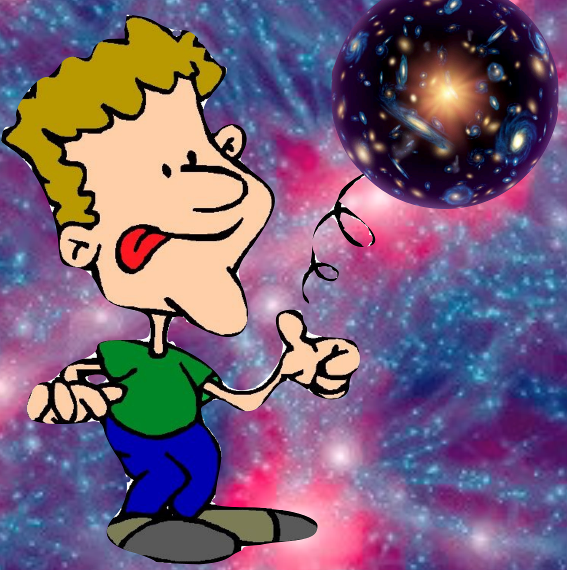
Modeling the birth and growth of the cosmic web in the quasi-linear regime

Sandrine Codis

- AIM/LCEG -

horizon-AGN

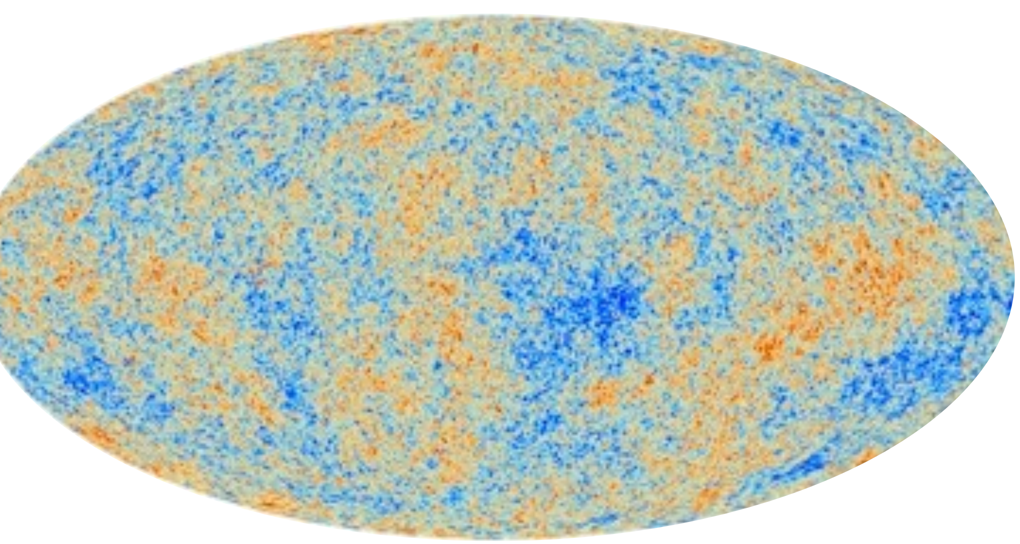


07/11/2022

EDSU 2022 - La Reunion

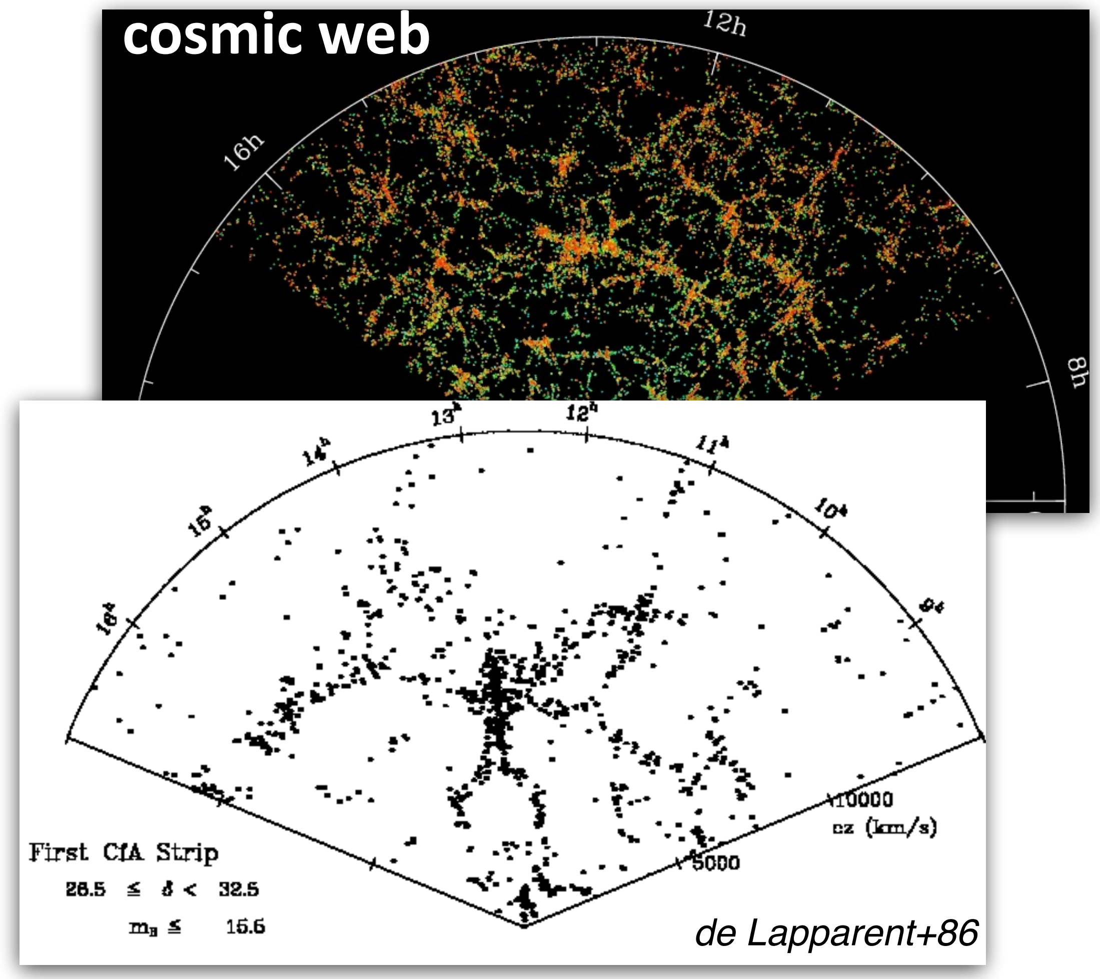
How is the cosmic web woven?

Gaussian primordial fluctuations



gravity

expansion



Vlasov-Poisson equations: dynamics of a self-gravitating collisionless fluid

Liouville theorem:

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} - m\nabla\phi\frac{\partial}{\partial \mathbf{p}}\right] f(\mathbf{x},\mathbf{p},\mathbf{t}) = \mathbf{0}$$

Poisson equation:

$$\Delta\phi = 4\pi a^2 G(\rho - \bar{\rho})$$

These highly non-linear equations can be solved using numerical simulations or analytically in some specific regimes. Exact solutions are crucial to understand the details of structure formation.

Before shell-crossing, moments>2 can be neglected (velocity dispersion,...)

continuity equation:

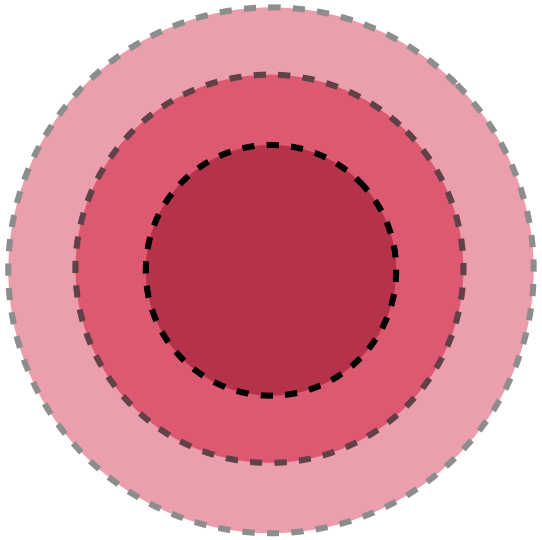
$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot [(1+\delta)\mathbf{u}] = 0$$

Euler equation:

$$\frac{\partial u_i}{\partial t} + \frac{\dot{a}}{a}u_i + \frac{u_j \partial_j u_i}{a} = -\frac{\partial_i \phi}{a} - \frac{\partial_j[\rho\sigma_{ij}]}{\rho a}$$

The spherical collapse dynamics

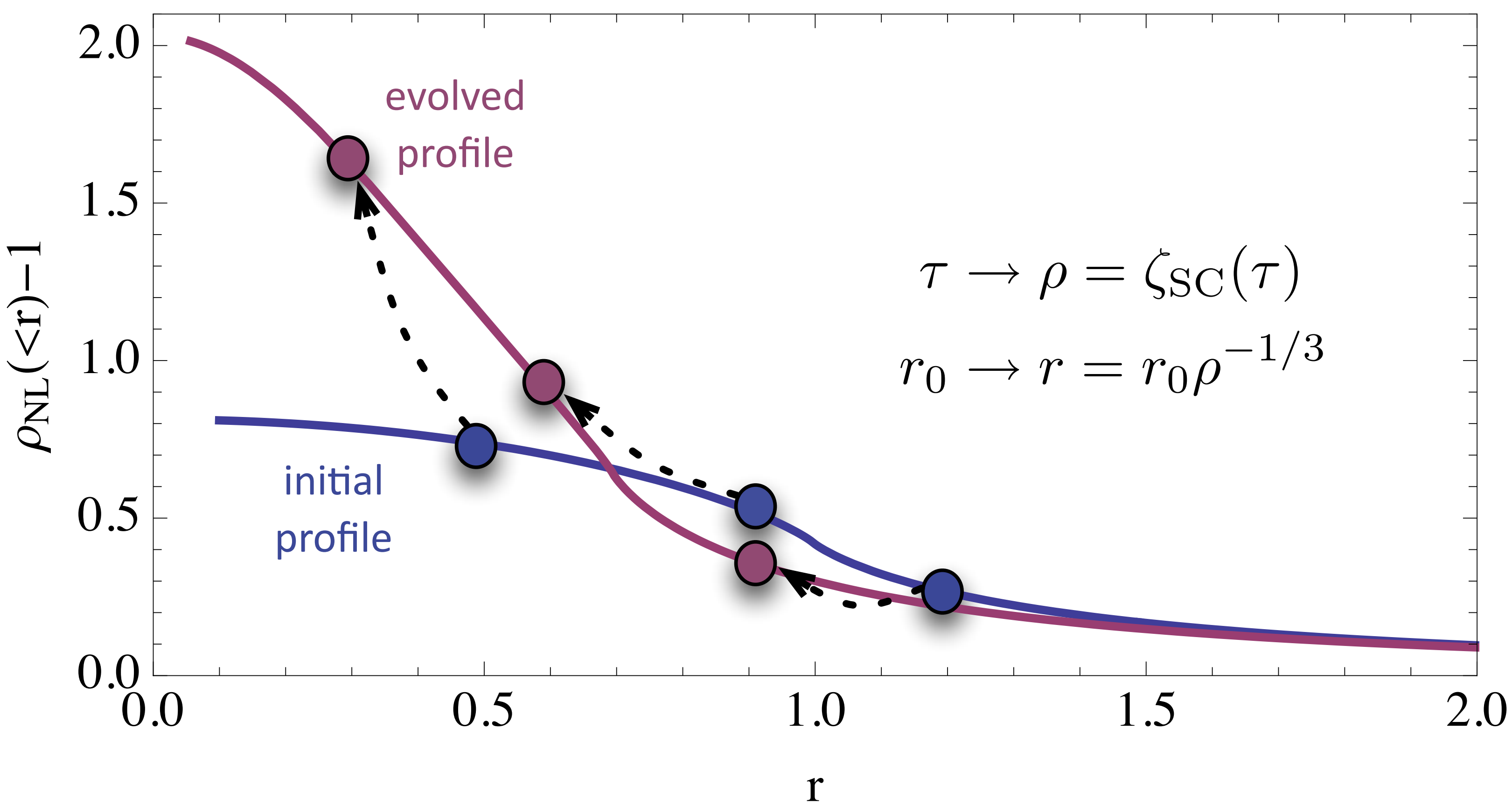
A solution is known for an initial spherically symmetric fluctuation thanks to Gauss theorem.



