Quantum effects in a Glauber model of interacting nuclei at high energies

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1 Modelling NN interaction

1.1 Unitarity constraints for NN amplitude in Glauber approach

In a Glauber approach we assume for the amplitudes of NN interaction:

$$\tilde{a}(q) = \int db\, e^{ibq}\, a(b)\ , \qquad a(b) = \int \frac{dq}{(2\pi)^2}\, e^{-ibq}\, \tilde{a}(q)\ . \tag{1}$$

The $a(b)$ and $\tilde{a}(q)$ are the amplitudes of NN interaction in the impact parameter and momentum space correspondingly (all integrations imply the integration over two-dimensional vectors). In accordance with the optical theorem:

$$\sigma^{tot} = 2\,\mathrm{Im}\,\tilde{a}(q=0) = \int db\ 2\,\mathrm{Im}\,a(b) = \int db\ \sigma^{tot}(b)\ , \tag{2}$$

where we have introduce $\sigma^{tot}(b) \equiv 2\,\mathrm{Im}\,a(b)$. The equation (2) also fixes the normalization of the amplitude. Under this normalization one has

$$\sigma^{el} = \int \frac{dq}{(2\pi)^2}\, |\tilde{a}(q)|^2 = \int db\, |a(b)|^2 = \int db\, \sigma^{el}(b)\ , \tag{3}$$

where $\sigma^{el}(b) \equiv |a(b)|^2$.

The condition

$$\sigma^{in} = \sigma^{tot} - \sigma^{el} \geq 0 \tag{4}$$

leads to the following constraint for the amplitude $a(b)$:

$$\int \left(2\,\mathrm{Im}\,a(b) - |a(b)|^2\right) db = \int \left(\sigma^{tot}(b) - \sigma^{el}(b)\right) db\ \geq\ 0\ . \tag{5}$$

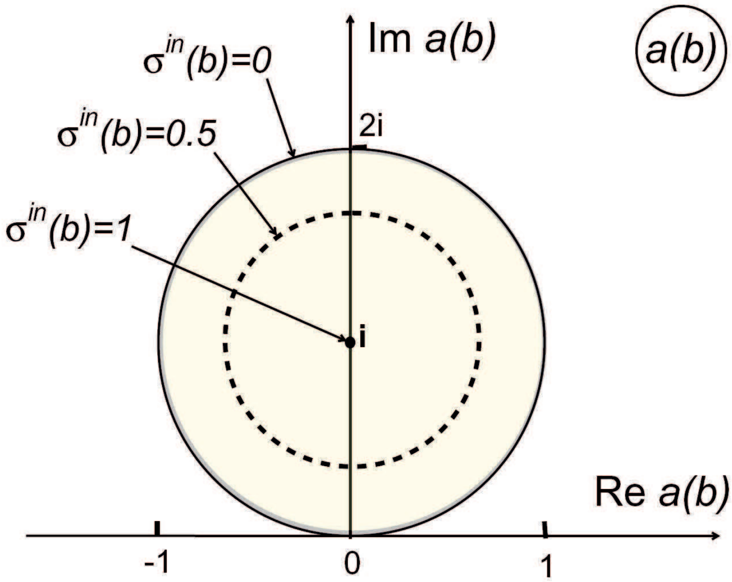

Figure 1: The constraints imposed on the amplitude $a(b)$ by the unitarity condition (6).

Note that at high energies in quasiclassical limit a partial wave quantum number, l, is proportional to an impact parameter, b. The unitarity condition is valid separately for each partial wave, l. This leads to the local validity of the constraint (5) at each value of impact parameter, b:

$$\sigma^{in}(b) = \sigma^{tot}(b) - \sigma^{el}(b) = 2\,\mathrm{Im}\,a(b) - |a(b)|^2 \geq 0 \ , \tag{6}$$

$$0 \leq \sigma^{tot}(b) \leq 4 \ , \qquad 0 \leq \sigma^{el}(b) \leq 4 \ , \qquad 0 \leq \sigma^{in}(b) \leq 1(!) \tag{7}$$

This follows from the unitarity relation for partial amplitudes in the high-energy limit.

