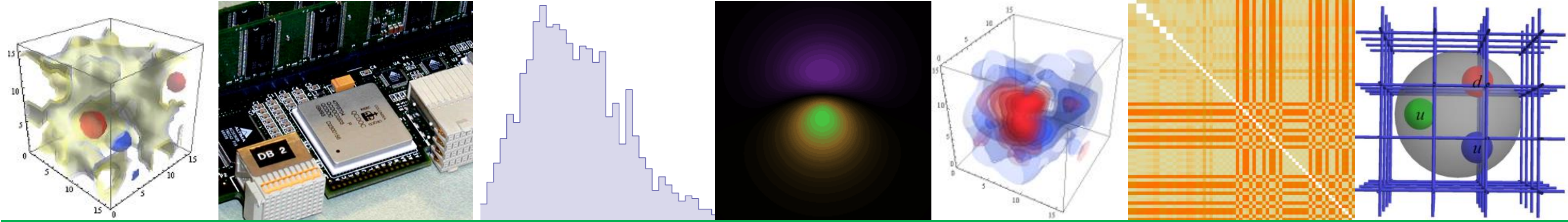
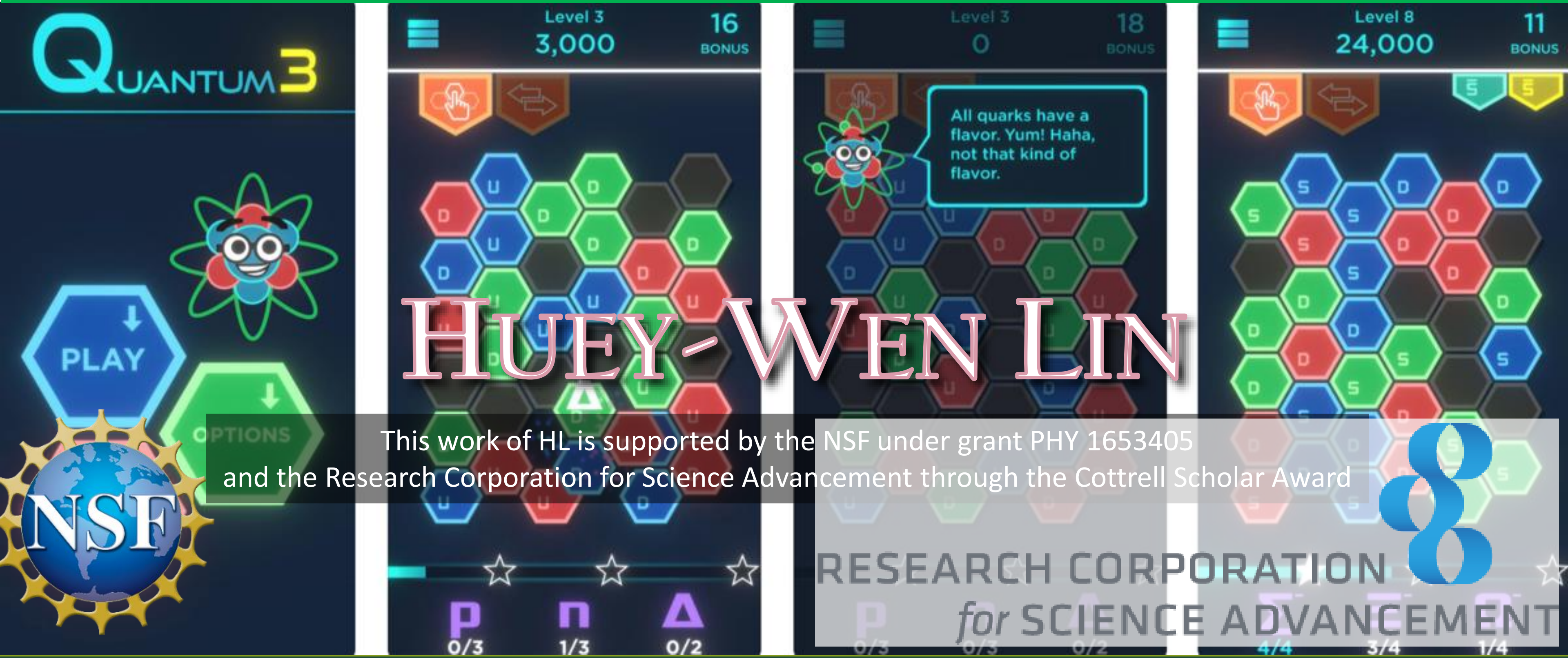


Overview of lattice results for hadron spectroscopy and structure (2)



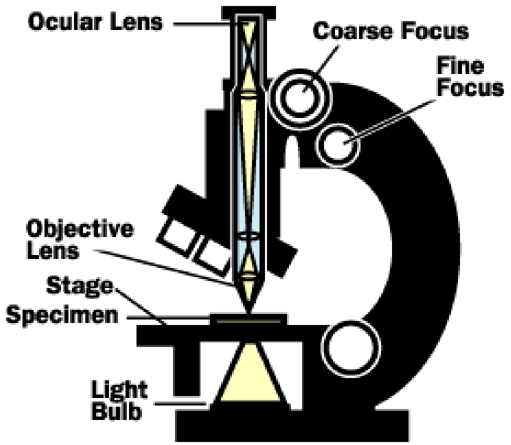
HUEY-WEN LIN

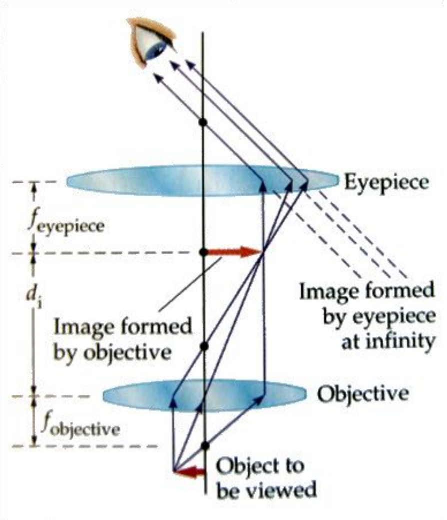
This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

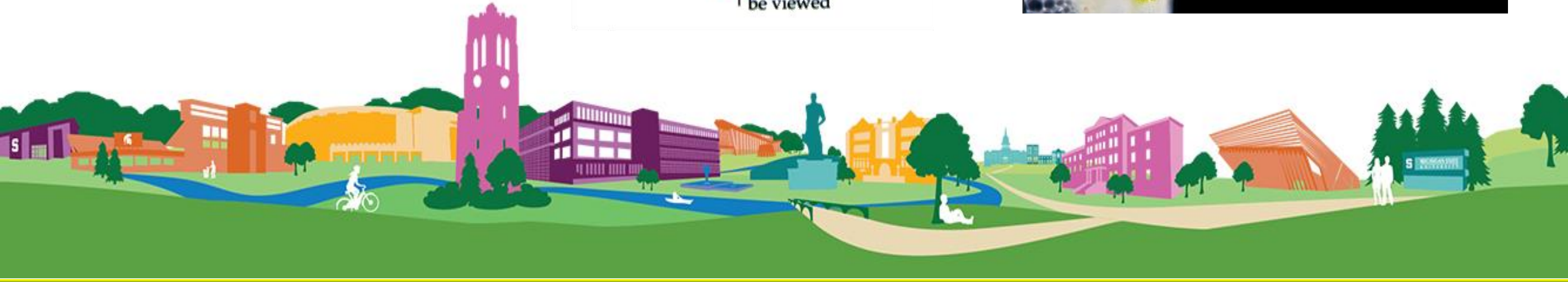
Topic in QCD: Structure

- What is the structure of the nucleon?

probing insights into nucleons







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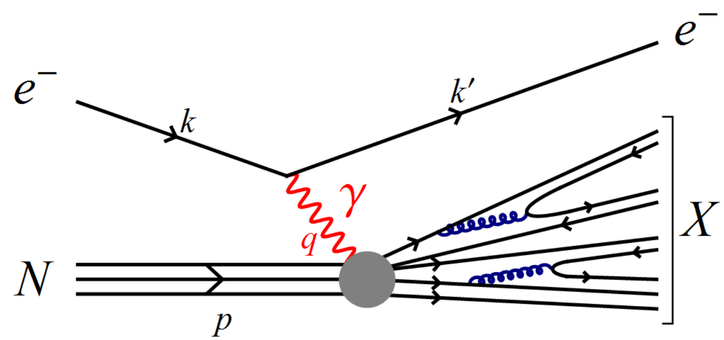
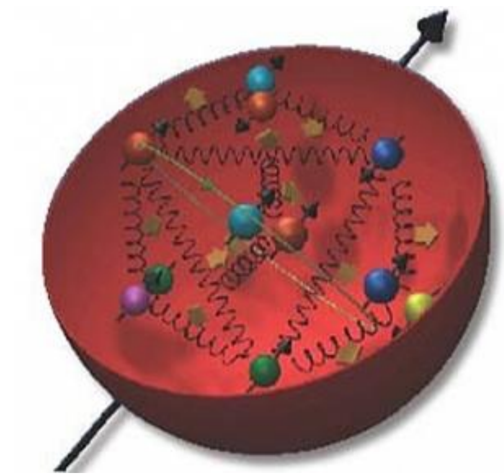
probing insights into nucleons

BNL

JLAB

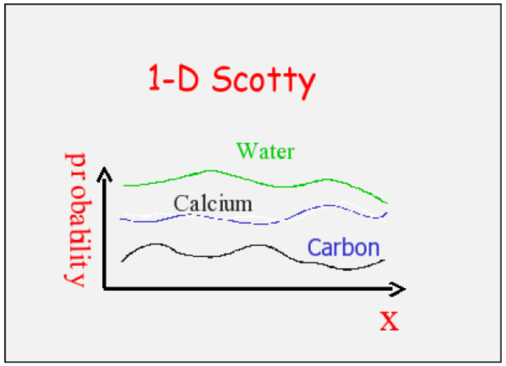




Hadron Structure

§ Structure function/distribution functions

- deep inelastic scattering
- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$



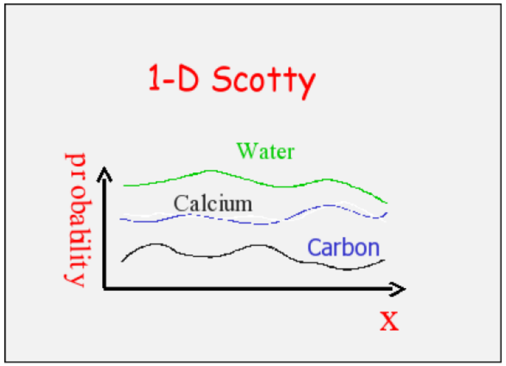
Hadron Structure

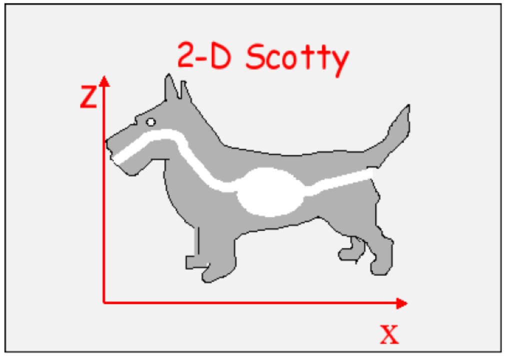
§ Structure function/distribution functions

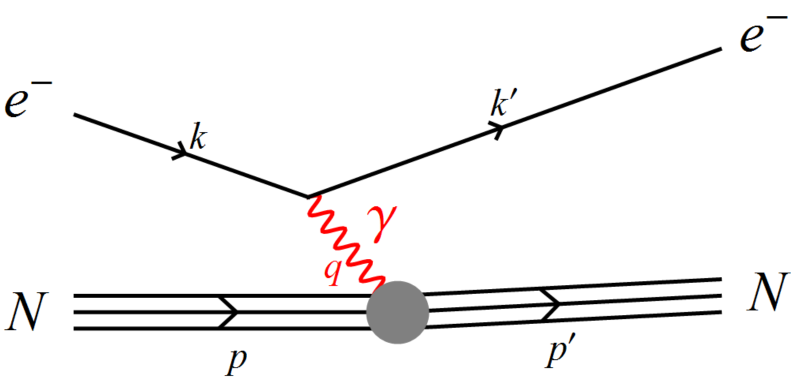
- deep inelastic scattering
- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$

§ Form factors

- elastic scattering
- $F_1(Q^2)$, $F_2(Q^2)$, $G_A(Q^2)$, $G_P(Q^2)$

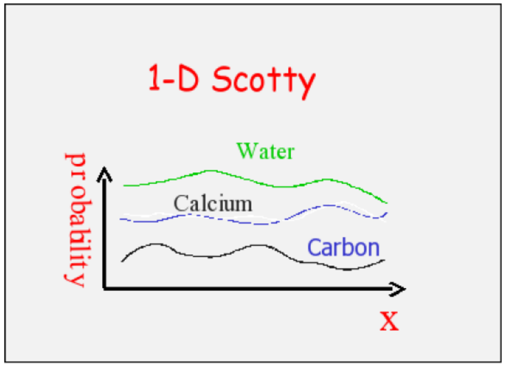


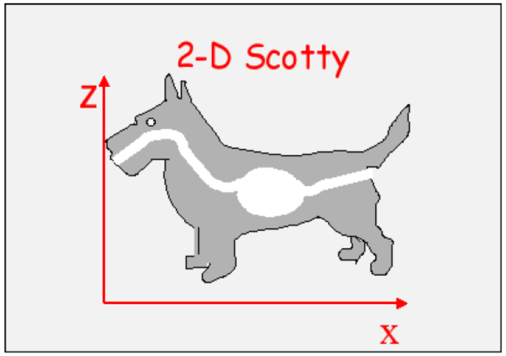


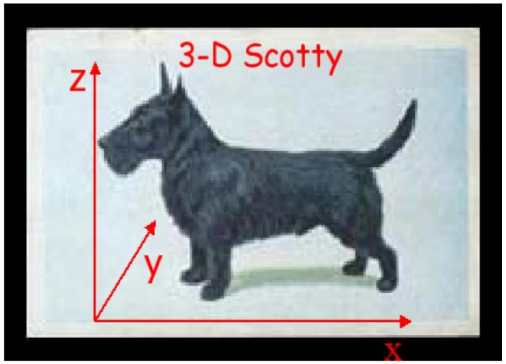


Hadron Structure

- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n\rangle_q$, $\langle x^n\rangle_{\Delta q}$, $\langle x^n\rangle_{\delta q}$
- Form factors
  - elastic scattering
  - $F_1(Q^2)$, $F_2(Q^2)$, $G_A(Q^2)$, $G_P(Q^2)$
- Generalized Parton Distribution
  - DVCS
  - $\langle x^{n-1}\rangle_q = A_{n0}(0)$, $\langle x^{n-1}\rangle_{\Delta q} = \tilde{A}_{n0}(0)$, $\langle x^n\rangle_{\delta q} = A_{Tn0}(0)$,
  - $F_1(Q^2) = A_{10}(Q^2)$, $F_2(Q^2) = B_{20}(Q^2)$, $G_A(Q^2) = \tilde{A}_{10}(Q^2)$, $G_P(Q^2) = \tilde{B}_{10}(Q^2)$
  - Nucleon spin







Outline

§ Consumer's Guide to Lattice Structure Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)
- PDF Moments

§ *x*-dependent Nucleon Structure

Biased selected results, highlighting work done by MSU students/postdocs

- Recent Lattice PDFs Progress
- Applications to Generalized Parton Distributions
- Future Prospects and Challenges

Missing meson structure, heavy quark structure, ... due to limited time

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

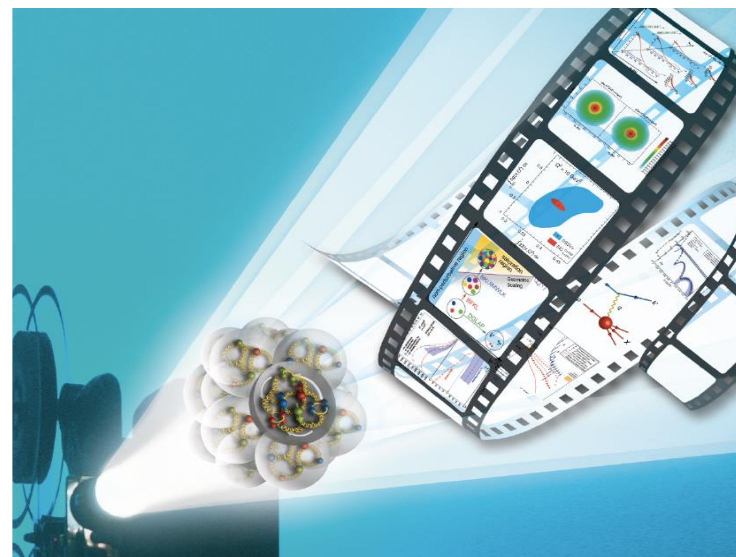
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

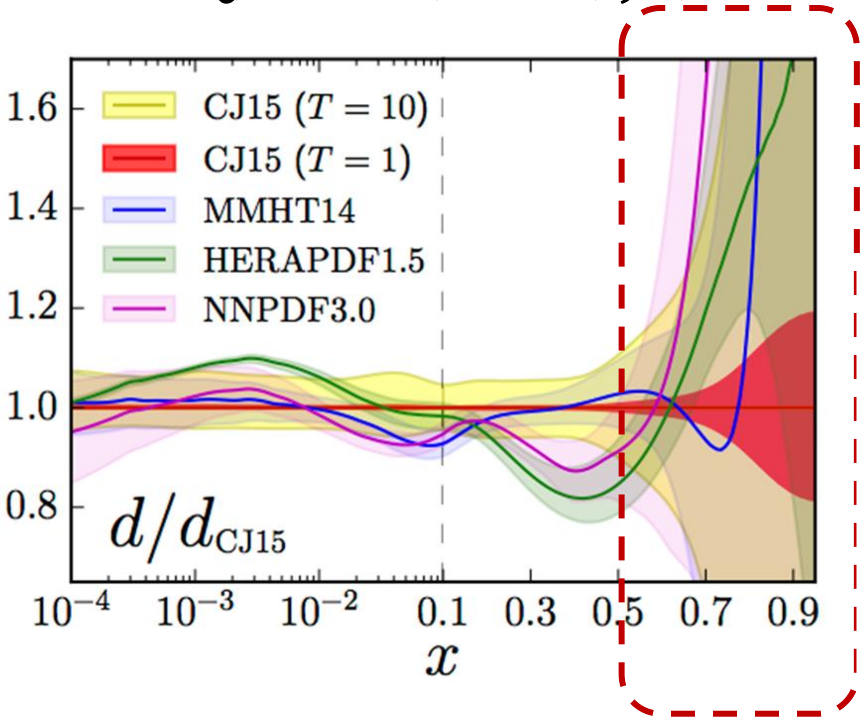
- Assumptions imposed

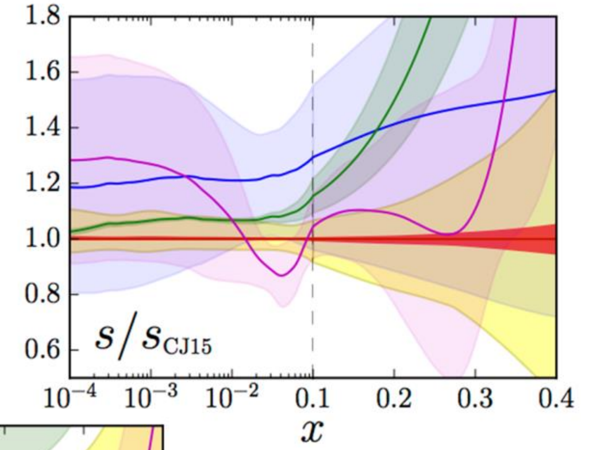
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

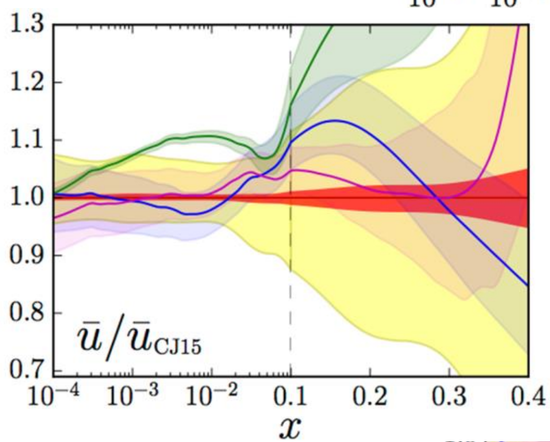
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

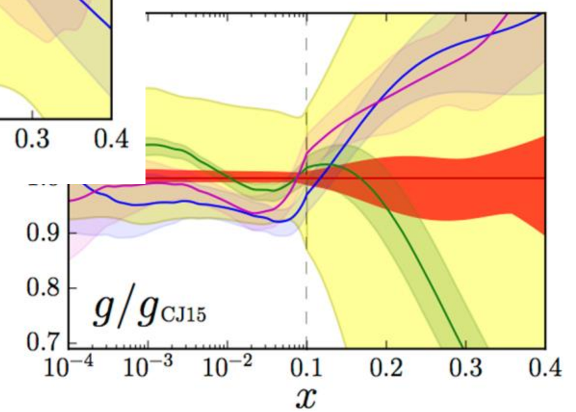
Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.