The evolution of the radius of the shell of mass M is given by

$$\ddot{R} = -\frac{GM}{R^2}$$

where M is $$M = \frac{4}{3}\pi R^3 \left(\rho - \frac{\Lambda}{8\pi G}\right).$$

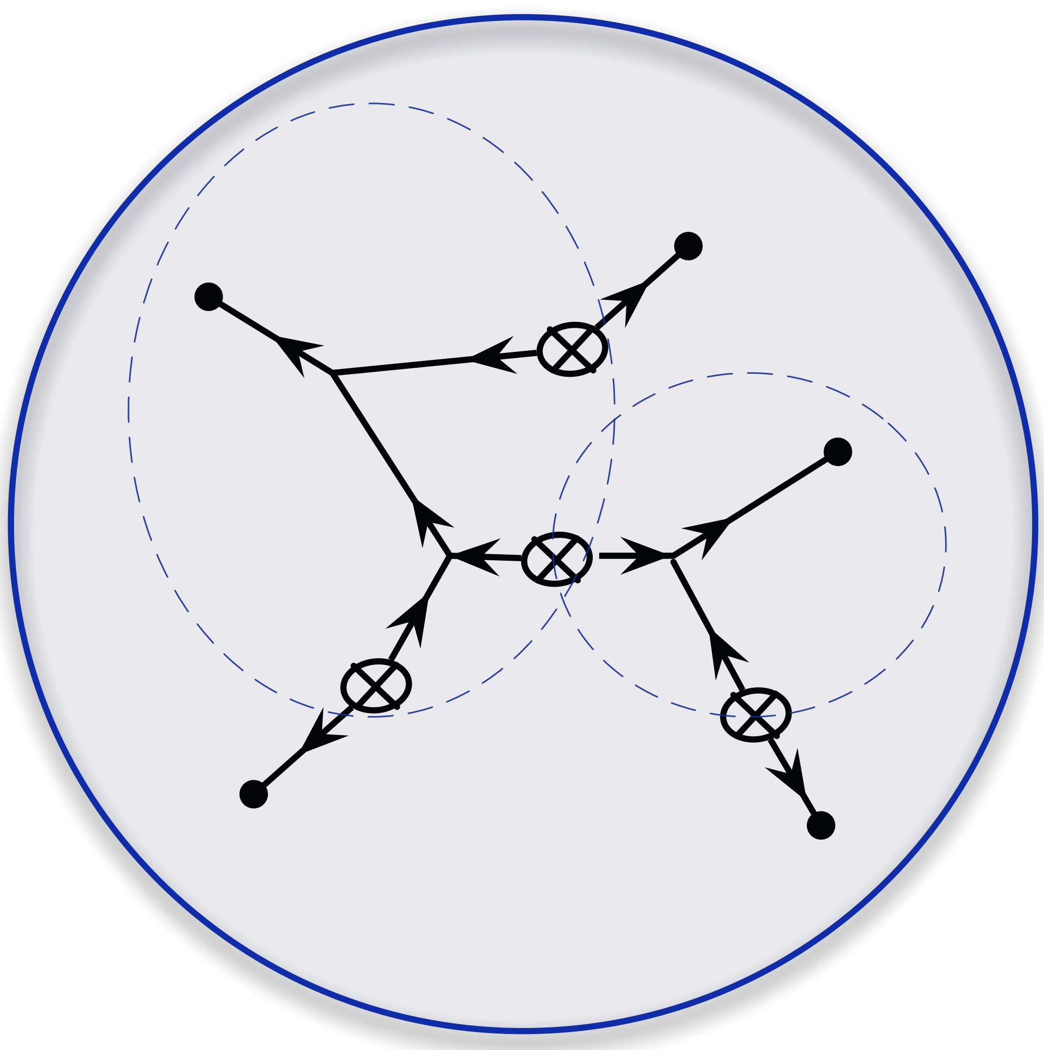


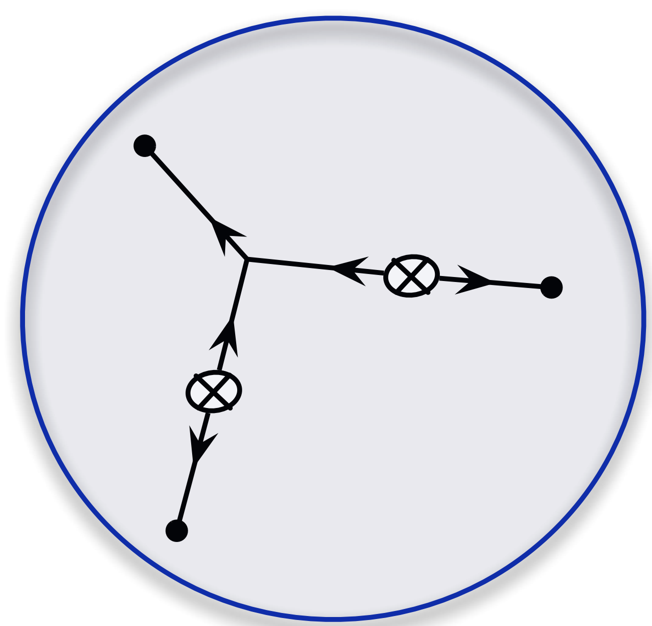
Perturbation theory

Assumption: cosmic fields can be expanded wrt initial fields $\delta(\mathbf{x},t) = \delta_1(\mathbf{x},t) + \delta_2(\mathbf{x},t) + \cdots$
All orders can then be computed hierarchically

$$\delta_n(\mathbf{k}) = \int \mathrm{d}^3\mathbf{q}_1 \ldots \int \mathrm{d}^3\mathbf{q}_n \, \delta_D(\mathbf{k} - \mathbf{q}_{1\ldots n}) F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n) \delta_1(\mathbf{q}_1) \ldots \delta_1(\mathbf{q}_n)$$

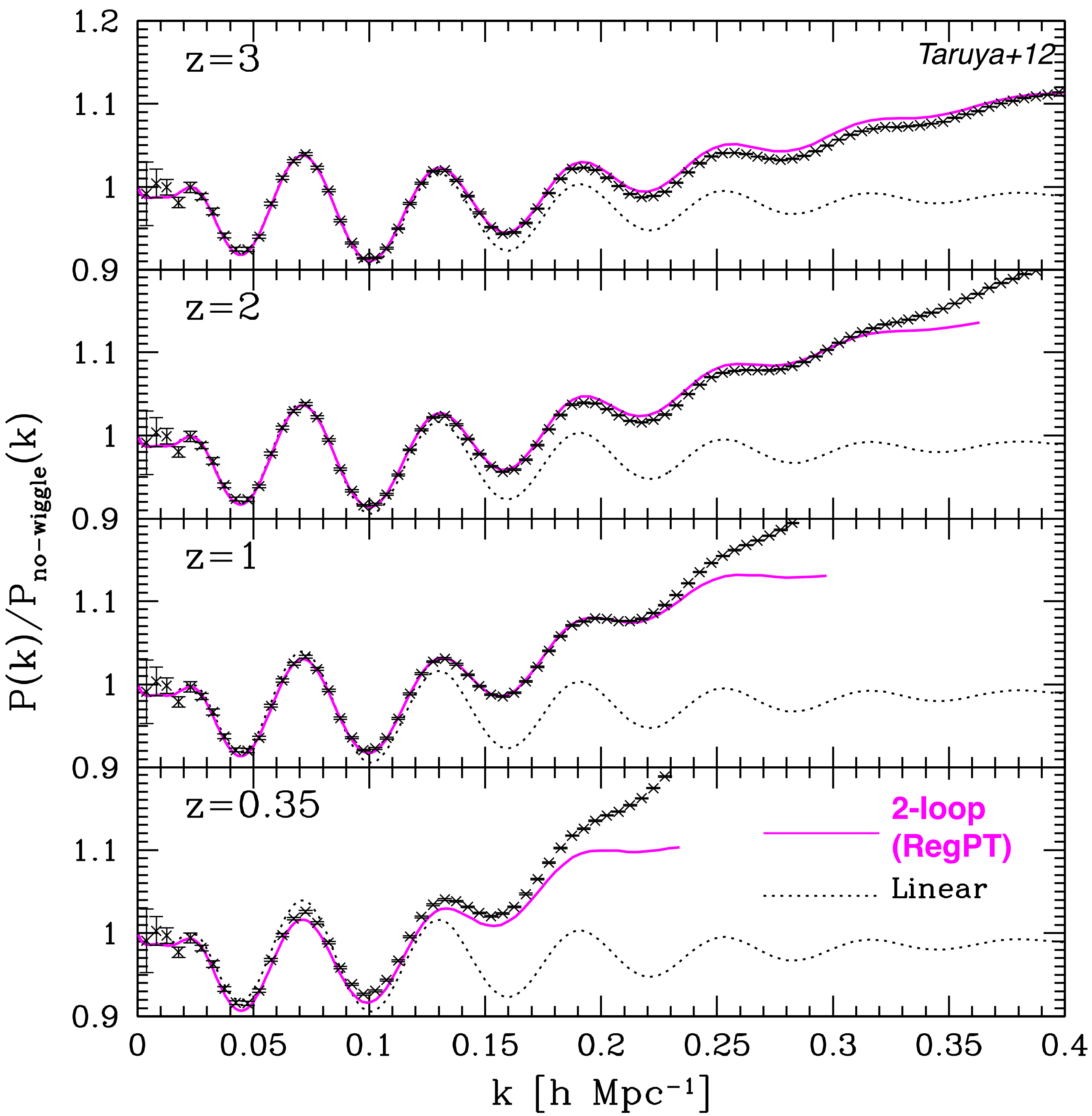
PT kernels





Perturbation theory

matter power spectrum:



This approach is valid in the weakly non-linear regime where $|\delta|<<1$ i.e at high redshift / large scale.

How to go beyond?

Perturbation theory

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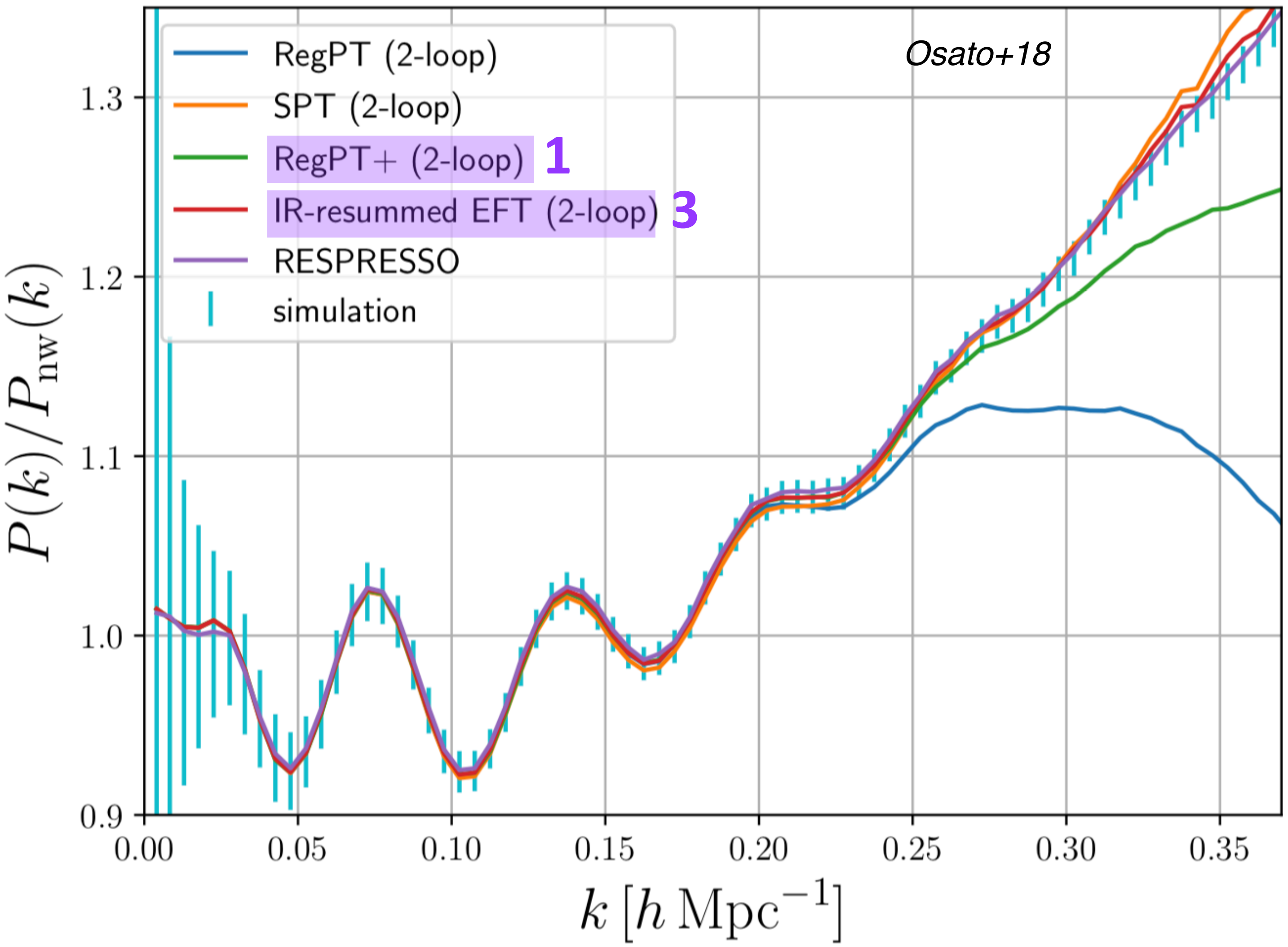
How to go beyond?
Adding **new degrees of freedom** in the modeling?

