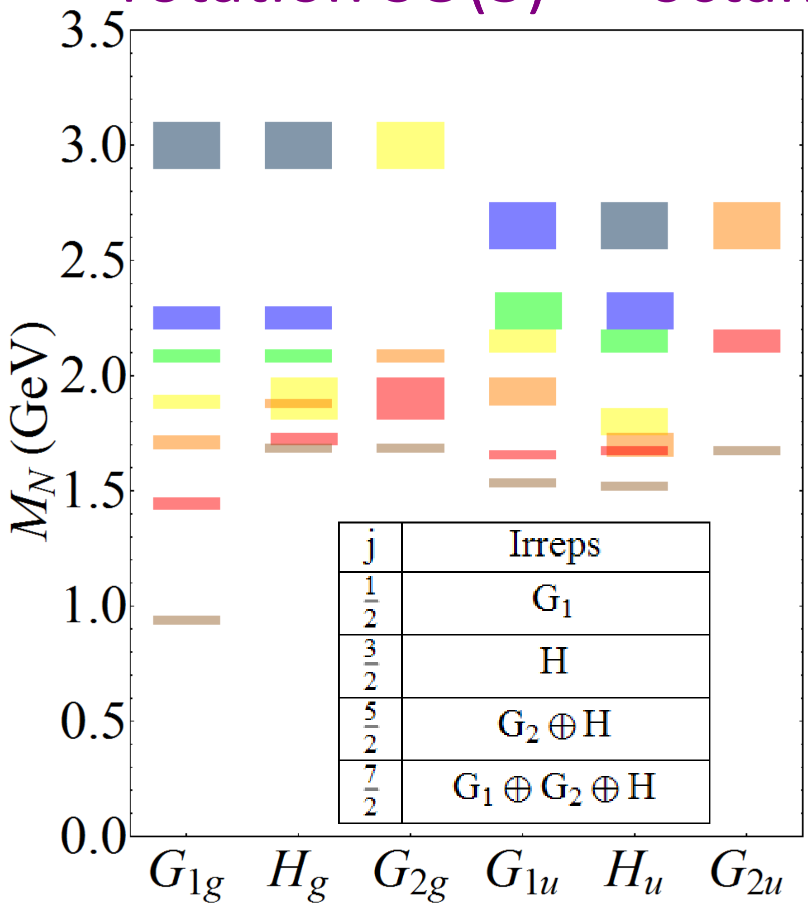

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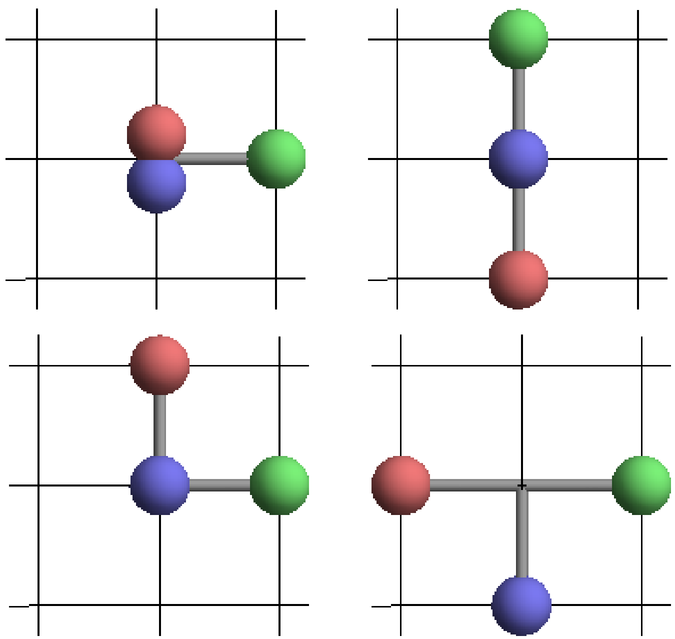


§ More details on operators: check out this YouTube video

Variational Method

§ Recall: still a coupling problem

lowest-mass state dominates $C(t) = \sum_n \langle O|n\rangle\langle n|O\rangle e^{-E_n t}$