For the complex amplitude, $a(b)$, the constraint (6) means that in the complex plane it lays inside the circle of the radius, $R = 1$, with the center at $a(b) = i$ (see Fig.1). In the center of the circle at $a(b) = i$ we have the maximum of inelastic scattering contribution:

$$\sigma^{in}(b) = 1 \ , \qquad \sigma^{in}(b) = \sigma^{el}(b) = \sigma^{tot}(b)/2 = 1 \ .$$

On the boarder of the circle we have the pure elastic scattering:

$$\sigma^{in}(b) = 0 \ , \qquad \sigma^{tot}(b) = \sigma^{el}(b) \ .$$

So, in the upper point of the circle in Fig.1 we have $a(b) = 2i$ and hence:

$$\sigma^{in}(b) = 0 \ , \qquad \sigma^{tot}(b) = \sigma^{el}(b) = 4 \ .$$

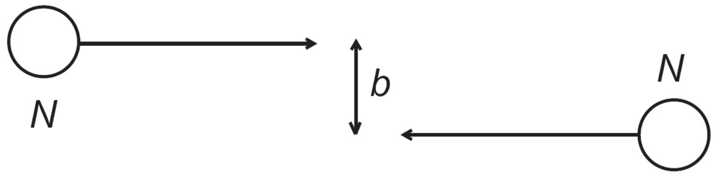

Figure 2: The NN interaction at impact parameter b.

This means that in the relation (6),

$$\sigma^{tot}(b) = \sigma^{el}(b) + \sigma^{in}(b) \ , \tag{8}$$

only the $\sigma^{in}(b) \in [0,\, 1]$ has a classical statistical interpretation (see Fig.2), as a probability of an inelastic NN interaction at the given value of the impact parameter b. (The portion of inelastic interactions at large number of scattering processes with the given impact parameter.) The $\sigma^{el}(b) \in [0,\, 4]$ and $\sigma^{tot}(b) \in [0,\, 4]$ have not such classical statistical interpretation and always have to be considered in the framework of the quantum scattering approach (see a discussion in the end of this paragraph).

1.2 Pure imaginary amplitude

First we consider the simple case of the pure imaginary amplitude: $a(b) = i\gamma(b)$. In this case

$$\sigma^{tot}(b) = 2\gamma(b) \ , \qquad \sigma^{el}(b) = \gamma^2(b) \ , \tag{9}$$

and the unitarity constraint (6) takes the following form:

$$\sigma^{in}(b) = \sigma^{tot}(b) - \sigma^{el}(b) = \gamma(b)[2 - \gamma(b)] \geq \ 0 \ , \tag{10}$$

which means that

$$0 \leq \gamma(b) \leq 2 \ . \tag{11}$$

By (10) we see that the $\sigma^{in}(b)$ reaches the maximum, which is equal to 1, at $\gamma(b) = 1$, and $0 \leq \sigma^{in}(b) \leq 1$. So it admits the statistical interpretation, as a probability of the NN inelastic interaction at given value of the impact parameter b. Whereas the $\sigma^{el}(b)$ and $\sigma^{tot}(b)$ have not such classical probabilistic interpretation in the quantum case, since $0 \leq \sigma^{el}(b) = \gamma^2(b) \leq 4$ and $0 \leq \sigma^{tot}(b) = 2\gamma(b) \leq 4$.

1.3 Gauss approximation for amplitude

In our calculations in the framework of a Glauber model for interacting nuclei we'll use the simple Gauss approximation for the amplitude $a(b) = i\gamma(b)$ of NN interaction, with

$$\gamma(b) = \gamma_0 \exp(-b^2/b_0^2) \ . \tag{12}$$

The amplitude of such form was used for example in the paper [1]. This amplitude depends only on two parameters γ_0 and b_0. The unitarity relation (10) leads to the following constraint for the parameter γ_0:

$$0 \le \gamma_0 \le 2 \ . \tag{13}$$

With the amplitude (12) we can easily to calculate all cross-sections.