Perturbation theory



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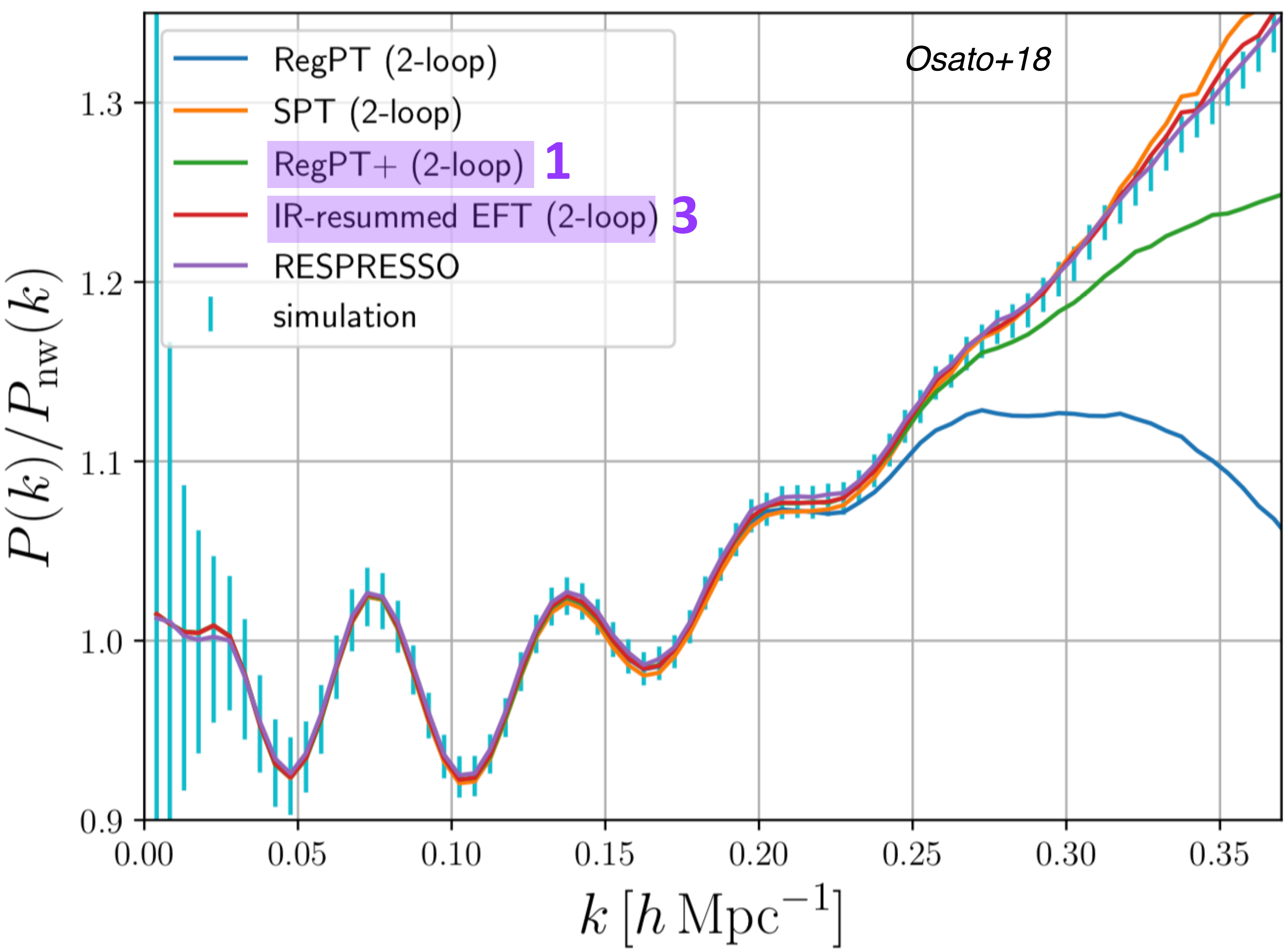

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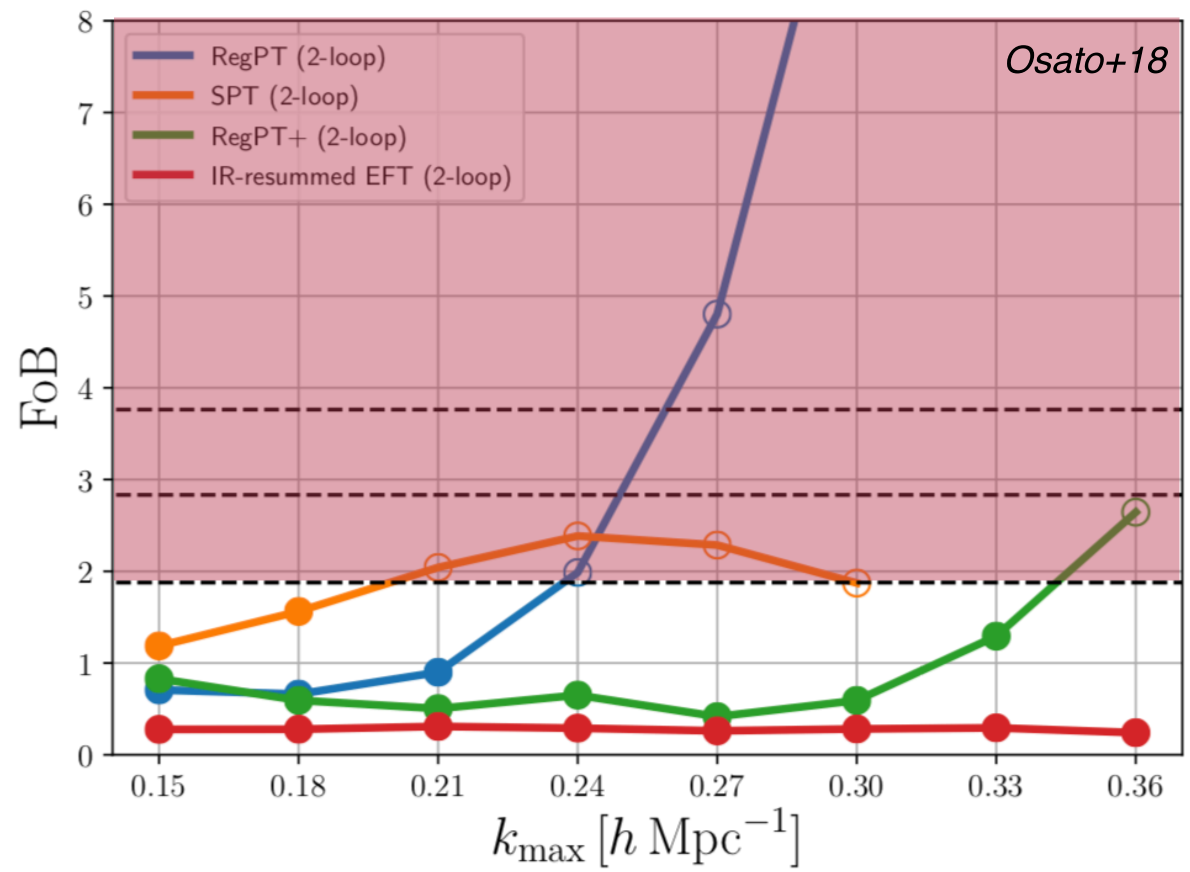
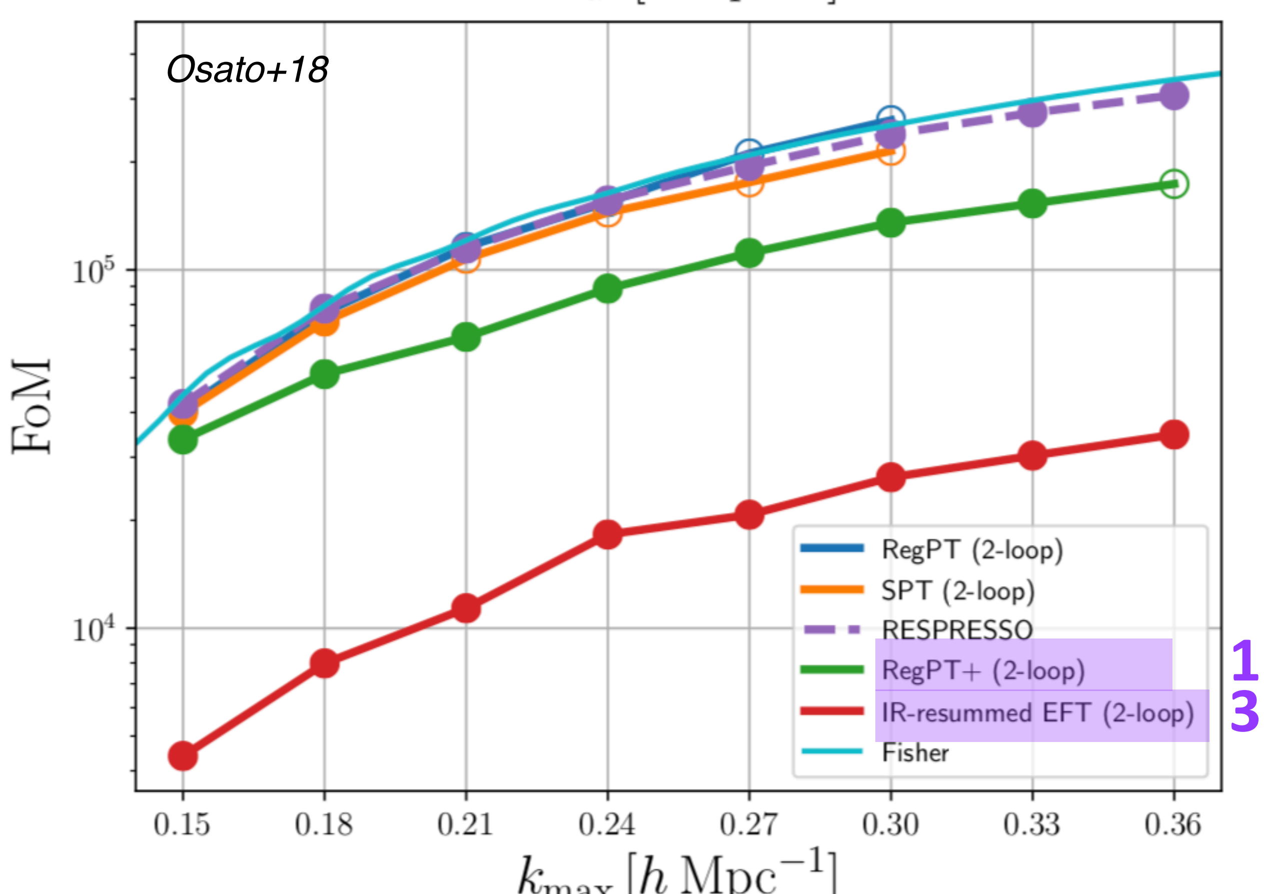
How to go beyond?
Adding **new degrees of freedom** in the modeling?

But… does it really allow us to get tighter cosmological constraints?

Perturbation theory





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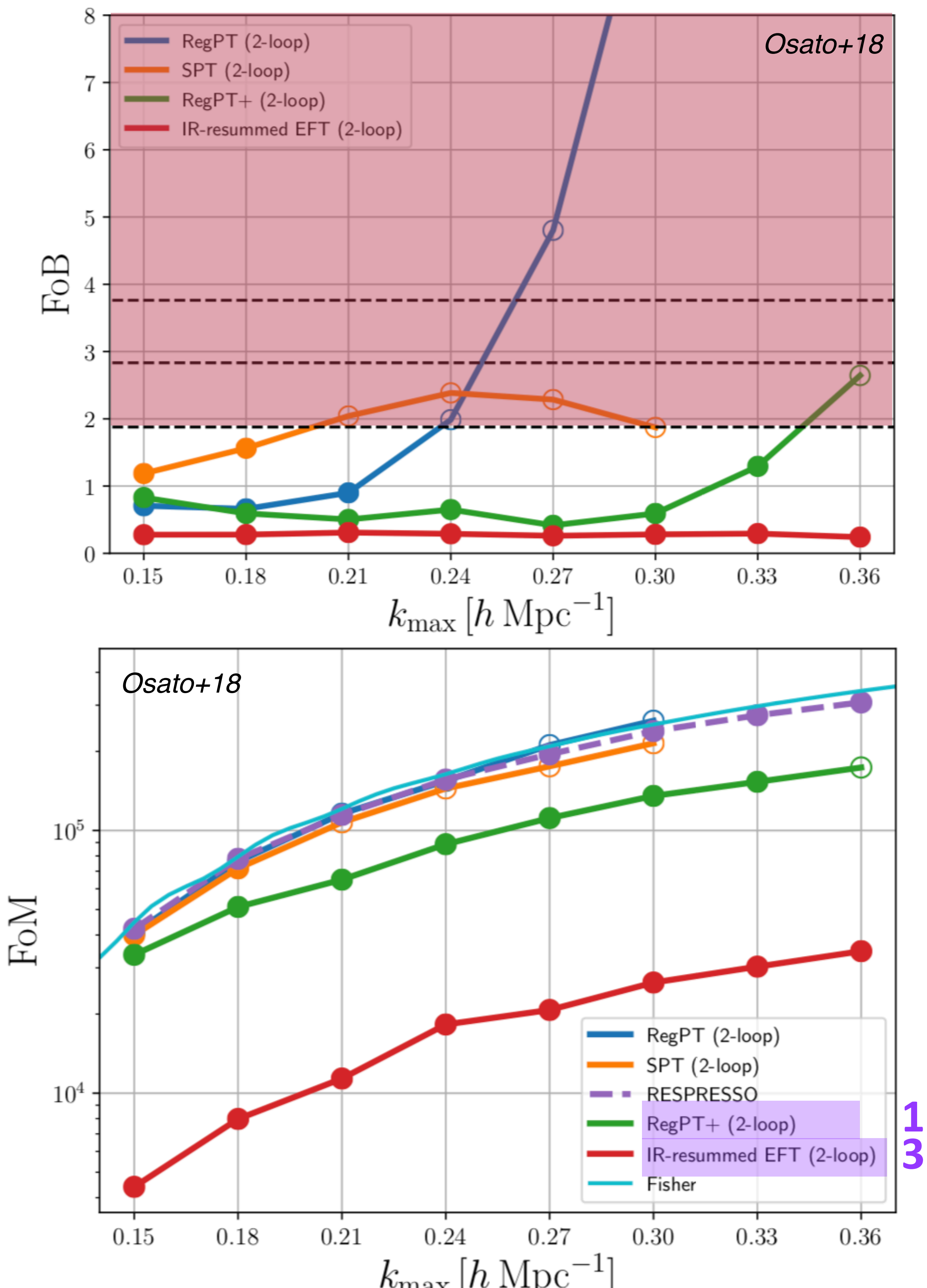