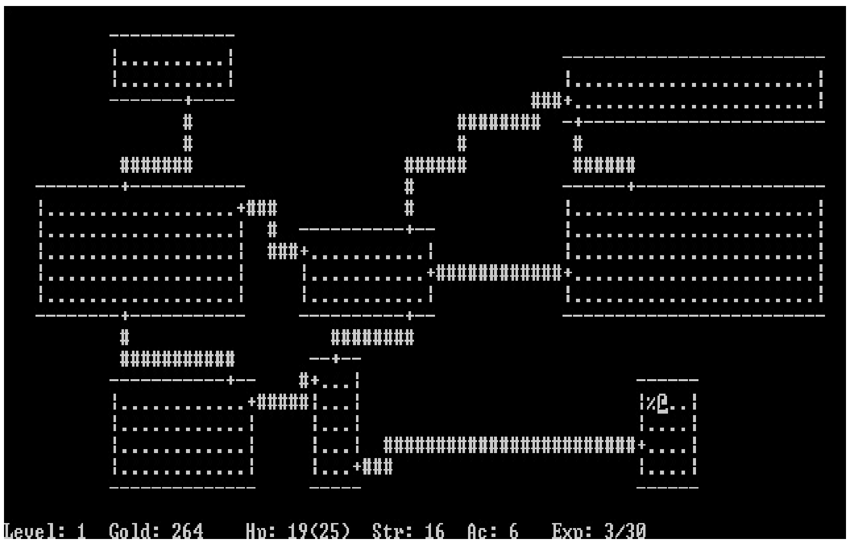


CTEQ-JLAB
https://www.jlab.org/theory/cj/

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

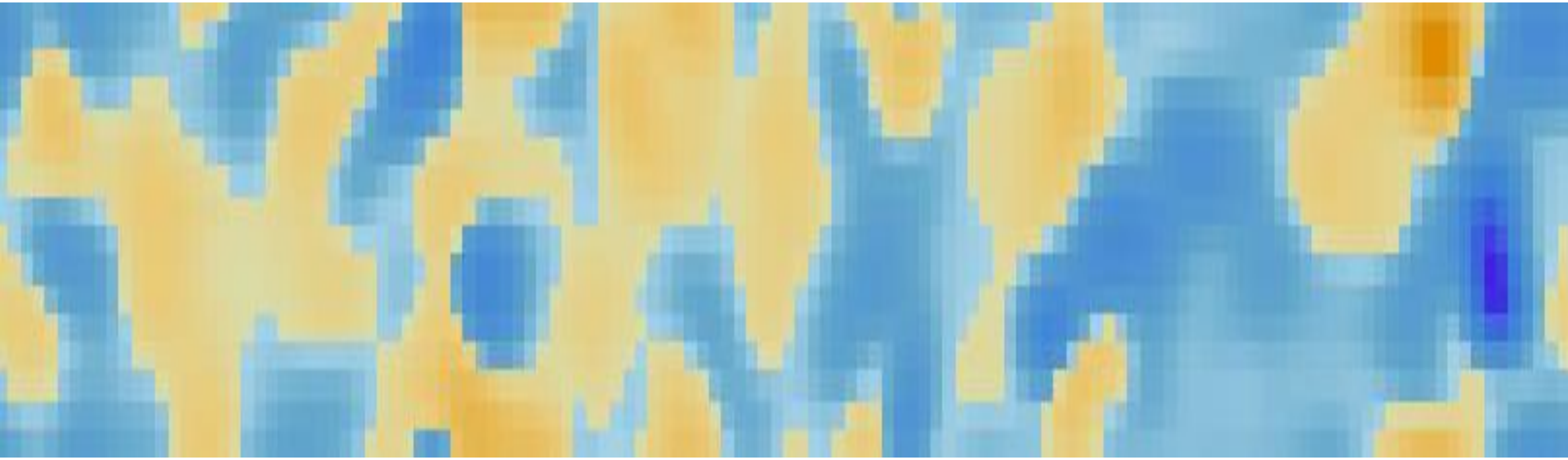
1980s



Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Nucleon Matrix Elements

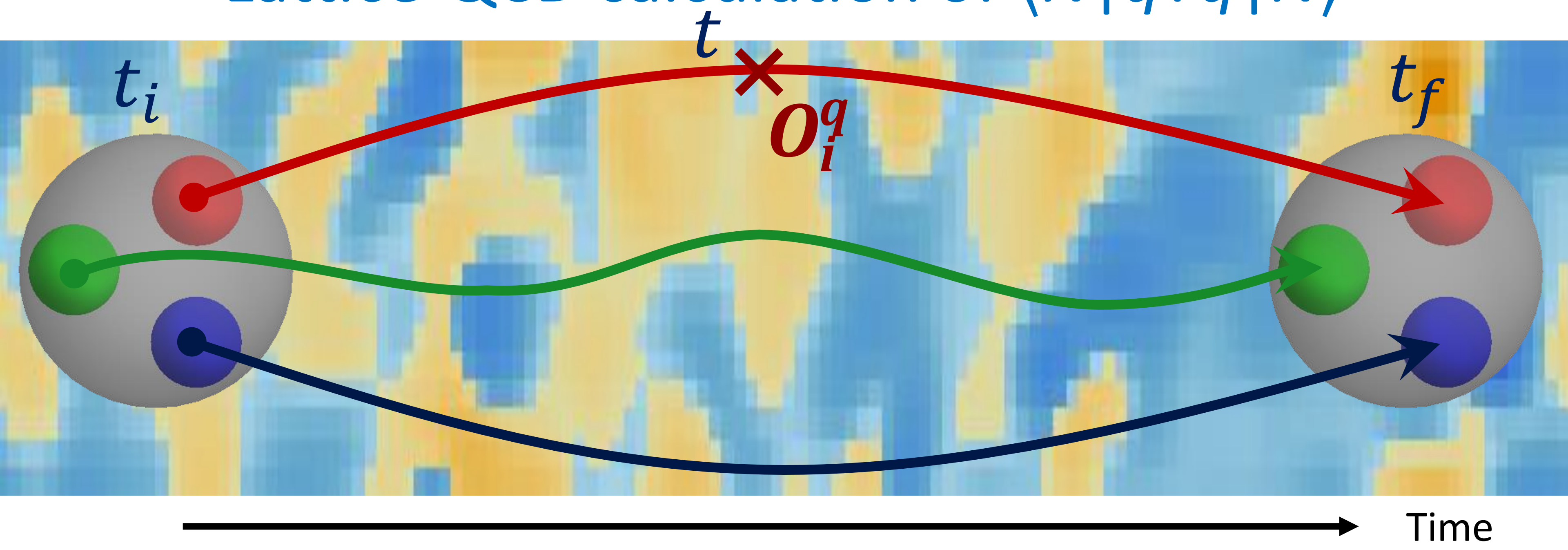


§ Pick a QCD vacuum

- Gauge/fermion actions, flavor (2, 2+1, 2+1+1), m_π, a, L, …

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



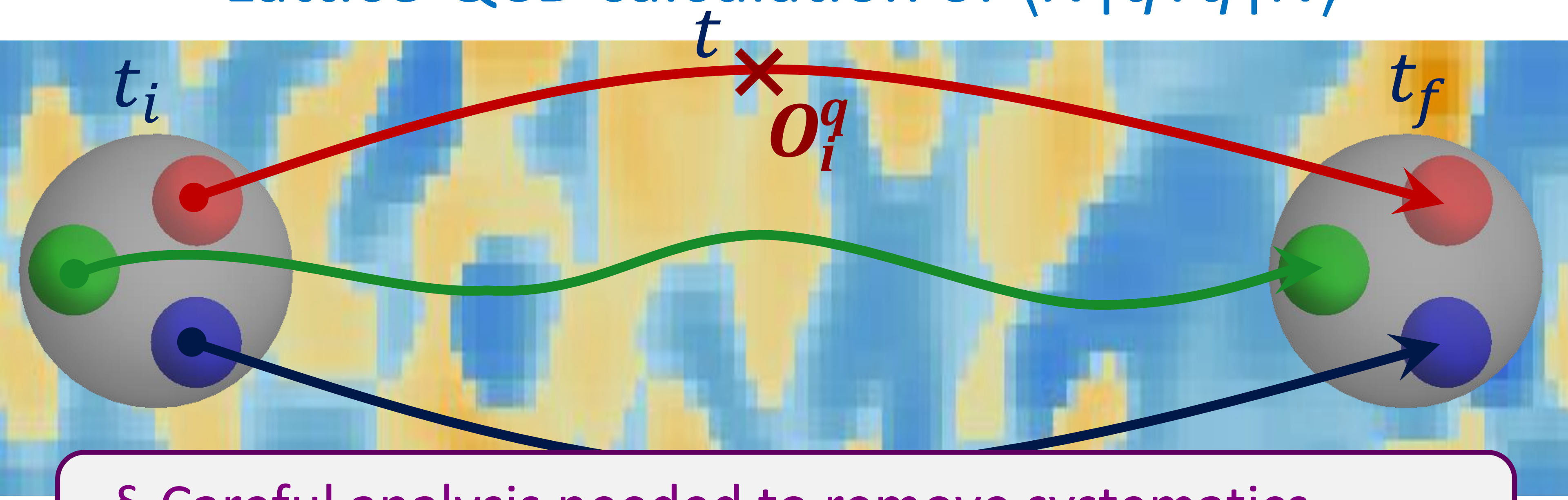
§ Construct correlators (hadronic observables)

- Requires "quark propagator"

  Invert Dirac-operator matrix (rank $O(10^{12})$)

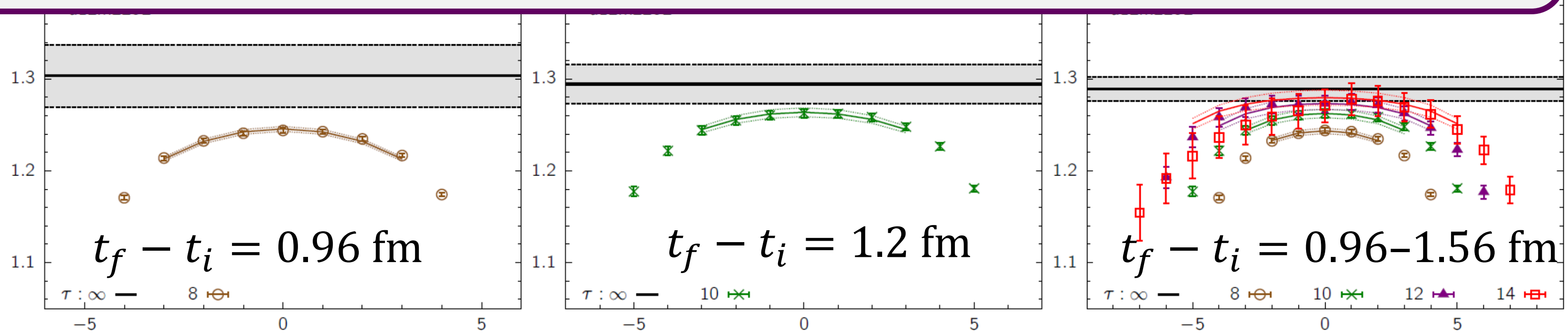
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



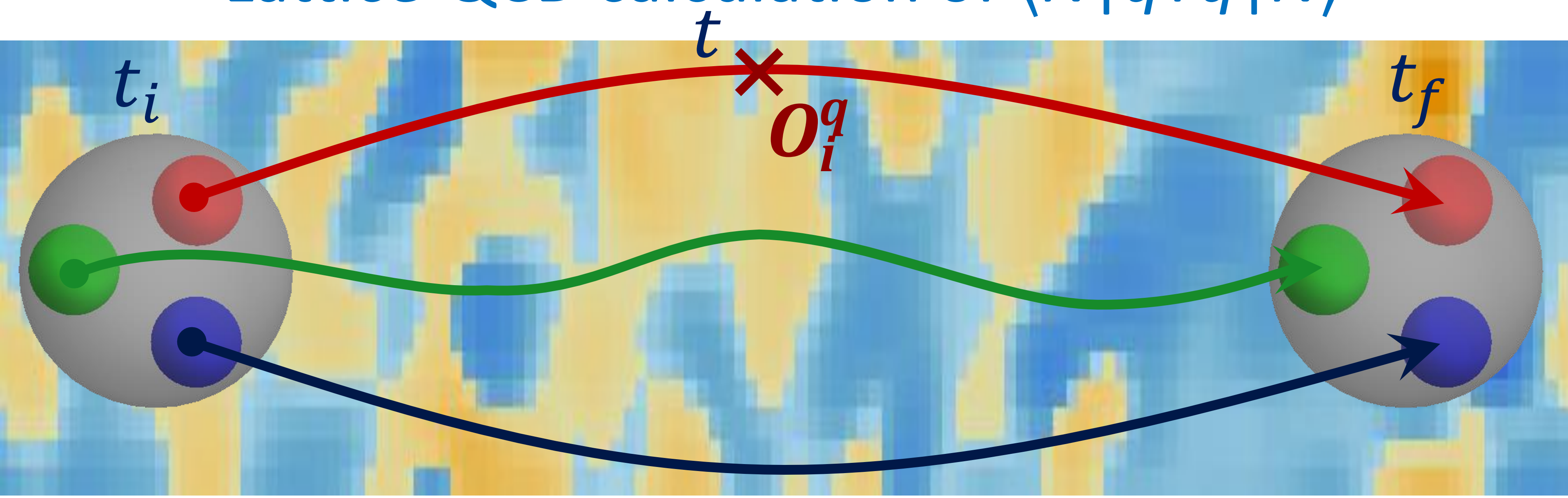
§ Careful analysis needed to remove systematics

- Wrong results if **excited-state systematic** is not under control



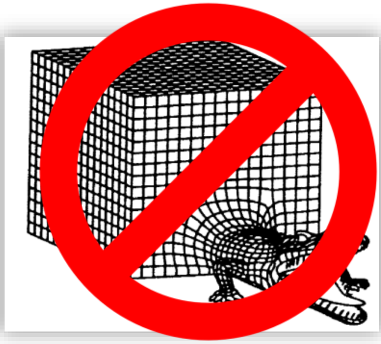
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



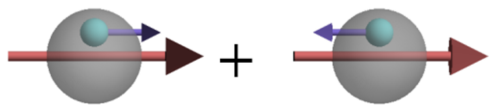
§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Moments of PDFs

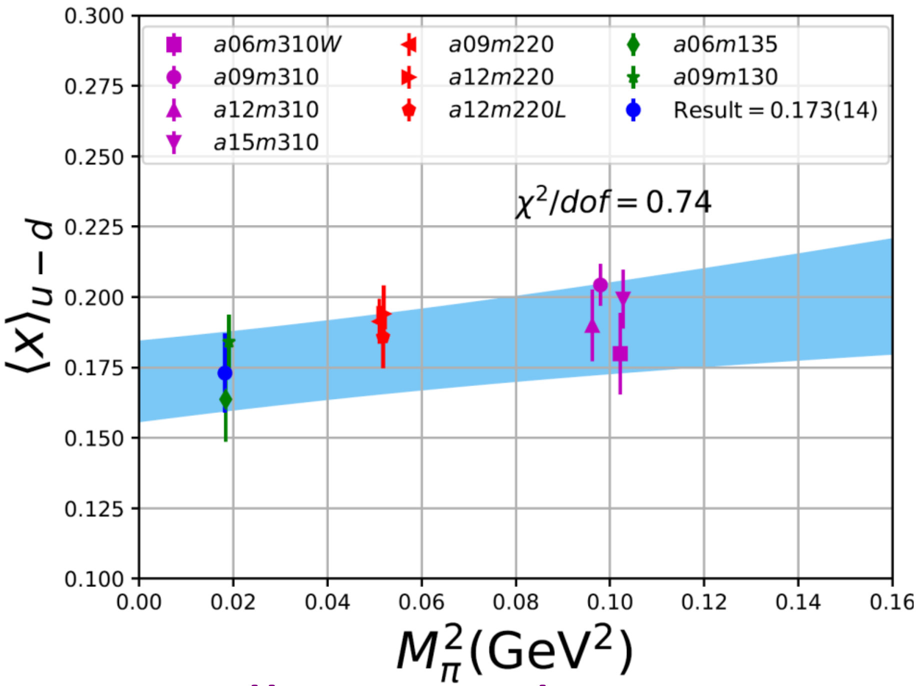
- First moments are most commonly done

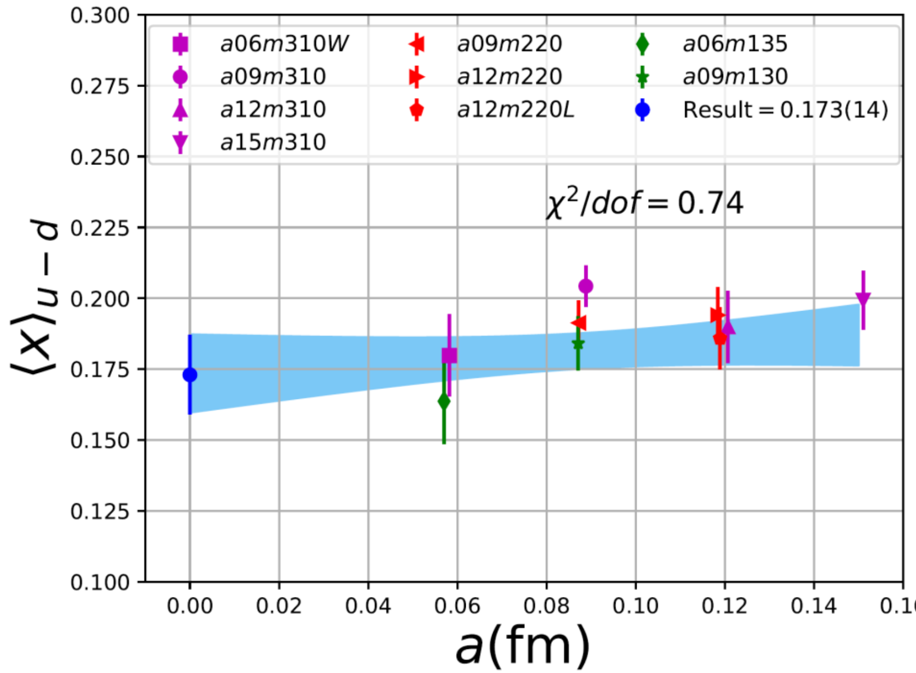


- State-of-the art example
  - Extrapolate to the physical limit

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

Santanu Mondal et al (PNDME collaboration), 2005.13779



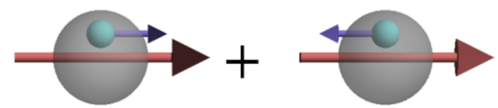


- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES	Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★	0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★	$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★	0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■	0.47(4)(11)	

** No quenching effects are seen.

Moments of PDFs

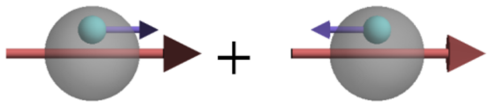
§ PDG-like rating system or average

§ LatticePDF Workshop

- Lattice representatives came together and devised a rating system

§ Lattice QCD/global fit status

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



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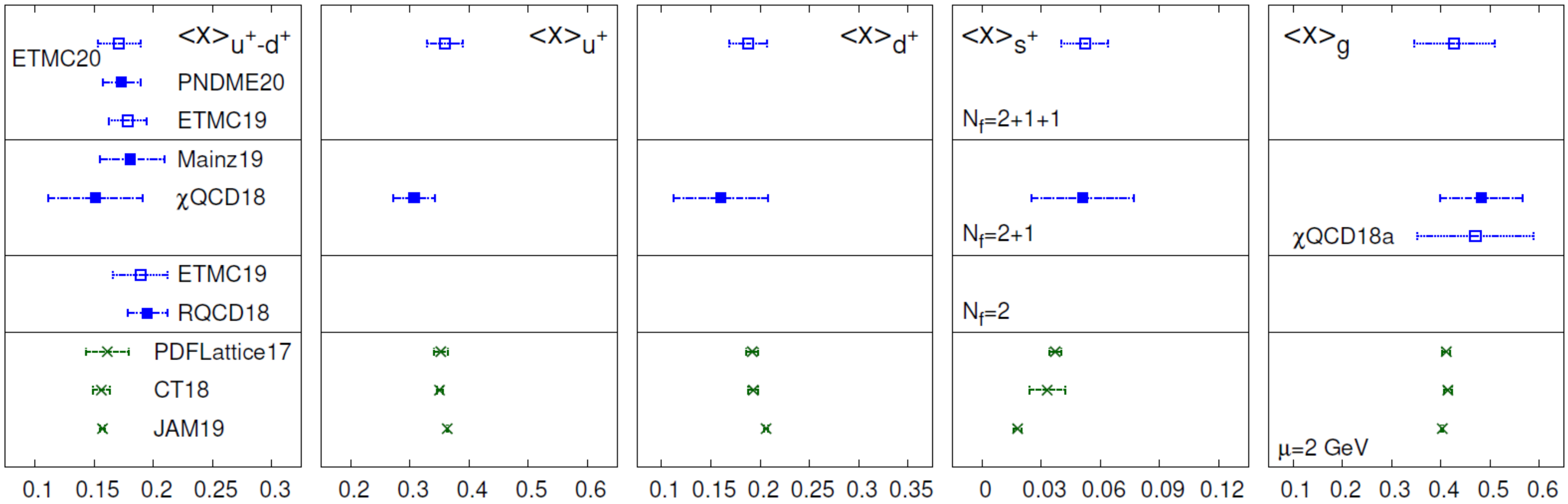

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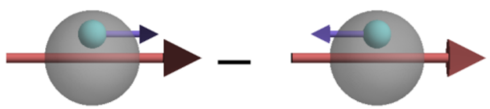
$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1}\Delta q(x)$$



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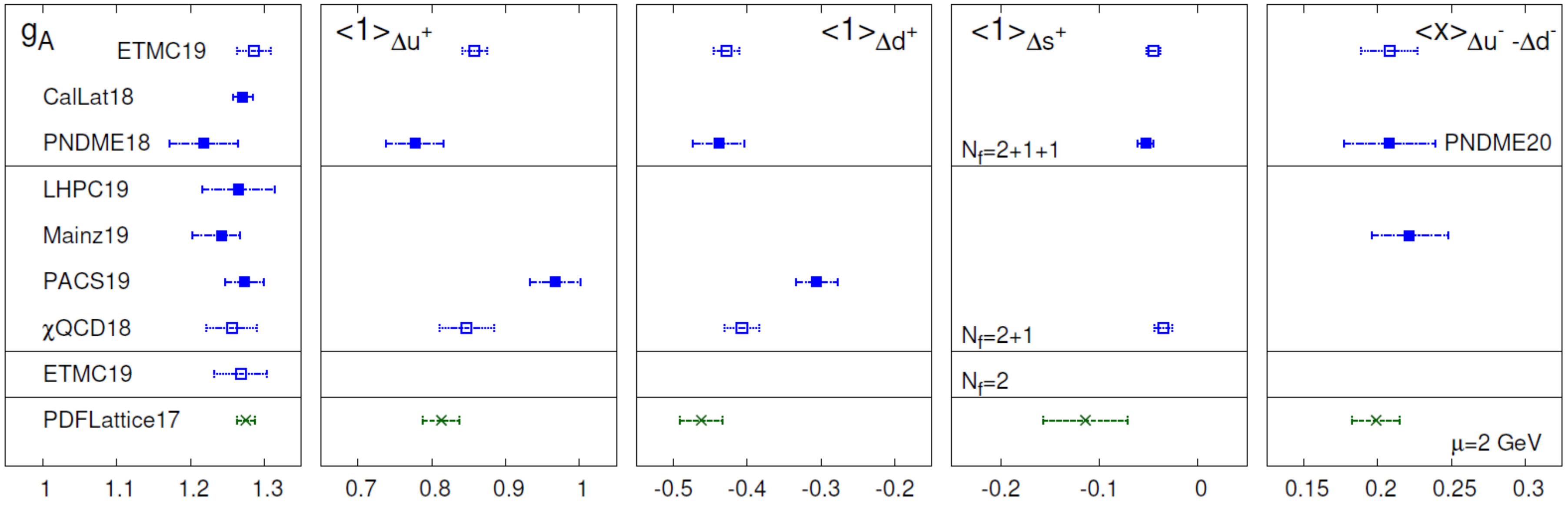

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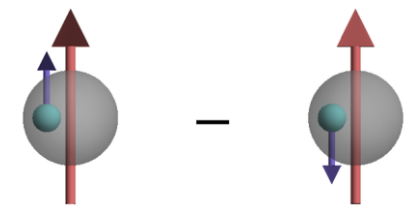

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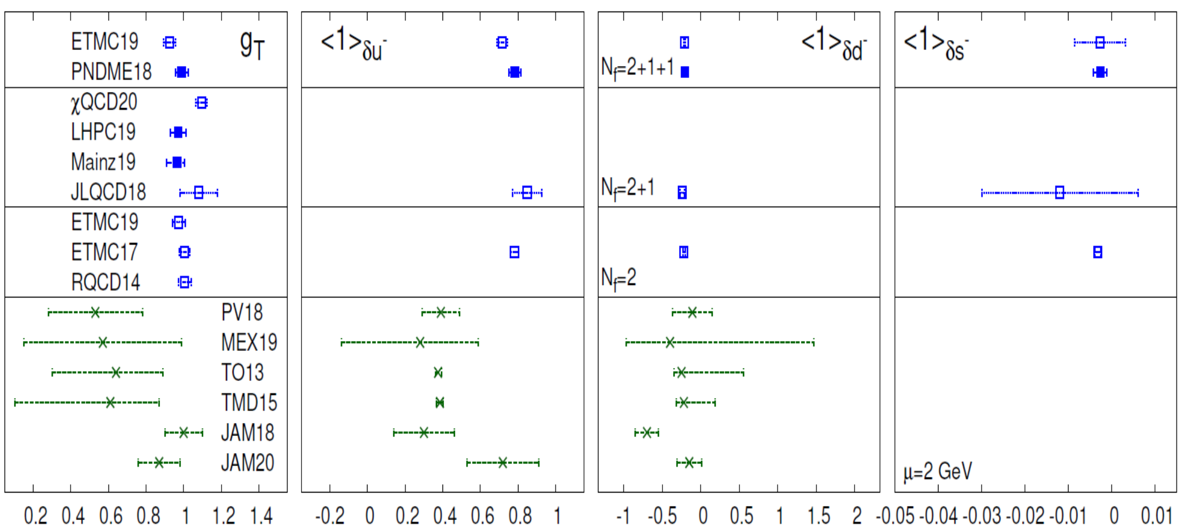
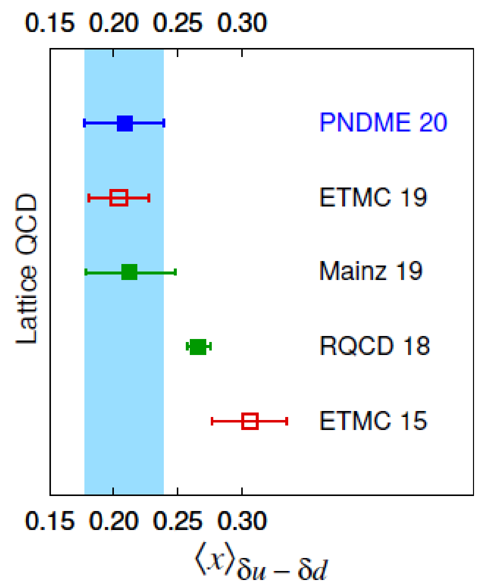