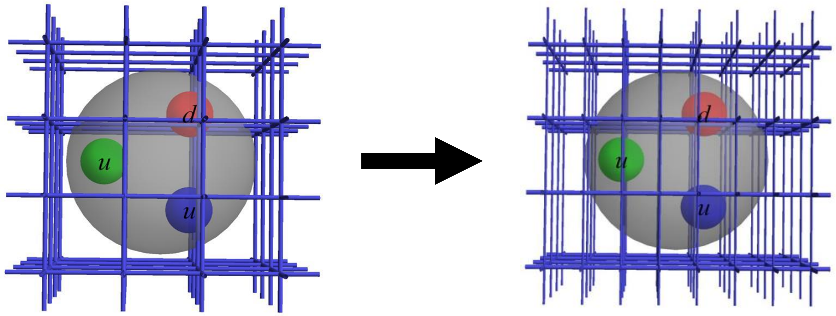
§ Decouple them:

1. Construct correlator matrix $C_{i\,j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0\rangle$
2. Solve the gen. eigensystem $C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}\psi = \lambda(t,t_0)\psi$
3. Simple analysis of eigenvalues (t-dependence) to get each excited state $\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1+\mathcal{O}(e^{-|\delta E|(t-t_0)}))$

C. Michael, Nucl. Phys. B 259, 58 (1985)

M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

§ Higher resolution (at least in time direction)



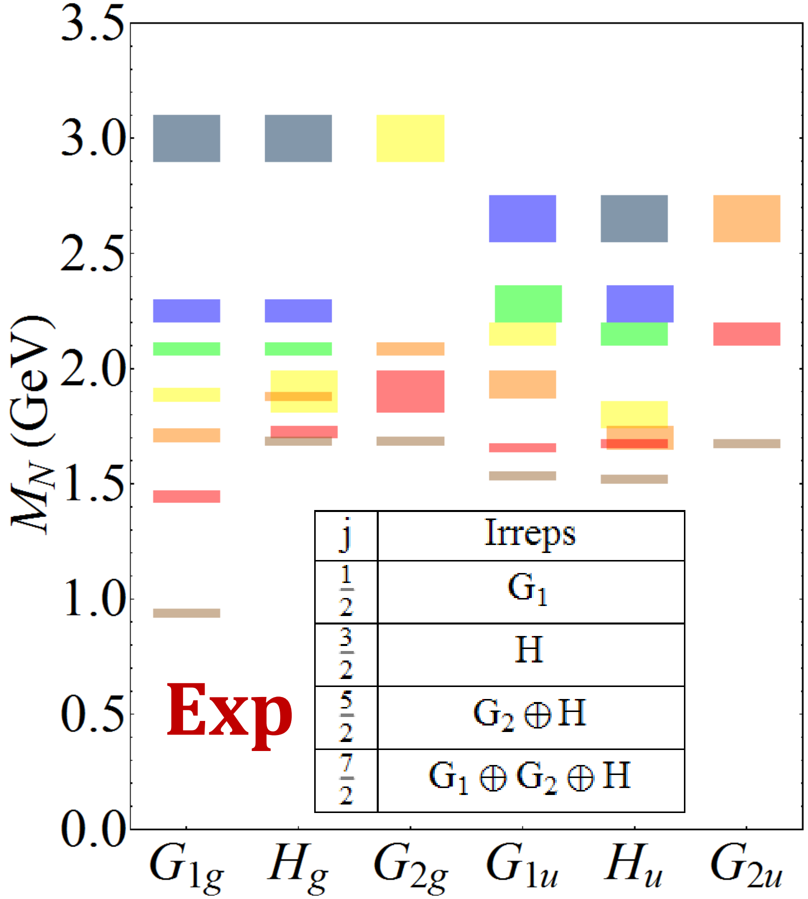
R. Edwards, B. Joo, HWL, Phys. Rev. D 78, 014505 (2008);
HWL et al., Phys. Rev. D 79, 034502 (2009)