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How to go beyond?
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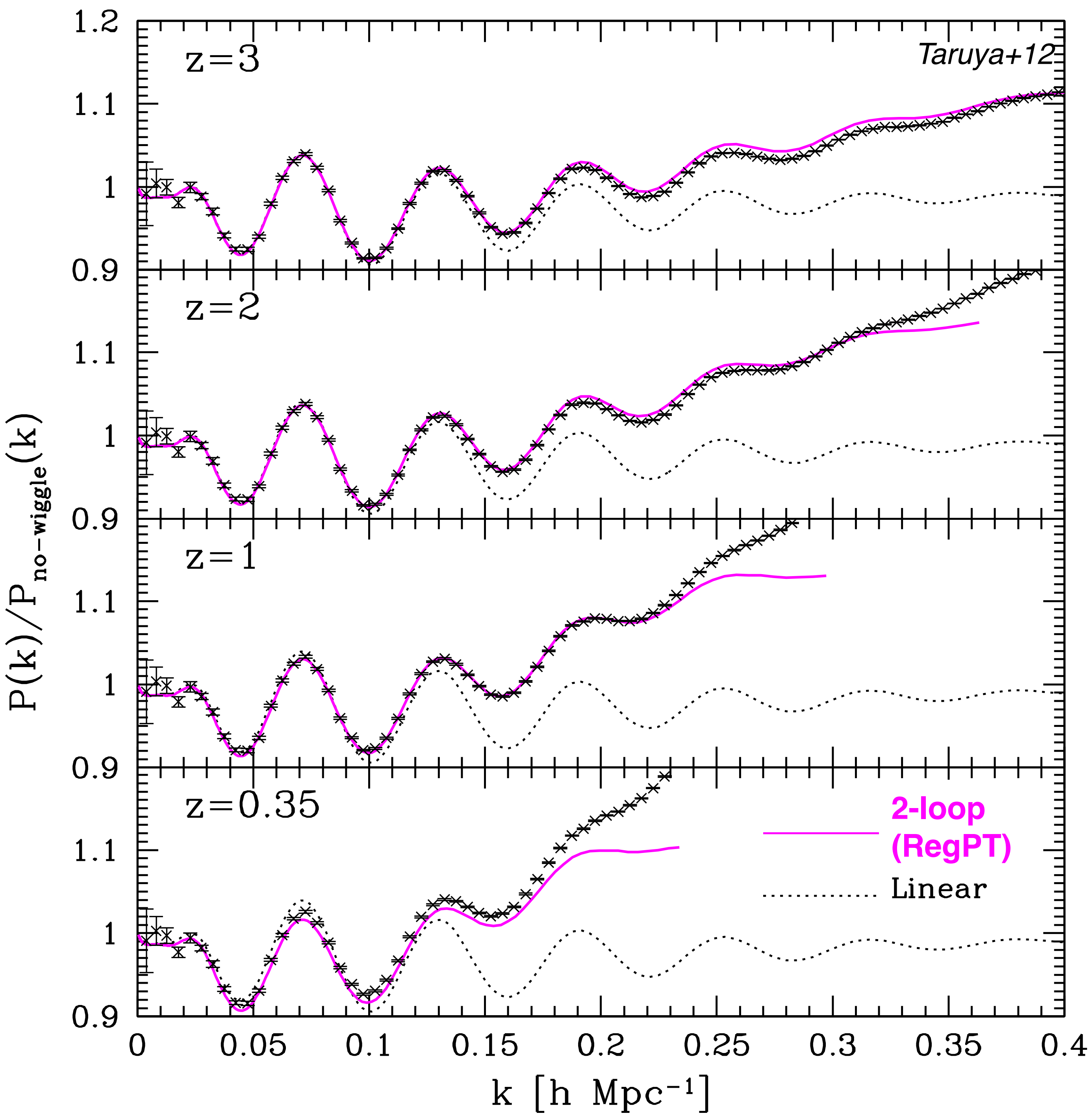
But… does it really allow us to get tighter cosmological constraints?

No!

Perturbation theory

matter power spectrum:



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How to go beyond **without introducing a myriad of free parameters**?

How to go beyond the weakly non-linear regime?

Need: configurations in which

-solutions from first principles can be found

-solutions are accurate as deep as possible in the non-linear regime

Motivation:

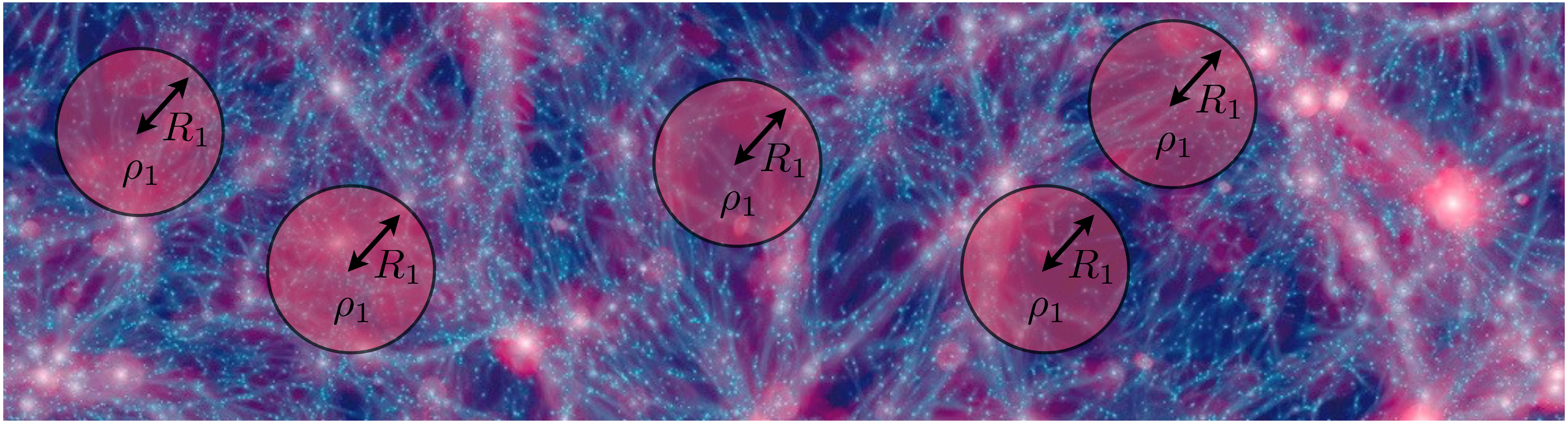
-theorists: we want to understand the physical processes driving structure formation!

-galaxy surveys: huge datasets that will need to be modelled very precisely to optimally extract the underlying cosmological information

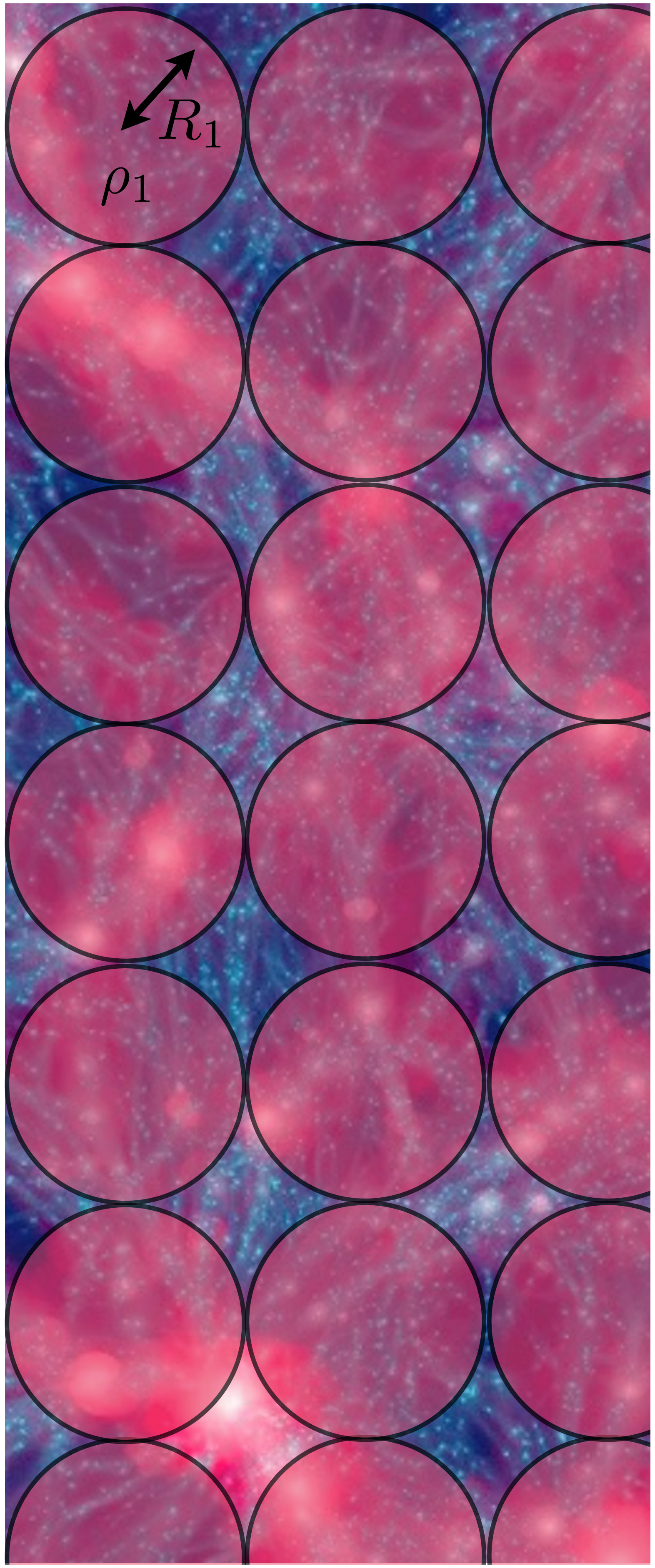
Idea: use the symmetry!

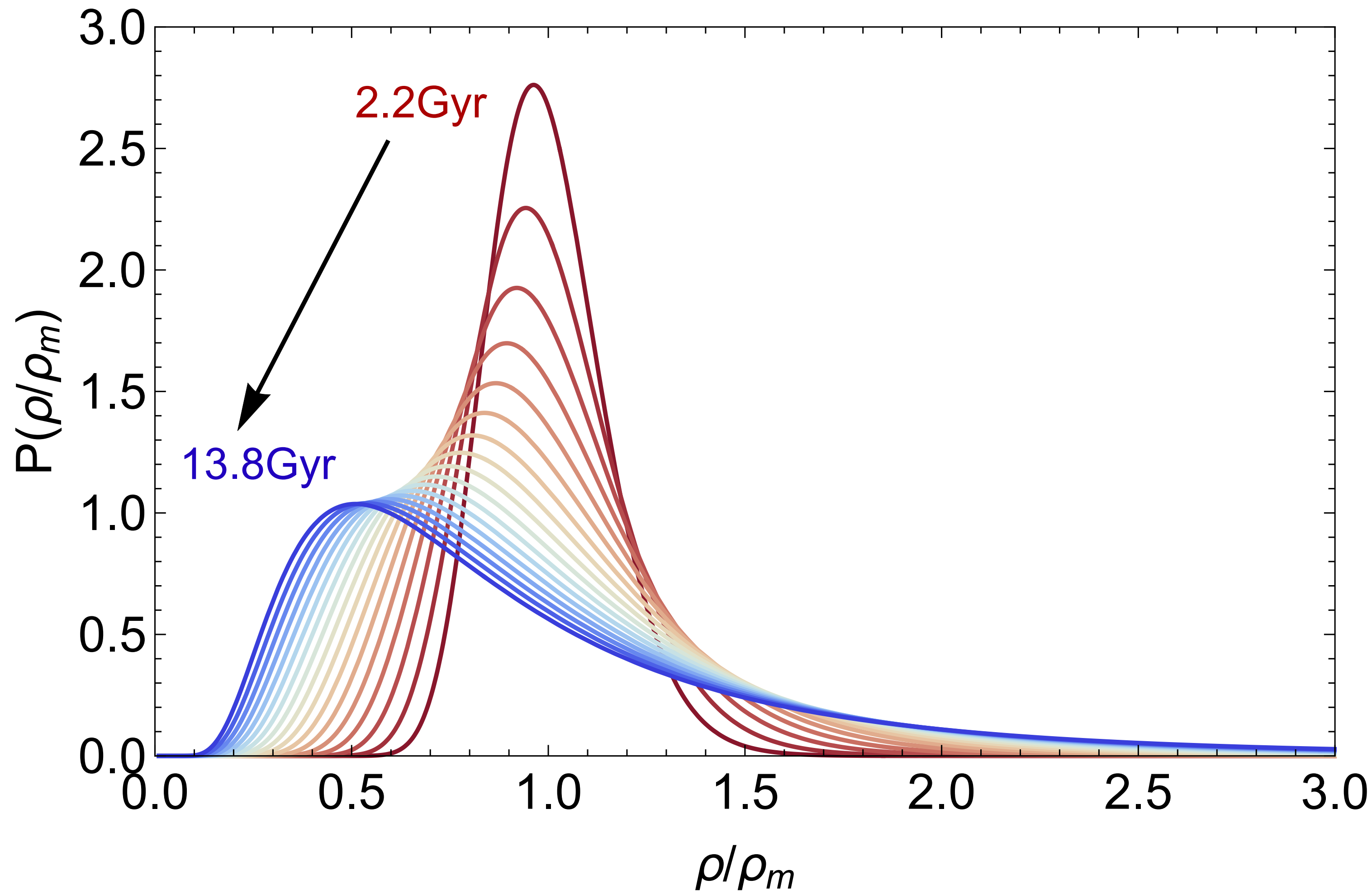
Proposed configurations: count-in-(spherical)cells