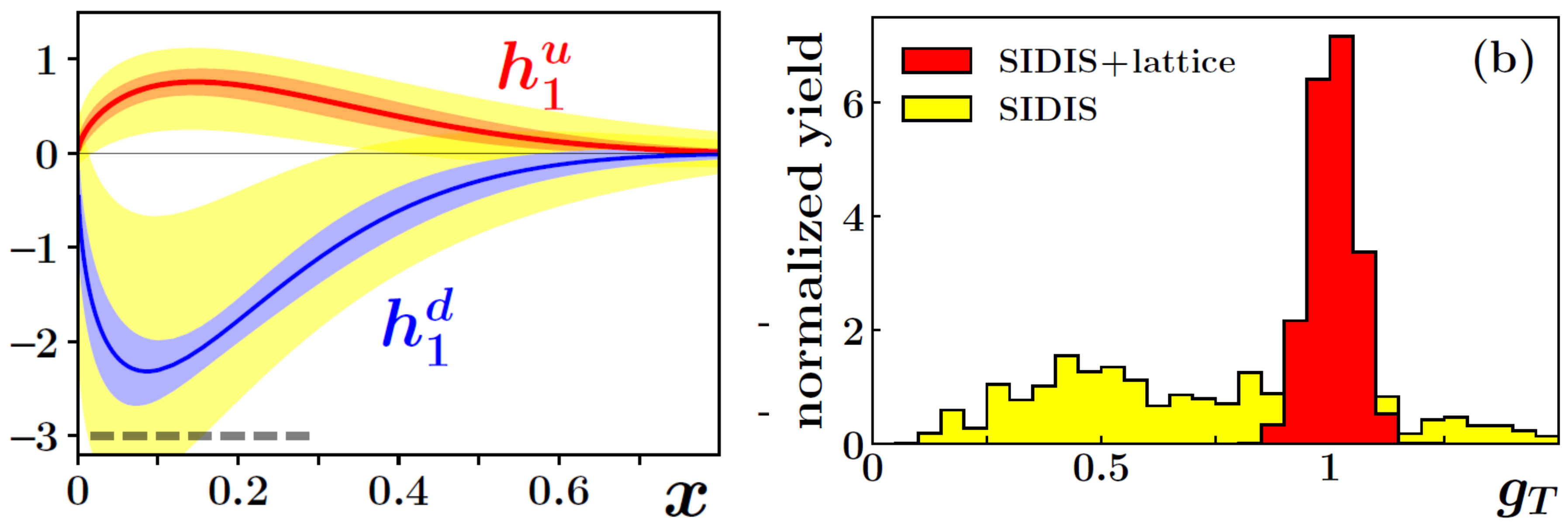


S. Mondal et al (PNDME), 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

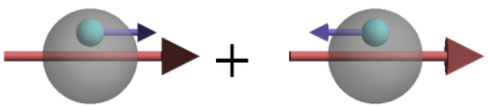


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Structure on the Lattice

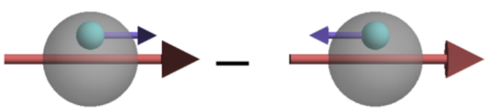
- Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



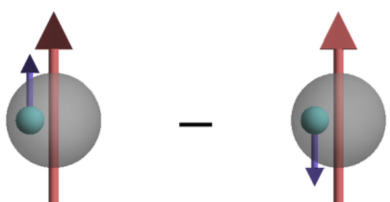
spin-averaged/unpolarized

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

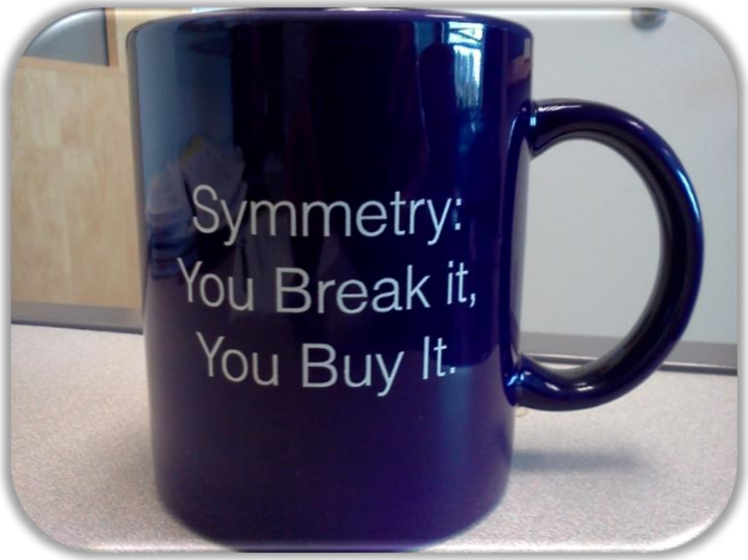
- True distribution can only be recovered with all moments

PDFs on the Lattice

- § Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Novel proposals to overcome this problem
- § Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea

W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Some novel proposal to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea
- New Strategy: Xiangdong Ji, PRL 111, 039103 (2013);
  - Adopt lightcone description for PDFs
  - Calculate finite-boost quark distribution
    - In $P_z \to \infty$ limit, parton distribution recovered
    - For finite P_z, corrections are applied through effective theory
  - Feasible with today's resources!

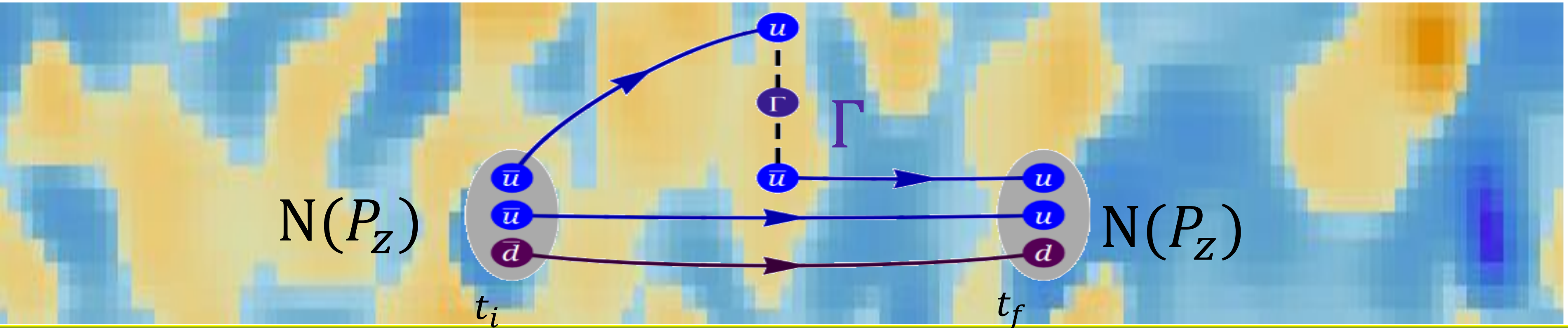
Bjorken-x Dependent Nucleon Structure

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = ∑ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (**LaMET**) (X. Ji, 2013; See 2004.03543 for review)



Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

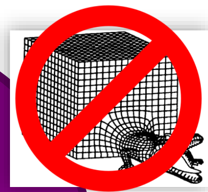
Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today



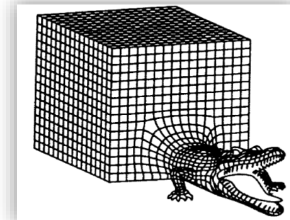
= Σ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

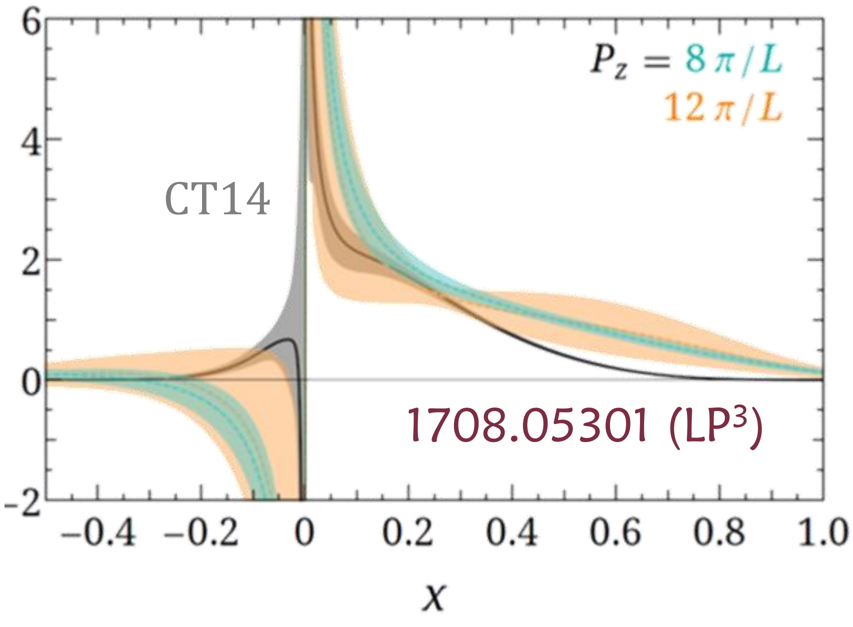
- Kernel is a complicated object;
  mostly only calculated up to one-loop level
- **Inverse problem to extract the wanted distribution**
  - Slightly different approaches from each group
  - Systematics vary
- Large momentum is needed in the lattice calculations in all methods to reach small-x region
  - Current projects focus on mid- to large-x

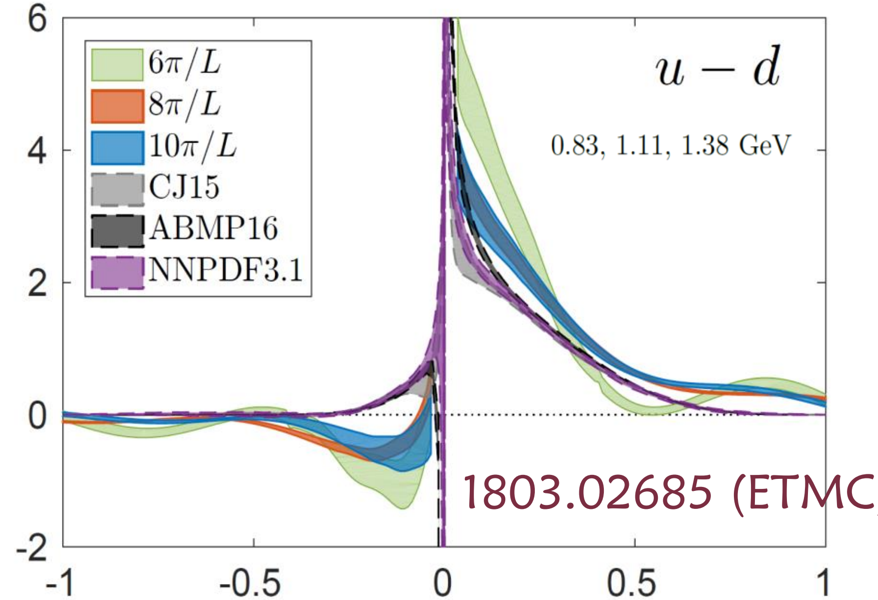
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





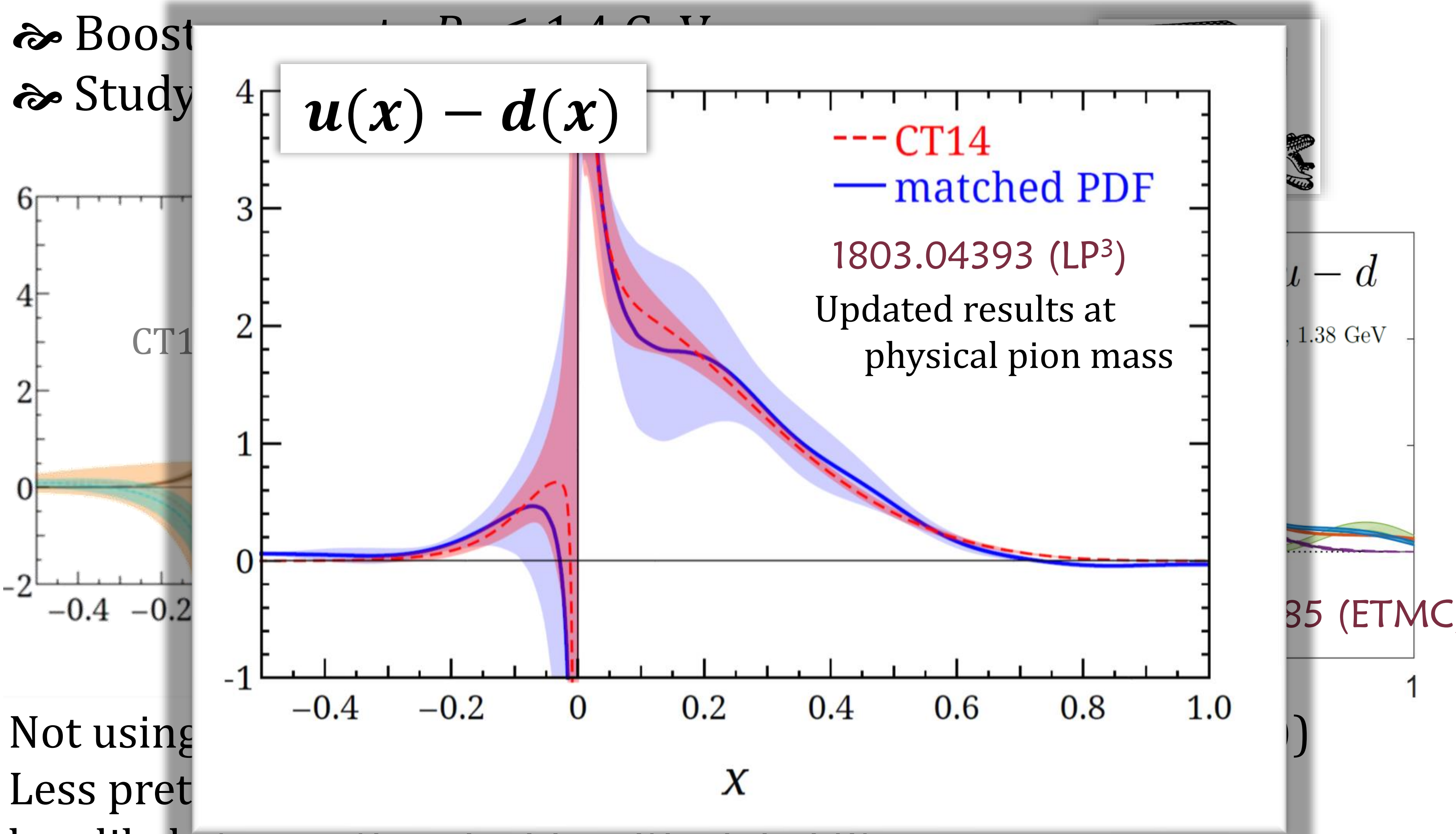


Not using parametrization (e.g. $xf(x, \mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost
- Study



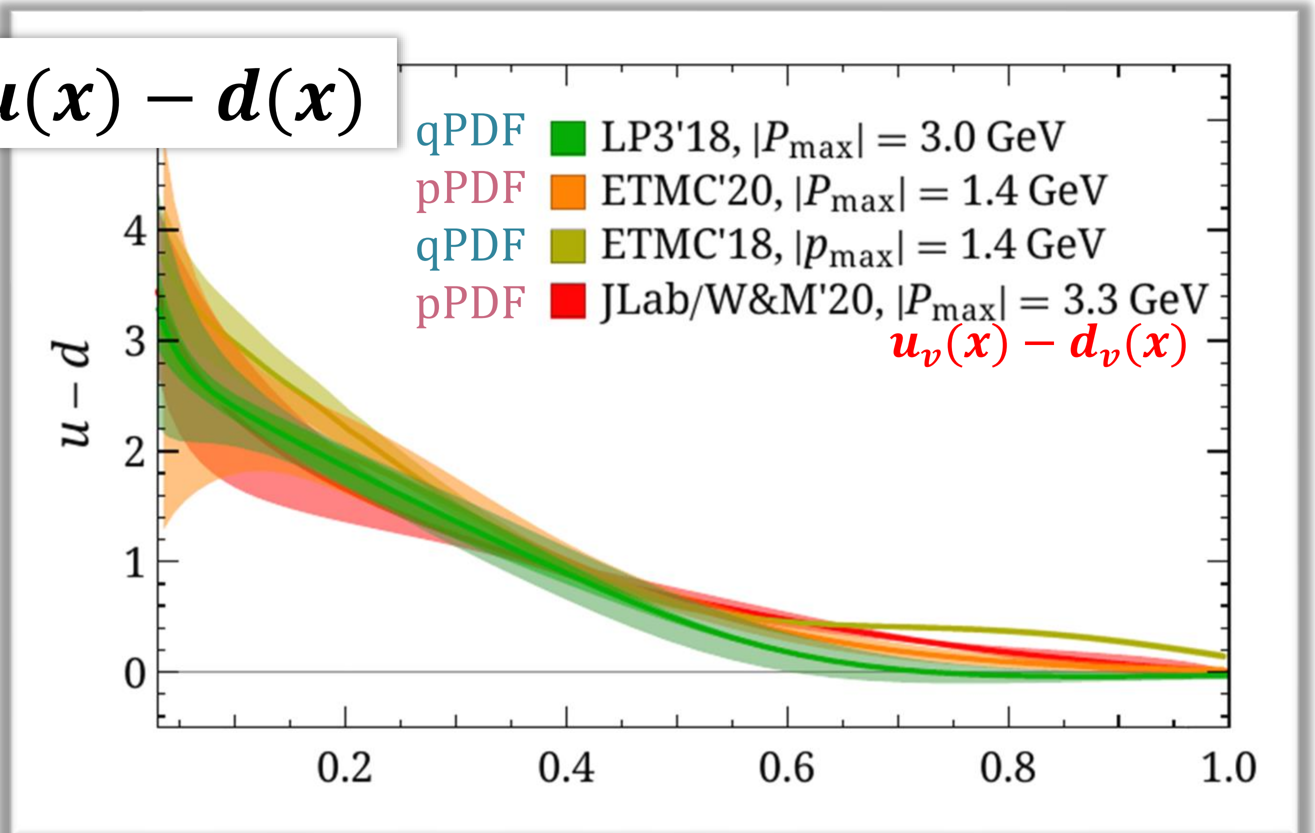
Not using
Less pret
less likely to exactly coincide with global fits.

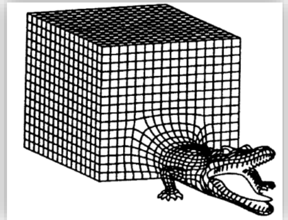
Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$





Finite volume,
Discretization,
…

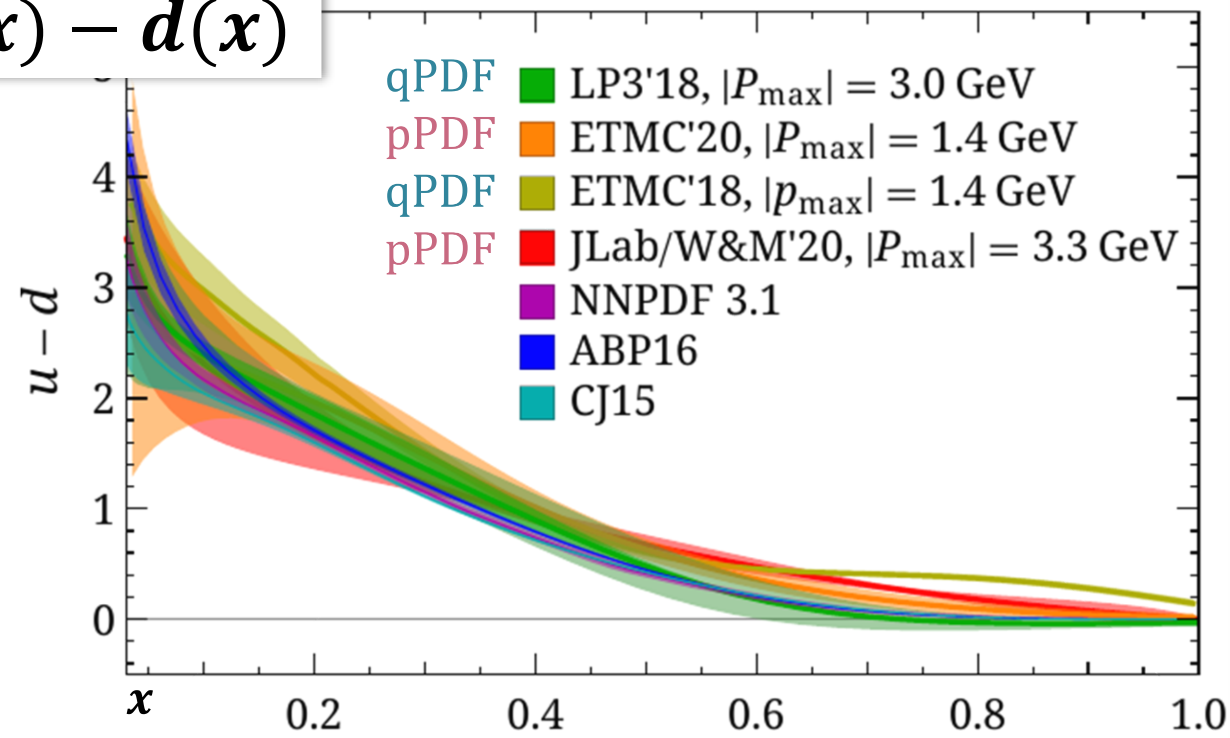
2006.08636, PDFLattice2019 report

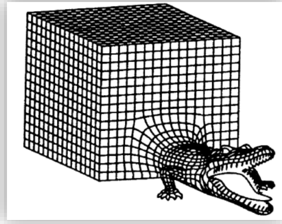
Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$