N_f = 2+1 Study: Nucleon

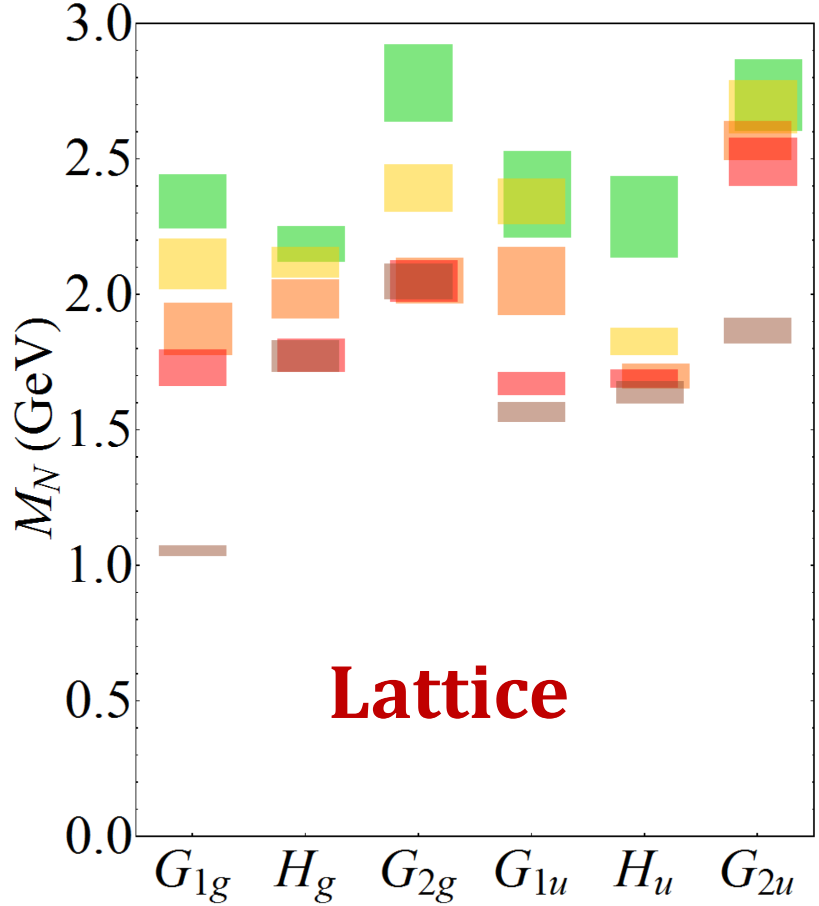
§ N_f = 2+1, anisotropic clover action

HSC, 1004.5072[hep-lat]

$V = 16^3 \times 128$, $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$, $M_\pi \approx 390$ MeV



j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$



Less Known Case

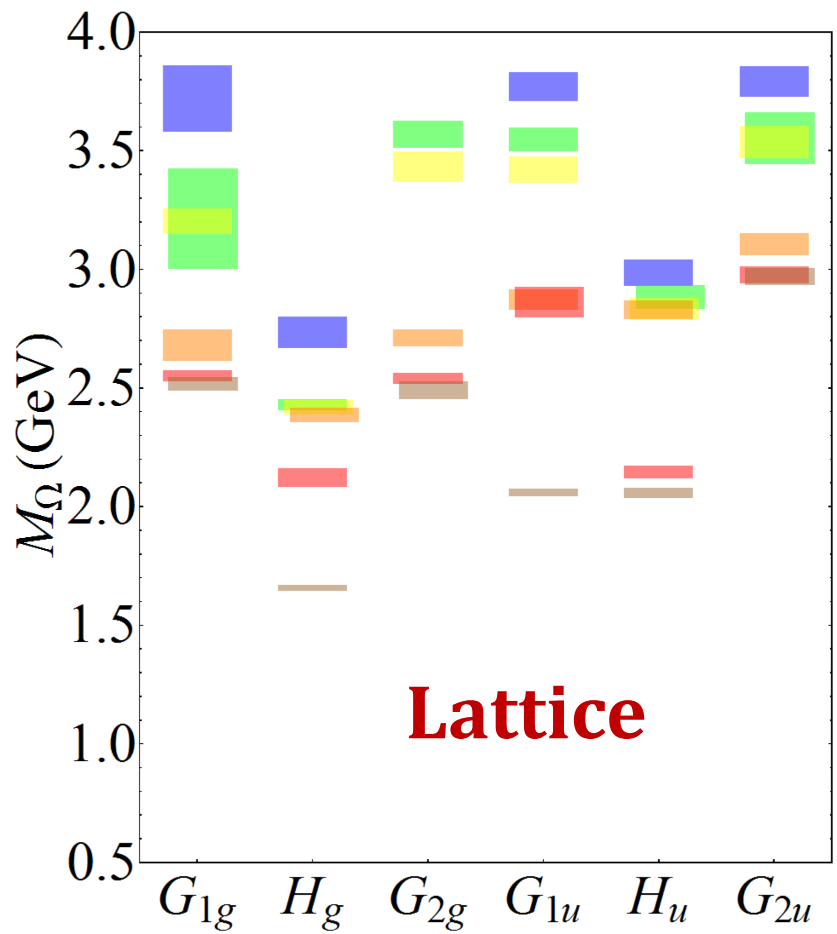
§ N_f = 2+1, anisotropic clover action

HSC, 1004.5072[hep-lat]