By (2) we have

$$\sigma^{tot}(b) = 2\gamma(b) = 2\gamma_0 \exp(-b^2/b_0^2) \ , \qquad \sigma^{tot} = 2\gamma_0 \int d^2b \, \exp(-b^2/b_0^2) = 2\pi\gamma_0 b_0^2 \ . \tag{14}$$

By (3) we have

$$\sigma^{el}(b) = \gamma^2(b) = \gamma_0^2 \exp(-2b^2/b_0^2) \ , \qquad \sigma^{el} = \gamma_0^2 \int d^2b \, \exp(-2b^2/b_0^2) = \pi\gamma_0^2 b_0^2/2 \ . \tag{15}$$

This by (10) leads to

$$\sigma^{in}(b) = \gamma(b)\,[2 - \gamma(b)] = \gamma_0 e^{-\frac{b^2}{b_0^2}}\,[2 - \gamma_0 e^{-\frac{b^2}{b_0^2}}] \tag{16}$$

By (1) we can also calculate

$$\tilde{a}(q) = i \int d^2b \, \gamma(b)\, e^{ibq} = i\pi\gamma_0 b_0^2 \, e^{-q^2\frac{b_0^2}{4}} \ , \qquad |\tilde{a}(q)|^2 = \pi^2\gamma_0^2 b_0^4 \, e^{-q^2\frac{b_0^2}{2}} \tag{17}$$

From the last formula, we see that parameter b_0 is simply connected with the slope of the diffraction cone B of the elastic NN scattering:

$$B = b_0^2/2 \ , \qquad b_0^2 = 2B \ . \tag{18}$$

Since the NN amplitude, (12), depends only on two parameters γ_0 and b_0, we can express the parameters of the model and all obtained results through two observable quantities: σ^{tot} and

$$\alpha \equiv \frac{\sigma^{el}}{\sigma^{tot}} \ . \tag{19}$$

$\sqrt{s}$ (GeV)	$\sigma^{tot}\,(mb)$	$\sigma^{el}\,(mb)$	σ^{el}/σ^{tot}	$\mathrm{Re}\,\tilde{a}(q=0)/\,\mathrm{Im}\,\tilde{a}(q=0)$
45 pp	41.9	7.5	0.179	0.062
53 pp	42.4	7.6	0.179	0.077
62 pp	43.6	7.7	0.177	0.095
53 $\mathrm{p\overline{p}}$	43.3	7.4	0.171	0.106
62 $\mathrm{p\overline{p}}$	44.1	7.5	0.170	
546 $\mathrm{p\overline{p}}$	61.3	12.9	0.210	0.135
900 $\mathrm{p\overline{p}}$	65.3			
1800 $\mathrm{p\overline{p}}$	72	16	0.222	0.132
7000 pp [2]	98.0±2.5	25.1±1.1	0.256	0.145
8000 pp [3, 4]	101.5±2.1	27.1±1.4	0.267	

Table 1: Total and elastic NN cross-sections data.

The α evolves with energy very slowly (see the Table 1). However as we'll see below this tiny increase of the α with energy has considerable influence on the profile function $\sigma^{in}(b)$, (10), of the inelastic NN interaction, which admits the probabilistic interpretation even in quantum case, as it was already mentioned above.

Using (14) and (15) we can express all quantities through σ^{tot} and $\alpha = \sigma^{el}/\sigma^{tot}$:

$$\gamma_0 = 4\frac{\sigma^{el}}{\sigma^{tot}} = 4\alpha\ , \qquad b_0^2 = 2B = \frac{\sigma^{tot\,2}}{8\pi\sigma^{el}} = \frac{\sigma^{tot}}{8\pi\alpha}\ . \tag{20}$$

By formula (16) we see that $\sigma^{in}(b)$ can reach its maximal value equal to 1 only at $\gamma(b) = 1$. By formula (12) the maximal value of $\gamma(b)$ is equal to $\gamma_0 = 4\alpha$. In Table 1 we see that at $\sqrt{s}$ <7 TeV the $\alpha < 1/4$ and hence $\gamma(b) \leq \gamma_0 < 1$. That by formula (16) leads to $\sigma^{in}(b) < 1$ and the $\sigma^{in}(b)$ reaches its maximum at zero impact parameter, $b = 0$, at that $\max[\sigma^{in}(b)] = \sigma^{in}(b=0) < 1$.