Finite volume,
Discretization,
…

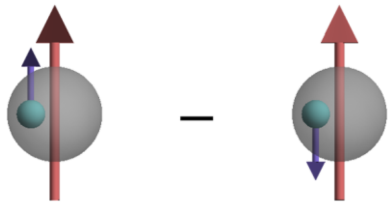
2006.08636, PDFLattice2019 report

Transversity

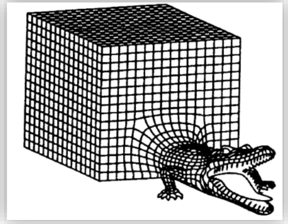
§ Summary of physical pion mass results

- Quasi-PDF method only

$\boldsymbol{\delta u(x) - \delta d(x)}$

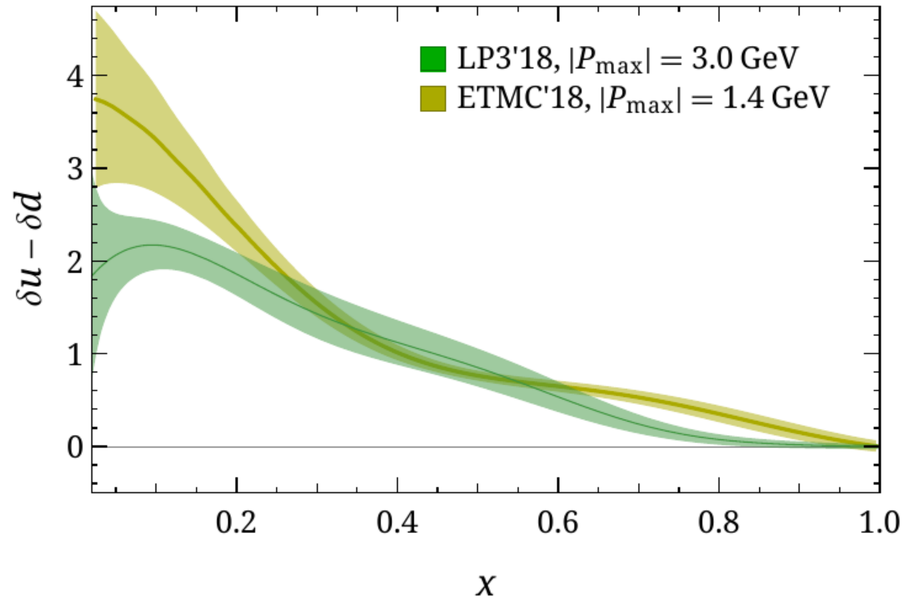


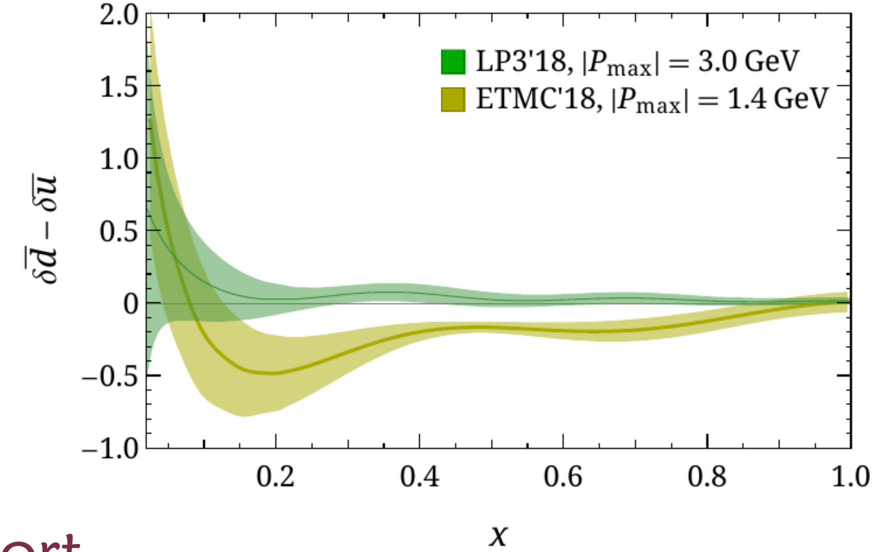
Transversity



Finite volume,
Discretization,
…

$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$





2006.08636, PDFLattice2019 report

§ Summary of physical pion mass results

- Quasi-PDF method only

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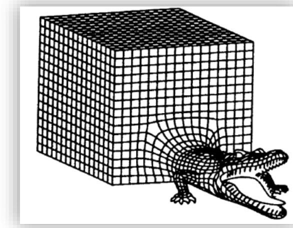
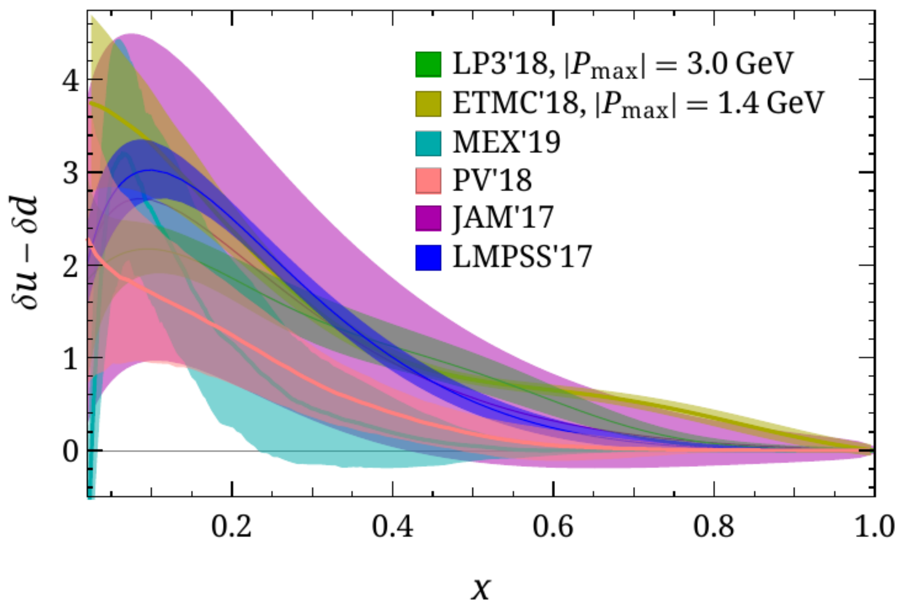
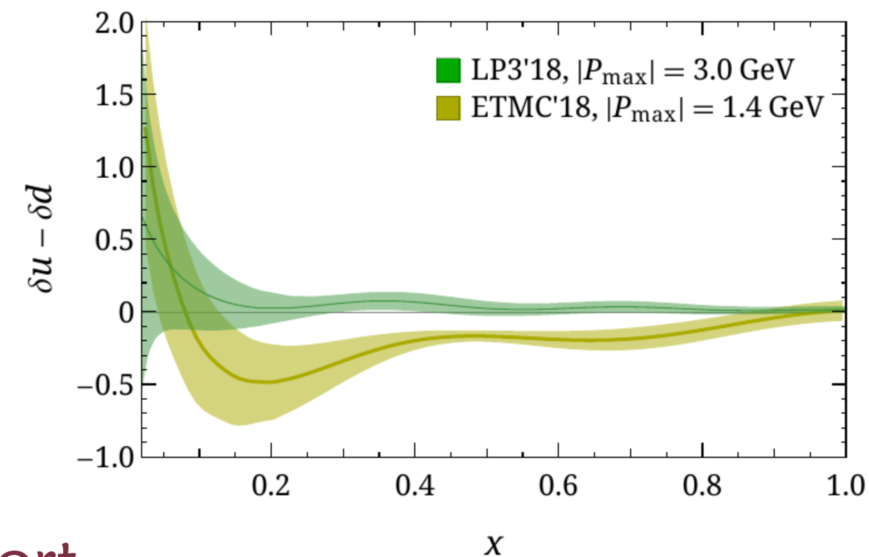
Transversity



Finite volume,
Discretization,
...

$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$





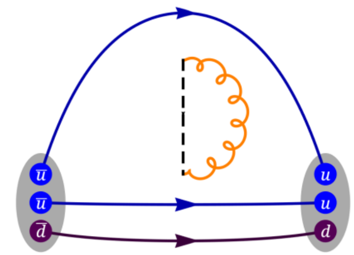
2006.08636, PDFLattice2019 report

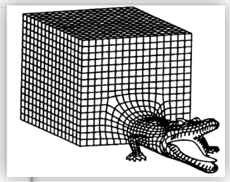
Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

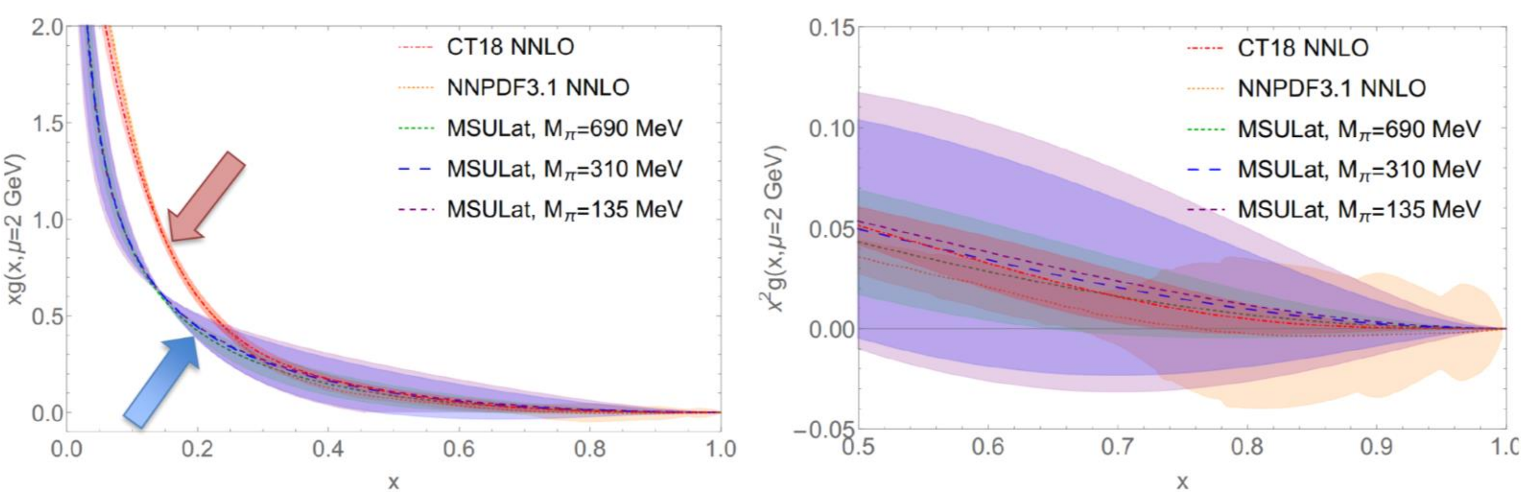
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

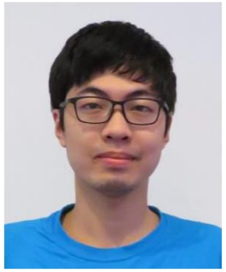
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\text{MS}}$ scheme.



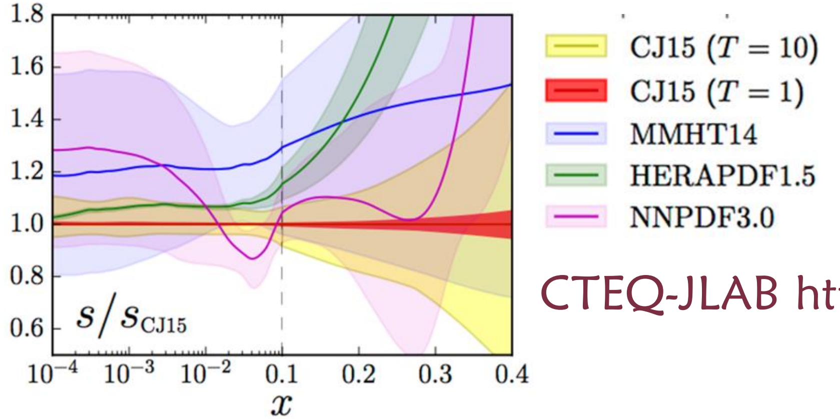


Zhouyou Fan (MSU)

Slide by Zhouyou Fan@DNP2020

First Lattice Strange PDF

§ Large uncertainties in global PDFs



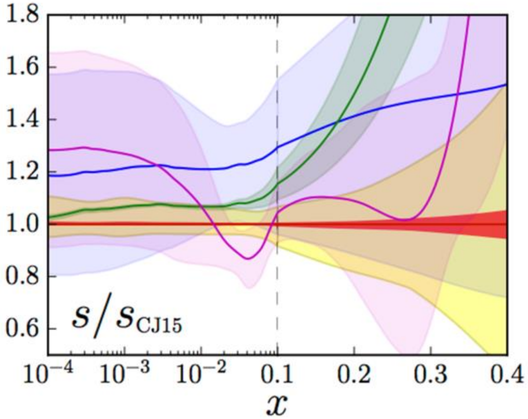
❧ Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

First Lattice Strange PDF

§ Large uncertainties in global PDFs



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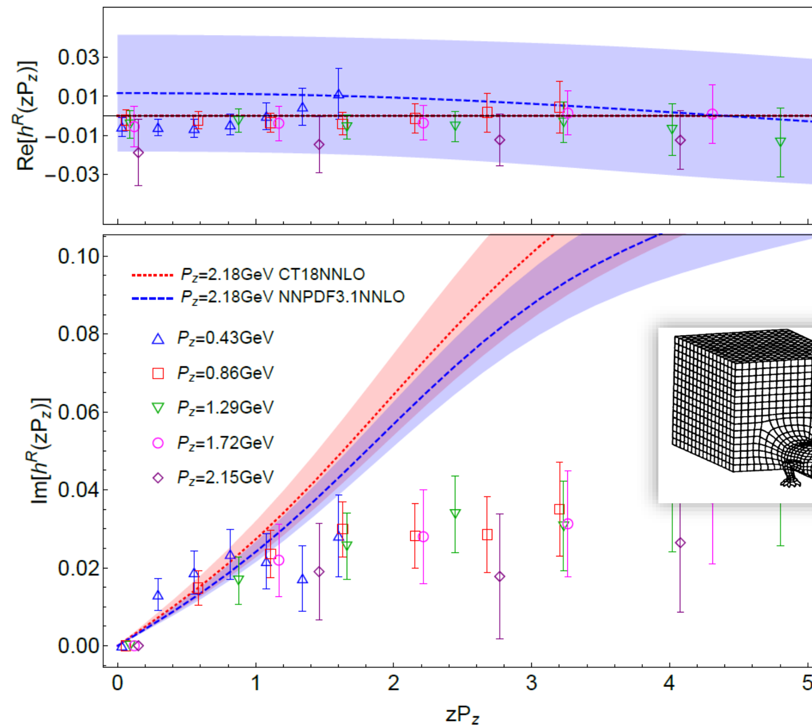
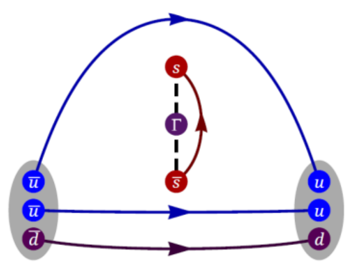


§ Results by MSULat/quasi-PDF method 2005.12015, Zhang, Lin, Yoon

- Clover on 2+1+1 HISQ, $a \approx 0.12$ fm extrapolated to $M_\pi \approx 140$ MeV



First Lattice Strange PDF

§ Large uncertainties in global PDFs

$$h^R(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx e^{ixzPz} \int_{-1}^{1} \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu R}{\mu}, \frac{\mu}{yP^z}, \frac{p_z^R}{yP^z}) q(y,\mu=2\text{ GeV})$$

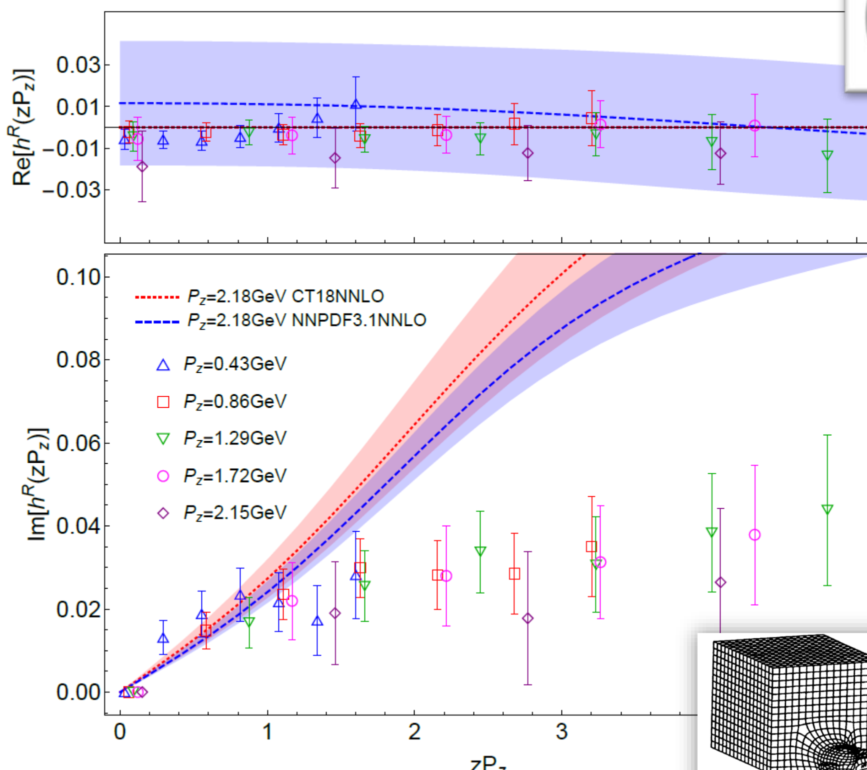
$\text{Re}[h(z)] \propto$

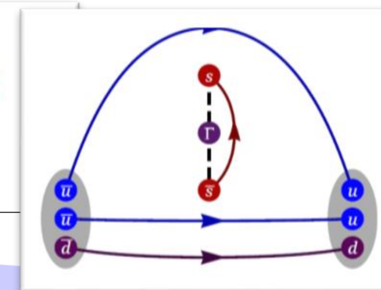
$$\int dx\,(s(x)-\bar{s}(x))\,cos(xzP_z)$$

$\text{Im}[h(z)] \propto$

$$\int dx\,(s(x)+\bar{s}(x))\,sin(xzP_z)$$

- symmetric $s-\bar{s}$ distribution.
- smaller momentum fraction.





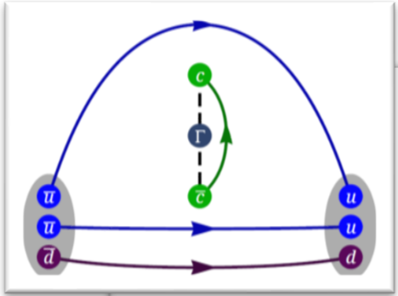
Rui Zhang (MSU)

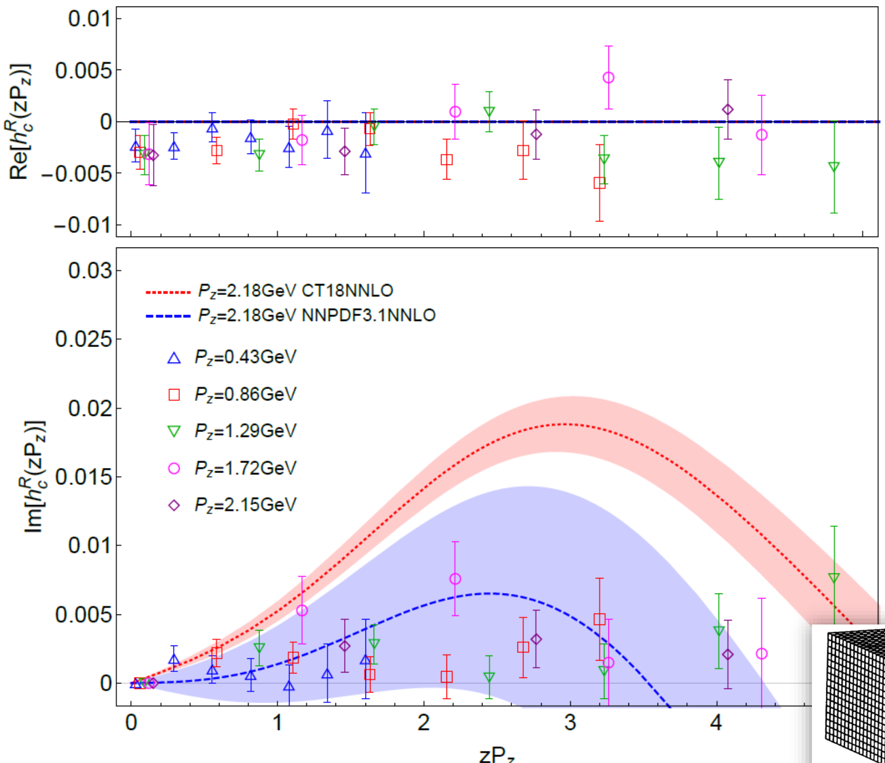
Slide by Rui Zhang @ DNP2020

First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method

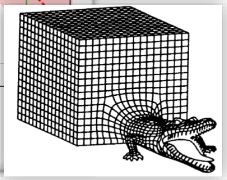
2005.12015, R. Zhang et al (MSULat)





- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF

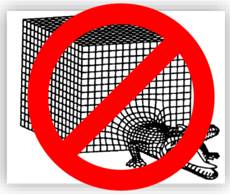
Rui Zhang (MSU)



First Continuum PDF

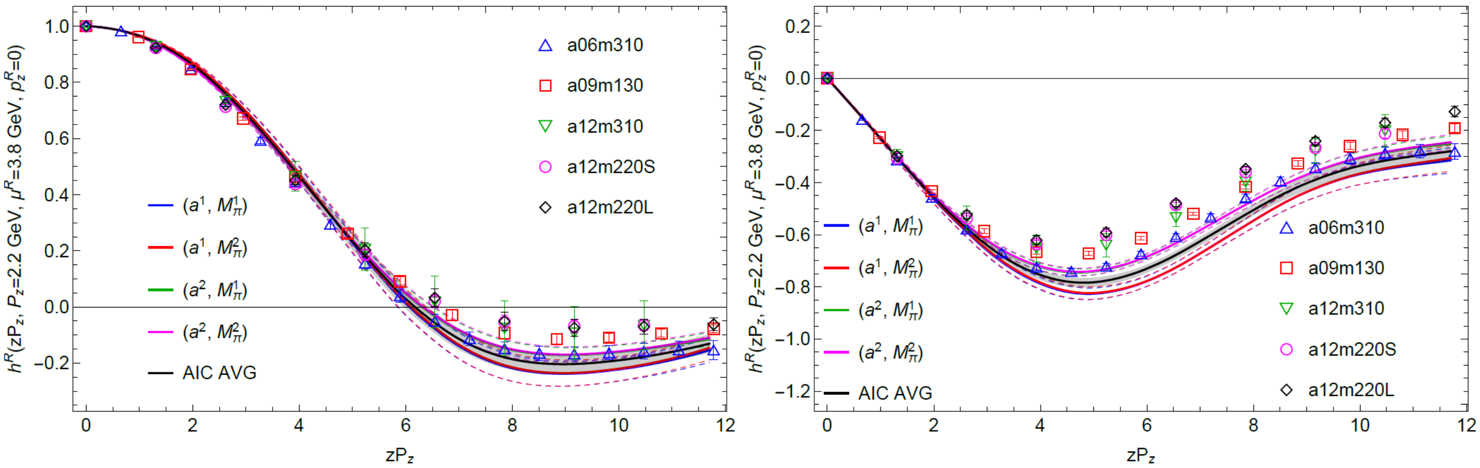
§ Nucleon PDFs using quasi-PDFs in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV



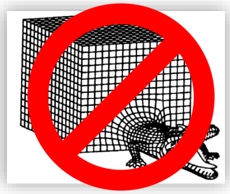
2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit



First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit



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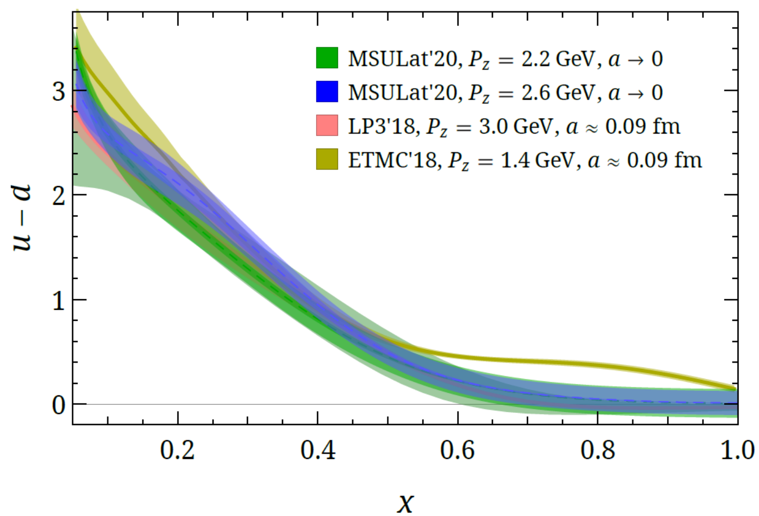
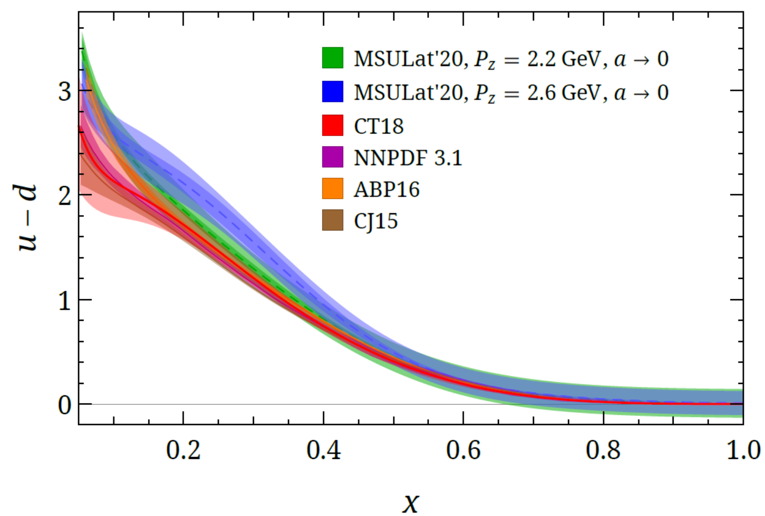
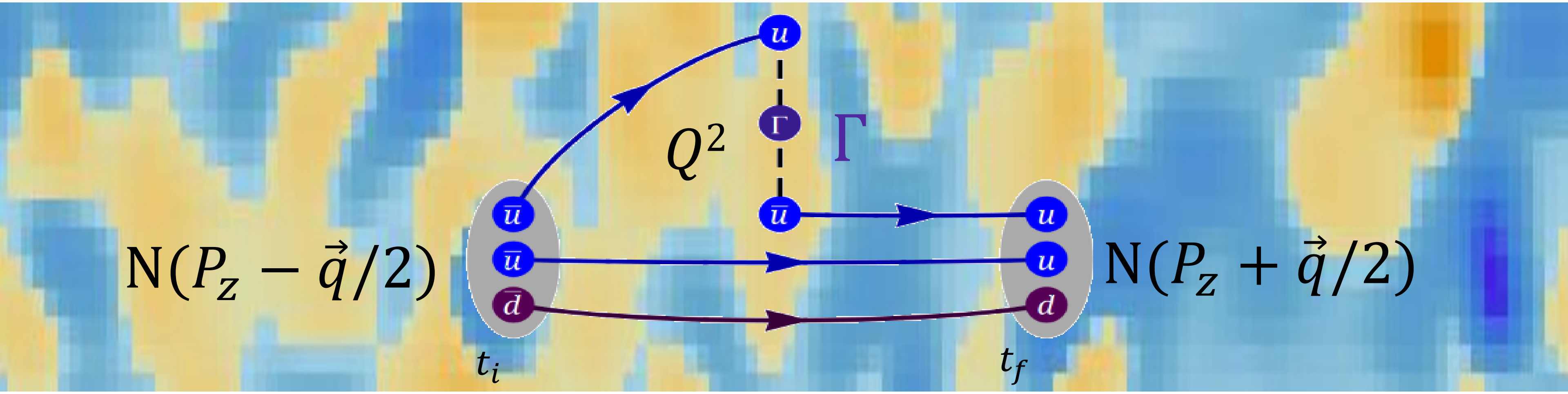




Bjorken-x Dependent GPDs

Generalized Parton Distributions

- On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_z)$$

$$= \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(\tilde{H}(x, \tilde{\xi}, t, \bar{P}_z) \gamma^0 + \tilde{E}(x, \tilde{\xi}, t, \bar{P}_z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

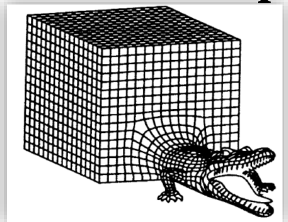
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

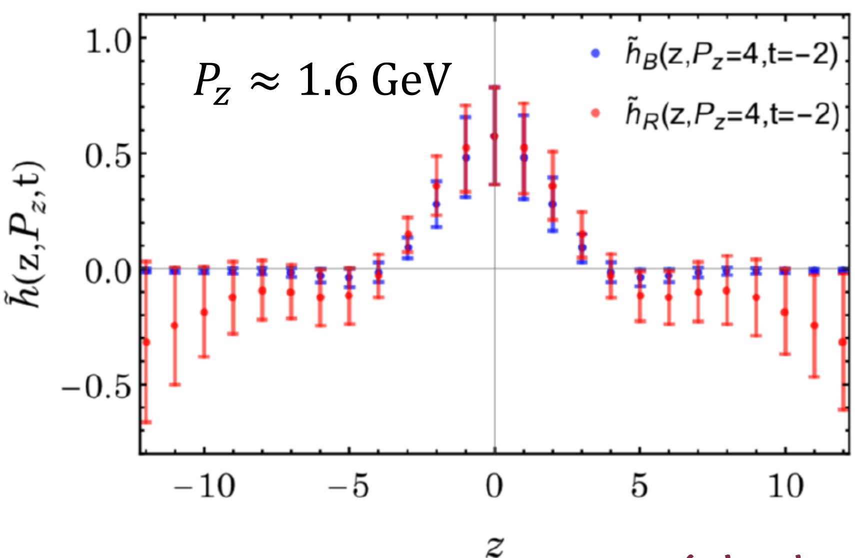
- Lattice details: clover/HISQ, 0.12fm, **310-MeV** pion mass

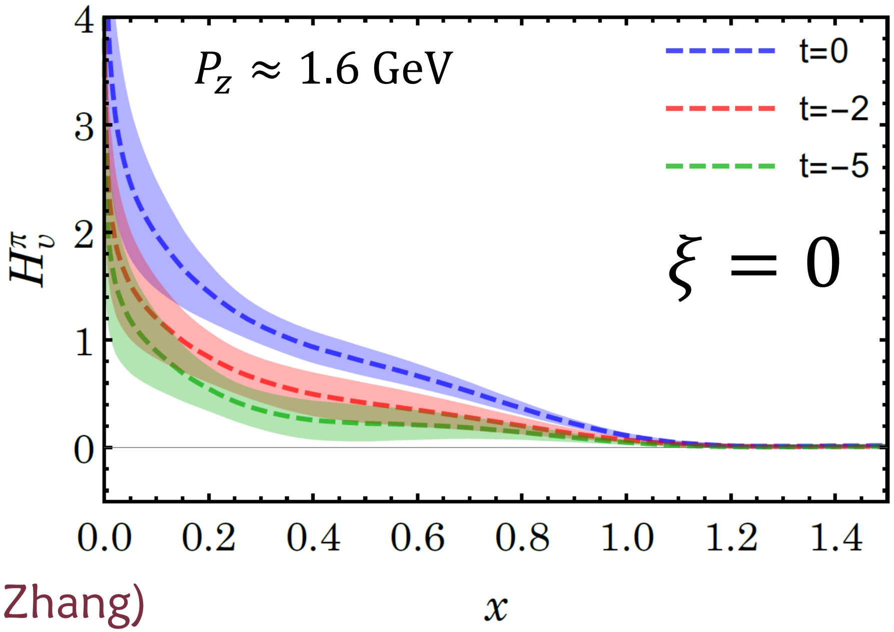
$P_z \approx 1.3, 1.6$ GeV

J. Chen, HL, J. Zhang, 1904.12376



$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+\Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$





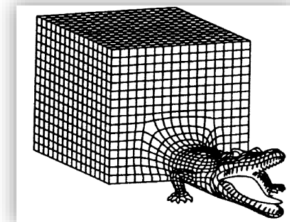
(plot by J. Zhang)

Isovector Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using quasi-PDFs

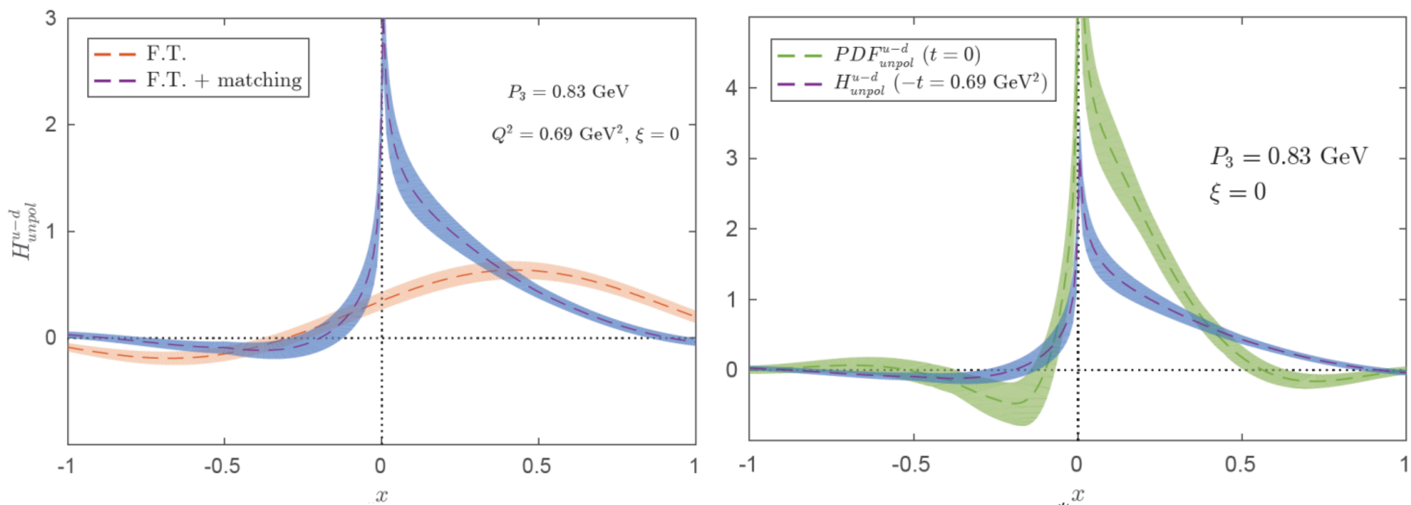
- Lattice details: twisted-mass fermions, 0.09fm, **270-MeV** pion mass, $P_z \approx 0.83$ GeV

$$F(x,\xi,t) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P'|O_{\gamma^+}(\zeta^-)|P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left\{H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right\}u(P)$$



nucleon $\xi = 0$ isovector results

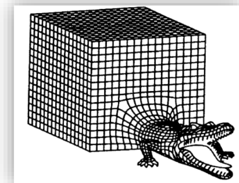
C. Alexandrou, (**ETMC**), 1910.13229 (Lattice 2019 Proceeding)



Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



finite-volume,
discretization,
…

$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \tilde{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

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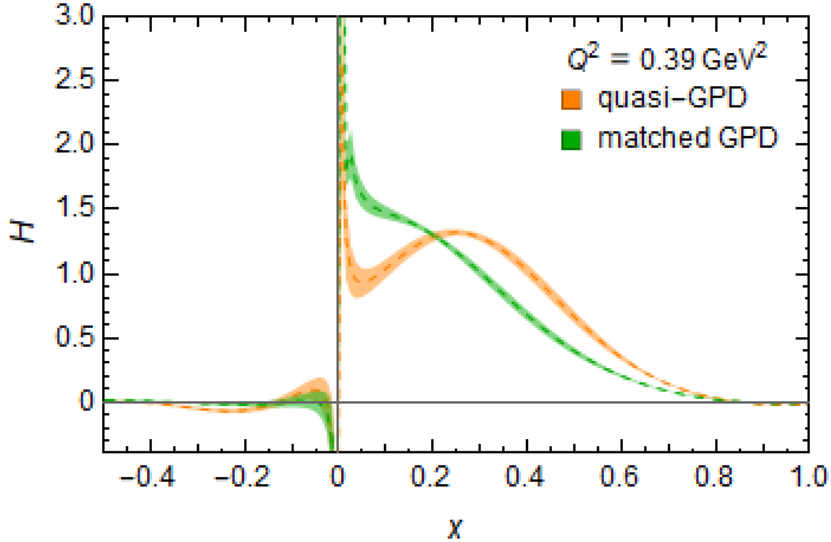
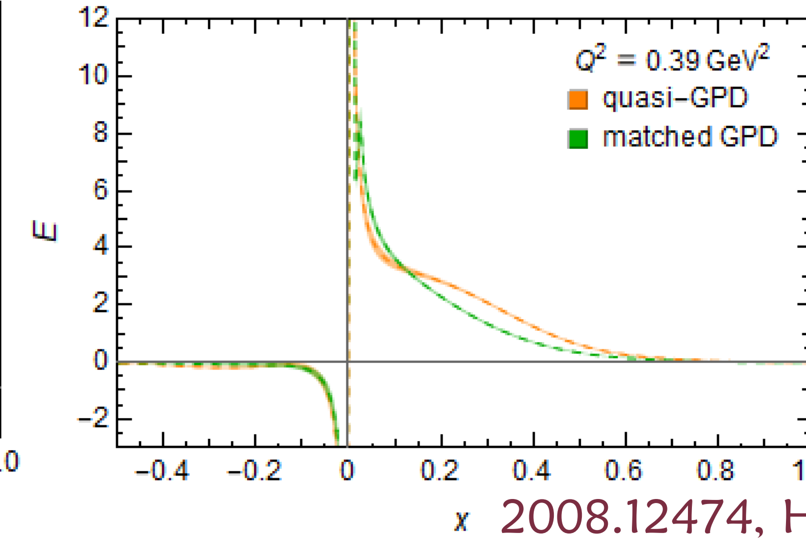




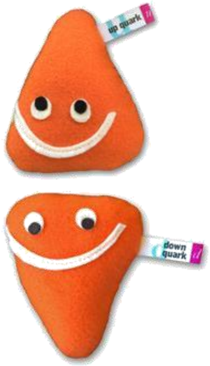
2008.12474, HL (MSULat)

Isovector Nucleon GPDs

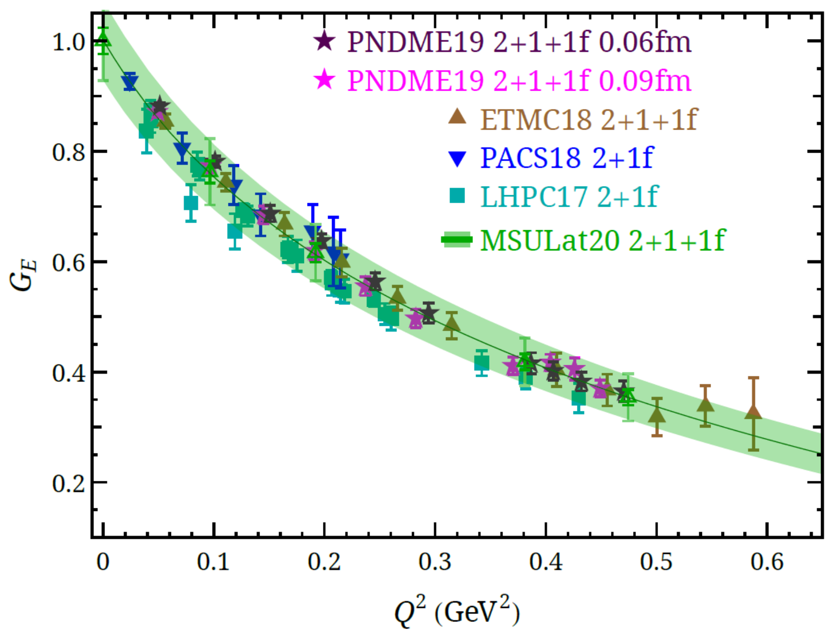
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} H^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 1$



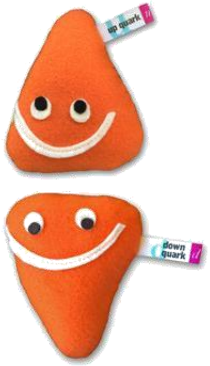
2008.12474, HL (MSULat)

Nucleon GPDs

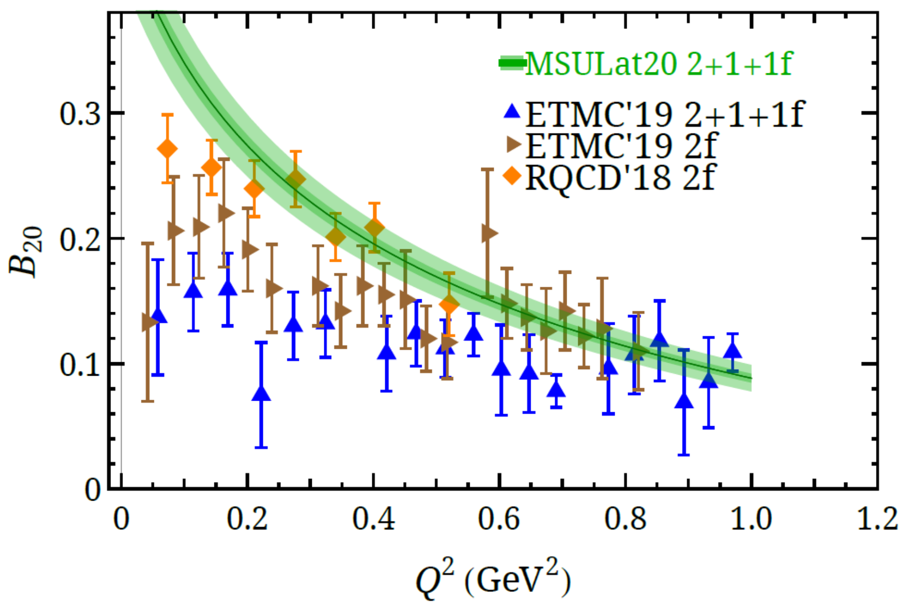
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} E^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n\ \text{even}}$$



$n = 2$



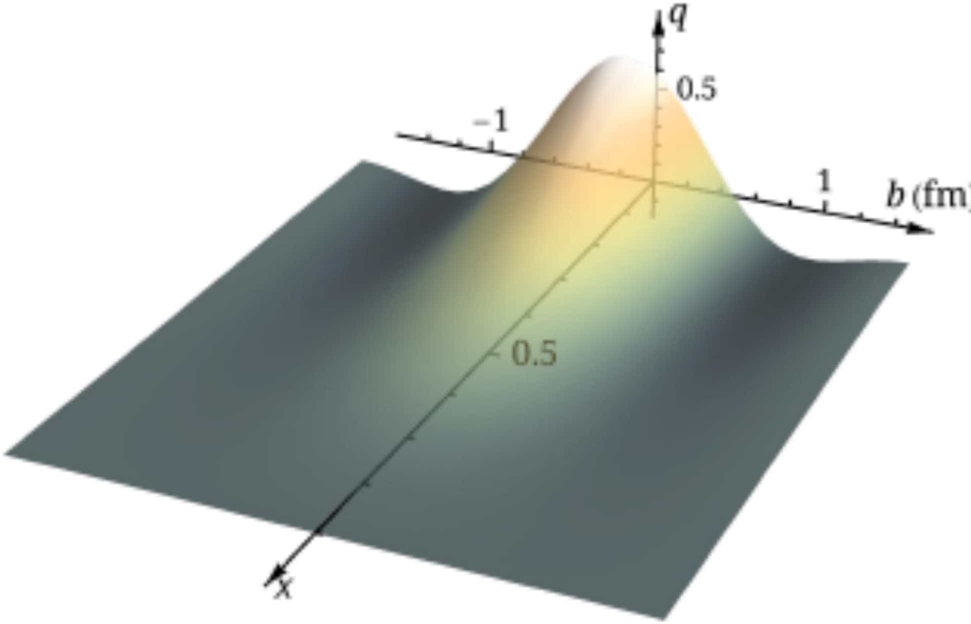
2008.12474, HL (MSULat)

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



2008.12474, HL (MSULat)

Nucleon Tomography

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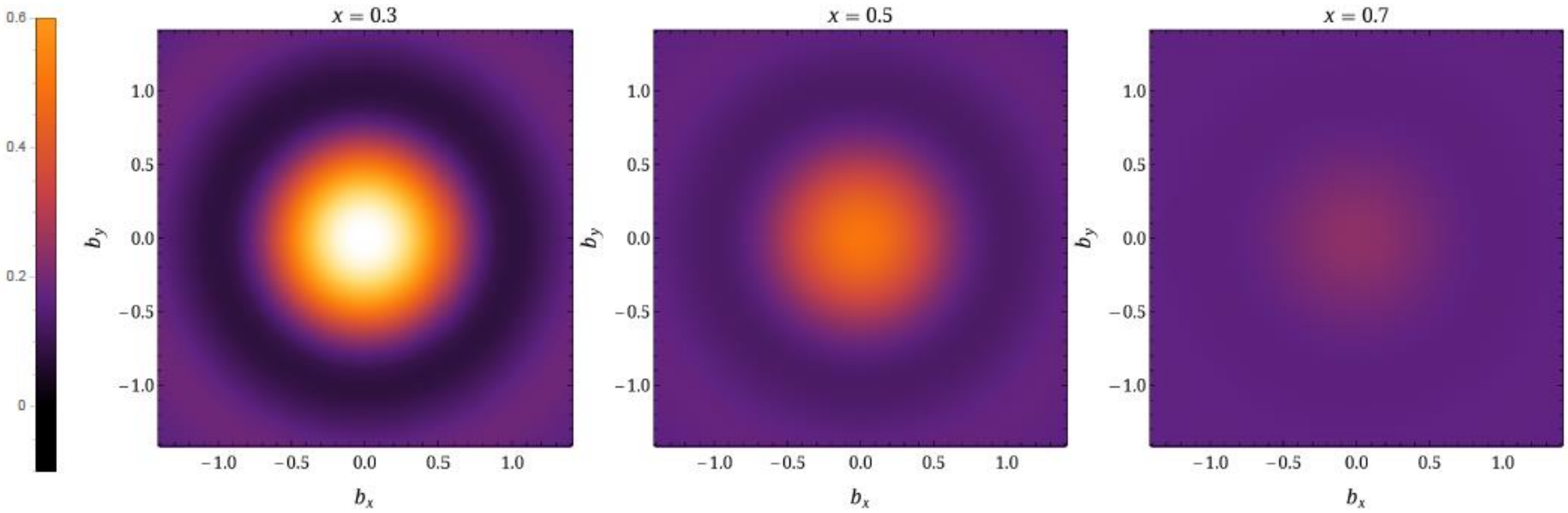
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



2008.12474, HL (MSULat)