$V = 16^3 \times 128$, $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$, $M_\pi \approx 390$ MeV

Ω BARYONS ($S = -3$, $I = 0$)		
Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

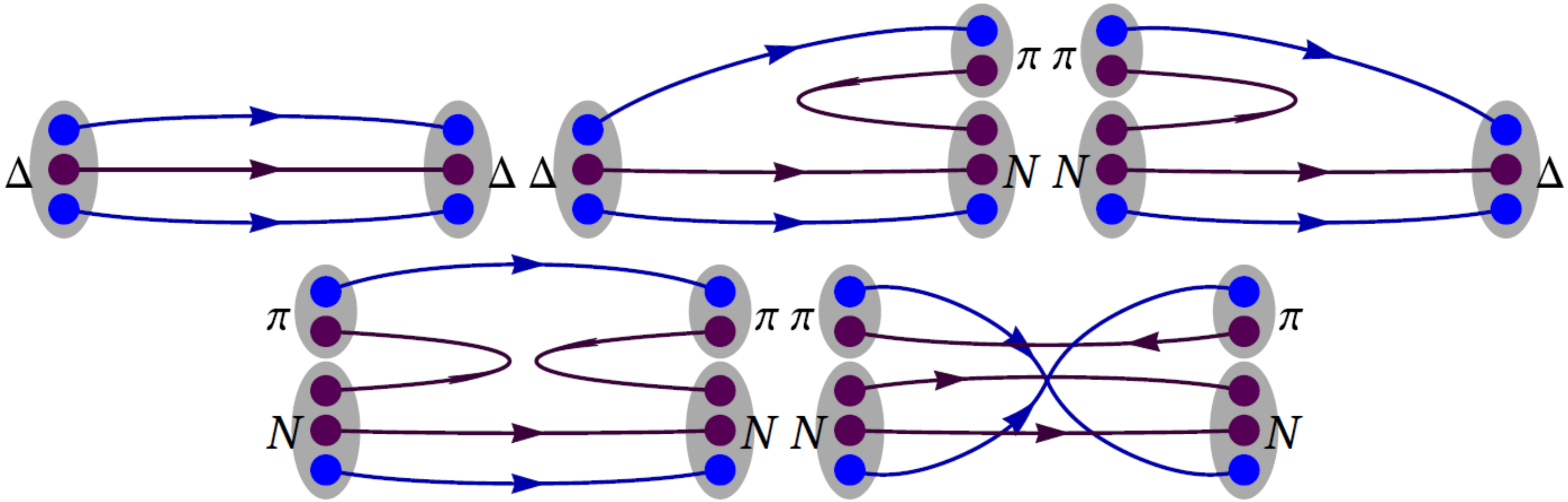
Exp



§ Predictive power

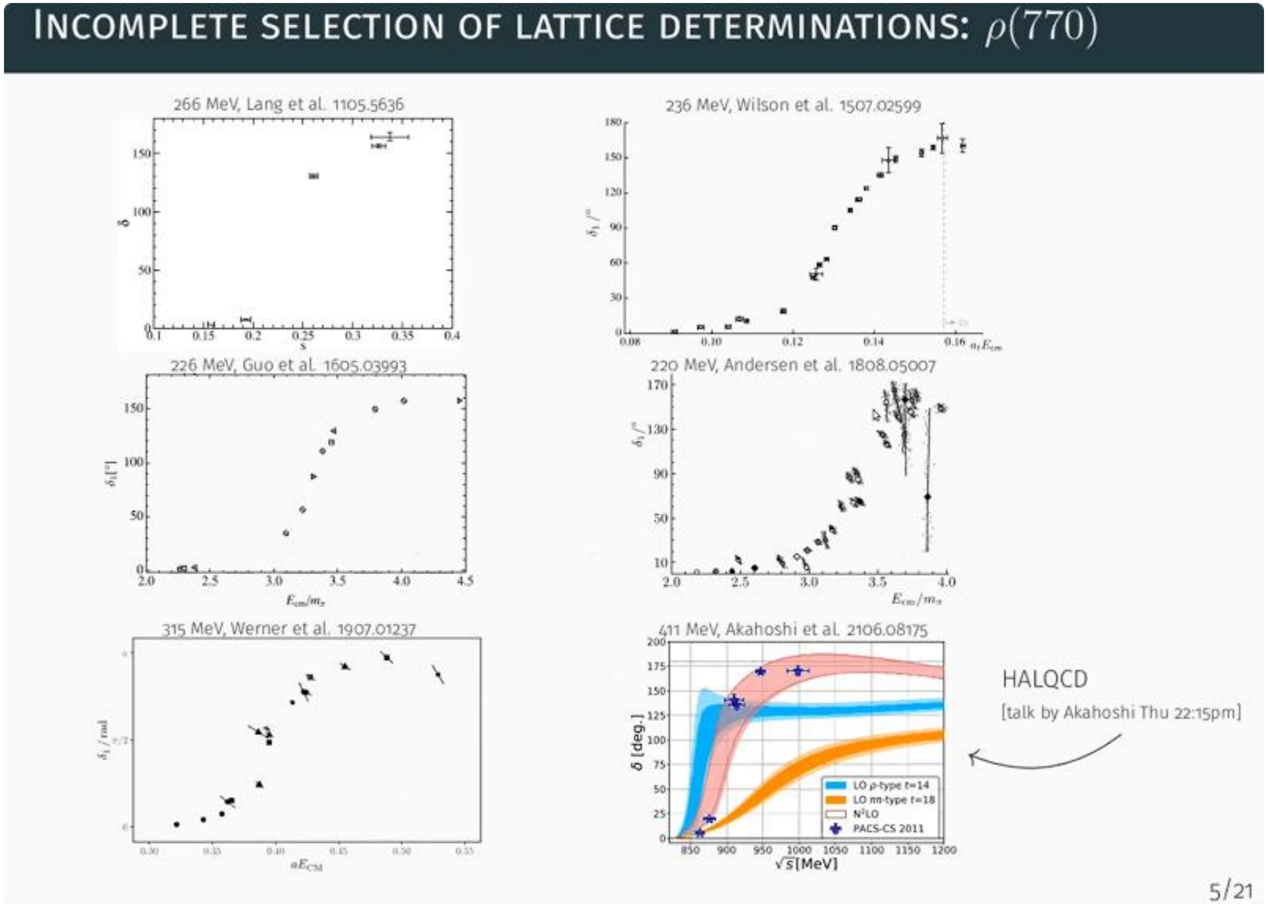
Challenge

- Resonances and multiple-particle final states
  - CAVER: Past lattice calculations have used $E_\pi + E_N > E_{\Delta,N*}$
- Delta example:



Challenge

§ Resonances and multiple-particle final states



Slide from Ben Hoerz at Lattice 2021

§ A lot of activities on-going on multiple-particle approaches

- Check out the annual lattice conference review; [Lattice2021 example]

Take a 5-min Coffee Break

Structure Next

Backup Slides

Meson Operator

- Euclidean two-point function