For example, at $\sqrt{s}$ =45 GeV the $\alpha = \sigma^{el}/\sigma^{tot} = 0.179$ and $\gamma_0 = 0.716 < 1$. In this case by (16) we have $\max[\sigma^{in}(b)] = \sigma^{in}(b=0) = \gamma_0(2-\gamma_0) = 0.92$ (see the dotted curve in Fig.3).

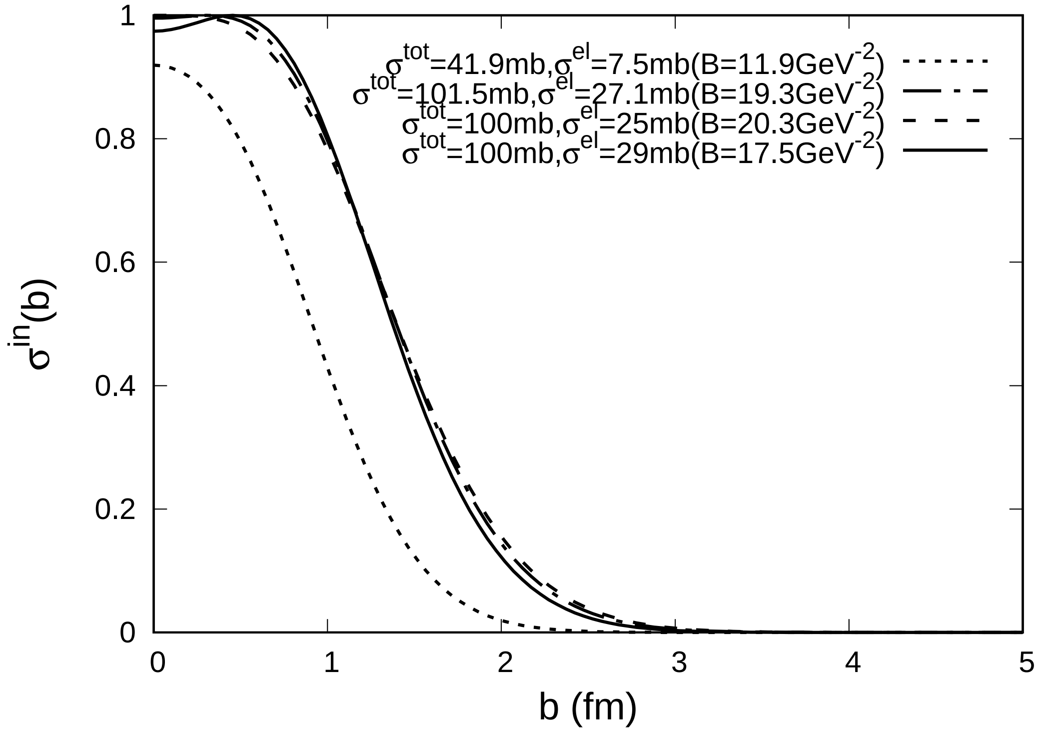

Figure 3: The profile function $\sigma^{in}(b)$, (10), of the inelastic NN interaction, admitting the probabilistic interpretation (even in the quantum case), calculated with Gauss approximation, (12), for the pure imaginary NN scattering amplitude, as a function of the impact parameter b. The dotted curve - for initial energy $\sqrt{s}$ =45 GeV, the dash-dotted curve - for initial energy $\sqrt{s}$ =8 TeV, the solid and dashed curves - with the small variation of parameters for illustration.