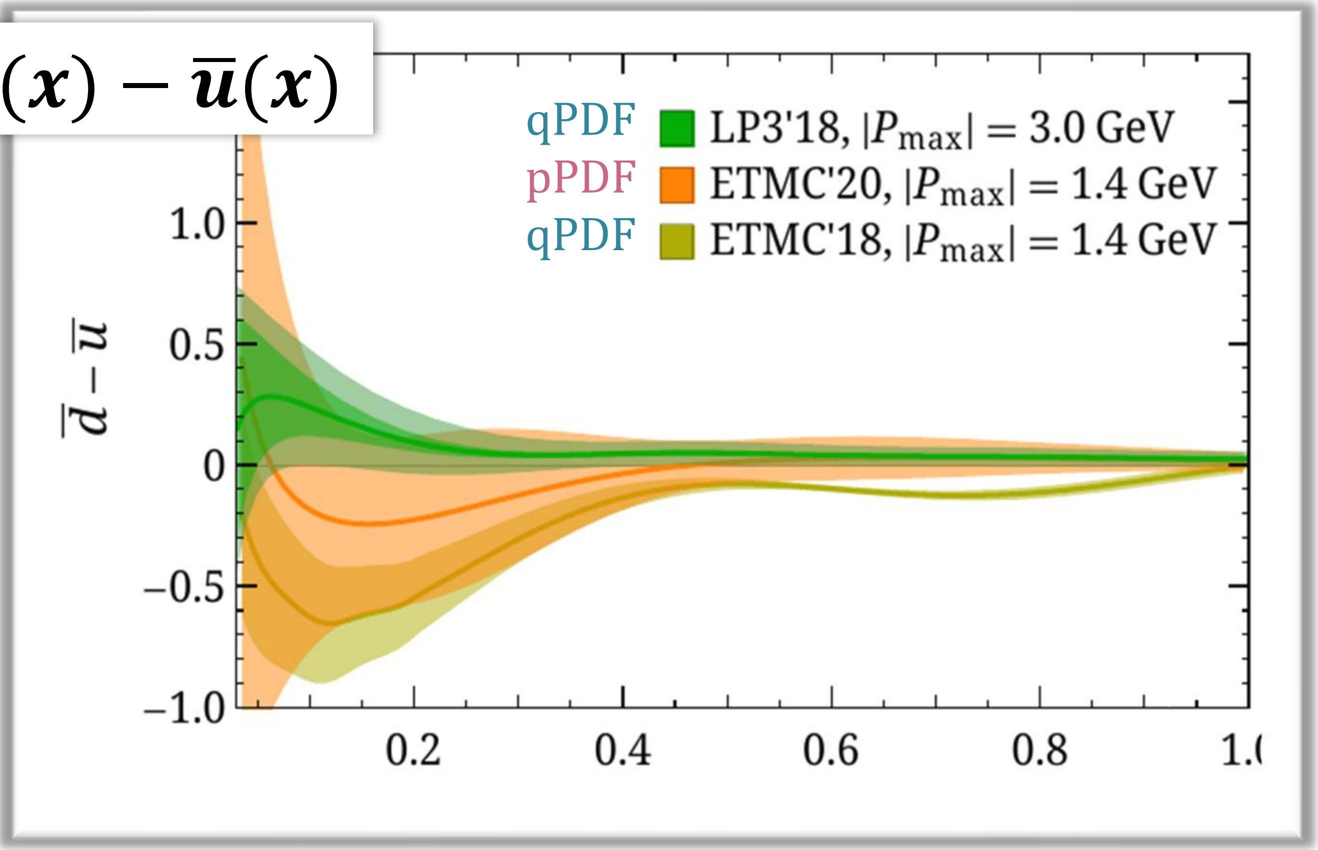
Future Prospects & Challenges

Physical Pion Mass Results

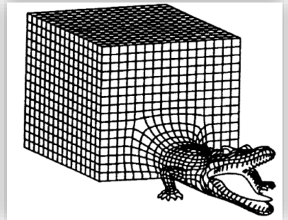
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



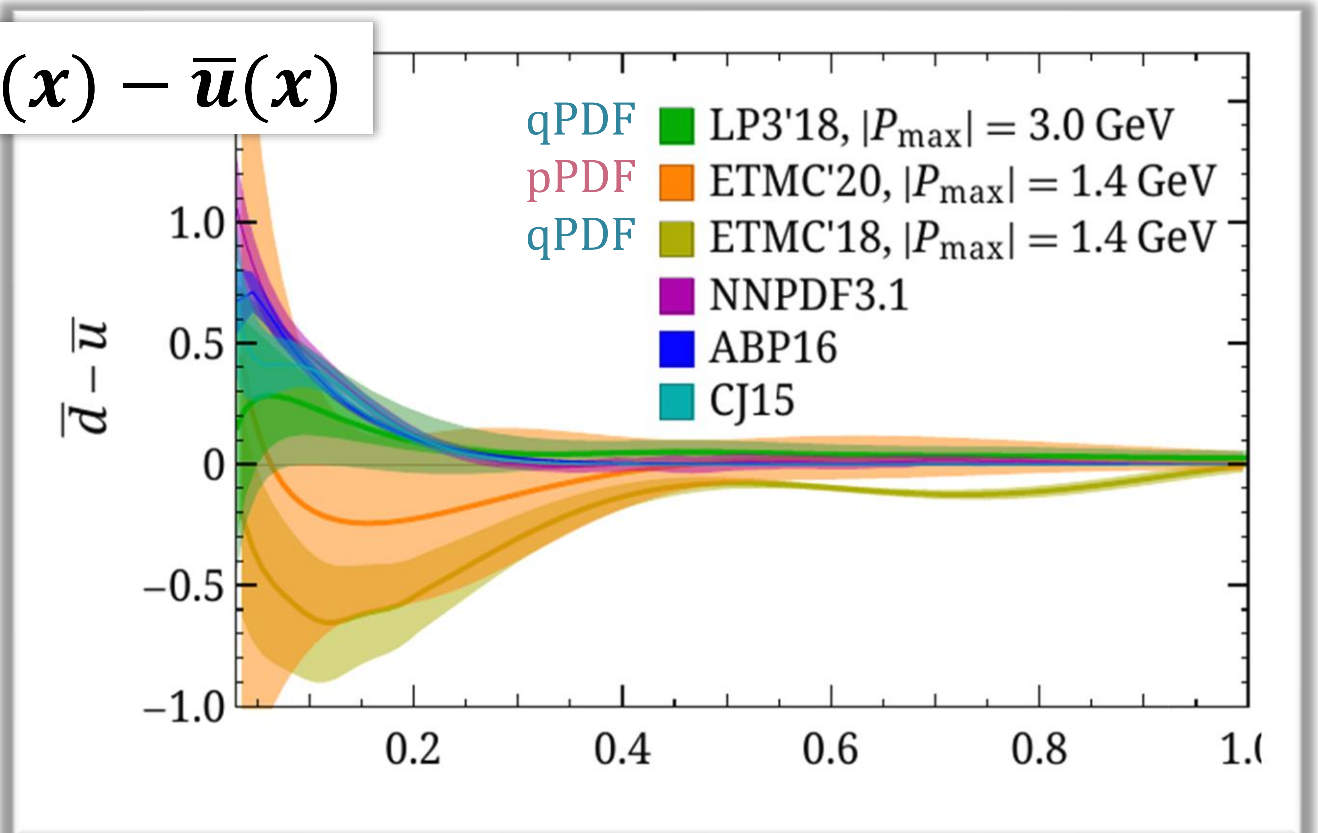
Finite volume,
Discretization,
…

Physical Pion Mass Results

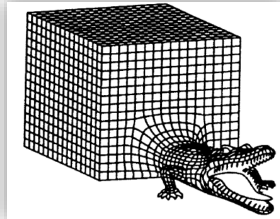
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



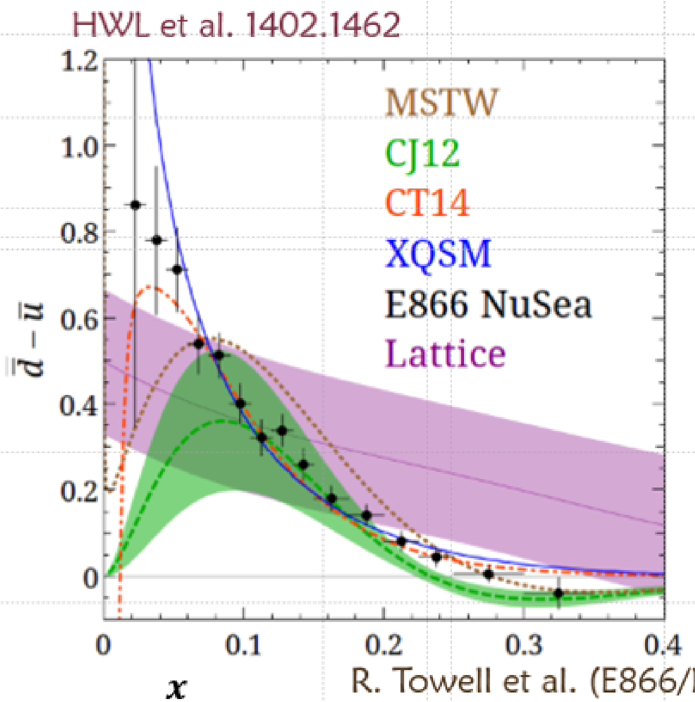
Finite volume,
Discretization,
…

Backstory

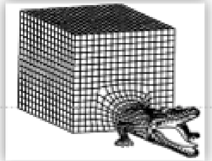
- Many of you are old enough to remember this:

Sea Flavor Asymmetry

- First time in LQCD history to study antiquark distribution!
  - $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**



$\bar{q}(x) = -q(-x)$



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

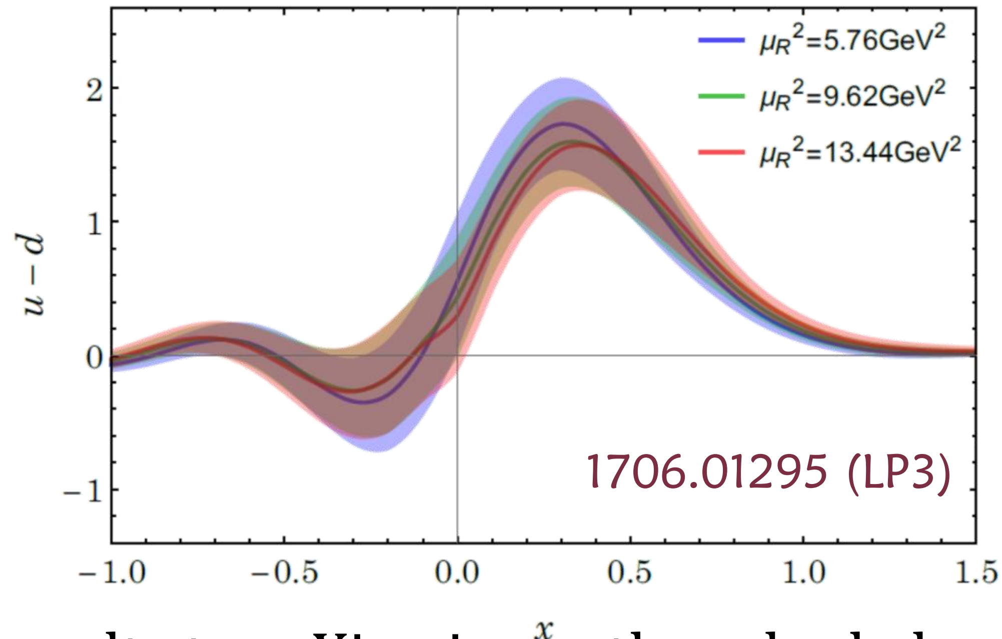
Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Caveat: These matrix elements are not properly renormalized

Backstory

- Efforts by multiple collaborations have been devoted into working on lattice renormalization
  - We finally obtained the renormalized ME, and the renormalized PDF results puzzled us for months!

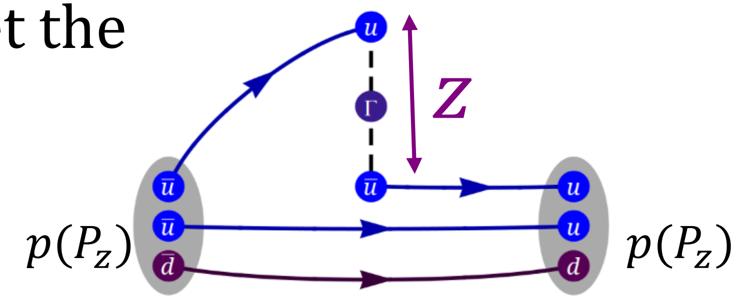


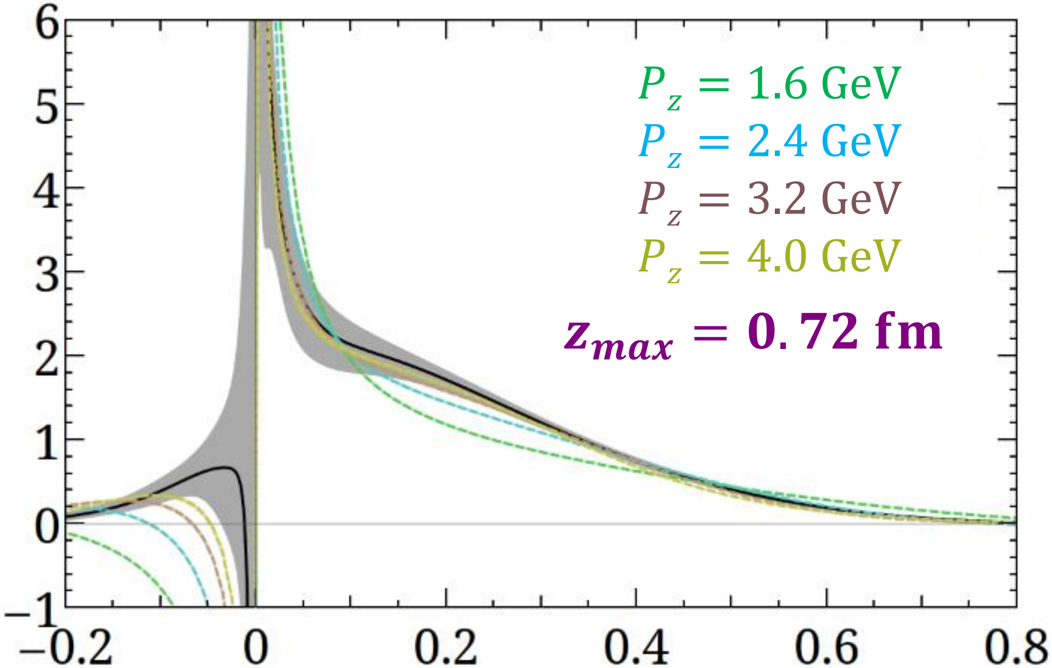
  - We finally posted the results to arXiv, since others had already posted their renormalized result

Continuum Toy Models

§ Impacts on antiquark and small-x regions?

- One needs large momentum just to get the sign of the antiquark correct!
- With small zP_z, one will miss over the majority of x





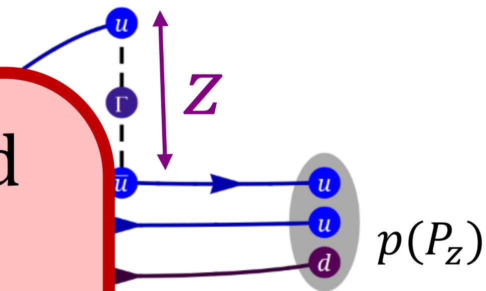
HL et al, 1708.05301

- Not just a quasi-PDF problem
- Going for large P_z is an unavoidable direction for all x-dependent methods
- Higher-loop matching kernel is not going to do much for it!

Continuum Toy Models

§ Impacts on antiquark and small-x regions?

- One needs large momentum just to get the

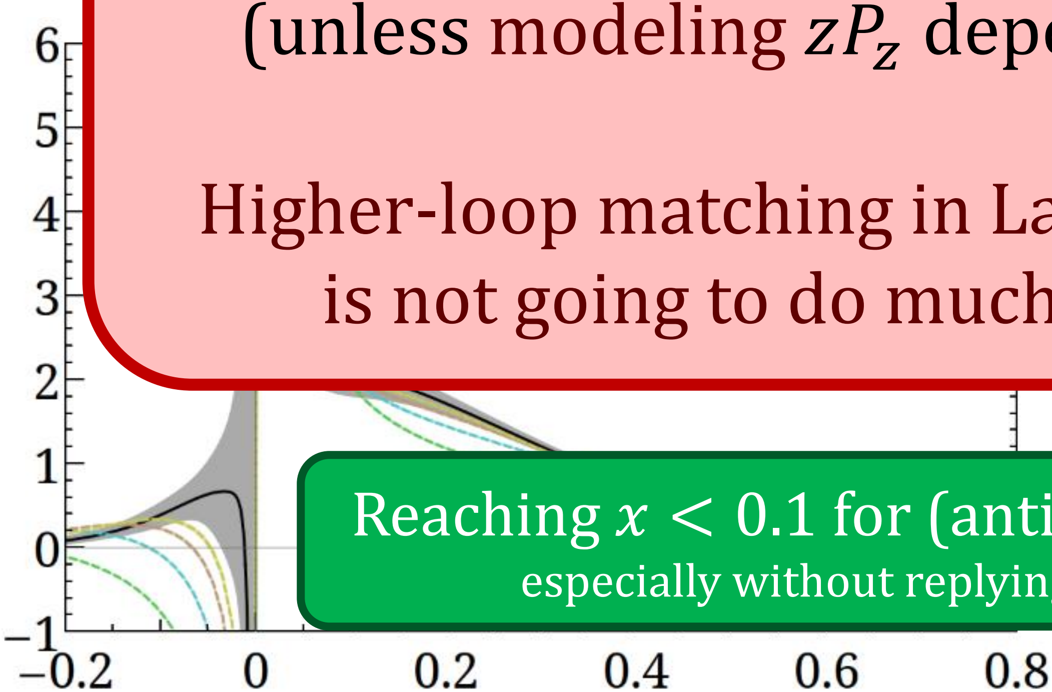


The x-dependent PDFs will be doomed by a bad choice of max zP_z!
(unless modeling zP_z dependence)

Higher-loop matching in LaMET later is not going to do much for it!

asi-PDF

ge P_z is an
direction for
all x-dependent methods



much for it!

Reaching $x < 0.1$ for (anti)quark remains challenging
especially without replying on an assumed parametrization

HL et al, 1708.05301

Antiquark and Small-x PDFs

Small-x parton physics on lattice

(Letter of Interest for Snowmass 2021)

Xiangdong Ji,[1] Luchang Jin,[2] Bo-Wen Xiao,[3] and Feng Yuan[4,*]

[1] *Department of Physics, University of Maryland, College Park, MD 20742, USA*
[2] *Department of Physics, University of Connecticut, Storrs, CT 06269, USA*
[3] *School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China*
[4] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

- Small-*x*: 165: Small-*x* parton physics on lattice (EF6)
  - Difficulties in reliably extracting small-*x* in current *x*-dependent methods
  - Bring together small-*x* and LQCD communities to explore ways in resolve this in near future

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

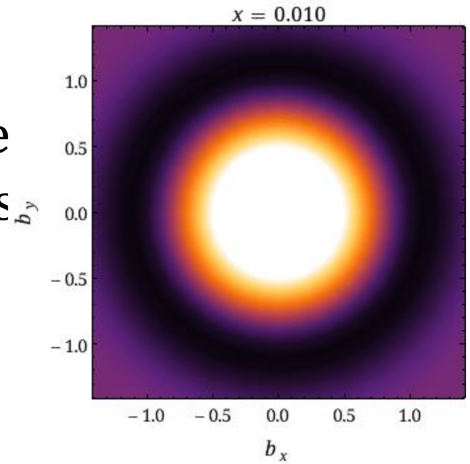
A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary

- § Exciting era using LQCD to study spectroscopy and structure of hadrons
- § Spectroscopy
  - For QCD-stable particles, lattice can make predictions
  - A lot of methods that's being developed in field, including scattering
  - Two-hadron and even three-hadrons interactions are in progress
- § Structure: overcoming longstanding limitations
  - More nucleon matrix elements with physical pion masses
  - Bjorken-*x* dependence of parton distributions are widely studied with LaMET and its variants
  - More study of systematics planned for the near future
  - Start to address neglected disconnected contributions obtaining flavor-dependent quantities



- § Stay tuned for more updates from LQCD

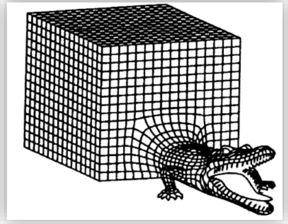
Backup Slides

Polarized PDFs

§ Summary of physical pion mass results

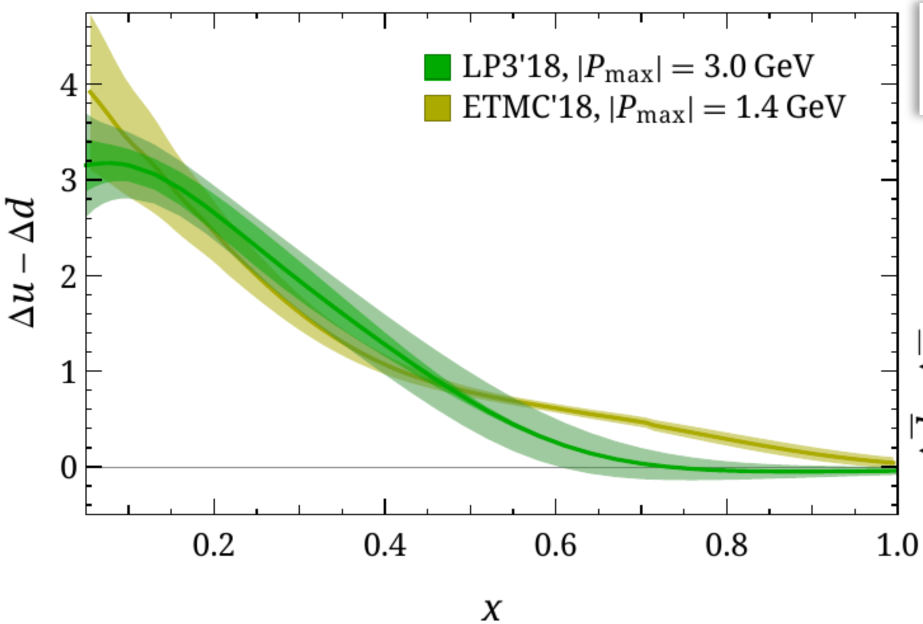
- Quasi-PDF method only

Helicity
long. polarized

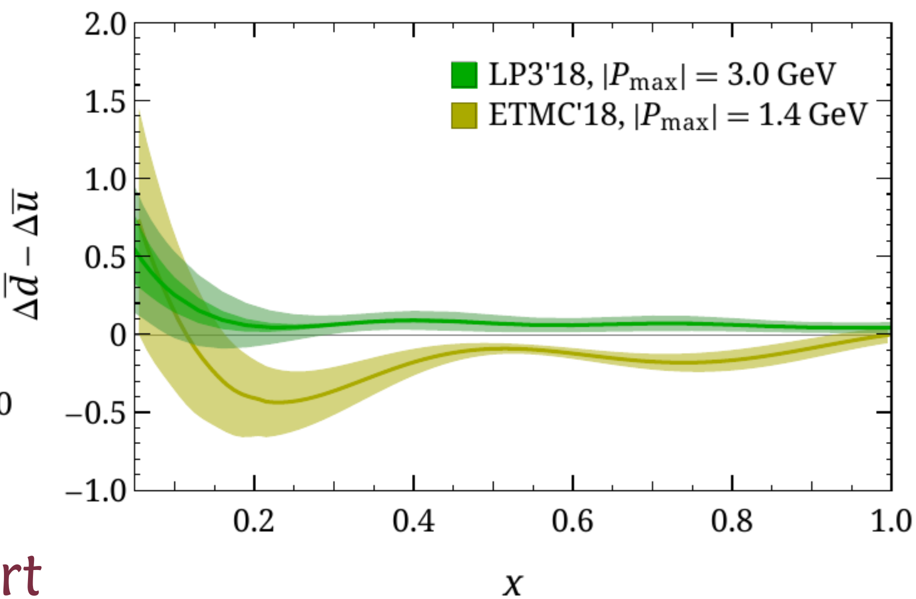


Finite volume,
Discretization,
…

$\Delta u(x) - \Delta d(x)$



$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$



2006.08636, PDFLattice2019 report

Polarized PDFs

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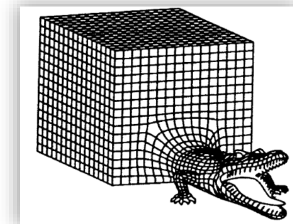
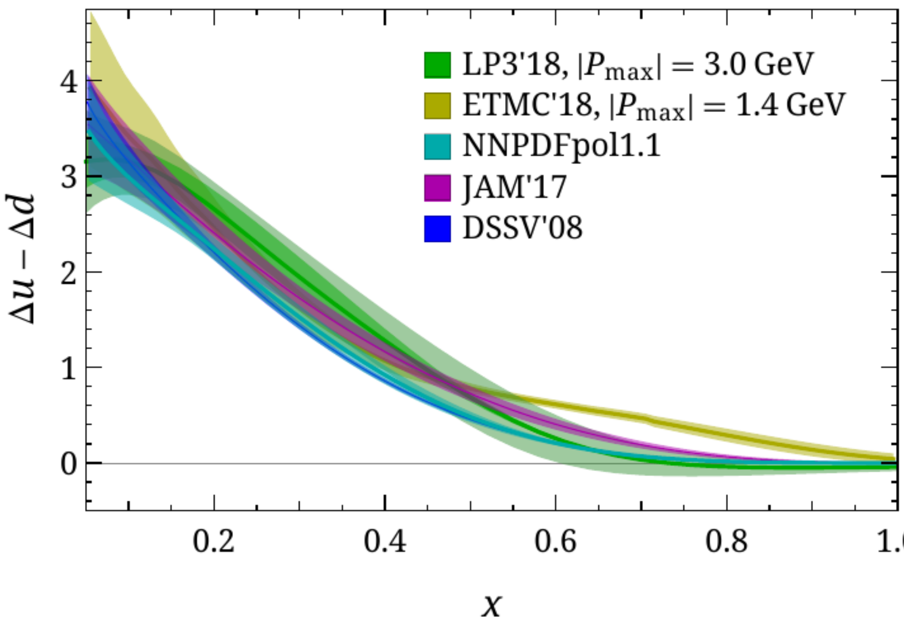
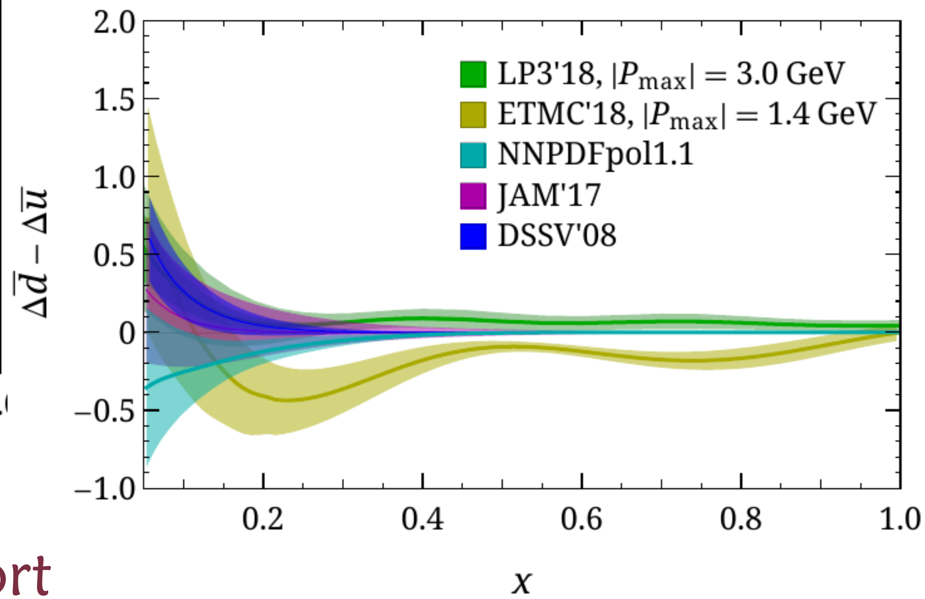