$$\mathcal{P}(\rho_1) = ?$$



Cosmic density PDF





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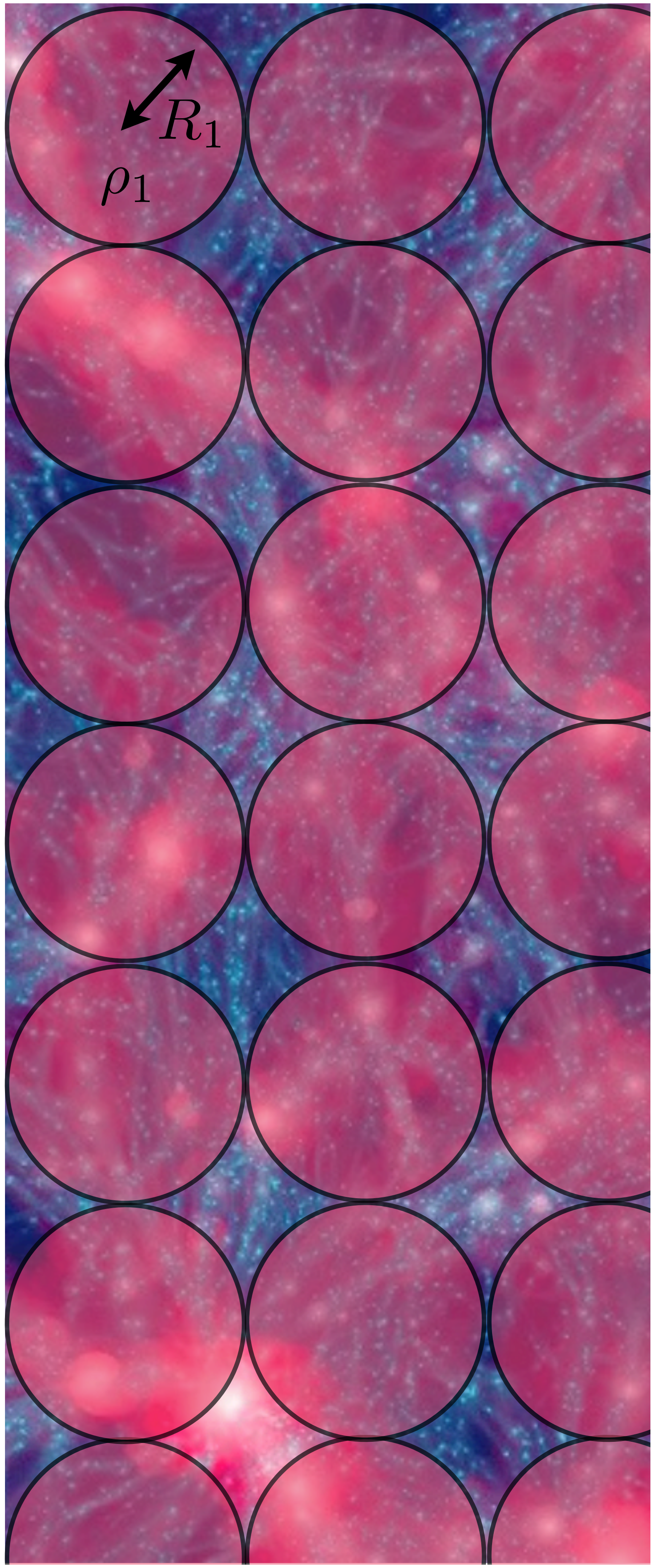
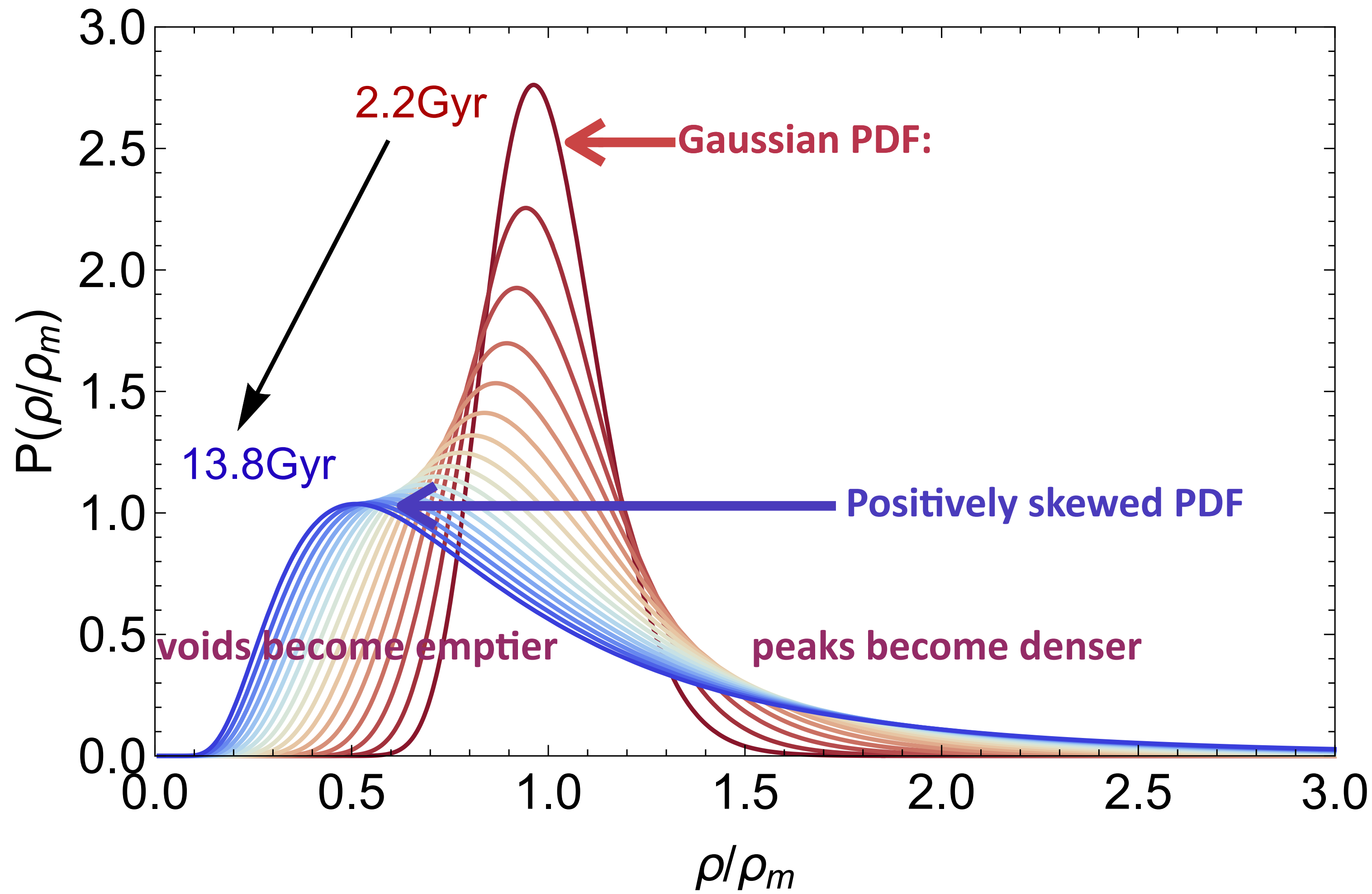




Driving parameter: variance σ (=amplitude of fluctuations)

From cumulants to PDF

PT can predict the n-th order cumulants whose ratios

$$S_n = \frac{\langle \delta^n \rangle_c}{\sigma^{2n-2}}$$

are almost z-independent.

Bernardeau 94

$$S_3 = \frac{34}{7} + \gamma_1,$$

$$S_4 = \frac{60712}{1323} + \frac{62\,\gamma_1}{3} + \frac{7\,\gamma_1{}^2}{3} + \frac{2\,\gamma_2}{3},$$

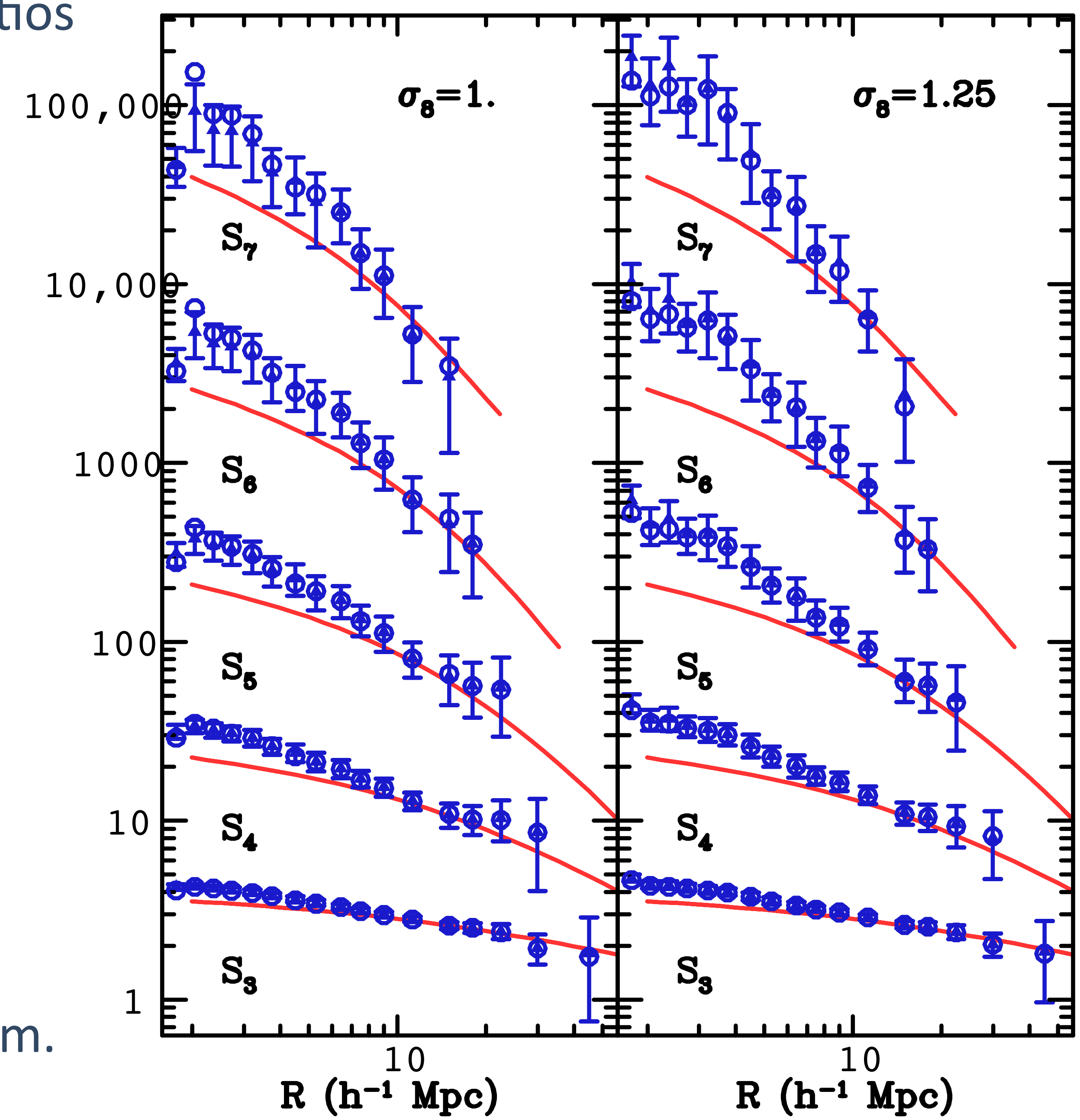
$$S_5 = \frac{200575880}{305613} + \frac{1847200\,\gamma_1}{3969} + \frac{6940\,\gamma_1{}^2}{63} + \frac{235\,\gamma_1{}^3}{27} + \frac{1490\,\gamma_2}{63} + \frac{50\,\gamma_1\,\gamma_2}{9} + \frac{10\,\gamma_3}{27},$$

where

$$\gamma_p = \frac{\mathrm{d}^p \log \sigma^2(R_0)}{\mathrm{d} \log^p R_0}$$

depends on the shape of the linear power spectrum.

Baugh & Gaztañaga 95



=> Hierarchy of cumulants: σ^2, $\langle\delta^3\rangle_c \propto \sigma^4$, $\langle\delta^4\rangle_c \propto \sigma^6$, ...

From cumulants to PDF

$$S_n = \frac{\langle \delta^n \rangle_c}{\sigma^{2n-2}}$$

The PDF of x=δ/σ can then be written as an Edgeworth expansion (in powers of σ):

$$P(x) = G(x) \left[1 + \sigma \frac{S_3}{3!} H_3(x) + \sigma^2 \left(\frac{S_4}{4!} H_4(x) + \frac{1}{2} \left(\frac{S_3}{3!} \right)^2 H_6(x) \right) + \cdots \right]$$

.