We can also find by (20) the value of B:

$$B = \frac{\sigma^{tot}}{16\pi\alpha} \ .$$

At this energy we have for the slope of the diffraction cone of the elastic NN scattering B =11.9GeV^{-2} .

As we see In Table 1 at the energy $\sqrt{s} \geq 7TeV$ the situation changes $\alpha > 1/4$ and hence $\gamma_0 = 4\alpha > 1$. In this case still by formula (16) we see that the $\sigma^{in}(b)$ reaches its maximal value equal to 1 at $\gamma(b) = 1$, but now this takes place at $b_{max} \neq 0$. We can fined the value of b_{max} from condition:

$$\gamma(b_{max}) = \gamma_0 \exp(-b_{max}^2/b_0^2) = 1 \ , \qquad b_{max} = b_0\sqrt{\ln\gamma_0} = b_0 \ln^{\frac{1}{2}}\gamma_0 \ . \tag{21}$$

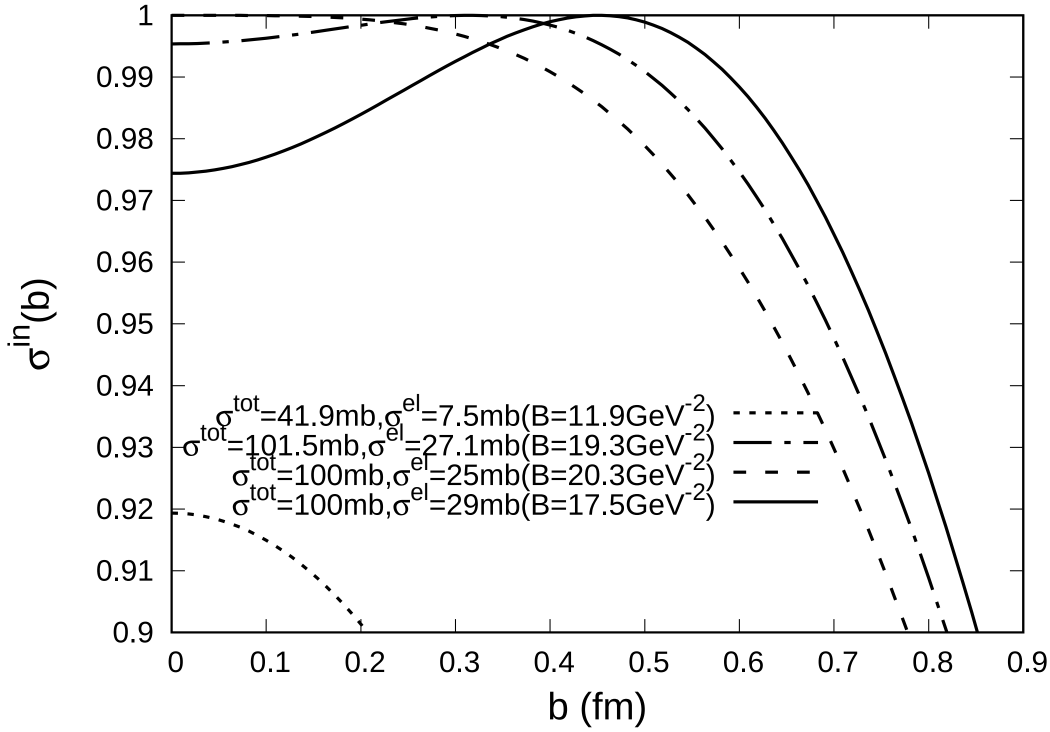

Figure 4: Enlarged view of the Fig.4

At $b = 0$ the $\gamma(b = 0) = \gamma_0 > 1$ and by (16) we see that $\sigma^{in}(b = 0) = \gamma_0(2 - \gamma_0) < 1$.