Finite volume,
Discretization,
…

$$\Delta u(x) - \Delta d(x)$$



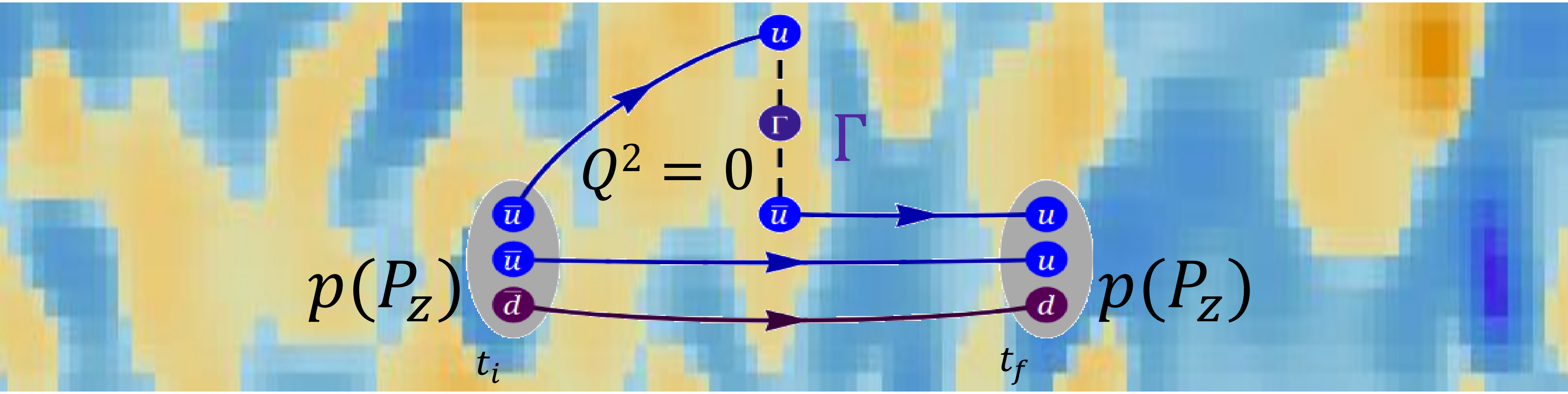
$$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$$



2006.08636, PDFLattice2019 report

Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

- No renormalization

$$\mathcal{M}\left(zP_z, z^2\right) = \frac{h(z,P_z)}{h(z,0)}$$

- FT zP_z-space to x-space at fixed z^2
  pseudo-PDF $\widetilde{\mathcal{M}}\left(x, z^2\right)$

§ Quasi-PDF

- Renormalization and ratios

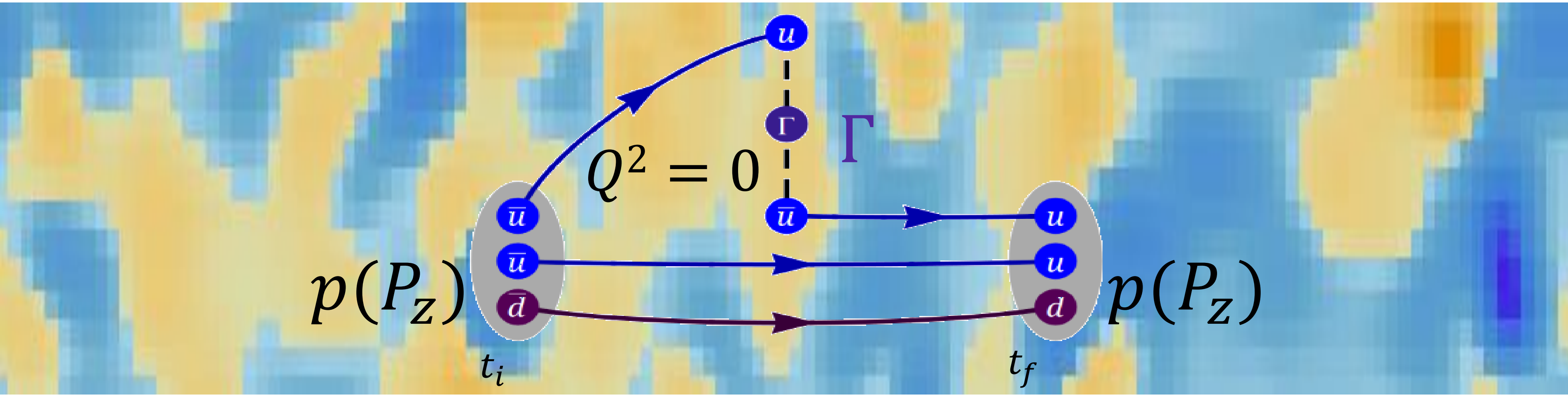
$$h^R\left(z, P_z, P^R\right) \text{ or } \frac{h\left(z,P_z,P^R\right)}{h\left(z=0,P_z,P^R\right)}$$

- FT z-space to x-space at fixed P_z
  quasi-PDF $\tilde{q}\left(x, P_z, P^R\right)$

See X. Ji, et al., NPB 964 (2021) and references on newer renormalization proposals

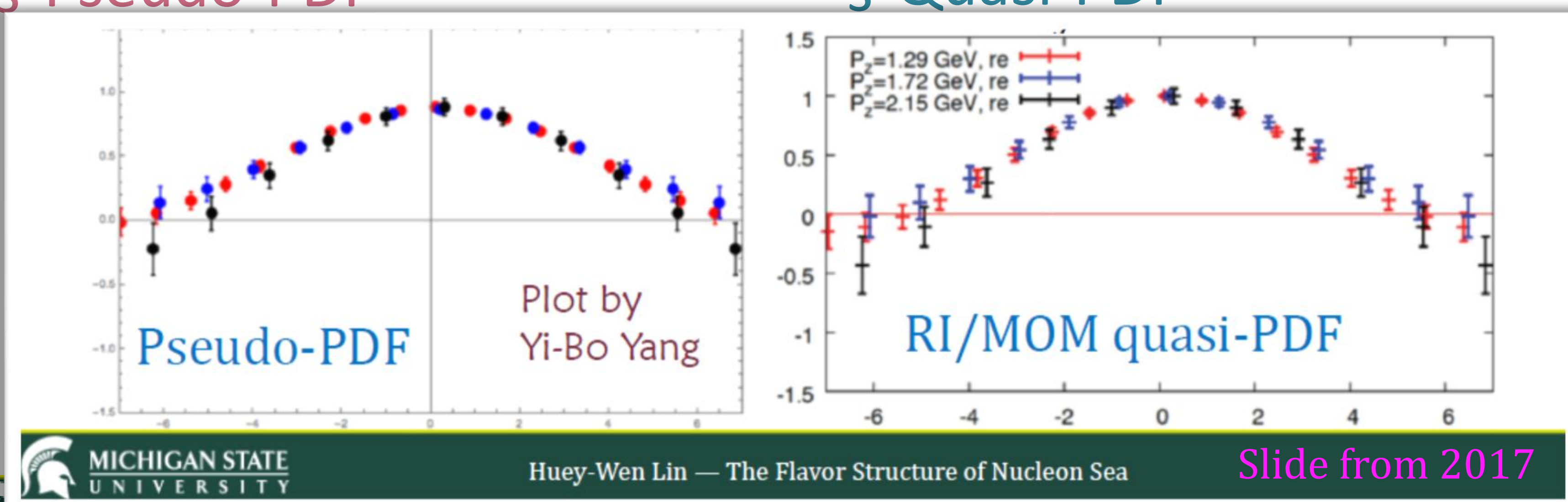
Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

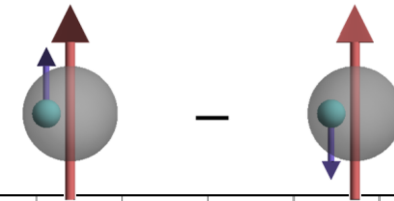
§ Quasi-PDF



Moments of PDFs

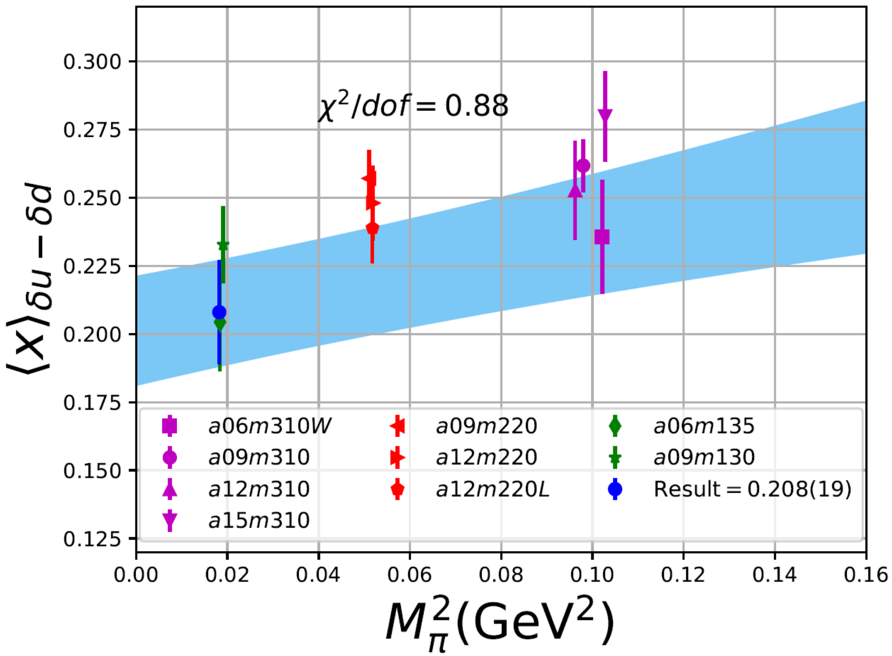
- Only lowest few moments

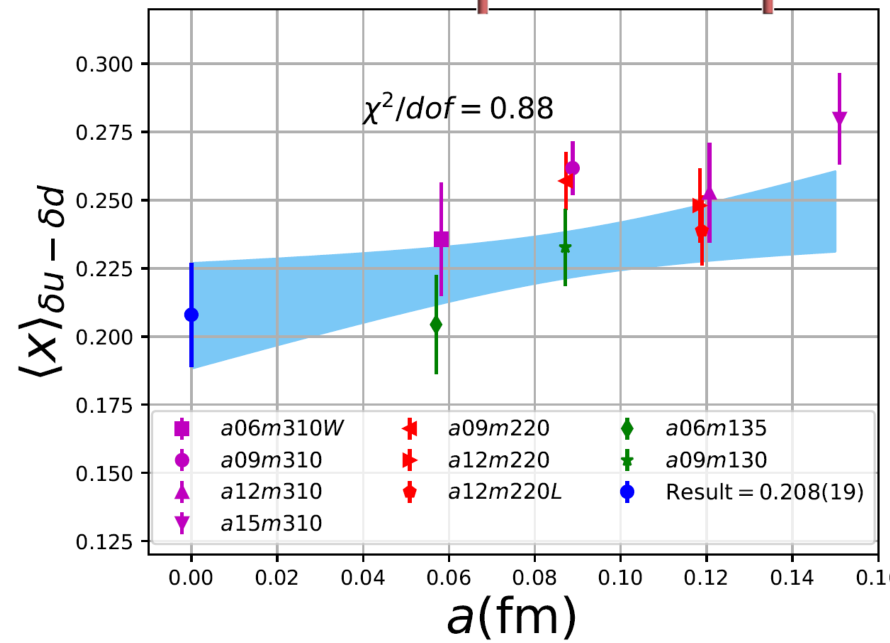
$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



- State-of-the art example
  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - ☙ Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636

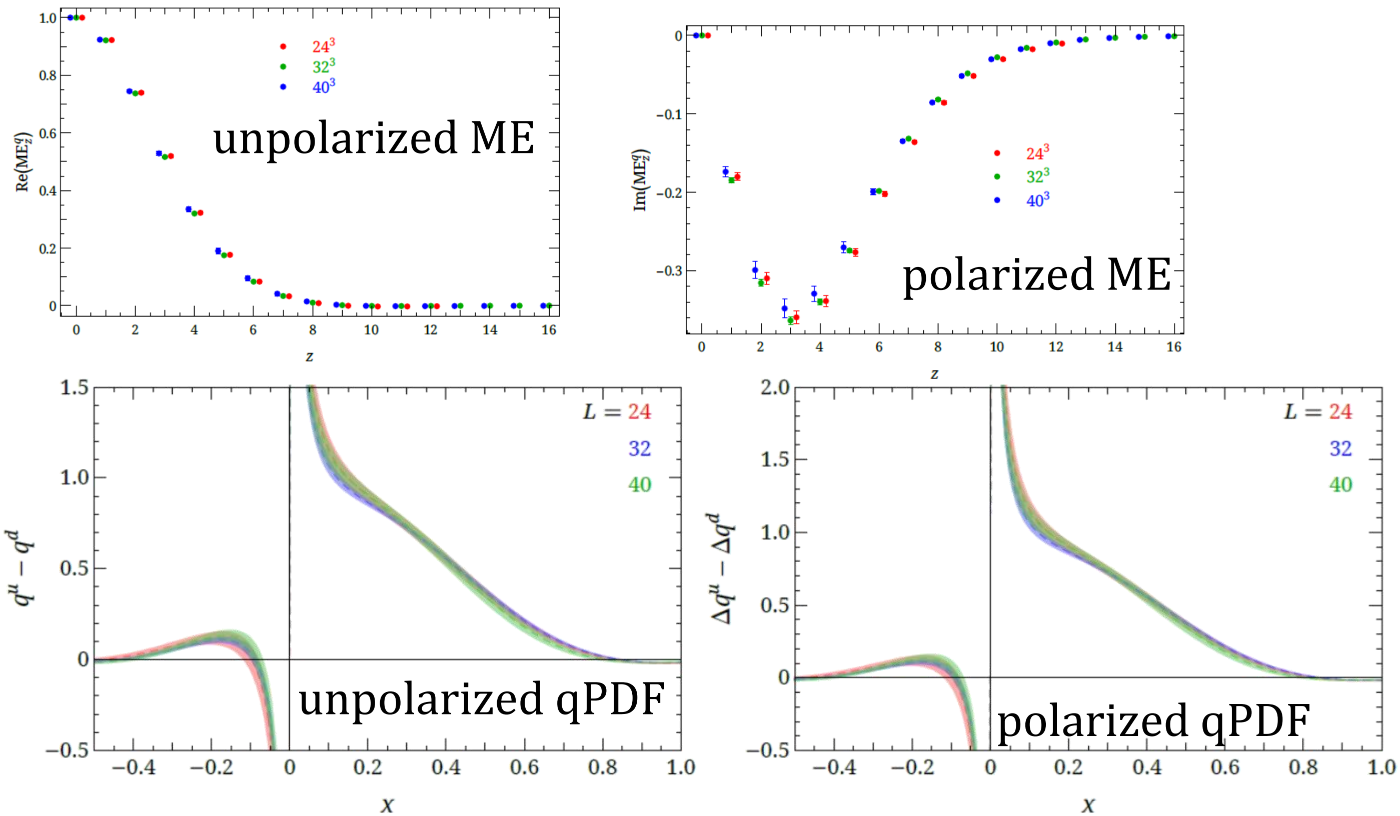
Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Systematics Study

§ First finite-volume study in quasi-PDFs

- Clover on 2+1+1 HISQ, $M_\pi \approx \mathbf{220}$ MeV, $a \approx 0.12$ fm
- $M_\pi L \approx 3.3, 4.4, 5.5$, $P_z \approx 1.3$ GeV

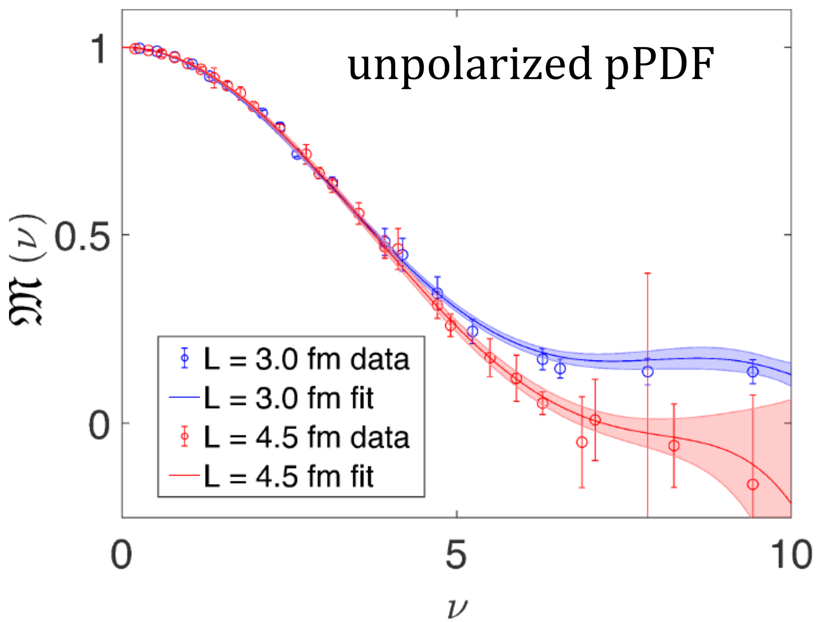
HL, R, Zhang, Phys.Rev.D 100 (2019) 7, 074502



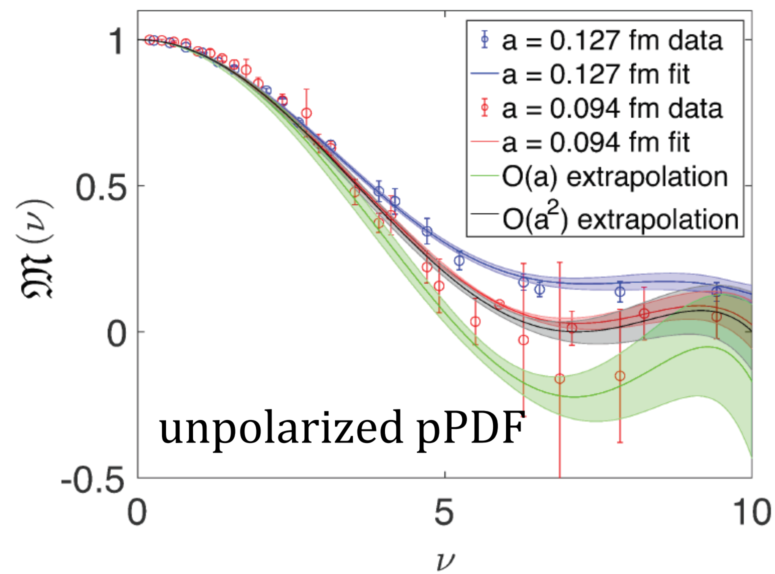
Systematics Study

- Finite-volume study in unpolarized pseudo-PDFs
  - 2+1f clover, $M_\pi \approx \mathbf{415}$ MeV, $a \approx 0.127$ fm
  - Two volumes used: $L \approx 3, 4.5$ fm

B. Joo et al (Jlab/W&M) 1908.09771



- Also see strong lattice-spacing dependence



- Lattice artifacts are sensitive to the simulated QCD vacuum
  - Each group will have to check their own systematics carefully