From cumulants to PDF

$$S_n = \frac{\langle \delta^n \rangle_c}{\sigma^{2n-2}}$$

The PDF of $x=\delta/\sigma$ can then be written as an Edgeworth expansion (in powers of σ):

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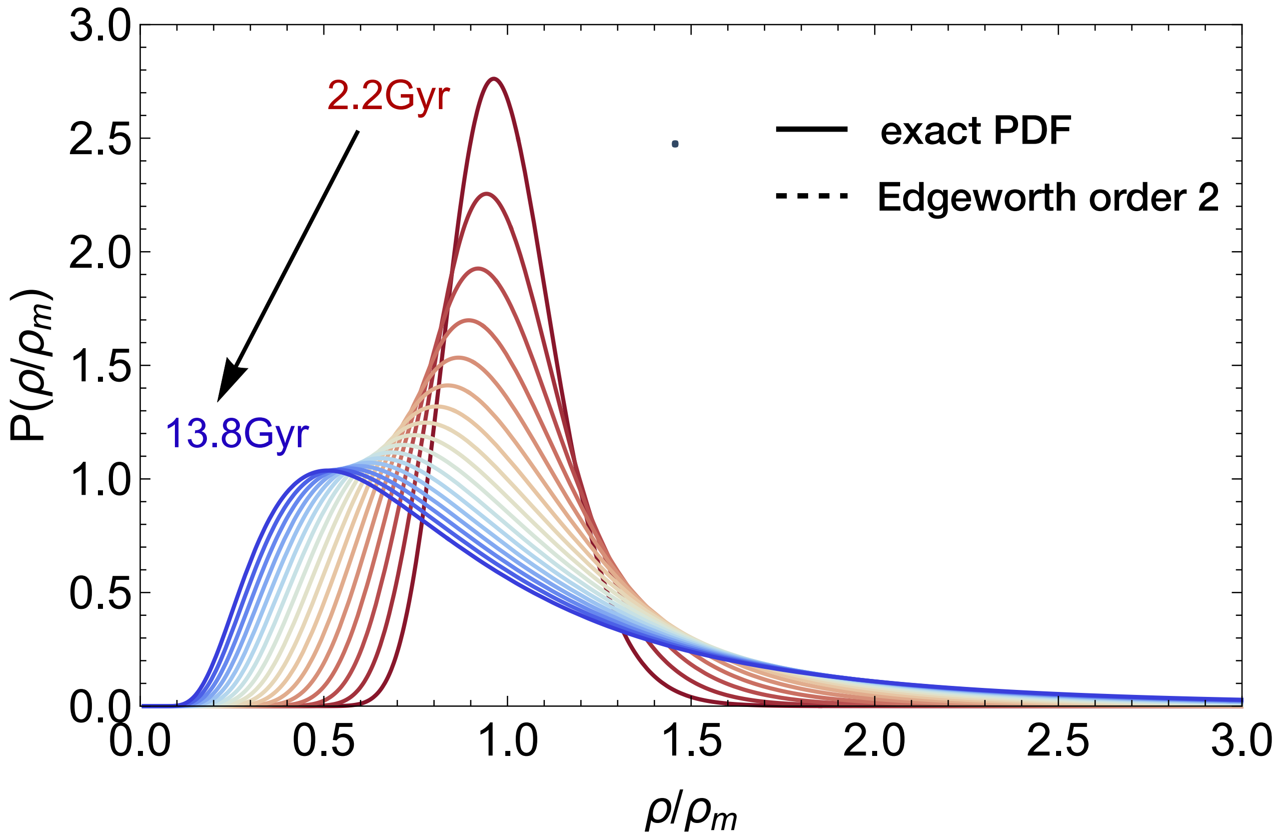

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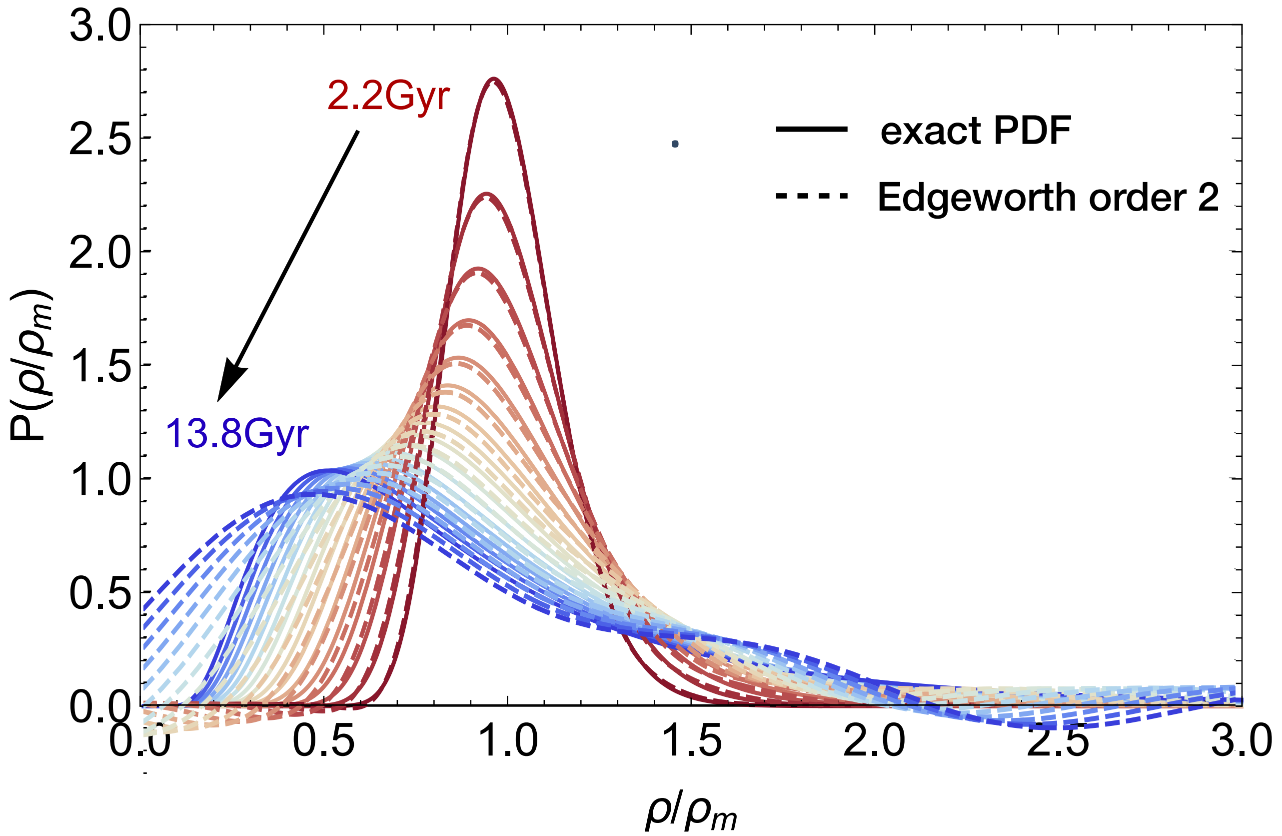


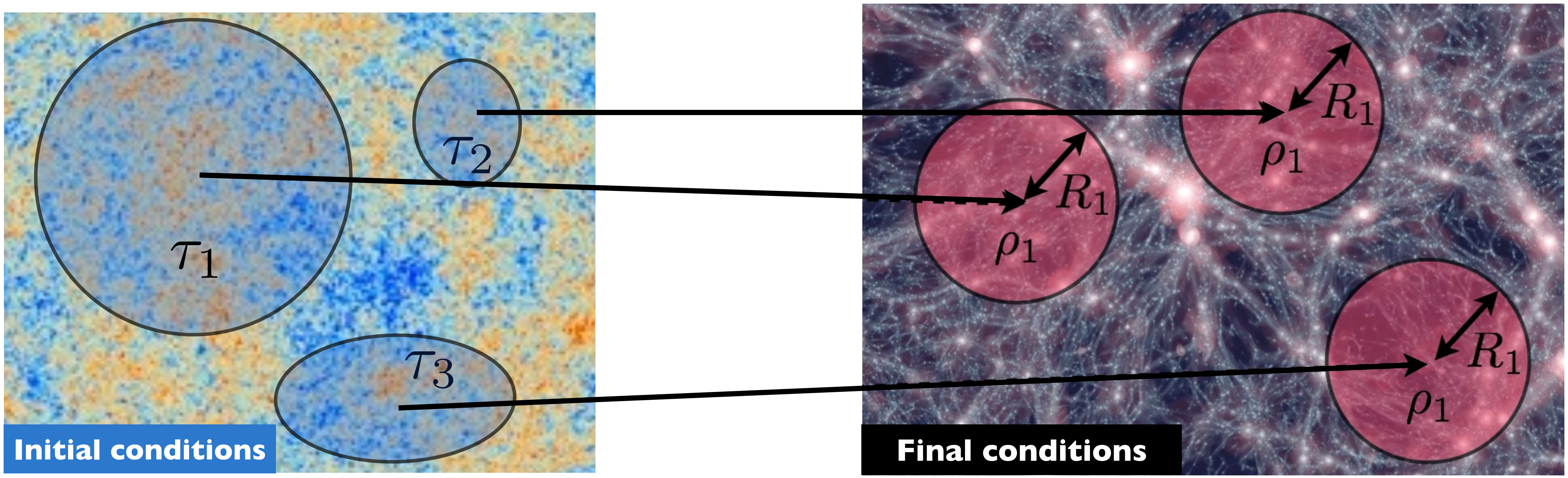
Problem : When this series is truncated at some orders, the PDF is unphysical : it is not normalised and can take negative values.

Solution : **large-deviation theory** provides us with a model for the PDF which does not suffer from those issues. All cumulants are exact at tree-order.

«An unlikely fluctuation is brought about by the least unlikely among all unlikely paths »

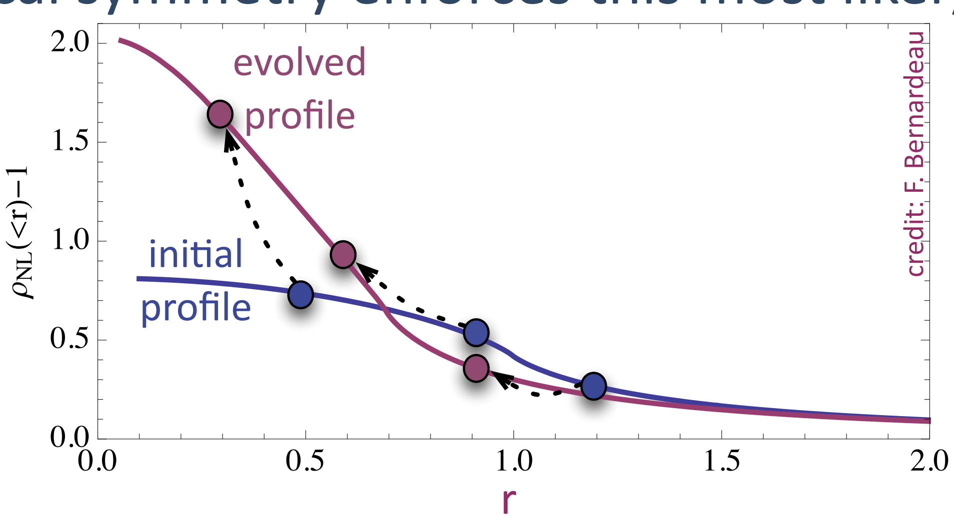
Large-deviation Theory: what is the most likely initial configuration a final density originates from?

In principle, one has to sum over all possible paths:



Contraction principle: different initial configurations can lead to the same final state! What is the most likely one?

Conjecture: Spherical symmetry enforces this most likely path to be the ***Spherical Collapse dynamics***.



$$\tau \rightarrow \rho = \zeta_{\rm SC}(\tau)$$

$$r_0 \rightarrow r = r_0 \rho^{-1/3}$$

Large-deviation Theory: in a nutshell

LDT tells us how to compute the **cumulant generating function** from the initial conditions using the spherical collapse as the « mean dynamics »:

$$\varphi(\{\lambda_k\}) = \sup_{\rho_i}(\lambda_i \rho_i - I(\rho_i))$$

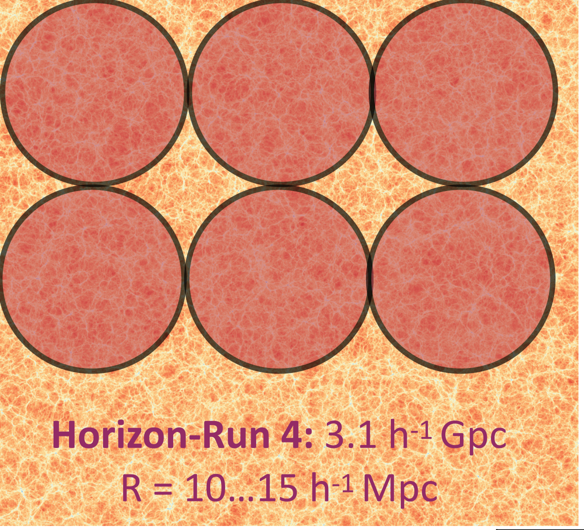
Varadhan's theorem

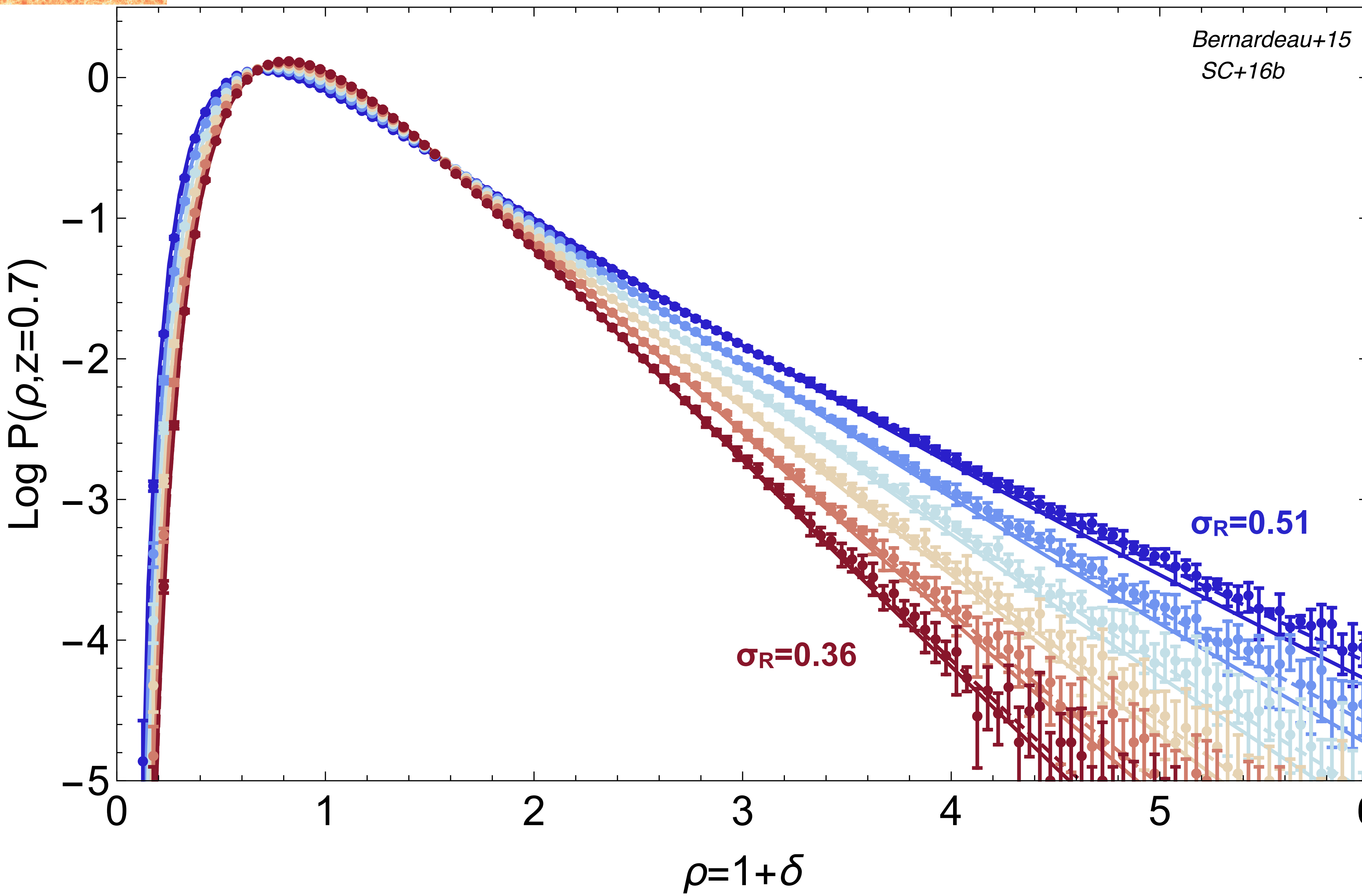
The density **PDF** is then obtained via an inverse Laplace transform of the CGF