For example, at $\sqrt{s}$ =8 TeV the $\alpha = \sigma^{el}/\sigma^{tot} = 0.267 > 1/4$ (see Table 1) and $\gamma_0 = 1.068 > 1$. In this case by (16) we have $\sigma^{in}(b = 0) = \gamma_0(2 - \gamma_0) = 0.995 < 1$ and the $\sigma^{in}(b)$ reaches its maximum value (equal to 1) at nonzero impact parameter b, so we obtain the so-called Halo effect (see the dash-dotted curve in Figs. 3 and 4). We can also find by (20) the value of B at this energy, $B = \sigma^{tot}/(16\pi\alpha) = 19.3\,\mathrm{GeV}^{-2}$.

For illustration we also performed some small variation of the model parameters near these experimental values (See the solid and dashed curves in Figs. 3 and 4.)

1.4 Possible further increase of the NN amplitude with energy

As we have found above, (11), the unitarity condition, (10), restricts the parameter γ_0 in the purely imaginary Gaussian NN amplitude, (12), by the value of 2, imposing no constraint on the value of the parameter b_0. Assume that at some very large energy the NN amplitude

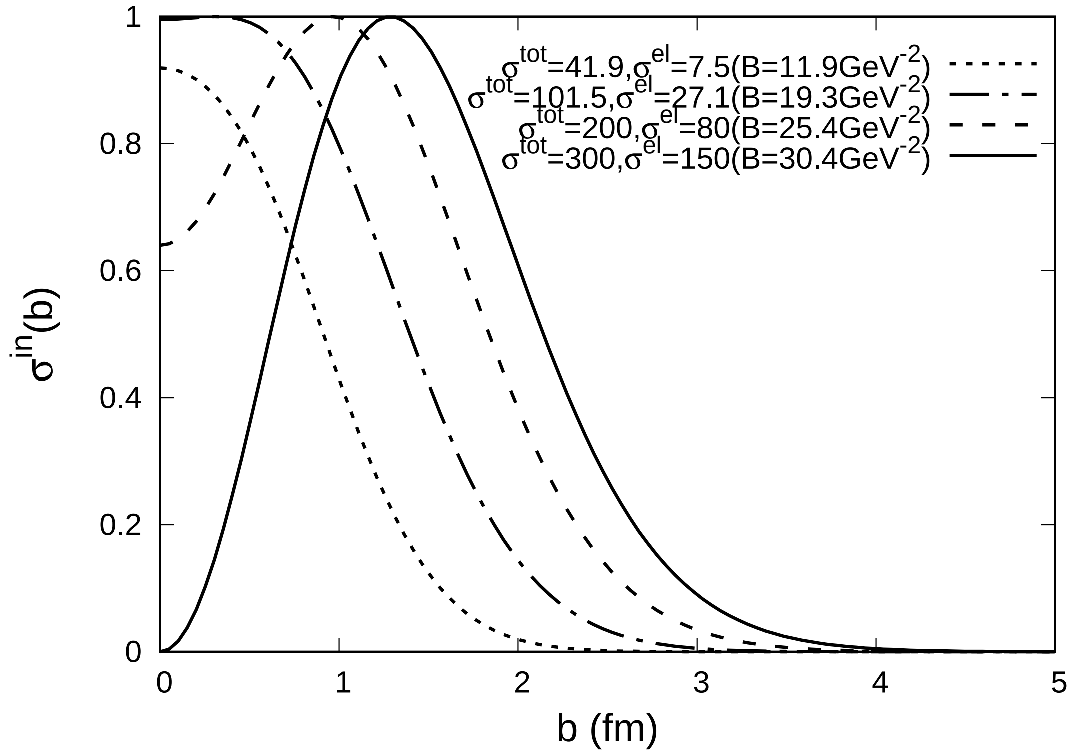

Figure 5: The profile function $\sigma^{in}(b)$, (10), of the inelastic NN interaction, calculated with Gauss approximation, (12), for the pure imaginary NN scattering amplitude, as a function of the impact parameter b. The dotted curve - for initial energy $\sqrt{s}$ =45 GeV, $\gamma_0 = 0.716$, the dash-dotted curve - for initial energy $\sqrt{s}$ =8 TeV, $\gamma_0 = 1.068$. The dashed and solid curves are the profile function, $\sigma^{in}(b)$, of the inelastic NN interaction in the case of a further increase in the amplitude of the NN scattering (12) with energy, for $\gamma_0 = 1.6$ and $\gamma_0 = 2$, respectively.