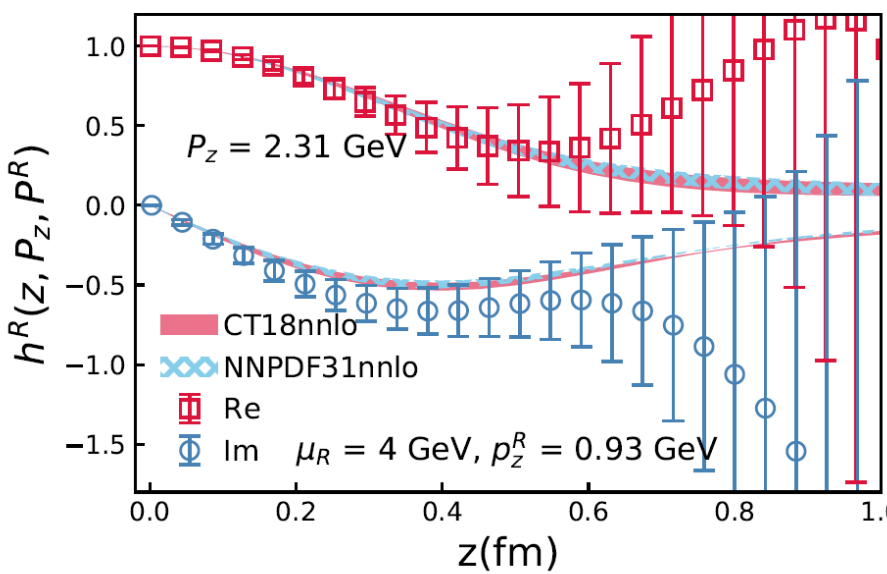
Superfine Lattice Spacing

§ Approaching continuum limit in quasi-PDFs

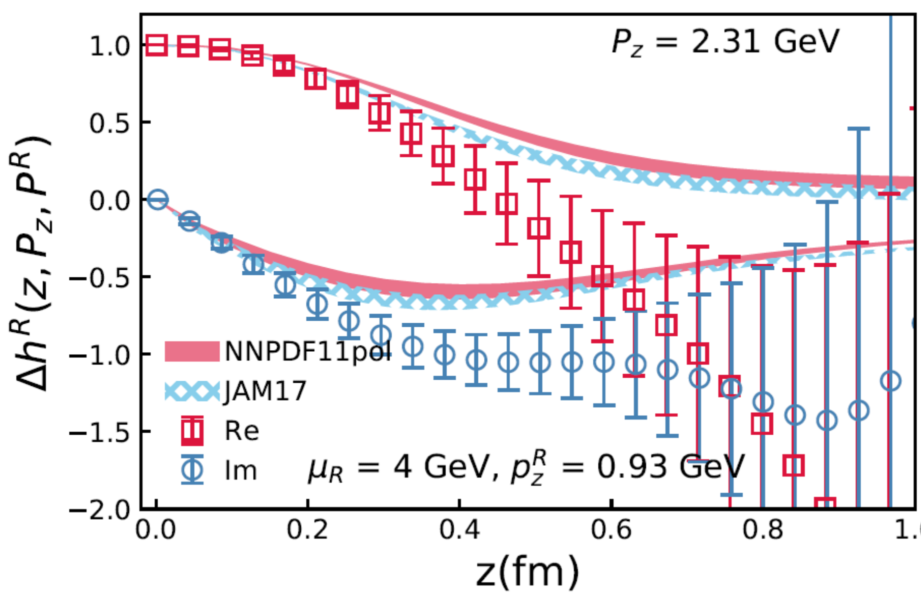
- Important for all x-dependent methods
- Large momentum required to reach $x < 0.1$ reliably
  $(aP_z)^n$ systematics should be small
- First work done with superfine lattice spacing, $a \approx 0.042$ fm

2005.12015, BNL/MSULat

Unpolarized ME



Polarized ME



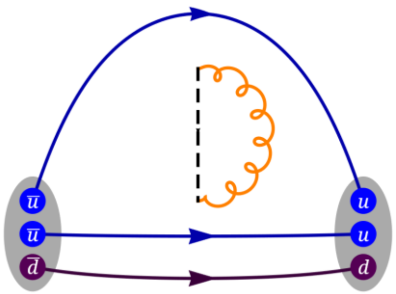
(plot by Xiang Goa)

Gluon PDF in Nucleon

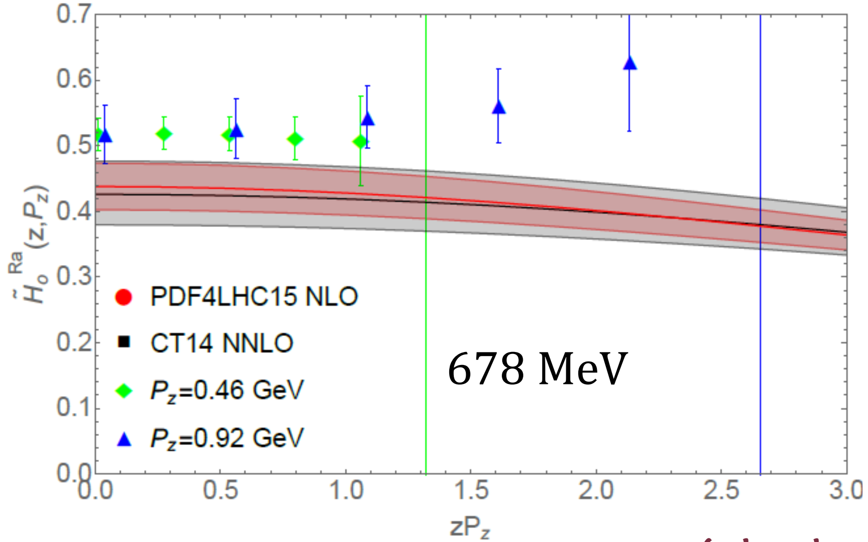
§ Pioneering first glimpse into gluon PDF using LaMET

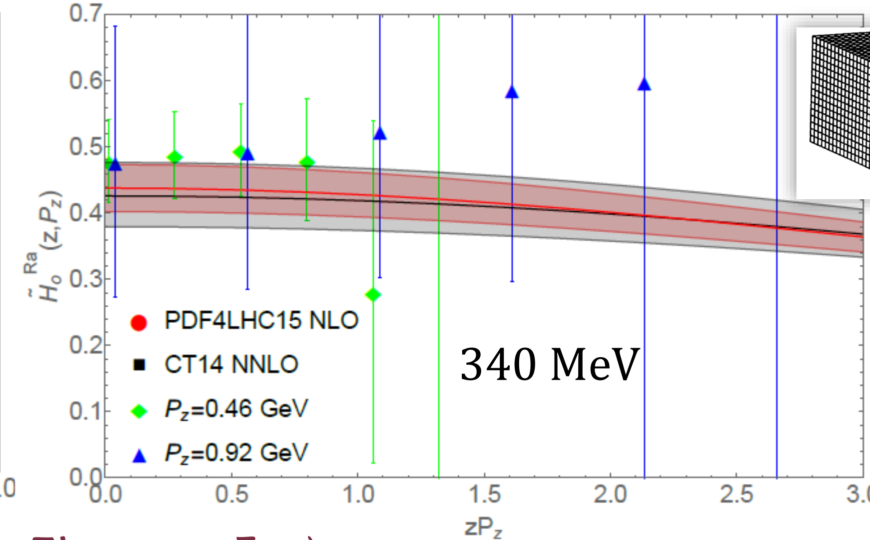
- Lattice details: overlap/2+1DWF, 0.16 fm, 340-MeV sea pion
- Study strange/light-quark

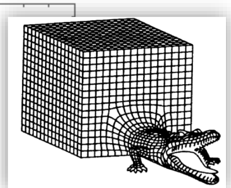
Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)



- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved



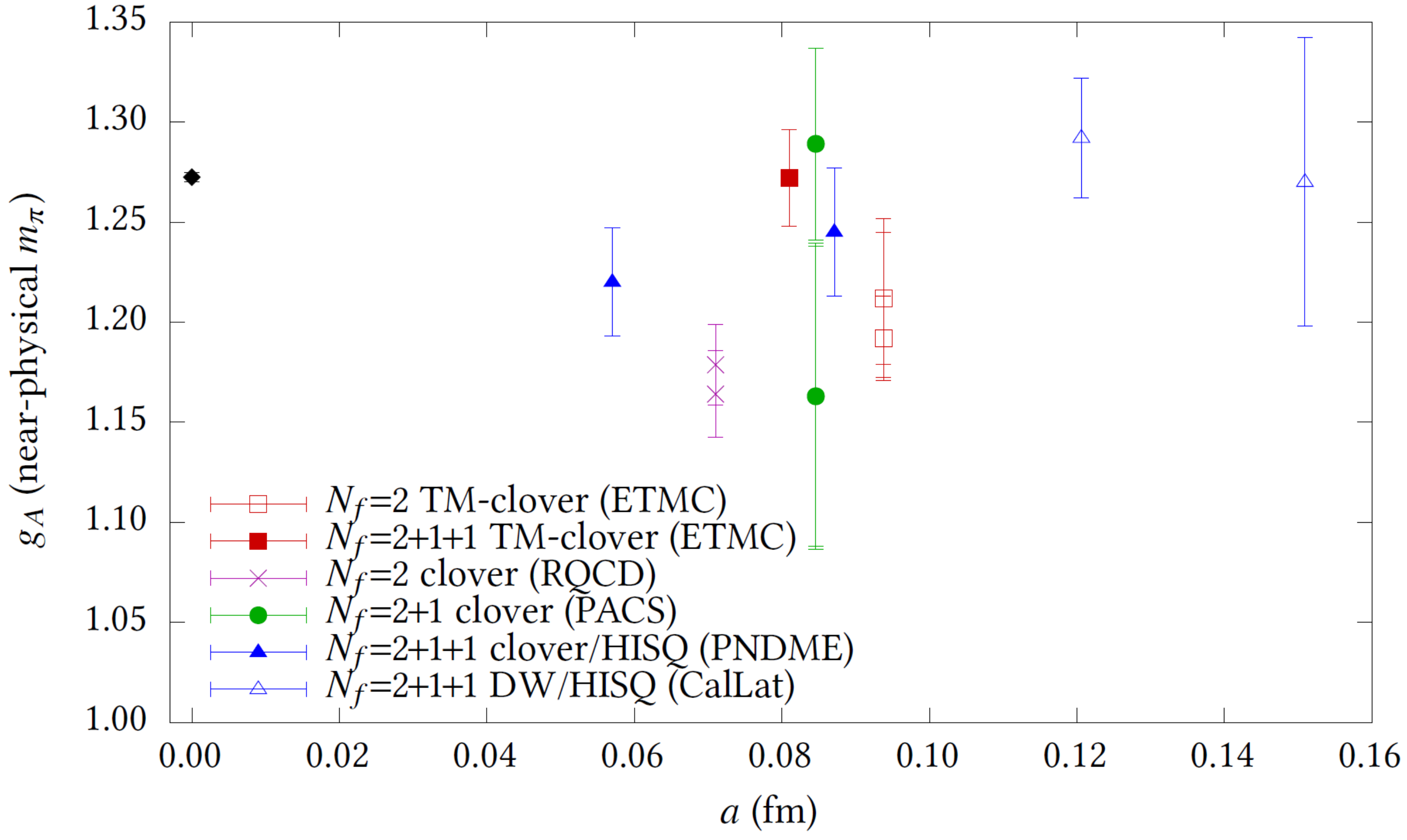




(plot by Zhouyou Fan)

Nucleon Axial Charge

§ Calculation near physical pion mass

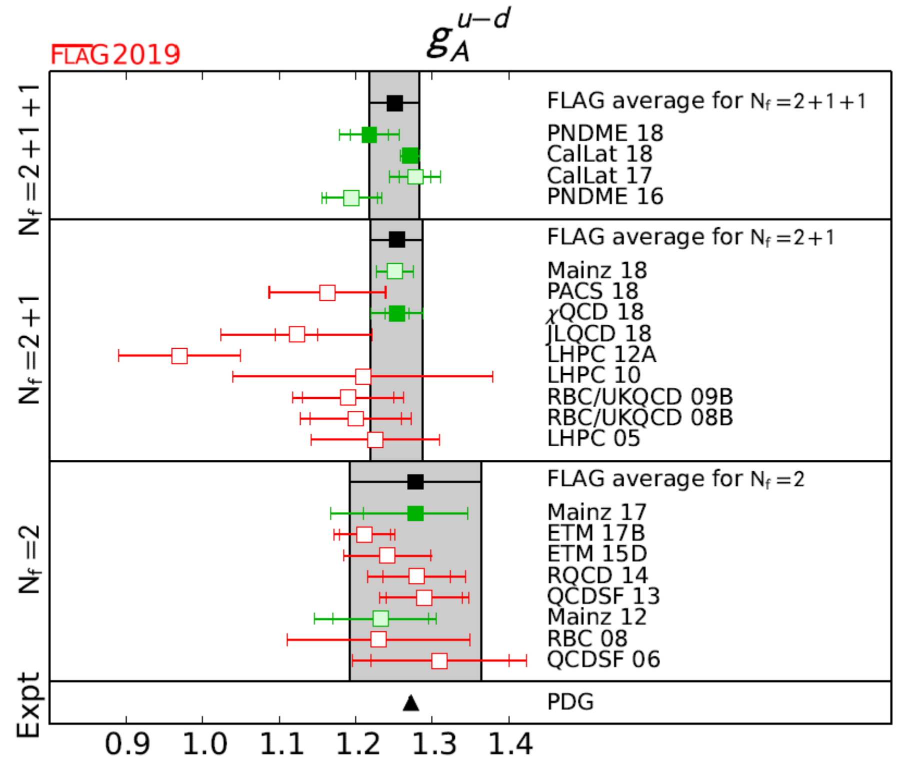


Plot by J. Green @Lattice 2018

Nucleon Axial Charge

§ Summary

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_A^{u-d}
PNDME 18[a]	[84]	2+1+1	A	★‡	★	★	★	★	1.218(25)(3(
CalLat 18	[85]	2+1+1	A	○	★	★	★	★	1.271(10)(7)
CalLat 17	[831]	2+1+1	P	○	★	★	★	★	1.278(21)(2(
PNDME 16[a]	[830]	2+1+1	A	○‡	★	★	★	★	1.195(33)(2(
Mainz 18	[915]	2+1	C	★	○	★	★	★	1.251(24)
PACS 18	[808]	2+1	A	■	■	★	★	■	1.163(75)(1
χQCD 18	[6]	2+1	A	○	★	★	★	★	1.254(16)(3(
JLQCD 18	[839]	2+1	A	■	○	○	★	★	1.123(28)(2(
LHPC 12A[b]	[916]	2+1	A	■‡	★	★	★	★	0.97(8)
LHPC 10	[846]	2+1	A	■	○	■	★	■	1.21(17)
RBC/UKQCD 09B	[833]	2+1	A	■	■	○	★	■	1.19(6)(4)
RBC/UKQCD 08B	[832]	2+1	A	■	■	○	★	■	1.20(6)(4)
LHPC 05	[917]	2+1	A	■	■	★	★	■	1.226(84)
Mainz 17	[86]	2	A	★	★	★	★	○	1.278(68)(±
ETM 17B	[824]	2	A	■	○	○	★	★	1.212(33)(2
ETM 15D	[822]	2	A	■	○	○	★	★	1.242(57)
RQCD 14	[819]	2	A	○	★	★	★	■	1.280(44)(46)
QCDSF 13	[400]	2	A	○	★	■	★	■	1.29(5)(3)
Mainz 12	[818]	2	A	★	○	○	★	○	$1.233(63)(^{+0.035}_{-0.060})$
RBC 08	[918]	2	A	■	■	■	★	■	1.23(12)
QCDSF 06	[817]	2	A	○	■	■	★	■	1.31(9)(7)



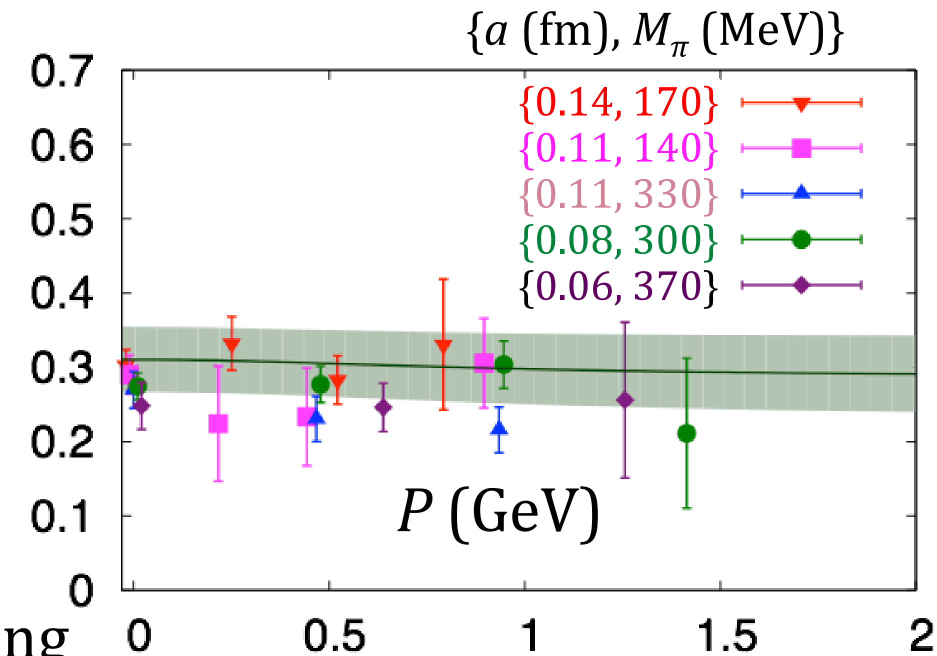
Gluon Helicity

§ Jaffe & Manohar, 1990 $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q^z + \mathcal{L}_g^z$

§ Can be calculated through large-momentum frame

X. Ji et al., PRL. 111 (2013) 112002; 110 (2013) 262002; PRD 89, 085030 (2014)

$$S_G(P)\,S_z = \frac{\left\langle PS \middle| \int d^3x\,\left(\vec{E}\times\vec{A}_{\text{phys}}\right)_z \middle| PS \right\rangle}{2E_P}$$



§ First results by **χQCD**

$$\Delta G\left(\mu^2 = 10\ \text{GeV}^2\right)$$
$$\approx S_G\left(\infty, \mu^2 = 10\ \text{GeV}^2\right)$$
$$= 0.287(55)(16)$$

Yang et al, Phys. Rev. Lett. 118 (2017) 102001

☙ Future improvement on matching

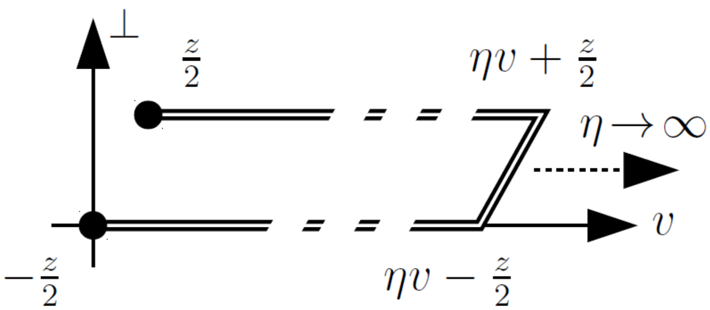
§ Current limit

☙ DSSV14 $\int_{0.05}^{1} dx\,\Delta G\left(10^2\ \text{GeV}, x\right) \approx [0.14, 0.24]$

Orbital Angular Momentum

§ Two definitions: Ji vs Jaffe & Manohar

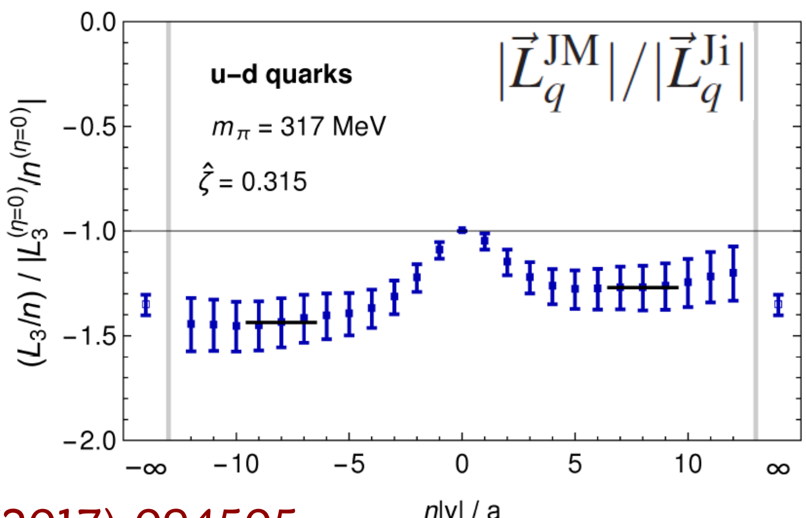
$$\vec{L}_q^{\text{Ji}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{D}\right] q \qquad \vec{L}_q^{\text{JM}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{\nabla}\right] q\,,$$



- Straight path $\eta = 0$ gives Ji's OAM
- Staple $\eta \to \infty$ gives Jaffe-Manohar OAM
- Difference is accumulated torque from final-state interaction

§ First result carried out by M. Engelhardt

2+1f clover at 518-MeV pion mass



Phys. Rev. D95 (2017) 094505

Machine-Learning Prediction

Input $X_i = (O_i^1, O_i^2, \dots)$ → ML Model → **Output** $\hat{O}_i$

R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

- **All Data**
  - Labeled (lb) Data
    - Training (tr) Data
    - Bias-Correction (BC) Data
  - Unlabeled (ul) Data

Prediction with bias correction Yoon et al., PRD 2018:

$$\langle C_{\text{pred,BC}} \rangle = \langle C_{\text{pred}} \rangle_{\text{ul}} + \langle C_{\text{BC}} - C_{\text{pred}} \rangle_{\text{BC}}$$

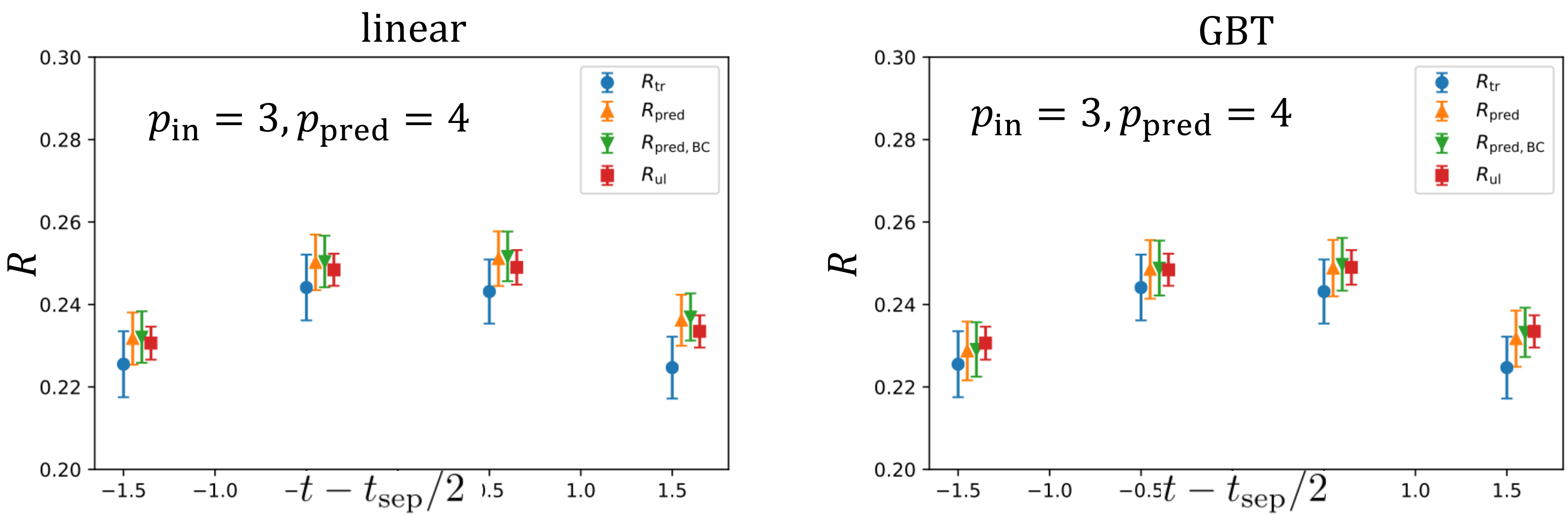
Machine-Learning Prediction

Input
$X_i = (O_i^1, O_i^2, \ldots)$ → ML Model → **Output**
$\hat{O}_i$

R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

§ Multiple quasi-PDF data sets studied (meson DA, gluon/kaon PDFs)

- Example kaon PDF at 220-MeV ensemble



p-prediction

(plot by Rui Zhang)

Supervised Regression Models

§ Gradient boosting tree (GBT)

Natekin and Knoll, Front. Neurorobot. 2013

$$f = f_{N_{\text{est}}} = \sum_{i}^{N_{\text{est}}} r_i h_i(x)$$

$$h_i(x) = \underset{h}{\operatorname{argmin}} \sum_j L\left(y_j, f_{i-1}(x_j) + h(x_j)\right)$$

$f_0 = r \cdot h_0$ a decision tree (DT) toward y

$f_1 = f_0 + r \cdot h_1$ a DT correcting f_0 toward y

$f_2 = f_1 + r \cdot h_2$ a DT correcting f_1 toward y

$\vdots$ $\vdots$

§ Linear regressor

$$f^{\text{lin}}(\vec{x}) = \theta_0 + \vec{\theta} \cdot \vec{x}$$

Kaon PDF Results