$$C^{(2)}(x,y) = \frac{1}{Z}\int [dU][d\psi][d\overline{\psi}]e^{-S_{\text{eff}}(U,\psi,\overline{\psi})}(\overline{\psi}(x)\Gamma\psi(x))(\overline{\psi}(y)\Gamma\psi(y)) \quad (1)$$

- "Non-derivative" bilinear operators only

Γ	$H(4)$	$^{2S+1}L_J$	J^{PC}	Charmonium	charm-strange
γ_5	A_1	$^1S_0$	0^{-+}	η_c	D_s
γ_i	A_1	$^3S_1$	1^{--}	J/Ψ	D_s^*
1	T_1	$^3P_0$	0^{++}	χ_{c0}	D_{s0}
$\gamma_5\gamma_i$	T_1	$^3P_1$	1^{++}	χ_{c1}	D_{s1}^*
$\gamma_i\gamma_j$	T_1	$^3P_3$	2^{++}	h_c	–

- Mass obtained from fitting meson correlators at large t

$$C^{(2)}(t) \sim A_\Gamma(e^{-m_\Gamma t} + e^{-m_\Gamma(T-t)}) \quad (2)$$

- Smeared source on the quark field

$$\psi^s(0) = \sum_{\vec{y}} F(\vec{y},0)\,\psi(\vec{y},0) \quad (3)$$

to improve the overlap with the ground-state signal