$$\exp \varphi(\lambda) = \int P(\rho) \exp(\lambda\rho) \leftrightarrow P(\rho) = \int_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\lambda}{2\imath\pi} \exp(\lambda\rho - \varphi(\lambda))$$

- This is exact in the zero variance limit. We then extrapolate to non zero values.
- **Parameter-free** theory which depends on cosmology through : the spherical collapse dynamics, the linear power spectrum and growth of structure.
- Predictions are fully **analytical**

One-cell density PDF



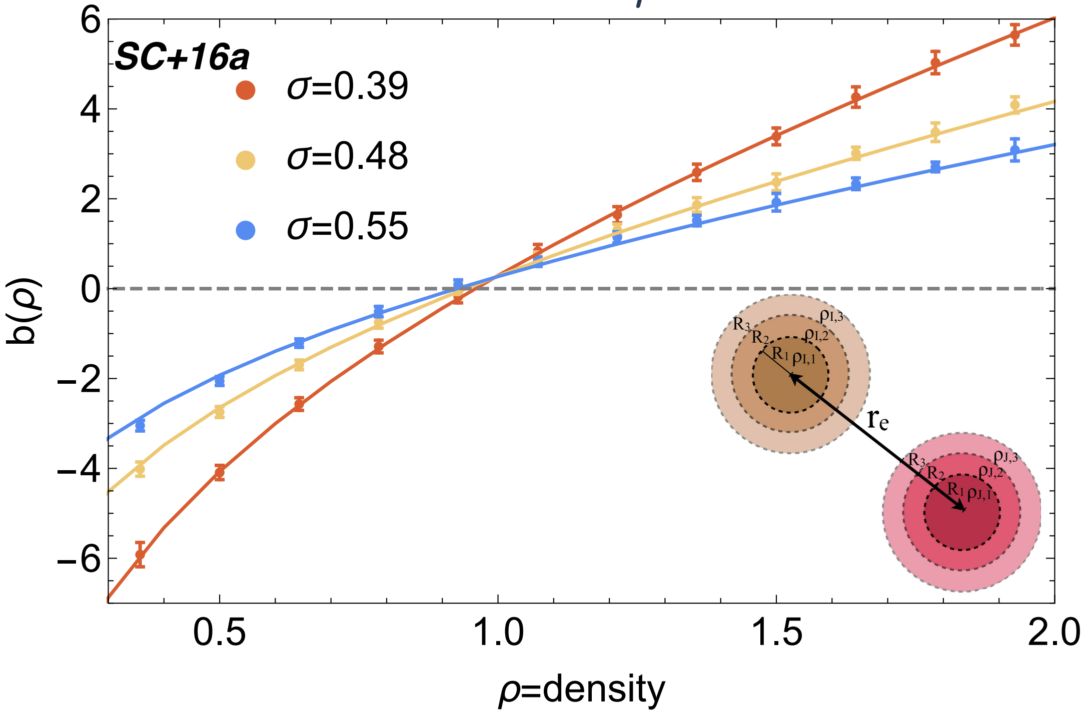


Many other predictions…

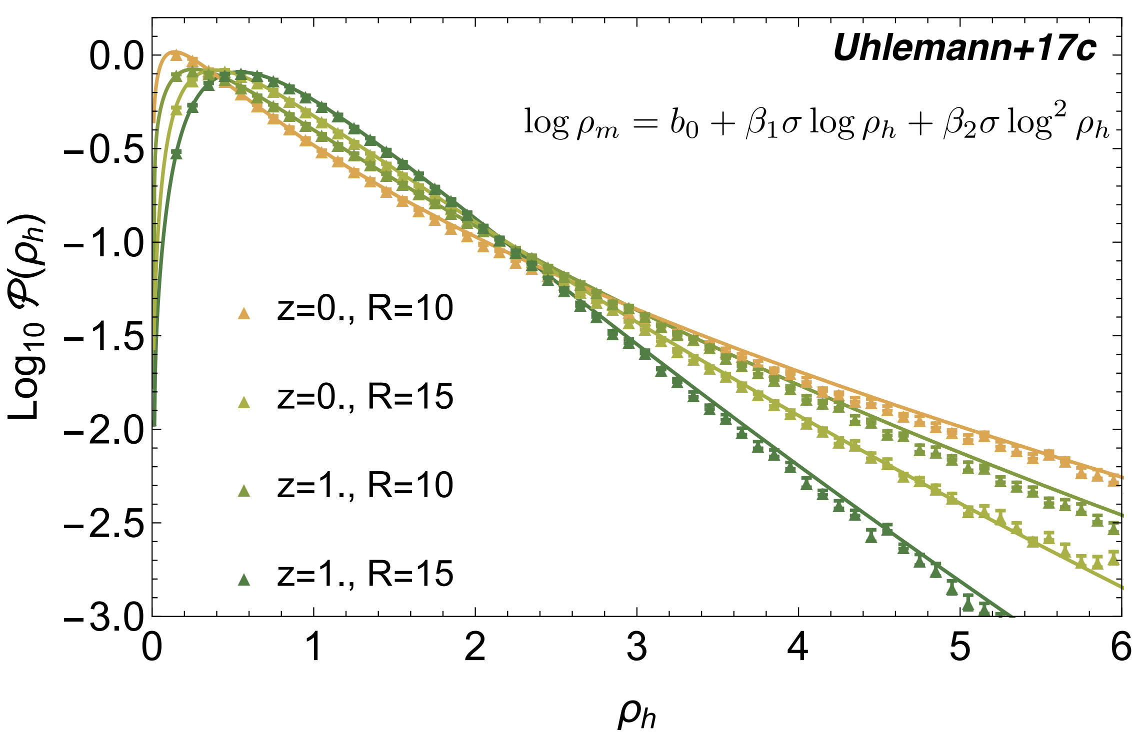
Error budget:

shot noise
finite volume error

$$\langle \hat{\mathcal{P}}^2(\rho)\rangle - \langle \hat{\mathcal{P}}(\rho)\rangle^2 = \frac{\mathcal{P}(\rho)}{N\Delta\rho} + \xi b^2(\rho)\mathcal{P}^2(\rho)$$

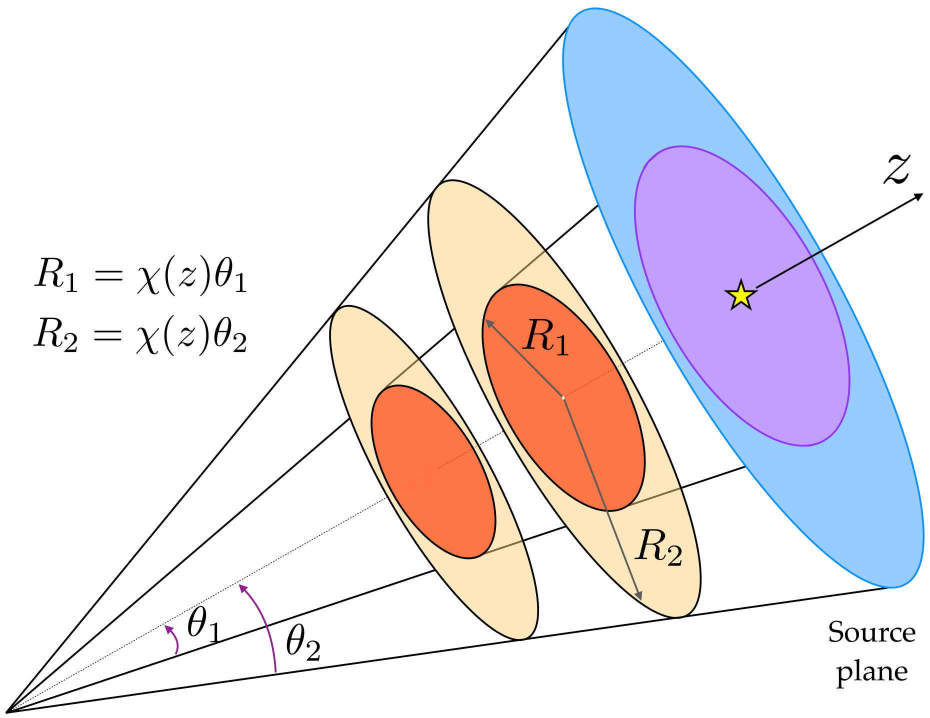


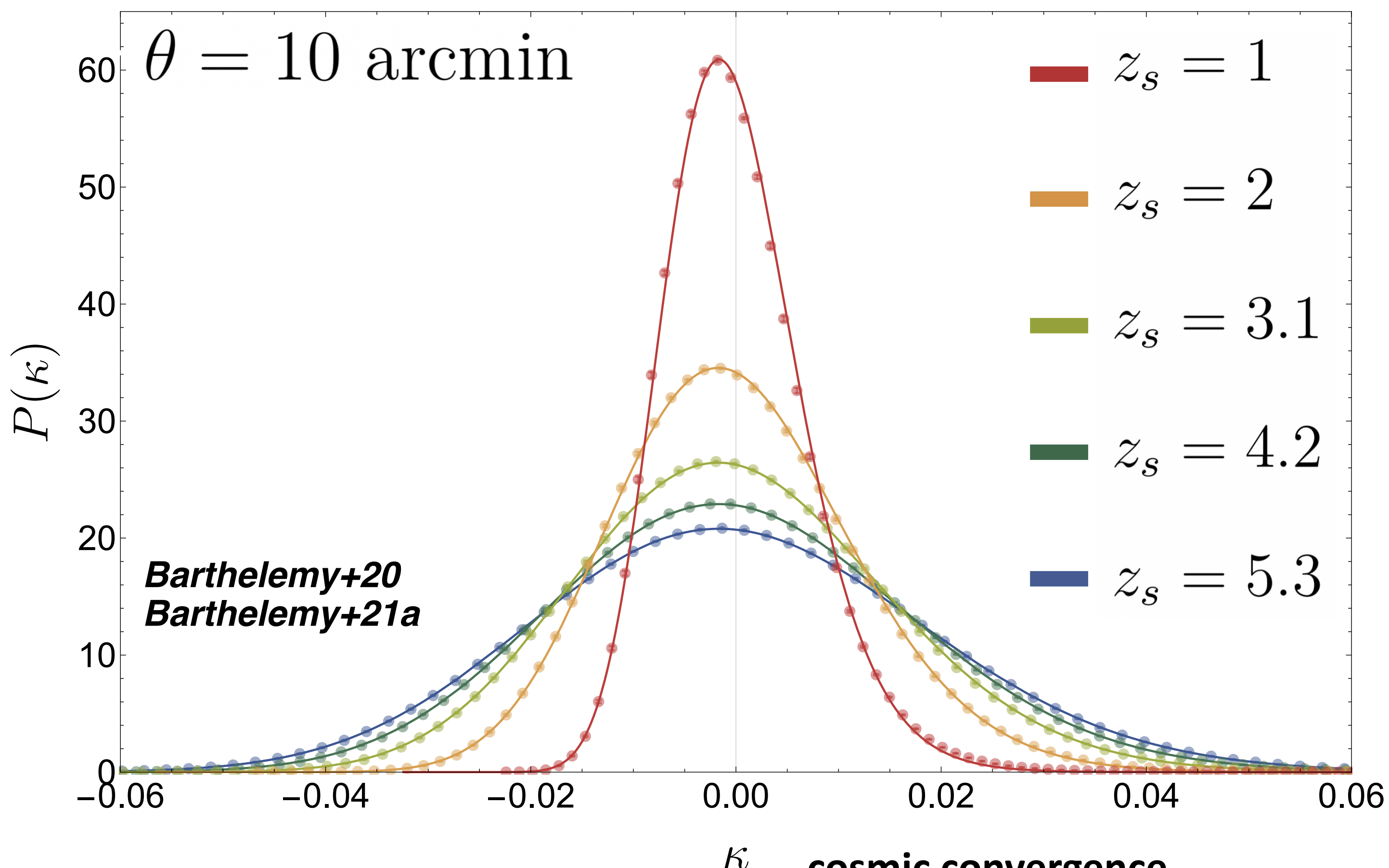
Dark matter halo density:



Cosmic shear PDF

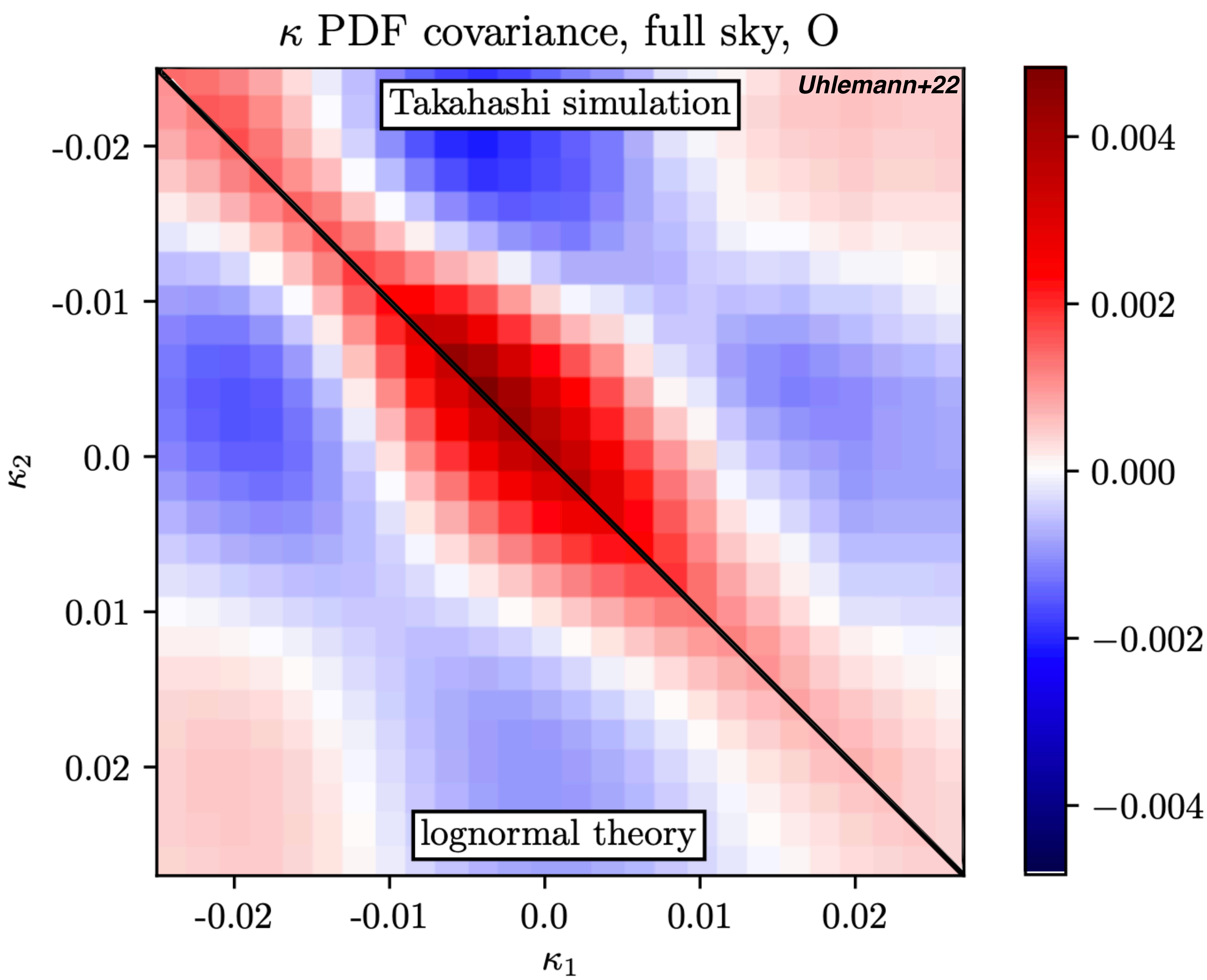
Alexandre Barthelemy
LMU-Munich





cosmic convergence

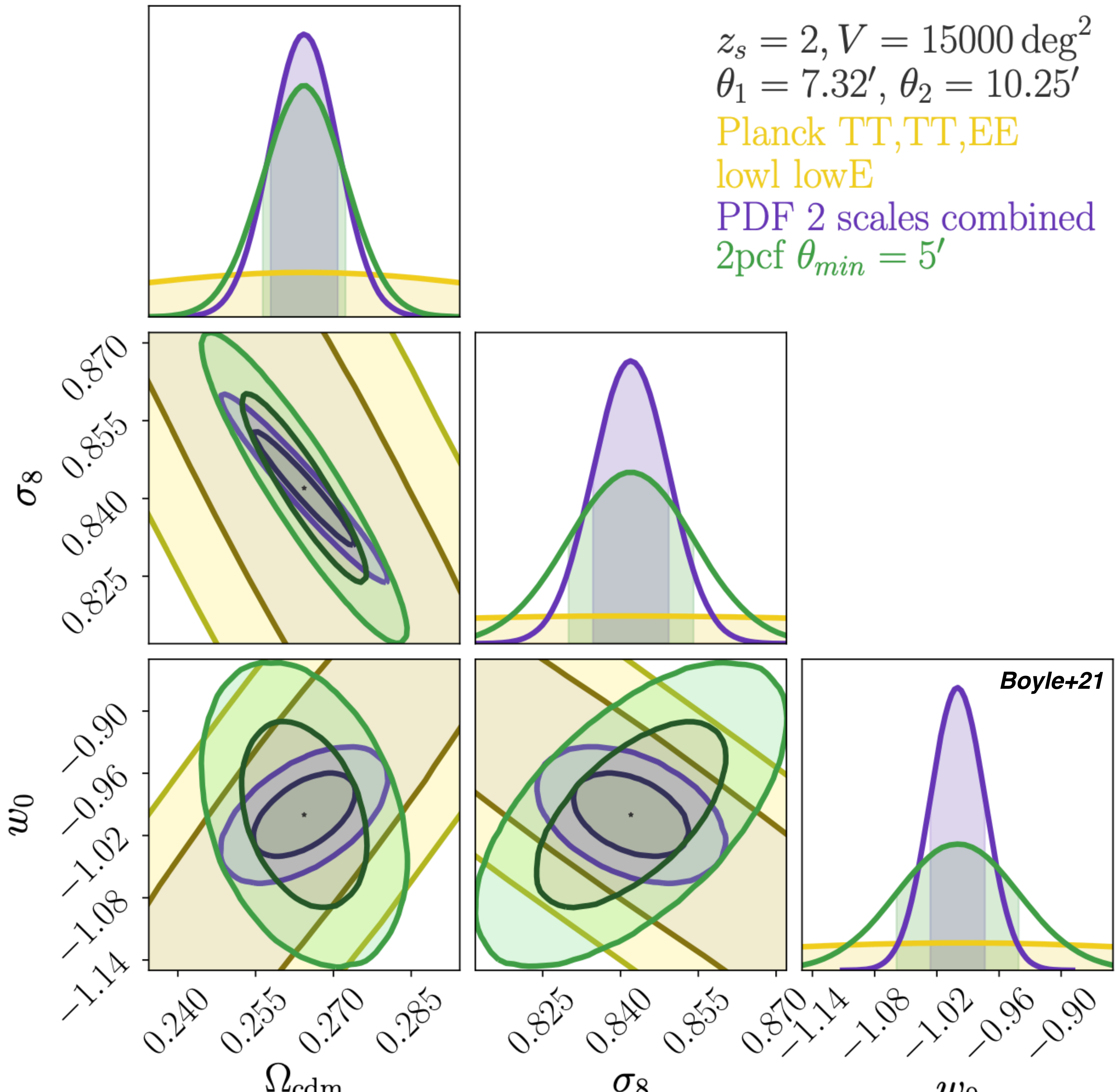
Modelling associated covariance matrices



Cora Uhlemann
Newcastle University

$\theta_s = 7.5'$ at source redshift $z_s = 2$

PDF as a cosmological probe…



Cold dark matter content **Amplitude of fluctuations** **Equation of state of dark energy**

Aoife Boyle / AIM

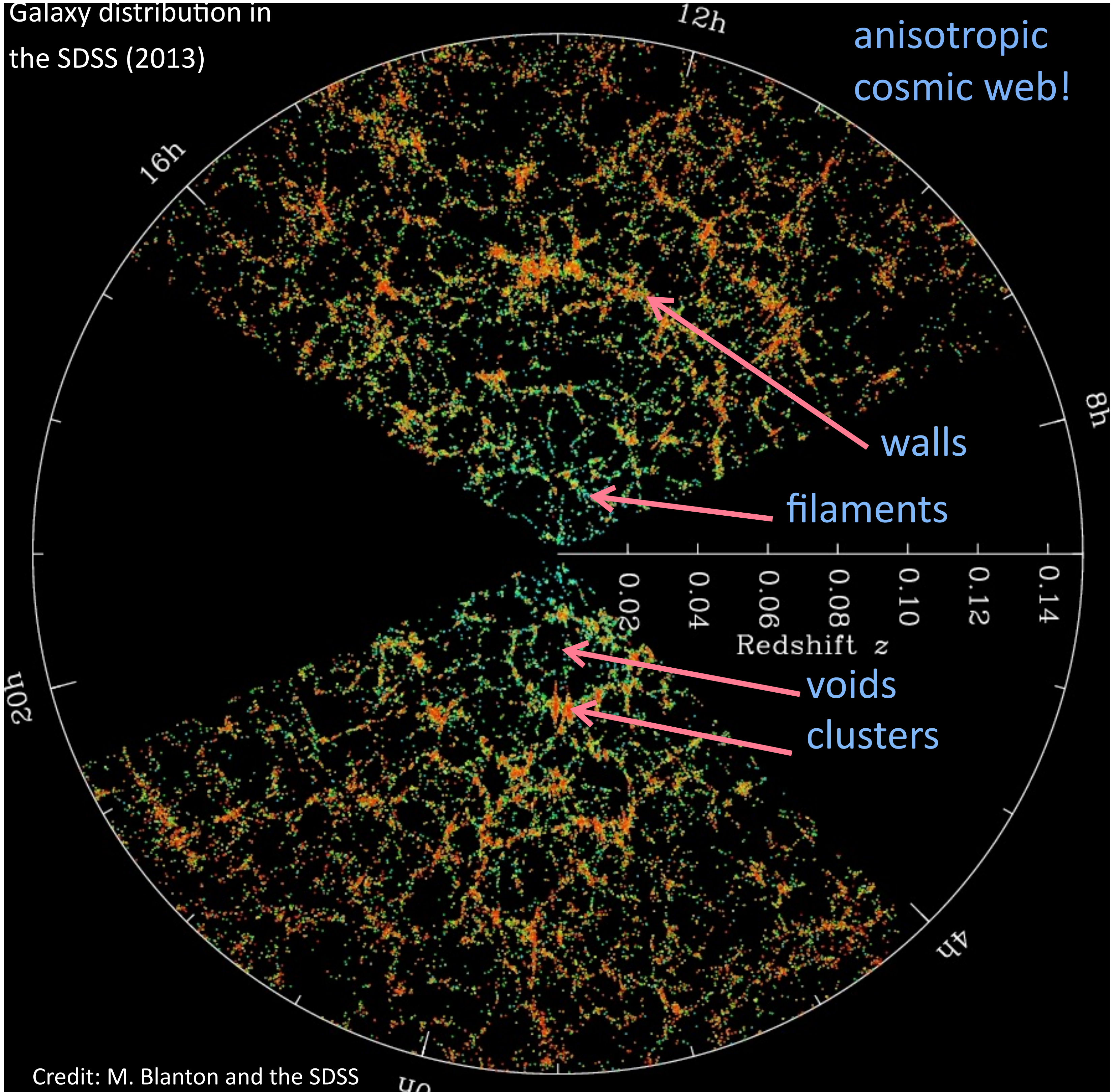
PDF as a cosmological probe…

Benefits:

- First principle **prediction** with explicit cosmology dependence which allows to track where the information comes from (and keep non linearities **under control** !)
- Easy to measure
- Gaussian likelihood

=> Proven to be a crucial tool in the landscape of « simulation-based » higher order estimators (Euclid paper under review by the consortium) !

Conclusion: very good understanding of cosmic fields on very large scales or in isotropic settings but …

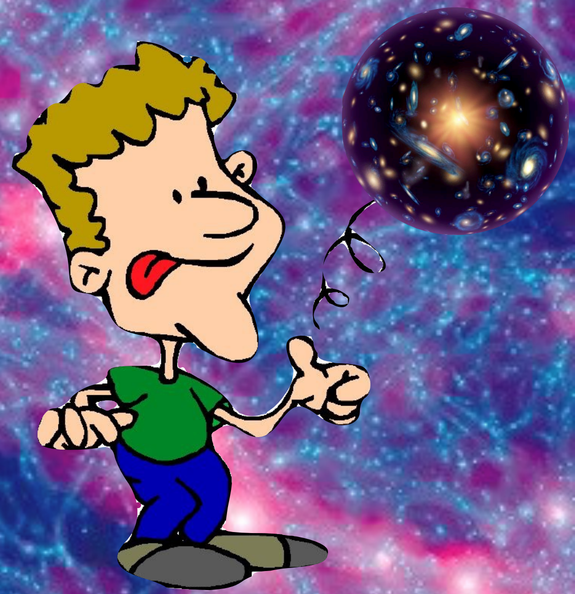


Conclusion

- The isotropic distribution of cosmic fields is successfully understood via large-deviation theory (accurate analytical predictions in the mildly non-linear regime)

- Crucial need for reliable large-scale structure estimators to analyse galaxy surveys (PDF is one such tool)

- The anisotropic geometry of the cosmic web carries important cosmological information and is key for galaxy evolution.



horizon-AGN