reaches this maximal value with the parameter $\gamma_0 = 2$:

$$a(b) = i\,\gamma(b)\ , \qquad \gamma(b) = 2\exp(-b^2/b_0^2)\ . \tag{22}$$

As it follows from formulas (20), in this case

$$\alpha = \frac{\sigma^{el}}{\sigma^{tot}} = \frac{\gamma_0}{4} = 0.5\ . \tag{23}$$

The parameter b_0 is not restricted by the unitarity condition, (10), and by the relation (20) it is connected with the value of the total cross-section, σ^{tot}, and the slope of the diffraction

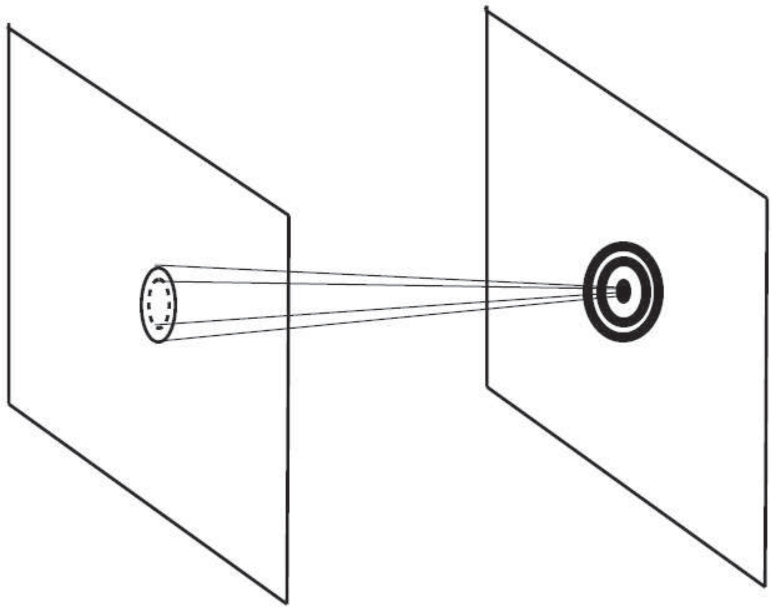

Figure 6: Black spot in the center of the image when light is diffracted by a round hole (The case of two Fresnel zones).

cone, B:

$$b_0^2 = 2B = \frac{\sigma^{tot}}{8\pi\alpha} = \frac{\sigma^{tot}}{4\pi} \ . \tag{24}$$

Only for an illustration, below we will put $\sigma^{tot} = 300\,\text{mb}$. In this case, by (23) and (24) we have $\sigma^{el} = 150\,\text{mb}$, $b_0 = 1.54\,\text{fm}$ and $B = 30.4\,\text{GeV}^{-2}$.

It is clear that for $\gamma_0 = 2$ we have $\sigma^{in}(b = 0) = 0$, see Fig.5. This is a purely quantum effect. It resembles the effect of the appearance of a dark spot in the center of the image when light is diffracted by a round hole of a dimension corresponding to an even number of Fresnel zones (see Fig.6).

1.5 Modifications for other forms of NN amplitude

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1.6 Conclusions

So, we come to the conclusions:

1. The simplest Gauss approximation, (12), for the pure imaginary NN scattering amplitude enables to describe the increase of the diffraction cone with energy from ISR to LHC energies.

2. It enables also to describe the so-called Halo effect in the profile function $\sigma^{in}(b)$, (10), of the inelastic NN interaction arising at very high ($\sqrt{s}$ $\geq$7 TeV) initial energies.

3. The last effect is a general quantum phenomenon, which do not connect with the Gauss approximation of the amplitude and will take place for large enough amplitude of any form.

References

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